Design of FIR Beamformer with Frequency Invariant Patterns via Jointly Optimizing Spatial and Frequency Responses

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ABSTRACT

An approach to the optimal design of FIR broadband beamformer with frequency invariant patterns via joint optimizing the spatial and frequency responses is proposed. The beam responses are jointly optimized to satisfy some spatial and frequency domain specifications by designing a bank of FIR filters corresponding to the input channels. It minimizes the maximum error between the designed beam patterns and the desired ones over the working frequencies within the mainlobe area while guaranteeing the sidelobes and the array patterns over the stopband to be below some given threshold values. White noise gain constraint is applied to improve the robustness of the beamformer against random errors. The beam response is expressed as a linear function of the FIR filters tap weights, and the design problem is formulated as the second-order cone programming, which can be solved efficiently via the well-established interior point methods. Results of computer simulation for a twelve-element semicircular array confirm satisfactory performance of the approach proposed in this paper.

1. INTRODUCTION

In broadband signal processing, the spectral characteristics of the target are used mostly in target detection and target classification. Since the radiated noise of the target generally spreads over a wide frequency band, distortion in the power spectrum might appear if the beamformer response varies with frequency. The so-called frequency invariant beamformer (FIB) technique is required as a remedy.

An FIB is a beamformer in which the response is approximately constant over some design bandwidth. An analog technique for FIB based on approximating an ideal continuous aperture was presented in [1-2] by exploring the relationship among the beam response, the frequency and the aperture of the arrays. However, the approach design weights to insure that the synthesized pattern approximates the reference one in both the mainlobe and the sidelobe areas. In fact, only the beam patterns within the mainlobe area are of interest. This approach leads to larger error since approximation in the sidelobe areas is unnecessary. The sidelobes only need to be guaranteed below some prescribed threshold value.

Recent advances in convex optimization have motivated a class of algorithms for the array pattern synthesis problem [3]. Broadband beamformers can be implemented by placing a tapped delay line or FIR filter at the output of each sensor [4]. Our group has presented an FIR filter structure for narrowband beamforming earlier in [5]. The narrowband FIR beamforming can be developed to a broadband one. That is, design a bank of FIR filters to ensure their responses approximately equal to the array weights over the working frequency band.

The FIR broadband FIB can be implemented by combining the optimal array pattern synthesis [6] and optimal FIR filters design [7], which is referred to as a separate optimization method. However, the separate optimization method can only obtain the optimal solution for both problems respectively. It is not clear how to obtain the optimal value of the stopband level of the desired FIR filters, and the sidelobes at the transition frequency band are difficult to be controlled. The global optimum solution for the FIR broadband FIB cannot be guaranteed. Joint optimization method proposed in this paper, however, can obtain the global optimum solution.

In this proposed paper, the broadband beam response is expressed as a linear function of the tap weights of a bank of FIR filters corresponding to the channels. Then, the joint optimization problem of the FIR filters design for broadband FIB is converted to a convex optimization form as the second-order cone programming (SOCP), which can be solved via the well-established interior point methods, for example, by SeDuMi [8].

2. PRINCIPLE OF FIR BEAMFORMER

Consider an array of N sensors, the array pattern (or beam response) in the direction θ at the frequency f is

$$p(f,\theta) = \mathbf{d}^T(f,\theta)\mathbf{w}(f), \tag{1}$$

where $\mathbf{d}(f,\theta) = \begin{bmatrix} d_1(f,\theta) & d_2(f,\theta) & \cdots & d_N(f,\theta) \end{bmatrix}^T$ is the array response vector of angle θ and frequency f, $\mathbf{w}(f) = \left[w_1(f), w_2(f), \cdots, w_N(f)\right]^T$ is the weight vector of frequency f, and $(\cdot)^T$ denotes the transpose operation.

Without loss of generality, we assume that the source sig-

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nals have finite bandwidth, i.e. the passband $F_{\mathrm{K}} \overset{\Delta}{=} [f_{\mathrm{L}}, f_{\mathrm{U}}] \subset F$. Where f_{L} and f_{U} is the low and up bound frequency respectively and $F \overset{\Delta}{=} [0, f_s/2)$ with f_s being the sampling frequency. The stopband is F_{P}, $F_{\mathrm{P}} \subset F$ and the transition band is F_{T}. Thus $F = F_{\mathrm{K}} \cup F_{\mathrm{P}} \cup F_{\mathrm{T}}$.

