Problem 1 (New MST)

We can add a large positive constant MAX INT to all of the edges in the original graph G, turning all of them positive. On the main graph G, we must set the weights to -1 for all the edges in subgraph F. We can now use the Kruskal method to determine the MST of the Graph. This will include all the edges of subgraph G and produce the required result.

Correctness: -

By first adding a large positive to the edges of the graph, we ensure that all edges are positive in the original graph G. We know from property of MST that , MST will stay unchanged because of applying a constant to all edges. When all the edges in a subgraph are positive and greater than zero, we set edges of subgraph F to -1 on graph G. This way we ensure that they are always included on the MST. Finally, if we run Kruskal method on this graph , it will include all of the edges of subgraph F while remaining MST because they have already minimum weight among the graph G.

Timing: -

Adding a positive weight to all edges on graph G is O(E). Next assigning -1 to all the subset of F belongs to E` on graph G is O(E) operation. Reason for constant run time here is because we are using adjacency list here. Finally running Kruskal algorithm on graph G is O (|E| LOG |V|). So total complexity of the algorithm is O(|E|) + O(|E|) + O (|E| LOG |V|) = O (|E| LOG |V|).

Problem 2 (New max flow)

From the original graph G , we can create a residual graph Gf. Now on the edge e we will increased its capacity by +1 . Now we can run the BFS algorithm on residual graph Gf and check if there is augmenting path exit from s to t . If there is such path exists then we will update the flow by 1 . If there is no such path exists, then flow is unchanged.

Correctness: -

By increasing the capacity of edged e by +1 on graph G , there are two case possible. Either max flow M(f) has been increased by M(f)+1 or max flow M(f) has been unchanged. If max flow has been increased by 1 then there should be a augmented path on residual network and max flow should be increased by 1 . But if there is no augmented path even after increasing edge capacity by 1 then it means current flow is maximum flow M(f) and there is no changed in that. So by running BFS algorithm on graph we have checked if there is augmented path from s to t on the graph which is linear in timing . So we will changed the flow M(f) accordingly if there is augmented path or not.

Timing:-

Creating a Residual graph is linear operation is max O|E| timing. Running BFS on checking augmented path is O(|V|+|E|) timing. Checking if max flow increased or not is O(1) operation . So total time of above algorithm will be O|E|+O(|V|+|E|) +O(1)= O(|V|+|E|) timing.