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Wideband Beamforming Based on Compressive Sensing

Rui Du, Yangyu Fan, and Jianshu Wang

***Abstract*—Wideband beamforming is of high significance in the field of array signal processing. However, most of the present methods would fail when some sensors of the receive array are not working and this situation occurs frequently in practical experiments. To address the problem, a new method of wideband beamforming based on Compressive Sensing(CS) is proposed. The received signal is first split into sub-band signals, then the full-array-signal is constructed on each subband and processed with beamforming technique. Finally signals are synthesized to get the wideband signal. Experimental result show the great performance of our proposed method and some additional examples are designed to further verify its effectiveness.**

I. INTRODUCTION

Wideband beamforming [1] is a technique that estimates the wideband signal of interest, which arrives from many directions with an array consisting of sensors in a noisy and interfering environment. In order to achieve a good performance, an array consisting of large number of sensors is usually adopted. However, the situation that some or even most of the sensors of the array are not working, occurs frequently in practice. In these occasions, most of the traditional methods cannot maintain their performances and fail to get beaforming results due to the lack of received signals.

Compressive Sensing (CS) theory [2,3,4] has received increasingly attention in these years. Recently, CS has been used to design a sparse array whose sensors' number is much less than an ordinary array to achieve high performance [5,6,7]. The adjacent sensor spacing could be larger than half of the minimum operating wavelength corresponding to the highest frequency of the signal of interest, which makes the design of the array much more flexible and feasible. According to the methods proposed in these papers, the numbers and locations of the sensors for the sparse array are determined by algorithms and cannot be changed once decided. Therefore, when some of the sensors break down, the beamformer fails to maintain its performance.

According to CS [2], sparse signals could be recovered precisely from fewer measurements than traditional methods. This idea has been employed in the estimation of Direction of Arrival (DOA) based on CS [8,9,10], which splits the entire spatial region into many small parts to estimate the DOA by recovering a spatial-sparse coefficient from the signals received with an array and judge the angle with the spatial-sparse coefficient. The reconstruction of sparse coefficient is the key in this idea where the coefficient could be recovered only after the sparse sampling procedure. Furthering the sparse sampling during CS, we find it is possible to recover the full array signal from the signal received with an array even when some of its sensors are inactive.

Therefore, we propose a novel beamforming algorithm based on CS to solve the aforementioned inactive sensor problem. The full-array signal is first recovered from the signal received with a partial active array on each sub-band. Then we reconstruct the full-array signals with narrow band beamforming techniques and synthesis all these sub-band signals to get the result.

The rest of this paper is structured as follows. Section Ⅱ presents our proposed method, including a simple sub-band beamforming structure, the signal model, the sparse reconstruction in CS and a beamforming technique. Finally, the examples are provided in Section Ⅲ and the conclusions are drawn in Section Ⅳ.

II. PROPOSED METHOD

A. *Subband Beamforming Structure*

The basic idea of sub-band beamforming is to split the received signals into sub-band signals and process them on each subband respectively with different beamfomers. The transformation of the wideband beamforming problem into a narrow band beamforming problem has been studied extensively to achieve good results.

Figure 1 shows a general subband beamforming structure, in which the array has M sensors which decomposes every signal into K subbands by an analysis filter bank. Then the K beamformers are set into each subband to fulfill the beamforming and finally get the fullband output $y(n)$ through a synthesis filter bank. In Figure 1, the blocks labeled by 'A' are analysis filter banks and the one labeled 'S' is synthesis filter bank.

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Rui Du is with the School of Electronics and Information, Northwestern Polytechnical University, Xi'an, Shaanxi, 710129, China (e-mail: rdu@mail.nwpu.edu.cn).

Yangyu Fan is with the School of Electronics and Information, Northwestern Polytechnical University, Xi'an, Shaanxi, 710129, China (e-mail:fan_yangyu@sina.com).

Jianshu Wang is with the School of Electronics and Information, Northwestern Polytechnical University, Xi'an, Shaanxi, 710129, China (e-mail:565563085@163.com).

