AN EIGENFILTER APPROACH TO THE DESIGN OF FREQUENCY INVARIANT BEAMFORMERS

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Abstract. An eigenfilter approach to the design of frequency invariant beamformers (FIBs) is proposed and its solution is provided by finding the minimum generalized eigenvector of two matrices. Several design examples are provided with satisfactory frequency invariant property and sidelobe attenuation. The proposed approach is general and can be applied to different array structures, such as linear arrays, circular arrays and rectangular arrays.

1. INTRODUCTION

Broadband beamforming has been studied extensively in the past and amongst them is a class of beamformers with frequency invariant responses [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12], which can form beams pointing to the signal of interest with a constant beamwidth. With the FIB technique, both adaptive broadband beamforming and broadband DOA (direction of arrival) estimation can be simplified greatly [13, 4, 14, 15]. In [4, 9, 10, 11, 12], the design is achieved based on simple multi-dimensional inverse Fourier transforms by exploiting the relationship between the array's spatial and temporal parameters and its beam pattern. More recently, a direct optimization approach was adopted using the convex optimization methods [16, 17, 18].

The least squares approach is a conventional and well-known method for the design of both FIR filters and broadband beamformers [19, 20, 21, 22, 23] and its solutions are normally obtained by matrix inversion based methods and eigenfilter based methods. Compared with the matrix inversion based methods, the eigenfilter based ones have the advantage of improved numerical stability. In this paper, we will extend the traditional eigenfilter based methods to the design of frequency invariant beamformers. The key to its success is to form a cost function enforcing the frequency invariance requirement in the design and one novel unconstrained least squares(ULS) formulation of the problem is proposed, with its solution based on the eigenfilter method and obtained by finding the minimum generalized eigenvector of two related matrices.

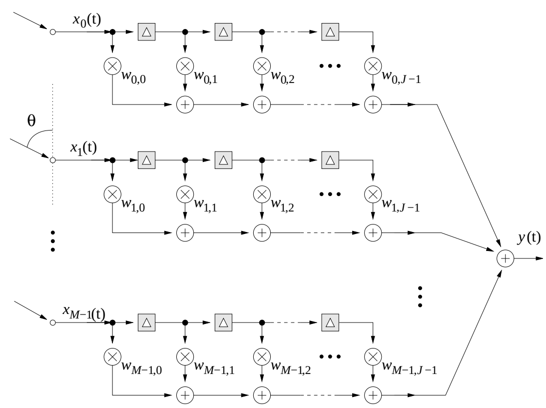

Fig. 1. A general broadband beamforming structure.

This paper is organized as follows. The general broadband beamforming structures are reviewed briefly in Section 2. The ULS formulation for the design of FIBs with its corresponding solution is given in Section 3. Several design examples based on different array structures are provided in Section 4. Conclusions are drawn in Section 5.

2. BROADBAND BEAMFORMING STRUCTURES

A general structure for broadband beamforming with tapped delay-lines (TDLs) or FIR/IIR filters is shown in Fig. 1, in which J is the number of delay elements associated with each of the M sensor channels. The beamformer with such a structure samples the propagating wave field in both space and time. Its response as a function of the signal angular frequency ω and the direction of arrival angle θ can be expressed as

$$\tilde{R}(\omega,\theta) = \sum_{m=0}^{M-1}\sum_{k=0}^{J-1} w_{m,k} e^{-j\omega(\tau_m + kT_s)}, \qquad (1)$$

where T_s is the delay between adjacent taps of the TDLs and τ_m is the spatial propagation delay between the $m-th$ sensor and a reference point. We can rewrite the response in a vector form

$$\tilde{R}(\omega,\theta) = \mathbf{w}^T \mathbf{s}(\omega,\theta) \qquad (2)$$

where $\mathbf{w}$ is the coefficient vector defined as

$$\mathbf{w} = [w_{0,0}, \cdots w_{M-1,0} \cdots w_{0,J-1} \cdots w_{M-1,J-1}]^T \ , \qquad (3)$$

