CAS 741: SRS
SpecSearch: A Numerical Search For The Spectrum Related To Travelling Periodic Waves

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1 Revision History

Date	Version	Notes
25-09-2018	1.0	Creation of the first draft
29-09-2018	1.1	Edit of 1.0. Added a GS, responsibilites, requirenments, definitions and verification of compatibility. Updated comments.tex. My comments are in red, SS comments are in blue.
01-10-2018	1.2	Edit of 1.1. Added more theory to support the instance model. Made a clearer distinction between user and reader charaterisitcs. Modified the goal statements.
03-10-2018	1.3	Edit of 1.2. Removed a redundant goal statement. Made some of Dr. Smith's suggested changes. Updated the traceability matrices.
04-10-2018	1.4	Edit of 1.3. Created IM2 and inserted tracebility graphs. Completed all of Dr. Smith's suggested changes. Performed a grammar and spelling check.
11-10-2018	1.5	Edit of 1.4. Draft 1.4 was peer reviewed by Hanane Zlitni. Most of her recommended changes were made.
28-11-2018	1.6	Edit of 1.5. Draft 1.5 was edited for the final documentation.
09-12-2018	1.7	Creation of final draft for final documentatation.

2 Reference Material

This section records information for easy reference.

2.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	**unit**	**SI**
m	length	metre
t	time	second

2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. Some of the symbols are unitless, they are marked with a dash. The choice of symbols was made to be consistent with the rogue wave and mathematical physics literature.

symbol	**unit**	**description**
λ	$\frac{1}{s}$	spectral parameter
u	m	wave amplitude
ϕ	-	eigen function
c	$\frac{m}{s}$	wave speed
ω	$\frac{1}{s}$	angular frequency
e	-	euler's Number
i	-	imaginary number
(g, d)	-	conserved quantities

2.3 Abbreviations and Acronyms

symbol	description
A	Assumption
$\mathbb{C}$	Complex numbers
cn	Elliptic cosine
DD	Data Definition
dn	Delta amplitude
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
NLS	Nonlinear Schrodinger
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PS	Physical System Description
$\mathbb{R}$	Real number line
R	Requirement
sn	Elliptic sine
SRS	Software Requirements Specification
progname	SpecSearch
T	Theoretical Model

2.4 Mathematical Notation

Let δ be a complex number of the form $\delta = m + ni$ $(m, n \in \mathbb{R})$. The real part of u is m and the imaginary part of u is n. i is the imaginary number such that $i = \sqrt{-1}$.

The complex conjugate of δ is defined to be $\bar{\delta} = m - ni$. The modulus of u is defined to be $|\delta| = \sqrt{m^2 + n^2}$.

Let $z(x,t) : \mathbb{R}^2 \to \mathbb{C}$. We will let z_t and z_x denote the partial derivatives of z with respect to t and x respectively. If z has one independent variable we will denote its derivative by z'.

$z_t = \lim_{h\to 0} \frac{z(x,t+t_0h)-z(x,t)}{h}$

Where z_t is differentiable if the above limit is the same regardless of the complex number (direction) t_0.

If H is a matrix then H_t is the matrix formed by taking the t derivative in each component of H.

$e^{i\omega t} = cos(\omega t) + isin(\omega t)$. So it is clear from the above definitions that

$|e^{i\omega t}| = \sqrt{sin^2(\omega t) + cos^2(\omega t)} = 1$

To see this consider the angle ωt for fixed t. Let the opposite side be a, denote the adjacent side by b and the hypotenuse by c. This forms a right angle triangle (see picture below). By definition, $sin(\omega t) = \frac{a}{c}$ and $cos(\omega t) = \frac{b}{c}$. Now, by pythagoras we have that

$\sqrt{sin^2(\omega t) + cos^2(\omega t)} = \sqrt{\frac{a^2+b^2}{c^2}} = \sqrt{\frac{c^2}{c^2}} = 1$

For each t. So $e^{i\omega t}$ stays on the complex unit circle.

