Compressive Sensing for Sparse Arrays

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***Abstract*—We assess the usefulness of compressive sensing (CS) for processing signals from sparse or highly thinned receive arrays. Compressive sensing consists of a theory and related algorithms for solving under determined linear equations with the assumption that solutions have only a few non-zero or dominant values and the remaining values are zero or small. Sparse arrays are a natural application of CS when (a) there are only a few array observations with each expressed as a linear combination of the received plane wave signal amplitudes and (b) there are a large number of possible angles of arrival of signals but only a few signals are actually present. Our approach employs linear beamforming of the sparse array signals to create data for compressive sensing that are a linear combination of the plane wave signal amplitudes. The beamforming matrix serves to create coefficients in this linear combination that nearly orthonormal, a requirement for successful application of CS. We evaluate CS for the problem of a five element sparse array located on a 90^o sector of a circular arc with large radius of curvature. Elements of the sparse array are themselves digitally beamformed phased arrays with main beams steered to a common direction. We found that compressive sensing works well for large signal to noise, reliably indicating the direction of arrival of a plane wave within the beamwidth of the phased array element and potentially reliable resolution of two equal intensity plane waves within one tenth of the beamwidth of the sparse array. False plane wave indications occur with significant probability when the signal to noise is small. However, an n or more of m indication algorithm with m statistically independent noise samples reduces significantly the probability of false indication.

I. Introduction

We investigate compressive sensing (CS) as an approach to processing signals from a sparse, receive array with the objective of assessing CS capabilities for enhanced estimates of signal strength and angle of arrival of received plane wave signals. Compressive sensing is an approach for solving a set of under determined, linear equations with the assumption that solutions have only a few non-zero or dominant values when the remaining values are zero or small. Specialized algorithms for the solution to the under determined equations are structured to emphasize the few dominant values of the solution [1]. The processing of signals from sparse, receive arrays seems to be a natural application of CS when (a) there are only a few elements in the array providing only a few observations expressed as a linear combination of the received plane wave signal amplitudes and (b) there are a large number of possible angles of arrival of signals but only a few signals are actually present. The application to sparse arrays is important since the larger aperture of the sparse array provides superior angular resolution and better estimates of the angle of arrival of received plane wave signals. The generally poor sidelobe structure of the highly thinned arrays may be compensated by the application of compressive sensing.

II. Formulation

For purposes of this analysis, we consider a collection of N_a planar, receive apertures located randomly and tangent to a large curved surface as shown in Figure 1. While the receive apertures are randomly located, it is assumed that their positions are surveyed and are accurately known after positioning.

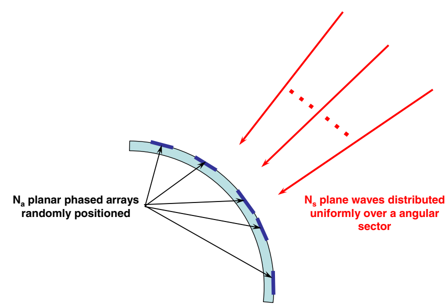

Fig. 1. N_a planar phased arrays located randomly on a circular arc excited by a spectrum of N_s plane waves distributed uniformly in an angular sector.

Each of the receive apertures is a digitally beamformed array with the ability to scan the main beam of the array pattern over a large angular sector within the field of view of the array. Each of the digitally beamformed arrays is assumed to be scanned to a common direction. The system is linear so that the received signal at each aperture is a linear combination of the signal amplitudes from each of the external plane wave excitations. As indicated in Figure 1, there are N_s possible angles of arrival of plane waves. We can write the signals at the outputs of the receive apertures as

$$\mathbf{e_a} = \mathbf{\Gamma}\, \mathbf{s},$$

where the N_s x 1 vector, $\mathbf{s}$, represents the plane wave complex amplitudes from each angle of arrival, s_n, $n = 1, 2, \ldots, N_s$, the N_a x 1 vector, $\mathbf{e_a}$, represents the complex valued receive signals at the aperture outputs, and the N_a x N_s sensor matrix, $\mathbf{\Gamma}$, with elements $\gamma_{i,k}$ represents the response at receive aperture i due to plane wave k. Here

$$\gamma_{i,k} = p_i(\theta_k - \theta_i)e^{jkr_i(\theta_k)},$$

where p_i is the complex valued pattern of the i^{th}aperture for a plane wave arriving at angle θ_k when the aperture is located at an angle θ_i and r_i is the distance from a common origin to the i^{th} aperture phase center along a path from the k^{th} plane wave excitation.