and $\mathbf{s}(\omega,\theta)$ is the $M \times J$ steering vector given by

$$\mathbf{s}(\omega,\theta) = \mathbf{s}_{T_s}(\omega) \otimes \mathbf{s}_{\tau_m}(\omega,\theta) \qquad (4)$$

where $\otimes$ denotes the Kronecker product, and

$$\mathbf{s}_{T_s}(\omega) = [1, e^{-j\omega T_s}, \cdots, e^{-j(J-1)\omega T_s}]^T \ , \qquad (5)$$

$$\mathbf{s}_{\tau_m}(\omega,\theta) = [e^{-j\omega\tau_0}, e^{-j\omega\tau_1}, \cdots, e^{-j\tau_{M-1}\omega}]^T \ . \qquad (6)$$

For a uniformly spaced linear array with an inter-element spacing d, we have the spatial propagation delay τ_m given by $\tau_m = m\tau_1 = m\frac{d}{c}\cos\theta$, with the first sensor position as the phase reference point. With the normalized angular frequency $\Omega = \omega T_s$, $\omega(m\tau_1 + kT_s)$ changes to $m\mu\Omega\cos\theta + k\Omega$ with $\mu = \frac{d}{cT_s}$. Then the steering vector changes to

$$\mathbf{s}(\Omega,\theta) = \mathbf{s}_{T_s}(\Omega) \otimes \mathbf{s}_{\tau_m}(\Omega,\theta) \qquad (7)$$

and

$$\mathbf{s}_{T_s}(\Omega) = [1, e^{-j\Omega}, \cdots, e^{-j(J-1)\Omega}]^T \qquad (8)$$

$$\mathbf{s}_{\tau_m}(\Omega,\theta) = [1, e^{-j\mu\Omega\cos\theta}, \cdots, e^{-j(M-1)\mu\Omega\cos\theta}]^T \ . \qquad (9)$$

Then we obtain the response as a function of Ω and θ

$$R(\Omega,\theta) = \mathbf{w}^T \mathbf{s}(\Omega,\theta) \ . \qquad (10)$$

For a uniform circular array with a circumferential spacing of d, as shown in Fig. 2, we have the spatial delay between the center of the circular array and the $m-th$ sensor, $\tau_m = \frac{r\cos(\theta-\theta_m)}{c}$, where $r = \frac{Md}{2\pi}$ is the radius of the circular

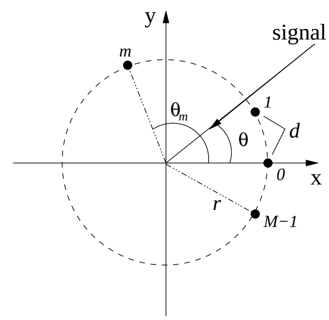

Fig. 2. A uniformly spaced circular array.

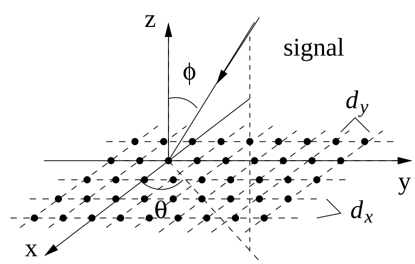

Fig. 3. A uniformly spaced rectangular array, which can be considered as a linear array with SDLs.

array and $\theta_m = m\frac{2\pi}{M}$ is the angle of the $m-th$ sensor. With the normalized angular frequency $\Omega = \omega T_s$, $\omega(\tau_m + kT_s)$ changes to $\Omega\beta\mu\cos(\theta-\theta_m) + k\Omega$ with $\beta = \frac{M}{2\pi}$. Then we obtain $\mathbf{s}_{\tau_m}$ for the uniform circular array case

$$\mathbf{s}_{\tau_m}(\Omega,\theta) = [e^{-j\Omega\beta\mu\cos(\theta-\theta_0)}, e^{-j\Omega\beta\mu\cos(\theta-\theta_1)}, \cdots, e^{-j\Omega\beta\mu\cos(\theta-\theta_{M-1})}]^T \ . \qquad (11)$$

