# Timed Trace Alignment with Metric Temporal Logic over Finite Traces

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*Trace alignment* (Aalst, van der 2013) is a prominent problem in Declarative Process Mining, which consists in minimally modifying, or *repairing*, a *log trace* to make it compliant with an input temporal specification. In its simplest formulation, traces are finite sequences of *events*, corresponding to the logs stored by a system under execution, and specifications capture properties that involve the mutual ordering of events, specified using the DECLARE language (van der Aalst, Pesic, and Schonenberg 2009), which is a fragment of the Linear-time Temporal Logic interpreted over finite traces $\text{LTL}_f$ (De Giacomo and Vardi 2013).

While the Business Process Management (BPM) community has devoted many efforts to this problem –see, e.g., promtools.org, (de Leoni, Maggi, and van der Aalst 2012), and (de Leoni, Maggi, and van der Aalst 2015), the best solution technique currently available comes from AI (De Giacomo et al. 2017) and is based on an automata-theoretic approach that takes advantage of the cost-optimal planning technology, e.g., (Helmert 2006; Torralba et al. 2014) to efficiently perform the search. Notably, such a technique is able to deal with any specification expressed in $\text{LTL}_f$, not only those allowed by DECLARE (which are defined in terms of $\text{LTL}_f$), a commonly adopted language in the BPM context.

A more sophisticated variant of the problem is *timed trace alignment*. In this variant, each event in a log trace is paired with a timestamp and the specifications can be expressed in a variety of forms, from pre-defined patterns (Lanz, Weber, and Reichert 2014), to formulas of more structured logics, such as the multi-perspective version of DECLARE, i.e., MP-DECLARE (Burattin, Maggi, and Sperduti 2016). MP-DECLARE is a fragment of Metric First-order Temporal Logic (Chomicki 1995) (MFOTL), a language that combines First-order Logic (FOL) and Metric Temporal Logic (Koymans 1990), both interpreted over finite traces, to express properties that concern event attributes and timestamps. Interestingly, for the timed version of trace alignment with specifications expressed in a rich language such as MP-DECLARE, a solution technique is not available yet, not even for the case where propositional formulas only are allowed in place of FO. Even more, it is not even known whether the existence of a solution is decidable.

In this talk, we present the timed version of trace alignment, introduced in (De Giacomo et al. 2021), with specifications expressed in metric temporal logic over finite traces ($\text{MTL}_f$), which is essentially a superlanguage of $\text{LTL}_f$. Importantly, $\text{MTL}_f$ is a strictly more expressive language than the propositional version of MP-DECLARE thus, from the temporal perspective, we are addressing the problem in its entire generality. In this setting, trace alignment requires to repair a trace not only by adding or removing events from a finite alphabet but, crucially, by also adding timestamps from the infinite, uncountable, real space. This results in a substantially more challenging setting than that of the basic variant (De Giacomo et al. 2017), as the presence of time makes the structures involved in the search (uncountably) infinite-state, which requires a more sophisticated technical machinery, based on alternating timed automata, as opposed to the standard finite-state automata sufficient for the untimed version.

The contribution is threefold. Firstly, it proposes an automata-based formalization of timed trace alignment which reduces the problem of finding a minimal-cost solution (in terms of changes made to the input trace) to the search for an accepting path in an alternating timed automaton. Secondly, based on the above formulation, it shows that the problem is solvable. Thirdly, by proving solvability in a constructive way, the work devises the first technique for actually computing a solution in the timed setting.

Technically, to achieve these results, we build on two works, namely (Ouaknine and Worrell 2007) and (Finkel and Schnoebelen 2001). The former proves decidability of $\text{MTL}_f$ by reducing the problem to non-emptiness of alternating timed automata and showing that this corresponds to the search for an accepting path in a *well-structured* transition system, while the latter provides the theoretical results on such transition systems to make the search effective. Broadly speaking, we generalize the approach described in (Ouaknine and Worrell 2007), which is aimed at showing the existence of a solution, to actually perform a search in the space of the solutions, and take advantage of the results from (Finkel and Schnoebelen 2001) to obtain termination of our technique.

We observe that, while the obtained technique is quite demanding from the complexity point of view, it solves the problem in its entire generality, as far as temporal aspects only are concerned, thus can serve as a reference for practical approaches that typically deal with less expressive fragments of $\text{MTL}_f$, such as the propositional fragment of MP-DECLARE.

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