An objective of this work is to design a beamforming matrix, $\mathbf{B}$, to estimate the complex amplitudes of the incident

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plane waves, $\hat{\mathbf{s}}$, as a linear combination of the N_a receive signals. That is, we require the N_s x N_a matrix $\mathbf{B}$ so that

$$\hat{\mathbf{s}} = \mathbf{B}\,\mathbf{\Gamma}\,\mathbf{s} = \mathbf{D}\,\mathbf{s} = \sum_{n=1}^{N_s} \mathbf{d_n} s_n, \tag{1}$$

where the N_s x 1 vectors, $\mathbf{d_n}$, are the columns of the matrix $\mathbf{D} = \mathbf{B}\,\mathbf{\Gamma}$. Compressive sensing requires that these vectors be as nearly orthogonal as possible, given, in our case, the constraints implicit in the sensor matrix, $\mathbf{\Gamma}$. This orthogonality requirement demands that $\mathbf{D^H D} \cong \mathbf{I}$, or as nearly diagonal as possible. Here $\mathbf{D^H}$ denotes the Hermitian or conjugate transpose of $\mathbf{D}$. Abolghasemi et. al. [2] describe an iterative, numerical approach for developing the matrix $\mathbf{D}$.

The beamforming matrix can be found from the solution to

$$\tilde{\mathbf{D}} = \mathbf{B}\,\mathbf{\Gamma}\,, \tag{2}$$

with $\tilde{\mathbf{D}}$ denoting the matrix with near orthogonal column vectors $\tilde{\mathbf{d}}_\mathbf{n}$. Equation (2) cannot be solved for the beamforming matrix directly since, in general, the sensor matrix is rank deficient and does not have an inverse. The pseudo inverse of $\mathbf{\Gamma}$ could be used in Equation (2) but is instructive to use instead the singular value decomposition of the sensor matrix with rank r, $\mathbf{\Gamma} = \mathbf{U}\,\mathbf{S}\,\mathbf{V^H}$. Here $\mathbf{U}$ represents the N_a x r matrix with columns representing the left singular vectors of $\mathbf{\Gamma}$, and $\mathbf{V}$ represents the N_s x r matrix with columns representing the right singular vectors of $\mathbf{\Gamma}$. The r x r diagonal matrix $\mathbf{S}$ has the non-zero singular values of $\mathbf{\Gamma}$ in decreasing value as its diagonal elements. The beamforming matrix is then the solution to the simultaneous equations

