Direction Estimation Using Compressive Sampling Array Processing

Ying Wang[⋆†], *Geert Leus*[†], *Ashish Pandharipande*[⋆]

[⋆]Philips Research Europe, 5656 AE Eindhoven, The Netherlands
Email: {ying.z.wang, ashish.p}@philips.com
[†]Department of EEMCS, Delft University of Technology, The Netherlands
Email: g.j.t.leus@tudelft.nl

ABSTRACT

We propose a new architecture, the compressive sampling (CS) array, for array based applications, by exploiting compressive sampling in the spatial domain. With random projections (or selections) of the array elements, we can transform a large size array into a small size array. We also propose two approaches of direction-of-arrival (DoA) estimation using our CS array, I. (joint) CS recovery, and II. CS beamformers. As a result we can greatly reduce the hardware complexity and software complexity while still maintaining the high resolution achieved as if a large size array were used.

Index Terms— Compressive sampling, direction-of-arrival estimation, beamforming.

1. INTRODUCTION

Estimating the direction-of-arrival (DoA) of propagating plane waves is a problem of broad interest in a variety of fields including wireless communications, radar and sonar systems, acoustic signal processing, medical imaging and seismology. It is common to use an array of multiple sensor/antenna elements that may be co-located or distributed in space to estimate DoAs of source signals. Figure 1 shows the block diagram of a receive antenna array, where each antenna element is connected to a separate receiver chain of frontend circuits as shown in Figure 2, that takes the received analog signal as input and then performs amplifying, down-converting, filtering, analog-to-digital convertor (ADC) sampling and finally outputs the digital baseband signal to a computer for further signal processing to estimate the DoAs of the sources.

Generally in array-based DoA estimation algorithms, the larger the number of antenna elements in the array (assume fixed antenna element spacing), the higher the angle resolution of the estimated directions. However the complexity increases significantly when the required resolution of DoA increases, in both hardware sense due to the large number of frontend circuit chains and in software sense due to the estimation algorithms involving manipulations of large size data matrices (e.g., eigenvalue decomposition of a matrix). A consequence is also that this solution is not energy-efficient.

Compressive sampling (CS) is a method for acquisition of sparse signals and reconstruction from compressed measurements. In [3], a compressive wireless array is proposed for bearing estimation. In [6], a compressive beamforming method is presented. Both approaches apply compressive sampling in the time domain to reduce the ADC sampling rate or the number of time samples for each element of the array.

In this paper we propose a new architecture for array-based applications by exploiting the sparsity in the angle spectrum, with fundamental differences from the approaches in [3] and [6]. Our CS array applies compressive sampling in the spatial domain, which compresses or transforms the array of a large number of elements into an array of a much smaller number of elements. Besides, the number of time samples/snapshots for each element can also be reduced using our approach. As a result we can greatly reduce hardware complexity (i.e., much fewer frontend circuit chains) and software complexity (i.e., less computations in the DoA estimation algorithms), while still maintaining the high resolution achieved as if a large size array were used. It is also interesting to relate the proposed CS array to the beamspace processing approach of [8]. In [8], a transform matrix is designed to effectively probe a spatial subband together with some filtering for anti-spatial-aliasing and then decimating. Our CS array uses random projections as a transform matrix to reduce the array dimension.

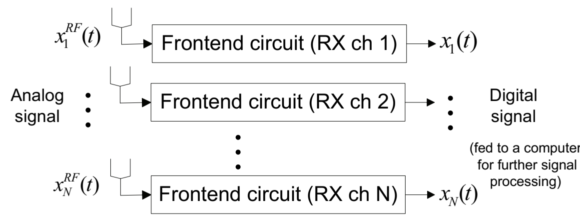

Fig. 1. Block diagram of a receive antenna array.

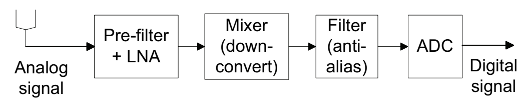

Fig. 2. Detailed blocks in each separate RX chain of frontend circuits connected to an antenna element.

The remainder of the paper is organized as follows. In Section 2, we briefly review the array based DoA estimation algorithms and the CS principle. In Section 3, we propose our CS array architecture together with two different approaches of estimating the angle spectrum for finding DoAs. Simulation results are shown in Section 4, and conclusions drawn in Section 5.

