ROBUST ADAPTIVE BEAMFORMING FOR LARGE STEERING ANGLE ERROR

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ABSTRACT

Standard Capon beamformer is sensitive to steering vector error and diagonal loading is an efficient approach to improve its robustness. Based on maximization of the output power, a varying diagonal loading algorithm has been proposed by Jian Li et.al. By analysis of this paper, it shows that the effect of antenna amplitude, phase and position errors to steering vector is generally less serious than that due to steering angle errors. For large steering angle error case, we propose to use pipe constraint instead of sphere one. Because optimization over a pipe is difficult, we propose to let a sequence of spheres overlay the pipe and Li's algorithm is subsequently applied to every sphere constraint. Simulation results demonstrate its superior performance.

1. INTRODUCTION

Array signal processing has been widely used in radar, sonar, communication etc[1]. For actual antenna arrays, there always exists gain, phase, position and steering angle errors. Conventional Capon beamformer is sensitive to these errors [2]. Robust beamformers has been explored for many years which mainly include: (1)Multiple constraints: Additional constraints including multi-angle and derivative constraints are imposed. But additional antenna elements are also needed to compensate the loss of the degree of freedom for interference reduction. (2) Eigen subspace projection algorithms (ESP)([3] and the references therein): For infinite snapshots case, the true steering vector is located in the signal subspace, so it is implied that the assumed steering vector projecting to the signal subspace can improve the robustness of beamforming. But for high signal-interference-ratio case, the performance of eigen-subspace projection algorithm is poor, this will be shown in our simulations. (3) Diagonal loading method (RCB)([4-8]and the references therein): If there are antenna's amplitude, phase and position errors, the deviation between nominal beam pattern and actual beam pattern is proportional to the norm of the weight. Hence, norm constraints and diagonal loading can improve the robustness of the beamformer. The diagonal loading quantum of the conventional diagonal loading methods is not directly related to the uncertainty of the steering vector. Until recently, varying diagonal loading method associated with the steering vector perturbation has been proposed[6,7, 8]. Although the start point of [7, 8] is from a reformulation of the Capon beamforming problem[9], we can understand it as follows. According to the idea that the output power of an optimum beamformer achieves a local maximum if the assumed steering vector coincides with the actual steering vector, the actual steering vector can then be obtained by maximizing the output power. It is also proven that this method is a varying diagonal loading method, and the diagonal loading quantum can be obtained efficiently[7, 8].

The influence of antenna amplitude, phase and position errors to the steering vector can be modelled as additive random perturbation, thus the actual steering vector can be regarded as located in a sphere centered on the assumed steering vector. But the influence of steering angle error is different, the steering vector forms a manifold with the change of steering angle error. Considering antenna amplitude, phase, position and steering angle errors simultaneously, the actual steering vectors can be thought as points located in a pipe, which is a tight constraint, compared with a big sphere constraint in order to include larger steering angle error. Direct optimization over a pipe is difficult, but optimization over a sphere is easy[8]. In this paper we propose to let a sequence of spheres to overlay the pipe and optimization is obtained over the sphere sequence. Simulation results show that the pipe constraint is superior over the big sphere constraint.

2. ROBUST CAPON BEAMFORMING ALGORITHM

In this paper, T,H and $*$ at the superscript denote matrix or vector transpose, conjugate transpose and conjugate, respectively. Assume there are M antenna elements, $\mathbf{s}_0$ and $\mathbf{w}$ are the assumed steering vector and weighting vector, respectively, $\mathbf{R}$ is the covariance matrix, Capon beamformer is formulated as follows[1,2]

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}\mathbf{w} \quad subject \quad to \quad \mathbf{w}^H\mathbf{s}_0 = 1. \qquad (1)$$

