Adaptive Digital Beamforming Algorithm of Circle Array Based on Compressed Sensing

Jian Wang, Wei-xing Sheng, Yu-bing Han, Xiao-feng Ma
School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China
Email: wangjian_sheshou@163.com; wxsheng@yahoo.com.cn; hanyb@njust.edu.cn; maxiaofeng@njust.edu.cn

***Abstract*—A new adaptive digital beamforming of circle array on the receiving radar station based on compressed sensing is proposed. According to the sparsity of targets in space, we design a digital beamforming by using the theory of compressed sensing. This algorithm greatly reduces the number of actual array elements with the same antenna aperture. In the case of sparse array, the echo information of the missing channels is restored firstly by using the theory of compressed sensing (CS), and then the restored signal is used to design the digital beamformer weights. Finally, the simulation results show the correctness of the proposed method.**

***Keywords*—compressed sensing; digital beamforming; sparse array; circle array; Orthogonal Matching Pursuit (OMP)**

I. INTRODUCTION

In bistatic radar systems, digital beamforming receiving array antenna is urgent needed so that the receiving antenna beams can cover the transmitting antenna beam flexibly. On receiving radar stations, to obtain high antenna gain and high angular measurement accuracy, an antenna array with a large number of antenna elements should be used. In radio astronomy systems, there is also a serious need in receiving antenna arrays with a large number of antenna elements. For a large-scale adaptive array, the high cost is a major drawback. Meanwhile, the computational burden and high-rate data transmission are two bottlenecks in the implementation of an adaptive beamforming algorithm. Sparse array is attractive because it can reduce number of elements as compared with a fully populated array. This is of particular interest in radar applications since a large aperture facilitates high performance with regard to angular accuracy, resolution, and detection of targets close to interference directions. By employing sparse arrays, it can be achieved at a reduced receiving channels, weight, power consumption, and cost as well as easier platform integration. The disadvantage of sparse arrays is their inherently high side-lobes, or near ambiguities, which may degrade performance in an interference environment. Traditional sparse array uses genetic algorithm or other optimization algorithms to optimize the position of elements, in order to reduce the side-lobes of the beam. However, it only optimizes static pattern and is difficult to guarantee the performance of the beam when we need to suppress interferences adaptively.

Recently, Candés and Donoho reported a novel sampling theory called compressed sensing, also known as compressive sampling (CS) [1] [2] [3] [4]. The theory is a new developed theoretical framework for information acquisition and processing, which is based on matrix analysis, statistical probability theory, topological geometry, optimization, functional analysis and so on. CS theory asserts that one can recover certain signals and images from far fewer samples or measurements than traditional methods use. To make this possible, CS relies on one principle: sparsity, which pertains to the signals of interest. The theory states that as long as the signal is sparse or compressible, you can reconstruct the original signal from a small number of samples by solving an optimization problem with high probability. The theory has been widely used in data acquisition, direction-of-arrival (DOA) estimation, medical imaging, radar [5], communication and many other fields.

An adaptive digital beamforming technique with compressed sensing (CS) on the receiving radar station of sparse circle array is proposed in this paper. Because of the angle sparseness of the arriving signal, CS theory can be adopted to sample receiving signals with a sparse array antenna. Then receiving signals from absence elements on the antenna aperture can be reconstructed by using orthogonal matching pursuit (OMP) algorithm [6]. Adaptive digital beamforming algorithms are then adopted to form antenna beams, whose main lobe is steered to the desired direction and nulls are steered to the directions of interferences. With the proposed adaptive digital beamforming technique, it greatly reduces the number of the elements; also the beam performance is similar with full arrays, which means that the beams have low side-lobes, deep nulls in the directions of interferences, and without grating lobes. Simulation results with the Monte Carlo method show that the beam performances of the proposed method are approaching to that of full array antenna. Actual antenna elements can be reduced greatly.

This paper is organized as follows. In section II, the principle and mathematical model of digital beamforming based on compressed sensing are discussed. The simulation results under different situations are shown to prove the correctness of the proposed algorithm in section III. Finally, a brief conclusion is given in section IV.

