Mimicking Behaviors in Separated Domains

Giuseppe De Giacomo

DEGIACOMO@DIAG.UNIROMA1.IT

 $Sapienza\ University\ of\ Rome$

DFRIED@OPENU.AC.IL

Dror Fried
The Open University of Israel

PATRIZI@DIAG.UNIROMA1.IT

Fabio Patrizi

ZHU@DIAG.UNIROMA1.IT

 $Sapienza\ University\ of\ Rome$

Shufang Zhu

Sapienza University of Rome

Abstract

Devising a strategy to make a system mimicking behaviors from another system is a problem that naturally arises in many areas of Computer Science. In this work, we interpret this problem in the context of intelligent agents, from the perspective of LTL_f , a formalism commonly used in AI for expressing finite-trace properties. Our model consists of two separated dynamic domains, \mathcal{D}_A and \mathcal{D}_B , and an LTL_f specification that formalizes the notion of mimicking by mapping properties on behaviors (traces) of \mathcal{D}_A into properties on behaviors of \mathcal{D}_B . The goal is to synthesize a strategy that step-by-step maps every behavior of \mathcal{D}_A into a behavior of \mathcal{D}_B so that the specification is met. We consider several forms of mapping specifications, ranging from simple ones to full LTL_f , and for each we study synthesis algorithms and computational properties.

1. Introduction

Mimicking a behavior from a system A to a system B is a common practice in Computer Science (CS) and Software Engineering (SE). Examples include a robot that has to real-time adapt a human behavior (Mitsunaga, Smith, Kanda, Ishiguro, & Hagita, 2008), or simultaneous interpretation of a speaker (Yarmohammadi, Sridhar, Bangalore, & Sankaran, 2013; Zheng, Liu, Zheng, Ma, Liu, & Huang, 2020). The challenge in behavior mimicking is twofold. Firstly, a formal specification of mimicking is needed; indeed, being potentially different, systems A and B may show substantially different behaviors, not directly comparable, thus a relationship, or map, between them must be formally defined to capture when a behavior from A is correctly mimicked by one from B. Secondly, since B ignores what A will do next, B must monitor the actions performed by A and perform its own actions, in such a way that the resulting behavior of B mimics that of A.

In this work, we look at the problem of devising a strategy for mimicking behaviors when the mapping specification is expressed in Linear Temporal Logic on finite traces (LTL_f) (De Giacomo & Vardi, 2013), a formalism commonly used in AI for expressing finite-trace properties. In our framework, systems A and B are modeled by two separated dynamic domains, \mathcal{D}_A and \mathcal{D}_B , in turn modeled as transition systems, over which there are agents A and B that respectively act, without affecting each other. The mapping specification is then a set of LTL_f formulas to be taken in conjunction, called mappings, that essentially

relate the behaviors of A to those of B. While B has full knowledge of both domains and their states, it has no idea which action A will take next. Nevertheless, in order to perform mimicking, B must respond to every action that A performs on \mathcal{D}_A by performing one action on \mathcal{D}_B . As this interplay proceeds, \mathcal{D}_A and \mathcal{D}_B traverse two respective sequences of states (traces) which we call the behaviors of A and B, respectively. The process carries on until either A or B (depending on the variant of the problem considered) decides to stop. The mimicking from A has been accomplished correctly, i.e., agent B wins, if the resulting traces satisfy the LTL $_f$ mapping specification. Our goal is to synthesize a strategy for B, i.e., a function returning an action for B given those executed so far by agent A, which guarantees that B wins, i.e., is able to mimic, respecting the mappings, every behavior of A. We call this the Mimicking Behavior in Separated Domains (MBSD) problem.

The mapping specifications can vary, consequently changing the nature of the mimicking, and consequently the difficulty of synthesizing a strategy for B. We study three different types of mappings. The first is the class of point-wise mappings, which establish a sort of local connection between the two separated domains. Point-wise mapping specifications have the form $\bigwedge_{i\leq k}\Box(\phi_i\to\psi_i)$ (see Section 2.2 for proper LTL_f definition) where each ϕ_i is a Boolean property over \mathcal{D}_A and each ψ_i is a Boolean property over \mathcal{D}_B . Point-wise mappings indicate invariants that are to be kept throughout the interaction between the agents. In Section 4.1 we give a detailed example of point-wise mappings from the Pac-Man world.

The second class is that of target mappings, which relate the ability of satisfying corresponding reachability goals (much in the same fashion as Planning) in the two separate domains. Target mapping specifications have the form $\bigwedge_{i\leq k}(\Diamond\phi_i\to\Diamond\psi_i)$, where ϕ_i and ψ_i are Boolean properties over \mathcal{D}_A and \mathcal{D}_B , respectively. Target mappings define objective for A and B and require that if A meets its objective then B must meet its own as well, although not necessarily at the same time. We give a detailed example of target mappings in Section 5.1, from the Rubik's cube world. The last class is that of general LTL $_f$ mappings. A general LTL $_f$ mapping specification has the form of an arbitrary LTL $_f$ formula Φ with properties over \mathcal{D}_A and \mathcal{D}_B .

Our objective is to characterize solutions for strategy synthesis for mimicking behaviors under the types of mapping specifications described above, from both the algorithmic and the complexity point of view. The input we consider includes both domains \mathcal{D}_A and \mathcal{D}_B , and the mapping specification. Since it is common to focus on problems in which either of the two is fixed (e.g. (De Giacomo & Rubin, 2018)), we provide solutions in terms of: combined complexity, where neither the size of the domain nor that of the mapping specification are fixed; mapping complexity, where domains' size are fixed but mapping specification's varies; and domain complexity, where the mapping specification's size is fixed but domains' vary.

For our analysis, we formalize the problem as a two-player game between agent A (Player 1) and agent B (Player 2) over a game graph that combines both domains \mathcal{D}_A and \mathcal{D}_B , with the winning objective varying in the classes discussed above. We start with *point-wise* mappings where A decides when to stop and derive a solution in the form of a winning strategy for a safety game in PTIME wrt combined, mapping and domain complexity. The scenario becomes more complex for target mappings, where the agent B decides when to stop, and where some objectives met during the agent's interplay must be recorded. We devise an algorithm exponential in the number of constraints, and show that the problem

is in PSPACE for combined and mapping complexity, and PTIME in domain complexity. To seal the complexity of the problem, we provide a PSPACE-hardness proof for combined complexity, already for simple acyclic graph structures. For domains whose transitions induce a tree-like structure, however, we show that the problem is still in PTIME for combined, mapping and domain complexity. Finally, we show that the problem with general LTL $_f$ mapping specifications is in 2EXPTIME for combined and mapping complexity, due to the doubly-exponential blowup of the DFA construction for LTL $_f$ formulas, and is PTIME in domain complexity.

The rest of the paper goes as follows. In Section 2 we give preliminaries, and we formalize our problem in Section 3. We give detailed examples and analyses of point-wise and target mapping specifications in Sections 4 and 5 respectively. We discuss solution for general mapping specifications in Section 6. Then we provide a more detailed discussion about related work in Section 7, and conclude in Section 8.

2. Preliminaries

We briefly recall preliminary notions that will be used throughout the paper.

2.1 Boolean Formulas

Boolean (or propositional) formulas are defined, as standard, over a set of propositional variables (or, simply, propositions) Prop, by applying the Boolean connectives \land (and), \lor (or) and \neg (not). Standard abbreviations are \rightarrow (implies), true (also denoted \top) and false (also denoted \bot). A proposition $p \in Prop$ occurring in a formula is called an atom, a literal is an atom or a negated atom $\neg p$, and a clause is a disjunction of literals. A Boolean formula is in Conjunctive Normal Form (CNF), if it is a conjunction of clauses. The size of a Boolean formula φ , denoted $|\varphi|$, is the number of connectives occurring in φ . A Quantified Boolean Formula (QBF) is a Boolean formula, all of whose variables are universally or existentially quantified. A QBF formula is in Prenex Normal Form (PNF) if all quantifiers occur in the prefix of the formula. True Quantified Boolean Formulas (TQBF) is the language of all QBF formulas in PNF that evaluate to true. TQBF is known to be PSPACE-complete.

2.2 LTL $_f$ Basics

Linear Temporal Logic over finite traces (LTL_f) is an extension of propositional logic to describe temporal properties on finite (unbounded) traces (De Giacomo & Vardi, 2013). LTL_f has the same syntax as LTL, one of the most popular logics for temporal properties on infinite traces (Pnueli, 1977). Given a set of propositions Prop, the formulas of LTL_f are generated by the following grammar:

$$\varphi ::= p \mid (\varphi_1 \land \varphi_2) \mid (\neg \varphi) \mid (\bigcirc \varphi) \mid (\varphi_1 \mathcal{U} \varphi_2)$$

where $p \in Prop$, O is the *next* temporal operator and \mathcal{U} is the *until* temporal operator, both are common in LTL_f . We use common abbreviations for eventually $\Diamond \varphi \equiv true \mathcal{U} \varphi$ and always as $\Box \varphi \equiv \neg \Diamond \neg \varphi$.

A word over Prop is a sequence $\pi = \pi_0 \pi_1 \cdots$, s.t. $\pi_i \subseteq 2^{Prop}$, for $i \geq 0$. Intuitively, π_i is interpreted as the set of propositions that are true at instant i. In this paper we deal only with finite, nonempty words, i.e., $\pi = \pi_0 \cdots \pi_n \in (2^{Prop})^+$. $last(\pi)$ denotes the last instant (index) of π .