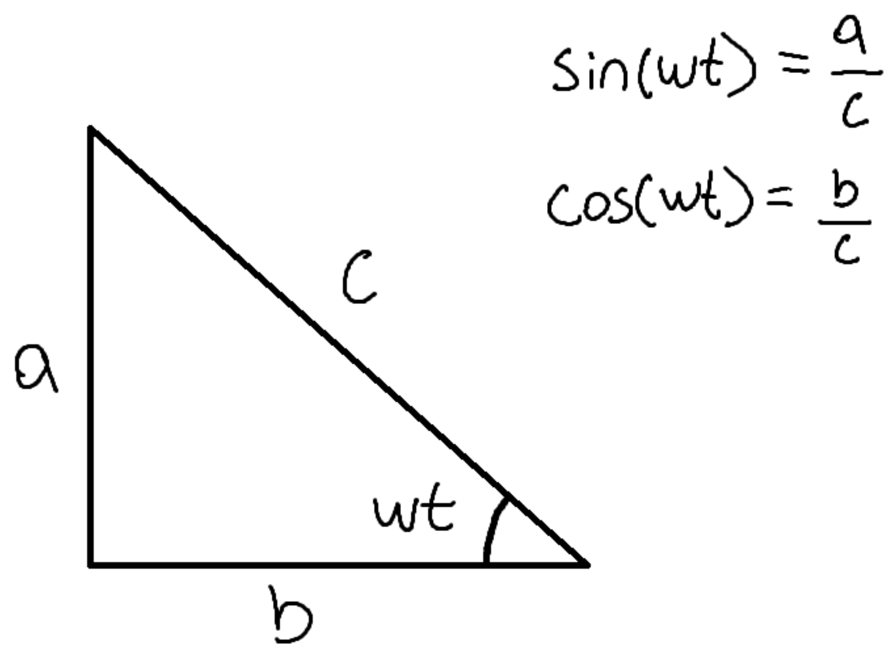

Figure 1: Trigonometric Ratios

Contents

3 Introduction

Rogue waves are large and unexpected waves that occur in nature. For example, giant water waves randomly appear on the ocean surface. These freak occurences pose a threat to ships and ocean liners. Rogue waves have also been documented in laser beam channels. This phenomenon has opened up many doors in research related to non-linear optics. (Kharif (2009))

Phycisists and mathematicans are interested in modeling and understanding rogues waves. They are particularly interested in classifying the stability of these waves and using equations to model their behaviour as a function of measurable physical parameters. My supervisor, Dr. Dmitry Pelinovsky, and I will use a particular matrix equation from a lax pair that is satisfied for wave solutions of the Non-Linear Schrödinger (NLS) equation in our investigation of rogue waves. The NLS equation is a good tool for modeling rogue waves and the spectrum of the previously mentioned matrix equation contains useful information regarding the behaviour of such waves. Further details can be read in section 5.1.

The purpose of the spectral search program (SpecSearch) is to use three reliable numerical differentiation algorithms to find the spectrum of the previously mentioned matrix equation for two particular wave solutions of the NLS equation corresponding to particular boundary value problems. We will try to get these spectral values to coincide with current work regarding the NLS equation. The next stage of our research will be to extend the program beyond the boundary conditions. This extension will provide insight into the next stage of our research - analytically finding the entire continuous spectrum of the operator.

3.1 Purpose of Document

The purpose of this document is to describe the requirements for SpecSearch. SpecSearch will search for the continuous spectrum of a lax pair that is compatible with solutions of the NLS equation.

This document will explain the mathematical models, assumptions, solution characterisitcs and goals of the software. After reading this document one should be able to understand the mathematical and physical context of the inputs and outputs. They should also recognize the connections and dependencies between these SRS topics.

The SRS is meant to be simaltaneously unambigious and abstract. It details the quality attributes of the software, such as functionality, without dicussing any code, numerical algorithms or scientific computing solutions. It is a precursor to the documents required in the development of scientific software. One should read and understand this document before the VnV, design or code itself.

3.2 Scope of Requirements

The scope of SpecSearch is limited to finding the spectrum of a particular linear lax equation for two boundary solutions of the NLS equation.

3.3 Characteristics of Intended Reader

The reader of this document should have taken an introductory course in partial differential equations and complex analysis. They should also have a first year undergraduate understanding of linear algebra.

