Sparse Microphone Array Design for Wideband Beamforming

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***Abstract*—The sparse wideband sensor/microphone array design problem is highly nonlinear and it is traditionally solved by genetic algorithms, simulated annealing or other similar optimization methods. This is an extremely time-consuming process and an optimum solution is not guaranteed. In this work, this problem is studied from the viewpoint of compressive sensing (CS) and a CS-based method is provided. Although there have been CS-based methods proposed for the design of narrowband arrays, its extension to the wideband case is not straightforward, as there are multiple coefficients associated with each sensor/microphone and it is not sufficient to simply minimize the l_1 norm of the weight coefficients to obtain a sparse array solution. To achieve this, a modified l_1 norm minimization method is derived and its effectiveness is verified by design examples.**

***Index Terms*—Sparse array, wideband beamforming, microphone array, compressive sensing.**

I. Introduction

Wideband beamforming has been studied extensively in the past, in particular in the area of microphone arrays [1], [2], [3]. It is well-known that in order to avoid the spatial aliasing problem for uniform linear arrays (ULAs), the adjacent sensor/microphone spacing has to be less than half the minimum operating wavelength corresponding to the highest frequency of the signal of interest. This can be problematic when considering arrays with a large aperture size, due to the cost associated with the number of sensors/microphones required. As a result, sparse arrays are a desirable alternative [4], which allow adjacent sensor separations greater than half a wavelength, while still avoiding grating lobes due to the randomness of sensor locations. Moreover, even with the same number of sensors and a similar aperture size, the nonuniform nature of a sparse array also provides more degrees of freedom to achieve a better beam response.

To optimize sensor/microphone locations, in [5], a hybrid approach combining a gradient based method and a genetic algorithm (GA) is used to design a sparse microphone array. This method achieves comparable results to standard array configurations. In [6], simulated annealing (SA) and intra block Monte Carlo (IBMC) algorithms are used to design microphone arrays for near and far field scenarios. Similar methods have also been used for the design of other types of sensor arrays. For example, GAs are used in the work of [7], [8], while SA is employed in [9]. The disadvantage of these types of method is the potentially extremely long computation time and the possibility of converging to a non-optimal solution.

More recently, the area of compressive sensing (CS) has been explored [10], and CS-based methods have been proposed in the design of narrowband sparse arrays [11], [12], [13], [14]. CS theory tells us that if certain conditions are met it is possible to recover some signals from fewer measurements than are used by traditional methods [10]. This can then form the basis of sparse array design methods by trying to attain an exact, or almost exact, match to a desired response while using as few sensors as possible. This is achieved by minimising the l_1 norm of the weight coefficients, subject to the error between the desired and designed responses being below a predefined level.

To the best of our knowledge, there has not been any work reported in the design of sparse wideband beamformers using CS-based methods. It is not straightforward to extend the narrowband design method to the wideband case, as there are tapped delay-lines (TDLs) or FIR/IIR filters associated with each received wideband signal, and for a wideband array to be sparse all coefficients along the TDL associated with an individual sensor have to be equal to zero. As a result it is not sufficient to simply minimize the l_1 norm of the weight

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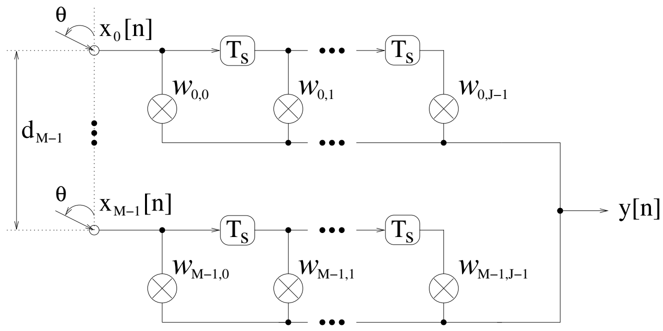

Fig. 1. A general wideband beamforming structure with a TDL length J.

coefficients. Instead all of the weight coefficients along a TDL have to be simultaneously minimized. In order to achieve this, a method similar to the technique employed in complex-valued l_1 norm minimization is proposed here [15].

The remainder of this paper is structured as follows. Section II gives details of the proposed design method, including the array model being used in Section II-A, and the CS-based process for designing sparse wideband arrays in Section II-B. Three design examples are provided in Section III and conclusions are drawn in section IV.