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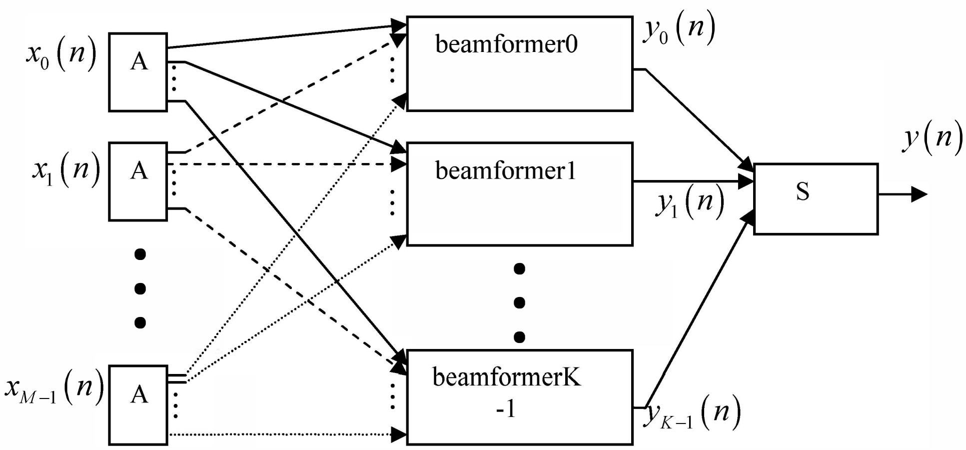

Fig. 1 A general subband beamforming structure

B. *Signal Model*

Considering an uniform linear array of M sensors with D signal sources placed in different positions in far field, the input signals can be regarded as plane-wave, as showed in Fig.

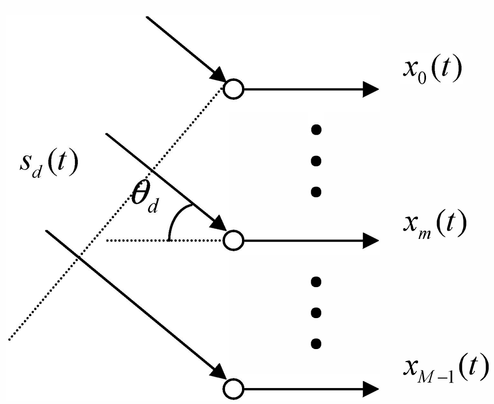

Fig. 2. A general array with $s_d(t)$ impinging from angle θ_d

The signal received at the mth sensor with D sources could be expressed by (1):

$$x_m(t) = \sum_{d=0}^{D-1} \beta_d s_d\left[t - \tau_m(\theta_d)\right] + v_m(t) \tag{1}$$

where $m = 0, \ldots, M-1$, β_d is the attenuation factor for source d, v_m is the noise received at the mth sensor.

One way to process wideband signal is to split the wideband signal into narrowband signals and process each narrowband signal with narrowband beamforming techniques. Hence, (1) could be transformed into a narrowband form at frequency ω as in (2).

$$x_m(t) = \sum_{d=1}^{D} \beta_d s_d(t) a_m(\theta_d) + v_m(t) \tag{2}$$

where $a_m(\theta_d) = e^{-j\omega\frac{2d\sin\theta_d}{c}}$, (2) could be expressed in matrix form, regardless of the attenuation factors as following:

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{v}(t) \tag{3}$$

where $\boldsymbol{x}(t) = [x_0(t), x_1(t), \cdots, x_{M-1}(t)]^{\mathrm{T}}$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_0), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_{D-1})]$$

$$\boldsymbol{a}(\theta_d) = [1, e^{-j\omega\frac{d\sin\theta_d}{c}}, \cdots, e^{-j\omega\frac{(M-1)d\sin\theta_d}{c}}]^{\mathrm{T}}$$

$$\boldsymbol{s}(t) = [s_0(t), s_1(t), \cdots, s_{D-1}(t)]^{\mathrm{T}}$$

$$\boldsymbol{v}(t) = [v_0(t), v_1(t), \cdots, v_{M-1}(t)]^{\mathrm{T}}$$

Equation (3) could be transformed into a discrete-time form with $t = n \times t_s$, t_s is time sampling interval:

$$\boldsymbol{x}(n) = \boldsymbol{A}\boldsymbol{s}(n) + \boldsymbol{v}(n) \tag{4}$$

C. *Compressive Sampling and Signal Reconstruction*

The received signals come from D different angles in far field. The angle region $[-90°, 90°]$ could be split into $2N$ parts and we suppose the signals are coming from D positions among all of these directions. As shown in Fig. 3, all the points represent potential positions and black points mean actual sources.