When the TDLs in the broadband linear array system shown in Fig. 1 are replaced by sensor delay-lines (SDLs), we will have a rectangular array system without TDLs [24]. Such a structure is shown in Fig. 3, which is an equally spaced rectangular array with $M \times J$ sensors and an inter-element spacing of d_x and d_y, respectively. Without loss of generality, we assume $d_x = d_y = d$ and the signals come from the direction $\varphi = \frac{\pi}{2}$. Since it is a broadband beamforming structure with spatial-only information, we replace kT_s in (1) by δ_k which represents the spatial propagation delay along the y axis and is given by $\delta_k = k\delta_1 = m\frac{d}{c}\sin\theta$, with the first sensor position as the phase reference point. With the normalized angular frequency $\Omega = \omega T_s$, $\omega(m\tau_1 + k\delta_1)$ changes to $m\mu\Omega\cos\theta+k\mu\Omega\sin\theta$. Then we obtain the steering vector for the rectangular array as

$$\mathbf{s}(\Omega,\theta)=\mathbf{s}_{\delta_k}(\Omega)\otimes\mathbf{s}_{\tau_m}(\Omega,\theta) \quad (12)$$

where

$$\mathbf{s}_{\delta_k}(\Omega,\theta)=[1,e^{-j\mu\Omega\sin\theta},\cdots,e^{-j(J-1)\mu\Omega\sin\theta}]^T\ . \quad (13)$$

3. THE EIGENFILTER APPROACH TO THE DESIGN OF FIBS

In the proposed method, the cost function for the design of FIBs consists of three parts. Firstly, the frequency invariance property is formulated by minimizing the Euclidean distance between the response at a fixed reference frequency Ω_r and those at all the other operating frequencies over a range of directions in which frequency invariance is considered. The cost function related to this property is given by

$$J_1=\int_{\Omega}\int_{\Theta_{FI}}\left|\mathbf{w}^T\mathbf{s}(\Omega,\theta)-\mathbf{w}^T\mathbf{s}(\Omega_r,\theta)\right|^2 d\Omega d\theta \quad (14)$$

where Θ_{FI} represents a direction range in which frequency invariance is considered. It can be either the main beam direction area or the whole angle range from $0°$ to $180°$ for the linear and planar arrays and from $-180°$ to $180°$ for the circular array. Here we will always consider the full angle range.

Secondly, we also need to minimize the response of the beamformer at the reference frequency Ω_r over the sidelobe region. This part of the cost function is defined as

$$J_2=\int_{\Theta_s}\left|\mathbf{w}^T\mathbf{s}(\Omega_r,\theta)\right|^2 d\theta \quad (15)$$

where Θ_s denotes the sidelobe region.

Thirdly, we need to maximize the spectrum energy at the reference frequency Ω_r over the mainlobe region.

Then, a complete formulation of the FIB design problem is obtained by combining the three elements together

$$J_{ULS}=\frac{\begin{array}{c}\int_{\Omega}\int_{\Theta_{FI}}\left|\mathbf{w}^T\mathbf{s}(\Omega,\theta)-\mathbf{w}^T\mathbf{s}(\Omega_r,\theta)\right|^2 d\Omega d\theta\\ +\,\alpha\int_{\Theta_s}\left|\mathbf{w}^T\mathbf{s}(\Omega_r,\theta)\right|^2 d\theta\end{array}}{\int_{\Theta_m}\left|\mathbf{w}^T\mathbf{s}(\Omega_r,\theta)\right|^2 d\Omega d\theta} \quad (16)$$

where α is a trade off parameter between the frequency invariant property and the sidelobe attenuation.

