CAS 741: SRS
A Numerical Search For The Spectrum Related To Travelling Periodic Waves

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1 Revision History

Date	Version	Notes
25-09-2018	1.0	Creation of the first draft
29-09-2018	1.1	Edit of 1.0. Added a GS, responsibilites, requirenments, definitions and verification of compatibility. Updated comments.tex. My comments are in red, SS comments are in blue.
01-10-2018	1.2	Edit of 1.1. Added more theory to support the instance model. Made a clearer distinction between user and reader charaterisitcs. Modified the goal statements.
03-10-2018	1.3	Edit of 1.2. Removed a redundant goal statement. Made some of Dr. Smith's suggested changes. Updated the traceability matrices.
04-10-2018	1.4	Edit of 1.3. Created IM2 and inserted tracebility graphs. Completed all of Dr. Smith's suggested changes. Performed a grammar and spelling check.
11-10-2018	1.5	Edit of 1.4. Draft 1.4 was peer reviewed by Hanane Zlitni. Most of her recommended changes were made.

2 Reference Material

This section records information for easy reference.

2.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	**unit**	**SI**
m	length	metre
t	time	second

2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. Some of the symbols are unitless, they are marked with a dash. The choice of symbols was made to be consistent with the rogue wave and mathematical physics literature.

symbol	**unit**	**description**
λ	$\frac{1}{s}$	spectral parameter
u	m	wave amplitude
ϕ	-	eigen function
c	$\frac{m}{s}$	wave speed
ω	$\frac{1}{s}$	angular frequency
e	-	euler's Number
i	-	imaginary number
(g, d)	-	conserved quantities

2.3 Abbreviations and Acronyms

symbol	description
A	Assumption
$\mathbb{C}$	Complex numbers
cn	Elliptic cosine
DD	Data Definition
dn	Delta amplitude
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
NLS	Nonlinear Schrodinger
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PS	Physical System Description
$\mathbb{R}$	Real number line
R	Requirement
sn	Elliptic sine
SRS	Software Requirements Specification
ProgName	SpecSearch
T	Theoretical Model

2.4 Mathematical Notation

Let δ be a complex number of the form $\delta = m + ni$ $(m, n \in \mathbb{R})$. The real part of u is m and the imaginary part of u is n. i is the imaginary number such that $i = \sqrt{-1}$.

The complex conjugate of δ is defined to be $\bar{\delta} = m - ni$. The modulus of u is defined to be $|\delta| = \sqrt{m^2 + n^2}$.

Let $z(x, t) : \mathbb{R}^2 \to \mathbb{C}$. We will let z_t and z_x denote the partial derivatives of z with respect to t and x respectively. If z has one independent variable we will denote its derivative by z'.

$z_t = \lim_{h \to 0} \frac{z(x,t+t_0h)-z(x,t)}{h}$

Where z_t is differentiable if the above limit is the same regardless of the complex number (direction) t_0.

If H is a matrix then H_t is the matrix formed by taking the t derivative in each component of H.

$e^{i\omega t} = cos(\omega t) + isin(\omega t)$. So it is clear from the above definitions that

$|e^{i\omega t}| = \sqrt{sin^2(\omega t) + cos^2(\omega t)} = 1$

To see this consider the angle ωt for fixed t. Let the opposite side be a, denote the adjacent side by b and the hypotenuse by c. This forms a right angle triangle (see picture below). By definition, $sin(\omega t) = \frac{a}{c}$ and $cos(\omega t) = \frac{b}{c}$. Now, by pythagoras we have that

$\sqrt{sin^2(\omega t) + cos^2(\omega t)} = \sqrt{\frac{a^2+b^2}{c^2}} = \sqrt{\frac{c^2}{c^2}} = 1$

For each t. So $e^{i\omega t}$ stays on the complex unit circle.