The solution of (1) can be obtained by using Lagrange multiplier easily[1]:

$$\mathbf{w} = \frac{\mathbf{R^{-1}s_0}}{\mathbf{s_0^H R^{-1} s_0}}. \tag{2}$$

Substituting (2) into (1), the output power can be obtained by

$$P = 1/(\mathbf{s_0^H R^{-1} s_0}). \tag{3}$$

For actual antenna elements, there always exist antenna amplitude, phase, position and steering angle errors. If the assumed steering vector is far away from the true steering vector, the desired signal may be cancelled and the output power will become small. Based on this observation, it is proposed in [8] to search the steering vector to maximize the output power under a spherical constraint, or equivalently, minimize the reciprocal of output power:

$$\min_{\mathbf{a}} \mathbf{a}^H \mathbf{R}^{-1} \mathbf{a}, \quad subject \quad to \quad ||\mathbf{a} - \mathbf{s}_0||^2 <= \varepsilon, \tag{4}$$

where ε is permitted deviation. The solution of (4) is

$$\hat{\mathbf{a}} = \mathbf{s}_0 - (\mathbf{I} + \lambda \mathbf{R})^{-1} \mathbf{s}_0, \tag{5}$$

where λ is the lagrange multiplier such that $(\mathbf{I}+\lambda\mathbf{R})^{-1}\mathbf{s}_0 = \varepsilon$ be satisfied. Replace $\mathbf{s}_0$ in (2) with $\hat{\mathbf{a}}$, the weight can be expressed as [8]

$$\mathbf{w} = \frac{(\mathbf{R} + \frac{1}{\lambda}\mathbf{I})^{-1}\mathbf{s}_0}{\mathbf{s}_0^H (\mathbf{R} + \frac{1}{\lambda}\mathbf{I})^{-1} \mathbf{R} (\mathbf{R} + \frac{1}{\lambda}\mathbf{I})^{-1} \mathbf{s}_0}. \tag{6}$$

This is a diagonal loading class method, and the diagonal loading quantum is associated with signal environments(R) and permitted error ε. Its super performance over conventional fixed diagonal loading method has been proven by simulations in [8]. But in [8], the relation between ε over amplitude, phase, position and steering angle errors is not considered. We will discuss this problem in the next section.

3. PIPE CONSTRAINT VARYING DIAGONAL LOADING ALGORITHM (PCVDL)

The influence of antenna amplitude, phase and position errors to the steering vector is different from that of steering angle error to the steering vector. We first consider that there are only amplitude, phase and position errors. Let $\mathbf{p}_i$, $\mathbf{p}_i^n$ denote the actual and nominal position vector of antenna i , respectively, the nominal gain and phase are one and zero, respectively. The actual gain, phase and position can be expressed as $g_i = 1 + \delta g_i$, $\varphi_i = 0 + \delta\varphi_i$, $\mathbf{p}_i = \mathbf{p}_i^n + \delta\mathbf{p}_i$. We assume that $\delta g_i, \delta\varphi_i$, $\delta p_{xi}, \delta p_{yi}$ and δp_{zi} are statistically independent, zero mean, Gaussian random variables with variance σ_g^2, σ_φ^2 and σ_p^2[1]. Let $\mathbf{k} = \frac{2\pi}{\lambda}\mathbf{v}$ be the

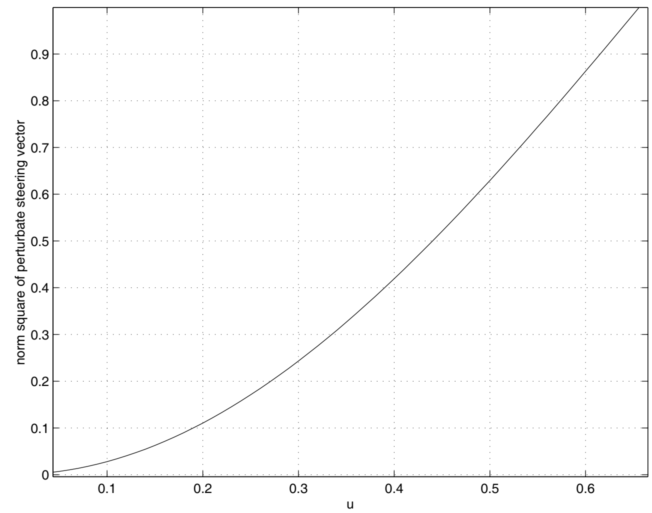

Fig. 1. The picture of $||\mathbf{a}(\theta_\mathbf{0}) - \mathbf{a}(\theta)||^2$ with $u = sin(\theta) - sin(\theta_0)$

