

# Recursive Steering Vector Estimation and Adaptive Beamforming under Uncertainties

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**Abstract**—The accurate determination of steering vector of a sensor array corresponding to a desired signal is often hindered by uncertainties due to array imperfections, such as the presence of direction-of-arrival (DOA) estimation error, mutual coupling, array sensor gain/phase uncertainties and sensor position perturbations. Consequently, the performance of conventional beamforming algorithms using the nominal steering vector may be significantly degraded. In this paper, a new method for recursively correcting possible deterministic errors in the estimated steering vector is proposed. It employs the subspace principle and estimates the desired steering vector using a convex optimization approach. We show that the solution can be obtained in closed form by using the Lagrange multiplier method. As the proposed method is based on an extended version of the conventional OPAST algorithm, it has low implementation complexity and moving sources can be handled. In addition, a robust beamformer with a new error bound using the proposed steering vector estimate are derived by optimizing the worst-case performance of the array after taking the uncertainties of the array covariance matrix into account. This gives a diagonally loaded Capon beamformer, where the loading level is related to the bound of the uncertainty in the array covariance matrix. Numerical results show that the proposed algorithm performs well especially at high signal-to-noise ratio (SNR) and in the presence of deterministic sensor uncertainties.

**Index Terms**—steering vector, subspace, robust beamforming, convex optimization, orthonormal PAST (OPAST).

## I. INTRODUCTION

ADAPTIVE beamforming using sensor arrays has been widely used in various fields such as radar, sonar, wireless communication and microphone array processing [1]. Basically, adaptive beamforming aims to enhance the desired signal received while suppressing the undesirable noise and interference. Adaptive beamforming can be achieved by embedding known training signals in the source signal transmitted or blindly by utilizing the estimated steering vectors of the sources. The steering vector is the gain vector of the emitting source at a given coordinate with respect to the array. It is therefore a function of the source coordinate and the geometry of the array. For a known array geometry, one can estimate the steering vectors of the far-field sources and therefore determine their direction-of-arrivals (DOAs). Based on the estimated steering vector of the desired signal, the interference can be efficiently suppressed by conventional adaptive beamforming algorithms, such as the Capon

beamformer [2]. However, the steering vector in real systems may not be determined accurately from the array geometry alone due to the presence of uncertainties such as sensor gain/phase uncertainties, position variations and mutual coupling [3], [4]. Previous works show that these distortions may dramatically degrade the performance of the conventional beamforming methods. Therefore, robust beamforming methods to address these uncertainties have received great attention over the last decades [5]–[10]. For instance, additional linear constraints on the beampattern have been proposed to better attenuate the interference and broaden the response around the nominal look direction [5], [6]. Unfortunately, these constraints may reduce the degree of freedom for suppressing undesired interference. This effect is especially significant for arrays with small number of sensors. Another problem is that these constraints are not explicitly related to the uncertainty of the array steering vector [7], [8]. In [9] and [10], quadratic constraint on the Euclidean norm of the beamformer weight vector or the uncertainty of the array steering vector has also been exploited. This leads to another popular class of robust beamforming techniques called diagonal loading. In these methods, the array covariance matrix is loaded with an appropriate multiple, called the loading level, of the identity matrix in order to satisfy the imposed quadratic constraint. However, it is somewhat difficult to relate the loading level with uncertainty bounds of the array steering vector, which may not be available in practice.

In this paper, instead of relying completely on the norm constraints in the beamforming algorithm, we focus on the problem of robust steering vector estimation for beamforming. A new algorithm for correcting possible deterministic error in the steering vector is proposed. Though the steering vector of the desired signal may be distorted by the imperfections of the array, it is shown that the proposed algorithm is capable of estimating the deterministic error in the steering vector resulting from say array gain/phase uncertainties. In order to estimate this error, a convex problem is formulated based on the subspace principle. We show that the problem can be solved in closed form, and hence an explicit expression of the robust steering vector can be derived. A sensitivity analysis of the derived robust beamformer to errors in steering vector is also performed. It is found that the variance of the beamformer weight vector is extremely sensitive to the eigenvalues of the array covariance matrix for a given error variance of the steering vector. Thus, an approach to determine the loading level of the robust Capon beamformer given the proposed steering vector estimation and perturbation bound of the array

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covariance matrix is proposed. The resultant robust beamformer is obtained by minimizing its worst-case performance. The proposed algorithm has an arithmetic complexity of  $O(N^3)$ , which is comparable to the conventional diagonally loaded Capon beamformer.

Another recent approach in [11] is to estimate the mismatch using sequential quadratic programming. The proposed method differs from this approach in that it focuses on adaptive and recursive implementations, and provides an analytic solution of the steering vector error with the help of subspace principle. Moreover, it is able to handle dynamic cases with moving sources, because it is developed based on an extended orthonormal PAST (OPAST) algorithm [12]–[14]. Alternatively, other efficient algorithms such as [22]–[25] may also be used. However, we shall only focus on the OPAST algorithms due to page limitation. Finally, computer simulation experiments are conducted to demonstrate the excellent performance and effectiveness of the proposed method over the conventional methods, especially at high signal-to-noise ratio (SNR) and in the presence of deterministic gain/phase uncertainties.

The paper is organized as follows. The problem formulation and standard Capon beamforming are briefly introduced in Section II. The proposed robust steering vector estimation for beamforming is given in section III. In section IV, numerical examples are conducted to demonstrate the excellent performance and effectiveness of the proposed methods, and finally, section V concludes the paper.

## II. PROBLEM FORMULATION

Consider an antenna array with  $N$  sensors impinged by  $K+1$  narrow-band uncorrelated signals including one desired signal and  $K$  interferences. Here, we assume that  $K+1 < N$ . The  $N \times 1$  array output  $\mathbf{x}(t)$  observed at the  $t$ th snapshot consists of the outputs of the  $N$  sensors, i.e.,  $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T$  with  $[\cdot]^T$  denoting the matrix transpose. More precisely, the array output can be written as

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{i}(t) + \mathbf{n}(t), \quad (1)$$

where  $\mathbf{s}(t) = \mathbf{a}(\theta_0)s_0(t)$ ,  $\mathbf{i}(t) = \sum_{k=1}^K \mathbf{a}(\theta_k)s_k(t)$ ,  $\mathbf{n}(t)$  are the desired signal, interference and noise components, respectively. Moreover,  $\mathbf{a}(\theta_0)$  and  $\{\mathbf{a}(\theta_k)\}_{k=1}^K$  are respectively the steering vectors of the desired signal and interferences. For an ideal uniform linear array,  $\mathbf{a}(\theta) = [1, e^{j2\pi\lambda^{-1}d\sin\theta}, \dots, e^{j2\pi\lambda^{-1}(N-1)d\sin\theta}]^T$  with  $\lambda$ ,  $d$  and  $\theta$  denoting the carrier wavelength, inter-sensor spacing and DOA, respectively. In this paper, the noise is considered to be an additive white Gaussian noise (AWGN) with zero mean and covariance matrix  $\sigma^2 \mathbf{I}$ , where  $\mathbf{I}$  is an identity matrix. The sensor outputs are linearly combined by a beamformer to form the desired output:

$$y(t) = \mathbf{w}^H \mathbf{x}(t), \quad (2)$$

where  $[\cdot]^H$  denotes Hermitian transposition, and  $\mathbf{w}$  is the  $N \times 1$  complex weight vector of the beamformer. The objective is to maximize the signal-to-interference-plus-noise ratio (SINR)

$$\text{SINR} = \frac{\sigma_0^2 |\mathbf{w}^H \mathbf{a}_0|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \quad (3)$$

where  $\mathbf{a}_0$  denotes  $\mathbf{a}(\theta_0)$  for simplicity,  $\sigma_0^2 = E[\mathbf{s}(t)\mathbf{s}^H(t)]$  is the power of the desired signal,  $\mathbf{R}_{i+n} = E[(\mathbf{i}(t) + \mathbf{n}(t))(\mathbf{i}(t) + \mathbf{n}(t))^H]$  is the covariance matrix of interference-plus-noise and  $E[\cdot]$  denotes the statistical expectation. Alternatively, the optimal weight vector is obtained by solving the following optimization problem [2]