In the spirit of the approach in [5], the FIR broadband beamformer can be stated as finding if there exists a bank of FIR filters with impulse response

$$\mathbf{h}_n = \left[h_{n0}, h_{n1}, \cdots, h_{n(L-1)}\right]^T, \; n = 1,2,\cdots,N, \quad (2)$$

corresponding frequency response at frequency f_k (k = 1,2, $\cdots$,K) is approximately equal to $w_n(f_k)$, i.e.

$$\widetilde{H}_{n,d}(f_k) = w_n(f_k), \; k = 1,2,\cdots,K, \; n = 1,2,\cdots,N, \quad (3)$$

where L in (2) is the length of filter tap weights. The tap weights are chosen to achieve the desired frequency response derived from the broadband array weights $\mathbf{w}$.

The inherent group delay (unit in taps) of an FIR filter of length L is nearly $\zeta = (L-1)/2$. The group delay of the desired FIR filter is not exactly equal to ζ in general, and can be decomposed into an integer part plus a decimal part. We assume the needed pre-steering delay (unit in taps) for channel n is ξ_n. The array weight can be thus rewritten as

$$\begin{aligned} w_n(f_k) &= \exp[-j2\pi f_k \operatorname{int}(\xi_n - \zeta)T_s] \\ &\cdot w_n(f_k)\exp[j2\pi f_k \operatorname{int}(\xi_n - \zeta)T_s], \end{aligned} \quad (4)$$

where $T_{\mathrm{s}} = 1/f_{\mathrm{s}}$ is the sampling interval and int(·) denotes round towards nearest integer. The first part of (4) can be implemented by integral delay of $\tau_n = \operatorname{int}(\xi_n - \zeta)$ taps (when it is minus, a plus integral number can be added for all channels), and the second part by an FIR filter. Thus, the desired frequency response of FIR filter corresponding to the input channel n can be expressed as

$$\begin{aligned} H_{n,d}(f_k) &= w_n(f_k)\exp(j2\pi f_k \tau_n T_s), \\ &k = 1,2,\cdots,K, \; n = 1,2,\cdots,N, \end{aligned} \quad (5)$$

As a result, the FIR broadband beamformer structure is shown in Fig.1.

Let $\kappa_{n,k} = \exp[-j2\pi f_k \tau_n T_s]$, we obtain

$$w_n(f_k) = H_{n,d}(f_k)\kappa_{n,k}, \; k = 1,2,\cdots,K, n = 1,2,\cdots,N. \quad (6)$$

For the FIR filter corresponding channel n, the complex frequency response of a FIR filter with impulse response $\mathbf{h}_n$ of length L is given by

$$H_{n,d}(f_k) = \sum_{l=0}^{L-1} h_{nl}\exp(-jl2\pi f_k/f_s) = \mathbf{e}^T(f_k)\mathbf{h}_n, \quad (7)$$

where $\mathbf{e}(f_k) = \left[1, e^{-j2\pi f_k/f_s}, \cdots, e^{-j(L-1)2\pi f_k/f_s}\right]^T$. Thus

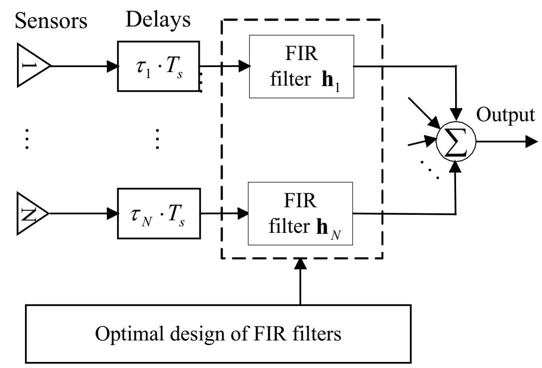

Fig. 1. FIR broadband beamformer structure.

$$\mathbf{w}(f_k) = \left[\mathbf{e}^T(f_k)\mathbf{h}_1\kappa_{1,k}, \cdots, \mathbf{e}^T(f_k)\mathbf{h}_n\kappa_{n,k}, \cdots, \mathbf{e}^T(f_k)\mathbf{h}_N\kappa_{N,k}\right]^T. \quad (8)$$

We obtain

$$\begin{aligned} p(f_k,\theta) &= \mathbf{d}^T(f_k,\theta)\mathbf{w}(f_k) \\ &= \mathbf{d}^T(f_k,\theta)\left[\mathbf{e}^T(f_k)\mathbf{h}_1\kappa_{1,k}, \cdots, \mathbf{e}^T(f_k)\mathbf{h}_n\kappa_{n,k}, \cdots, \mathbf{e}^T(f_k)\mathbf{h}_N\kappa_{N,k}\right]^T \\ &= \left[d_1(f_k,\theta)\mathbf{e}^T(f_k)\kappa_{1,k}, \cdots, d_n(f_k,\theta)\mathbf{e}^T(f_k)\kappa_{n,k}, \cdots,\right. \\ &\quad \left. d_N(f_k,\theta)\mathbf{e}^T(f_k)\kappa_{N,k}\right] \cdot \left[\mathbf{h}_1^T, \cdots, \mathbf{h}_n^T, \cdots, \mathbf{h}_N^T\right]^T. \end{aligned} \quad (9)$$