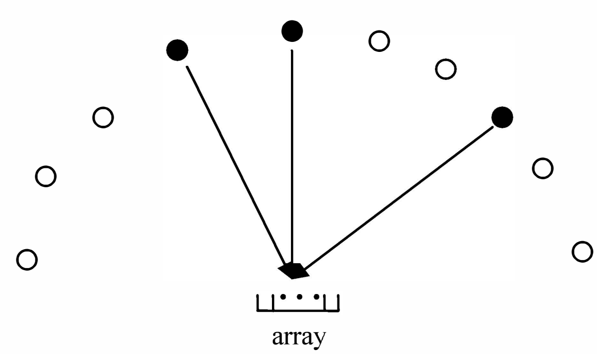

Fig. 3 Spatial sparsity of the signal

Normally we can get $D \ll 2N$ if N is large enough, which means there are only D sources located among these $2N$ positions. The signal received with the array thus could be recovered to a spatial sparse signal.

The relationship between the received signal and the spatial sparse signal could be expressed with the equation:

$$\boldsymbol{x}(n) = \boldsymbol{Hs}(n) + \boldsymbol{v}(n) \tag{5}$$

where $\boldsymbol{x}(n) = [x_0(n), x_1(n), x_{M-1}(n)]^{\mathrm{T}}$ is the signal received with an array of M sensors, $\boldsymbol{H} = [a(\theta_0), a(\theta_1), \cdots, a(\theta_{2N-1})]$ is a transformation matrix consisting of the steering vectors on the whole angle region and $\boldsymbol{s}(n)$ is a sparse projection coefficient with only a few nonzero elements, $\boldsymbol{v}(n)$ is the noise vector.

According to CS, $\boldsymbol{s}(n)$ is a sparse signal and could be reconstructed precisely through compressive sampling $\boldsymbol{x}(n)$.

The idea of Compressive Sampling is to sample $\boldsymbol{x}(n)$ with a sparse sampling matrix $\boldsymbol{\Phi}$, which is uncorrelated to transformation matrix $\boldsymbol{H}$, and get $z(n)$ with (6).

$$\begin{aligned} z(n) &= \boldsymbol{\Phi x}(n) = \boldsymbol{\Phi}\left[\boldsymbol{Hs}(n) + \boldsymbol{v}(n)\right] \\ &= \boldsymbol{Ps}(n) + \boldsymbol{v}'(n) \end{aligned} \tag{6}$$

where $\boldsymbol{P} = \boldsymbol{\Phi H}$ is measurement matrix and $\boldsymbol{v}'(n) = \boldsymbol{\Phi v}(n)$ is the sampled noise which will be omitted in the reminder of this paper for simplification.

In CS, $\boldsymbol{s}(n)$ can be recovered from $z(n)$ if the measurement matrix $\boldsymbol{P}$ satisfies the Restricted Isometry Property.

If we consider the signal received with partial active array as $z(n)$ and figure out the right measurement matrix $\boldsymbol{P}$, we can easily obtain the sparse coefficient $\boldsymbol{s}(n)$ with CS.

The sampling matrix $\boldsymbol{\Phi}$ is very important in CS and several sampling matrices could be chosen such as: Gaussian Random Matrix, Hadamard Matrix, Sparse Random Matrix and Partial Fourier Matrix etc.

