Problem 1 (Almost-SAT)

Solution:

To check and verify if Almost-SAT is NP-Complete, we will check and loop across each clause and check if its True. There are n literals and it take max O(n) time to verify one clause. Total there are m clause so max it will take O(mn) time to verify all clauses. This is polynomial in time. This proved Almost-SAT is NP-Complete.

Now we will prove that Almost-SAT is NP hard. For that , we already knows that SAT is NP-Hard. So we will reduced SAT -→Almost SAT. For this , first we will consider that there is formula F of (x1,x2,x3….xn) literals which is input to SAT. Now to this formula F we will add two clauses x1 and (Not x1) and called this formula F'. Since F satisfy exactly m clauses, this new formula F' will satisfy new added constraints. Reason since x1 can be true or false so either of this constraint will be true. This process SAT→Almost SAT satisfy this .
Now we will proved opposite way, for this let's assume there is assignment which satisfy function F' . So now only unsatisfied clause will be two new added constraint x and (not x1) . Rest m clauses will be satisfying. This provides that output from F' will satisfy SAT problem. Now it takes O(1) time for this reduction to add these clauses. This proved that Almost SAT is Np-hard.

Problem 2 (Flower)

Solution:

First, we will verify that this is NP-Completed. For this we can check if we can check in polynomial time if give graph is flower.
If we have give a graph G(V,E) , where we know 4 vertices form independent set among each other which represent as start. Where only one of the edge of Independent Set is connected to clique of size k in graph G. It takes n2(n square) time to check if there is a clique on graph with size k . We will check each pair of vertices of clique of size k are connected to each other. Then it takes O(n2 square) time to check no. of pair of vertices in graph if they are connected by independent set of size 4. If there are n+4 total vertices in the graph where k=n and we have clique of size k , then given graph is flower otherwise it is not. As we can verify in polynomial time that if a given graph is flower or not so this is Np-Completed problem.

Now we will prove that flower is NP-hard problem. For this we will convert known Clique Np-hard problem to Flower problem of Clique and Independent set. We will create a copy of input graph G'(V,E) and we can add 4 new vertices to input graph G such that they are connected to each other in a clique of size k, but they are not connected to no other vertices in input graph G. It take O(4) time to add 4 new vertices. So if there exist a solution S which has Clique of size k in graph G it will also exist in Graph G'. As we have only added 4 new independent set vertices which as connected to each other so they cannot be part of Clique. So solution to flower problem exist if solution to Clique problem exists.

We will prove now opposite way, If there exist a solution of flower problem then we can drop independent set of size 4 vertices in graph . If we have Clique of size k=n in new graph G', then it will also have solution of Clique size of k in original input Graph G. It takes O(4) time to drop 4 vertices of independent set and O(n2) to check if graph has clique of size k. As this time is polynomial, this proved Flower problem is Np-hard.

References: -

http://anmolkapoor.in/2020/09/22/Proving-CLIQUE-IS-problems-as-NP-Complete/