II. Proposed Design Method

A. Array Model

A general linear array structure for wideband beamforming with a TDL length J is shown in Fig. 1, where T_s is the sampling period or temporal delay between adjacent signal samples [16], [17]. Here we assume that all of the sensors or microphones are omnidirectional and have the same response. The beamformer output $y[n]$ is a linear combination of differently delayed versions of the received array signals $x_m[n]$, $m = 0, \cdots, M-1$. A plane-wave signal model is assumed, i.e. all the speech signals impinge upon the array from the far field. The distance from the zeroth microphone to the subsequent microphones is denoted by d_m for $m = 0, \cdots, M-1$, where $d_0 = 0$ as it is the distance from the zeroth microphone to itself. Fig. 1 also shows an incident signal arriving at an angle θ.

The steering vector of the array as a function of the normalized frequency $\Omega = \omega T_s$ and the angle of arrival θ is

$$\begin{aligned} \mathbf{s}(\Omega,\theta) = & [1, \cdots, e^{-j\Omega(J-1)}, \\ & e^{-j\Omega\mu_1\cos(\theta)}, e^{-j\Omega(\mu_1\cos(\theta)+1)}, \\ & \cdots, e^{-j\Omega(\mu_1\cos(\theta)+(J-1))}, \\ & \cdots, e^{-j\Omega(\mu_{M-1}\cos(\theta)+(J-1))}]^T. \end{aligned} \quad (1)$$

where $\mu_m = \frac{d_m}{cT_s}$ for $m = 0, 1, \cdots, M-1$ and $\{\cdot\}^T$ indicates the transpose operation.

The response of the array is then given by

$$P(\Omega,\theta) = \mathbf{w}^H \mathbf{s}(\Omega,\theta), \quad (2)$$

where $\{\cdot\}^H$ is the Hermitian transpose and the coefficient vector $\mathbf{w}$ is given by

$$\mathbf{w} = [\mathbf{w}_0^T \ \ \mathbf{w}_1^T \ \ \ldots \ \ \mathbf{w}_{M-1}^T]^T \ , \quad (3)$$

with

$$\mathbf{w}_m = [\mathrm{w}_{m,0} \ \ \mathrm{w}_{m,1} \ \ \ldots \ \ \mathrm{w}_{m,J-1}]^T. \quad (4)$$

B. CS-Based Design

As previously mentioned, CS has been employed in the design of sparse narrowband arrays by trying to match the array's response to a desired/reference one, $P_r(\Omega,\theta)$. Extending the design to the wideband case, we first consider Fig. 1 as being a grid of potential active microphone locations. In this instance d_{M-1} is the maximum aperture of the array and the values of d_m, for $m = 1, 2, \ldots, M-2$, are selected to give a uniform grid, with M being a large enough number to cover all potential locations of the microphones. Sparseness is then introduced by selecting the set of weight coefficients to give as few active locations as possible, while still having a designed response close to the desired one.

In the first instance, this problem can be formulated as

$$\begin{aligned} & \min && ||\mathbf{w}||_0 \\ & \text{subject to} && ||\mathbf{p}_r - \mathbf{w}^H\mathbf{S}||_2 \leq \alpha \ , \end{aligned} \quad (5)$$

where $||\mathbf{w}||_0$ is the number of nonzero weight coefficients in $\mathbf{w}$, $\mathbf{p}_r$ is the vector holding the desired beam response at sampled frequency points Ω_k and angle θ_l, $k = 0, 1, \cdots, K-1$, $l = 0, 1, \cdots, L-1$, $\mathbf{S}$ is the matrix composed of the steering vectors at the corresponding frequency Ω_k and angle θ_l, and α places a limit on the allowed difference between the desired and the designed responses. In this constraint $||\cdot||_2$ denotes the l_2 norm.

In detail, $\mathbf{p}_r$ and $\mathbf{S}$ are respectively given by

$$\begin{aligned} \mathbf{p}_r = & [P_r(\Omega_0,\theta_0), \cdots, P_r(\Omega_0,\theta_{L-1}), \\ & P_r(\Omega_1,\theta_0), \cdots, P_r(\Omega_1,\theta_{L-1}) \\ & ..., P_r(\Omega_{K-1},\theta_{L-1})] \end{aligned}$$

and

$$\begin{aligned} \mathbf{S} = & [\mathbf{s}(\Omega_0,\theta_0), \cdots, \mathbf{s}(\Omega_0,\theta_{L-1}), \\ & \mathbf{s}(\Omega_1,\theta_0), \cdots, \mathbf{s}(\Omega_1,\theta_{L-1}), \cdots, \mathbf{s}(\Omega_{K-1},\theta_{L-1})]. \end{aligned}$$

Here the desired response $P_r(\Omega,\theta)$ can be obtained from that of a traditional uniform linear array, or simply assumed to be an ideal response, i.e., one at the mainlobe area and zero for the sidelobe area.