II. PRINCIPLE OF THE ALGORITHM

A. Signal model

Now we consider a circle array with the aperture of $2R$. Fig. 1 shows the elements' distribution of the full array. The distance between the neighboring elements is $\lambda/2$, where λ is the radar's working wavelength. For convenience, we consider the circle array as a linear array. All array elements are arranged and numbered from left to right and from top to down. Mark the coordinate of the ith element as (x_i, y_i).

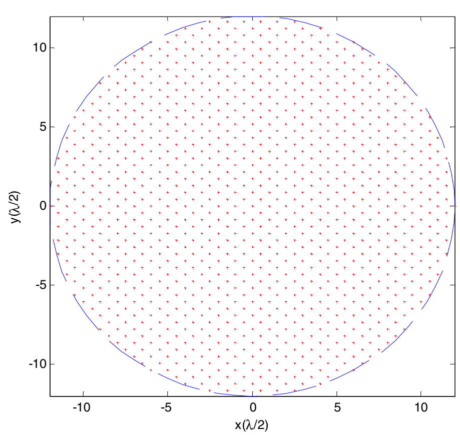

Fig. 1 Elements' distribution of the full array

Assume that K echo signals with the complex amplitude of $s_k(t)$ incident to the antenna array. The incident angles are written as $(u_k,v_k)=[\sin(\theta_k)\cos(\varphi_k),\sin(\theta_k)\sin(\varphi_k)]$, $k=1,2,\ldots,K$, where (θ_k,φ_k) means the pitch angle and the azimuth angle. One of these signals is the desired signal, the others are interferences. The received signal can be expressed as $\boldsymbol{X}(t)=[x_1(t), x_2(t),\ldots, x_N(t)]^{\mathrm{T}}$. Regardless of the receiver noise, it is written as

$$\boldsymbol{X}(t)=\sum_{k=1}^{K} s_k(t)\,\boldsymbol{a}(\omega_{kk}) \tag{1}$$

where $\boldsymbol{a}(\omega_{kk})$ means the directional vector of kth incident signal and $\omega_{kk}=(u_k,v_k)$.

$$\boldsymbol{a}(\omega_{kk})=\left(e^{j2\pi(x_1u_k+y_1v_k)/\lambda},e^{j2\pi(x_2u_k+y_2v_k)/\lambda},\cdots,e^{j2\pi(x_Nu_k+y_Nv_k)/\lambda}\right)^{\mathrm{T}} \tag{2}$$

We decompose the entire UV-space into Γ^2 parts. Then we get $\omega_{ij}=(u_i,v_j)$, where $i,j=1,2,\ldots, \Gamma$. The transformation matrix $\boldsymbol{H}$ is defined

$$\boldsymbol{H}=\left[\boldsymbol{a}(\omega_{11}),\ldots,\boldsymbol{a}(\omega_{1\Gamma}),\boldsymbol{a}(\omega_{21}),\ldots,\boldsymbol{a}(\omega_{2\Gamma}),\cdots,\boldsymbol{a}(\omega_{\Gamma 1}),\ldots\boldsymbol{a}(\omega_{\Gamma\Gamma})\right] \tag{3}$$

Then the received signal $\boldsymbol{X}(t)$ is achieved by

$$\boldsymbol{X}(t)=\boldsymbol{HS}(t) \tag{4}$$

where $\boldsymbol{S}(t)=[0,0,\ldots,s_1(t),0,\ldots,0,\ldots,s_K(t),0,\ldots,0]^{\mathrm{T}}$. It is clear that $\boldsymbol{S}(t)$ is sparse which has only few nonzero elements. Therefore, as the CS theory said, the received signal $\boldsymbol{X}(t)$ can be recovered accurately by using the CS reconstruction algorithm.

When considering the receiver noise, Eq. (4) can be rewritten as

$$\boldsymbol{X}(t)=\boldsymbol{HS}(t)+\boldsymbol{V}(t) \tag{5}$$

where $\boldsymbol{V}(t)=[v_1(t), v_2(t),\ldots, v_N(t)]^{\mathrm{T}}$ is a Gaussian white noise vector.

