ADAPTIVE BEAMFORMING WITH JOINT ROBUSTNESS AGAINST SIGNAL STEERING VECTOR ERRORS AND INTERFERENCE NONSTATIONARITY

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ABSTRACT

Adaptive beamforming methods are known to degrade in the presence of both signal steering vector errors and interference nonstationarity. In this paper, we develop a new approach to adaptive beamforming which is jointly-robust against these two phenomena. Our approach is based on the optimization of the worst-case beamforming performance. A computationally efficient convex optimization based algorithm is proposed to compute the beamformer weights. Computer simulations compare the performance of our algorithm with other robust adaptive beamforming techniques.

1. INTRODUCTION

The performance of traditional adaptive beamforming methods is known to degrade severely if the array weights are not able to adapt sufficiently fast to nonstationary interferers [1]-[5]. Such nonstationarity can be caused by interferer and antenna motion, antenna vibration, as well as propagation channel variability. Moving interferences represent an especially serious problem in the case of large aperture arrays [3]. As a result, robust adaptive beamforming techniques are required in these cases.

Recently, a few methods have been developed to improve the performance of adaptive beamforming in nonstationary environments. Several authors exploited the idea of artificial broadening of the beampattern nulls in unknown interfering directions. Two different approaches have been independently developed using this idea. The first one exploits the so-called *data-dependent derivative constraints* (DDCs) [2]-[3], whereas the second one is based on *matrix tapers* (MTs) [4]-[5]. As shown in [6], the DDC and MT approaches are closely related to each other. Unfortunately, both of them are *ad hoc* techniques and, hence, their performance can be unpredictable in scenarios with rapidly moving interferers. Moreover, the DDC and MT methods are applicable to interfering sources with the plane wavefronts only. Clearly, the assumption of plane-wavefront interferers may become violated in practical scenarios with wavefront distortions resulting from environmental inhomogeneities, multipath propagation, local scattering and fading, as well as near-field interference [7].

In this paper, we develop a new approach to robust adaptive beamforming. Our approach provides *joint robustness* against the signal steering vector errors and nonstationary interferers with arbitrary wavefronts to prevent signal self-nulling and ensure interference nulling, respectively. It represents a further generalization of the method of [8] and [9] where only the robustness against signal steering vector errors has been considered. Similar to [8]-[9], our beamformer is based on the concept of the worst-case performance optimization but uses it in a more general form. The parameters of our algorithm can be optimally chosen based on known levels of uncertainty of the signal steering vector and the array data matrix. We show that the weights of the proposed robust beamformer can be efficiently computed by means of a convex optimization-based algorithm using second-order cone (SOC) programming.

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2. BACKGROUND

The output of a narrowband beamformer is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \tag{1}$$

where k is the time index, $\mathbf{x}(k)$ is the $M \times 1$ complex vector of the array data, $\mathbf{w}$ is the $M \times 1$ complex weight vector, M is the number of array sensors, and $(\cdot)^H$ is the Hermitian transpose. The snapshot vector can be modeled as

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \\ &= s(k)\mathbf{a} + \mathbf{i}(k) + \mathbf{n}(k) \end{aligned} \tag{2}$$

where $\mathbf{s}(k)$, $\mathbf{i}(k)$, and $\mathbf{n}(k)$ are the desired signal, interference, and noise components, respectively. Here, $s(k)$ is the signal waveform, and $\mathbf{a}$ is the signal steering vector (which is assumed to be precisely known in this section). The weight vector can be found from the maximum of the Signal-to-Interference-plus-Noise Ratio (SINR)

$$\mathrm{SINR} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}|^2}{\mathbf{w}^H \mathbf{R}_{\mathrm{i+n}} \mathbf{w}} \tag{3}$$

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where $\mathbf{R}_{i+n} = \mathrm{E}\{[\mathbf{i}(k)+\mathbf{n}(k)][\mathbf{i}(k)+\mathbf{n}(k)]^H\}$ is the $M \times M$ interference-plus-noise covariance matrix, σ_s^2 is the signal power, and $\mathrm{E}\{\cdot\}$ denotes the statistical expectation. It is easy to find the solution for the weight vector by maintaining the so-called *distortionless response* towards the desired signal and minimizing the output interference-plus-noise power. Hence, the maximization of (3) is equivalent to

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \qquad \text{subject to} \qquad \mathbf{w}^H \mathbf{a} = 1 \tag{4}$$