3.4 Organization of Document

This document follows a template provided by Dr. Spencer Smith at McMaster University (Smith and Lai (2005),Smith et al. (2007)). It begins by introducing the document. This introduction is followed by a general overview of the system and then an outline of the goals and mathematical theory. The document continues by describing the behaviour between inputs and outputs, judging output and then foreshadows changes to the software. It concludes with useful graphs and figures for tracebility.

The latex template is available in Dr. Spencer Smith's gitlab repository CAS 741. https://gitlab.cas.mcmaster.ca/smiths/cas741

4 General System Description

This section identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

4.1 System Context

Diagram 2 shows the system context. The circle represents the user of the scientific software. The rectangle resembles the software system. The arrows indicate the flow of data between software and environment. The inputs are the physical wave parameters and the output is the spectrum of the matrix operator.

- User Responsibilities:
  - Ensure that the input variables resemble the wave that you intend to analyze.
  - Ensure that the assumptions imposed on waves in this software are reasonable for your reasearch.
  - Ensure that the software is used in a legal and ethical manner.
- SpecSearch Responsibilities:
  - Detect data type mismatch.
  - Find the spectrum of the lax equation for two particular wave solutions.
  - Display the spectrums on the complex plane.

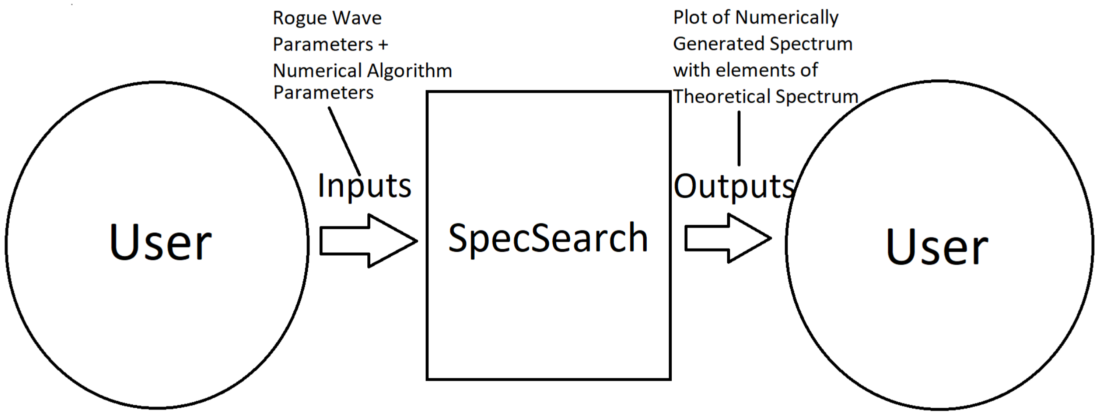

Figure 2: System Context

4.2 User Characteristics

The user should have a basic understanding of wave mechanics (wave speed, amplitude and angular frequency). They should also understand the concept of conserved quantities.

4.3 System Constraints

My supervisor requires that software will be created with MATLAB.

5 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

5.1 Problem Description

A lax pair is a set of matrices or operators that satisfy differential equations. The NLS equation, a PDE used to model rogue waves and modulated wave packets, appears as a compatibility condition of a particular lax pair of equations. One equation is a spectral problem, with matrix U, and the other is a time evolution problem, with matrix V. See 5.2.5 for more details. The spectral problem is important because the spectral parameter within it contains information about the stability of solutions.

SpecSearch will attempt to produce a numerical approximation of the continuous spectrum for the previously mentioned spectral problem. In particular, it will find the spectrum for two general travelling periodic wave solutions of the focusing NLS equation.