Define $\mathbf{v}(f_k,\theta) = \left[\mathbf{v}_1^T(f_k,\theta), \cdots, \mathbf{v}_n^T(f_k,\theta), \cdots \mathbf{v}_N^T(f_k,\theta)\right]^T$ with $\mathbf{v}_n^T(f_k,\theta) = d_n(f_k,\theta)\mathbf{e}^T(f_k)\kappa_{n,k}$ and $\mathbf{h} = \left[\mathbf{h}_1^T, \cdots, \mathbf{h}_n^T, \cdots, \mathbf{h}_N^T\right]^T$, Eq. (9) takes the form

$$p(f_k,\theta) = \mathbf{v}^T(f_k,\theta)\mathbf{h}. \quad (10)$$

3. DESIGN OF FIB

The FIB problem is to design a bank of FIR filters for each channel to insure that the resulting beam responses approximate the reference responses over the working frequency band within the mailobe areas while controlling the sidelobes and the array patterns over the stopband.

Let $p_{\mathrm{d}}(\theta)$ be the desired array response at the direction θ, Θ_{M} and Θ_{S} be the mainlobe and sidelobe areas, respectively. The broadband array pattern synthesis problem is then stated as follows:

Find the FIR filter impulse response $\mathbf{h}_n$ such that

$$p(f,\theta) \approx p_{\mathrm{d}}(\theta) \text{ for all } \theta \in \Theta_{\mathrm{M}} \text{ and } f \in F_{\mathrm{K}},$$

$$\text{and } |p(f,\theta)| \le 10^{DL/20} \text{ for } \theta \in \Theta_{\mathrm{S}} \text{ or } f \in F_{\mathrm{P}},$$

where DL is controlling sidelobe level in dB.

We discretize the passband, stopband and transition band using a finite number of frequencies $f_k \in F_{\mathrm{K}}$ ($k = 1,2,\cdots,K$), $f_p \in F_{\mathrm{P}}$ ($p = 1,2,\cdots,P$) and $f_t \in F_{\mathrm{T}}$ ($t = 1,2,\cdots,T$). The frequencies are often uniformly spaced. Let θ_i ($i=1,\cdots,I$) be chosen grid that approximates the whole visible areas Θ over the stopband, and $\theta_s \in \Theta_S$ ($s=1,\cdots,S$) and $\theta_m \in \Theta_M$ ($m=1,\cdots,M$) be chosen grid that approximates the sidelobe areas Θ_S and the mainlobe areas Θ_M over the working frequency band using a finite number of angles respectively. There is a compromise between design precision and the number of discretization.

At the same time, white noise gain constraint (WNC) can be used to improve the beamformer robustness against random errors in array characteristics [9]. We specify white noise gain for a desired level of insensitivity to random errors. For a FIR broadband beamformer, white noise gain constraint is to bound the norm of FIR filter impulse response to some known constant $\gamma > 0$:

$$\sum_{n=1}^{N} \|\mathbf{h}_n\|^2 \le \gamma \text{ or } \|\mathbf{h}\|^2 \le \gamma , \tag{11}$$

where $\|\cdot\|$ denotes the Euclidean norm.

How well $p(f_k,\theta)$ approximates $p_d(\theta)$ can be measured in peak error across Θ_M, the broadband array pattern synthesis problem can be represented as a minimax optimization problem

$$\begin{aligned}
\min_{\mathbf{h}} \quad & \max\left(\lambda_m \left| p_d(\theta_m) - p(f_k,\theta_m) \right|\right), \\
& \qquad m=1,\cdots,M,\ k=1,2,\cdots,K, \\
\text{subject to } & \left| p(f_k,\theta_s) \right| \le \delta_s,\ s=1,\cdots,S,\ k=1,2,\cdots,K, \\
& \left| p(f_t,\theta_s) \right| \le \delta_s,\ s=1,\cdots,S,\ t=1,2,\cdots,T, \\
& \left| p(f_p,\theta_i) \right| \le \xi_i,\ i=1,\cdots,I,\ p=1,2,\cdots,P, \\
& \|\mathbf{h}\| \le \sqrt{\gamma},
\end{aligned} \tag{12}$$

where λ_m is a non-negative weighting coefficient, δ_s and ξ_i control the sidelobes and the array patterns at the stopband.