2. PRELIMINARIES

2.1. Direction estimation using sensor/antenna array

At time instant t, $t = 1, 2, \ldots, T$, where T is the total number of snapshots taken, the received signal at an array of N sensor/antenna

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elements can be written as an $N \times 1$ vector $\mathbf{x}(t)$ of the form

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k) s_k(t) + \mathbf{w}(t) \tag{1}$$

where $s_k(t), k = 1, 2, \ldots, K$, is a narrowband planar wavefront impinging on the array from a direction θ_k, K is the number of source waveforms, $\mathbf{w}(t)$ is an $N \times 1$ vector representing the additive noise at the array, and $\mathbf{a}(\theta_k)$ is an array steering/response vector of size $N \times 1$ corresponding to the source from a direction θ_k. If a uniform linear array (ULA) consisting of N elements spaced equally by a distance d is used to estimate the DoA of a source from a direction θ (w.r.t. the array boresight), the corresponding $N \times 1$ array steering vector is

$$\mathbf{a}(\theta) = [1 \; e^{-j\alpha} \; \cdots \; e^{-j\alpha(N-1)}]^T, \tag{2}$$

where $\alpha = 2\pi(d/\lambda)\sin(\theta)$ represents the phase shift from element to element along the ULA, with λ being the wavelength of the received signal. The received signal vector $\mathbf{x}(t)$ of (1) can also be expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t) \tag{3}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1) \; \mathbf{a}(\theta_2) \; \cdots \; \mathbf{a}(\theta_K)]$ is an $N \times K$ matrix of steering/response vectors, and $\mathbf{s}(t) = [s_1(t) \; s_2(t) \; \cdots \; s_K(t)]^T$ is a $K \times 1$ vector representing the sources. The spatial correlation matrix of the observed signal vector $\mathbf{x}(t)$ is $\mathbf{R}_x = E[\mathbf{x}(t)\mathbf{x}^H(t)]$, where $E[\cdot]$ is the expectation operator, which can be estimated by taking an average over T snapshots. The DoAs of the source signals are estimated by looking for the directions that give the maximum response in the angle spectrum, which can be estimated using some well-known beamformers, e.g., the minimum variance distortionless response (MVDR) and the multiple signal classification (MUSIC) [7] beamformers. The MVDR angle spectrum estimate is given by

$$P_x(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{R}_x^{-1}\mathbf{a}(\theta)} \tag{4}$$

which requires calculating the inverse of the $N \times N$ matrix $\mathbf{R}_x$.

2.2. Compressive sampling

We first briefly introduce the CS framework [4]. Suppose that $\mathbf{x} \in \mathbb{C}^N$ is a signal vector of size $N \times 1$, which can be written as $\mathbf{x} = \mathbf{\Psi}\mathbf{z}$, where $\mathbf{\Psi}$ is the $N \times N$ sparsity basis matrix and $\mathbf{z}$ an $N \times 1$ vector with $K \ll N$ non-zero (and large enough) entries. The CS theory states that $\mathbf{x}$ can be recovered using $M = K\mathcal{O}(\log N)$ non-adaptive linear projection measurements on to an $M \times N$ basis matrix $\mathbf{\Phi}$ that is incoherent with $\mathbf{\Psi}$. An example construction of $\mathbf{\Phi}$ is given by choosing elements that are drawn independently from a random distribution, e.g., Gaussian, Bernoulli. The measurement vector $\mathbf{y}$ can be written as

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{z}. \tag{5}$$

Reconstruction is achieved by solving the following l_1-norm optimization problem

$$\hat{\mathbf{z}} = \arg\min_{\mathbf{z}} \|\mathbf{z}\|_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{z}. \tag{6}$$

Linear programming techniques, e.g., basis pursuit [1], or iterative greedy algorithms [9] can be used to solve (6).

Distributed versions of CS have been considered in [2] and [5] in order to exploit the underlying correlation structures in the signal observations. The specific signal observation model of interest to us is the joint sparsity model JSM-2 [5], where the received signals have a common sparsity basis support but with different basis coefficient values. Recovery in distributed compressive sampling can be done using algorithms like simultaneous orthogonal matching pursuit (SOMP) [5], Multiple measurement vectors FOCal Underdetermined System Solver (M-FOCUSS) [2].