In practice, the cost function in (5) will be replaced by the l_1 norm [10],

$$\begin{array}{ll} \min & ||\mathbf{w}||_1 \\ \text{subject to} & ||\mathbf{p}_r - \mathbf{w}^H\mathbf{S}||_2 \leq \alpha \ . \end{array} \tag{6}$$

The above formulation is effective in the design of narrowband arrays, where the TDL length $J = 1$ (i.e. each sensor has only one weight coefficient attached) and the number of nonzero coefficients will be the same as the number of active sensors. In other words, any coefficient with a zero value will mean that the associated sensor is inactive. However, in the wideband case, solving (6) will not guarantee a sparse solution. This is due to there being a TDL length of $J > 1$, with multiple weight coefficients associated with each microphone location. The minimization problem in (6) just looks to have as few nonzero weight coefficients as possible without considering which TDL they are on.

In order to guarantee a sparse solution, the weight coefficients along a TDL have to be simultaneously minimized. When all coefficients along a TDL are zero-valued, we can then consider the corresponding location to be inactive and sparsity is introduced. To achieve this, similar to the technique used in complex-valued l_1 norm minimization [15], we minimize a modified l_1 norm as follows,

$$\begin{array}{ll} \min & t \ \epsilon \ \mathbb{R}^+ \\ \text{subject to} & ||\mathbf{p}_r - \mathbf{w}^H\mathbf{S}||_2 \leq \alpha \\ & |\langle\mathbf{w}\rangle|_1 \leq t \end{array} \tag{7}$$

where

$$|\langle\mathbf{w}\rangle|_1 = \sum_{m=0}^{M-1} \left|\left| \begin{bmatrix} w_{m,0} \\ \vdots \\ w_{m,J-1} \end{bmatrix} \right|\right|_2 . \tag{8}$$

Now we decompose t to $t = \sum_{m=0}^{M-1} t_m$, $t_m \epsilon \ \mathbb{R}^+$. In vector form, we have

$$t = [1, \cdots, 1] \begin{bmatrix} t_0 \\ \vdots \\ t_{M-1} \end{bmatrix} = \mathbf{1}^T\mathbf{t}. \tag{9}$$

Then (7) can be rewritten as

$$\begin{array}{ll} \min\limits_{\mathbf{t}} & \mathbf{1}^T\mathbf{t} \\ \text{subject to} & ||\mathbf{p}_r - \mathbf{w}^H\mathbf{S}||_2 \leq \alpha. \\ & \left|\left| \begin{bmatrix} w_{m,0} \\ \vdots \\ w_{m,J-1} \end{bmatrix} \right|\right|_2 \leq t_m, \quad m = 0, \cdots, M-1. \end{array} \tag{10}$$

Define

$$\hat{\mathbf{w}} = [t_0, w_{0,0}, \cdots, w_{0,J-1}, t_1, \cdots, w_{M-1,J-1}]^T, \tag{11}$$

$$\hat{\mathbf{c}} = [1, \mathbf{0}_J, 1, \mathbf{0}_J, \cdots, \mathbf{0}_J]^T \tag{12}$$

and

$$\begin{aligned} \hat{\mathbf{s}}(\Omega,\theta) \quad = \quad & [0, 1, \cdots, e^{-j\Omega(J-1)}, \\ & 0, e^{-j\Omega\mu_1\cos(\theta)}, e^{-j\Omega(\mu_1\cos(\theta)+1)}, \cdots, \\ & e^{-j\Omega(\mu_1\cos(\theta)+(J-1))}, \\ & \cdots, e^{-j\Omega(\mu_{M-1}\cos(\theta)+(J-1))}]^T, \end{aligned} \tag{13}$$

where $\mathbf{0}_J$ is an all-zero $1 \times J$ row vector. A matrix $\hat{\mathbf{S}}$ similar to $\mathbf{S}$ can be created from $\hat{\mathbf{s}}$, given by

$$\begin{aligned} \hat{\mathbf{S}} \quad = \quad & [\hat{\mathbf{s}}(\Omega_0,\theta_0), \cdots, \hat{\mathbf{s}}(\Omega_0,\theta_{L-1}), \\ & \hat{\mathbf{s}}(\Omega_1,\theta_0), \cdots, \hat{\mathbf{s}}(\Omega_1,\theta_{L-1}), \cdots, \hat{\mathbf{s}}(\Omega_{K-1},\theta_{L-1})]. \end{aligned}$$

