Problem 1 [DPV] Problem 7.18 (max-flow variants)

Solution:
Part (c):

For a given graph G(V,E) where we have lower bound le and total capacity ce is given . Now we have to find max flow for this graph using linear programming.

So our objective function is :-
Maximize(sum of total flow across all vertexes from source to sink)

i.e max(sum of flow across Fi from start vertex s to sink vertex t) where Fi is the flow along edge i.

which is subject to

le<=fe<=ce

In max-flow, linear equation we have a constraint which tell us how much flow can be flow through it using fe for each edge e. Also, we know that max Incoming flow across each edge should be equal to max outgoing flow of that edge. For each edge of flow , we have changed the constraint to lower bound. If lower bound be le then constraint will be le<=fe<=ce . Where ce is max capacity of each edge which it can carry.

Part (d):

Now here, outgoing flow across each node u is differ by (1-Eu) from its incoming flow. where Eu is the loss coefficient.

Maximize (sum of total flow across all edges from source to sink)
i.e max (sum of flow across Fi from start vertex s to sink vertex t) where Fi is the flow along edge i.
which is subject to

(1-Eu) * Sum of Total flows of into vertex(total in) = Sum of Total flows of into vertex (total out)

For this , for node u , we have changed the total incoming flow for node u . Where total flow out of vertex is (1-Eu) less than incoming flow for the same vertex .

Problem 2 [DPV] Problem 8.9 (hitting set)

Solution:

First, we show that hitting set is NP problem. For a given hitting set H , we will loop through each element of H across all sets {S1,S2....Sn} and check if element exist an element in set Si . It takes O(1) time to check is there is a element in each set. There are total m elements in set H and there are total n sets. So we can verify in O(mn) time to verify hitting set solution. As this is in polynomial time so Hitting set problem is in NP Class.

Now we will prove that it is in Class of NP hard. For this we will reduce known Vertex Cover problem to Hitting set. Given a Graph G(V,E) , we have all vertices of G(V) belongs to all element of family of sets (V(G)) . Each set of this family of sets, Si is the edges of graph (u,v). Now if vertex cover of graph G is of size k , this means for every edge(u,v) either u or v belong to Vertex Cover. So elements of VC will formed hitting set which will belong to at least one set of family of sets S and will intersect each set Si. So it's similar to finding a VC of size most b. Now we will convert hitting set problem to VC Cover problem. As each element of hitting set belongs to family of sets, so this hitting set must be VC which covered all edges of graphs G(V, E) . So this proved that hitting set is NP-hard also.

References :-

https://www.geeksforgeeks.org/hitting-set-problem-is-np-complete/