STEERING INVARIANT ROBUST SIDELOBE CANCELLATION IMPLEMENTATION FOR LARGE ARRAYS

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ABSTRACT - Adaptive Beamforming (ABF) using a Conventional Beamformer (CBF) output interference estimator-subtractor (ES) is known as sidelobe cancellation (SC). SC forms a very low dimensional data vector, called the auxiliary vector, as an input to the adaptive ES function. After a treatment of robust and steering invariant SC background, the frequency dependent auxiliary vector formation from geometrically thinned, sparse array apertures is considered. The corresponding auxiliary aperture spans the full array aperture and consequently does not sacrifice the potential for mainlobe interference rejection while still maintaining essentially optimum SC. Robust steering invariant (SI) SC is compared to a full element space ABF.

1. 0 INTRODUCTION

Element space adaptivity for Adaptive Beamforming (ABF) is not a computationally efficient approach to sensor arrays with a very large number of sensors, $100 < N < 10{,}000$, and multiple octave bandwidths. Previous research [OT03-1,2] has focused on Steering Invariant Sidelobe Cancellation (SISC) ABF methods wherein the frequency of operation alone was used as the justification for efficient selection of the so-called reduced dimension auxiliary array data (vector). The auxiliary array is used to adaptively form an estimate of the coherent noise "interference" referenced to the output of a conventional time delay-and-sum beamformer (CBF). The CBF component of the SC uses all N sensors and ensures that the full array signal gain is provided at each frequency. This frequency-only approach results in using the outputs of summed groups of adjacent sensors as auxiliary array data where the group phase center separation is either equal to or less than a half wave-length at the frequency of operation. For linear array broadband systems, adaptive channel count reductions on the order of thirty-percent can be achieved relative to broadband element space adaptation.

In this paper, additional computational efficiency is achieved by using a sparse auxiliary array aperture of grouped sensors that is geometrically thinned before adaptive ES processing. Spatially linear-thinned composite auxiliary array aperture design methods are explored that allow the average spacing between groups to exceed one-half wave-length. Significant auxiliary array data vector dimension reduction is achieved and noise gain performance loss relative to the fully-populated group approach is minimal. Sparse array aperture designs are considered that provide the same potential for spatial resolution improvement as element space and contain an adequate dimensionality to counter situations with a large number of discrete interferers. The method described gives essentially full element space performance because the CBF component of the SISC provides robust maximum Array Signal Gain (ASG) and the reduced dimension SISC simply has to null the strong interference content to minimize Array Noise Gain (ANG).

The next section discusses a simple CBF-ABF convex blending approach to achieving robustness to ABF signal model mismatch. Section 3.0 reviews the highlights of robust steering invariant sidelobe cancellation (RSISC). Section 4.0 discusses auxiliary array selection for SC as a minimally redundant array [A00] aperture of thinned sensor groups. Next, the thinned auxiliary array RSISC as a function of auxiliary channel count is compared with full element space adaptivity in asymptotic performance.

2.0 ROBUST CBF-ABF BLENDING

In a minimum variance distortionless response (MVDR) ABF, robustness to model error in the steering vector, $\mathbf{d}(\theta)$, at frequency f can be achieved by iteratively adjusting an added diagonal loading term, α, in the sample estimate of the inter-sensor cross-spectral density matrix (CSDM), $\hat{\mathbf{R}}$. Explicitly, for beamformer weight vector

$$\mathbf{w}_{\mathrm{MVDR}}(\alpha) = \frac{[\hat{\mathbf{R}} + \alpha \mathbf{I}_{\mathrm{N}}]^{-1} \mathbf{d}(\theta)}{\mathbf{d}(\theta)' [\hat{\mathbf{R}} + \alpha \mathbf{I}_{\mathrm{N}}]^{-1} \mathbf{d}(\theta)}, \quad (2.1)$$

