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Least Squares Frequency Invariant Beamforming Robust Against Microphone Mismatches

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***Abstract*—The frequency invariant beamforming for microphone arrays is of great interest in practice for distortionless acoustic signal acquisition. Several design approaches for frequency invariant beamformers (FIBs) have been proposed in the literature. However, most of these approaches assume the ideal microphone characteristics, whose performance deteriorates greatly in the presence of microphone mismatches. To deal with this problem, a robust FIB design approach using least squares criterion is proposed in this paper, which takes the microphone statistical characteristics into consideration. Design examples are shown to demonstrate the effectiveness of the proposed design approach.**

I. Introduction

MICROPHONE arrays have found wide applications in hands-free telephony, teleconferencing, hearing aids, and speech recognition systems, etc. [1]. One of the important functions of microphone arrays is to acquire acoustic signals using broadband beamforming methods. Several broadband beamforming methods have been proposed in the literature [1]. Amongst them, the frequency invariant beamformers (FIBs), which can achieve a beampattern independent of frequency, is particularly interested [2]–[5]. As it is known, when the conventional delay-and-sum beamformer is used in practice other than the FIBs, the broadband acoustic signals acquired by microphone arrays will be low-pass filtered when the steering direction of the beamformer is different from the actual signal incident angle [1], which can lead to the distortion of the acoustic signal of interest. Therefore, FIBs have found important applications in distortionless acoustic signal acquisition [3] or direction-of-arrival estimation [6].

Recently, the FIB design approaches using the least squares criterion have been proposed [4], [5]. Compared with the conventional methods for FIB design [3], the least squares based method is easy to implement since it leads to a closed-form solution, moreover, it is applicable to arbitrary array geometries. It should be pointed out that the FIB design approaches using least squares assume the ideal microphone characteristics [4], [5]. As it is well known, there usually exist microphone mismatches in practice, i.e., microphone gain and phase errors [7]–[9]. Moreover, microphone array characteristics are usually not exactly available to the designers and can even change over time [7]. As shown in this paper, the least squares based FIB design approaches are highly sensitive to the errors in microphone mismatches. Therefore, efficient and robust FIB design for microphone arrays is practically required. Motivated by this fact, in this paper, we proposed a robust FIB design approach by taking into consideration of the statistical characteristics of microphones.

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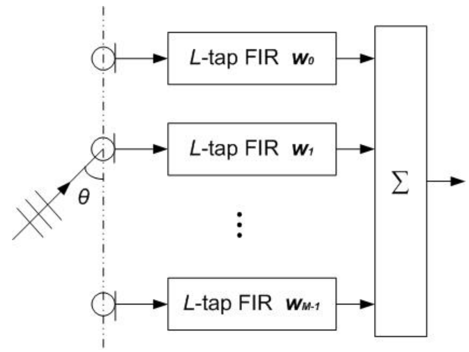

Fig. 1. Broadband beamformer configuration for microphone arrays.

This paper is organized as follows. The mathematical data model for broadband beamforming is presented in Section II. Section III gives a brief review of the nonrobust FIB design using least squares criterion. Then, our proposed robust FIB design approach is developed in Section IV. Design examples are presented in Section V to demonstrate the effectiveness of the proposed method. Finally, Section IV concludes the paper.

II. Mathematical Data Model

Consider a broadband linear array consisting of M microphones in the far-field, as shown in Fig. 1. Let an L-tap finite impulse response (FIR) filter $\mathbf{w}_m \in \mathbb{R}^{L\times 1}$ $(m = 0, 1, \cdots, M-1)$ be used behind each microphone. The microphone characteristics of the mth microphone can be represented as [7]

$$A_m(f,\theta) = a_m(f,\theta)e^{-j\gamma_m(f,\theta)}, \quad m = 0, \cdots, M-1 \quad (1)$$

