CAS 741: Problem Statement
A numerical search for the spectrum related to travelling periodic waves

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Table 1: Revision History

Date	Developer(s)	Change
2018-09-12	Robert White	Grammar revision
2018-09-13	Dmitry Pelinovsky	Clarification of terminology
2018-09-18	Robert White	Wrapping at 80 char

A spectrum is a set of admissible eigenvalues. These eigenvalues express physical properties of their respective systems. For instance, there exists a countable number of frequencies associated with the quantum harmonic oscillator and Laplacian operator in periodic boundary conditions. Physicists are interested in using spectra to characterize physical systems. Mathematicians are interested in analytical and numerical behaviour of spectral problems.

The Non-Linear Schrödinger (NLS) equation is ubiquitous in fluid dynamics and optics. It can be used to model rogue waves and modulated wave packets. Rogue waves are significantly large amplitude waves that appear unexpectedly. They are particularly dangerous for ships in the ocean. There are recorded events of these freak waves breaking large ships and boats. Rogue waves are also important for people studying optics as they have been observed in light.

The NLS equation appears as a compatibility condition of a particular Lax pair of linear equations. One equation is a spectral problem and the other is a time evolution problem. We are interested in finding the continuous spectrum of the spectral problem.

The purpose of this software is to find the location of the continuous spectrum for general travelling periodic waves. Given certain physical parameters it will search for admissible values of the spectral parameter and then display eigen-values and associated eigen-functions. The final goal will be to use this spectrum to better understand rogue waves, in the NLS equation.

Matlab will be used to generate the code for this project. The software will be made under the windows operating system. It should be able to run on windows, mac OS and Linux machines with an appropriate MATLAB licence. The resulting software will be posted on the webpage of Dr. Pelinovsky and will

be used by a community of researchers working with rogue waves and integrable systems.

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