an α is selected to satisfy the white noise gain inequality constraint $|\mathbf{w}_{\mathrm{MVDR}}|^2 \le G$. In Eq. (2.1), $\mathbf{I}_N$ is the N-by-N identity matrix. This constraint limits signal suppression resulting from mismatch by forcing $\mathbf{w}_{\mathrm{MVDR}}$ to approach the more robust, uniformly spatially weighted CBF beamformer weight vector $\mathbf{d}(\theta)$ as α becomes large. It is observed that for larger values of α, which correspond to smaller values of G, the spatial response of the constrained, less "aggressive" adaptive beamformer has high side lobes beginning at -13.2 dB below the main beam maximum response. If there are strong interfering sources in this sidelobe region of the constrained quiescent beam pattern, the signal-to-interference-plus-noise-ratio, SINR, may be unacceptable.

As an alternative to diagonal loading, the convex [C02] beamformer weighting vector linear combination [O02]

$$\mathbf{w} = (1-\beta)\,\mathbf{h}(\theta) + \beta\,\mathbf{w}_{\mathrm{MVDR}}(\alpha=0) \qquad (2.2)$$

defines a blend of a spatially windowed, low sidelobe CBF, $\mathbf{h}(\theta)$, and a fully aggressive MVDR ABF. The blending control rule on a beam-to-beam basis is

$\beta = 1$, for $|\mathbf{w}_{\mathrm{MVDR}}(\alpha=0) - \mathbf{h}(\theta)|^2 < G_{\mathrm{mse}}$

and

$$\beta = \left\{ \frac{G_{\mathrm{mse}}}{|\mathbf{w}_{\mathrm{MVDR}}(\alpha=0) - \mathbf{h}(\theta)|^2}, \right. \quad \text{for } |\mathbf{w}_{\mathrm{MVDR}}(\alpha=0) - \mathbf{h}(\theta)|^2 \ge G_{\mathrm{mse}} \qquad (2.3)$$

For comparison of diagonal loading (DL) and convex blending (CB), consider an array of N = 48 sensors that are uniformly spaced at one-half wave length at frequency f = 1.0 Hz. There is a desired signal with an SNR = 3 db at azimuth cosine -0.5 and an interference with INR = 40 is swept from azimuth cosine -1.0 to 1.0. Steering vector mismatch imposes a -50 db Gaussian complex, additive perturbation to the steering vector $\mathbf{d}(\theta)$. The adapted beam output power due to interference alone, Figure 1, and SINR, Figure 2, for a Hann spatial window CBF, unconstrained MVDR, DL and CB versus interference arrival angle cosine are presented. In Figures 1 and 2, the quiescent sidelobe levels can be compared because the constraint parameters, G and G_{mse} are such that the width of the mainlobe varies over approximately the same range. It can be seen that the sidelobe response

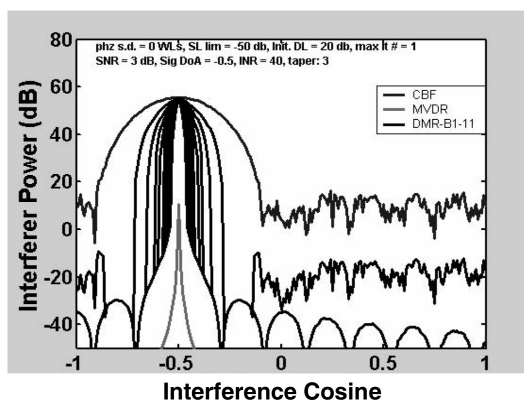

Figure 1a. Desired source beam interference output power versus interference arrival angle cosine for windowed CBF, unconstrained MVDR and robust MVDR with convex blending (CB) robustness.