In practice, the true interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ is unavailable. Therefore, this matrix is replaced in (4) by the sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{N} \mathbf{X}\mathbf{X}^H \tag{5}$$

where $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_N]$ is the $M \times N$ array data matrix and N is the number of snapshots. After such a replacement, the problem (4) can be rewritten as

$$\min_{\mathbf{w}} \|\mathbf{X}^H \mathbf{w}\| \qquad \text{subject to} \qquad \mathbf{w}^H \mathbf{a} = 1 \tag{6}$$

From (6), the following well-known solution can be found for the weight vector:

$$\mathbf{w} = \alpha \hat{\mathbf{R}} \mathbf{a} \tag{7}$$

where $\alpha = (\mathbf{a}^H \hat{\mathbf{R}}^{-1} \mathbf{a})^{-1}$ is the norm constant which does not affect the output SINR (3). The solution (7) is commonly referred to as the sample matrix inverse (SMI) based minimum variance distortionless response (MVDR) beamformer.

In [10]-[12] (also, see references therein), the diagonal loading approach to robust beamforming has been studied. It has been shown that this approach can greatly improve the robustness against errors due to steering vector mismatch and small sample size.

3. ROBUST BEAMFORMING USING WORST-CASE PERFORMANCE OPTIMIZATION

Let us now assume that both the signal steering vector and the data matrix are known imprecisely and introduce the following *actual* (mismatched) steering vector and data matrix as

$$\tilde{\mathbf{a}} = \mathbf{a} + \boldsymbol{\delta} \tag{8}$$

$$\tilde{\mathbf{X}} = \mathbf{X} + \boldsymbol{\Delta} \tag{9}$$

respectively, where $\boldsymbol{\delta}$ is the $M \times 1$ error vector and $\boldsymbol{\Delta}$ is the $M \times N$ error matrix whose norms are assumed to be bounded by some known constraints ε and γ, i.e.,

$$\|\boldsymbol{\delta}\| \le \varepsilon, \qquad \|\boldsymbol{\Delta}\|_{\mathcal{F}} \le \gamma \tag{10}$$

where $\|\cdot\|_{\mathcal{F}}$ is the Frobenius norm of a matrix. Note that the errors $\boldsymbol{\delta}$ and $\boldsymbol{\Delta}$ can occur because of a steering vector mismatch and data nonstationarity, respectively. Indeed, if the array data is nonstationary at an interval of N samples, the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ becomes time-dependent and (5) can no longer serve as a proper estimate of this matrix. The SINR expression must be then rewritten as

$$\mathrm{SINR}(t) = \frac{\sigma_s^2 |\mathbf{w}^H \tilde{\mathbf{a}}|^2}{\mathbf{w}^H \mathbf{R}_{i+n}(t) \mathbf{w}} \tag{11}$$

To provide the robustness against both the signal steering vector errors and data nonstationarity, we will obtain the beamformer weights by solving the following optimization problem:

$$\min_{\mathbf{w}} \max_{\|\boldsymbol{\Delta}\|_{\mathcal{F}} \le \gamma} \|\tilde{\mathbf{X}}^H \mathbf{w}\| \quad \text{subject to} \quad |\mathbf{w}^H \tilde{\mathbf{a}}| \ge 1 \quad \forall \ \|\boldsymbol{\delta}\| \le \varepsilon \tag{12}$$

Note that the beamformer weights in (12) are obtained by means of minimizing the worst-case output power (i.e., the power that corresponds to the worst-case norm-bounded mismatch $\boldsymbol{\Delta}$) subject to the distortionless response constraint which must be satisfied for the steering vector with the worst-case norm-bounded error $\boldsymbol{\delta}$.

Equivalently, the problem (12) can be written as

$$\begin{aligned} &\min_{\mathbf{w}} \max_{\|\boldsymbol{\Delta}\|_{\mathcal{F}} \le \gamma} \|\mathbf{X}^H \mathbf{w} + \boldsymbol{\Delta}^H \mathbf{w}\| \\ &\text{subject to} \quad \min_{\|\boldsymbol{\delta}\| \le \varepsilon} |\mathbf{w}^H (\mathbf{a} + \boldsymbol{\delta})| \ge 1 \end{aligned} \tag{13}$$

The following results can be proven for the constraint and the objective function in (13).