Now we arrive at the final formulation for the sparse microphone array design problem

$$\begin{array}{ll} \min\limits_{\hat{\mathbf{w}}} & \hat{\mathbf{c}}^T\hat{\mathbf{w}} \\ \text{subject to} & ||\mathbf{p}_r - \hat{\mathbf{w}}^H\hat{\mathbf{S}}||_2 \leq \alpha. \\ & \left|\left| \begin{bmatrix} w_{m,0} \\ \vdots \\ w_{m,J-1} \end{bmatrix} \right|\right|_2 \leq t_m, \quad m = 0, \cdots, M-1. \end{array} \tag{14}$$

This problem can be solved using cvx, a package for specifying and solving convex programs [18], [19].

III. Design Examples

In this section three design examples will be presented in order to verify the effectiveness of the proposed method. This includes two broadside mainbeam examples and one off-broadside example. In all examples the value of λ is the wavelength associated with a normalized frequency of $\Omega = \pi$. If the highest frequency of interest is 10KHz and the sampling frequency is 20KHz, it gives a wavelength of 3.4cm at a speed of 340m/s.

TABLE I
MICROPHONE LOCATIONS FOR THE FIRST DESIGN EXAMPLE.

n	d_n/λ	n	d_n/λ
1	0	9	5.81
2	0.81	10	5.96
3	1.62	11	6.72
4	2.42	12	7.58
5	3.28	13	8.38
6	4.09	14	9.19
7	4.24	15	10
8	5.00		

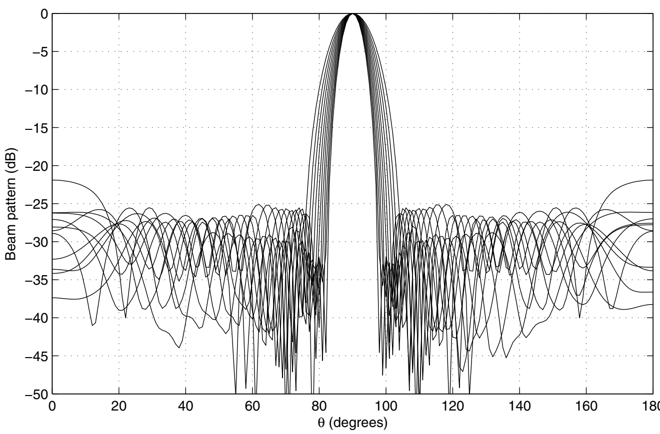

Fig. 2. Response of the first designed array.

A. Design Example 1: Broadside Mainbeam

In this example, an ideal array response is used as the desired one, with the main beam located at $\theta_m = 90^0$ and sidelobe region at $\theta_s = [0^0, 80^0] \bigcup [100^0, 180^0]$, which is sampled every 1° in the design. The normalized frequency range of interest is $\Omega_I \in [0.5\pi, \pi]$ and is sampled every 0.05π with a TDL length of $J = 15$. The grid of 100 potential microphone locations is spread over an aperture of 10λ or 34 cm. With a value of $\alpha = 10$, the resultant 15 active locations obtained by the proposed design are shown in Tab. I.

Fig. 2 shows its beam response at sampled normalized frequencies. For each frequency, the main beam appears at the desired location $\theta_m = 90^0$ and there is also sufficient attenuation in the sidelobe regions.

B. Design Example 2: Broadside Mainbeam

For this example, the normalized frequency range has been increased to $[0.3\pi, \pi]$, with 15 sampled frequency points and a TDL length of $J = 20$ is used. Again the desired mainlobe is at $\theta_m = 90^0$, with the sidelobe region at $\theta_s = [0^0, 80^0] \bigcup [100^0, 180^0]$, being sampled every 1°. The grid of 200 potential microphone locations is spread over an aperture of 20λ or 68 cm. The value of α is 13. As a result, a 32-microphone array is obtained and their locations are shown in Tab. II. Its beam response is shown in Fig. 3, with a desired main beam direction and sufficient attenuation for the sidelobe areas.