The convex optimizations can be set up for solution by SeDuMi, an interior-point solver that can accept both second-order cone and linear constraints. In SeDuMi, the dual standard form of convex conic optimization problem is defined as

$$\max \mathbf{b}^T\mathbf{y},\ \text{subject to } \mathbf{c} - \mathbf{A}^T\mathbf{y} \in \mathrm{K}, \tag{13}$$

where $\mathbf{y}$ is a vector containing the designed variables; $\mathbf{A}$ is an arbitrary matrix; $\mathbf{b}$ and $\mathbf{c}$ are arbitrary vectors; K is a symmetric cone consisting of Cartesian products of elementary cones (each corresponding to a constraint); Note that $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{c}$ can be complex-valued and must have matching dimensions.

The standard or unit SOC of dimension α is defined as

$$SOC^{\alpha} \stackrel{\Delta}{=} \left\{ \begin{bmatrix} u \\ \mathbf{x} \end{bmatrix} \middle| u \in \Re, \mathbf{x} \in C^{\alpha-1}, \|\mathbf{x}\| \le u \right\} \tag{14}$$

where u is a real scalar; $\mathbf{x}$ is a complex $\alpha-1$ dimensional vector.

Introduce a new scalar nonnegative variable ε. Define $\mathbf{y} = \left[\varepsilon, \mathbf{h}^T\right]^T$ and $\mathbf{b} = \left[-1, \mathbf{0}_{1\times LN}\right]^T$ so that $-\varepsilon = \mathbf{b}^T\mathbf{y}$. Where $\mathbf{0}_{1\times LN}$ is a $1\times LN$ dimensional zero vector. The optimization problem of (12) becomes

$$\begin{aligned}
& \max_{\mathbf{y}} \mathbf{b}^T\mathbf{y},\ \text{subject to} \\
& \lambda_m \left| p_d(\theta_m) - \left[0 \quad \mathbf{v}^T(f_k,\theta_m)\right]\mathbf{y} \right| \le \left[1 \quad \mathbf{0}_{1\times LN}\right]\mathbf{y}, \\
& \qquad\qquad m=1,\cdots,M,\ k=1,2,\cdots,K, \\
& \left| \left[0 \quad \mathbf{v}^T(f_k,\theta_s)\right]\mathbf{y} \right| \le \delta_s,\ s=1,\cdots,S,\ k=1,2,\cdots,K, \\
& \left| \left[0 \quad \mathbf{v}^T(f_t,\theta_s)\right]\mathbf{y} \right| \le \delta_s,\ s=1,\cdots,S,\ t=1,2,\cdots,T, \\
& \left| \left[0 \quad \mathbf{v}^T(f_p,\theta_i)\right]\mathbf{y} \right| \le \xi_i,\ i=1,\cdots,I,\ p=1,2,\cdots,P, \\
& \left\| \left[\mathbf{0}_{LN\times 1} \quad \mathbf{I}_{LN\times LN}\right]\mathbf{y} \right\| \le \sqrt{\gamma},
\end{aligned} \tag{15}$$

where $\mathbf{I}_{LN\times LN}$ is an LN dimensional identity matrix.

The convex optimization problem (15) can be reformulated in the dual standard form (13) of SOCP and solved by SeDuMi MATLAB toolbox easily. After solving the optimization problem (15), the vector $\mathbf{y}$ contains both the achievable maximum synthesized error ε and the optimal filters tap weights $\mathbf{h}$. And the resulting FIR filters of our broadband beamformer corresponding to channel n is given by it subvector $\mathbf{h}_n$.

4. DESIGN EXAMPLES

Consider a semicircular hydrophone array of N=12 sensors equally spaced on the circumference with 15° apart. The radius is 1.5 m. The diagram of the array is shown in Fig. 2.

Assume that the sampling frequency is f_s = 6400Hz, and the frequency band $\left[0, f_s/2\right]$ is discretized with uniform frequency bin of 64Hz. The working frequency band is F_{K} = [960Hz, 1920Hz] that cover one-octave frequency band, the stopband is F_{P} = [0Hz, 768Hz] $\cup$ [2112Hz, 3200Hz] and the transition band F_{T} = [832Hz, 896Hz] $\cup$ [1984Hz, 2048Hz].