B. Compressed sampling and reconstruction

Now we design an $M\times N$ dimension sampling matrix $\boldsymbol{\Phi}$ with $M<<N$ which is uncorrelated with the transformation matrix $\boldsymbol{H}$. The measurement matrix $\boldsymbol{Y}$ can be obtained by

$$\begin{aligned}\boldsymbol{Y}(t)&=\boldsymbol{\Phi X}(t)\\&=\boldsymbol{\Phi}\left[\boldsymbol{HS}(t)+\boldsymbol{V}(t)\right]\\&=\boldsymbol{PS}(t)+\boldsymbol{V}'(t)\end{aligned} \tag{6}$$

where $\boldsymbol{V}'(t)=\boldsymbol{\Phi V}(t)$ is the noise of the system after compressed sampling. Sampling matrix $\boldsymbol{\Phi}$ means a method of compressed sampling of the space signals, which can be achieved by randomly selecting M rows from an identity matrix of size $N \times N$. That is to say, we can select M elements from the array to sample space signals.

The matrix $\boldsymbol{P}=\boldsymbol{\Phi H}$ of size $M \times \Gamma^2$ in Eq. (6) is called observation matrix. Theoretical studies in [4] [7] have shown that we can accurately reconstruct the receiver signal $\boldsymbol{X}(t)$ from the compressed vector $\boldsymbol{Y}(t)$ if the observation matrix $\boldsymbol{P}$ satisfies the condition of Restricted Isometry Property (RIP). Therefore the design of the sample matrix $\boldsymbol{\Phi}$ is very important. There are many sample matrixes have been used, such as, Hadamard matrix, Gaussian random matrix, sparse random matrix, part of the Fourier matrix and so on [8]. Here $\boldsymbol{P}=\boldsymbol{\Phi H}$ has been proved satisfied the condition of RIP.

A way of selecting M elements from the full array of N elements is described in details below. Firstly, generate N random numbers which is distributed in [0,1] uniformly. Then arrange these numbers from small to large. Finally, the first M random numbers are selected as the number of the array elements.

We write the output of the M elements as $\boldsymbol{Y}(t)=[y_1(t), y_2(t), \ldots, y_M(t)]^{\mathrm{T}}$.

Now we discretize the time-domain signal, and get L snapshots of the echo signal in time domain. We rewrite the Eq. (6) as

$$\boldsymbol{Y}_{M\times L}=\boldsymbol{\Phi}_{M\times N}\boldsymbol{X}_{N\times L}=\boldsymbol{P}_{M\times\Gamma^2}\boldsymbol{S}_{\Gamma^2\times L}+\boldsymbol{V}'_{M\times L} \tag{7}$$

After achieving the compressed samples of the M elements $\mathbf{Y}_{M\times L}$, we can estimate the projection coefficient vector $\boldsymbol{S}_{\Gamma^2\times L}$ by using the OMP algorithm which is discussed in [6] [9] [10]. Then we can reconstruct the receiver signal $\boldsymbol{X}_{N\times L}$

$$\boldsymbol{X}=\boldsymbol{HS} \tag{8}$$

Details of the OMP algorithm:

a. Initialize the residual $\boldsymbol{r}_0=\boldsymbol{y}$, the number of iterations $n=1$, the index matrix $\boldsymbol{\Lambda}=\varnothing$, $\boldsymbol{J}=\varnothing$.

b. Compute the correlation coefficient vector $\boldsymbol{u}=\left[u_1,u_2,\ldots,u_{\Gamma^2}\right]$ by

$$u_j=\left\|\boldsymbol{P}_j^{\mathrm{H}}\boldsymbol{r}\right\|_2,\ j=1,2,\ldots,\Gamma^2 \tag{9}$$

and put the number of the maximum value of $\boldsymbol{u}$ into the matrix $\boldsymbol{J}$.

c. Update the support matrix $\boldsymbol{P}_{\Lambda}$, with $\boldsymbol{\Lambda}=\boldsymbol{\Lambda}\,\Box\,\boldsymbol{J}$.

d. Compute $\hat{\boldsymbol{S}}$ by

$$\hat{\boldsymbol{S}}=\arg\min\left\|\boldsymbol{Y}-\boldsymbol{P}_{\Lambda}\hat{\boldsymbol{S}}\right\|_2 \tag{10}$$

and update the residual by

$$\boldsymbol{r}_{\mathrm{new}}=\boldsymbol{Y}-\boldsymbol{P}_{\Lambda}\hat{\boldsymbol{S}} \tag{11}$$

e. If $\|\boldsymbol{r}_{new}\text{-}\boldsymbol{r}\|\geq\varepsilon$, then $\boldsymbol{r}=\boldsymbol{r}_{new}$ and $n=n+1$, jump to step *b*. Otherwise, stop.