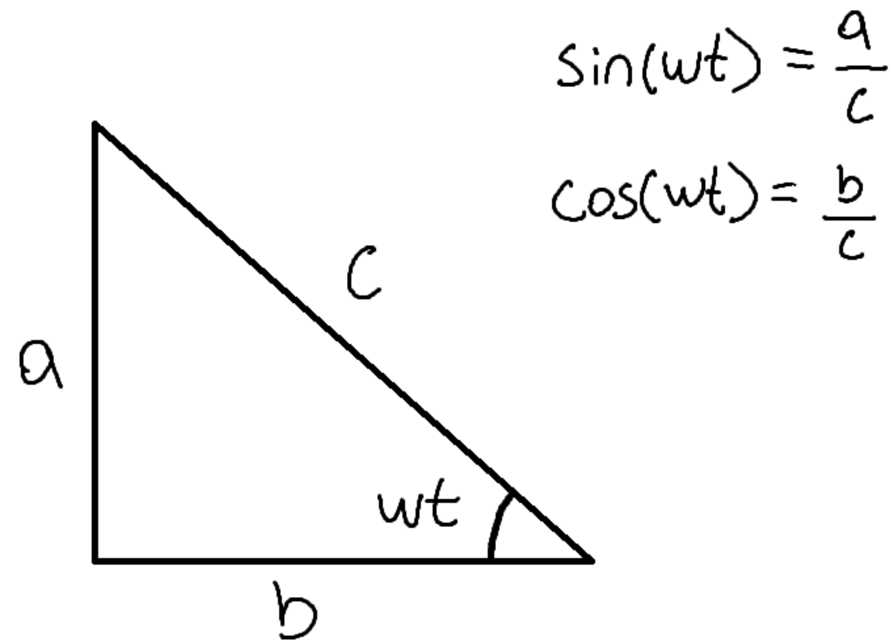

Figure 1: Trigonometric Ratios

Contents

3 Introduction

3.1 Purpose of Document

The purpose of this document is to describe the requirements for the spectral search program (SpecSearch). SpecSearch will search for the continuous spectrum of a lax pair that is compatible with solutions of the NLS equation.

This document will explain the mathematical models, assumptions, solution characterisitcs and goals of the software. After reading this document one should be able to understand the mathematical and physical context of the inputs and outputs. They should also recognize the connections and dependencies between these SRS topics.

The SRS is meant to be simaltaneously unambigious and abstract. It details the quality attributes of the software, such as functionality, without dicussing any code, numerical algorithms or scientific computing solutions. It is a precursor to the documents required in the development of scientific software. One should read and understand this document before the VnV, design or code itself.

3.2 Scope of Requirements

The scope of SpecSearch is limited to finding the spectrum of a particular linear lax equation and determining the stability of the corresponding waves. The waves being investigated are solutions to the focusing NLS equation.

3.3 Characteristics of Intended Reader

The reader of this document should have taken an introductory course in partial differential equations and complex analysis. They should also have a first year undergraduate understanding of linear algebra.

3.4 Organization of Document

This document follows a template provided by Dr. Spencer Smith at McMaster University. It begins by introducing the document. This introduction is followed by a general overview of the system and then an outline of the goals and mathematical theory. The document continues by describing the behaviour between inputs and outputs, judging output and then foreshadows changes to the software. It concludes with useful graphs and figures for traceability.

More details of the template can be found in Smith et al (2005) and Smith et al (2007). The latex template is available in Dr. Spencer Smith's gitlab repository CAS 741.

4 General System Description

This section identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

4.1 System Context

Diagram 1 shows the system context. The circle represents the user of the scientific software. The rectangle resembles the software system. The arrows indicate the flow of data between software and environment.