where $a_m(f,\theta)$ and $\gamma_m(f,\theta)$ are the gain and phase response at frequency f and impinging angle θ, respectively. The array steering vector can be expressed as

$$\overline{\mathbf{g}}(f,\theta) = \mathbf{A}(f,\theta)\cdot\mathbf{g}(f,\theta) \quad (2)$$

where $\mathbf{A}(f,\theta)$ is an $ML\times ML$ diagonal matrix containing microphone characteristics, and $\mathbf{g}(f,\theta)$ is the ideal array steering vector. With the given array configuration, we have

$$\mathbf{A}(f,\theta) = \mathbf{I}_L \otimes \mathrm{diag}\left[A_0(f,\theta), A_1(f,\theta), \cdots, A_{M-1}(f,\theta)\right] \tag{3}$$

where $\mathbf{I}_L$ is the $L \times L$ identity matrix, $\otimes$ denotes the Kronecker product, $\mathrm{diag}[\cdot]$ represents the diagonal matrix, and

$$\mathbf{g}(f,\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi f/f_s} \\ \vdots \\ e^{-j2\pi(L-1)f/f_s} \end{bmatrix} \otimes \begin{bmatrix} e^{-j2\pi f\tau_0} \\ e^{-j2\pi f\tau_1} \\ \vdots \\ e^{-j2\pi f\tau_{M-1}} \end{bmatrix} \tag{4}$$

where f_s represents the sampling frequency, and $\tau_m = d_m \cos\theta/c$ $(m = 0, \cdots, M-1)$ are the time delays from the mth microphone to the reference point being chosen as the center of the microphone array, where d_m is the distance between the mth microphone and the center of the array, and c is the sound speed in air.

From (2), the beamformer response can be expressed as

$$P(f,\theta) = \mathbf{w}^T \overline{\mathbf{g}}(f,\theta) \tag{5}$$

where $(\cdot)^T$ represents transpose, and $\mathbf{w} = [\mathbf{w}_0^T, \mathbf{w}_1^T, \cdots, \mathbf{w}_{M-1}^T]^T \in \mathbb{R}^{ML\times 1}$ is the beamformer weight vector.

III. Nonrobust FIB Design Using Least Squares

To facilitate our new algorithm development, the nonrobust FIB design using least squares method [5] is briefly reviewed.

Following the idea of spatial response variation [10], the FIB design is based on the constrained least squares criterion [11], assuming the ideal microphone characteristics, i.e. $A_m(f,\theta) = 1$ in (1). The cost function is given by

$$\begin{aligned} J_{\mathrm{cls}}(\mathbf{w}) = & \sum_{n=0}^{N-1}\sum_{k=0}^{K-1} \left|\mathbf{w}^T\mathbf{g}(f_n,\theta_k) - \mathbf{w}^T\mathbf{g}(f_r,\theta_k)\right|^2 \\ & + \beta \sum_{\theta_k\in\Theta_s} \left|\mathbf{w}^T\mathbf{g}(f_r,\theta_k)\right|^2 \end{aligned} \tag{6}$$

where f_r is the predefined reference frequency of interest, N and K are the numbers of discretization points in the frequency and the angle of interest, repectively, and β is to control the trade-off of the frequency-invariant characteristics and the sidelobe level in sidelobe range Θ_s.

The nonrobust FIB design problem can be formulated as

$$\min_{\mathbf{w}} J_{\mathrm{cls}}(\mathbf{w}) \qquad \text{s.t. } \mathbf{w}^T\mathbf{g}(f_r,\theta_l) = 1 \tag{7}$$

where the constraint is to ensure that the beamfomer response at f_r in the look direction θ_l to be unity.