C. Adaptive digital beamforming

After reconstructing the echo signals by the reconstruction algorithm, we design a beamformer based on the iterative linearly constrained minimum variance (LCMV) with the recovered data. It can receive the desired signal and suppress the interferences.

The LCMV algorithm minimizes the power of the output signals and gets the weight $\boldsymbol{w}$ by solving the linear constraint equation [11]

$$\min\left(E\left[\left|\boldsymbol{y}(t)\right|^2\right]=\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{w}\right)s.t.\quad \boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\omega_s)=1 \tag{12}$$

To simplify the problem, we constrained in the desired direction only. $\boldsymbol{y}(t)=\boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(t)$ is the output of the array. $\boldsymbol{R}$ is the covariance matrix of output signals. $\boldsymbol{a}(\omega_s)$ is the directional vector in the desired direction. $\boldsymbol{w}$ is the weight vector. We can get the optimized weight vector by

$$\boldsymbol{w}=\boldsymbol{R}^{-1}\boldsymbol{a}(\omega_s)\left[\boldsymbol{a}^{\mathrm{H}}(\omega_s)\boldsymbol{R}^{-1}\boldsymbol{a}(\omega_s)\right]^{-1} \tag{13}$$

For online implementation, we can also develop an iterative formula for beamforming weight calculation [12].

$$\begin{gathered}\boldsymbol{w}(k+1)=\boldsymbol{A}\left[\boldsymbol{w}(k)-\mu\boldsymbol{x}(k)\boldsymbol{y}^{\mathrm{H}}(k)\right]+\boldsymbol{F},\\ k=1,2,3\ldots\end{gathered} \tag{14}$$

where μ *is* iteration step size, k is the iteration index and $\boldsymbol{A}=\boldsymbol{I}\text{-}\boldsymbol{a}(\omega_s)[\boldsymbol{a}^{\mathrm{H}}(\omega_s)\mathbf{a}(\omega_s)]^{-1}\boldsymbol{a}^{\mathrm{H}}(\omega_s)$, $\boldsymbol{F}=\boldsymbol{a}(\omega_s)[\boldsymbol{a}^{\mathrm{H}}(\omega_s)\boldsymbol{a}(\omega_s)]^{-1}$.

After reconstructing the receiver signal $\boldsymbol{X}(t)$ by equation (8), we can achieve the weight coefficients by using Eq. (14). When $|\|\boldsymbol{w}(k+1)\|\text{-}\|\boldsymbol{w}(k)\||<\varepsilon$ is satisfied, where ε is a pre-set error, the iteration stops. Then we can get the most optimized weight coefficients of the LCMV algorithm.

III. SIMULATION RESULTS

A. The comparison under different compression ratios

The theory of compressed sensing points out that, the original signal can be recovered with a small amount of sampling data. But different compression ratios have different recovery errors. Now we give the analysis of signal recovery errors of different compression ratios below.

Assume the radius of the circle array is $R=6\lambda$. Elements are arranged as described in section II, so $N=1051$. Assume $\Gamma=25$, so the entire UV-space is decomposed into 625 parts. We select 100, 200, 500 and 1000 elements from the N array elements randomly, and analyze the signal recovery errors of the situation with a signal and two interferences respectively. Data is recovered snapshot by snapshot with SNR=-10dB~40dB and SIR=-30dB.

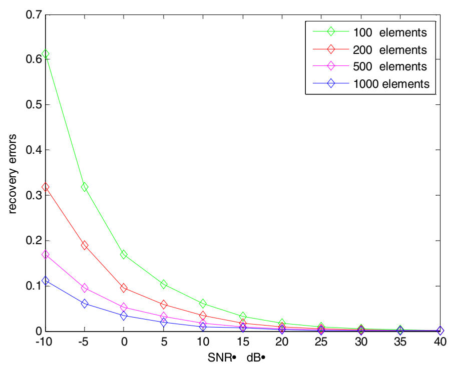

Fig. 2 Signal recovery errors under different compression ratios

Fig.2 shows the curves of recovery errors versus signal-to-noise ratios (SNRs) under different compression ratios. From Fig. 2, we can see that, the greater the SNR is, the smaller the recovery error is. And the recovery error is decreased when the number of the elements increases. When SNR>0dB, the data recovered by the outputs of 100 elements is precise enough to be used for digital beamforming. Considering both the radar cost and the recovery errors, we select 100 elements.