$$\mathbf{B} = \tilde{\mathbf{D}}\mathbf{V}\mathbf{S^{-1}}\mathbf{U^H}.$$

III. Analysis

We have developed the beamforming matrix and investigated the orthogonality of the $\tilde{\mathbf{d}}_\mathbf{n}$, which are the column vectors of $\tilde{\mathbf{D}}$, and the response of the beamforming process for several cases. We assume that $N_a = 5$ sparse array elements are located at random but known positions on one quarter of a circular arc of radius $\rho = 500\lambda$, where λ is the wavelength of the received signal. Each element array is itself a fully filled phased array which is 32λ in size and located in the common plane of the arriving signals. Each element array is digitally beamformed with patterns that are either maximum gain or uniformly low sidelobes. We use the terminology “element array” to denote that each “element” of the sparse array is itself a phased array. The main beam of each element array is pointed in the common angular direction assumed for the received plane wave if that direction is within the field of view of the element array. For each element array configuration, we determine the distributions of correlations between the $\tilde{\mathbf{d}}_\mathbf{n}$ as a measure of plane wave detectability and potential ambiguity in determining the angle of arrival of a plane wave. Since each $\tilde{\mathbf{d}}_\mathbf{n}$ is associated with one of N_s possible angles of arrival, a highly auto correlated $\tilde{\mathbf{d}}_\mathbf{n}$ with small cross correlation to other $\tilde{\mathbf{d}}_\mathbf{m}, m \neq n$, would indicate the presence of a plane wave at a specific angle of arrival with amplitude well determined from Equation (1) using this strong orthogonality and the CS algorithms. Figure 2 illustrates the distribution of correlations, $\left|\tilde{\mathbf{d}}_\mathbf{n}^\mathbf{H}\tilde{\mathbf{d}}_\mathbf{m}\right|$, between the response vectors of $\tilde{\mathbf{D}}$ for the case when element arrays with maximum gain are employed. Self or auto correlations occur when $n = m$ and are denoted by solid blue triangles while cross correlations occur when $n \neq m$ and are denoted by open red triangles. The correlation index denotes each of the possible correlation pairs, ranging from 1 to $N_s(N_s+1)/2$ for $N_s = 21$ possible plane wave directions distributed uniformly over an 8^o sector. We observe in this Figure that the first five plane wave samples within the broadside -3dB broadside beamwidth of an element array are highly correlated while the cross correlation between adjacent samples is 7 dB to 9 dB smaller. Even plane wave samples at the -8 dB point on the beamwidth are correlated with the cross correlation to adjacent samples being 6 dB to 7 dB smaller.

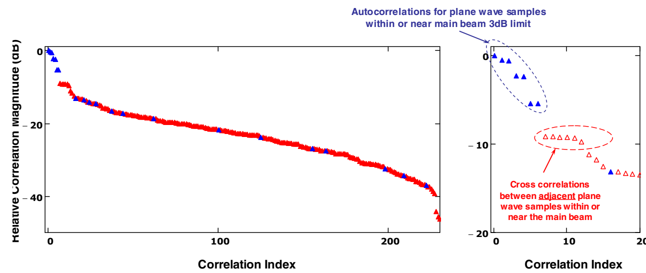

Fig. 2. Distribution of signal amplitude correlations with five element sparse array with each element array being a 32λ maximum gain array; Solid, blue triangles denote auto or self correlations and open, red triangles denote cross correlations.

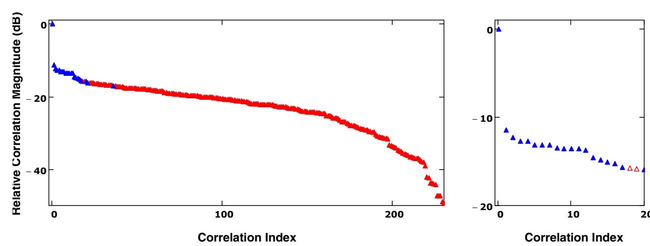

Fig. 3. Distribution of signal amplitude correlations with five element sparse array with each element array having -30 dB uniform sidelobe patterns; Solid, blue triangles denote auto or self correlations and open, red triangles denote cross correlations.

To indicate the theoretical potential for this process, it is interesting to compare these results with those of Figure 3 for an idealized, not necessarily realizable element array with a small beamwidth containing only one plane wave sample and sidelobes at the -30 dB level for all other samples. Only the single, main beam sample is highly correlated with cross correlation to an adjacent sample being nearly 15 dB smaller.