Lemma 1, [8]: If

$$|\mathbf{w}^H \mathbf{a}| \ge \varepsilon \|\mathbf{w}\| \tag{14}$$

then

$$\min_{\|\boldsymbol{\delta}\| \le \varepsilon} |\mathbf{w}^H (\mathbf{a} + \boldsymbol{\delta})| = |\mathbf{w}^H \mathbf{a}| - \varepsilon \|\mathbf{w}\| \tag{15}$$

Proof: Applying the triangle and Cauchy-Schwarz inequalities along with $\|\boldsymbol{\delta}\| \le \varepsilon$, we have

$$\begin{aligned} |\mathbf{w}^H \mathbf{a} + \mathbf{w}^H \boldsymbol{\delta}| &\ge |\mathbf{w}^H \mathbf{a}| - |\mathbf{w}^H \boldsymbol{\delta}| \\ &\ge |\mathbf{w}^H \mathbf{a}| - \varepsilon \|\mathbf{w}\| \end{aligned} \tag{16}$$

It can be readily verified that the minimum of $|\mathbf{w}^H \mathbf{a} + \mathbf{w}^H \boldsymbol{\delta}|$ is achieved with

$$\boldsymbol{\delta} = -\frac{\mathbf{w}}{\|\mathbf{w}\|} \varepsilon e^{j\phi} \tag{17}$$

$$\phi = \text{angle}\left\{\mathbf{w}^H \mathbf{a}\right\} \tag{18}$$

and is equal to the right-hand side of (16) if (14) holds true. □

Lemma 2: Let

$$f(\mathbf{w}) = \max_{\|\boldsymbol{\Delta}\|_{\mathcal{F}} \le \gamma} \|\mathbf{X}^H \mathbf{w} + \boldsymbol{\Delta}^H \mathbf{w}\| \tag{19}$$

Then

$$f(\mathbf{w}) = \|\mathbf{X}^H \mathbf{w}\| + \gamma \|\mathbf{w}\| \tag{20}$$

Proof: We first show that

$$f(\mathbf{w}) \le \|\mathbf{X}^H \mathbf{w}\| + \gamma \|\mathbf{w}\| \tag{21}$$

For any matrix $\boldsymbol{\Delta}$, $\|\boldsymbol{\Delta}\| \le \|\boldsymbol{\Delta}\|_{\mathcal{F}}$ (where $\|\cdot\|$ is the matrix 2-norm). Hence, for any $\boldsymbol{\Delta}$, we have

$$\begin{aligned} \|\mathbf{X}^H \mathbf{w} + \boldsymbol{\Delta}^H \mathbf{w}\| &\le \|\mathbf{X}^H \mathbf{w}\| + \|\boldsymbol{\Delta}^H \mathbf{w}\| \\ &\le \|\mathbf{X}^H \mathbf{w}\| + \|\boldsymbol{\Delta}\| \|\mathbf{w}\| \\ &\le \|\mathbf{X}^H \mathbf{w}\| + \|\boldsymbol{\Delta}\|_{\mathcal{F}} \|\mathbf{w}\| \\ &\le \|\mathbf{X}^H \mathbf{w}\| + \gamma \|\mathbf{w}\| \end{aligned} \tag{22}$$

and (21) is proved.

Next, we show that

$$f(\mathbf{w}) \geq \|\mathbf{X}^H\mathbf{w}\| + \gamma\|\mathbf{w}\| \tag{23}$$

Introduce

$$\mathbf{\Delta}_* \triangleq \frac{\gamma \mathbf{w}\mathbf{w}^H\mathbf{X}}{\|\mathbf{w}\|\|\mathbf{X}^H\mathbf{w}\|} \tag{24}$$

Using the property $\|\mathbf{\Delta}_*\|_{\mathcal{F}}^2 = \mathrm{trace}\{\mathbf{\Delta}_*^H\mathbf{\Delta}_*\}$, it is easy to verify that $\|\mathbf{\Delta}_*\|_{\mathcal{F}} = \gamma$. Therefore,