5.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- **Spectrum**: The set of allowable values for the spectral parameter of a matrix or operator.
- **Operator**: A mapping that transforms elements of a space into other elements of the same space.
- **Traveling Periodic Wave**: A periodic and one-dimensional wave that travels with a constant speed, c, and angular frequency, ω.
- **Compatibility Condition**: Conditions for which the lax pair of equations is guaranteed.
- **Stability**: A solution is stable if slight perturbations in initial conditions lead to slight perturbations in the entire solution.
- **Initial Data**: The graph of a wave (or its derivative) at a fixed point in time, tyically t=0.
- **Rogue Wave**: A wave is considered rogue if its amplitude is more than double of the average of the upper third wave amplitudes in its surroundings.
- **PDE**: A partial differential equation.An equation containing one or more partial derivatives. The partial derivative of a multivariable function is a derivative with respect to one of its arguments.
- **Countable:** A set that is finite or has the same cardinality as the natural numbers (1,2,3,4,5 ....).
- **Focusing wave:** When the nonlinear Schrödinger equation adds the nonlinear term $(+|u|^2u)$.
- **Hamiltonian:** Is an operator corresponding to the total energy of a system.

5.1.2 Physical System Description

The physical system of SpecSearch includes the following elements:

PS1: Unspecified body of water or laser channel with a traveling periodic wave.

5.1.3 Goal Statements

Given physical parameters of the rogue wave and numerical differentiation parameters, the goal statement(s) are:

GSlocate: Locate elements in the spectrum of the lax pair matrix for particular two wave solutions(see 5.2.5)

GStheor: Compare theoretical results to the approximated spectrum.

5.2 Solution Characteristics Specification

The instance model that governs SpecSearch is presented in Subsection 5.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

Aham: The wave equation is a complex hamiltonian evolution equation.

Amom: The linear momentum densities are proportional to the integrands of the corresponding velocity functionals

Anls: The wave is a solution to the NLS equation.

Afoc: The system only deals with focusing waves.

Aham, Amom, Anls and Afoc are assumptions that are used in the derivation of the focusing NLS Equation 5.2.2. This equation is the foundation for all of the general definitions, data definitions and instance models.

Awav: All waves in the model are general traveling perioidic waves with constant speed and frequency.

This assumption allows us to reduce TM1 5.2.2 to GD1 5.2.3.

Astat: The system will start by assuming that the wave is stationary and then use the stationary solution to build the moving solution.

This assumption allows us to set $c = 0$ in DD1 5.2.4. The equation in DD2 5.2.4 follows from this simplification.

Asmooth: The eigenfunctions of the spectrum are two times differentiable.

This assumption is necessary for the compatibility condition in IM1 5.2.5.

5.2.2 Theoretical Models

This section focuses on the Theoretical Models that SpecSearch is based on.

Number	TM1
Label	**Focusing NLS Equation**
Equation	$iq_t + q_{xx} + 2\|q\|^2 q = 0$
Description	The NLS equation appears as one of the fundamental equations for describing the evolution of waves and wave packets in weakly nonlinear media that have dispersion. Here t represents time, x represents position and q is the wave.
Source	Deconinck and L.Segal
Ref. By	5.2.4, 5.2.3
Assumptions	Aham, Amom, Anls, Afoc

5.2.3 General Definitions

This section focuses on the general equations and laws that SpecSearch is based on.

Number	GD1
Label	**Focusing NLS for General travelling periodic waves**
Equation	$u'' + 2\|u\|^2 u + 2icu' = \omega u$
Description	General traveling periodic waves that satisfy the Schrödinger equation will have this form. c is the wave speed and ω is the frequency. A derivation of this equation follows this table.
Source	Deconinck and L.Segal
Ref. By	5.2.4, 5.2.5
Assumptions	Aham, Amom, Anls, Afoc, Awav

Derivatition of the Focusing NLS for General Traveling Periodic Waves

We start with the focusing NLS equation: $iq_t + q_{xx} + 2|q|^2 q = 0$ (5.2.2)

The NLS equation is a PDE meant to model modulated wave packets and rogue waves in physics. u is the complex envelope of the wave, t is time and x is the position in one dimensional space.

A general traveling periodic wave has the form $z(x,t) = u(x+2ct)e^{i\omega t}.(u : \mathbb{R} \to \mathbb{C})$. We will now search for general traveling periodic waves that are solutions to the NLS equation. This is done by setting z=q in the NLS equation. This yields:

$$
\begin{gathered}
iz_t + z_{xx} + 2|z|^2 z = 0 \\
\Rightarrow i(2cu'e^{i\omega t} + i\omega u e^{i\omega t}) + u''e^{i\omega t} + 2|u|^2|e^{i\omega t}|^2 u e^{i\omega t} = 0 \\
\Rightarrow e^{i\omega t}(2icu' + i^2\omega u + u'' + 2|u|^2 u) = 0 \\
\Rightarrow 2icu' - \omega u + u'' + 2|u|^2 u = 0
\end{gathered}
$$

Which is the result in GD1.