wavenumber, where $\mathbf{v}$ is the unit direction vector of signal propagation. The ith element of nominal steering vector and actual steering vector are $s_i = exp(-j\mathbf{k}^T\mathbf{p}_i^n)$ and $a_i = (1+\delta g_i)exp(j\delta\varphi_i - j\mathbf{k}^T\mathbf{p}_i)$, respectively. The deviation between $\mathbf{a}$ and $\mathbf{s}$ can be expressed as

$$\begin{aligned} ||\mathbf{a} - \mathbf{s}||^2 &= \sum_{i=1}^{M} (a_i - s_i)^*(a_i - s_i) \\ &= \sum_{i=1}^{M} (1+\delta g_i)^2 + M \\ &- \sum_{i=1}^{M} 2Re\{(1+\delta g_i)exp(j\delta\varphi_i - j\mathbf{k}^T\delta\mathbf{p}_i)\}, \end{aligned} \tag{7}$$

where $||\mathbf{a} - \mathbf{s}||^2$ is a random variable. According to [1], we have

$$E\{||\mathbf{a} - \mathbf{s}||^2\} \approx M(\sigma_g^2 + \sigma_\varphi^2 + \sigma_\lambda^2), \tag{8}$$

where $\sigma_\lambda = \frac{2\pi}{\lambda}\sigma_p$.

We assume the array is uniform linear array with half-wavelength elements spacing, there are no amplitude, phase and position errors, the assumed and actual directions are θ_0 and θ, respectively, then

$$||\mathbf{s}(\theta) - \mathbf{s}(\theta_0)||^2 = 2M - 2Re\left(\mathbf{s}(\theta)^H \mathbf{s}(\theta_0)\right). \tag{9}$$

It is easy to obtain

$$\begin{aligned} ||\mathbf{s}(\theta) - \mathbf{s}(\theta_0)||^2 =& 2M - 2cos\left(\pi\frac{M-1}{2}\left(sin\theta - sin\theta_0\right)\right) \\ &\times \frac{sin\left(\pi\frac{M}{2}\left(sin(\theta) - sin(\theta_0)\right)\right)}{sin\left(\pi\frac{1}{2}\left(sin(\theta) - sin(\theta_0)\right)\right)}. \end{aligned} \tag{10}$$

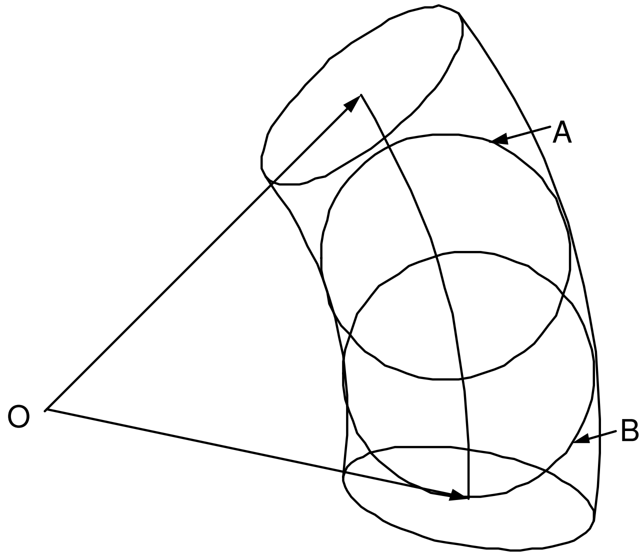

Fig. 2. The constraint set of steering vector(pipe).