B. The analysis of the beam performance

We select $M=100$ elements from the array with the same size above. Fig. 3 shows the distribution of the 100 elements. It is assumed that uv values of the desired signal and the interferences are (0.44, 0.52), (0.60,-0.58) and (-0.36,-0.36) respectively with SNR=10dB, INR=40dB.

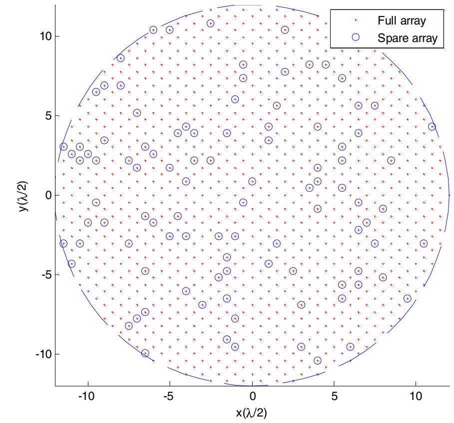

Fig. 3 The distribution of the sparse array element

We can reconstruct the received signals of full arrays by using the OMP algorithm. Then we use the recovery data to form antenna beams by the LCMV algorithm. And compare it with other two kinds of beams which are formed by the received data of 100 elements and the received data of full array respectively.

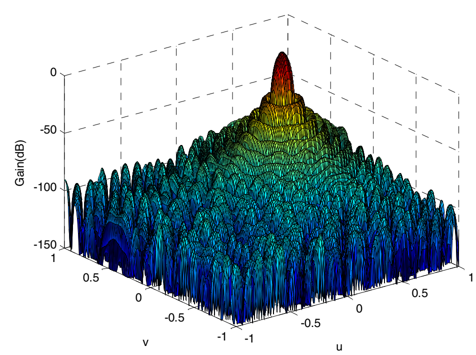

Fig. 4 The beam formed by the recovery data (SNR=10dB, INR=40dB)

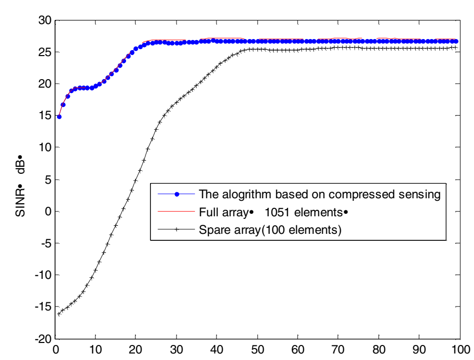

Fig. 5 Convergence in different methods (SNR=10dB, INR=40dB)

The beam formed by the recovery data is shown in

Fig. 4. It is shown that, when the number of the elements reduces from 1051 to 100 without reducing the antenna aperture, the side-lobe of the beam formed by the recovery data is -40dB, and the depth of nulls in the direction of the interferences are -94dB and -105dB. It has the similar performance with the beam formed by the full array (1051 elements) data. The curves in Fig. 5 show the output signal-to-interference-noise ratio (SINR) changes with snapshots in three different ways. We can see that, after the output SINR is convergence, the SINR achieved by the method proposed in this paper with the recovered data is smaller than it achieved with the original data of the full array, but much larger than it achieved with the original data of 100 elements. So the method is also applied to the iterative LCMV algorithm.

C. *The Monte Carlo analysis*

In order to verify the correctness of the algorithm in the cases of different SNRs, different INRs, and different directions of signals, we randomly select the directions of the desired signal and interferences, and perform 100 Monte Carlo simulations under the situations given below. The directions of the interferences are out of the main lobe.

Situation 1: SNR= 0~30dB，SIR= -30dB;

Situation 2: SNR= 10dB，SIR= -70~-10dB;

From situation 1, we find that, the recovery errors decrease when the input SNR increases. In situation 2, no matter how large the interferences are, the signal can be recovered accurately with SNR=10dB.