Given a finite word π and an LTL_f formula φ , we inductively define when φ is true on π at instant $i \in \{0, \ldots, last(\pi)\}$, written $\pi, i \models \varphi$, as follows:

- $\pi, i \models p \text{ iff } p \in \pi_i \text{ (for } p \in Prop);$
- $\pi, i \models \varphi_1 \land \varphi_2 \text{ iff } \pi, i \models \varphi_1 \text{ and } \pi, i \models \varphi_2;$
- $\pi, i \models \neg \varphi \text{ iff } \pi, i \not\models \varphi;$
- $\pi, i \models \bigcirc \varphi \text{ iff } i < last(\pi) \text{ and } \pi, i + 1 \models \varphi;$
- $\pi, i \models \Box \varphi \text{ iff } \forall j.i \leq j \leq last(\pi) \text{ and } \pi, j \models \varphi;$
- $\pi, i \models \Diamond \varphi \text{ iff } \exists j.i \leq j \leq last(\pi) \text{ and } \pi, j \models \varphi;$
- $\pi, i \models \varphi_1 \mathcal{U} \varphi_2$ iff $\exists j.i \leq j \leq last(\pi)$ and $\pi, j \models \varphi_2$, and $\forall k.i \leq k < j$ we have that $\pi, k \models \varphi_1$.

In this paper, we make extensive use of $\Box \varphi$ and $\Diamond \varphi$.

We say that $\pi \in (2^{Prop})^+$ satisfies an LTL_f formula φ , written $\pi \models \varphi$, if $\pi, 0 \models \varphi$. For every LTL_f formula φ defined over Prop, we can construct a Deterministic Finite Automaton (DFA) \mathcal{F}_{φ} that accepts exactly the traces that satisfy φ (De Giacomo & Vardi, 2013). More specifically, $\mathcal{F}_{\varphi} = (2^{Prop}, Q, q_0, \eta, acc)$, where 2^{Prop} is the alphabet of the DFA, Q is the finite set of states, $q_0 \in Q$ is the initial state, $\eta: Q \times 2^{Prop} \to Q$ is the transition function, and $acc \subseteq Q$ is a set of accepting states.

2.3 Two-player Games

A (turn-based) two-player game models a game between two players, Player 1 (P1) and Player 2 (P2), formalized as a pair $\mathcal{G} = (\mathcal{A}, W)$, with \mathcal{A} the game arena and W the winning objective. The arena $\mathcal{A} = (U, V, u_0, \alpha, \beta)$ is essentially a bipartite-graph, where:

- *U* is a finite set of *P*1 nodes;
- V is a finite set of P2 nodes:
- $u_0 \in U$ is the initial node;
- $\alpha \subseteq U \times V$ is the transition relation of P1;
- $\beta \subseteq V \times U$ is the transition relation of P2.

Intuitively, a token initially in u_0 is moved in turns from nodes in U to nodes in V and vice-versa. P1 moves when the token is in a node $u \in U$, by choosing a destination node $v \in V$ for the token, such that $(u,v) \in \alpha$. P2 acts analogously, when the token is in a node $v \in V$, by choosing a node $v \in U$ according to $v \in V$. Thus, $v \in V$ and $v \in V$ alternate their

moves, with P1 playing first, until at some point, after P2 has moved, the game stops. As the token visits the nodes of the arena, it defines a sequence of alternating U and V nodes called play. If, when the game stops, the play meets W, then P2 wins, otherwise P1 wins.

Formally, a play (of A) $\rho = \rho_0 \cdots \rho_n \in (U \cup V)^+$ is a finite, nonempty sequence of nodes such that:

- $\rho_0 = u_0;$
- $(\rho_i, \rho_{i+1}) \in \alpha$, for i even;
- $(\rho_i, \rho_{i+1}) \in \beta$, for i odd;
- n is even (which implies, by α and β , that $\rho_n \in U$).

Let $Plays_{\mathcal{A}}$ be the set of all plays of \mathcal{A} and let $last(\rho) = n$ be the last position (index) of play ρ . $\rho|_{U} = \rho_{0}\rho_{2}\cdots\rho_{n}$ is the projection of ρ on U. and $\rho|_{V} = \rho_{1}\rho_{3}\cdots\rho_{n-1}$ is the projection of ρ on V. The prefix of ρ ending at the *i*-th state is denoted as $\rho^{i} = \rho_{0}\cdots\rho_{i}$.

The winning objective W is a (compact) representation of a set of plays, called winning plays. P2 wins if the game produces a winning play, otherwise P1 wins. A strategy for P2 is a function $\sigma: V^+ \to U$, which returns a P1 node $u \in U$, given a finite sequence of P2 nodes. A strategy σ is said to be memory-less if, for every two sequences of nodes $w = w_0 \cdots w_n$ and $w' = w'_0 \cdots w'_m \in V^+$, whenever $w_n = w_m$, it holds that $\sigma(w) = \sigma(w')$; in other words, the move returned by σ is a function of the last node in the sequence. A play ρ is compatible with a P2 strategy σ if $\rho_{i+1} = \sigma(\rho^i|_V)$, for $i = 0, \ldots, last(\rho) - 1$. A P2 strategy σ is winning in $\mathcal{G} = (\mathcal{A}, W)$, if every play ρ compatible with σ is winning.

In this paper we consider two classes of games. The first class is that of reachability games in which for a set $g \subseteq U$ of P1 nodes, W = Reach(g), where Reach(g) (reachability objective) is the set of plays containing at least one node from g. Formally $\text{Reach}(g) = \{ \rho \in Plays_{\mathcal{A}} \mid \text{there exists } k.0 \leq k \leq last(\rho) : \rho_k \in g \}$.

The second class is that of safety games, in which again for a set $g \subseteq U$ of P1 nodes, $W = \operatorname{Safe}(g)$, where $\operatorname{Safe}(g)$ (safety objective) is the set of plays where all P1 nodes are from g. Formally, $\operatorname{Safe}(g) = \{ \rho \in Plays_{\mathcal{A}} \mid \text{ for all even } k.0 \leq k \leq last(\rho) : \rho_k \in g \}$. Both reachability and safety games can be solved in PTIME in the size of \mathcal{G} , and if there is a winning strategy for P2 in \mathcal{G} then, and only then, there is a winning memory-less strategy for P2 in \mathcal{G} (Martin, 1975).

3. Mimicking Behaviors in Separated Domains

The problem of mimicking behaviors involves two agents, A and B, each operating in its own domain, \mathcal{D}_A and \mathcal{D}_B respectively, and requires B to "correctly" mimic in \mathcal{D}_B , the behavior (i.e., a trace) exhibited by A in \mathcal{D}_A . The notion of "correct mimicking" is formalized by a mapping specification, or simply mapping, which is an LTL_f formula, specifying when a behavior of A correctly maps into one of B. The agents alternate their moves on their respective domains, with A starting first, until one of the two decides to stop. Only one agent A and B, designated as the stop agent, has the power to stop the process, and can do so only after both A and B have moved in the last turn. The mapping constraint is evaluated only when the process has stopped.

The dynamic domains where agents operate are modeled as labelled transition systems.

Definition 1 (Dynamic Domain). A dynamic domain over a finite set Prop is a tuple $\mathcal{D} = (S, s_0, \delta, \lambda), s.t.$:

- S is the finite set of domain states;
- $s_0 \in S$ is the initial domain state;
- $\delta \subseteq \mathcal{S} \times \mathcal{S}$ is the transition relation;
- $\lambda: \mathcal{S} \mapsto 2^{Prop}$ is the state-labeling function.

With a slight abuse of notation, for every state $s \in S$, we define the set of possible successors of s as $\delta(s) = \{s' \mid (s,s') \in \delta\}$. \mathcal{D} is deterministic in the sense that given s, the agent operating in \mathcal{D} can select the transition leading to the next state s' from those available in $\delta(s)$. Without loss of generality, we assume that \mathcal{D} is serial, i.e., $\delta(s) \neq \emptyset$ for every state $s \in \mathcal{S}$. A finite trace of \mathcal{D} is a sequence of states $\tau = s_0 \cdots s_n$ s.t. $s_{i+1} \in \delta(s_i)$, for $i = 0, \ldots, n-1$. Infinite traces are defined analogously, except that $i = 0, \ldots, \infty$. By $|\tau|$ we denote the length of τ , i.e., the (possibly infinite) number of states it contains. In the following, we simply use the term trace for a finite trace, and explicitly specify when it is infinite.

We next model the problem of mimicking behaviors by two dynamic systems over disjoint sets of propositions, together with an LTL_f formula specifying the mapping, and the designation of the stop agent.

Definition 2. An instance of the Mimicking Behaviors in Separated Domains (MBSD) problem is a tuple $\mathcal{P} = (\mathcal{D}_A, \mathcal{D}_B, \Phi, Ag_{stop})$, where:

- $\mathcal{D}_A = (S, s_0, \delta^A, \lambda^A)$ is a dynamic domain over $Prop^A$;
- $\mathcal{D}_B = (T, t_0, \delta^B, \lambda^B)$ is a dynamic domain over $Prop^B$, with $Prop^A \cap Prop^B = \emptyset$;
- Φ is the mapping specification, i.e., an LTL_f formula over $Prop^A \cup Prop^B$;
- $Ag_{stop} \in \{A, B\}$ is the designated stop agent.

Intuitively, a solution to the problem is a strategy for agent B that allows B to step-by-step map the observed behavior of agent A into one of its behaviors, in such a way that the mapping specification is satisfied, according to the formalization provided next.

Formally, a strategy for agent B is a function $\sigma:(S)^+ \to T$ which returns a state of \mathcal{D}_B , given a sequence of states of \mathcal{D}_A . Observe that this notion is fully general and is defined on all \mathcal{D}_A 's state sequences, even non-traces. Among such strategies, we want to characterize those that allow B to satisfy the mapping specification by executing actions only on \mathcal{D}_B .

We say that a strategy σ is executable in \mathcal{P} if:

- $\sigma(s_0) = t_0$;
- $\sigma(\tau^A)$ is defined on every trace τ^A of \mathcal{D}_A ;
- for every trace $\tau^A = s_0 \cdots s_n$ of \mathcal{D}_A , the sequence $\tau^B = \sigma(s_0)\sigma(s_0s_1)\cdots\sigma(s_0s_1\cdots s_n)$ is a trace of \mathcal{D}_B (of same length as that of τ^A).

When σ is executable, the trace τ^B as above is called the trace induced by σ on τ^A , and denoted as $\tilde{\sigma}(\tau^A)$.