A. Compressive Sensing Algorithm

In order to assess the usefulness of CS for sparse array signal detection and estimation, we have implemented a CS algorithm proposed by Çetin [3] and used by Varshney [4] for synthetic aperture processing. In general, compressive sensing requires that we minimize with respect to $\mathbf{s}$

$$\min_{\mathbf{s}}\ J(\mathbf{s}) = \left\|\mathbf{r} - \tilde{\mathbf{D}}\mathbf{s}\right\|_2^2 + \alpha \sum_{n=1}^{N_s} |s_n|^p\,, \tag{3}$$

when $p < 1$ and $\|x\|_2^2 \equiv \sum_{n=1}^{N_s} |x_n|^2$. The regularization term, $\alpha \sum_{n=1}^{N_s} |s_n|^p$, serves to reward large elements and penalize small elements of the solution in the minimization process. In our case, $\mathbf{r} = \mathbf{B}\,\mathbf{\Gamma_e}\,\mathbf{s_o} + \mathbf{Bn}$ where $\mathbf{s_o}$ denotes the true signal vector to be determined, $\mathbf{\Gamma_e}$ represents the actual sensor matrix in contrast to the sensor matrix model used to determine the beamforming maxtrix, and $\mathbf{n}$ represents the N_a x 1 vector of complex, additive receiver noise at each element array output. Each entry of $\mathbf{n}$ is an independent, complex Gaussian random variant.

A contribution of Çetin is to introduce a small constant, $\varepsilon \geq 0$ into the regularization term in (3) to give

$$\min_s \; J_\varepsilon(\mathbf{s}) = \left\|\mathbf{r} - \tilde{\mathbf{D}}\mathbf{s}\right\|_2^2 + \alpha \sum_{n=1}^{N_s} (|s_n|^2 + \varepsilon)^{\frac{p}{2}}. \qquad (4)$$

Unlike the objective function in Equation (3), this objective function is differentiable and thus supports iterative, numerical minimization with quasi-Newton approaches using complex data. The iterative solution is [3, pp 73 - 76]

$$\mathbf{s}^{n+1} = \mathbf{s}^n - \gamma \mathbf{H}(\mathbf{s}^n)^{-1} \nabla_s J_\varepsilon(\mathbf{s}^n), \qquad (5)$$

where γ denotes the iterative step size between estimates of the signal vector, $\mathbf{s}^{n+1}$, at stage $n+1$ and the signal vector $\mathbf{s}^n$ at stage n . Here

$$\mathbf{H}(\mathbf{s}) = 2\tilde{\mathbf{D}}^H\tilde{\mathbf{D}} + p\alpha\mathbf{\Lambda}(\mathbf{s}),$$

and

$$\mathbf{\Lambda}(\mathbf{s}) = \mathrm{diag}\left\{(|s_n|^2 + \varepsilon)^{\frac{p}{2}-1}\right\},$$

giving for the gradient

$$\nabla_s J_\varepsilon(\mathbf{s}) = \mathbf{H}(\mathbf{s})\mathbf{s} - 2\tilde{\mathbf{D}}^H\mathbf{r}. \qquad (6)$$

Substituting this expression for the gradient into Equation (5) gives the iterative solution

$$\mathbf{H}(\mathbf{s}^n)\mathbf{s}^{n+1} = (1-\gamma)\mathbf{H}(\mathbf{s}^n)\mathbf{s}^n + 2\gamma\tilde{\mathbf{D}}^H\mathbf{r}, \qquad (7)$$

which can be solved at each iteration for $\mathbf{s}^{n+1}$. Iterations continue until incremental changes in the solution vector are small.

B. *Performance*

This iterative approach has been applied to the sparse array model described above. Figures 4 and 5 illustrate results of the CS minimization for large signal to noise. Figure 4 shows the response of the Çetin compressive sensing algorithm when a single plane wave excites the sparse array. Initially (symbol ▲), the algorithm shows no sensitivity to the plane wave but after 227 iterations with $\gamma = .1$ the solution (symbol ▲) clearly and accurately indicates the presence of the plane wave but with a bias in the unit amplitude of the plane wave. The algorithm converges monotonically and achieves nearly the final solution state after approximately 50 iterations. The residual level of the converged solution (≈-70 dB) is determined largely by the selection of the additive constant ε discussed above and the signal to noise.

Figure 5 shows the convergence of the compressive sensing algorithm when two plane waves excite the sparse array. The plane waves are separated by 0.1^o, which is 0.056 of the beamwidth of the element array.