$$\begin{aligned} \min \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s. t.} \quad & \mathbf{w}^H \mathbf{a}_0 = 1, \end{aligned} \quad (4)$$

where  $\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)]$  is the covariance matrix of the array output. It is known that the solution of (4) is given by

$$\mathbf{w}_{\text{MVDR}} = \frac{\mathbf{R}^{-1} \mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}^{-1} \mathbf{a}_0}, \quad (5)$$

which is called minimum variance distortionless response (MVDR) beamformer or the Capon beamformer. It should be noted that this beamformer is obtained based on the assumption that the array response or the steering vector of the desired signal, i.e.,  $\mathbf{a}_0$ , is known accurately. However, as mentioned earlier,  $\mathbf{a}_0$  is subject to uncertainties due to various imperfections of the array and DOA estimation error. Hence, the true steering vector  $\mathbf{a}$  of the desired signal should be written as

$$\mathbf{a} = \mathbf{a}_0 + \Delta, \quad (6)$$

where  $\Delta$  denotes the uncertainty in  $\mathbf{a}_0$ . Once  $\Delta$  is known, one just needs to replace  $\mathbf{a}_0$  in (5) by  $\mathbf{a}$  to get the optimal MVDR beamformer. In practice, the uncertainty  $\Delta$  is generally unknown to users and the performance of the beamformer (5) will degrade considerably when  $\Delta$  is simply ignored. Hence, a number of robust methods have been proposed to take this uncertainty into account. For instance, by assuming that the true steering vector lies within an ellipsoid centered at  $\mathbf{a}_0$ , the robust Capon beamforming (RCB) [7] or the robust minimum variance beamforming (RMVB) [8] algorithm can be employed to solve for  $\mathbf{a}$ . However, both of these methods require *a priori* knowledge of the ellipsoid, such as its norm bound.

In general, the uncertainty  $\Delta$  in (6) consists of two error components, namely i) deterministic error which changes only slowly with time as a result of sensor gain/phase uncertainties and location errors, etc and ii) stochastic error which results from other stochastic variations such as sensor noise on the initial DOA estimation. In the following section, a new approach is introduced to estimate a correction to  $\mathbf{a}_0$  by taking advantage of the subspace principle.

### III. ROBUST STEERING VECTOR ESTIMATION FOR BEAMFORMING

#### A. Robust Steering Vector Estimation

In practice, the nominal steering vector  $\mathbf{a}_0$  is usually obtained by a DOA estimation algorithm given the array geometry. Due to uncertainties of the array, such as aforementioned sensor locations or gain/phase uncertainties, the steering vector computed from the given array geometry may deviate from the true one. Therefore, the nominal steering vector  $\mathbf{a}_0$  may be subject to a deterministic error  $\Delta$  from the true steering vector  $\mathbf{a}$ .

Since subspace principle is an effective approach in high resolution DOA estimation, we shall propose a new method to determine this deterministic correction  $\Delta$  using the subspace approach. In general, the uncertainty  $\Delta$  should lie inside a hypersphere with radius  $\varepsilon$ :

$$\|\mathbf{a} - \mathbf{a}_0\|^2 = \|\Delta\|^2 \leq \varepsilon. \quad (7)$$

Conventionally, the error bound parameter  $\varepsilon$  is assumed to be known, e.g., [7]. Since this knowledge may not be accurately available in practice, we propose to estimate the uncertainty  $\Delta$  directly without the prior knowledge of  $\varepsilon$ .

Based on the subspace principle, we know that the true steering vector  $\mathbf{a}$  is orthogonal to the  $N \times (N - K - 1)$  noise subspace  $\mathbf{U}_n$ , i.e.,

$$\mathbf{U}_n^H \mathbf{a} = \mathbf{U}_n^H (\mathbf{a}_0 + \Delta) = \mathbf{0}. \quad (8)$$

Generally, it is assumed that  $K + 1 < N$  and the ambient noise is AWGN, so that  $\mathbf{U}_n$  can be obtained from the eigenvalue decomposition (EVD) of the covariance matrix  $\mathbf{R}$ , and  $\mathbf{U}_n$  consists of the  $N - K - 1$  eigenvectors corresponding to the  $N - K - 1$  smallest eigenvalues. However, the computational complexity of EVD may be prohibitive for some real-time applications. Therefore, subspace tracking algorithm will be employed in this paper to reduce the arithmetic complexity and handle the scenarios involving moving sources.

Though the true steering vector is unknown, it usually lies within a small region around the nominal steering vector. Therefore, it is natural to choose the smallest  $\Delta$  such that (8) is satisfied. On the other hand, since the noise subspace is estimated, say by subspace tracking algorithms, slight tracking errors are inevitable and it will depend on the speed of the moving sources and other stochastic errors such as sensor noises. To address this issue, it is assume that true subspace is given by  $\mathbf{U}_n = \hat{\mathbf{U}}_n + \delta\mathbf{U}_n$ , where  $\hat{\mathbf{U}}_n$  is the estimated noise subspace and  $\delta\mathbf{U}_n$  is the estimation error due to sensor noise or other stochastic errors.

Consequently, equation (8) becomes

$$(\hat{\mathbf{U}}_n + \delta\mathbf{U}_n)^H (\mathbf{a}_0 + \Delta) = \mathbf{0}. \quad (9)$$

It should be noted that  $\Delta$  represents the deterministic part of the errors which arise from say gain/phase mismatch and location errors which are assumed to be invariant. On the other

hand, the estimation error  $\delta\mathbf{U}_n$ , which may arise from sensor noise etc, is a random matrix. Though its exact value is unknown, for well-designed systems, it is reasonable to assume that it is zero mean. Moreover, we shall show that the determination of  $\Delta$  will be benefited from the knowledge of its covariance

$$\mathbf{C}_{\delta\mathbf{U}} = E[\delta\mathbf{U}_n \delta\mathbf{U}_n^H]. \quad (10)$$

Furthermore, since  $\mathbf{U}_n$  is estimated by the subspace tracking algorithm, the stochastic error will appear as instantaneous variation of the subspace and hence its covariance matrix can be approximately estimated from the subspace tracking algorithm. This will be explained in detail later in sub-Section III-B when the tracking of the subspace is investigated. Therefore, the proposed method is particularly useful for DOA tracking scenario, where the subspace can be continuously tracked. This is, however, different from conventional robust beamforming methods, which usually do not take the subspace tracking into account.

Next, we shall focus on the determination of  $\Delta$ . First of all, we rearrange (9) as

$$\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta) = -\delta\mathbf{U}_n^H (\mathbf{a}_0 + \Delta). \quad (11)$$

By taking the Euclidean norm on both sides of (11), one gets

$$\|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 = (\mathbf{a}_0 + \Delta)^H (\delta\mathbf{U}_n \delta\mathbf{U}_n^H) (\mathbf{a}_0 + \Delta). \quad (12)$$

Moreover, by taking expectation over  $\delta\mathbf{U}_n$  on both sides of (12), we have

$$\|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 = (\mathbf{a}_0 + \Delta)^H \mathbf{C}_{\delta\mathbf{U}} (\mathbf{a}_0 + \Delta). \quad (13)$$

Since both  $\mathbf{C}_{\delta\mathbf{U}}$  and  $\Delta$  are typically small, the right hand side can be approximated by omitting the terms involving the product of  $\mathbf{C}_{\delta\mathbf{U}}$  and  $\Delta$  as follows

$$(\mathbf{a}_0 + \Delta)^H \mathbf{C}_{\delta\mathbf{U}} (\mathbf{a}_0 + \Delta) \approx \mathbf{a}_0^H \mathbf{C}_{\delta\mathbf{U}} \mathbf{a}_0 \stackrel{\Delta}{=} \zeta. \quad (14)$$