For two traces $\tau^A = s_0 \cdots s_n$ and $\tau^B = t_0 \cdots t_n$ of \mathcal{D}_A and \mathcal{D}_B , respectively, we define their joint trace label, denoted $\lambda(\tau^A, \tau^B)$ as the word over $2^{Prop^A \cup Prop^B}$ s.t. $\lambda(\tau^A, \tau^B) = (\lambda^A(s_0) \cup \lambda^B(t_0)) \cdots (\lambda^A(s_n) \cup \lambda^B(t_n))$. In words, $\lambda(\tau^A, \tau^B)$ is the word obtained by joining the labels of the states of τ_A and τ_B at same positions.

We can now characterize solution strategies.

Definition 3. A strategy σ is a solution to an MBSD problem instance $\mathcal{P} = (\mathcal{D}_A, \mathcal{D}_B, \Phi, Ag_{stop})$, if σ is executable in \mathcal{P} and either:

- 1. $Ag_{stop} = A$ and every trace τ^A of \mathcal{D}_A is s.t. $\lambda(\tau^A, \tilde{\sigma}(\tau^A)) \models \Phi$; or
- 2. $Ag_{stop} = B$ and every infinite trace τ_{∞}^{A} of \mathcal{D}_{A} has a finite prefix τ^{A} s.t. $\lambda(\tau^{A}, \tilde{\sigma}(\tau^{A})) \models \Phi$.

The definition requires that the strategy σ be executable in \mathcal{P} , i.e., that σ returns an executable move for B, whenever A performs an executable move. Then, two cases are identified, which correspond to the possible designations of the stop agent. In case 1, the stop agent is A. In this case, since A can stop at any time point (unknown in advance by B), B must be able to continuously (i.e., step-by-step) mimic A's behavior, otherwise A could stop at a point where B fails to mimic. Case 2 is slightly different, as B can choose when to stop. In this case, σ must prescribe a sequence of moves, in response to A's, such that Φ is eventually (as opposed to continuously) satisfied, at which point B can stop the execution. Seen differently, σ must prevent A from moving indefinitely, over an infinite horizon (without B ever being able to mimic A).

4. Mimicking Behaviors with Point-wise Mapping Specifications

In this section, we explore mimicking specifications that are of *point-wise* nature. This setting requires that B, while mimicking A, constantly satisfies certain conditions, which can be regarded as *invariants*. Such a requirement is formally captured by the following specification, where φ_i and ψ_i are Boolean formulas over \mathcal{D}_A and \mathcal{D}_B , respectively:

$$\varphi = \bigwedge_{i=1}^k \square(\varphi_i \to \psi_i).$$

We first provide an illustrative example that demonstrates the use of point-wise mappings, then explore algorithmic and complexity results.

4.1 Point-wise Mapping Specifications in the Pac-Man World

In the popular game Pac-Man, the eponymous character moves in a maze to eat all the candies. Four erratic ghosts, Blinky, Pinky, Inky and Clyde, wander around, threatening Pac-Man, which cannot touch them or looses (we neglect the special candies with which Pac-Man can fight the ghosts). The ghosts cannot eat the candies. In the real game, the maze is continuous but, for simplicity, we consider a grid model where cells are identified by

two coordinates. Also, we imagine a variant of the game where the ghosts can walk through walls. Pac-Man wins the stage when it has eaten all the candies. The ghosts end the game when this happens.

We model this scenario as an MBSD problem $\mathcal{Q} = (\mathcal{G}, \mathcal{P}, \Phi, A)$, with domains $\mathcal{P}(\text{ac-Man}, \text{agent } B)$ and $\mathcal{G}(\text{hosts}, \text{agent } A)$. In \mathcal{P} , states model Pac-Man's and candies's position, while transitions model Pac-Man's move actions. Pac-Man cannot walk through walls. A candy disappears when Pac-Man moves on it. Similarly, states of \mathcal{G} model (all) ghosts' position, and transitions model ghosts' movements through cells. Each transition corresponds to a move of all ghosts at once. \mathcal{G} does not model candies or walls, as they do not affect nor are affected by ghosts.

Assuming an $N \times N$ grid with some cells occupied by walls, domain $\mathcal{P} = (S, s_0, \delta^p, \lambda^p)$ is as follows, where C is the set of cells (x, y) not containing a wall:

- for every $(x, y) \in C$, introduce the Boolean propositions $p_{x,y}$ (Pac-Man at (x, y)) and $c_{x,y}$ (candy at (x, y)), and let $Prop^p$ be the set of all such propositions;
- $S \subseteq 2^{(Prop^p)}$ is the set of all interpretations over $Prop^p$ (represented as subsets of $Prop^p$), such that:
 - every $s \in S$ contains exactly one proposition $p_{x,y}$ (Pac-Man occupies exactly one cell);
 - for every $s \in S$, if $p_{x,y} \in s$ then $c_{x,y} \notin s$ (if Pac-Man is in (x,y) the cell contains no candy);
- let $s_0 = \{p_{0,0}\} \cup \{c_{x,y} \mid (x,y) \in C \setminus (0,0)\}$ (Pac-Man in (0,0); cells without Pac-Man or walls contain a candy);
- δ^p is such that $(s, s') \in \delta^p$ iff, for all $(x, y) \in C$:
 - if $p_{x,y} \in s$ then $p_{x',y'} \in s'$, with $(x,y) \in \{(x,y), (x,y+1), (x,y-1), (x+1,y), (x-1,y)\}$ (Pac-Man moves at most by one cell, either horizontally or diagonally);
 - if $c_{x,y} \in s$ and $p_{x,y} \notin s'$ then $c_{x,y} \in s'$ (all candies available in s remain so if not eaten by Pac-Man).
- $\lambda^p(s) = s$.

Domain $\mathcal{G} = (T, t_0, \delta^g, \lambda^g)$ is defined in a similar way (we omit the formal details): we use propositions $bk_{x,y}, pk_{x,y}, ik_{x,y}, cd_{x,y}$ for Blinky, Pinky, Inky and Clyde's position, respectively; T is the set of interpretations where each ghost occupies exactly one cell (possibly containing a wall; many ghosts may be in the same cell); the ghosts start at (N/2, N/2) (t_0) ; δ^g models a 1-cell horizontal or diagonal move for all ghosts at once; λ^g is the identity.

Pac-Man's primary goal (besides eating all candies) is to stay alive, which we formalize with the following point-wise mapping:

$$\Phi = \bigwedge_{(x,y) \in C} \Box((bk_{x,y} \lor pk_{x,y} \lor ik_{x,y} \lor cl_{x,y}) \to \neg p_{x,y}).$$

Any strategy σ that is a solution to $\mathcal{Q} = (\mathcal{G}, \mathcal{P}, \Phi, B)$ keeps Pac-Man alive. To enforce Φ , Pac-Man needs a strategy that prevents ending up in a cell where a ghost is. Notice that,

to compute σ , one cannot proceed greedily by considering only one step at a time, but must plan over all future evolutions, to guarantee that Pac-Man does not eventually get trapped. With such σ , no matter when the ghosts end the game, Pac-Man will never lose (and, in fact, it will win, if the ghosts stop when all candies on the maze have been eaten).

4.2 Solving MBSD with Point-wise Mapping Specifications

We show how to solve an MBSD instance \mathcal{P} by reduction to the problem of finding a winning strategy in a two-player game, for which algorithms are well known (Martin, 1975). Specifically, we construct a two-player game $\mathcal{G}_{\mathcal{P}} = (\mathcal{A}, W)$ that has a winning strategy iff \mathcal{P} has a solution.

Given an MBSD instance $\mathcal{P} = (\mathcal{D}_A, \mathcal{D}_B, \Phi, Ag_{stop})$, with $\mathcal{D}_A = (S, s_0, \delta^A, \lambda^A)$ and $\mathcal{D}_B = (T, t_0, \delta^B, \lambda^B)$, we construct the game arena $\mathcal{A} = (U, V, u_0, \alpha, \beta)$, where:

- $U = S \times T$;
- $V = S \times T$:
- $u_0 = (s_0, t_0);$
- $\alpha = \{(s, t), (s', t) \mid (s, s') \in \delta^A\};$
- $\beta = \{(s,t), (s,t') \mid (t,t') \in \delta^B\}.$

Intuitively, the nodes of \mathcal{A} represent joint state configurations of both \mathcal{D}_A and \mathcal{D}_B (initially in their respective initial states), while the transition functions account for the moves A (modeled by P1) and B (modeled by P2) can perform, imposing, at the same time, their strict alternation.

As for the winning objective W, the key idea is that, since in point-wise mappings the temporal operator \Box (always) distributes over conjunction, and since $Ag_{stop} = A$, the conjuncts of the mapping are in fact propositional formulae to be guaranteed all along the agent behaviors, captured by plays of A. This can be easily expressed as a safety objective on A, as shown below.

Let $\Phi = \bigwedge_{i=1}^k \Box(\varphi_i \to \psi_i)$ be the (point-wise) mapping specification. We have that $\Phi \equiv \Box \Phi'$, where $\Phi' \equiv \bigwedge_{i=1}^k (\varphi_i \to \psi_i)$ is a Boolean formula where every φ_i is over Prop^A only and every ψ_i over Prop^B only. Therefore, in order to solve \mathcal{P} , we need to find a strategy σ such that for every trace τ^A of \mathcal{D}_A , $\lambda(\tau^A, \tilde{\sigma}(\tau^A)) \models \Box \Phi'$, that is, $\lambda^A(s_j) \cup \lambda^B(t_j) \models \Phi'$ for $j = 0, \ldots, |\tau^A|$. Thus we can set $W = \operatorname{Safe}(g)$, with $g = \{(s,t) \in U \mid \lambda^A(s) \cup \lambda^B(t) \models \Phi'\}$. As a consequence of the above construction, we obtain the following result.

Lemma 1. There is a solution to \mathcal{P} if and only if there is a solution to the safety game $\mathcal{G}_{\mathcal{P}}$.