Figure 6 shows the response when the signal to noise at the element array is degraded. In this work, the signal to noise is defined at the output of an element array with the assumption that the signal is coherently received at the peak of the main beam of a maximum gain array. In the top illustration, the algorithm converges to the correct plane wave direction but includes a false indication at approximately 47.5^o when the element array signal to noise is 6 dB. Decreasing the signal to noise to -4 dB results in no detection of the single plane wave and two false indications. The residual background level has increased to approximately -50 dB due now to the increased noise level in the element array observations.

We investigated the probability of correct plane wave indication and the probability of false indications as a function

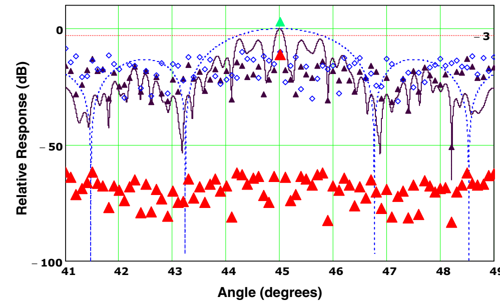

Fig. 4. Response of Çetin compressive sensing algorithm applied to a sparse array with a single plane wave excitation. Sparse array has five elements, each of which is a 64 element phased array with main beam pointed at 45^o. $N_s = 81$ possible plane waves are assumed uniformly distributed over an 8^o sector and the signal to noise at the element array is assumed to be 66 dB. The signal to noise is observed at the output of each array assuming that the incident signal experiences maximum array gain. The algorithm step size, γ, was taken to be 0.1.
▲ Final response of algorithm
▲ Initial state of solution
▲ Angular location of plane wave
◇ Response with pseudo inverse of D
- - - - Element array pattern
——— Maximum gain sparse array pattern .

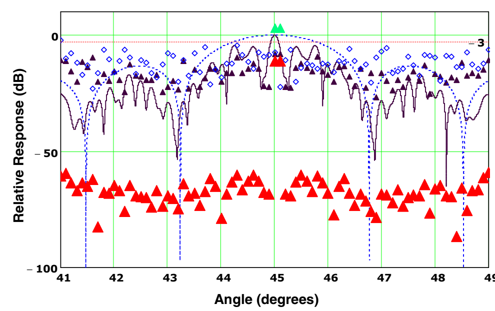

Fig. 5. Response of Çetin compressive sensing algorithm applied to a sparse array with a two plane waves exciting the array. The plane waves are separated by 0.1^o which is 0.056 of the beamwidth of the element array. All other conditions are the same as those of Figure 4.

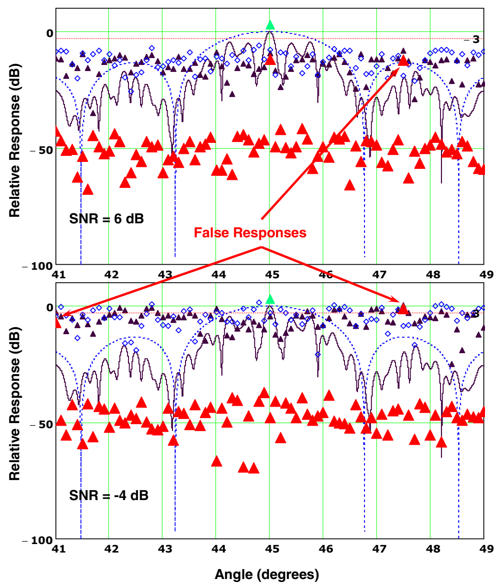

Fig. 6. Response of Çetin compressive sensing algorithm applied to a sparse array with a single plane wave and degraded signal to noise. The top illustrates the response with 6 dB signal to noise with a false alarm encountered at 47.5^o. The bottom illustrates the response with -4 dB signal to noise with two false alarms and missed detection of the single plane wave. All other conditions are the same as those of Figure 4.

of signal to noise and position of the plane wave with respect to the pointing direction of the sparse array. Figure 7 shows simulation results as a function of signal to noise for the conditions given in Figure 4. The blue curve denotes correct indication of the plane wave exciting the sparse array and the red curve denotes estimates of the probability of one or more false indications among the remaining 80 possible plane wave directions. We have provided 100 simulation trials, each with independently selected positions of the five planar arrays and independently selected complex noise samples at each array output. The bands denote 0.95 confidence intervals for each simulation sample.