3. COMPRESSIVE SAMPLING (CS) ARRAY

The proposed CS array architecture is illustrated in Figure 3. The analog signal picked up by the antenna elements of the receiver array is first 'compressed' (details will be given below) in the analog domain and then passed through a fewer number of frontend circuit chains to obtain the digital baseband signal for further processing. At time instant t, $t = 1, 2, \ldots, T$, the array with N sensor/antenna

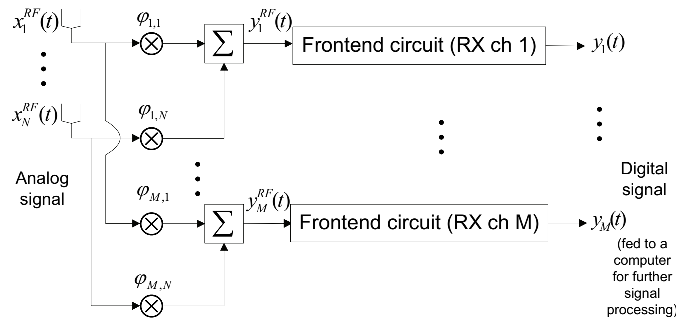

Fig. 3. The compressive sampling (CS) receiver array architecture.

elements collects a signal vector $\mathbf{x}(t)$ of the form given in (1). A CS measurement matrix or compression matrix $\mathbf{\Phi}$ is applied on the array such that the original signal vector $\mathbf{x}(t)$ of size $N \times 1$ is transformed to a compressed signal vector $\mathbf{y}(t)$ of size $M \times 1$.

$$\mathbf{y}(t) = \mathbf{\Phi}\mathbf{x}(t) \tag{7}$$

For notational simplification, here we used the digital baseband version to describe all the signals. Note that the compression is actually applied on the analog RF signals. The down-conversion and digitizing processes do not affect the compression processing. The compression matrix $\mathbf{\Phi}$ is of size $M \times N$, with the element $\phi_{m,n}$, $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, drawn independently from a random distribution. Denote the m-th element of the compressed signal vector $\mathbf{y}(t)$ as $y_m(t)$, which is obtained by summing the original N branches of signals $x_1(t), x_2(t), \ldots, x_N(t)$ weighted by $\phi_{m,1}, \phi_{m,2}, \ldots, \phi_{m,N}$ respectively, and then fed to the m-th frontend chain to obtain the digitized version y_m. The multiplication of the original signal $x_n(t)$ with a weight $\phi_{m,n}$ is implemented by an attenuator on each branch, or simply a phase shifter ($e^{j0}, e^{j\pi}$) in case of a Bernoulli distribution (± 1 with probability 0.5). If we expect the number of sources to locate is K, then the number of compressed branches can be selected as $M = K\log(N)$ to ensure that embedding of a sparse signal into a compressed subspace $\mathbf{\Phi}$ does not destroy the essential information of the original signal.

We calculate the spatial correlation matrix $\mathbf{R}_y$ of the compressed signal vector $\mathbf{y}(t)$ by averaging over T snapshots, i.e., $\mathbf{R}_y = E[\mathbf{y}(t)\mathbf{y}^H(t)]$. Applying an eigenvalue decomposition on $\mathbf{R}_y$ we can obtain eigenvalues in descending order ($\lambda_{y,1} \geq \lambda_{y,2} \geq \ldots \geq \lambda_{y,M} \geq 0$), among which $\lambda_{y,1}, \lambda_{y,2}, \ldots, \lambda_{y,K}$ are K signal eigenvalues corresponding to the sources and the remaining are $M - K$ noise eigenvalues. The number of sources K can be estimated using methods such as Akaike [10].