Proof. As an intuition, notice that once computed, a winning strategy for $\mathcal{G}_{\mathcal{P}}$ is essentially a solution to \mathcal{P} . This, indeed, can be obtained by projecting away the V component of all the nodes in a play ρ , thus transforming ρ into a trace of \mathcal{D}_A .

We now show the proof in detail. We first show that if there is a solution to \mathcal{P} then there is a solution to $\mathcal{G}_{\mathcal{P}}$. For that, we first show that if σ is an executable strategy for

 \mathcal{P} then σ can be reduced to a strategy σ' for $G_{\mathcal{P}}$. To this end, consider a play $\rho = \rho_0 \rho_1 \cdots \rho_n$, with $\rho_i = (s_i, t_i)$ and a state (s_{n+1}, t_n) such that $((s_n, t_n), (s_{n+1}, t_n)) \in \alpha$. Let $\tau = s_0 s_1 s_3 \cdots s_{n-1} s_{n+1} \in V^{n/2+1}$. By the definition of $\mathcal{G}_{\mathcal{P}}$, τ is a trace of \mathcal{D}_A . Therefore, since σ is executable, σ is defined on τ . Thus, for $\rho' = \rho \circ (s_{n+1}, t_n)$, where \circ denotes concatenation, we can define $\sigma'(\rho') = (s_{n+1}, \sigma(\tau))$. Note that this is a proper definition since the trace $\tilde{\sigma}(\tau)$ induced by σ on τ is a trace in \mathcal{D}_B , hence $(t_n, \sigma(\tau)) \in \delta^B$. Thus σ' is a proper strategy for $\mathcal{G}_{\mathcal{P}}$.

Next, we need the following claim that describes the correspondence between σ and σ' .

Claim 1. A sequence $\rho = \rho_0 \cdots \rho_n \in (U \cup V)^+$ is a play of $\mathcal{G}_{\mathcal{P}}$ compatible with σ' iff there exist a trace $\tau^A = s_0 \cdots s_n$ of \mathcal{D}_A and a trace τ^B of \mathcal{D}_B such that $\tau^B = \tilde{\sigma}(\tau^A) = t_0 \cdots t_n$ and $\rho = (s_0, t_0)(s_1, t_0) \cdots (s_n, t_{n-1})(s_n, t_n)$.

For a proof of Claim 1, given a trace $\tau^A = s_0 \cdots s_n$ of \mathcal{D}_A , let $\tau^B = \tilde{\sigma}(\tau^A) = t_0 \cdots t_n$ be the trace of \mathcal{D}_B induced by σ on τ^A . By the definition of $\mathcal{G}_{\mathcal{P}}$ and that of σ' provided above, it follows that the sequence $\rho = (s_0, t_0)(s_1, t_0) \cdots (s_n, t_{n-1})(s_n, t_n)$ is a play of $\mathcal{G}_{\mathcal{P}}$ compatible with σ' . On the other hand, for a play $\rho = (s_0, t_0) \cdots (s_n, t_n)$ compatible with σ' , again by the definition of $\mathcal{G}_{\mathcal{P}}$ and σ' , we have that the sequences $\tau_A = s_0 \cdots s_n$ and $\tau_B = t_0 \cdots t_n$ are traces of, respectively \mathcal{D}_A and \mathcal{D}_B , such that $\tau_B = \tilde{\sigma}(\tau_A)$.

Back to proving Lemma 1, since σ is a solution, every trace τ^A in \mathcal{D}_A is such that $\lambda(\tau^A, \tilde{\sigma}(\tau^A)) \models \Phi$. For $\tau^A = s_0 \cdots s_n$, let $\tau^B = \tilde{\sigma}(\tau^A) = t_0 \cdots t_n$. Because $\Phi = \Box \Phi'$ is a point-wise mapping, for every i, we have that $(\lambda^A(s_i), \lambda^B(t_i)) \models \Phi'$, that is, in $\mathcal{G}_{\mathcal{P}}$, $(s_i, t_i) \in g$.

Now, let $\rho = (s_0, t_0)(s_1, t_0) \cdots (s_n, t_{n-1})(s_n, t_n)$ be a play in $\mathcal{G}_{\mathcal{P}}$ compatible with σ' (recall $(s_n, t_n) \in U$). By Claim 1, the sequences $\tau^A = s_0 \cdots s_n$ and $\tau^B = t_0 \cdots t_n$ are traces of \mathcal{D}_A and \mathcal{D}_B , respectively, with $\tau^B = \tilde{\sigma}(\tau^A)$. Then $(\lambda^A(s_n), \lambda^B(t_n) \models \Phi'$, that is $(s_n, t_n) \in g$. Since ρ is arbitrary, every play in $G_{\mathcal{P}}$ compatible with σ' ends in a g node, hence σ' is a winning strategy for P2 in Safe(g). That completes the first direction of the theorem.

For the other direction, assume that σ' is a strategy for $\mathcal{G}_{\mathcal{P}}$. Define a strategy σ'' for \mathcal{P} as follows. Define first $\sigma''(s_0) = t_0$. Then, For a play $\rho = \rho_0 \rho_1 \cdots \rho_n$, with $\rho_i = (s_i, t_i)$, and a state (s_{n+1}, t_n) such that $((s_n, t_n), (s_{n+1}, t_n)) \in \alpha$, note that $\tau = s_0 s_1 s_3 \cdots s_{n-1} \cdots s_{n+1} \in V^{n/2+1}$ is a trace of \mathcal{D}_A , and define $\sigma''(\tau) = \sigma'(\rho \circ (s_{n+1}, t_n))$. By the definition of $\mathcal{G}_{\mathcal{P}}$, it follows that $\tau' = t_0 t_2 \cdots t_n \sigma''(\tau)$ is a trace in \mathcal{D}_B , thus σ'' is an executable strategy in \mathcal{P} .

To describe the correspondence between σ' and σ'' we make the next claim, completely analogous to Claim 1.

Claim 2. A sequence $\rho = \rho_0 \cdots \rho_n \in (U \cup V)^+$ is a play of $\mathcal{G}_{\mathcal{P}}$ compatible with σ' iff there exist a trace $\tau^A = s_0 \cdots s_n$ of \mathcal{D}_A and a trace τ^B of \mathcal{D}_B such that $\tau^B = \tilde{\sigma}''(\tau^A) = t_0 \cdots t_n$ and $\rho = (s_0, t_0)(s_1, t_0) \cdots (s_n, t_{n-1})(s_n, t_n)$.

For a proof, given a trace $\tau^A = s_0 \cdots s_n$ of \mathcal{D}_A , let $\tau^B = \tilde{\sigma''}(\tau^A) = t_0 \cdots t_n$ be the trace of \mathcal{D}_B induced by σ'' on τ^A . By the definition of σ'' provided above, it follows that the sequence $\rho = (s_0, t_0)(s_1, t_0) \cdots (s_n, t_{n-1})(s_n, t_n)$ is a play of $\mathcal{G}_{\mathcal{P}}$ compatible with σ' . On the other hand, for a play $\rho = (s_0, t_0) \cdots (s_n, t_n)$ compatible with σ' , again by the definition of σ'' , we have that the sequences $\tau_A = s_0 \cdots s_n$ and $\tau_B = t_0 \cdots t_n$ are traces of, respectively \mathcal{D}_A and \mathcal{D}_B , such that $\tau_B = \tilde{\sigma}''(\tau_A)$.

Now to conclude Lemma 1, assume that σ' is a winning strategy for P2 in $\mathcal{G}_{\mathcal{P}}$, with winning objective $W = \operatorname{Safe}(g)$. For a trace $\tau^A = s_0 \cdots s_n$ of \mathcal{D}_B , let $\tau^B = \tilde{\sigma''}(\tau^A) = t_0 \cdots t_n$. By Claim 2, we have that the sequence $\rho = (s_0, t_0)(s_1, t_0) \cdots (s_n, t_{n-1})(s_n, t_n)$ is a play of $\mathcal{G}_{\mathcal{P}}$ compatible with σ' . Moreover, since σ' is winning, for $i = 1, \ldots, 2n$, $\rho_i \in g$. But then, for all pairs (s, t) in ρ , we have that $(\lambda^A(s), \lambda^B(t)) \models \phi'$, that is $\lambda(\tau^A, \tau^B)$ satisfies $\Box \Phi$. Since τ^A is arbitrary, it follows that σ'' is a solution for \mathcal{P} , which completes the proof. \Box

Finally, the construction of the safety game $\mathcal{G}_{\mathcal{P}}$ together with Lemma 1 gives us the following result.

Theorem 1. Solving MBSD for point-wise mapping specifications is in PTIME for combined complexity, mapping complexity and domain complexity.

Proof. Given an MBSD instance \mathcal{P} , we construct the safety game $\mathcal{G}_{\mathcal{P}}$ as shown. Observe that the construction of $\mathcal{G}_{\mathcal{P}}$ requires constructing the game arena \mathcal{A} , which can be done in time polynomial in $|\mathcal{D}_A| + |\mathcal{D}_B|$, and setting the set of states g, which takes at most time $\mathcal{O}(|\Phi'|)$ for each state in \mathcal{A} . Finally by Lemma 1 we have that \mathcal{P} has a solution if and only if $\mathcal{G}_{\mathcal{P}}$ has a solution, where solving a safety game takes linear time in the size of $\mathcal{G}_{\mathcal{P}}$ (Martin, 1975).

Observe that if \mathcal{D}_A and \mathcal{D}_B are represented compactly (logarithmically) using, e.g., logical formulas or PDDL specifications (Haslum, Lipovetzky, Magazzeni, & Muise, 2019), then the domain (and hence the combined) complexity becomes EXPTIME, and mapping complexity remains PTIME. Similar considerations hold also for the other cases that we analyze throughout the paper.