As a result, the linear equality (8) is modified to  $\|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 \approx \zeta$ , which is then relaxed to the following quadratic inequality

$$\|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 \leq \zeta, \quad (15)$$

since typically only the bounds on the uncertainties are required. Consequently, the problem at hand is to minimize the Euclidean norm of  $\Delta$  while satisfying (15):

$$\begin{aligned} \min \quad & \|\Delta\|^2 \\ \text{s. t.} \quad & \|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 \leq \zeta. \end{aligned} \quad (16)$$

It is noted that (16) is a convex quadratically constrained quadratic programming problem and hence an optimal solution does exist. We now employ the Lagrange multiplier method to solve for this solution. The *Lagrangian*  $L$  associated with (16) is given by

$$L(\Delta, \lambda) = \|\Delta\|^2 + \lambda (\|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 - \zeta), \quad (17)$$

where  $\lambda > 0$  is the Lagrange multiplier and we have excluded

the trivial solution  $\Delta = \mathbf{0}$ . By setting the partial derivative of (17) with respect to  $\Delta$  to zero, one gets the first order necessary condition for optimality as follows

$$\Delta + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \Delta + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 = \mathbf{0}.$$

On the other hand, since the problem is convex and the objective function is differentiable, any stationary point is also the global solution. Hence, the optimal solution  $\hat{\Delta}$  to (16) is given by

$$\hat{\Delta} = -\lambda (\mathbf{I} + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H)^{-1} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0. \quad (18)$$

To determine  $\lambda$ , a common way is to substitute (18) back to the equation  $\|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 = \zeta$  and it will in general give rise to a nonlinear equation in  $\lambda$ .

Fortunately, we shall show below that a closed form solution of  $\lambda$  can be obtained. Firstly, we assume that the noise subspace  $\hat{\mathbf{U}}_n$  is orthogonally obtained, i.e.,  $\hat{\mathbf{U}}_n$  satisfies

$$\hat{\mathbf{U}}_n^H \hat{\mathbf{U}}_n = \mathbf{I}.$$

Then, the term  $(\mathbf{I} + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H)^{-1}$  on the right side of (18) can be simplified to

$$\begin{aligned} (\mathbf{I} + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H)^{-1} &= \mathbf{I} - \lambda \hat{\mathbf{U}}_n (\mathbf{I} + \lambda \hat{\mathbf{U}}_n^H \hat{\mathbf{U}}_n)^{-1} \hat{\mathbf{U}}_n^H \\ &= \mathbf{I} - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H, \end{aligned} \quad (19)$$

with the help of the matrix inverse lemma that  $(\mathbf{I} + \mathbf{A}\mathbf{B})^{-1} = \mathbf{I} - \mathbf{A}(\mathbf{I} + \mathbf{B}\mathbf{A})^{-1}\mathbf{B}$ . Substituting (19) into (18), one gets

$$\begin{aligned} \hat{\Delta} &= -\lambda \left( \mathbf{I} - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \right) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 \\ &= -\lambda \left( \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \right) \mathbf{a}_0 \\ &= -\frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0. \end{aligned} \quad (20)$$

Then, substituting (20) into the constraint of the problem in (16), one gets the following equation on  $\lambda$ :

$$\begin{aligned} \left\| \hat{\mathbf{U}}_n^H \left( \mathbf{a}_0 - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 \right) \right\|^2 &= \left\| \hat{\mathbf{U}}_n^H \mathbf{a}_0 - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n^H \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 \right\|^2 \\ &= \left\| \frac{1}{1 + \lambda} \hat{\mathbf{U}}_n^H \mathbf{a}_0 \right\|^2 = \zeta. \end{aligned} \quad (21)$$

Consequently,  $\lambda$  is given by

$$\lambda = \alpha^{-1} - 1, \quad (22)$$

where  $\alpha$  is defined as

$$\alpha = \left( \zeta^{-1} \mathbf{a}_0^H \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 \right)^{-1/2}. \quad (23)$$

Finally, by substituting (23) into (21), we obtain the following close form solution to the problem in (16) as

$$\hat{\Delta} = (\alpha - 1) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0. \quad (24)$$

From (10) and (14), it can be seen that when  $\delta \mathbf{U}_n \rightarrow 0$ , we

TABLE I  
THE EXTENDED OPAST ALGORITHM FOR NOISE SUBSPACE AND ITS COVARIANCE TRACKING

Initialize $\mathbf{P}(0)$ , $\mathbf{U}_s(0)$ , $\mathbf{U}_n(0)$ and $\mathbf{C}_{\delta \mathbf{U}}(t)$
For $t = 1, 2, \dots$ , do
$\mathbf{y}(t) = \hat{\mathbf{U}}_s^H(t-1)\mathbf{x}(t)$
$\mathbf{h}(t) = \mathbf{P}(t-1)\mathbf{y}(t)$
$\mathbf{g}(t) = \mathbf{h}(t) / [\beta + \mathbf{y}^H(t)\mathbf{h}(t)]$
$\mathbf{P}(t) = \beta^{-1} \text{Tr} \{ \mathbf{P}(t-1) - \mathbf{g}(t)\mathbf{h}^H(t) \}$
$\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{U}}_s(t-1)\mathbf{y}(t)$
$\tau(t) = \ \mathbf{g}(t)\ ^{-2} \left( (1 + \ \mathbf{e}(t)\ ^2 \ \mathbf{g}(t)\ ^2)^{-2} - 1 \right)$
$\tilde{\mathbf{e}}(t) = \tau(t) \hat{\mathbf{U}}_s(t-1)\mathbf{g}(t) + (1 + \tau(t) \ \mathbf{g}(t)\ ^2) \mathbf{e}(t)$
$\hat{\mathbf{U}}_s(t) = \hat{\mathbf{U}}_s(t-1) + \tilde{\mathbf{e}}(t)\mathbf{g}^H(t)$
$\delta \tilde{\mathbf{U}}_n(t) = -\hat{\mathbf{U}}_s(t-1)\mathbf{g}(t)\tilde{\mathbf{e}}^H(t)$
$\quad - \mathbf{e}(t)\mathbf{g}^H(t)\hat{\mathbf{U}}_s^H(t-1) - \ \mathbf{g}(t)\ ^2 \tilde{\mathbf{e}}(t)\tilde{\mathbf{e}}^H(t)$
$\tilde{\mathbf{U}}_n(t) = \tilde{\mathbf{U}}_n(t-1) + \delta \tilde{\mathbf{U}}_n(t)$
$\mathbf{C}_{\delta \mathbf{U}}(t) = \beta \mathbf{C}_{\delta \mathbf{U}}(t-1) + (1 - \beta) \delta \tilde{\mathbf{U}}_n(t) \delta \tilde{\mathbf{U}}_n^H(t)$
end t

have  $\mathbf{C}_{\delta \mathbf{U}} \rightarrow 0$  and  $\zeta \rightarrow 0$ . Consequently, the value of  $\alpha$  will be approximately zero and the solution in (24) will reduce to  $\hat{\Delta} = -\hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0$ . Careful examination shows that this is the solution of the conventional projection approach [20], [21]. Therefore, the proposed approach offers an alternative interpretation to the conventional projection approach and extends it further to include possible uncertainties arising from tracking or other stochastic errors. One of its main advantages is that simple analytical solution is available which greatly simplifies the implementation. We now discuss the recursive tracking of the subspace and the determination of  $\mathbf{C}_{\delta \mathbf{U}}$ .

### B. Noise Subspace Tracking and Robust Beamforming

As mentioned previously, the noise subspace  $\mathbf{U}_n$  can be estimated by EVD of the array covariance matrix  $\mathbf{R}$ . However, the complexity of EVD may be prohibitive in some practical implementations especially for antenna arrays with large number of elements. More importantly, EVD may not be feasible for dynamic environments where moving sources are involved. A number of subspace tracking algorithms have been proposed to deal with this problem in the last decades [12]–[14].