3.1. Angle spectrum estimation via CS recovery

We can calculate the angle spectrum by finding a sparse basis of the array signal and formulating the DoA estimation problem in terms of a CS problem and then solving it with CS recovery algorithms. Rewrite the signal model of (3) to group T snapshots as

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{W}, \tag{8}$$

where $\mathbf{X} = [\mathbf{x}(1)\ \mathbf{x}(2)\ \cdots\ \mathbf{x}(T)]$ is a matrix of size $N \times T$, $\mathbf{A}$ is a matrix of size $N \times K$ as in (3), $\mathbf{S} = [\mathbf{s}(1)\ \mathbf{s}(2)\ \cdots\ \mathbf{s}(T)]$ is a matrix of size $K \times T$, and $\mathbf{W} = [\mathbf{w}(1)\ \mathbf{w}(2)\ \cdots\ \mathbf{w}(T)]$ is a matrix of size $N \times T$. Let $\theta_1, \theta_2, \ldots, \theta_{N_s}$ be a set of angles, N_s being the total number of angles we want to scan. Using the ULA steering/response vector given by (2), we define an angle scanning matrix of size $N \times N_s$ by

$$\mathbf{\Psi} = [\mathbf{a}(\theta_1)\ \mathbf{a}(\theta_2)\ \cdots\ \mathbf{a}(\theta_{N_s})] \tag{9}$$

Also define an $N_s \times 1$ sparse vector

$$\mathbf{z}(t) = [z_{\theta_1}(t)\ z_{\theta_2}(t)\ \cdots\ z_{\theta_{N_s}}(t)]^T, \tag{10}$$

with K nonzero coefficients $z_\theta(t) = s_k(t)$ at positions corresponding to the source angles $\theta = \theta_k, k = 1, 2, \ldots, K$, and zero coefficients at the remaining $N_s - K$ positions. Then the signal model of (8) can be rewritten as a CS problem

$$\mathbf{X} = \mathbf{\Psi}\mathbf{Z} + \mathbf{W}, \tag{11}$$

where $\mathbf{Z} = [\mathbf{z}(1)\ \mathbf{z}(2)\ \cdots\ \mathbf{z}(T)]$ is a matrix of size $N_s \times T$. Notice that $\mathbf{z}(1), \mathbf{z}(2), \ldots, \mathbf{z}(T)$ are jointly sparse since they have the nonzero coefficients at the same positions. The angle scanning matrix $\mathbf{\Psi}$ is therefore a sparse basis of the received array signals. The compressed array signal of (7) can be rewritten as

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{X} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{Z} + \mathbf{\Phi}\mathbf{W}, \tag{12}$$

where $\mathbf{Y} = [\mathbf{y}(1)\ \mathbf{y}(2)\ \cdots\ \mathbf{y}(T)]$. Utilizing the joint sparse structure, we can solve $\mathbf{Z}$ efficiently using joint CS recovery algorithms. Let $\hat{\mathbf{Z}} = [\hat{\mathbf{z}}(1)\ \hat{\mathbf{z}}(2)\ \cdots\ \hat{\mathbf{z}}(T)]$ be the recovered solution, the angle spectrum is calculated by

$$P_y(\theta) = \frac{1}{T}\sum_{t=1}^{t=T} \|\hat{z}_\theta(t)\|^2, \theta = \theta_1, \theta_2, \ldots, \theta_{N_s} \tag{13}$$

When there are multiple time snapshots of the received array signals, we use joint CS recovery algorithms to estimate the angle spectrum as described above. When there is only a single time snapshot, i.e., $T = 1$, we can use the single measurement vector based CS recovery algorithms, e.g., FOCUSS.

3.2. Angle spectrum estimation via CS beamformers

We can calculate the angle spectrum by deriving beamformers for the CS array. Following the MVDR requirement we can easily derive an estimate of the angle spectrum of the CS array as

$$P_y(\theta) = \frac{1}{\mathbf{b}^H(\theta)\mathbf{R}_y^{-1}\mathbf{b}(\theta)} \tag{14}$$

$$\mathbf{b}(\theta) = \mathbf{\Phi}\mathbf{a}(\theta) \tag{15}$$

where $\theta = \theta_1, \theta_2, \ldots, \theta_{N_s}$, $\mathbf{R}_y$ is the spatial correlation matrix of the compressed array signals, $\mathbf{b}(\theta)$ is the compressed array steering/response vector, and $\mathbf{a}(\theta)$ is the original array steering/response vector depending on the array geometry, e.g., given by (2) if a ULA is used. The DoAs of the source signals are estimated by looking for the directions that give the maximum response in the compressed angle spectrum $P_y(\theta)$. If we use other beamformers like MUSIC, the compressive angle spectrum estimate can be obtained in a similar way as the MVDR discussed here.