$$\begin{aligned} f(\mathbf{w}) &= \max_{\|\mathbf{\Delta}\|_{\mathcal{F}}\leq\gamma} \|(\mathbf{X}+\mathbf{\Delta})^H\mathbf{w}\| \\ &\geq \|(\mathbf{X}+\mathbf{\Delta}_*)^H\mathbf{w}\| \\ &= \left\|\mathbf{X}^H\mathbf{w} + \frac{\gamma\mathbf{X}^H\mathbf{w}\mathbf{w}^H}{\|\mathbf{w}\|\|\mathbf{X}^H\mathbf{w}\|}\mathbf{w}\right\| \\ &= \left\|\mathbf{X}^H\mathbf{w} + \frac{\gamma\|\mathbf{w}\|}{\|\mathbf{X}^H\mathbf{w}\|}\mathbf{X}^H\mathbf{w}\right\| \\ &= \|\mathbf{X}^H\mathbf{w}\| + \gamma\|\mathbf{w}\| \end{aligned} \tag{25}$$

and, with (25), equation (23) is proved. Comparing (21) and (23), we finally prove Lemma 2. □

Assuming $|\mathbf{w}^H\mathbf{a}| \geq \varepsilon\|\mathbf{w}\|$ and applying Lemmas 1 and 2 to (13), we can rewrite the latter problem as

$$\min_{\mathbf{w}} \|\mathbf{X}^H\mathbf{w}\| + \gamma\|\mathbf{w}\| \quad \text{subject to} \quad |\mathbf{w}^H\mathbf{a}| \geq \varepsilon\|\mathbf{w}\| + 1 \tag{26}$$

Note that the assumption $|\mathbf{w}^H\mathbf{a}| \geq \varepsilon\|\mathbf{w}\|$ is relevant here because the parameter ε should be small [9] while the array *white noise gain* should be sufficient [11].

The constraint in (26) is still nonconvex due to the absolute value operation on the left-hand side. To make it convex, note that the cost function in (26) remains unchanged when $\mathbf{w}$ undergoes an arbitrary phase rotation and, therefore, without any loss of generality, we can choose $\mathbf{w}$ such that

$$\Re e\{\mathbf{w}^H\mathbf{a}\} \geq 0 \tag{27}$$

$$\Im m\{\mathbf{w}^H\mathbf{a}\} = 0 \tag{28}$$

where (27)-(28) should be used in (26) as additional constraints [9]. Then, (26) can be equivalently rewritten as

$$\min_{\mathbf{w}} \|\mathbf{X}^H\mathbf{w}\| + \gamma\|\mathbf{w}\| \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a} \geq \varepsilon\|\mathbf{w}\| + 1 \tag{29}$$

Note that from the form of the constraint in (29) it follows that the conditions (27)-(28) are satisfied automatically and, therefore, there is no need to add them as additional constraints to (29). Another important observation is that the inequality constraint in (29) can be replaced by the equality $\mathbf{w}^H\mathbf{a} = \varepsilon\|\mathbf{w}\| + 1$. However, we will use this constraint in its original inequality form which is suitable for the SOC implementation of (29).

The problem (29) is already written in the form of a SOC program. It can be solved using computationally efficient interior point algorithms with the complexity $O(M^3)$, for example, using the SeDuMi MATLAB toolbox [13].

It is worth noting that the SOC programming based beamformer proposed in [8]-[9] is a particular case of (29), because if we set $\gamma = 0$ then (29) transforms to the SOC problem derived in [8]-[9].

4. SIMULATIONS

In our simulations, we assumed a uniform linear array (ULA) of $M = 20$ omnidirectional sensors spaced half a wavelength apart. In both our examples, we assumed three interferers with plane wavefronts and the interference-to-noise ratio (INR) equal to 30 dB in a single sensor and a desired signal with the signal-to-noise ratio (SNR) equal to -5 dB in a single sensor. Without loss of generality, the noise power is assumed to be equal to one in each sensor and the desired signal is always present in the data snapshots. We assume that the presumed signal steering vector is the plane wave impinging from the angle of 7° relative to broadside, while the actual signal steering vector is modeled as a distorted copy of the presumed signal steering vector. Such a distortion may occur, for example, because of wave propagation effects in an inhomogeneous medium. We assume independent-increment phase distortions of the desired signal wavefront (see [14] and references therein). Each of these phase distortions (which remain fixed for all snapshots) is independently drawn from a Gaussian random generator with the variance equal to 0.04.