5.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

Number	DD1
Label	**Conservation Equations**
Symbols	g, c, d, ω
SI Units	-
Equation(1) Equation(2)	$\bar{u}u' - u\bar{u}' + 2ic\|u\|^2 = 2ig$ $\|u'\|^2 + \|u\|^4 + d = \omega\|u\|^2$
Description	These are conserved quantities in the NLS equation. For fixed g and d the above relations are true for all space and time. A derivation of these quantities follows this table.
Sources	Deconinck and L.Segal
Ref. By	5.2.5
Assumptions	Aham, Amom, Anls, Afoc, Awav

Justification of a Conserved Quantity

I will first multiply T1 by $\bar{u}$.

$$u''\bar{u} + 2|u|^2 u\bar{u} + 2icu'\bar{u} = \omega u\bar{u} \text{ (*)}$$

Taking the complex conjugate yields:

$$\bar{u''}u + 2|u|^2\bar{u}u - 2ic\bar{u'}u = \omega\bar{u}u \text{ (**)}$$

Subtraction of * with ** yields:

$$u''\bar{u} + 2|u|^2u\bar{u} + 2icu'\bar{u} - (\bar{u''}u + 2|u|^2\bar{u}u - 2ic\bar{u'}u) = \omega u\bar{u} - (\omega\bar{u}u)$$
$$\Rightarrow u''\bar{u} - \bar{u''}u + 2ic(u'\bar{u} + \bar{u'}u) = 0$$
$$\Rightarrow \int u''\bar{u} - \int \bar{u''}u + 2ic\int(u'\bar{u} + \bar{u'}u) = 2ig$$

Noticing that the term in the integral attached to 2ic is a product rule.

$$\Rightarrow \int u''\bar{u} - \int \bar{u''}u + 2icu\bar{u} = 2ig$$
$$\Rightarrow \bar{u}u' - \int \bar{u'}u' - u\bar{u'} + \int \bar{u'}u' + 2icu\bar{u} = 2ig$$
$$\Rightarrow \bar{u}u' - u\bar{u'} + 2ic|u|^2 = 2ig$$

The second conserved quantity is obtained by multiplying $\bar{u'}$ by T1, adding the complex conjugate and integrating.

Number	DD2
Label	**Solutions of GD1**
Symbols	k, cn, sn, dn, a, b
SI Units	-
Equation	$\rho(x) = b - a^2k^2sn^2(ax; k)$
Description	This is a solution to the focusing NLS for General Traveling Periodic Waves. $k \in (0, 1)$, sn is the jacobi elliptic function and a, b are related to the conserved quantities. The constraints on a, b are $b \leq a^2$ and $b \geq a^2k^2$. Along two boundaries of this triangular region and with $a = 1$ the equation simplifies to $\rho_1(x) = dn(x; k)$ and $\rho_2(x) = kcn(x; k)$. The derivation of these formulae are extremely long and will be omitted from this document. **We will only concern ourselves with the spectrum for these two equations, $\rho_1(x)$ and $\rho_2(x)$.**
Sources	Deconinck and L.Segal
Ref. By	5.2.5
Assumptions	Aham, Amom, Anls, Afoc, Awav

5.2.5 Instance Models

This section transforms the problem defined in Section 5.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 5.2.4 to replace the abstract symbols in the models identified in Sections 5.2.2 and 5.2.3.

The goal GSlocate is solved by IM1.