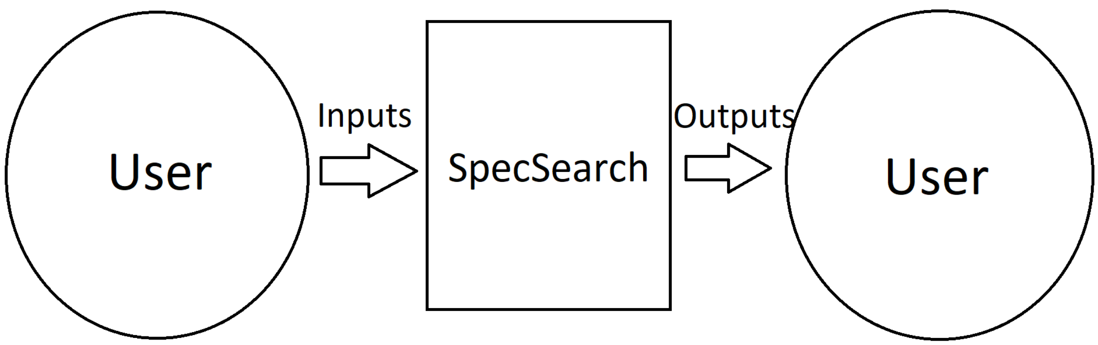

Figure 2: System Context

- User Responsibilities:
  - Ensure that the input variables resemble the wave that you intend to analyze.
  - Ensure that the assumptions imposed on waves in this software are reasonable for your reasearch.
  - Ensure that the software is used in a legal and ethical manner.
- SpecSearch Responsibilities:
  - Detect data type mismatch.
  - Solve the lax equation associated with the given wave parameters.
  - Connect the discrete spectrum in order to form a continuous spectrum.
  - Display the continuous spectrum on the complex plane.
  - Determine whether or not the waves are stable.

4.2 User Characteristics

The user should have a basic understanding of wave mechanics (wave speed, amplitude and angular frequency). They should also understand the concept of conserved quantities.

4.3 System Constraints

My supervisor requires that software will be created with MATLAB.

5 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

5.1 Problem Description

A lax pair is a set of matrices or operators that satisfy differential equations. The NLS equation, a PDE used to model rogue waves and modulated wave packets, appears as a compatibility condition of a particular lax pair of equations. One equation is a spectral problem, with matrix U, and the other is a time evolution problem, with matrix V. See instance model 1 for more details.

The spectral problem is important because the spectral parameter within it contains information about the stability of solutions. See instance model 2 for more details regarding stability.

SpecSearch will attempt to produce a numerical approximation of the continuous spectrum for the previously mentioned spectral problem. In particular, it will find the spectrum for general travelling periodic wave solutions of the focusing NLS equation.

Previous attempts have used an algebraic method to calculate the spectral parameter (Pelinovsky 2018)(Deconinck 2017). These attempts have only been successful at finding a countable number of points on the spectrum. This software will search for more points and "connect" the points in an attempt to approximate and search for the continuous spectrum.

5.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- **Spectrum**: The set of allowable values for the spectral parameter of a matrix or operator.

- **Operator**: A mapping that transforms elements of a space into other elements of the same space.
- **Traveling Periodic Wave**: A periodic and one-dimensional wave that travels with a constant speed, c, and angular frequency, ω.
- **Compatibility Condition**: Conditions for which the lax pair of equations is guaranteed.
- **Stability**: A solution is stable if slight perturbations in initial conditions lead to slight perturbations in the entire solution.
- **Initial Data**: The graph of a wave (or its derivative) at a fixed point in time, tyically t=0.
- **Rogue Wave**: A wave is considered rogue if its amplitude is more than double of the average of the upper third wave amplitudes in its surroundings.
- **PDE**: A partial differential equation.An equation containing one or more partial derivatives. The partial derivative of a multivariable function is a derivative with respect to one of its arguments.
- **Countable:** A set that is finite or has the same cardinality as the natural numbers (1,2,3,4,5 ....).
- **Focusing wave:** When the nonlinear Schrödinger equation adds the nonlinear term $(+|u|^2u)$.
- **Hamiltonian:** Is an operator corresponding to the total energy of a system.

5.1.2 Physical System Description

The physical system of SpecSearch includes the following elements:

PS1: Unspecified body of water or laser channel with a traveling periodic wave.

5.1.3 Goal Statements

Given the constant wave speed,c , angular frequency ,ω, conserved quantities, (g,d), and the intial profile of a general traveling periodic wave, the goal statements are:

GSlocate: Locate elements of the continuous spectrum on the complex plane.

GSstable: Determine the stability of the solutions.

5.2 Solution Characteristics Specification

The instance model that governs SpecSearch is presented in Subsection 5.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

Aham: The wave equation is a complex hamiltonian evolution equation.

Amom: The linear momentum densities are proportional to the integrands of the corresponding velocity functionals

Awav: All waves in the model are general traveling perioidic waves with constant speed and frequency.

Anls: The wave is a solution to the NLS equation.

Afoc: The system only deals with focusing waves.

Astat: The system will start by assuming that the wave is stationary and then use the stationary solution to build the moving solution.

5.2.2 Theoretical Models

This section focuses on the general equations and laws that SpecSearch is based on.