These simulation results indicate that for large signal to noise (greater than 20-25 dB as defined above) there is virtually certain indication of a correct plane wave direction with a very small probability of false indication in other directions. In contrast, for the noise dominant environment (small signal to noise) there is virtual certainty that false indications will occur. In fact, the probability of indication in the direction of the exciting plane wave is approximately 0.2 which, we hypothesize, is due to noise excitation rather than the presence of the plane wave. Indeed, if the 80 indications away from the plane wave excitation direction are due to noise and are statistically independent, then the probability of one or more indications can be given from the binomial distribution as 1 - probability of zero indications = $1 - 0.2^0(1-0.2)^{80} \cong 1$, which is consistent with the red curve in Figure 7 for small signal to noise.

Figure 8 shows simulation results as the direction of the plane wave excitation deviates from broadside for large signal to noise (SNR = 46 dB). The plane wave is indicated correctly so long as the plane wave is within the broadside beamwidth

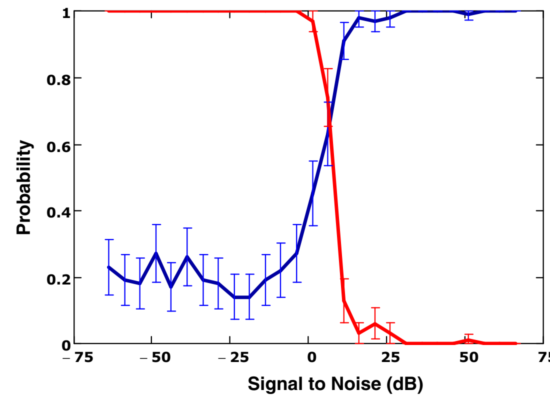

Fig. 7. Simulation results for probability of correct plane wave indication (—-) and probability of one or more incorrect indications (——-) as a function of signal to noise. Vertical bars denote 0.95 confidence intervals.

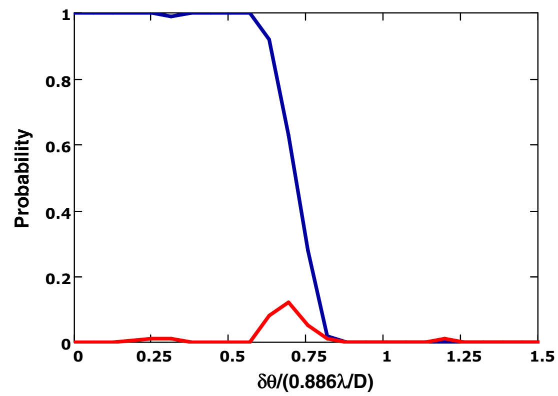

Fig. 8. Simulation results for probability of correct plane wave indication (—-) and probability of one or more incorrect indications (——-) as a function of the deviation of the angle of arrival from broadside of the plane wave, $\delta\theta$, expressed as a fraction of the broadside element array beamwidth, $0.886\lambda/D$.

of the element array, given by $0.886\lambda/D$ where D is the dimension of the element array. Because of the large signal to noise, the probability of one or more incorrect indications is small.

Returning to the results of Figure 7, we observed that in the noise dominant case there appears to be an approximate probability of 0.2 of a false indication in any plane wave direction due to the noise. This suggests that the probability of multiple indications from any plane wave direction resulting from repeated trials of the CS algorithm with statistically independent noise samples might be small. That is, an algorithm requiring n or more indications in $m \geq n$ trials would be useful to reduce false indications. Figure 9 shows the results of an investigation of this n or more of m algorithm.