Number	IM1
Label	**Searching for the continuous spectrum**
Input	Boundary wave solutions to NLS equation (5.2.4)
Output	λ such that: $\phi_x = U(u, \lambda)\phi$
Description	Let u be a solution to 5.2.3. $U(u,\lambda) = \begin{pmatrix} \lambda & u \\ -\bar{u} & -\lambda \end{pmatrix}$ T1 is a compabililty condition for the lax pair comprised of the above row's ODE and $\phi_t = V(u,\lambda)\phi$ $V(u,\lambda) = i\begin{pmatrix} 2\lambda^2 + \|u\|^2 & u_x + 2\lambda u \\ \bar{u}_x - 2\lambda\bar{u} & -2\lambda^2 - \|u\|^2 \end{pmatrix}$ $\phi \in \mathbb{C}$ A reduction of IM1 to an eigenvalue problem is provided below this table.
Sources	Deconinck and L.Segal
Ref. By	
Assumptions	Aham, Amom, Anls, Afoc, Asmooth

Justification of Compatibility Conditions and Reduction to Spectral Problem

For a smooth C^2 function ,ϕ, we have that $\phi_{tx} = \phi_{xt}$.
From IM1 $\Rightarrow \phi_{xt} = \delta_t U\phi + UV\phi$ and $\phi_{tx} = \delta_x V\phi + VU\phi$.
Combining the above two equations :

$$\Rightarrow \delta_t U\phi + UV\phi = \delta_x V\phi + VU\phi$$

$$\Rightarrow \delta_t U + UV = \delta_x V + VU \ (-)$$

$$\Rightarrow \begin{pmatrix} 0 & u_t \\ -\bar{u}_t & 0 \end{pmatrix} + i\begin{pmatrix} \lambda & u \\ -\bar{u} & -\lambda \end{pmatrix}\begin{pmatrix} 2\lambda^2 + |u|^2 & u_x + 2\lambda u \\ \bar{u}_x - 2\lambda\bar{u} & -2\lambda^2 - |u|^2 \end{pmatrix} = i\begin{pmatrix} u_x\bar{u} + u\bar{u}_x & u_{xx} + 2\lambda u_x \\ \bar{u}_{xx} - 2\lambda\bar{u}_x & -u_x\bar{u} - u_x\bar{u}_x \end{pmatrix} +$$

$$i\begin{pmatrix} 2\lambda^2 + 2|u|^2 & u_x + 2\lambda u_x \\ \bar{u}_x - 2\lambda\bar{u} & -2\lambda^2 - |u|^2 \end{pmatrix}\begin{pmatrix} \lambda & u \\ -\bar{u} & -\lambda \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} i(2\lambda^3 + \lambda|u|^2 + u\bar{u}_x - 2\lambda|u|^2) & u_t + i(\lambda u_x + 2\lambda^2 u - 2\lambda^2 u - u|u|^2) \\ -\bar{u}_t + i(-2\lambda^2\bar{u} - \bar{u}|u|^2 - \lambda\bar{u}_x + 2\lambda^2\bar{u}) & i(-\bar{u}u_x - 2\lambda u\bar{u} + 2\lambda^3 + \lambda|u|^2) \end{pmatrix} =$$

$$
i\begin{pmatrix} u_x\bar{u}+u\bar{u}_x+2\lambda^3+\lambda|u|^2-u_x\bar{u}-2\lambda|u|^2 & u_{xx}+2\lambda u_x+2\lambda^2 u+u|u|^2-2|u|^2-\lambda u_x \\ \bar{u}_{xx}-2\lambda\bar{u}_x+\lambda\bar{u}_x-2\lambda^2\bar{u}+2\lambda^2\bar{u}+\bar{u}|u|^2 & -u_x\bar{u}-u_x\bar{u}_x+u\bar{u}_x-2\lambda|u|^2+2\lambda^3+\lambda|u|^2 \end{pmatrix}
$$
$$
\Rightarrow \begin{pmatrix} 0 & u_t-iu|u|^2 \\ -\bar{u}_t-i\bar{u}|u|^2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & iu_{xx}+iu|u|^2 \\ i\bar{u}_{xx}+i\bar{u}|u|^2 & 0 \end{pmatrix}
$$
$$
\Rightarrow \begin{pmatrix} 0 & iu_t+2u|u|^2+u_{xx} \\ iu_t+2u|u|^2+u_{xx} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
$$

This is true if u satisfies the NLS equation (T1). Thus, the NLS equation is a compatibility condition for the lax pair in IM1.