Considering the fact that a quadratic form of an antisymmetric matrix is always zero, (7) can be further recast as the following real-valued form

$$\min_{\mathbf{w}} \mathbf{w}^T \mathrm{Re}\{\mathbf{Q}_{\mathrm{cls}}\}\mathbf{w} \qquad \text{s.t. } \mathbf{C}^T\mathbf{w} = \mathbf{f} \tag{8}$$

where

$$\begin{aligned} \mathbf{Q}_{\mathrm{cls}} = & \sum_{n=0}^{N-1}\sum_{k=0}^{K-1} [\mathbf{g}(f_n,\theta_k) - \mathbf{g}(f_r,\theta_k)][\mathbf{g}(f_n,\theta_k) \\ & - \mathbf{g}(f_r,\theta_k)]^H + \beta \sum_{\theta_k\in\Theta_s} \mathbf{g}(f_r,\theta_l)\mathbf{g}(f_n,\theta_k)^H \end{aligned} \tag{9}$$

and $\mathbf{C} = [\mathrm{Re}\{\mathbf{g}(f_r,\theta_l)\}, \mathrm{Im}\{\mathbf{g}(f_r,\theta_l)\}]^T$ with $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denoting the real and imaginary parts, respectively, $(\cdot)^H$ represents the conjugate transpose, and $\mathbf{f} = [1, 0]^T$.

By using the Lagrange multiplier approach, we can obtain the closed-form solution to (9)

$$\mathbf{w} = (\mathrm{Re}\{\mathbf{Q}_{\mathrm{cls}}\})^{-1}\mathbf{C}\left(\mathbf{C}^T(\mathrm{Re}\{\mathbf{Q}_{\mathrm{cls}}\})^{-1}\mathbf{C}\right)^{-1}\mathbf{f}. \tag{10}$$

IV. Proposed Robust FIB Design

As demonstrated in Section V below, the above-mentioned FIB design approach will degrade in the presence of microphone mismatches. To deal with this problem, we hereby propose a robust FIB design approach by taking into consideration of microphone statistical characteristics [7]. To proceed, we define the joint probability density function (pdf) of the microphone gain a and phase γ as $f_{\mathcal{A}}(A) = f_{\alpha,\mathcal{G}}(a,\gamma)$, with $A = ae^{-j\gamma}$. Furthermore, the following assumptions are made

- A1) $f_{\mathcal{A}}(A)$ is independent of frequency and angle or available for different frequency-angle regions, such that the problem can be easily split up.
- A2) Without loss of generality, assume that all microphone characteristics A_m, $m = 0, \cdots, M-1$, have the same pdf $f_{\mathcal{A}}(A)$.
- A3) The microphone gain a and phase γ are independent, i.e., $f_{\mathcal{A}}(A) = f_\alpha(a)f_{\mathcal{G}}(\gamma)$, where $f_\alpha(a)$ is the pdf of the gain a, and $f_{\mathcal{G}}(\gamma)$ is the pdf of the phase γ.

Following the above assumptions, the proposed robust FIB design using constrained least squares can be formulated as

$$\min_{\mathbf{w}} J_{\mathrm{tot}}(\mathbf{w}) \qquad \text{s.t. } \mathbf{w}^T\overline{\mathbf{g}}_{\mathrm{m}}(f_r,\theta_l) = 1 \tag{11}$$

where the total constrained least squares cost function is

$$\begin{aligned} J_{\mathrm{tot}}(\mathbf{w}) = & \int_{A_0}\cdots\int_{A_{M-1}} \{\sum_{n=0}^{N-1}\sum_{k=0}^{K-1} \left|\mathbf{w}^T\overline{\mathbf{g}}(f_n,\theta_k)\right. \\ & \left. - \mathbf{w}^T\overline{\mathbf{g}}(f_r,\theta_k)\right|^2 + \beta \sum_{\theta_k\in\Theta_s} \left|\mathbf{w}^T\overline{\mathbf{g}}(f_r,\theta_k)\right|^2\} \\ & \times f_{\mathcal{A}}(A_0)\cdots f_{\mathcal{A}}(A_{M-1})\mathrm{d}A_0\cdots\mathrm{d}A_{M-1} \\ = & \mathbf{w}^T\mathbf{Q}_{\mathrm{tot}}\mathbf{w} \end{aligned} \tag{12}$$