TABLE II
MICROPHONE LOCATIONS FOR THE SECOND DESIGN EXAMPLE.

n	d_n/λ	n	d_n/λ
1	0	17	10.10
2	0.80	18	10.90
3	1.71	19	11.06
4	2.61	20	11.81
5	3.52	21	11.96
6	4.37	22	12.71
7	5.28	23	12.86
8	5.43	24	13.72
9	6.28	25	14.62
10	7.19	26	14.77
11	7.34	27	15.63
12	8.09	28	16.48
13	8.24	29	17.39
14	9.00	30	18.29
15	9.14	31	19.20
16	9.90	32	20

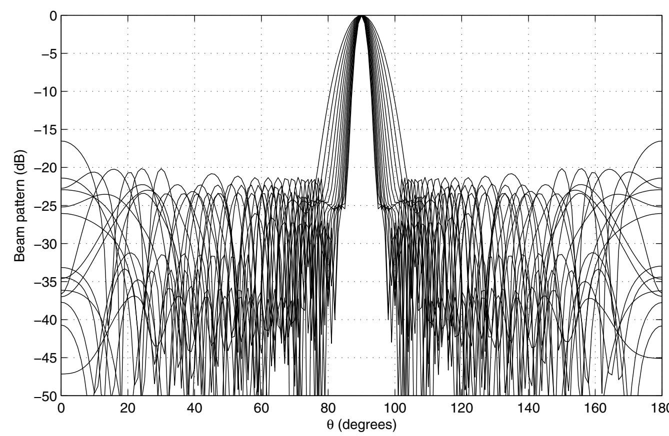

Fig. 3. Response of the second designed array.

C. Design Example 3: Off-Broadside Mainbeam

For this example, the normalized frequency range is $[0.4\pi, 0.9\pi]$, with 11 sampled frequency points and a TDL length of $J = 25$ is used. The desired mainlobe direction is at $\theta_m = 125^0$ and the sidelobe region is given by $\theta_s = [0^0, 115^0] \bigcup [135^0, 180^0]$, sampled every 1° in the design. The grid of 100 potential microphone locations is spread over an aperture of 10λ or 34 cm. The value of α is 9. An 18-microphone array is obtained and their locations are shown in Tab. III. Its beam response is shown in Fig. 4. The desired main

TABLE III
MICROPHONE LOCATIONS FOR THE THIRD DESIGN EXAMPLE.

n	d_n/λ	n	d_n/λ
1	0	10	5.15
2	0.61	11	5.61
3	1.26	12	6.31
4	1.87	13	6.92
5	2.47	14	7.53
6	3.08	15	8.13
7	3.74	16	8.79
8	4.39	17	9.44
9	5.00	18	10

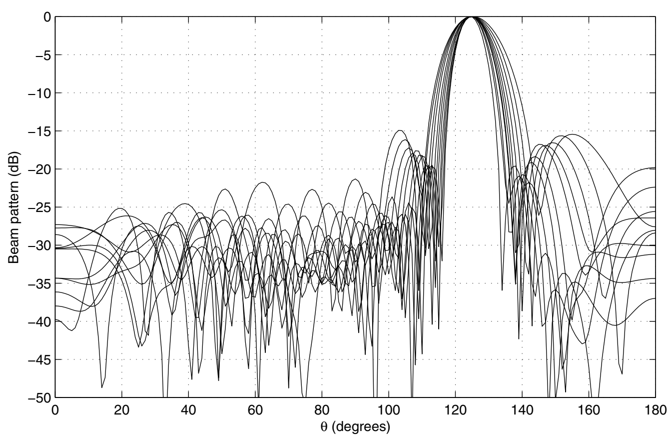

Fig. 4. Response of the third designed array.

beam direction has been achieved at all normalised frequencies other than $\Omega = 0.9\pi$ where the mainbeam is roughly at $124°$, but still very close to the designed main beam direction. At all normalised frequencies sufficient sidelobe attenuation has been reached, as in previous two examples.

IV. CONCLUSIONS

In this paper, a compressive sensing based method for the design of wideband sparse sensor/microphone arrays has been proposed. Although CS-based methods have been proposed for the design of narrowband arrays, their extension to the wideband case is not straightforward, as there are multiple coefficients along a TDL associated with each sensor and it is not sufficient to simply minimize the l_1 norm of the weight coefficients to obtain a sparse array solution. Instead all the weight coefficients along a TDL have to be simultaneously minimized. In order to achieve this, a modified l_1 norm minimization method has been derived. Three design examples have been provided to verify the effectiveness of the method.

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