Now let $\phi = (\phi_1, \phi_2)^T$. The lax operator in the output row of the IM1 table 5.2.5 can be rewritten as follows:

$$
\lambda \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \frac{d}{dx} & -u \\ -u & -\frac{d}{dx} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}
$$

Where u is a boundary general traveling periodic wave solution to the focusing NLS equation from the description of 5.2.4 and multiplication with operator $\frac{d}{dx}$ is differentation. Thus, finding λ is equivalent to finding the eigenvalues of IM1.

5.2.6 Data Constraints

There are no data constraints.

5.2.7 Properties of a Correct Solution

A correct solution will be a list of complex numbers. There should be at least four purely complex elements of the spectrum for each cn instance. There should also be at least two purely real elements of the spectrum for each dn instance.

6 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

6.1 Functional Requirements

Rin: SpecSearch shall take in quantity k and numerical constants N, P as input. (5.2.3)

Rfind: SpecSearch will find values of λ. (5.2.5)

Rplt: SpecSearch will plot the spectrum on the complex plane for the cn and dn solutions for three different numerical algorithms. The x-axis is the real part and the y-axis is the imaginary part. Certain spectral elements from (segal) will be plotted on the appropriate figures.

6.2 Nonfunctional Requirements

NFR1: The software should be maintainable and manageable as it will be modified and continually updated during the research process. Adding a new numerical method should simply involve modifying a single matrix definition in the MATLAB code. The elements of the matrix should be thouroughly commented. Modification of this matrix should take at most $\frac{1}{4}$ of the development time of the original development of the matrix.

NFR2: The software should be accurate within the standards of my supervisor and reliable for researchers studying rogue waves. This means that the numerical algorithms implemented should produce minor error terms and no sporous elements of the spectrum.

7 Likely Changes

LC1: We might add constraints or bounds to the input variables.

8 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 1 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 2 shows the dependencies of instance models, requirements, and data constraints on each other. Table 3 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	T1	GD1	DD1	DD2	IM1
T1	X	X	X	X	
GD1		X	X	X	
DD1			X	X	
DD2				X	
IM1					X

Table 1: Traceability Matrix Showing the Connections Between Items of Different Sections

	IM1	Rin	Rfind	Rplt
IM1	X			
Rin		X		
Rfind	X	X	X	
Rplt	X	X	X	X

Table 2: Traceability Matrix Showing the Connections Between Requirements and Instance Models

	Aham	Amom	Anls	Afoc	Awav	Astat	Asmooth
TM1	X	X	X	X			
GD1	X	X	X	X	X		
DD1	X	X	X	X	X	X	
DD2	X	X	X	X	X	X	
IM1	X	X	X	X	X	X	X

Table 3: Traceability Matrix Showing the Connections Between Assumptions and Other Items

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is dependent on the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Figure 3 shows the dependencies of theoretical models, general definitions, data definitions and instance models on each other. Figure 4 shows the dependencies of instance models, requirements, and data constraints on each other.