Number	T1
Label	**Focusing NLS for General travelling periodic waves**
Equation	$u'' + 2\|u\|^2 u + 2icu' = \omega u$
Description	General traveling periodic waves that satisfy the Schrödinger equation will have this form.
Source	Deconinck (2017)
Ref. By	DD1, IM1

Derivatition of the Focusing NLS for General Traveling Periodic Waves

We start with the focusing NLS equation: $iq_t + q_{xx} + 2|q|^2 q = 0$

The NLS equation is a PDE meant to model modulated wave packets and rogue waves in physics. u is the complex envelope of the wave, t is time and x is the position in one dimensional space.

A general traveling periodic wave has the form $z(x,t) = u(x+2ct)e^{i\omega t}.(u : \mathbb{R} \to \mathbb{C})$. We will now search for general traveling periodic waves that are solutions to the NLS equation. This is done by setting z=q in the NLS equation. This yields:

$$
\begin{gathered}
iz_t + z_{xx} + 2|z|^2 z = 0 \\
\Rightarrow i(2cu'e^{i\omega t} + i\omega u e^{i\omega t}) + u''e^{i\omega t} + 2|u|^2|e^{i\omega t}|^2 u e^{i\omega t} = 0 \\
\Rightarrow e^{i\omega t}(2icu' + i^2\omega u + u'' + 2|u|^2 u) = 0 \\
\Rightarrow 2icu' - \omega u + u'' + 2|u|^2 u = 0
\end{gathered}
$$

Which is the result in T1.

5.2.3 General Definitions

There are no general definitions.

5.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

Number	DD1
Label	**Conservation Equations**
Symbols	g, c, d, ω
SI Units	-
Equation(1) Equation(2)	$\bar{u}u' - u\bar{u}' + 2ic|u|^2 = 2ig$ $|u'|^2 + |u|^4 + d = \omega|u|^2$
Description	These are conserved quantities in the NLS equation. For fixed g and d the above relations are true for all space and time.
Sources	Deconinck (2017)
Ref. By	IM1

Justification of a Conserved Quantity

I will first multiply T1 by $\bar{u}$.

$$u''\bar{u} + 2|u|^2 u\bar{u} + 2icu'\bar{u} = \omega u\bar{u} \ (*)$$

Taking the complex conjugate yields:

$$\bar{u}''u + 2|u|^2 \bar{u}u - 2ic\bar{u}'u = \omega \bar{u}u \ (**)$$

Subtraction of * with ** yields:

$$\begin{gathered} u''\bar{u} + 2|u|^2 u\bar{u} + 2icu'\bar{u} - (\bar{u}''u + 2|u|^2\bar{u}u - 2ic\bar{u}'u) = \omega u\bar{u} - (\omega\bar{u}u) \\ \Rightarrow u''\bar{u} - \bar{u}''u + 2ic(u'\bar{u} + \bar{u}'u) = 0 \\ \Rightarrow \int u''\bar{u} - \int \bar{u}''u + 2ic\int(u'\bar{u} + \bar{u}'u) = 2ig \end{gathered}$$

Noticing that the term in the integral attatched to 2ic is a product rule.

$$\begin{gathered} \Rightarrow \int u''\bar{u} - \int \bar{u}''u + 2icu\bar{u} = 2ig \\ \Rightarrow \bar{u}u' - \int \bar{u}'u' - u\bar{u}' + \int \bar{u}'u' + 2icu\bar{u} = 2ig \\ \Rightarrow \bar{u}u' - u\bar{u}' + 2ic|u|^2 = 2ig \end{gathered}$$

The second conserved quantity is obtained by multiplying $\bar{u}'$ by T1, adding the complex conjugate and integrating.

5.2.5 Instance Models

This section transforms the problem defined in Section 5.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 5.2.4 to replace the abstract symbols in the models identified in Sections 5.2.2 and 5.2.3.

The goals GSlocate and GSstable are solved by IM1 and IM2 respectively.