5. Mimicking Behaviors with Target Mapping Specifications

We now explore mimicking specifications that are of target nature. In this setting, B has to mimic A in such a way that whenever A reaches a certain target, so does B, although not necessarily at the same time step: B is free to reach the required target at the same time, later, or even before A does. For this to be possible, B must have the power to stop the game, which is what we assume here. Formally, target mapping specifications are formulas of the following form, where φ_i and ψ_i are Boolean properties over \mathcal{D}_A and \mathcal{D}_B , respectively:

$$\varphi = \bigwedge_{i=1}^{k} (\Diamond \varphi_i) \to (\Diamond \psi_i)$$

As before, we first give an illustrative example that demonstrates the use of target mappings, then we explore algorithmic and complexity results.

5.1 Target Mapping Specifications in Rubik's Cube

Two agents, teacher H and learner L are provided with two Rubik's cubes of different sizes: H has edge of size 4 whereas L has one of size 3. L wants to learn from H the main steps to solve the cube; to this end, H shows L how to reach certain milestone configurations on the cube of size 4 and asks L to replicate them on the cube of size 3, even in a different

order. Milestones are simply combinations of solved faces, e.g., red and green, white and blue and yellow, or simply white. Obviously, L cannot blindly replicate H's moves, as the cubes are of different sizes and the actual sequences to solve the faces are different; thus, L must find its way to reach the same milestones as H, possibly in a different order. When L is tired, it can stop the learning process.

We model this scenario as an MBSD problem instance $\mathcal{R} = (\mathcal{H}, \mathcal{L}, \Phi, B)$, where \mathcal{H} and \mathcal{L} model, respectively, H's and L's dynamic domain, i.e., the two cubes. The two domains are conceptually analogous but, modeling cubes of different sizes, they feature different sets of states and transitions, which correspond to cube configurations and possible moves, respectively. We model such domains parametrically wrt the size E of the edge.

Fix the cube in some position, name the faces as U(p), D(own), L(eft), R(ight), F(ront), B(ack), let $Fac = \{U, D, L, R, F, B\}$, and associate a pair of integer coordinates to each position in a face, so that every position is identified by a triple $(f, x, y) \in Pos = Fac \times \{0, \ldots, E-1\}^2$. To model the color assigned to tile (f, x, y), we use propositions of the form $c_{f,x,y}$, with $c \in Col = \{white, green, red, yellow, blue, orange\}$. Let Prop be the set of all such propositions. Finally, index the horizontal and vertical "slices" of the cube from 0 to E-1.

The (parametric) dynamic domain for a Rubik's cube with edge of size E is the domain $\mathcal{D}(E) = (S, s_0, \delta, \lambda)$, where:

- $S \subseteq 2^{Prop^E}$ is the set of all admissible (i.e., reachable) cube's configurations; among other constraints, omitted for brevity, this requires that, for every $s \in S$:
 - for every $(f, x, y) \in Pos$, there exists exactly one $c \in C$ such that $c_{f,x,y} \in s$ (every position has exactly one color);
- s_0 is an arbitrary state from S;
- δ allows a transition from s to s' iff s' models a configuration reachable from s by a 90° (clockwise or counter-clockwise) rotation of one of its 2 * E slices;
- $\lambda(s) = s$.

We then define $\mathcal{H} = \mathcal{D}(4)$ and $\mathcal{L} = \mathcal{D}(3)$. To distinguish the elements of \mathcal{H} from those of \mathcal{L} , we use a primed version in the latter, e.g., Pos' for positions, $c'_{f,x,y}$ for propositions, and so on.

As said, L's goal is to replicate the milestones shown by H. For every face $f \in Fac$, we define formula $C_f = \bigwedge_{(f,x,y)\in Pos} c_{f,x,y}$ to express that the tiles of face f have all the same color c. For \mathcal{L} , we correspondingly have $C'_f = \bigwedge_{(f,x,y)\in Pos'} c'_{f,x,y}$.

We report below an example of target mappings:

$$\begin{array}{l} (\lozenge blue_R) \to (\lozenge blue_R') \\ (\lozenge (red_U \land white_L)) \to (\lozenge (red_U' \land white_L')) \\ (\lozenge (red_U \land \neg white_L)) \to (\lozenge (red_U' \land \neg white_L')). \end{array}$$

Observe that L has many ways to fulfill H's requests: for instance, by reaching a configuration where $blue'_R \wedge red'_U \wedge white'_L$ holds, it has fulfilled the first and the second request, even if the configuration was reached before H showed the milestones. Obviously, however,

the last request cannot be fulfilled at the same time as the second one, as $white'_L$ clearly excludes $\neg white'_L$, thus an additional effort by L is required to satisfy the specification.

5.2 Solving MBSD with Target Mapping Specifications

For target mappings as well, we reduce MBSD to strategy synthesis for a two-player game. To this end, assume an MBSD instance $\mathcal{P} = (\mathcal{D}_A, \mathcal{D}_B, \Phi, B)$ with mapping specification $\Phi = \bigwedge_{i=1}^k (\Diamond \varphi_i) \to (\Diamond \psi_i)$. To solve \mathcal{P} , we must find a strategy σ such that for every infinite trace $\tau_\infty^A = s_0 s_1 \cdots$ of \mathcal{D}_A and every conjunct $(\Diamond \varphi_i) \to (\Diamond \psi_i)$ of Φ , if there exists an index j_i such that $\lambda^A(s_{j_i}) \models \varphi_i$, then there exist a finite prefix $\tau_A = s_0 \cdots s_n$ of τ_∞^A and an index l_i such that, for $\sigma(\tau) = t_0 \cdots t_n$, we have that $\lambda^B(t_{l_i}) \models \psi_i$ (recall φ_i and ψ_i are Boolean formulae over $Prop^A$ only and $Prop^B$ only, respectively). As per Definition 3, this is equivalent to requiring that $\lambda(\tau^A, \tilde{\sigma}(\tau^A)) \models (\Diamond \varphi_i) \to (\Diamond \psi_i)$.

The challenge in constructing σ is that the index l_i may be equal, smaller or larger then j_i . Thus σ needs to record which φ_i or ψ_i were already met during the trace, up to the current point. Since the number of possible traces to the current state may be exponential, keeping count of all possible options may be expensive. We first discuss general domain structure, then in Section 5.2.2 we explore a very specific tree-like structure.

For general domains, there may exist many traces ending in a given state, and each such trace contains states that satisfy, in general, different sub-formulas φ_i and ψ_i occurring in the mappings. Thus satisfaction of sub-formulas cannot be associated to states as done before, but must be associated to traces. In fact, to check whether a target mapping is satisfied, it is enough to remember, for every $i = 1, \ldots, k$, whether A has satisfied φ_i and/or B has satisfied ψ_i , along a trace. This observation suggests to introduce a form of memory to record satisfaction of sub-formulas along traces. We do so by augmenting the game arena constructed in Section 4. In particular, we extend each node in the arena with an array of bits of size 2k to keep track of which sub-formulas φ_i and ψ_i were satisfied, along the play that led to the node, by some of the domain states contained in the nodes of the play.

Formally, let $M = (\{0,1\}^2)^k$ and let $[cd] = ((c_1, d_1), \dots, (c_k, d_k))$ denote the generic element of M. Given an MBSD instance $\mathcal{P} = (\mathcal{D}_A, \mathcal{D}_B, \Phi, B)$, where $\mathcal{D}_A = (S, s_0, \delta^A, \lambda^A)$ and $\mathcal{D}_B = (T, t_0, \delta^B, \lambda^B)$, we define the game arena $\mathcal{A} = (U, V, u_0, \alpha, \beta)$ as follows:

- $U = S \times T \times M$:
- $V = S \times T \times M$:
- $u_0 = (s_0, t_0, [cd])$ such that, for every $i \leq k$, $c_i = 1$ iff $\lambda^A(s_0) \models \phi_i$ and $d_i = 1$ iff $\lambda^B(t_0) \models \psi_i$;
- $((s,t,[cd]),(s',t,[c'd])) \in \alpha$ iff $(s,s') \in \delta^A$, and for $i=1,\ldots,k$, if $\lambda^A(s') \models \phi_i$ then $c'_i = 1$, otherwise $c'_i = c_i$;
- $((s,t,[cd]),(s,t',[cd'])) \in \beta$ iff $(t,t') \in \delta^B$, and for $i=1,\ldots,k$, if $\lambda^B(t') \models \psi_i$ then $d_i'=1$, otherwise $d_i'=d_i$.

We then define the game structure $\mathcal{G}_{\mathcal{P}} = (\mathcal{A}, W)$, where W = Reach(g), with $g = \{u \in U \mid u = (s, t, [cd]), \text{ where } [cd] \text{ is s.t. } c_i = 0 \text{ or } d_i = 1, \text{ for every } i = 1, \dots, k\}$. Intuitively, g is the set of all nodes reached by a play such that if ϕ_i is satisfied in the play (by a state

of \mathcal{D}_A in some node of the play), then so is ψ_i , for i = 1, ..., k (by a state of \mathcal{D}_B in some node of the play). Thus, if a play contains a node from g then the corresponding traces of \mathcal{D}_A and \mathcal{D}_B , combined, satisfy all the mapping's conjuncts.

As a consequence of this construction, we obtain the following result, the full proof of which is in line of Lemma 1.

Lemma 2. There is a solution to \mathcal{P} if and only if there is a winning strategy for the reachability game $\mathcal{G}_{\mathcal{P}}$.

Then, Lemma 2 gives us the following.

Theorem 2. MBSD with target mapping specifications can be solved in time polynomial in $|\mathcal{D}_A \times \mathcal{D}_B| \times |\Phi| \times 4^k$, with Φ the mapping specification and k the number of its conjuncts.

Proof. Given an MBSD instance \mathcal{P} with target mapping specifications, we construct a reachability game $\mathcal{G}_{\mathcal{P}}$ as shown above, which has size $|\mathcal{D}_A \times \mathcal{D}_B| \times 4^k$ and construction time polynomial in $|\mathcal{D}_A \times \mathcal{D}_B| \times |\Phi| \times 4^k$. Result then follows from Lemma 2 and from the fact that reachability games can be solved in linear time in the size of the game.