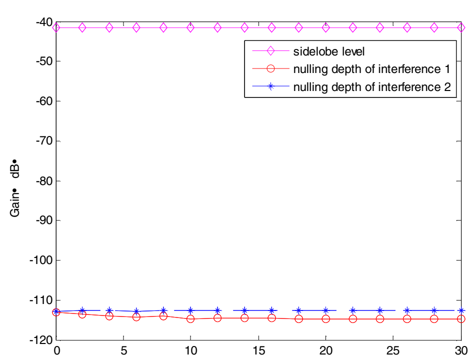

Fig. 6 The levels of sidelobe and depth of nulls in the direction of interference under different method

Then we do the digital beamforming with the recovery data. Fig. 6 shows the simulation results with SIR=-30dB. We can observe that, the side-lobes are low and the nulls are deep. When the SNR is low, the beam pattern is still good. But the recovered data may lose the desired signal. The recovered data can’t be used for signal processing. When the SNR is high, the data is accurate enough. As a result, the algorithm proposed is correct when SNR>0dB.

IV. CONCLUSION

A new adaptive digital beamforming in receiving end based on compressed sensing is presented. It is a new way to reduce the elements of the array without reducing the antenna aperture and deteriorating the performance of the beams. It greatly reduces the number of RF front-ends. According to the sparsity of the target in space, the algorithm accurately reconstructs the data of the whole channels of the full array by the method of compressed sensing, with the data received by the sparse array. And form the digital beams with the recovered data. Simulation results prove the effects of the algorithm. Simulation results show that the new adaptive digital beamforming algorithm is much better than the ordinary method with 100 elements and has the similar beam performance with the full array. When the signal is too weak, the algorithm fails because the desired signal is lost in the recovered data, so it can't be used for digital beamforming. As a result, the algorithm can only be used when signal-to-noise ratio is greater than 0 dB.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grants 11273017.

REFERENCES

[1] Candès E J, Wakin M B. An introduction to compressive sampling[J]. IEEE Signal Processing Mgazine, 2008, 25(2): 21-30.

[2] Ender J H G. On Compressive sensing applied to radar[J]. Signal Processing, 2010, 90: 1402-1414.

[3] Li Zhuo-fan, Yan Jing-wen. Theory and application of compressive sensing[J]. Microcomputer Applications, 2010, 31(3): 12-16.

[4] Dai Qiong-hai, Fu Chang-jun, Ji Xiang-yang. Research on compressed sensing[J], Chinese Journal of Computers, 2011, 34(3): 425-434.

[5] Herman M and Strohmer T. Compressed sensing radar[C]. IEEE International Conference on Acoustics Speech and Processing, Caesars Palace, Las Vegas, Nevada, USA. March 31-April 4, 2008, 1509-1512.

[6] Joel A. Tropp, and Anna C. Gilbert. Signal recovery from random measurements via orthogonal matching pursuit[J], IEEE Transactions on Information Theory, 2007, 53(12): 4655-4666.

[7] Richard Baraniuk, Mark Davenport, Ronald DeVore, Michael Wakin. A simple proof of the restricted isometry property for random matrices[J]. Constructive Approximation, 2008, 28(3): 253–263.

[8] Li Xiao-bo. The research of measurement matrix based on compressed sensing [D]. Beijing Jiaotong University, 2010.6.

[9] Zhou Chan-mei. Matching pursuit algorithm for signal reconstruction based on compressive sensing[D]. Beijing Jiaotong University, 2010.6.

[10] Yang Hai-rong, Zhang Cheng, Ding Da-wei, Wei Sui. The theory of compressed sensing and reconstruction algorithm[J]. Acta Eelctronic Sinica, 2011, 39(1): 142-148.

[11] Lloyd J. Griffiths, Hhiarles W. JIM. An alternative approach to linearly constrained adaptive beamforming[J], IEEE Transactions on Antennas and Propagation, 1982, 30(1): 27-34.

[12] Huang Fei, Sheng Wei-xing, Ma Xiao-feng, Wang Wei. Robust adaptive beamforming for large-scale arrays[J], Signal Processing, 2010, 90(1): 165-172.