Number	IM1
Label	**Searching for the continuous spectrum**
Input	c, ω, g, d, initial wave profile
Output	λ such that: $\phi_x = U(u,\lambda)\phi$
Description	$U(u,\lambda) = \begin{pmatrix} \lambda & u \\ -\bar{u} & -\lambda \end{pmatrix}$ T1 is a compabililty condition for the lax pair comprised of the above row's ODE and $\phi_t = V(u,\lambda)\phi$ $V(u,\lambda) = i\begin{pmatrix} 2\lambda^2 + \|u\|^2 & u_x + 2\lambda u \\ \bar{u}_x - 2\lambda\bar{u} & -2\lambda^2 - \|u\|^2 \end{pmatrix}$ $\phi \in \mathbb{C}$
Sources	Deconinck (2017)
Ref. By	IM2

Justification of Compatibility Conditions

For a smooth C^2 function ,ϕ, we have that $\phi_{tx} = \phi_{xt}$.
From IM1 $\Rightarrow \phi_{xt} = \delta_t U\phi + UV\phi$ and $\phi_{tx} = \delta_x V\phi + VU\phi$.
Combining the above two equations :

$$\Rightarrow \delta_t U\phi + UV\phi = \delta_x V\phi + VU\phi$$

$$\Rightarrow \delta_t U + UV = \delta_x V + VU \ (-)$$

$$\Rightarrow \begin{pmatrix} 0 & u_t \\ -\bar{u}_t & 0 \end{pmatrix} + i\begin{pmatrix} \lambda & u \\ -\bar{u} & -\lambda \end{pmatrix}\begin{pmatrix} 2\lambda^2 + |u|^2 & u_x + 2\lambda u \\ \bar{u}_x - 2\lambda\bar{u} & -2\lambda^2 - |u|^2 \end{pmatrix} = i\begin{pmatrix} u_x\bar{u} + u\bar{u}_x & u_{xx} + 2\lambda u_x \\ \bar{u}_{xx} - 2\lambda\bar{u}_x & -u_x\bar{u} - u_x\bar{u}_x \end{pmatrix} +$$

$$i\begin{pmatrix} 2\lambda^2 + 2|u|^2 & u_x + 2\lambda u_x \\ \bar{u}_x - 2\lambda\bar{u} & -2\lambda^2 - |u|^2 \end{pmatrix}\begin{pmatrix} \lambda & u \\ -\bar{u} & -\lambda \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} i(2\lambda^3 + \lambda|u|^2 + u\bar{u}_x - 2\lambda|u|^2) & u_t + i(\lambda u_x + 2\lambda^2 u - 2\lambda^2 u - u|u|^2) \\ -\bar{u}_t + i(-2\lambda^2\bar{u} - \bar{u}|u|^2 - \lambda\bar{u}_x + 2\lambda^2\bar{u}) & i(-\bar{u}u_x - 2\lambda u\bar{u} + 2\lambda^3 + \lambda|u|^2) \end{pmatrix} =$$

$$i\begin{pmatrix} u_x\bar{u} + u\bar{u}_x + 2\lambda^3 + \lambda|u|^2 - u_x\bar{u} - 2\lambda|u|^2 & u_{xx} + 2\lambda u_x + 2\lambda^2 u + u|u|^2 - 2|u|^2 - \lambda u_x \\ \bar{u}_{xx} - 2\lambda\bar{u}_x + \lambda\bar{u}_x - 2\lambda^2\bar{u} + 2\lambda^2\bar{u} + \bar{u}|u|^2 & -u_x\bar{u} - u_x\bar{u}_x + u\bar{u}_x - 2\lambda|u|^2 + 2\lambda^3 + \lambda|u|^2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & u_t - iu|u|^2 \\ -\bar{u}_t - i\bar{u}|u|^2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & iu_{xx} + iu|u|^2 \\ i\bar{u}_{xx} + i\bar{u}|u|^2 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & iu_t + 2u|u|^2 + u_{xx} \\ iu_t + 2u|u|^2 + u_{xx} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

This is true if u satisfies the NLS equation (T1). Thus, the NLS equation is a compatibility condition for the lax pair in IM1.

Number	IM2
Label	**Stability**
Input	λ, ω, g, d
Output	Stable or Unstable
Description	$\Omega = \pm 2i(\lambda^4 - \frac{1}{2}\omega\lambda^2 - ig\lambda + \frac{1}{16}\omega^2 - \frac{1}{4}d)^{\frac{1}{2}}$ If the real part of $\Omega > 0$ then the solution, u, is unstable.
Sources	Deconinck (2017)
Ref. By	IM2

Validation of Stability Criteria

Without loss of generality it is possible to set $c = 0$.

We can write $\phi(x,t) = M(t)\chi(x,t)$. Here M(t) is the diagonal matrix with the first entry as $e^{\frac{i\omega t}{2}}$ and second entry as $e^{\frac{-i\omega t}{2}}$ and σ_3 is the diagonal matrix with top entry 1 and bottom entry -1.