An immediate consequence of Theorem 2 is that, for mappings of fixed size, the domain-complexity of the problem is in PTIME. For combined complexity, note that the memory-keeping approach adopted in $\mathcal{G}_{\mathcal{P}}$ is of a monotonic nature, i.e., once set, the bits corresponding to the satisfaction of ψ_i and ϕ_i cannot be unset. We use this insight to tighten our result and show that the presented construction can be in fact carried out in PSPACE.

Theorem 3. MBSD for target mapping specifications is in PSPACE for combined complexity and mapping complexity, and in PTIME for domain complexity.

Proof. Having shown PTIME membership for domain-complexity in Theorem 2, it remains to show membership in PSPACE for combined-complexity. Assume that P2 wins the game $\mathcal{G}_{\mathcal{P}}$ and let $\sigma_{\mathcal{P}}$ be a memory-less winning strategy for P2. First see that every play ρ in $\sigma_{\mathcal{P}}$ is finite. Therefore, since $\sigma_{\mathcal{P}}$ is memory-less then every play ρ in $\sigma_{\mathcal{P}}$ does not hold two identical V nodes. That means that wlog in every play, the [cd] index in every game node changes after at most $2 \times |\mathcal{D}_A \times \mathcal{D}_B|$ steps (since there are two copies in $\mathcal{G}_{\mathcal{P}}$ of the domains product- for P1 and P2). Next, we use the monotonicty property in $\mathcal{G}_{\mathcal{P}}$. Specifically, between every two consecutive game nodes $\rho_i = (s, t, [cd]), \, \rho_{i+1} = (s', t', [c'd'])$ for some i in ρ , every index in [cd] can only remain as is or change from 0 to 1, therefore the bit index changes at most 2k times throughout the play.

Thus, we reduce $\mathcal{G}_{\mathcal{P}}$ to an identical game $\mathcal{G}'_{\mathcal{P}}$ that terminates either when reaching an accepting state (then P2 wins), or after $2 \times |\mathcal{D}_A \times \mathcal{D}_B| \times 2k$ moves (then P1 wins). Standard Min-Max algorithms (e.g. (Russell & Norvig, 2020)) that work in space size polynomial to maximal strategy depth can be deployed to verify a winning strategy for P2 in $\mathcal{G}'_{\mathcal{P}}$. Then on one hand if there is a winning strategy for $\mathcal{G}_{\mathcal{P}}$ then there is a winning strategy for $\mathcal{G}_{\mathcal{P}}$ then there is a memory-less winning strategy for $\mathcal{G}_{\mathcal{P}}$ that terminates after at most $2 \times |\mathcal{D}_A \times \mathcal{D}_B| \times 2k$ moves, which means that there is a winning strategy for P2 in $\mathcal{G}'_{\mathcal{P}}$.

We continue our analysis of the case of MBSD target mapping specifications by exploring whether memory-keeping is avoidable and a more effective solution approach can be found. As the following result implies, this is, most likely, not the case.

Theorem 4. MBSD for target mapping specifications is PSPACE-hard in combined complexity (even for \mathcal{D}_A , \mathcal{D}_B as simple DAGs).

Proof Outline. We give a proof sketch, see Section 5.2.1 below for the detailed proof.

A QBF-CNF-1 formula is a QBF formula in a CNF form in which every clause contains at most one universal variable. The language TQBF-CNF-1, of all true QBF-CNF-1 formulas, is also PSPACE-complete. See Proposition 1 below for completion. We show a polynomial time reduction to MBSD from a TQBF-CNF-1.

Given a QBF-CNF-1 formula F, assume wlog that each alternation holds exactly a single variable. Construct the following MBSD instance \mathcal{P}_F . Intuitively, the domains \mathcal{D}_A and \mathcal{D}_B are directed acyclic graphs (DAG) where \mathcal{D}_A controls the universal variables and \mathcal{D}_B controls the existential variables, see Figure 1 for a rough sketch of the domains graph for a QBF Formula with universal variables x_1^A, x_2^A and existential variables x_1^B, x_2^B . The initial states are s_1^A for agent A and s_1^B for agent B. By traversing the domains in alternation, each agent can choose at every junction node depicted as s_i^A for D_A or s_i^B for D_B , between either a true path through \top depicted nodes, or false path through \bot depicted nodes, thus correspond to setting assignments to propositions that are analogue to universal (agent A) or existential (agent B) variables. For example, by visiting $s_{1\tau}^A$, agent A satisfies a proposition called $p_{1_{\perp}}^{A}$ that corresponds to assign the universal variable $x_{1}^{A} = true$. The mapping Φ is set according to F where each clause corresponds to a specific conjunct. For example a clause $(x_1^A \vee x_2^B)$ becomes a conjunct $\Diamond(p_{1_+}^A) \to \Diamond(p_{2_+}^B)$ of Φ , where $p_{1_+}^A, p_{2_+}^B$ are propositions in $Prop^A$ and $Prop^B$ respectively. An additional conjunct is added to ensure that Agent B does not stop ahead of time. Then a strategy for agent B of which path to choose at every junction node corresponds to a strategy of which existential variable to assign for F. As such, F is true if and only if there is a solution to the MBSD \mathcal{P}_F .

5.2.1 Detailed proof of Theorem 4

We first provide a detailed proof of Theorem 4. Then for completness we prove that the language TQBF-CNF-1, used in the proof, is PSPACE-complete.

Given a QBF-CNF-1 formula F with n universal variables $x_1^A, \cdots x_n^A$ and n existential variables $x_1^B, \cdots x_n^B$, assume wlog that each alternation holds exactly a single variable. Construct the following MBSD instance \mathcal{P}_F . Intuitively for $H \in \{A, B\}$, the separate \mathcal{D}_H domains are DAGs, each composed of n+1 major states $s_1^H, s_2^H, \cdots s_{n+1}^H$ respectively. Let $Prop^H = \{p_{1_{\top}}^H, p_{1_{\bot}}^H, \cdots p_{n_{\top}}^H, p_{n_{\bot}}^H, p_*^H\}$. From s_i^H , for $1 \leq i \leq n$, Agent H can move only to s_{i+1}^H through exactly one of the following paths: a directed true path that visits a vertex s_{i+1}^H labeled by $\{p_{i+1}^H\}$ or a directed false path that visits a vertex s_{i+1}^H labeled by $\{p_{i+1}^H\}$. From s_{n+1}^H there is only directed self-loop. Thus the choice of which path to take means whether the subformula $\Diamond(p_{i+1}^H)$ is satisfied (that corresponds to setting $x_i = true$), or $\Diamond(p_{i+1}^H)$ isn't satisfied (that corresponds to setting $x_i = false$). Finally label s_{n+1}^A with $\{p_*^A\}$ and s_{n+1}^B with $\{p_*^B\}$. Then $\Diamond(p_*^H)$ is true in every game played.

For the mapping specification Φ , note that every clause C at F is of the form $(l_{x^A} \vee C_B)$ or (C_B) , where l_{x^A} is a literal of a universal variable (universal literal) and (C_B) is a disjunction of literals of existential constraint (existential literals). For every such C_B define C_B^{Prop} to be a disjunction of propositions from $Prop^B$ in which every negated (resp. un-negated) literal $l_{x_i^B}$ is replaced with $p_{i_\perp}^B$ (resp. $p_{i_\perp}^B$). Next, for every clause C of F, add to Φ a conjunct ν_C as follows. If C is of the form $(l_{x_i^A} \vee C_B)$, set $\nu_C = (\lozenge(p_{i_\perp}^A) \to \lozenge(C_B^{Prop}))$ if $l_{x_i^A}$ is un-negated, and $\nu_C = (\lozenge(p_{i_\perp}^A) \to \lozenge(C_B^{Prop}))$ if $l_{x_i^A}$ is negated (note that the negation has switched for p^A). If C is of the form (C_B) , set $\nu_C = (\lozenge(p_*^A) \to \lozenge(C_B^{Prop}))$. Since a clause $(x^A \vee C_B)$ is logically equivalent to $(\neg x^A \to C_B^{Prop})$ and the clause (C_B) is logically equivalent to (∇C_B^{Prop}) , the construction of Φ mirrors a clause C with its corresponding conjunct ν_C . To complete Φ , add a final conjunct $(\lozenge(p_*^A) \to \lozenge(p_*^B))$ called the stopping-constraint. Note that the stopping-constraint is true only when both agents reach s_{n+1}^H . Thus, the role of the stopping-constraint is to ensure that agent B does not stop the game before reaching its end. Finally set $Ag_{stop} = B$ to finish the construction of P as an MBSD problem with target mapping specification as required.

We give an example of the construction. let F be the QBF input as follows.

$$F = \forall x_1^A \exists x_1^B \forall x_2^A \exists x_2^B ((x_1^A \lor x_1^B \lor x_2^B)$$

$$\land (\neg x_2^A \lor \neg x_1^B)$$

$$\land (x_1^B \lor \neg x_2^B))$$

Then the MBSD \mathcal{P}_F is constructed as follows. The domains $\mathcal{D}_A, \mathcal{D}_B$ are in Figure 1 where s_1^A, s_1^B are the initial state for agents A, B respectively. s_3^A has a proposition $\{p_*^A\}$ and s_3^B has a proposition $\{p_*^B\}$. For every $H \in \{A, B\}$ and $i \in \{1, 2\}$ every node $s_{i_{\perp}}^H$ has a proposition $p_{i_{\perp}}^H$ and every node $s_{i_{\perp}}^H$ has a proposition $p_{i_{\perp}}^H$. The stop agent Ag_{stop} is set to B. The mapping specification is as follows:

$$\begin{split} \Phi = & (\Diamond(p_{1_{\perp}}^{A}) \rightarrow \Diamond(p_{1_{\top}}^{B} \vee p_{2_{\top}}^{B})) \\ & \wedge (\Diamond(p_{2_{\top}}^{A}) \rightarrow \Diamond(p_{1_{\perp}}^{B})) \\ & \wedge (\Diamond(p_{*}^{A}) \rightarrow \Diamond(p_{1_{\top}}^{B} \vee p_{2_{\perp}}^{B})) \\ & \wedge (\Diamond(p_{*}^{A}) \rightarrow \Diamond(p_{*}^{B})) \end{split}$$

Back to the proof, obviously the construction of \mathcal{P} is time-polynomial wrt |F|. Note that while the agents move in $\mathcal{D}_A, \mathcal{D}_B$, the only choices that the agent has are at every s_i^H , to decide whether to move through the true-path or the false-path. Also note that both agents always progress at the same pace. That is: agent A is in s_i^A iff agent B is in s_i^B . Also note that in every path that the agents take their respective domains, exactly one of $s_{i_{\perp}}^H$ or $s_{i_{\perp}}^H$ can be visited, thus at every trace formed either $p_{i_{\perp}}^H$ or $p_{i_{false}}^H$ are satisfied but not both. That means that $\Diamond(p_{i_{\perp}}^H) \leftrightarrow \Diamond(p_{i_{\perp}}^H)$ is always true.