Gaussian Random Matrix is adopted in this paper since it is easy to get: set a Gaussian Random Matrix $\boldsymbol{Q}$ whose dimension is $M \times M$ and generate a sparse sampling matrix $\boldsymbol{\Phi}$ whose dimension is $L \times M$ with L rows selected from matrix $\boldsymbol{Q}$ where the rows are chosen according to the positions of active sensors. Once $\boldsymbol{\Phi}$ is set, it is easy for us to get the measurement matrix $\boldsymbol{P}$ where $\boldsymbol{P} = \boldsymbol{\Phi H}$.

$$\begin{gathered} \min \|\boldsymbol{s}(n)\|_1 \\ \text{subject to } z(n) = \boldsymbol{Ps}(n) \end{gathered} \tag{7}$$

We can reconstruct the spatial-sparse coefficient $\boldsymbol{s}(n)$ from sparse sampled signal $z(n)$ with (7) to get the original full-array signal $\boldsymbol{x}(n)$ through the following equation:

$$\boldsymbol{x}(n) = \boldsymbol{Hs}(n) \tag{8}$$

When $\boldsymbol{x}(n)$, the full-array signal, is achieved, it is easy for us to process it with beamforming techniques on each subband.

D. Beamforming

When all the subband signals are obtained, the signals could be computed with beamforming techniques and beam pattern could also be achieved.

The mathematical expression of the output of a beamformer can be showed as follows where $\boldsymbol{w}$ is the weight vector from a beamformer.

$$\boldsymbol{y}(n) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(n) \tag{9}$$

With some information about DOA of signals of interest, the result can be achieved by minimizing the output variance $E = \{\boldsymbol{y}(n) * \boldsymbol{y}(n)^{\mathrm{H}}\}$, subject to a main lobe constraint $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta) = 1$. θ is the angle of signal of interest and the 1 at the right hand side of the equation indicates the distortionless towards direction θ at frequency ω. If there are several linearly independent constraints imposed on $\boldsymbol{w}$, this linearly constrained minimum variance problem can be formulated as

$$\begin{gathered} \min \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{xx}\boldsymbol{w} \\ \text{subject to } \boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta) = 1 \\ \left\|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_{\Theta_s})\right\| \le \delta \\ \left\|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_{\Theta_I})\right\| \le \varepsilon \end{gathered} \tag{10}$$

where $\boldsymbol{R}_{xx} = E\{\boldsymbol{xx}^{\mathrm{H}}\}$ is correlation matrix of the received array signals which assumes to be positive definite. Θ_s is the angle region of sidelobe. Θ_I is the direction of interfering signals. δ and ε is a very small positive value.

Given the whole band of signal has been split into K subbands, it's easy for us to compute $\boldsymbol{w}$ on each subband with the full-array signal reconstructed from (8).

III. Design Examples

Design examples will be presented in this section. The cvx toolbox[11][12], a package for solving convex problems, is employed here to achieve the reconstructed signal and solutions of weight vector $\boldsymbol{w}$.

For all the examples in this section, the speed of wave is 340(m/s) which is the speed of sound wave. The sampling rate has been set to 8000(Hz). An uniform linear array consists of 12 sensors is used in this part, and the space between adjacent sensors is half of the wavelength corresponding to the highest frequency which is 340/8000(m). δ is set to 0.1 which means the depression for side lobe region is -20 dB and ε is set to 0.01, representing the null for interfering signal is -40dB.

Beam patterns with the original full array signal are also computed and shown as a baseline in this part.

A. Design Example 1: Active Sensors with Random Locations

For this example, 2 sources are placed in far field, one's direction is $0°$ and another's is $30°$, the main lobe will be set up towards $0°$. The frequency of interest is $[0.2\pi, 0.6\pi]$ and sampled every 0.025π. Assume only 4 of these sensors are active during the experiment and the locations of these four sensors are chosen randomly.

We reconstruct the full-array signals from the received signals with these 4 active sensors and then beamform the signal with (10). The result is shown in Fig. 4.

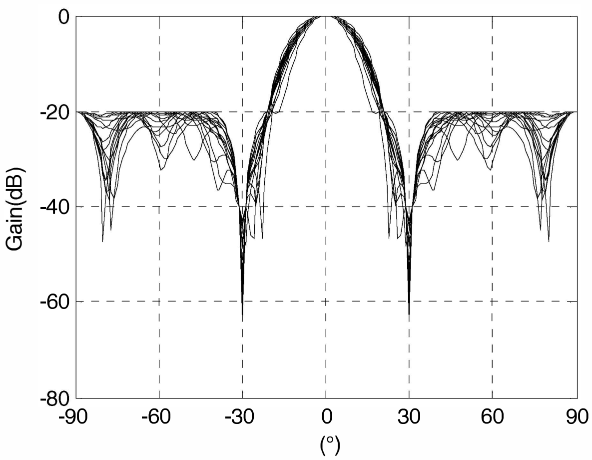

Fig. 4a. The beam patterns through the proposed method with 4 sensors chosen randomly