Though most of these algorithms focus on signal subspace tracking, they can also be extended to noise subspace tracking since the noise and signal subspaces are related by

$$\tilde{\mathbf{U}}_n(t) = \mathbf{I} - \hat{\mathbf{U}}_s(t) [\hat{\mathbf{U}}_s^H(t) \hat{\mathbf{U}}_s(t)]^{-1} \hat{\mathbf{U}}_s^H(t), \quad (25)$$

where  $\hat{\mathbf{U}}_s(t)$  is the estimated signal subspace. It can be seen that the subspaces are now functions of the time index  $t$  since the subspaces are tracked continuously. Despite the simple relationship, the complexity of (25) is still  $O(N^3)$  due to the matrix inversion operation. Fortunately, with the use of OPASt algorithm [14], the signal subspace  $\hat{\mathbf{U}}_s(t)$  estimated is orthogonal and hence  $\hat{\mathbf{U}}_s^H(t)\hat{\mathbf{U}}_s(t) = \mathbf{I}$ . Consequently, (25) can be reduced to

$$\tilde{\mathbf{U}}_n(t) = \mathbf{I} - \hat{\mathbf{U}}_s(t)\hat{\mathbf{U}}_s^H(t), \quad (26)$$

which provides an more efficient mean for computing the noise subspace. Furthermore, it is known that  $\hat{\mathbf{U}}_n(t)\hat{\mathbf{U}}_n^H(t) + \hat{\mathbf{U}}_s(t)\hat{\mathbf{U}}_s^H(t) = \mathbf{I}$ . This implies that  $\tilde{\mathbf{U}}_n(t) = \hat{\mathbf{U}}_n(t)\hat{\mathbf{U}}_n^H(t)$ . Therefore, if the noise subspace is obtained as (26), the uncertainty of the steering vector at time  $t$  can be estimated according to (24) as follows:

$$\hat{\Delta}(t) = (\alpha(t) - 1)\tilde{\mathbf{U}}_n(t)\mathbf{a}_0(t), \quad (27)$$

where  $\alpha(t) = (\zeta^{-1}(t)\mathbf{a}_0^H(t)\tilde{\mathbf{U}}_n(t)\mathbf{a}_0(t))^{-1/2}$  and  $\zeta(t) = \mathbf{a}_0^H(t)\mathbf{C}_{\delta U}(t)\mathbf{a}_0(t)$ .

We now extend the OPASt algorithm to recursively track the noise subspace  $\tilde{\mathbf{U}}_n(t)$  and the covariance  $\mathbf{C}_{\delta U}$  required by our robust steering vector estimation algorithm. According to the extended OPASt algorithm shown in Table I, the orthogonal signal subspace  $\hat{\mathbf{U}}_s(t)$  is recursively updated as

$$\hat{\mathbf{U}}_s(t) = \hat{\mathbf{U}}_s(t-1) + \tilde{\mathbf{e}}(t)\mathbf{g}^H(t), \quad (28)$$

where  $\tilde{\mathbf{e}}(t)$  and  $\mathbf{g}(t)$  are defined in Table I. Substituting  $\hat{\mathbf{U}}_s(t)$  into (26), one gets

$$\tilde{\mathbf{U}}_n(t) = \tilde{\mathbf{U}}_n(t-1) + \delta\tilde{\mathbf{U}}_n(t), \quad (29)$$

where

$$\begin{aligned} \delta\tilde{\mathbf{U}}_n(t) = & -\hat{\mathbf{U}}_s(t-1)\mathbf{g}(t)\tilde{\mathbf{e}}^H(t) \\ & - \tilde{\mathbf{e}}(t)\mathbf{g}^H(t)\hat{\mathbf{U}}_s^H(t-1) - \|\mathbf{g}(t)\|^2 \tilde{\mathbf{e}}(t)\tilde{\mathbf{e}}^H(t). \end{aligned} \quad (30)$$

It can be seen that the noise subspace can now be estimated recursively from the signal subspace with a low arithmetic complexity. We also notice that (30) provides us with the instantaneous perturbation of the noise subspace from which its covariance can be efficiently estimated. More precisely, we propose to estimate the covariance of the noise subspace, i.e.,  $\mathbf{C}_{\delta U}(t)$ , recursively as follows

$$\mathbf{C}_{\delta U}(t) = \beta\mathbf{C}_{\delta U}(t-1) + (1-\beta)\delta\tilde{\mathbf{U}}_n(t)\delta\tilde{\mathbf{U}}_n^H(t), \quad (31)$$

where  $0 < \beta \leq 1$  is a forgetting factor. Once the noise subspace  $\tilde{\mathbf{U}}_n(t)$  and the covariance  $\mathbf{C}_{\delta U}(t)$  are obtained, the value of the bound  $\zeta(t)$  and the uncertainty of steering vector  $\hat{\Delta}(t)$  can be estimated according to (14) and (27), respectively. Accordingly, the steering vector can be updated as  $\hat{\mathbf{a}}(t) = \mathbf{a}_0(t) + \hat{\Delta}(t)$ . The conventional MVDR beamformer can

thus be invoked to obtain a new robust beamformer by replacing  $\mathbf{a}_0(t)$  in (5) by  $\hat{\mathbf{a}}(t)$ . Hence, the following robust MVDR (R-MVDR) beamformer is proposed as:

$$\mathbf{w}_{R-MVDR}(t) = \frac{\mathbf{R}^{-1}(t)(\mathbf{a}_0(t) + \hat{\Delta}(t))}{(\mathbf{a}_0(t) + \hat{\Delta}(t))^H \mathbf{R}^{-1}(t)(\mathbf{a}_0(t) + \hat{\Delta}(t))}, \quad (32)$$

It should be noted that for online implementations and moving sources, the covariance matrix  $\mathbf{R}(t)$  should also be recursively estimated say by the popular formula

$$\mathbf{R}(t) = \beta\mathbf{R}(t-1) + (1-\beta)\mathbf{x}(t)\mathbf{x}^H(t). \quad (33)$$

### C. Sensitivity Analysis and Modification of the R-MVDR Beamformer

So far, it has been shown that a new robust MVDR beamformer can be obtained by exploiting the OPASt algorithm. In this section, we briefly analyze its sensitivity to the error in the steering vector, and we shall show that the proposed beamformer can be extended further to take the error of the array covariance matrix into account. To begin with, we assume that  $\mathbf{R}(t)$  is nonsingular and denote its EVD by  $\mathbf{U}(t)\mathbf{\Lambda}(t)\mathbf{U}^H(t)$ , where  $\mathbf{\Lambda}(t)$  and  $\mathbf{U}(t)$  compose of the eigenvalues and eigenvectors, respectively. Moreover,  $\mathbf{U}(t)$  is an orthogonal matrix satisfying  $\mathbf{U}(t)\mathbf{U}^H(t) = \mathbf{I}$ . Here, it is considered that the stochastic error in  $\mathbf{a}(t)$  is  $\delta\mathbf{a}(t)$  with zero mean, then (32) can be rewritten as

$$\begin{aligned} \tilde{\mathbf{w}}_{R-MVDR}(t) = & \frac{\mathbf{U}(t)\mathbf{\Lambda}^{-1}(t)\mathbf{U}^H(t)(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))}{(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))^H \mathbf{U}(t)\mathbf{\Lambda}^{-1}(t)\mathbf{U}^H(t)(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))} \\ = & \frac{\alpha_0(t)\mathbf{w}_{R-MVDR} + \mathbf{U}(t)\mathbf{\Lambda}^{-1}(t)\mathbf{U}^H(t)\delta\mathbf{a}(t)}{(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))^H \mathbf{U}(t)\mathbf{\Lambda}^{-1}(t)\mathbf{U}^H(t)(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))}, \end{aligned} \quad (34)$$

where  $\hat{\mathbf{a}}_0(t) = \mathbf{a}_0(t) + \hat{\Delta}(t)$ ,  $\alpha_0(t) = \hat{\mathbf{a}}_0^H(t)\mathbf{R}^{-1}(t)\hat{\mathbf{a}}_0(t)$ . Define

$$\tilde{\mathbf{W}}_{R-MVDR}(t) = \mathbf{U}^H(t)\tilde{\mathbf{w}}_{R-MVDR}(t), \quad (35a)$$

$$\mathbf{W}_{R-MVDR}(t) = \mathbf{U}^H(t)\mathbf{w}_{R-MVDR}(t), \quad (35b)$$

$$\hat{\mathbf{A}}_0(t) = \mathbf{U}^H(t)\hat{\mathbf{a}}_0(t), \quad (35c)$$

$$\delta\mathbf{A}(t) = \mathbf{U}^H(t)\delta\mathbf{a}(t). \quad (35d)$$

We have

$$\begin{aligned} \tilde{\mathbf{W}}_{R-MVDR}(t) = & \frac{\alpha_0(t)\mathbf{W}_{R-MVDR}(t) + \mathbf{\Lambda}^{-1}(t)\delta\mathbf{A}(t)}{(\hat{\mathbf{A}}_0(t) + \delta\mathbf{A}(t))^H \mathbf{\Lambda}^{-1}(t)(\hat{\mathbf{A}}_0(t) + \delta\mathbf{A}(t))}. \end{aligned} \quad (36)$$