with

$$\begin{aligned} \mathbf{Q}_{\mathrm{tot}} = & \int_{A_0}\cdots\int_{A_{M-1}} \{\sum_{n=0}^{N-1}\sum_{k=0}^{K-1} [\overline{\mathbf{g}}(f_n,\theta_k) - \overline{\mathbf{g}}(f_r,\theta_k)] \\ & \times[\overline{\mathbf{g}}(f_n,\theta_k) - \overline{\mathbf{g}}(f_r,\theta_k)]^H + \beta \sum_{\theta_k\in\Theta_s} \overline{\mathbf{g}}(f_r,\theta_l) \\ & \times \overline{\mathbf{g}}(f_r,\theta_l)^H\} f_{\mathcal{A}}(A_0)\cdots f_{\mathcal{A}}(A_{M-1})\mathrm{d}A_0\cdots\mathrm{d}A_{M-1} \end{aligned} \tag{13}$$

and the mean steering vector $\overline{\mathbf{g}}_{\mathrm{m}}(f_r,\theta_l)$ is given by

$$\begin{aligned} \overline{\mathbf{g}}_{\mathrm{m}}(f_r,\theta_l) = & \int_{A_0}\cdots\int_{A_{M-1}} \overline{\mathbf{g}}(f_r,\theta_l) f_{\mathcal{A}}(A_0)\cdots \\ & \times f_{\mathcal{A}}(A_{M-1})\mathrm{d}A_0\cdots\mathrm{d}A_{M-1} \\ = & \left[\mu_a\mu_\gamma^c\mathbf{g}_{\mathrm{R}}(f_r,\theta_l) + \mu_a\mu_\gamma^s\mathbf{g}_{\mathrm{I}}(f_r,\theta_l)\right] \\ & + j\left[\mu_a\mu_\gamma^c\mathbf{g}_{\mathrm{I}}(f_r,\theta_l) - \mu_a\mu_\gamma^s\mathbf{g}_{\mathrm{R}}(f_r,\theta_l)\right] \end{aligned} \tag{14}$$

where $\mathbf{g}_\mathrm{R}(f_r, \theta_l)$ and $\mathbf{g}_\mathrm{I}(f_r, \theta_l)$ denote the real and imaginary parts of $\mathbf{g}(f_r, \theta_l)$, respectively, and

$$\mu_a = \int_a a f_\alpha(a)\mathrm{d}a \tag{15}$$

$$\mu_\gamma^c = \int_\gamma \cos\gamma f_\mathcal{G}(\gamma)\mathrm{d}\gamma \tag{16}$$

$$\mu_\gamma^s = \int_\gamma \sin\gamma f_\mathcal{G}(\gamma)\mathrm{d}\gamma. \tag{17}$$

To facilitate the robust FIB design, we further evaluate $\mathbf{Q}_\mathrm{tot}$ in (13). Substituting (2) and (9) into (13), we have

$$\begin{aligned}\mathbf{Q}_\mathrm{tot} &= \int_{A_0}\cdots\int_{A_{M-1}} \mathbf{A}\mathbf{Q}_\mathrm{cls}\mathbf{A}^H f_\mathcal{A}(A_0)\cdots \\ &\quad \times f_\mathcal{A}(A_{M-1})\mathrm{d}A_0\cdots\mathrm{d}A_{M-1}.\end{aligned} \tag{18}$$