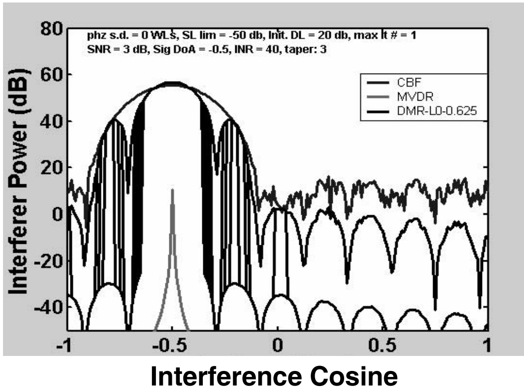

Figure 1b. Desired source beam interference output power versus interference arrival angle cosine for windowed CBF, unconstrained MVDR and robust MVDR with diagonal loading (DL) robustness.

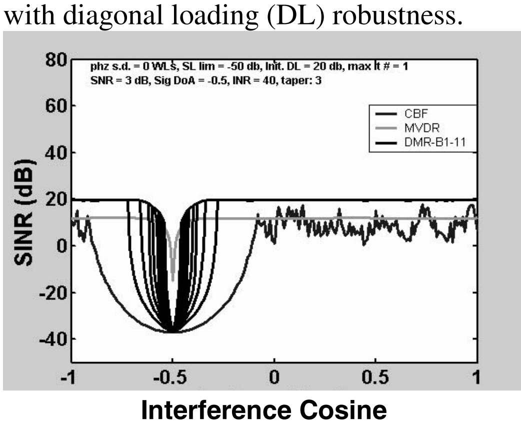

Figure 2a. Desired source beam SINR versus interference arrival angle cosine for windowed CBF, unconstrained MVDR and robust MVDR with convex blending (CB).

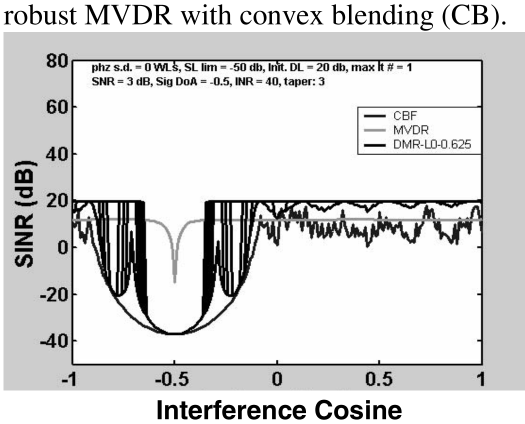

Figure 2b. SINR versus interference arrival angle cosine comparison of MVDR and robust MVDR with diagonal loading.

region is consistently lower and the SINR consistently higher for CB robustness relative to MVDR ABF with DL.

3.0 STEERING-INVARIANT (SI) SIDELOBE CANCELLATION (SC)

A functional diagram of the CB robust SISC ABF considered is illustrated in Figure 3. A four octave band coverage resulting in at greater than an 8:1 dimensionality reduction is achieved at the frequency dependent grouping output **z** relative to the N-dimensional element "snapshot" vector **x**. This reduction is obtained, first, by a simple frequency dependent grouping that sums outputs of adjacent elements in $\mathbf{S}_a$ into **u** and, second, sparse selection of the grouped sensor outputs by $\mathbf{S}_t$ summed into **z**. The adaptive ES function is performed on the auxiliary array data vector, **z**, and the matrix inversion required, $\mathbf{R}_{zz}^{-1}$, is applied to all steering locations. Hence, the reference to steering invariant sidelobe cancellation is adopted. A signal-free noise reference for a desired signal with steering vector $\mathbf{d}_0$ for ES noise cancellation is provided by the steering dependent single point constraint nulling matrix, $\mathbf{P}_0$. Additional robustness is achieved by linear convex blending of the SISC output with the output of a conventional beamformer (CBF) having a desired quiescent beam pattern determined by the spatial window matrix **G**. The mainlobe maintenance (MLM) function of blending a CBF and ABF output described in Section 2.0 is only activated when a high level source is within the mainlobe and is mismatched with respect to the desired maximum response axis. The vector $\mathbf{r}_{zy\_0} = E\{\mathbf{z}y_{\_0}^*\}$ where E{ } and * are the statistical expectation and complex conjugation operators respectively. All other matrix-vector quantities are defined in Figure 3. The sidelobe cancellation vector, $\mathbf{w}_{sc}$, minimizes the variance of the noise cancelled output y [OT03-1,2].