Hence, the mean of the weight vector is approximately given by

$$E[\tilde{\mathbf{W}}_{R-MVDR}(t)] \approx \frac{\alpha_0(t)\mathbf{W}_{R-MVDR}(t)}{E[(\hat{\mathbf{A}}_0(t) + \delta\mathbf{A}(t))^H \mathbf{\Lambda}^{-1}(t)(\hat{\mathbf{A}}_0(t) + \delta\mathbf{A}(t))]}$$

$$= \frac{\alpha_0(t) \mathbf{W}_{R-MVDR}(t)}{\hat{\mathbf{A}}_0^H(t) \Lambda^{-1}(t) \hat{\mathbf{A}}_0(t) + \text{trace}(\Lambda^{-1}(t) \mathbf{C}_{\delta A})}, \quad (37)$$

where  $\mathbf{C}_{\delta A}(t) = E[\delta \mathbf{A}(t) \delta \mathbf{A}^H(t)]$  and we have truncated the higher order terms in the expansion of the delta method [27] so that  $E[\tilde{\mathbf{W}}_{R-MVDR}(t)]$  can be approximated by evaluating the expectation of its numerator and denominator separately. Furthermore, if  $\mathbf{C}_{\delta A}(t)$  is small so that the second term in the denominator is small compared with the first term, which is the usual case, then

$$\begin{aligned} E[\tilde{\mathbf{W}}_{R-MVDR}(t)] &\approx \frac{\alpha_0(t) \mathbf{W}_{R-MVDR}(t)}{\hat{\mathbf{A}}_0^H(t) \Lambda^{-1}(t) \hat{\mathbf{A}}_0(t)}, \\ &= \mathbf{W}_{R-MVDR}(t) \end{aligned} \quad (38)$$

since  $\alpha_0(t) = \hat{\mathbf{a}}_0^H(t) \mathbf{R}^{-1}(t) \hat{\mathbf{a}}_0(t) = \hat{\mathbf{A}}_0^H(t) \Lambda^{-1}(t) \hat{\mathbf{A}}_0(t)$ . The perturbation of  $\mathbf{W}_{R-MVDR}(t)$  due to  $\delta \mathbf{A}(t)$  is thus

$$\delta \mathbf{W}_{R-MVDR}(t) \approx \alpha_0^{-1}(t) \Lambda^{-1}(t) \delta \mathbf{A}(t). \quad (39)$$

Let  $\mathbf{C}_{\delta W}(t) = E[\delta \mathbf{W}_{R-MVDR}(t) \delta \mathbf{W}_{R-MVDR}^H(t)]$ , we have

$$\begin{aligned} \text{tr}(\mathbf{C}_{\delta W}(t)) &\approx \text{tr}(E[\alpha_0^{-2}(t) \Lambda^{-1}(t) \delta \mathbf{A}(t) \delta \mathbf{A}^H(t) \Lambda^{-1}(t)]) \\ &= \text{tr}(\alpha_0^{-2}(t) \Lambda^{-1}(t) \mathbf{C}_{\delta A} \Lambda^{-1}(t)) \\ &= \alpha_0^{-2}(t) \sum_{i=1}^N \lambda_i^{-2}(t) \mathbf{C}_{\delta A, i}(t), \end{aligned} \quad (40)$$

where  $\mathbf{C}_{\delta A, i}(t)$  and  $\lambda_i(t)$  are the  $i$ th diagonal entry of  $\mathbf{C}_{\delta A}(t)$  and  $\Lambda^{-1}(t)$ , respectively. It is noted that  $\text{trace}(\mathbf{C}_{\delta W}(t))$  will increase with the variance  $\mathbf{C}_{\delta A}(t)$ . More importantly, it can be seen that  $\mathbf{w}_{R-MVDR}(t)$  is extremely sensitive to eigenvalues of  $\mathbf{R}(t)$ , especially when  $\mathbf{R}(t)$  is ill-conditioned. Hence, even if  $\mathbf{C}_{\delta A}(t)$  is not very large, the perturbation in  $\mathbf{R}(t)$  will result in significant variation of  $\mathbf{w}_{R-MVDR}(t)$ . In this case, one cannot obtain a proper beamformer even though the true steering vector is known.

An effective method is to employ robust beamforming approaches. For instance, the diagonal loading method, which is closely related to ridge regression in reducing the variance of the estimator while scarifying slightly the bias, is commonly used. The regularized solution is given by adding a small diagonal matrix to  $\mathbf{R}(t)$ :

$$\bar{\mathbf{w}}_{R-MVDR}(t) = \frac{(\mathbf{R}(t) + \mu \mathbf{I})^{-1} \hat{\mathbf{a}}(t)}{\hat{\mathbf{a}}^H(t) (\mathbf{R}(t) + \mu \mathbf{I})^{-1} \hat{\mathbf{a}}(t)}, \quad (41)$$

where  $\mu \geq 0$  is the diagonal loading level. Though the beamformer is now biased, the variance is:

$$\text{trace}(\mathbf{C}_{R-\delta W}(t)) \approx \sum_{i=1}^N \frac{\mathbf{C}_{\delta A, i}(t) (\lambda_i(t) + \mu)^{-2}}{(\mathbf{A}_0^H(t) (\Lambda(t) + \mu \mathbf{I})^{-1}(t) \mathbf{A}_0(t))^2}, \quad (42)$$

which decreases with increasing value of  $\mu$ . However, it is well known that the diagonal loading level is somewhat difficult to determine in practice. One conventional way to deal

TABLE II  
MAIN STEPS OF THE PROPOSED ALGORITHM

- 
- Step 1)** Update the covariance matrix  $\mathbf{R}(t)$  recursively as  $\mathbf{R}(t) = \beta \mathbf{R}(t-1) + (1-\beta) \mathbf{x}(t) \mathbf{x}^H(t)$ .
- Step 2)** Update the signal subspace using the OPAST algorithm as  $\hat{\mathbf{U}}_s(t) = \hat{\mathbf{U}}_s(t-1) + \tilde{\mathbf{e}}(t) \mathbf{g}^H(t)$ .
- Step 3)** Compute the noise subspace with the estimated signal subspace as  $\tilde{\mathbf{U}}_n(t) = \mathbf{I} - \hat{\mathbf{U}}_s(t) \hat{\mathbf{U}}_s^H(t)$  and the covariance matrix  $\mathbf{C}_{\delta U}(t)$  based on the extended OPAST algorithm as  $\mathbf{C}_{\delta U}(t) = \beta \mathbf{C}_{\delta U}(t-1) + (1-\beta) \delta \tilde{\mathbf{U}}_n(t) \delta \tilde{\mathbf{U}}_n^H(t)$ . Calculate  $\zeta(t)$  as (14).
- Step 4)** Use the estimated noise subspace  $\tilde{\mathbf{U}}_n(t)$ ,  $\zeta(t)$  and the given nominal steering vector  $\mathbf{a}_0(t)$  to compute the uncertainty in the steering vector as (27).
- Step 5)** Update the steering vector as  $\hat{\mathbf{a}}(t) = \mathbf{a}_0(t) + \hat{\Delta}(t)$ .
- Step 6)** Compute the beamformer as (50) with the estimated  $\hat{\mathbf{a}}(t)$  and  $\gamma(t)$ .
- 

with this problem is to estimate the noise power  $\sigma_n^2$  and select the regularization parameter  $\mu$  as  $\min(\kappa \sigma_n^2, \sigma_{\min}^2)$ , where  $\kappa$  is a user defined constant and  $\sigma_{\min}^2$  is the minimum loading level. Usually,  $\kappa$  is chosen as 10 to combat the uncertainties of steering vector and covariance matrix. Unfortunately, it has been shown in many literatures that such a fixed diagonal level cannot provide a satisfactory performance. A number of robust beamforming algorithms have therefore been proposed by assuming that the steering vector and/or covariance matrix are known imprecisely and lie within certain bounds [7]–[9], [15]–[17].