3.3. Advantages of the CS array over the conventional array

Compared with a conventional array having a large number of antenna elements, our CS array in effect transforms/compresses a large size array into a small size array. First, the hardware complexity is greatly reduced because of a much smaller number of frontend circuit chains. Second, the software (estimation algorithms) complexity is also reduced because of a smaller dimension of the array data. More specifically, the DoA estimation algorithms involve an inverse and eigenvalue decomposition of the spatial correlation matrix with computational complexity of $\mathcal{O}(n^3)$ for an $n \times n$ matrix. The proposed CS array architecture with CS beamformers needs to work with $\mathbf{R}_y$ of size $M \times M$, while the conventional array needs to work with $\mathbf{R}_x$ of size $N \times N$. Besides, the conventional array processing usually requires the number of time snapshots $T > N$ to avoid an ill-conditioned $\mathbf{R}_x$ of size $N \times N$. The CS array can work with $T = 1$ using the CS recovery approach or $T > M$ using the CS beamformer approach. With such reduced complexity, our CS array can still achieve similar high resolution in the DoA estimation as a conventional large size array, which will be shown later in the simulation section.

3.4. Some implementation issues

Different compression matrix - random sub-sampling: One design of practical interest is constructing the $\mathbf{\Phi}$ of size $M \times N$ by randomly selecting M rows from an identity matrix of size $N \times N$. As a result, the compression procedure is performing a random sub-sampling/selection of the antenna elements of the array, which further simplifies the frontend circuits compared to the case random Gaussian entries are used.

Different array geometry than ULA: In the discussion we used ULA as an example of array geometry. For other array geometries, e.g., circular array or square array, the CS array can be built using the same architecture as shown in Figure 3. The compression matrix contains randomly generated weights that are independent of the array geometry.

4. SIMULATION RESULTS

The original (or conventional) array is an ULA with $N = 36$ antenna elements. The CS array uses $M = 8$. There are $K = 2$ source signals, with binary phase shift keying (BPSK) modulation. The signal power of the k-th ($k = 1, 2, \ldots, K$) source is $E[|s_k(t)|^2] = 1$. The noise power at the n-th ($n = 1, 2, \ldots, N$) array element is $E[|w_n(t)|^2] = 0.01$.The DoAs of the two sources are respectively $\theta = 0^\circ$ and $\theta = 3^\circ$ w.r.t. the ULA boresight. The number of time snapshots is $T = 60$. The number of scanning angles is $N_s = 360$, searching from -90° to 90° w.r.t. the ULA boresight. The compression matrix $\mathbf{\Phi}$ has entries drawn from an i.i.d. Gaussian distribution (mean=0, variance=$\sqrt{1/M}$). The CS recovery algorithm used is M-FOCUSS with 16 iterations and $p = 0.8$ [2].

We compare the estimated angle spectrum using the conventional array and the proposed CS array in Figure 4. The two conventional arrays use MVDR beamformers. The CS array uses the two proposed approaches, I. CS-recovery (solid line with dot) and II. CS-MVDR (solid line with circle). The conventional array with $N = 8$ (dashed line with asterisk) cannot provide a good enough angle resolution to identify the two sources, and the conventional array with $N = 36$ (solid line with asterisk) can clearly identify the two sources. Using the proposed CS array architecture, we can achieve similar angle resolution as an $N = 36$ conventional array, while requiring similar hardware complexity and software complexity as an $N = 8$ conventional array.

We also used random sub-sampling as a special form of 'compression' as shown in Figure 5, which produces a similar angle spectrum estimate as a random Gaussian $\mathbf{\Phi}$, and with simpler implementation by just selecting randomly M out of N elements of the antenna array.