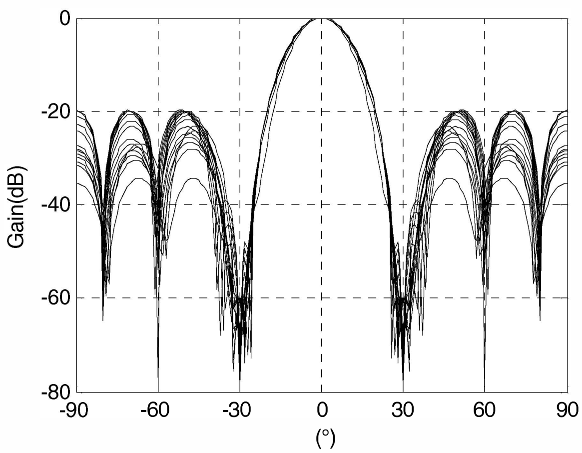

Fig. 4b. The beam patterns from the original full array signals

Fig. 4a shows the beam patterns through proposed method from the signal received with 4 active sensors and Fig. 4b shows the result with original full array signals. On each subband, the main lobe is set towards $0°$ while the interfering signal is at direction $30°$ and the side lobe region is $\Theta_s = [-90°, -20°] \cup [20°, 90°]$

Compared with the beam pattern in Fig. 4b, we can find that very good beam patterns have been achieved with the signals from 4 random active sensors through the proposed method. For each subband, main lobe aims at the desired angle without deflection, the nulls for interference direction are lower than -40 dB, and the depression for the side lobe reaches -20dB.

B. Design Example 2: Active Sensor with Given Locations

2 sources are located in far field at angle $-40°$ and $0°$ respectively in this example. Suppose a situation that some sensors of the array are not working during the experiment, and it would be easy for us to know the working states of these sensors, in this example the sensors' working states are $[1,0,0,1,0,0,1,0,1,0,1,0]$ where '1' represents the locations of active sensors and '0' represents the inactive ones.

Frequency of interest has been changed to $[0.3\pi, 0.8\pi]$ in this experiment and sampled with a step 0.025π. The received signals are processed with the proposed algorithm. The result is shown in Fig. 5.

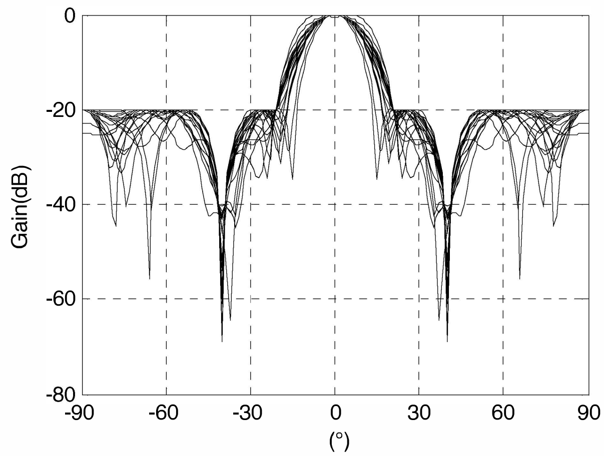

Fig. 5a. The beam pattern through the proposed method with 5 given active sensors

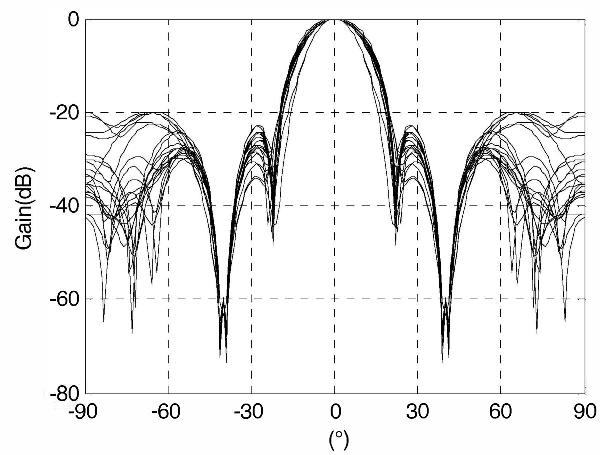

Fig. 5b. The beam pattern with original full array signal

Fig. 5a shows the beam patterns through the proposed method and Fig. 5b shows the result with full array signal, all the main lobes in these two figures are set to $0°$, interference signal come from $-40°$ and the side lobe region is $\Theta_s = [-90°, -20°] \cup [20°, 90°]$.