In this paper, we shall estimate the perturbation bound of the array covariance matrix  $\mathbf{R}(t)$  so that it can be utilized in these robust beamforming algorithms. Moreover, it will be adopted to determine the loading level of the conventional diagonal loading method, which yields a simple but robust beamformer. Firstly, let the true and mismatched array covariance matrices be  $\tilde{\mathbf{R}}(t)$  and  $\mathbf{R}(t)$ , respectively. Hence, we have

$$\tilde{\mathbf{R}}(t) = \mathbf{R}(t) + \delta \mathbf{R}(t), \quad (43)$$

where  $\delta \mathbf{R}(t)$  is the error matrix due to the perturbation of  $\mathbf{R}(t)$  and it is assumed to be bounded by a certain known or estimated parameter  $\gamma(t)$ , i.e.,

$$\|\delta \mathbf{R}(t)\| \leq \gamma(t). \quad (44)$$

Consequently, the problem in (4) can be rewritten as

$$\begin{aligned} \min \mathbf{w}^H(t)(\mathbf{R}(t) + \delta\mathbf{R}(t))\mathbf{w}(t) \\ \text{s.t. } \mathbf{w}^H(t)\hat{\mathbf{a}}(t) = 1, \quad \|\delta\mathbf{R}(t)\| \leq \gamma(t), \end{aligned} \quad (45)$$

which can further be rewritten as the following problem of minimizing the worst-case output power:

$$\begin{aligned} \min_{\mathbf{w}} \max_{\|\delta\mathbf{R}(t)\| \leq \gamma} \mathbf{w}^H(t)(\mathbf{R}(t) + \delta\mathbf{R}(t))\mathbf{w}(t) \\ \text{s.t. } \mathbf{w}^H(t)\hat{\mathbf{a}}(t) = 1. \end{aligned} \quad (46)$$

In order to solve (46), we firstly solve the problem

$$\begin{aligned} \max_{\delta\mathbf{R}(t)} \mathbf{w}^H(t)(\mathbf{R}(t) + \delta\mathbf{R}(t))\mathbf{w}(t) \\ \text{s.t. } \|\delta\mathbf{R}(t)\| \leq \gamma(t), \end{aligned} \quad (47)$$

whose solution is given by [17]

$$\delta\mathbf{R}(t) = \gamma(t) \frac{\mathbf{w}(t)\mathbf{w}^H(t)}{\|\mathbf{w}(t)\|^2}. \quad (48)$$

After some manipulation, the problem (46) can finally be reformulated as

$$\begin{aligned} \min \mathbf{w}^H(t)(\mathbf{R}(t) + \gamma(t)\mathbf{I})\mathbf{w}(t) \\ \text{s.t. } \mathbf{w}^H(t)\hat{\mathbf{a}}(t) = 1. \end{aligned} \quad (49)$$

Apparently, the solution of the problem in (49) is given by

$$\mathbf{w}_{R-MVDR-WC}(t) = \frac{(\mathbf{R}(t) + \gamma(t)\mathbf{I})^{-1}\hat{\mathbf{a}}(t)}{\hat{\mathbf{a}}^H(t)(\mathbf{R}(t) + \gamma(t)\mathbf{I})^{-1}\hat{\mathbf{a}}(t)}. \quad (50)$$

Comparing the worst-case solution (50) with that in (41), the loading level  $\mu$  is directly related to the perturbation bound of the covariance matrix  $\gamma(t)$  in (44). In real systems, it may be able to select the perturbation bound  $\gamma(t)$  based on prior information. In this paper, the instantaneous variation of the array covariance matrix, i.e.,  $\mathbf{R}(t) - \mathbf{R}(t-1)$ , will be adopted to estimate the perturbation bound. More precisely, such bound is assumed to be proportional to the norm of the instantaneous variation  $\gamma(t) = k \|\mathbf{R}(t) - \mathbf{R}(t-1)\|$ . In fact, it is found experimentally that  $\gamma(t)$  can be chosen from a wide range, with  $k$  between 1% to 20%, without affecting significantly the performance. Hence, the choice of  $\gamma(t)$  is not a crucial problem if the value of  $\gamma(t)$  is not too large. For illustrative purpose, we choose  $\gamma(t) = k \|\mathbf{R}(t) - \mathbf{R}(t-1)\|$  with  $k = 10\%$  in Section IV. Finally, the proposed robust steering vector estimation and diagonally loaded MVDR beamformer based on worst-case performance optimization (R-MVDR-WC) is summarized in Table II.

We now briefly discuss the arithmetic complexity of the proposed algorithm. In step 1, the covariance matrix can be efficiently updated in  $O(N^2)$  complexity. In step 2, the signal subspace can be updated in  $O(N(K+1))$  complexity. The complexity in step 3 is  $O(N^3)$  which is larger than the previous two steps due to the matrix product  $\delta\tilde{\mathbf{U}}_n(t)\delta\tilde{\mathbf{U}}_n^H(t)$ . The complexity in step 6 is also  $O(N^3)$  flops due to the

required matrix inversion process. Hence, the proposed method is of the same order as other conventional algorithms such as the Capon beamforming, robust Capon beamforming (RCB) and diagonal loading (DL).

#### IV. NUMERICAL EXAMPLE

In order to evaluate the performance of the proposed algorithm, a ULA with  $N=10$  sensors separated by half wavelength is considered. The noise is assumed to be AWGN with a power of 0dB. One desired signal and two interferences are assumed to impinge on the array from far-field. In the first two examples, the DOA of the desired signal is assumed to be fixed at  $0^\circ$ , whereas in the last two examples the DOA of the desired signal is considered to be time-varying and is given by  $10^\circ \times 10^{-3}t$ ,  $0 \leq t \leq 1000$ , where  $t$  is the index of snapshots. For all simulations, the DOAs of the two interferences are fixed to be  $40^\circ$  and  $60^\circ$ . The powers of the interferences are fixed to be 30dB, i.e., interference-to-noise ratio (INR) is 30dB. The noise subspace is obtained using the extended OPAST subspace tracking method as shown in Table I, where the forgetting factor is  $\beta = 0.99$ . The perturbation bound of the array covariance matrix is estimated as  $\gamma(t) = \|\mathbf{R}(t) - \mathbf{R}(t-1)\| \times 10\%$ . In all examples, the signal-plus-interference number and hence the signal subspace rank used in the extended OPAST are assumed to be known, and are equal to 3. In practice, the subspace rank can be estimated using say the minimum description length (MDL) algorithm.

For comparison, the following conventional algorithms are tested: 1) the conventional diagonal loading (DL) beamformer with a fixed loading level of 10; 2) robust Capon beamforming (RCB) [7] with the error bound equal to  $\varepsilon = 3.2460$ , which corresponds to a  $2^\circ$  DOA mismatch when the DOA of the desired signal is  $0^\circ$ , and 3) the worst-case method [18]. In the simulations, the DOA of the desired signal is first estimated using the conventional ESPRIT algorithm [26] with the tracked signal subspace  $\hat{\mathbf{U}}_s(t)$ . Then, the proposed robust beamforming as well as other conventional algorithms are invoked based on the estimated DOA. The performances of all these methods are compared in terms of the output SINR.