The visible areas $\Theta = [0°, 180°]$ is discretized with a uniform grid of 2.5°. Take the conventional (delay-and-sum) beam patterns at the frequency of 960 Hz and the look direction of 90° as the reference beam patterns, and 5 beam response points at the angles of $\{\theta_m\}_{m=1}^{5}$ = {72.5°, 80°, 90°, 100°, 107.5°} are chosen as the desired beam response. The sidelobe areas are Θ_{S} = [0°, 65°] $\cup$ [115°, 180°] and the prescribed sidelobe level is set to be –25dB, i.e. $\delta_s = 0.0562$ in (15). The weighting coefficients are chosen to be $\lambda_m = 1$ ($m=1,2,\cdots,5$) and the length of each FIR filter is L=64.

We first design the FIR broadband FIB using the separate

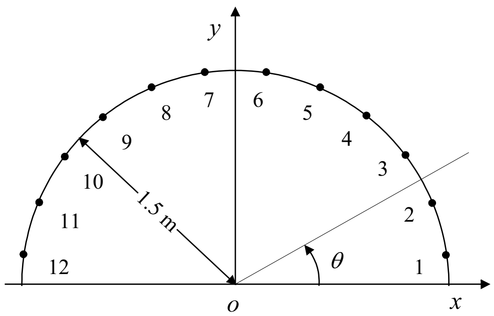

Fig. 2. The semicircular hydrophone array.

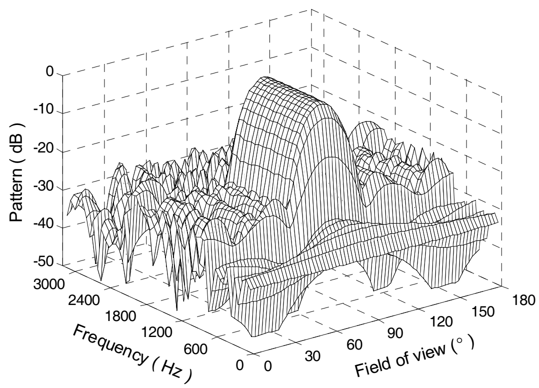

Fig. 3. Array patterns via separate optimization method.

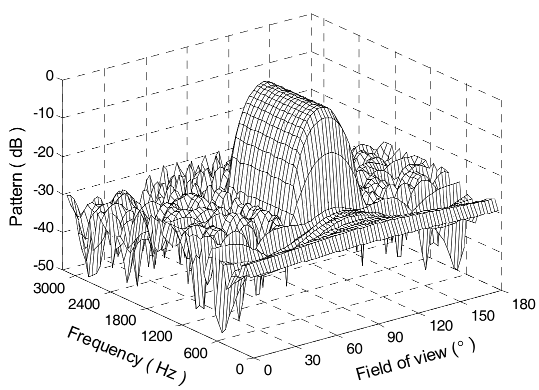

Fig. 4. Array patterns via joint optimization method.

optimization method. The optimization criterion of the array pattern synthesis at the 16 frequencies is: minimize the maximum synthesis error and guarantees the sidelobes to be below –25dB, and the norm of the array weights below 0.7. The optimization criterion of the FIR filters design is: minimize the mean-square error over the passband while guaranteeing the stopband level to be below –40 dB.

The achieved array patterns by the separate optimization method are shown in Fig. 3. The maximum synthesized error of the FIR broadband FIB is 0.031. The sidelobes at the passband, the transition band and the stopband are –23dB, –21dB and –26dB, respectively. The norm of total 12 FIR filters tap weights is 0.31.

The prescribed pattern magnitude is set to be –25 dB, and the constraint on the norm of FIR filters impulse response is $\sqrt{\gamma} = 0.3 < 0.31$. The achieved array patterns as a function of frequency and angle by our joint optimization method are shown in Fig. 4. The maximum synthesized error of the resulting FIR broadband FIB is 0.012, which is far below that achieved by the separate optimization method. The sidelobes and the beam response magnitude at stopband by joint optimization method are strictly below the prescribed value (-25 dB). By comparing Fig. 4 with Fig. 3, it can be seen that the performance improvement is significant.

5. CONCLUSION

An approach to the optimal design of FIR broadband beamformer with frequency invariant patterns via joint optimization of spatial and frequency responses has been proposed. The relationship between the beam responses and the impulse response of the FIR filters are deduced. Convex SOCP-based formulation of our design problem for broadband frequency invariant beamformer is derived. The advantage of the joint optimization method proposed in this paper is that it can find the global optimum solution to the broadband FIB. Results of computer simulations of FIR broadband frequency invariant beamforming for a semicircular hydrophone array show good performance of the proposed method. The obtained array pattern is approximately constant over one-octave frequency bandwidth, while guaranteeing the sidelobes and pattern at the stopband are below the given threshold value.

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