Substituting this into the lax pair in IM1 yields:

$$\chi_x = U\chi \text{ and } \chi_t = V\chi - \tfrac{i\omega}{2}\sigma_3\chi$$
$$\text{Let } \chi(x,t) = e^{t\Omega}\phi(x).$$
$$\Rightarrow \Omega\phi = V(u,\lambda)\phi - \tfrac{i\omega}{2}\sigma_3\phi$$

This matrix system admits a non zero solution when the determinant of the following:

$$R(\Omega,\lambda) = \begin{pmatrix} i(2\lambda^2 + |u|^2 - \frac{\omega}{2}) - \Omega & i(u_x + 2\lambda u) \\ i(\bar{u}_x - 2\lambda\bar{u}) & -i(2\lambda^2 + |u|^2 - \frac{\omega}{2}) - \Omega \end{pmatrix}$$

Is nonzero. Expanding the determinant of R it follows that:

$$|R| = \Omega^2 + 4(\lambda^4 - \tfrac{1}{2}\omega\lambda^2 + \tfrac{1}{16}\omega^2 - \tfrac{1}{4}d) = \Omega^2 + 4P(\lambda) = 0$$
$$\Rightarrow \Omega^2 = -4P(\lambda)$$
$$\Rightarrow \Omega = \pm\sqrt{-4P(\lambda)}$$

By Theorem 5.1 of (Deconinck, 2017) any Ω with real part larger than zero implies instability of the periodic wave u.

5.2.6 Data Constraints

There are no data constraints.

5.2.7 Properties of a Correct Solution

A correct solution will be an array of numbers that appear on the continuous spectrum. The plot will connect these points in an attempt to model the entire continuous spectrum. The stability of solutions will be displayed as booleans (stable or unstable).

6 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

6.1 Functional Requirements

Rin: SpecSearch shall take in conserved quantities (d, g), wave parameters (c, ω), and initial wave data as input.

Rfind: SpecSearch will find values of λ.

Rcon: SpecSearch will connect the points in Rfind in an attempt to approximate the continuous spectrum.

Rplt: SpecSearch will plot the spectrum on the complex plane.

Rstl: SpecSearch will determine the stability of solutions.

6.2 Nonfunctional Requirements

NFR1: The software should be maintainable and manageable as it will be modified and continually updated during the research process. It should be simple to add new numerical methods for approximating the derivatives of ϕ.

NFR2: The software should be accurate within the standards of my supervisor and reliable for researchers studying rogue waves.

7 Likely Changes

LC1: We might add constraints or bounds to the input variables.

8 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 1 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 2 shows the dependencies of instance models, requirements, and data constraints on each other. Table 3 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	T1	DD1	IM1	IM2
T1	X	X	X	X
DD1		X	X	X
IM1			X	X
IM2				X

Table 1: Traceability Matrix Showing the Connections Between Items of Different Sections

	IM1	IM2	Rin	Rfind	Rcon	Rplt	Rstl
IM1	X	X	X	X			
IM2		X	X				X
Rin	X	X	X	X			
Rfind	X	X	X	X	X	X	
Rcon					X	X	
Rplt						X	
Rstl							X

Table 2: Traceability Matrix Showing the Connections Between Requirements and Instance Models

	Aham	Amom	Awav	Anls	Afoc	Astat
T1	X	X	X	X	X	X
DD1	X	X	X	X	X	X
IM1	X	X	X	X	X	X
IM2	X	X	X	X	X	X
LC1						

Table 3: Traceability Matrix Showing the Connections Between Assumptions and Other Items

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is dependent on the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Figure 3 shows the dependencies of theoretical models, general definitions, data definitions and instance models on each other. Figure 4 shows the dependencies of instance models, requirements, and data constraints on each other.