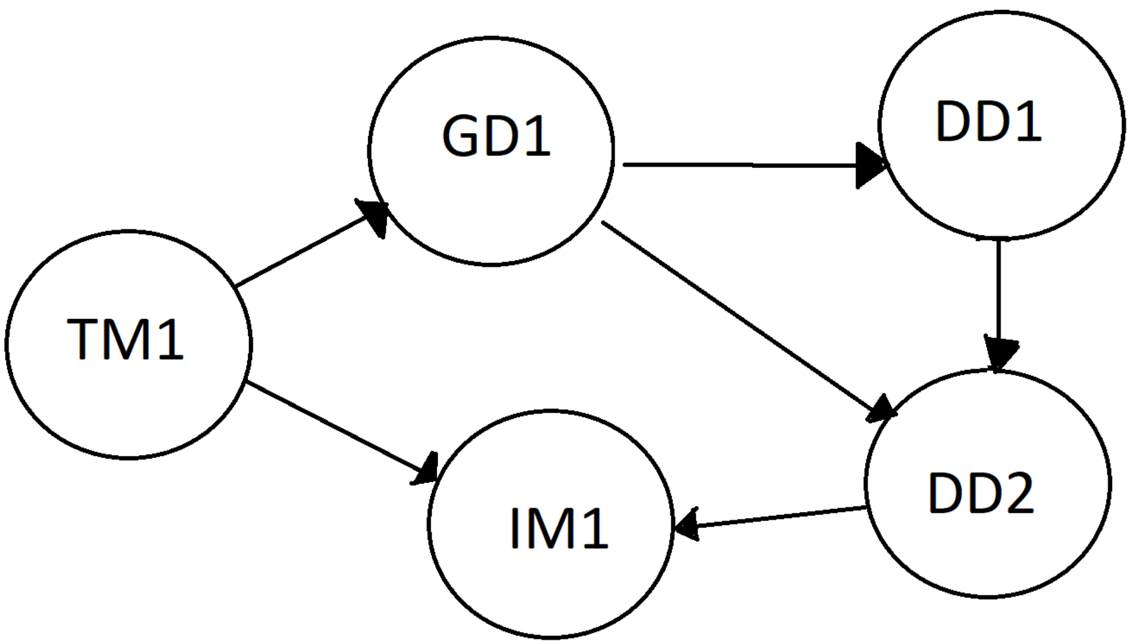

Figure 3: Traceability Graph Showing the Connections Between Items of Different Sections

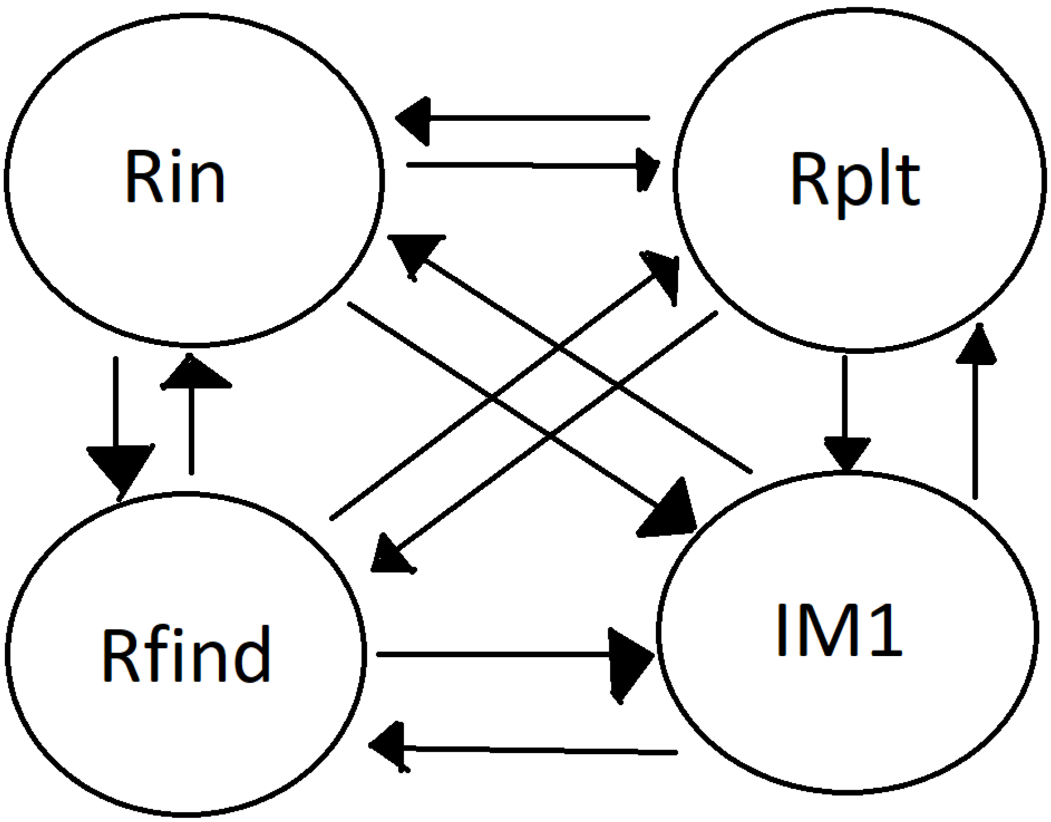

Figure 4: Traceability Graph Showing the Connections Between Items of Different Sections

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9 Appendix

9.1 Symbolic Parameters

There are no symbolic parameters.