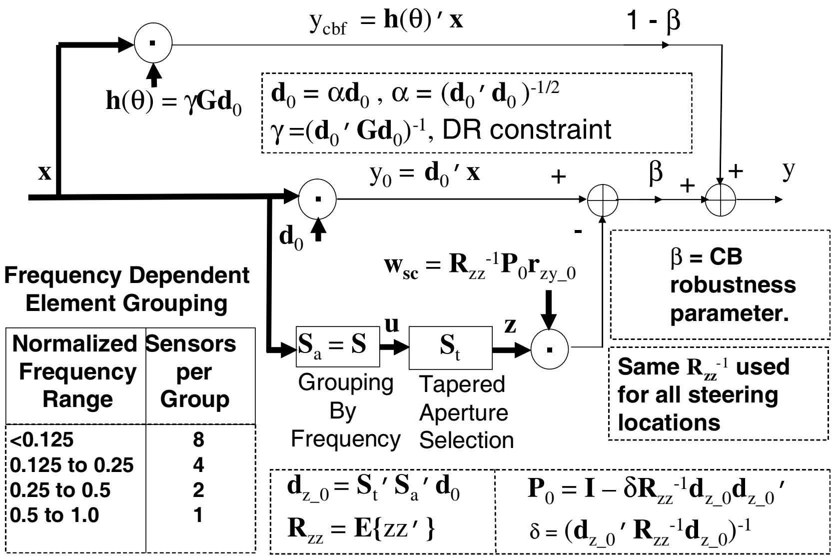

Figure 3. Functional diagram of Robust Steering Invariant Sidelobe Cancellation (RSISC) based Adaptive Beamformer (ABF)

4.0 THINNED AUXILIARY ARRAY

Consider a linear array of N sensors that are uniformly spaced at a distance of one-half wave length at frequency f = 1.0. Two types of minimally redundant sparse aperture, thinned auxiliary arrays are presented: (1) M_i linearly spaced sensor groups inside and $M_0/2$ geometrically thinned sensor groups outside designated as LI-M_i/TO-M_0 and (2) $M_0/2$ linearly spaced sensor groups on the outside and M_i sensor groups symmetrically and geometrically thinned on the inside designated as LO-M_0/TI-M_i. Examples of these two types of thinned auxiliary apertures are illustrated in Figure 4. Even though steered beams are not actually formed in the adaptive processing of the SISC auxiliary aperture, the beam patterns for broadside steered beams for each of these two thinning procedures at frequency 0.0625 are shown in Figure 5. Using the same total number of sensor groups, the LI/TO design exhibits lower side lobes than the LO/TI sparse group design which has a more narrow mainlobe. These characteristics would make the LI/TO auxiliary aperture more effective in nulling an interference in the sidelobe and LO/TI more effective in the

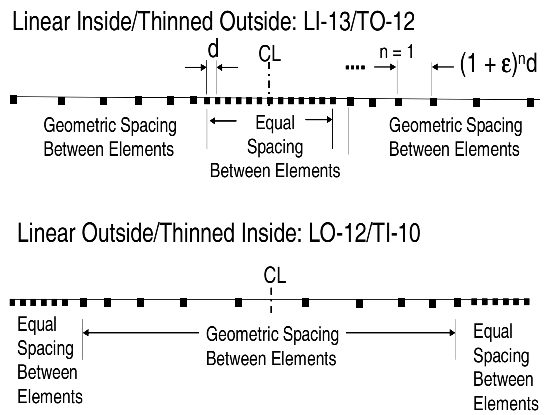

Figure 4. (top) LI-13/TO-12 and (bottom) LO-12/TI-10 auxiliary designs

mainlobe. Figure 6 shows the asymptotic, i.e. infinite time average, beam power response versus beam steering azimuth cosine for an N = 384 example and twelve sources at frequency f = 0.0625. The dimensionality reduction consists of first summing sensors into 48 groups of 8 (8:1). In Figure 6, an LI-24/TO-16 auxiliary array aperture design is used. The locations of the twelve sources are indicated by the arrows and the SNRs referenced to a single sensor are given beside the arrow. This LI/TO design is seen to give resolution equivalent