For this investigation, a simulation with 100 trials was performed. Each trial consisted of a single distribution of five planar arrays independently selected and placed on one quarter of a circular arc as described previously. For each array distribution, four independently selected complex noise samples were added to the plane wave signal at each array output and the compressive sensing algorithm applied to each of the four signal and noise configurations. From the results, we found estimates of (a) the probability of two or more, three or more or four indications of a plane wave in the desired direction and (b) the probability of one or more false indications in the other 80 directions, where each false indication results from two or more, three or more or four compressive sensing indications in a given plane wave direction.

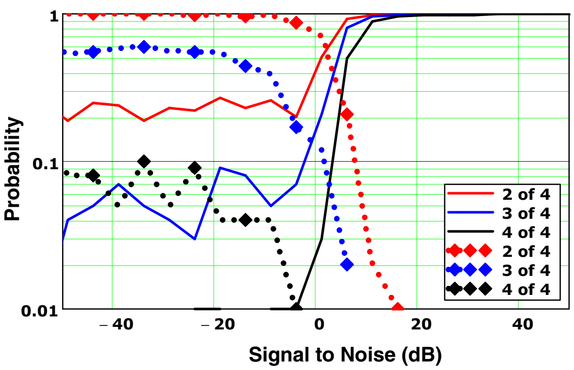

Fig. 9. Simulation results for probability of correct plane wave indication (—— (2 or more of 4);—— (3 or more of 4);—— (4 of 4)) and probability of one or more incorrect indications (-◆--◆- (2 or more of 4);-◆--◆- (3 or more of 4);-◆--◆- (4 of 4)) as a function of signal to noise.

Results of this simulation are given in Figure 9. The solid curves (—— (2 or more of 4);—— (3 or more of 4);—— (4 of 4)) give estimates of the probability of correct plane wave indication when n = 2, 3, 4 CS algorithm indications occur in $m = 4$ trials. The dashed curves (-◆--◆- (2 or more of 4);-◆--◆- (3 or more of 4);-◆--◆- (4 of 4)) give estimates of the probability of one or more false plane wave indications when n = 2, 3, 4 simultaneous CS algorithm indications occur in the remaining 80 possible plane wave positions. The probability of one or more incorrect indications in the 80 directions where no plane wave exists and noise is dominant is reduced substantially using the n or more of m algorithm. The probability is near one when two of four CS indications are present and is reduced to less than 0.1 when four of four CS indications are required.

The probability of correct plane wave indication at small signal to noise is reduced to below 0.01 by requiring four of four CS indications (—— (4 of 4)). This appears consistent with the binomial distribution where the probability of four CS indications in four trials is $0.2^4 = 1.6 \cdot 10^{-3}$ when the probability of each CS indication is 0.2 as observed in Figure 7. However, this requires an additional 5 dB signal to noise at the probability of plane wave indication of 0.5.

Finally, we consider the resolution of the sparse array with CS processing as an approach for separating two closely spaced plane wave excitations of the same amplitude. Figure 10 shows the results from 100 simulations of the CS algorithm with independently selected random positions for the five elements of the sparse array and $m = 4$ independently selected sets of additive noise at the sparse array elements. The angular separation between the plane waves is taken to be 0.1 of the effective of the beamwidth of the sparse array, given by $0.886\lambda/D_{sa}$ where the effective aperture of the sparse array is $D_{sa} = \sqrt{2}\rho$ with ρ giving the radius of the circular arc supporting the sparse array. The n or more of m algorithm was applied to designating each plane wave with $n = 2, 3$ and 4. The top of Figure 10 shows estimates of the probability as a function of signal to noise as defined previously that (a) two plane waves are observed (—— (4 of 4); – – – (3 or more of 4); • • • • (2 or more of 4)) and (b) only one plane wave is observed (—— (4 of 4); – – – (3 or more of 4); • • • • (2 or more of 4)). The bottom of Figure 10 provides estimates that the CS algorithm falsely designates a plane wave at any one of the remaining 79 possible plane wave positions that are not excited by a source (—— (4 of 4); – – – (3 or more of 4); • • • • (2 or more of 4)).