Now assume that F is true. Therefore there is a strategy σ_F for the existential player that sets F to be true. Then we construct the following strategy $\sigma_{\mathcal{P}}$ for agent B: whenever agent A is at s_i^A and takes the true-path and thus satisfies $p_{i_{\perp}}^A$ (resp. false-path to satisfy $p_{i_{\perp}}^A$, assign $x_i^A = true$ (resp. $x_i^A = false$) in σ_F . If the result is $x_i^B = true$ (resp. $x_i^B = false$)

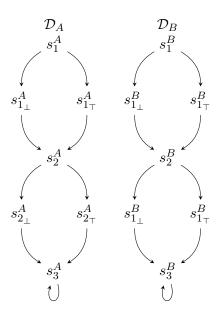


Figure 1: A rough sketch of the domains in the reduction construction in Theorem 4. The initial state for agent A is s_1^A and for agent B is s_1^B .

then set agent B to take the true-path and thus satisfy $p_{i_{\perp}}^{B}$ (resp. false-path to satisfy $p_{i_{\perp}}^{B}$. Due to the mirroring between Φ and F, it follows that when both agents reach s_{n+1}^{H} (and therefore the stopping-constraint is true), we have that every clause C in F is true and thus so is its corresponding conjunct ν_{C} (recall that the subformula $\Diamond(p_{*}^{A})$ is always true).

Next, assume that there is a winning strategy $\sigma_{\mathcal{P}}$ for \mathcal{P} . Then similarly we construct a strategy σ_F as follows. At every point s_i^B , whenever agent B takes the true-path (resp. false-path) set $x_i^B = true$ (resp. $x_i^B = false$). Following $\sigma_{\mathcal{P}}$ ensured all the conjuncts of Φ are true. Note that since the stopping-constraint is satisfied, Agent B reaches s_{n+1}^B which guarantees that σ_F is well defined for all variables. In addition, every clause C corresponding to a conjunct ν_C must also be true. For example, if $\Diamond(p_*^A) \to \Diamond(C_B^{Prop})$ is true then since $\Diamond(p_*^A)$ is always true, it means that $\Diamond(C_B^{Prop})$ must be true, which means that a proposition in C_B^{Prop} is satisfied which means that a variable in C_B is set true in σ_F , hence C_B is true. That completes the proof. \square

The PSPACE-hardness of TQBF-CNF-1 is not a hard exercise, for completion we bring a full proof.

Proposition 1. TQBF-CNF-1 is PSPACE-complete.

Proof. TQBF-CNF is known to be PSPACE-complete (Garey & Johnson, 1979). Obviously TQBF-CNF-1 is in PSPACE, we show PSPACE-hardness. Given a QBF-CNF formula F, we transform F to a QBF-CNF-1 formula F' such that F is true if and only if F' is true. For that, we construct a formula F' from F as follows. We first add a fresh existential variable z_i for every universal variable x_i . In addition, conjunct F with clauses $(x_i \vee \neg z_i)$

and $(\neg x_i \lor z_i)$ that their conjunction is logically equivalent to $(x_i \leftrightarrow z_i)$. Finally, in every original clause C of F we replace every literal x_i with z_i and every literal $\neg x_i$ with $\neg z_i$. For the alternation order, we place the z_i anywhere after x_i (we can add dummy universal variables to keep the alternation interleaving order, as standard in such reductions). Since every original clause in F contains now only existential variables, we have that F' is indeed in the QBF-CNF-1 form that we described. Moreover, note that in F' every clause that holds a universal literal is of a size of 2.

Obviously, constructing F' from F is of polynomial time to |F|. Assume that F is true. Then there is a strategy σ_F for choosing existential variables such that F is true. Then define a strategy $\sigma_{F'}$ that copies σ_F , and for every choice for z_i , echos the assignment for x_i . That is set $z_i = true$ iff x_i was set to true. Since every x_i precedes z_i , this can be done. Then such a strategy sets F' to be true. Next assume F' is true. Then there is a strategy $\sigma_{F'}$ for choosing existential variables such that F is true. Then set a strategy for σ_F that just repeats $\sigma_{F'}$ while completely ignoring the assignment for z variables (this can be done since every assignment for z_i in $\sigma_{F'}$ has to be the same assignment that was set for x_i). Again, it follows that such a strategy sets F to be true. Thus, TQBF-CNF-1 is PSPACE-complete as well.

5.2.2 MBSD for Tree-like Domains

We conclude this section by discussing a very specific tree-like domain structure. We say that a dynamic domain $\mathcal{D} = (S, s_0, \delta, \lambda)$ is tree-like if the transition relation δ induces a tree structure on the states, except for some states which may admit self-loops as their only outgoing transition (therefore such states would be leaves, if self-loops were not present). For this class of domains, the exponential blowup on the number of traces does not occur, as for every state s there exists only a unique trace ending in s (modulo a possible suffix due to self-loops).

Theorem 5. Solving MBSD for target mapping specifications and tree-like \mathcal{D}_A and \mathcal{D}_B is in PTIME for combined complexity, domain complexity, and mapping complexity.

Proof. Given an MBSD instance \mathcal{P}_{tree} with tree-like \mathcal{D}_A and \mathcal{D}_B , consider the two-player game structure $\mathcal{G}_{\mathcal{P}_{tree}} = (\mathcal{A}, W)$ where the game arena \mathcal{A} is as described in Section 4.2. It is immediate to see that since \mathcal{D}_A and \mathcal{D}_B are tree-like, so is \mathcal{A} , if we consider the edges defined by α and β (which reflect those in \mathcal{D}_A and \mathcal{D}_B).

Now, note that for every node $(s,t) \in U$ in the arena \mathcal{A} and for $i=1,\ldots,k$, we can easily check whether the unique play ρ of \mathcal{A} that ends in (s,t) contains two (possibly distinct) nodes with indices j_i and l_i , such that $\lambda^A(s_{j_i}) \models \varphi_i$ and $\lambda^B(t_{l_i}) \models \psi_i$. If that is the case, we call (s,t) an i-accepting node. Then, we define the set of accepting states as $g = \{u \in U \mid u \text{ is } i\text{-accepting, for } i=1,\ldots,k\}$, and the winning condition as W = Reach(g). In this way, $\mathcal{G}_{\mathcal{P}_{tree}}$ is a reachability game, constructed in time polynomial in the size of \mathcal{P}_{tree} , and solvable in linear time in the size of $\mathcal{G}_{\mathcal{P}_{tree}}$. Result then follows since \mathcal{P}_{tree} has a solution if and only if there is a solution to $\mathcal{G}_{\mathcal{P}_{tree}}$.

As before, the combined and domain complexities are EXPTIME, for \mathcal{D}_A and \mathcal{D}_B described succinctly.

6. Solving MBSD with General Mapping Specifications

The final variant of mapping specifications that we study is of the most general form, where Φ can be any arbitrary LTL_f formula over $Prop^A \cup Prop^B$. For this, we exploit the fact that for every LTL_f formula Φ , there exists a DFA \mathcal{F}_{Φ} that accepts exactly the traces that satisfy Φ (De Giacomo & Vardi, 2013). Depending on which agent stops, the problem specializes into one of the following:

- if A stops: find a strategy for B such that every trace always visits an accepting state of \mathcal{F}_{Φ} ;
- if B stops: find a strategy for B such that every trace eventually reaches an accepting state of \mathcal{F}_{Φ} .

To solve this variant, we again reduce MBSD to a two-player game structure $\mathcal{G}_{\mathcal{P}} = (\mathcal{A}, W)$, as in our previous constructions, then solve a safety game, if A stops, and a reachability game, if B stops. To follow the mapping as the game proceeds, we incorporate \mathcal{F}_{Φ} into the arena. This requires a careful synchronization, as the propositional labels associated with the *states* of dynamic domains affect the *transitions* of the automaton.

Formally, given an MBSD instance $\mathcal{P} = (\mathcal{D}_A, \mathcal{D}_B, \Phi, Ag_{stop})$, where $\mathcal{D}_A = (S, s_0, \delta^A, \lambda^A)$ and $\mathcal{D}_B = (T, t_0, \delta^A, \lambda^A)$, we construct the DFA $\mathcal{F}_{\Phi} = (\Sigma, Q, q_0, \eta, acc)$ as in (De Giacomo & Vardi, 2013), where $\Sigma = 2^{Prop^A \cup Prop^B}$ is the input alphabet.

Then, we define a two-player game arena $\mathcal{A} = (U, V, u_0, \alpha, \beta)$ as follows:

- $U = S \times T \times Q$:
- $V = S \times T \times Q$;
- $u_0 = (s_0, t_0, q'_0)$, where $q'_0 = \eta(q_0, \lambda(s_0) \cup \lambda(t_0))$;
- $\alpha = \{(s, t, q), (s', t, q) \mid (s, s') \in \delta^A\};$
- $\beta = \{(s, t, q), (s, t', q') \mid (t, t') \in \delta^B \text{ and } \eta(q, \lambda(s) \cup \lambda(t')) = q'\}.$

Intuitively, \mathcal{A} models the synchronous product of the arena defined in Section 4, with the DFA \mathcal{F}_{Φ} . As such, the DFA first needs to make a transition from its own initial state q_0 to read the labelling information of both initial states s_0 and t_0 of \mathcal{D}_A and \mathcal{D}_B , respectively. This is already accounted for by q'_0 , in the initial state u_0 of the arena. At every step, from current node u = (s, t, q), P1 first chooses the next state s' of \mathcal{D}_A , then P2 chooses a state t' of \mathcal{D}_B , both according to their transition relation, and finally \mathcal{F}_{Φ} progresses, according to its transition function η and by reading the labeling of s' and t', from q to $q' = \eta(q, \lambda^A(s') \cup \lambda^B(t'))$.