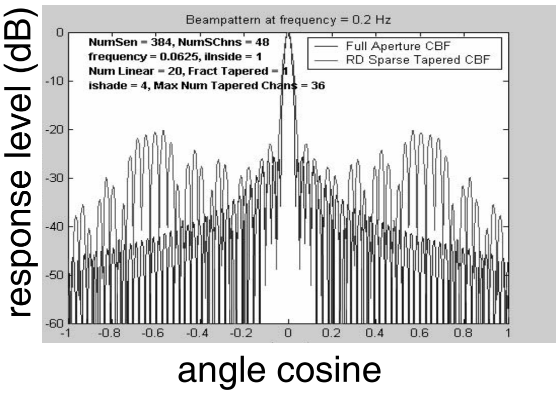

Figure 5a. Broadside steered beam pattern for LI-20/TO-16 sparse auxiliary formed from 40 groups of 8 sensors per group from a linear array of 384 sensors.

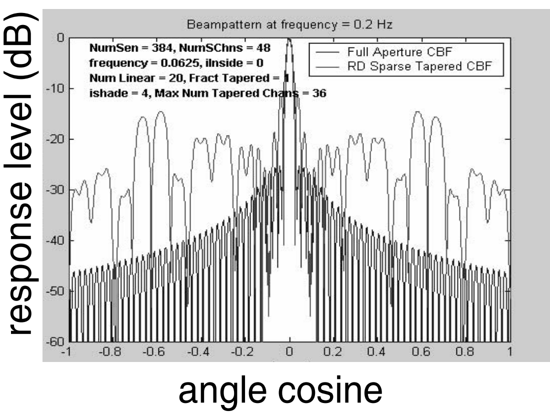

Figure 5b. Broadside steered beam pattern for LO-20/TI-16, sparse auxiliary array formed from 40 groups of 8 sensors per group from a linear array of 384 sensors.

to the full element space MVDR with an order of magnitude fewer adaptive channels and nearly three orders of magnitude reduction in computational burden required for CSM matrix inversion. The highest level (30 dB) source at azimuth cosine 0.5 is less than one beam width from the low level (10 dB) source.

5.0 SUMMARY

The method of Robust Steering Invariant Sidelobe Cancellation (RSISC) has been extended to include the use of a sparse auxiliary aperture to estimate-subtract the

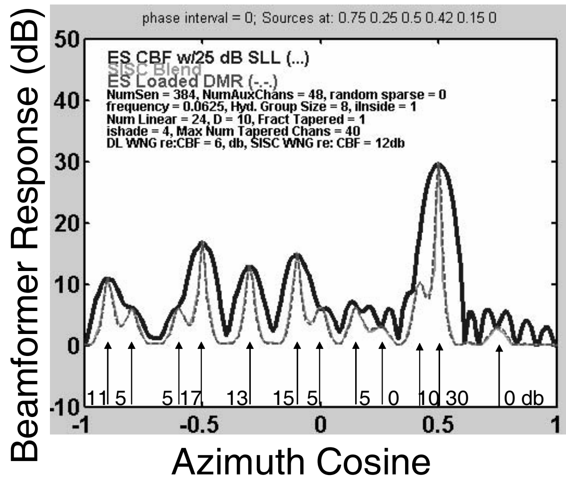

Figure 6. RSISC azimuthal response to 12 sources with a LI-24/TO-16 auxiliary aperture.

dominant interfering source waveforms in the output of a beam steered at a desired location. This extension further enhances the efficiency of the RSISC technique and need not compromise interference nulling performance and array gain.

References

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