It can be seen that when $||\mathbf{s}(\theta)-\mathbf{s}(\theta_0)||^2 = M$, $|sin(\theta)-sin(\theta_0)| \approx \frac{0.6672}{M}$ for large M, such as $M >= 10$. $||\mathbf{s}(\theta)-\mathbf{s}(\theta_0)||^2/M$ with $u = sin(\theta) - sin(\theta_0)$ as a variable for $M = 10$ is shown in Fig.1. For a typical larger amplitude, phase and position errors case, such as $\sigma_g^2+\sigma_\varphi^2+\sigma_\lambda^2 = 0.1$, $E\{||\mathbf{a}-\mathbf{s}||^2/M\} = 0.1$, according to Fig.1, which corresponds to steering angle error of $\frac{1}{5}$ resolution cell of conventional DFT beamformer. So the influence of amplitude, phase and position errors is weaker relative to that of steering angle error. If we allow a larger steering angle error, we should choose a larger ε. We shall show in the next section that for a small steering vector error, the larger the ε chosen in (4), the poorer the performance is, so we should choose ε as small as possible to fit the influence of amplitude, phase and position errors. In order to tolerate a larger steering angle error without performance degradation, we can modify the constraint so that $\mathbf{a}$ is located in a pipe as illustrated in Fig.2. Optimization over a pipe is difficult, but optimization over a sphere is easy, so we can let K spheres to overlay the pipe, optimization is obtained over the sequence of spheres.

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^H\mathbf{R}^{-1}\mathbf{a} \\ s.t. \quad & ||\mathbf{a}-\mathbf{s}(\theta_i)||^2 <= \varepsilon, \\ & for \quad particular \quad i, \quad i = 1, \cdots, K \end{aligned} \tag{11}$$

The new algorithm can be summarized as follows:

Step 1) Compute the eigendecomposition of R.

Step 2) For $i = 1, \cdots, K$, solve λ and get the $\hat{\mathbf{a}}_i$ using (5)[8].

Step 3) Choose i' such that $i' = \min_i \hat{\mathbf{a}}_i\mathbf{R}^{-1}\hat{\mathbf{a}}_i$.

Step 4) Replace $\mathbf{s}_0$ with $\mathbf{s}(\theta_{i'})$ in (6), the beamforming weights can be obtained.

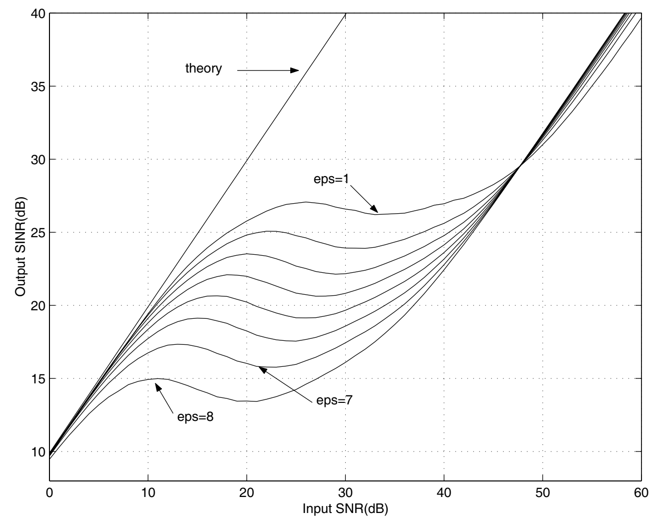

Fig. 3. Output SINR vs input SNR for different ε using RCB; first example.

It should be noted that the computation load is mainly on eigendecomposition of R in Step 1), the additional computation in Step 2) and Step 3) are small compared with that of step 1)[8]. For example, let antenna numbers be 10, 20, 40 and 80, respectively. In one simulation conditions, it costs 0.594, 2.141, 9.765 and 67.78 time unit to do matrix eigendecomposition in Step 1), but it only costs 0.5, 0.531, 0.875 and 2.109 time unit to solve λ in step 2). Hence, if K is not very big, the additional computation load is moderate, but the performance is improved greatly.

4. SIMULATION RESULTS

In all of the simulations below, we consider a uniform linear array with M=10 and half-wavelength sensors spacing. The desired and interference signals are located in far field and the desired signal's direction is 0°(broadside). When the snapshots are finite, the results are the average of 100 Monte Carlo simulations.

Example 1. The assumed signal direction is 0.5°, then $||\mathbf{a}-\mathbf{s}_0||^2 = 0.21348$. There are no amplitude, phase and position errors. Three interferences located at directions -25°, 30°, 50° has interference-noise-ratio 15dB, 30dB and 40dB, respectively. The snapshots is infinite. $\varepsilon = 1, \cdots, 8$ are chosen in Li's method and output SINR with input SNR are shown in Fig.3. It shows that the SINR decrease with the increase of ε, so we should choose ε as near as the true deviation.