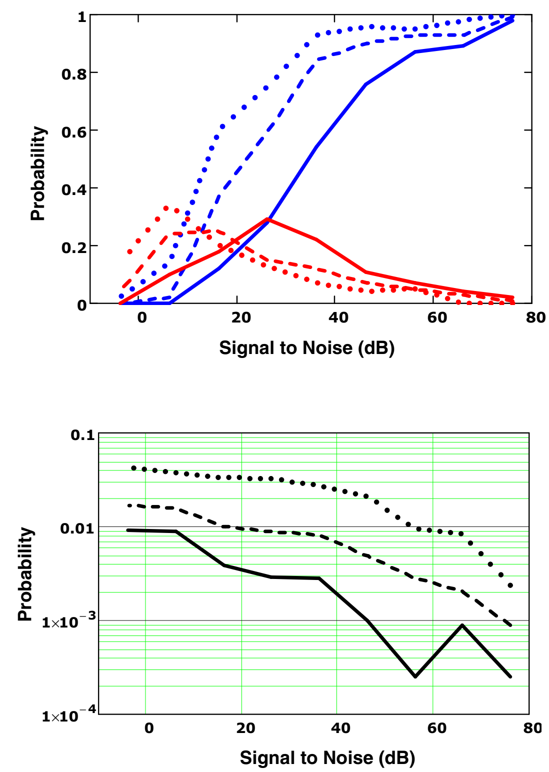

Fig. 10. The probability as a function of signal to noise that (top illustration) two plane waves are observed (—— (4 of 4); – – – (3 or more of 4); • • • • (2 or more of 4)) and only one plane wave is observed (—— (4 of 4); – – – (3 or more of 4); • • • • (2 or more of 4)) and (bottom illustration) false estimates of a plane wave are designated at any one of the remaining 79 possible plane wave positions that are not excited by a source (—— (4 of 4); – – – (3 or more of 4); • • • • (2 or more of 4)).

It is clear that the CS algorithm applied to the sparse array with large signal to noise successfully designates both plane waves separated in angle by one tenth of the beamwidth of the sparse array. However, performance degrades rapidly at moderate signal to noise, requiring an additional 10-30 dB signal to noise for indication of two plane waves over that required for a single plane wave (Figure 9). This performance degradation is not explained by the assumption of two independent plane wave indications with the probability of an indication of each plane wave given by the estimates of Figure 9.

IV. CONCLUSIONS

We investigated compressive sensing for processing signals from sparse or highly thinned receive arrays. Sparse arrays are a natural application of CS when (a) there are only a few array observations expressed as a linear combination of the received plane wave signal amplitudes and (b) there are a large number of possible angles of arrival of signals but only a few signals are actually present. The approach described employs linear beamforming of the sparse array signals to create data for compressive sensing that are a linear combination of the plane wave signal amplitudes. The beamforming matrix is designed to create coefficients in this linear combination that are approximately orthonormal, a requirement for successful application of CS.

We evaluated CS for the problem of a five element sparse array located on a 90^o sector of a circular arc with large radius of curvature when compared to the wavelength of the incident radiation. Each element of the sparse array is itself a digitally beamformed phased array with its beam steered to a common direction. Signals from beamformed array are linearly combined by the beamforming matrix to provide data for the compressive sensing algorithm. The algorithm used here was proposed and evaluated by Çetin [3].

We found that compressive sensing works well when the signal at each beamformed array is large when compared to receiver noise at the array output. There was reliable indication of the direction of arrival of a plane wave within the beamwidth of the phased array element and potentially reliable resolution of two plane waves within one tenth of the beamwidth of the sparse array for large signal to noise. False plane wave indications occur with significant probability when the signal to noise is small. In this case, an n or more of m detection algorithm with m statistically independent noise samples at the array outputs reduces significantly the probability of false indication.

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