For the winning objective W, define the set of goal nodes $g = \{u \in U \mid u = (s, t, q) \text{ such that } q \in acc\}$. That is, g consists of the nodes in the arena where \mathcal{F}_{Φ} is in an accepting state. Then, we define W = Safe(g) (to play a safety game), if $Ag_{stop} = A$, and W = Reach(g) (to play a reachability game), if $Ag_{stop} = B$.

The following theorem states the correctness of the construction.

Theorem 6. There is a solution to \mathcal{P} if and only if there a solution to $\mathcal{G}_{\mathcal{P}}$.

Proof. Let $Ag_{stop} = A$ (the case for $Ag_{stop} = B$ is similar), thus $\mathcal{G}_{\mathcal{P}} = (\mathcal{A}, \mathrm{Safe}(g))$. By Definition 3, \mathcal{P} has a solution σ iff for every trace τ^A of \mathcal{D}_A , we have that $\lambda(\tau^A, \tilde{\sigma}(\tau^A)) \models \Phi$. That is, $\lambda(\tau^A, \tilde{\sigma}(\tau^A))$ is accepted by \mathcal{F}_{Φ} , i.e., the run on \mathcal{F}_{Φ} of $\lambda(\tau^A, \tilde{\sigma}(\tau^A))$ ends at an accepting state $q \in acc$. Due to the strict one-to-one correspondence between the transitions of $\mathcal{G}_{\mathcal{P}}$ with those of \mathcal{D}_A , \mathcal{D}_B and \mathcal{F}_{Φ} , we can simply transform σ to be such that $\sigma: V^+ \to U$. Hence, every play $\rho = \rho_0 \rho_1 \cdots \rho_n$ of \mathcal{A} compatible with σ is such that $\rho_k \in g$ for every even $k.0 \leq k \leq last(\rho)$. By definition of safety game, this holds iff σ is a winning strategy of $\mathcal{G}_{\mathcal{P}} = (\mathcal{A}, \mathrm{Safe}(g))$.

Clearly, the constructed winning strategy σ from the reduced game $\mathcal{G}_{\mathcal{P}}$ is a solution to \mathcal{P} .

Finally, we obtain the following complexity result for the problem in its most general form.

Theorem 7. Solving MBSD for general mapping specifications can be done in 2EXPTIME in combined complexity and mapping complexity, and in PTIME in domain complexity.

Proof. Constructing the DFA \mathcal{F}_{Φ} from the mapping specification Φ is in 2EXPTIME in the number of sub-formulas of Φ (De Giacomo & Vardi, 2013). Once \mathcal{F}_{Φ} is constructed, observe that the game arena \mathcal{A} is the product of \mathcal{D}_A , \mathcal{D}_B and the DFA \mathcal{F}_{Φ} , which requires, to be constructed, polynomial time in the size of $|\mathcal{D}_A| + |\mathcal{D}_B| + |\mathcal{F}_{\Phi}|$. Moreover, both safety and reachability games can be solved in linear time in the size of \mathcal{A} , from which it follows that the MBSD problem for general mappings is in 2EXPTIME in combined complexity, PTIME in domain complexity, and 2EXPTIME in mapping complexity.

7. Related Work

Linear Temporal Logic on finite traces (LTL_f) (De Giacomo & Vardi, 2013) has been widely adopted in different areas of CS and AI, as a convenient way to specify finite-trace properties, due to the way it finely balances expressive power and reasoning complexity. It has been used, e.g., in Machine Learning to encode a-priori knowledge (Camacho, Icarte, Klassen, Valenzano, & McIlraith, 2019; De Giacomo, Iocchi, Favorito, & Patrizi, 2019; Xie, Zhou, & Soh, 2021); in strategy synthesis to specify desired agent tasks (De Giacomo & Vardi, 2015; Zhu, Tabajara, Li, Pu, & Vardi, 2017; Camacho, Baier, Muise, & McIlraith, 2018); in Business Process Management (BPM) as a specification language for process execution monitoring (Pesic, Schonenberg, & van der Aalst, 2007; De Giacomo, De Masellis, Grasso, Maggi, & Montali, 2014; Di Ciccio, Maggi, Montali, & Mendling, 2017). It has also found application as a natural way to capture non-Markovian rewards in Markov Decision Processes (MDPs) (Brafman, De Giacomo, & Patrizi, 2018), MDPs policy synthesis (Wells, Lahijanian, Kavraki, & Vardi, 2020), and non-Markovian planning and decision problems (Brafman & De Giacomo, 2019). Here we show yet another use of LTL_f. We use it to relate the behaviors in two separated domains through mapping specifications so as to control the mimicking between the two domains.

Mimicking has been recently studied in Formal Methods (Amram, Bansal, Fried, Tabajara, Vardi, & Weiss, 2021). In (Amram et al., 2021), the notion of *mimicking* is specified in separated GR(k) formulas, a strict fragment of LTL. This makes the setting there not suitable for specifying mimicking behaviors of intelligent agents, since an intelligent agent will not keep acting indefinitely long, but only for a finite (but unbounded) number of steps. Moreover, the distinctions between the two systems and the mimicking specification were not singled out. This makes it difficult to provide a precise computational complexity analysis with respect to the systems, and the mimicking specification, separately.

A strictly related work, though more specific, is *Automatic Behavior Composition* (De Giacomo, Patrizi, & Sardiña, 2013), where a set of available behaviors must be orchestrated in order to mimic a desired, unavailable, target behavior. That work deals with a specific mapping specification over actions, corresponding to the formal notion of *simulation* (Milner, 1971). This current work devises a more general framework and a solution approach for a wider spectrum of mapping specifications, in a finite-trace framework.

Finally, we want to notice that our framework is similar to what studied in data integration and data exchange (Lenzerini, 2002; Fagin, Kolaitis, Miller, & Popa, 2005; Giacomo, Lembo, Lenzerini, & Rosati, 2007; Kolaitis, 2018), where there are source databases, target databases, and mapping between them that relate the data in one with the data in the other. While similar concepts can certainly be found in our framework, here we do not consider data but dynamic behaviors, an aspect which makes the technical development very different.

8. Conclusion and Discussion

We have studied the problem of mimicking behaviors in separated domains, in a finite-trace setting where the notion of mimicking is captured by LTL_f mapping specifications. The problem consists in finding a strategy that allows an agent B to mimic the behavior of another agent A. We have devised an approach for the general formulation, based on a reduction to suitable two-player games, and have derived corresponding complexity results. We have also identified two specializations of the problem, based on the form of their mappings, which show simpler approaches and better computational properties. For these, we have also provided illustrative examples.

A question that naturally arises, for which we have no conclusive answer yet, is to what extent domain separation and possibly separated types of conditions can be exploited to obtain complexity improvements in general, not only on the problems analyzed here. In this respect, we take the following few points for discussion.

We first note that the framework in (Amram et al., 2021) can be adapted to an infinite-trace variant of MBSD, with target mapping specifications of the form $\Phi = \bigwedge_{l=1}^k (\bigwedge_{i=1}^{n_l} \Box \Diamond (\varphi_{l,i}) \rightarrow \bigwedge_{j=1}^{m_l} \Box \Diamond (\psi_{l,j}))$. The results in (Amram et al., 2021), which build heavily on domain separation, can be tailored to obtain a polynomial-time algorithm for (explicit) separated domains in combined complexity. In contrast, Theorem 4 in this paper shows that the finite variant is PSPACE-hard already for much simpler mappings. This gap seems to suggest that domain separation cannot prevent the book-keeping that is possibly mandatory for the finite case. Note however that Theorem 2 of this paper can be easily extended to specifications of the form $\Phi' = \bigwedge_{l=1}^k (\bigwedge_{i=1}^{n_l} \Diamond (\varphi_{l,i}) \rightarrow \bigwedge_{j=1}^{m_l} \Diamond (\psi_{l,j}))$, yielding an algorithm of time polynomial in the domain size but exponential in the number of Boolean subformulas in Φ' .

A second point of observation is the following. While the result in Section 6 provides an upper bound for mappings expressed as general LTL_f formulas, one can consider a more relaxed form $\Phi = \bigwedge_{i \le k} (\phi_i \to \psi_i)$ where each ϕ_i (resp. ψ_i) is an LTL_f formulas over $Prop^A$ (resp. $Prop^B$) only. While still PSPACE-hard (see Theorem 4), it is tempting to use some form of memory keeping as done in Theorem 2 to avoid the 2EXPTIME complexity. The challenge, however, is that every attempt to monitor satisfaction for even a single LTL_f sub-formula, whether ϕ_i or ψ_i , seems to require an LTL_f to DFA construction that already yields the 2EXPTIME cost. Another approach could be to construct a DFA separately for each LTL_f sub-formula, then combine them along with the product of the domains and continue as in Section 6. This however involves a game with a state space to explore that is the (non-minimized) product of the respective DFAs, and is typically much larger than the (minimized) DFA constructed directly from Φ (as observed in (Tabajara & Vardi, 2019; Zhu, Tabajara, Pu, & Vardi, 2021)). Moreover, in practice, state-of-the-art tools for translating LTL_f to DFAs (Bansal, Li, Tabajara, & Vardi, 2020; De Giacomo & Favorito, 2021) tend to take maximal advantage of automata minimization. How to avoid the DFA construction in such separated mappings to gain computational complexity advantage is yet to be explored.

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