From (18), we have the property: for the (m,n)-th elements of $\mathbf{Q}_\mathrm{tot}$ and $\mathbf{Q}_\mathrm{cls}$, i.e., $\mathbf{Q}_\mathrm{tot}^{(m,n)}$ and $\mathbf{Q}_\mathrm{cls}^{(m,n)}$, it holds that

$$\mathbf{Q}_\mathrm{tot}^{(m,n)} = \sigma_a^2\mathbf{Q}_\mathrm{cls}^{(m,n)}, \qquad \text{if } m = n \tag{19}$$

where $\sigma_a^2 = \int_a a^2 f_\alpha(a)\mathrm{d}a$, and

$$\mathbf{Q}_\mathrm{tot}^{(m,n)} = \mu_a^2\sigma_\gamma^c\mathbf{Q}_\mathrm{cls}^{(m,n)}, \qquad \text{if } m \neq n \tag{20}$$

where $\sigma_\gamma^c = (\mu_\gamma^c)^2 + (\mu_\gamma^s)^2$.

Combining (19) and (20), we have

$$\begin{aligned}\mathbf{Q}_\mathrm{tot} &= \left\{\left[\mathrm{diag}\left((\sigma_a^2 - \mu_a^2\sigma_\gamma^c)\mathbf{1}_{M\times 1}\right) + (\mu_a^2\sigma_\gamma^c)\mathbf{1}_{M\times M}\right]\right. \\ &\quad \left.\otimes \mathbf{1}_{L\times L}\right\} \odot \mathbf{Q}_\mathrm{cls},\end{aligned} \tag{21}$$

where $\mathbf{1}_{m\times n}$ is $m \times n$ matrix with all elements equal to 1.

Considering that the imaginary part of $\mathbf{Q}_\mathrm{tot}$ is antisymmetric, the robust FIB design problem (11) now becomes

$$\min_{\mathbf{w}} \mathbf{w}^T\mathrm{Re}\{\mathbf{Q}_\mathrm{tot}\}\mathbf{w} \qquad \text{s.t. } \mathbf{C}_\mathrm{m}^T\mathbf{w} = \mathbf{f} \tag{22}$$

where

$$\mathbf{C}_\mathrm{m} = \begin{bmatrix} \mu_a\mu_\gamma^c\mathbf{g}_\mathrm{R}(f_r,\theta_l) + \mu_a\mu_\gamma^s\mathbf{g}_\mathrm{I}(f_r,\theta_l) \\ \mu_a\mu_\gamma^c\mathbf{g}_\mathrm{I}(f_r,\theta_l) - \mu_a\mu_\gamma^s\mathbf{g}_\mathrm{R}(f_r,\theta_l) \end{bmatrix}^T. \tag{23}$$

Solving (22), one can obtain the robust FIB weight vector

$$\mathbf{w} = (\mathrm{Re}\{\mathbf{Q}_\mathrm{tot}\})^{-1}\mathbf{C}_\mathrm{m}\left(\mathbf{C}_\mathrm{m}^T(\mathrm{Re}\{\mathbf{Q}_\mathrm{tot}\})^{-1}\mathbf{C}_\mathrm{m}\right)^{-1}\mathbf{f}. \tag{24}$$

V. Design Examples

In this section, design examples are presented to illustrate the performance of the proposed robust FIB design. A 12-element uniform linear microphone array with inter-element spacing 4.25 cm is used. Behind each microphone, an FIR filter with $L = 25$ taps is used. The sample frequency is set to $f_s = 8$ kHz. The frequency band of interest is $[2000, 3600]$Hz, where the reference frequency is set to $f_r = 2800$ Hz. The sidelobe range is defined as $\Theta_s = [0°, 75°] \cup [105°, 180°]$, and the broadside angle is chosen as the look direction angle, i.e., $\theta_l = 90°$. The trade-off parameter β is set to 0.5.

As a comparison, we first consider the performance of the nonrobust FIB design for ideal microphones, i.e., $a_m(f,\theta) = 1$ and $\gamma_m(f,\theta) = 0$. Fig. 2 shows the beampattern by using the nonrobust FIB design approach. It can be seen that the nonrobust FIB design approach can achieve perfect frequency invariant beampattern within the frequency band of interest for ideal microphones.