In the first example, we simulate the moving interferer case. The interference angles change as

$$\begin{aligned} \theta_1(k) &= 40^\circ + 10^\circ \sin(k/15) \\ \theta_2(k) &= -40^\circ - 15^\circ \cos(k/15) \\ \theta_3(k) &= -25^\circ + 10^\circ \cos(k/5) \end{aligned}$$

respectively, where k is the time index. Three methods have been compared: our robust beamformer (29), the diagonally loaded sample matrix inversion (LSMI) beamformer [10], and the MT-based beamformer [4]-[5]. To make the latter technique robust against signal steering vector errors, an additional diagonal loading was introduced. That is, the MT was applied to the diagonally loaded sample covariance matrix rather than to the conventional sample covariance matrix. The diagonal load of 15 is used in the LSMI beamformer and the diagonally loaded MT method. In our robust beamformer, the parameters $\varepsilon = 2.7$ and $\gamma = 180$ are chosen. These parameter values provide nearly optimal performance of the methods tested. The parameters of the MT were optimized to provide the best performance in this example as well. For all the beamformers tested, the training data cell (i.e., the data sliding window) is followed by the test cell (the so-called beamforming snapshot). The sliding window of 50 snapshots is used in all examples. Fig. 1 shows the output SINR of the beamformers tested versus the sliding window index. The optimal SINR curve is also displayed in this figure. From Fig. 1, we see that the proposed beamformer outperforms the LSMI technique and has quite similar performance with the diagonally loaded MT beamformer. Interestingly, the relative performance of the proposed and MT beamformers is different in different parts of the plot. This means that the performance of postbeamforming signal detection algorithms can be substantially improved by using jointly the outputs of these two beamformers instead of using only one of them.

In our second example, we study the SINR losses in the stationary case which we have to accept as a price for the improved robustness in the nonstationary case. In this example, we test all methods with the same parameters as in the first example. The only difference between the first and second examples is that in the second example we model the interferers as sources with the fixed angles $\theta_1 = 40^\circ$, $\theta_2 = -40^\circ$, and $\theta_3 = -25^\circ$. Fig. 2 displays the output SINR of the beamformers tested versus the sliding window index in this case. From this figure, we see that the proposed

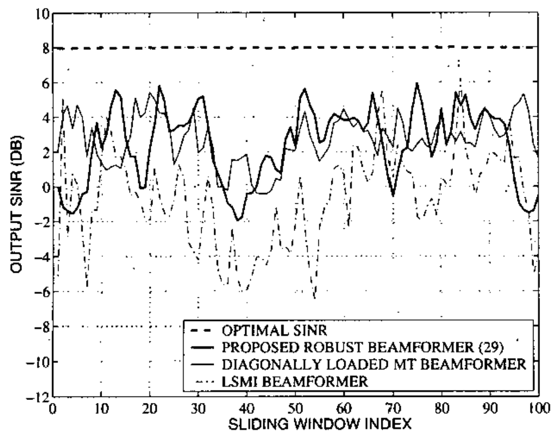

Fig. 1. Output SINR versus the sliding window index. First example.

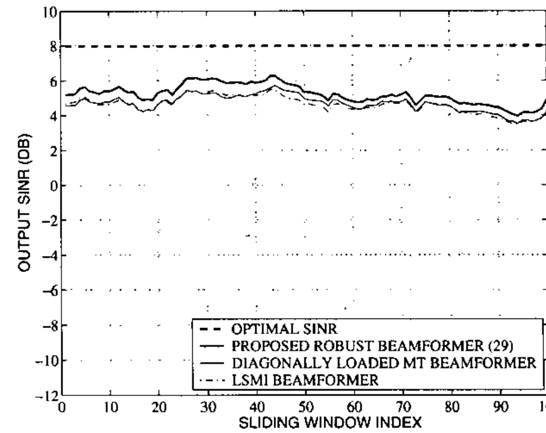

Fig. 2. Output SINR versus the sliding window index. Second example.

beamformer has better performance than the LSMI and diagonally loaded MT beamformers in this case.

5. CONCLUSIONS

A new approach to robust adaptive beamforming has been developed. Our beamformer is based on the optimization of the worst-case performance and is shown to provide joint robustness against signal steering vector errors and interference nonstationarity. The parameters of our technique can be optimally chosen based on known levels of uncertainty in the signal steering vector and the array data matrix. A convex optimization based formulation of the underlying robust beamforming problem has been derived to compute the beamformer weights. It uses SOC programming and can be efficiently implemented by means of interior point algorithms.

Computer simulations have demonstrated the relationship between the performance of our algorithm and other popular robust beamforming techniques.

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