With the signals sampled by 5 active sensors and their location information, we could get a result by our proposed method. Comparing Fig. 5a with Fig 5b, we can find that the main lobes in Fig. 5a aim toward the desired direction constantly without any distortion where the nulls for interfering signal reach -40dB stably and the suppression for the side lobe region is about -20dB.

C. *Design Example 3: One Active Sensor*

One source is put at angle $0°$ and another is put at $50°$ in this experiment. In order to test the proposed method in extreme situation, we suppose a special occasion that a array consists of 12 sensors is used but only one sensor is working well in the experiment and the location of this sensor is chosen randomly.

The frequency of interest is $[0.3\pi, 0.7\pi]$ and sampled every 0.025π. The proposed method is tested and the result is shown in Fig. 6.

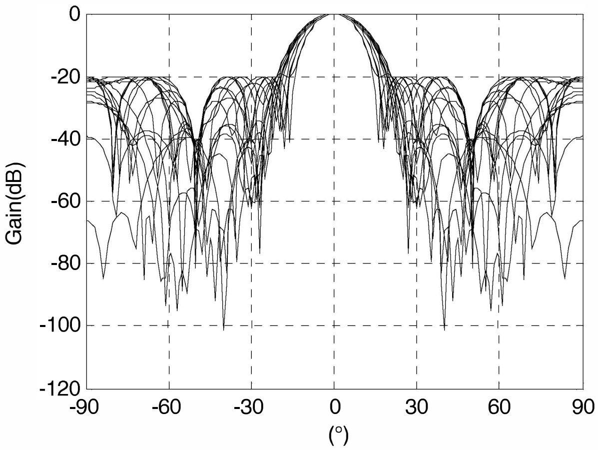

Fig. 6a. The beam pattern with the proposed method with 1 active sensors

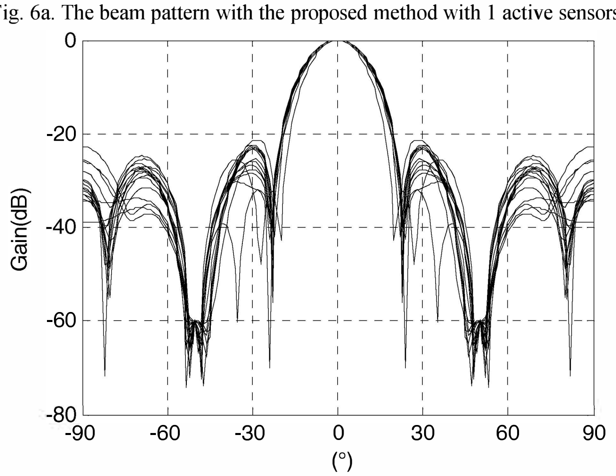

Fig. 6b. The beam pattern with original full array signal

Figure 6a shows the beam patterns by the proposed method and Figure 6b shows the result from full array signals with main lobe aimed at $0°$ while interference is at angle $40°$ and the side lobe region is $\Theta_s = [-90°, -20°] \cup [20°, 90°]$.

Only one sensor is active in this example and from the result shown in Fig.6a and Fig. 6b, we can find that the main lobe produced by our proposed method aims at desired direction constantly without distortion while the nulling points for interfering signal reaches -40 dB and the depression for side lobe region is about -20dB.

IV. CONCLUSION

In this paper, a new method for wideband beamforming based on CS has been proposed. First, the full-array signals could be recovered with CS on each subband. Then it is processed with beamforming technique and the final result is obtained with a synthesis filter bank. The proposed algorithm is of highly practical value that a very good beam pattern can be achieved with fewer active sensors or even only one active sensor, as shown by the simulations.

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