Next, we consider the beamformer design in the presence of microphone mismatches. Suppose the microphone gains $a_m(f,\theta)$ have a uniform distribution in $[0.9, 1.1]$, and the phases $\gamma_m(f,\theta)$ have a uniform distribution in $[-10°, 10°]$. The beampatterns of the nonrobust design and the proposed robust design are shown in Figs. 3 and 4, respectively. Each figure is an average of 100 simulation runs. As can be seen from Fig. 3, the nonrobust FIB design approach degrades greatly in the presence of microphone mismatches, especially in the lower frequency band. As shown in Fig. 4, in comparison, our proposed FIB design is robust against microphone mismatches and generates the satisfactory result.

VI. Conclusion

A least squares FIB design approach robust against microphone mismatches has been proposed in this paper, which takes into consideration of the statistical characteristics of microphones. The proposed approach leads to a closed-form solution and thus is computationally efficient. Moreover, it can be applicable to arbitrary microphone arrays. The effectiveness of the proposed approach is evaluated by the design examples.

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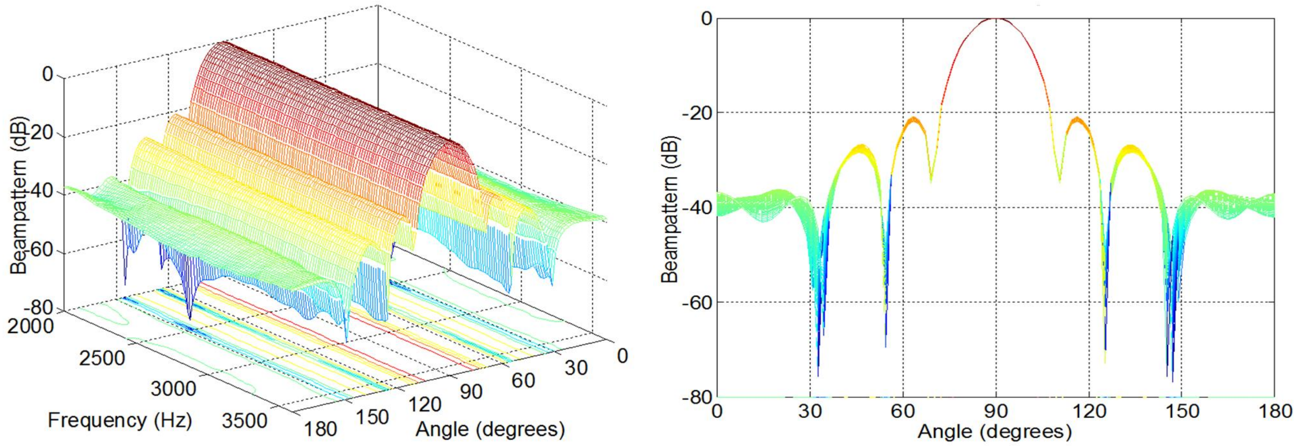
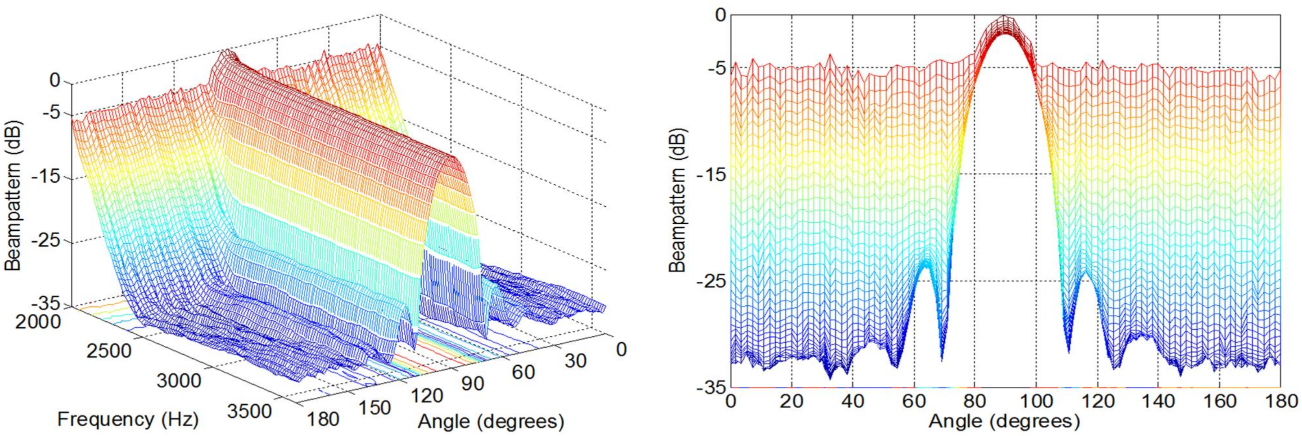