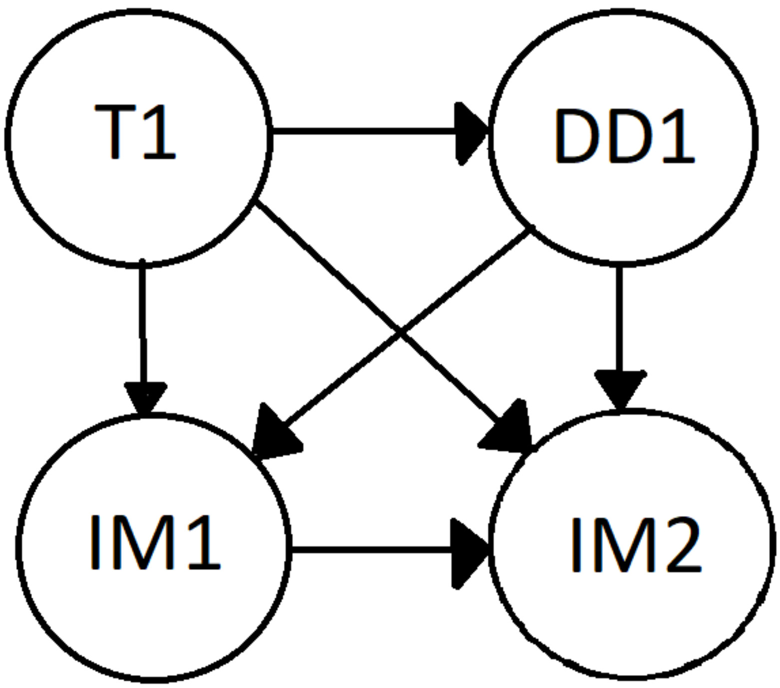

Figure 3: Traceability Graph Showing the Connections Between Items of Different Sections

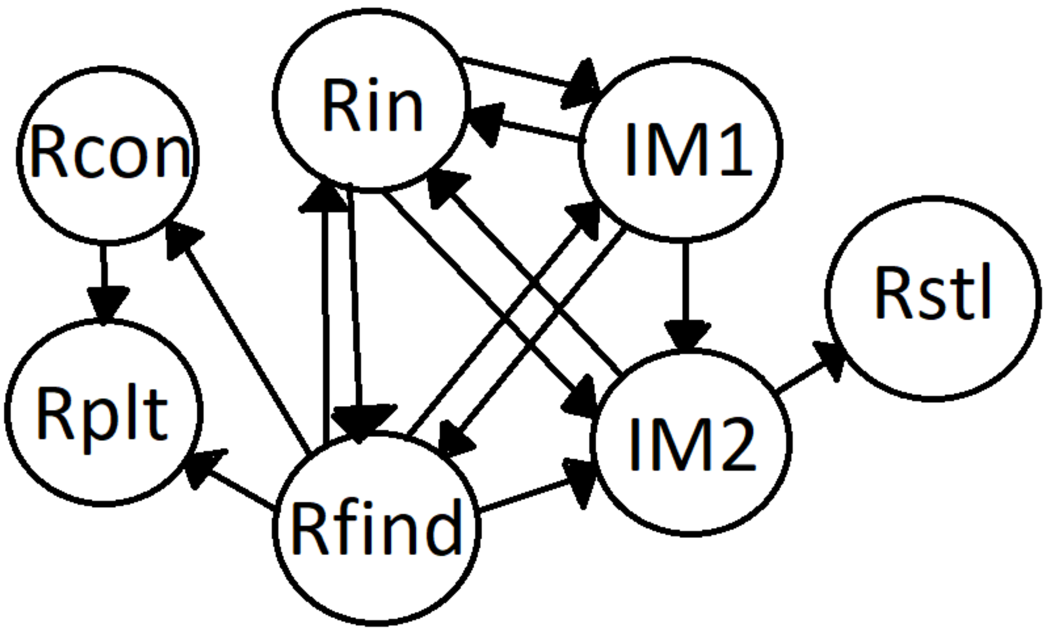

Figure 4: Traceability Graph Showing the Connections Between Items of Different Sections

References

[1] Spencer Smith and Lei Lai. A New Requirements Template for Scientific Computing. Proceedings of SREP'05, Paris, France 2005.

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[3] Dmitry E. Pelinovsky. Localization in Periodic Potentials. Cambridge University Press, 2011.

[4] Bernard Deconinck and Benjamin L.Segal. The stability spectrum for elliptic solutions to the focusing NLS equation. PhysicaD, 2017.

[5] J. Chen and D.E. Pelinovksy. Rogue periodic waves in the focusing nonlinear Schrodinger equation. Proceeding A of Roy.Soc. Lond., 2018.

9 Appendix

9.1 Symbolic Parameters

There are no symbolic parameters.