##### Example 1: Stationary Case.

In the first example, we test the performance of the proposed method in a stationary case, i.e., the desired signal has a fixed DOA, which is assumed to be  $0^\circ$ . We assume that there is no other uncertainties except the DOA mismatch due to the accuracy of DOA tracking algorithm. The output SINR at each time instant is calculated according to (3). Figs. 1 and 2 show the tracked DOA of the desired signal and the output SINR of various beamformers with a low SNR of -5dB and a relatively high SNR of 5dB, respectively. From these two figures, it can be seen that when the SNR is low, the tracking algorithm converges slowly. Hence, there will be large DOA mismatch before convergence. Also, it can be seen that the proposed beamformer gives a better performance when there is a large

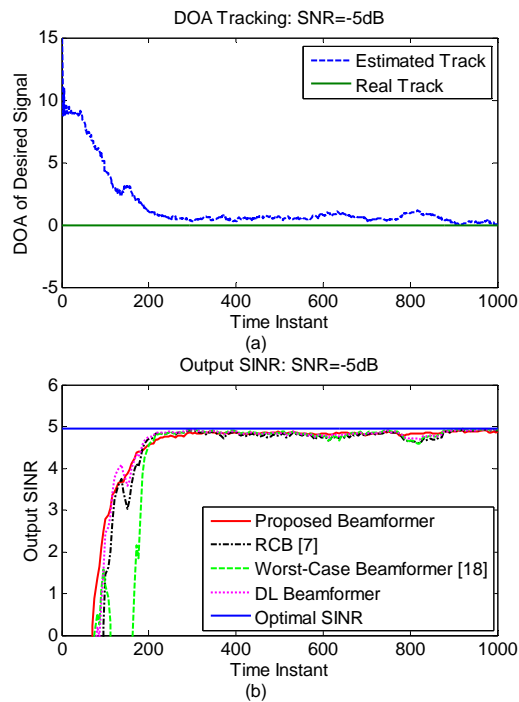


Fig.1. (a) DOA tracking using the extended OPASt in a stationary case with SNR=-5dB. (b) Output SINR of various beamformers in a stationary case with SNR=-5dB.

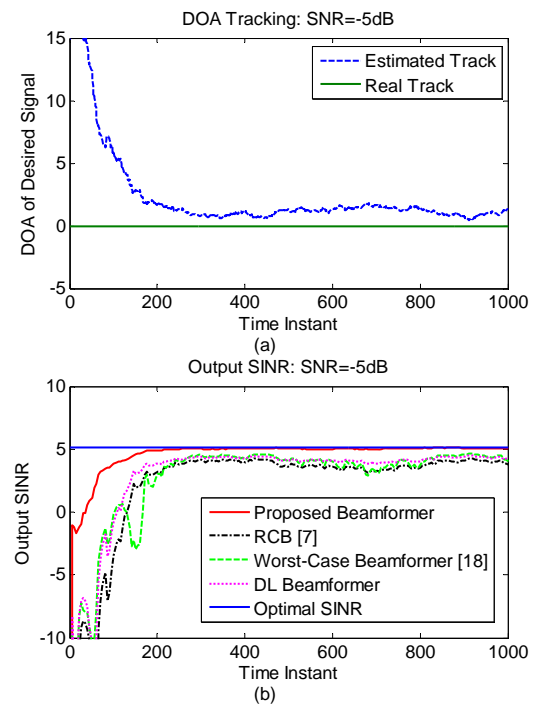


Fig.3(a) DOA tracking using the extended OPASt in a stationary case with gain/phase uncertainties and SNR=-5dB. (b) Output SINR of various beamformers in a stationary case with gain/phase uncertainties and SNR=-5dB.

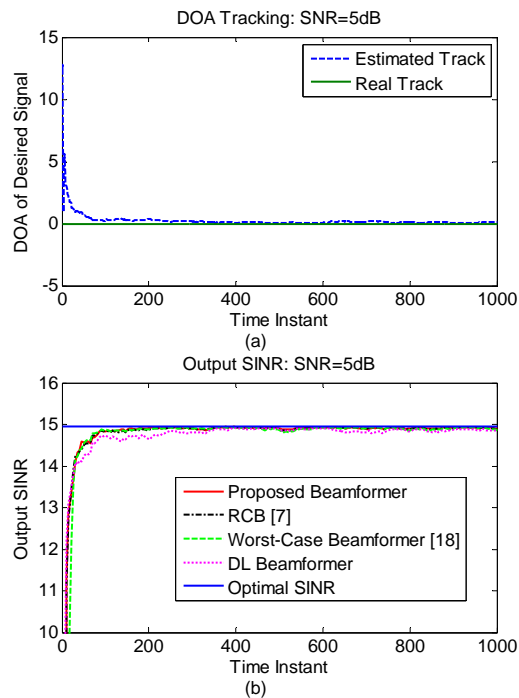


Fig. 2 (a) DOA tracking using the extended OPASt in a stationary case with SNR=5dB. (b) Output SINR of various beamformers in a stationary case with SNR=5dB.

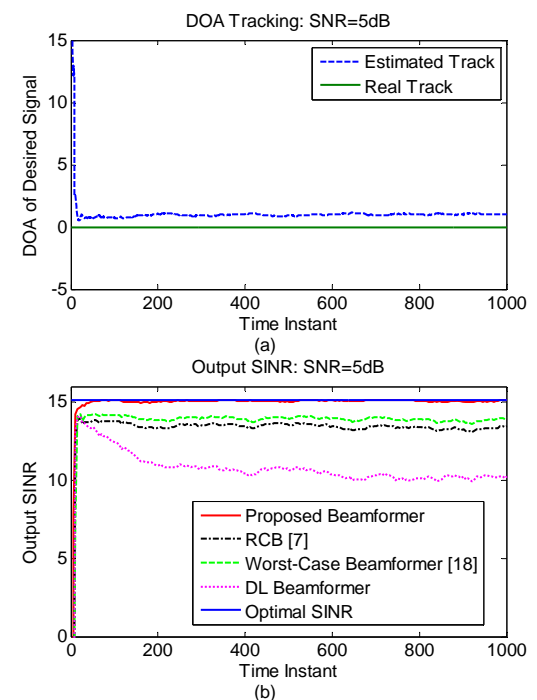


Fig.4(a) DOA tracking using the extended OPASt in a stationary case with gain/phase uncertainties and SNR=5dB. (b) Output SINR of various beamformers in a stationary case with gain/phase uncertainties and SNR=5dB.

DOA mismatch. On the other hand, when the tracking algorithm converges, the DOA of the desired signal can be estimated with a high accuracy. Therefore, all beamformers can

give excellent performance, which is almost identical to the optimal one.



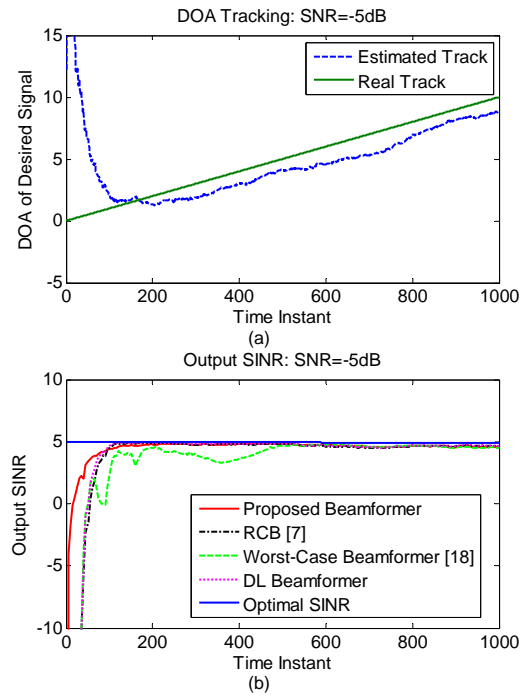


Fig. 5 DOA tracking using the extended OPAST in a dynamic case with SNR=-5dB. (b) Output SINR of various beamformers in a dynamic case with SNR=-5dB.