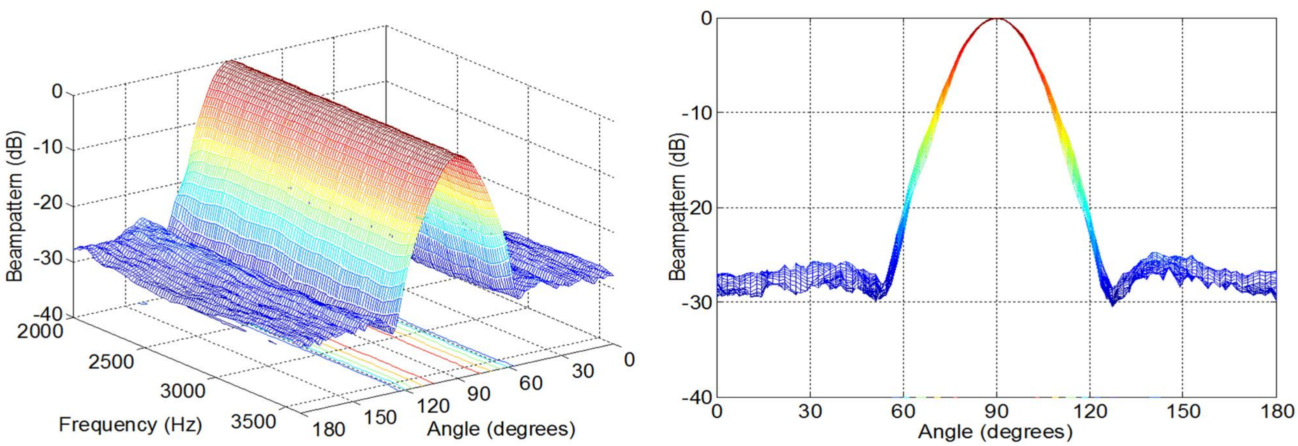
(a) (b)

Fig. 2. Beampattern of the nonrobust FIB design with the ideal microphone characteristics. (a) 3-D view. (b) Side view.



(a) (b)

Fig. 3. Beampattern of the nonrobust FIB design in the presence of microphone mismatches. Average of 100 Monte Carlo runs. Microphone gains $a_m(f,\theta)$ have a uniform distribution in $[0.9, 1.1]$, and the phases $\gamma_m(f,\theta)$ have a uniform distribution in $[-10^\circ, 10^\circ]$. (a) 3-D view. (b) Side view.



(a) (b)

Fig. 4. Beampattern of the proposed robust FIB design in the presence of microphone mismatches. Average of 100 Monte Carlo runs. Microphone gains $a_m(f,\theta)$ have a uniform distribution in $[0.9, 1.1]$, and the phases $\gamma_m(f,\theta)$ have a uniform distribution in $[-10^\circ, 10^\circ]$. (a) 3-D view. (b) Side view.