Example 2. The assumed signal direction is 5°, then $||\mathbf{a}-\mathbf{s}_0||^2 = 15.23$. The snapshots are 200. Three interferences located at directions -30°, -20°, 30° have interference-noise-ratio 30dB, 35dB and 40dB, respectively. The an-

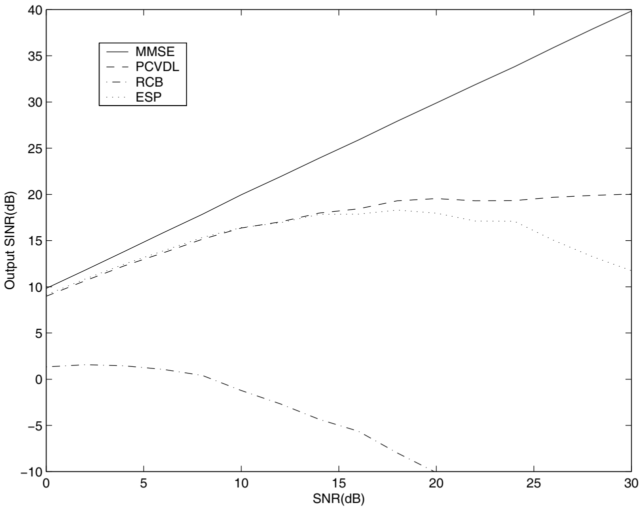

Fig. 4. Output SINR vs input SNR; second example.

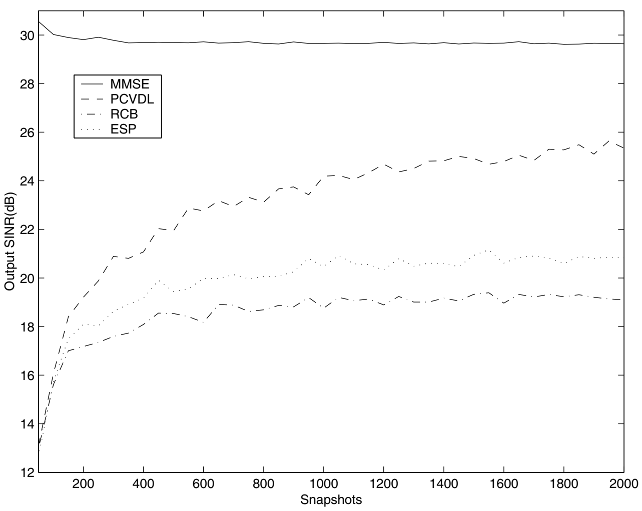

Fig. 5. Output SINR vs Snapshots ,third example.

tenna elements have amplitude, phase and position errors with variance 0.0025, 0.0025 and $0.0025(\lambda/2\pi)^2$. We choose $\varepsilon = 0.5436$ in PCVDL method and $\varepsilon = 3.246$ in RCB method. The results of PCVDL, RCB, ESP (The signal and interference number is known) and MMSE[1](The desired signal is known) are shown in Fig.4.

Example 3. The simulation conditions are the same as that in example 2 except that the $\varepsilon = 6.8227$ in RCB method, the SNR is fixed 20dB and the snapshots varying from 50 to 2000. The results of PCVDL, RCB, ESP and MMSE are shown in Fig.5. Fig.4, and Fig.5 show that for large steering angle error, the PCVDL method is superior than the RCB and ESP methods in our simulation conditions.

5. CONCLUSIONS

Varying diagonal loading is a robust beamforming method and the diagonal loading quantum is decided by the signal environment and permitted steering vector error. Generally, the influence of steering angle error is more serious than that of antenna amplitude, phase and position errors. We propose a pipe constraint instead of the sphere constraint. Optimization over a pipe is difficult, we propose to let a sequence of spheres to overlay the pipe and optimization is obtained over the sequence of sphere constraints. Simulation results show the superior performance of this proposed method.

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