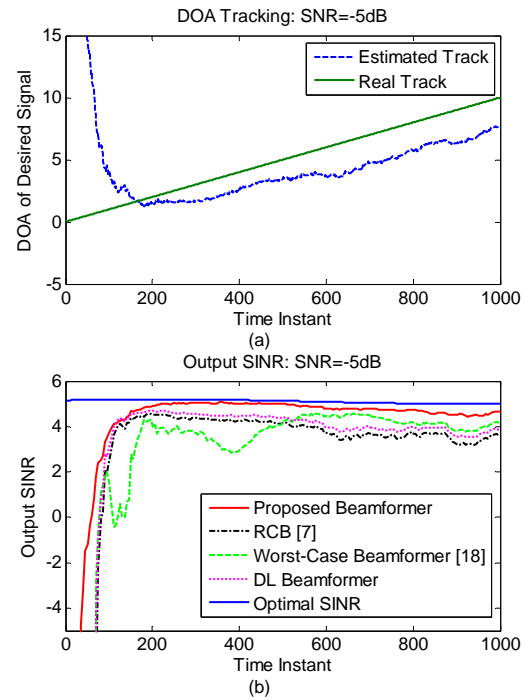


Fig. 7. DOA tracking using the extended OPAST in a dynamic case with gain/phase uncertainties and SNR=-5dB. (b) Output SINR of various beamformers in a dynamic case with gain/phase uncertainties and SNR=-5dB.

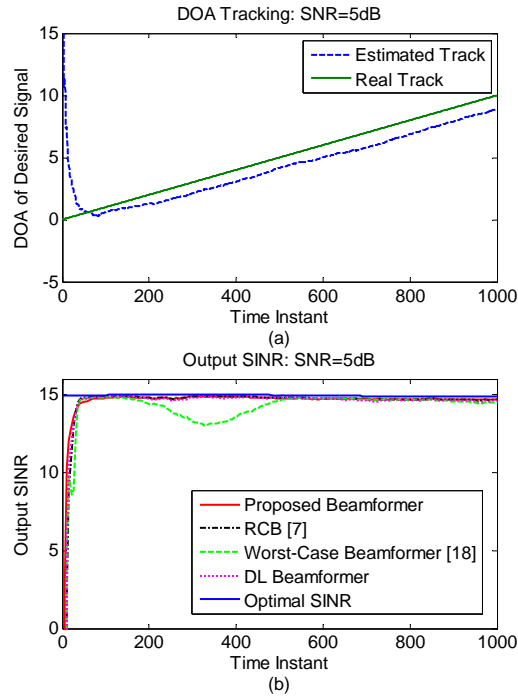


Fig. 6 DOA tracking using the extended OPAST in a dynamic case with SNR=5dB. (b) Output SINR of various beamformers in a dynamic case with SNR=5dB.

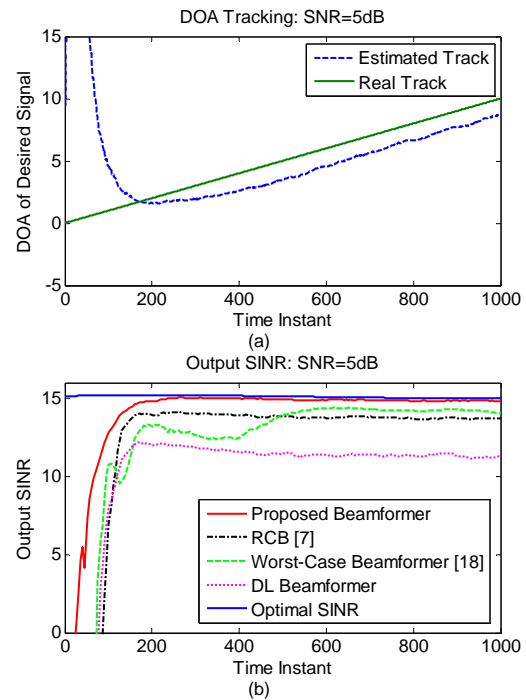


Fig. 8 DOA tracking using the extended OPAST in a dynamic case with gain/phase uncertainties and SNR=5dB. (b) Output SINR of various beamformers in a dynamic case with gain/phase uncertainties and SNR=5dB.

*Example 2: Stationary Case with Array Gain/phase Uncertainties.*

In order to test the robustness of the proposed method against

array imperfections, in this example, the array gain/phase uncertainties are considered. It is known that these uncertainties usually lead to a degradation of DOA estimation

and beamforming performance. Following the last example, in this simulation, each sensor (except the first reference sensor) is further assumed to be suffered from a gain/phase uncertainty of the form  $\rho_i e^{j\phi_i}$ ,  $2 \leq i \leq N$ . Both the gain and phase uncertainties are assumed to be uniformly distributed as  $\rho_i \sim U(0.8, 1.2)$  and  $\phi_i \sim U(-\pi/5, \pi/5)$ . For simulation, a fixed set of the gain/phase uncertainties is taken as:  $\{\rho_i\}_{i=2}^N = \{1.0369, 0.9695, 1.0033, 1.0176, 1.0560, 1.0309, 0.9665, 1.0718, 1.0690\}$  and  $\{\phi_i\}_{i=2}^N = \{-0.2916, -0.2947, 0.2547, 0.4015, 0.1534, 0.3667, -0.3193, -0.4652, -0.1343\}$ . The resultant DOA tracking and output SINR are shown in Fig. 3 and Fig. 4. Obviously, we can notice that the accuracy of DOA tracking is considerably degraded due to the existence of array gain/phase uncertainties. As can be seen in Fig. 3(a) and Fig. 4(a), there is a larger DOA mismatch even after the convergence of the tracking algorithm compared with the case without array gain/phase uncertainties. However, it can be seen that the proposed method outperforms the conventional ones and nearly achieves the optimal performance. Careful examination also shows that the performance of the conventional RCB deteriorates due to such uncertainties. Since the worst-case beamformer [18] takes the uncertainties in the array covariance matrix into account, it is able to achieve a better performance than that of RCB.

#### Example 3: Dynamic Case.

The settings in this example is identical to those in Example 1, except that the DOA of the desired signal is time-varying and given by  $10^\circ \times 10^{-3}t$ ,  $0 \leq t \leq 1000$ , where  $t$  is the index of snapshots. Fig. 5 and Fig. 6 show the DOA tracking results and output SINR with SNRs of  $-5\text{dB}$  and  $5\text{dB}$ , respectively. Compared with the stationary case, it can be noticed that there is a much larger DOA mismatch due to the dynamic of the desired signal. However, we can find that after convergence, all the methods can still successfully suppress the undesired interference and achieve excellent performance.

#### Example 4: Dynamic Case with Array Gain/phase Uncertainties.

It has been shown in Example 2 that when there are array gain/phase uncertainties, the DOA cannot be well tracked even in a stationary case. In this example, we will show the performance of the proposed method in a time-varying case with array gain/phase uncertainties. Again, the dynamic model of the desired signal is assumed to be the same as that in Example 3. The DOA tracking results and output SINRs at  $-5\text{dB}$  and  $5\text{dB}$  SNRs are shown respectively in Fig. 7 and Fig. 8. As expected, the DOA tracking performance degrades due to the array gain/phase uncertainties. Furthermore, it can be seen that the conventional methods are significantly influenced by such uncertainties, especially at higher SNRs. On the contrary, the proposed method can still achieve an excellent performance.

## V. CONCLUSIONS

A new method for correcting possible deterministic errors in the steering vector due to sensor uncertainties is presented. It uses the subspace principle and the resulting problem can be formulated as a convex problem and solved in closed form. Using an extended OPA algorithm, the algorithm is further extended to handle scenarios involving moving sources while requiring a low complexity. An analysis on the perturbation of beamforming weights due to the DOA estimation errors is also performed and it suggests that the former is also highly sensitive to the eigenvalues of the estimated covariance matrix. Hence, a new adaptive beamformer, which minimizes the worst-case performance of the array subject to covariance matrix uncertainties, is also presented. The resultant beamformer resembles the diagonally loaded Capon beamformer with the loading level given by a bound on the uncertainties in the array covariance matrix, which can be estimated recursively. Simulation results show that the proposed algorithm can offer satisfactory performance, especially at high SNR levels and in the presence of deterministic sensor uncertainties.

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