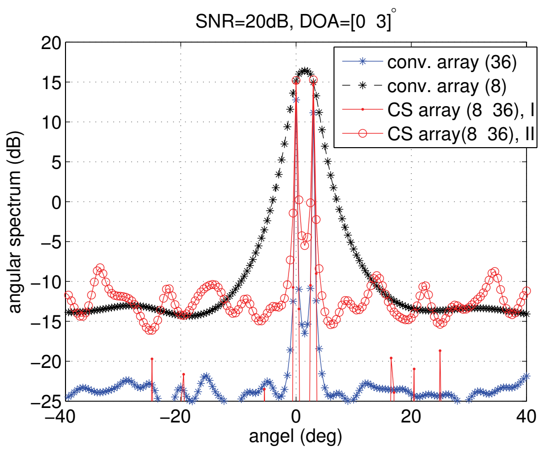

Fig. 4. Angle spectrum of the conventional array ($N = 36$), the conventional array ($N = 8$), and the proposed CS array ($M = 8, N = 36$). Compression matrix: random Gaussian.

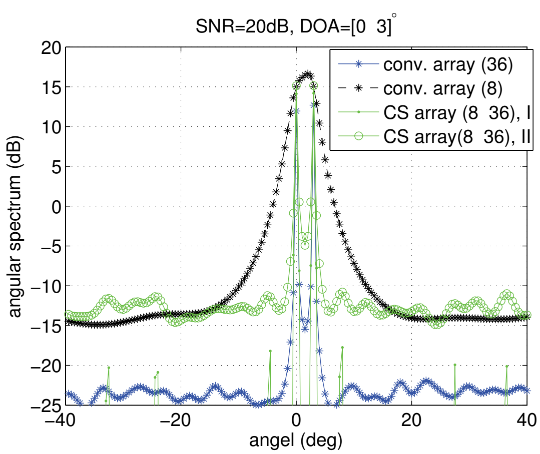

Fig. 5. Angle spectrum of the conventional array ($N = 36$), the conventional array ($N = 8$), and the proposed CS array ($M = 8, N = 36$). Compression matrix: random sub-sampling.

5. CONCLUSIONS

We have proposed a CS array for array based applications, by exploiting compressive sampling in the spatial domain. Specifically, the acquisition is done by compressively sampling the received array signals and then estimating the DoAs of sources using two different approaches: I. CS recovery - we propose a sparse basis for the array signals and then formulate the DoA estimation problem in terms of a CS recovery problem that can be solved using known CS recovery algorithms; II. CS beamformer: we propose a CS beamformer following the MVDR requirement to estimate the angle spectrum (other beamformers like MUSIC can be derived similarly). Our CS array can achieve similar angle resolution as if using a conventional array of large size, while the hardware complexity and software complexity are comparable to a conventional array of much smaller size.

6. REFERENCES

[1] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM Journal on Scientific Computing*,vol. 43, No. 1, pp. 129159, 2001.

[2] S. F. Cotter, B. D. Rao, K. Engan, and K. K-Delgado, "Sparse solutions to linear inverse problems with multiple measurement vectors," *IEEE Trans. on Signal Processing*, vol. 53, pp. 2477-2488, July 2005.

[3] V. Cevher, A. Gurbuz, J. McClellan, and R. Chellappa, "Compressive wireless arrays for bearing estimation", ICASSP 2008, pp. 2497-2500, March 31 2008-April 4, 2008.

[4] D. Donoho, "Compressed sensing", *IEEE Trans. on Information Theory*, pp. 1289-1306, Apr. 2006.

[5] M. F. Duarte, S. Sarvotham, D. Baron, M. B. Wakin and R. G. Baraniuk, "Distributed compressed sensing of jointly sparse signals," *Asilomar Conference on Signals, Systems and Computers*, pp. 1537-1541, 2005.

[6] A. Gurbuz, J. McClellan, and V. Cevher, "A compressive beamforming method", ICASSP 2008, pp. 2617-2620, March 31 2008-April 4, 2008.

[7] L. C. Godara, "Application of antenna arrays to mobile communications, Part II: beam-forming and direction-of-arrival considerations", *Proceedings of the IEEE*, vol. 85, pp. 1195-1245, Aug. 1997.

[8] G. M. Kautz and M. D. Zoltowski, "Beamspace DOA estimation featuring multirate eigenvector processing", *IEEE Trans. on Signal Processing*, vol. 44, NO. 7, July 1996.

[9] C. La and M. Do, "Signal reconstruction using sparse tree representations," *SPIE Wavelets XI*, vol. 5914, pp. 59140W.1-59140W.11, Sept 2005.

[10] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, pp. 387-392, Apr. 1985.