Now let (Ω_n,θ_k) be the grid chosen from the continuous frequency and angle ranges above. Then the least squares formulation for the FIB design becomes

$$J_{ULS}=\frac{\begin{array}{c}\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}\left|\mathbf{w}^T\mathbf{s}(\Omega_n,\theta_k)-\mathbf{w}^T\mathbf{s}(\Omega_r,\theta_k)\right|^2\\ +\,\alpha\sum_{\theta_k\in\Theta_s}\left|\mathbf{w}^T\mathbf{s}(\Omega_r,\theta_k)\right|^2\end{array}}{\sum_{\theta_k\in\Theta_m}\left|\mathbf{w}^T\mathbf{s}(\Omega_r,\theta_k)\right|^2} \quad (17)$$

where N and K are the number of samples chosen over the frequency and the angle ranges considered for frequency invariance, respectively. Note that the spectrum energy used here is just the summation of energy over the mainlobe at the reference frequency θ_r, which leads to a much more reduced computational complexity than the conventional methods in the design of broadband beamformers [22]. We can rewrite (17) as

$$J_{ULS}=\frac{\mathbf{w}^T\mathbf{Q}_1\mathbf{w}}{\mathbf{w}^T\mathbf{Q}_2\mathbf{w}} \quad (18)$$

where

$$\begin{aligned}\mathbf{Q}_1=&\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}|\mathbf{s}(\Omega_n,\theta_k)-\mathbf{s}(\Omega_r,\theta_k)|^2\\ &+\alpha\sum_{\theta_k\in\Theta_s}\mathbf{s}(\Omega_r,\theta_k)\mathbf{s}(\Omega_r,\theta_k)^H\end{aligned} \quad (19)$$

and

$$\mathbf{Q}_2=\sum_{\theta_k\in\Theta_m}\mathbf{s}(\Omega_r,\theta_k)\mathbf{s}(\Omega_r,\theta_k)^H \quad (20)$$

As $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are clearly Hermitian, we can obtain $\mathbf{w}^T\mathbf{Q}_1\mathbf{w}=\mathbf{w}^T\mathbf{Q}_{1R}\mathbf{w}$ and $\mathbf{w}^T\mathbf{Q}_2\mathbf{w}=\mathbf{w}^T\mathbf{Q}_{2R}\mathbf{w}$, where $\mathbf{Q}_{1R}$ and $\mathbf{Q}_{2R}$ are the real symmetric matrices of $\mathbf{Q}_1$ and $\mathbf{Q}_2$, respectively. Thus Equation (18) changes to

$$J_{ULS}=\frac{\mathbf{w}^T\mathbf{Q}_{1R}\mathbf{w}}{\mathbf{w}^T\mathbf{Q}_{2R}\mathbf{w}} \quad (21)$$

Thus the optimal $\mathbf{w}$ can be obtained by finding the generalized eigenvector corresponding to the minimum eigenvalue of $\mathbf{Q}_{1R}$ and $\mathbf{Q}_{2R}$. Note that the optimum weight vector obtained in this way will not necessarily have a unity response at the look direction θ_r. To have the desired unity response, we need to normalize the resultant weight vector $\mathbf{w}$. To avoid this normalisation and also have a direct control of the array response to some specific directions, such as a unity response to the desired signal direction, we can follow the total least squares (TLS) approach in the design of broadband beamformers [22] and obtain the following formulation of the problem

$$J_{CTLS}=\frac{\mathbf{w}^T\mathbf{Q}_{1R}\mathbf{w}}{\mathbf{w}^T\mathbf{Q}_{2R}\mathbf{w}+1}\ \text{ subject to } \mathbf{C}^T\mathbf{w}=\mathbf{f}, \quad (22)$$

where $\mathbf{C}^T\mathbf{w} = \mathbf{f}$ provides the desired constraints on the array coefficients. This formulation can be transformed to

$$J_{CTLS} = \frac{\hat{\mathbf{w}}^T\hat{\mathbf{Q}}_1\hat{\mathbf{w}}}{\hat{\mathbf{w}}^T\hat{\mathbf{Q}}_2\hat{\mathbf{w}}} \text{ subject to } \hat{\mathbf{C}}\hat{\mathbf{w}} = \mathbf{0} \tag{23}$$

with $\hat{\mathbf{w}}$, $\hat{\mathbf{Q}}_1$, $\hat{\mathbf{Q}}_2$ and $\hat{\mathbf{C}}$ defined as

$$\begin{aligned} \hat{\mathbf{w}} &= [\mathbf{w}^T, -1]^T, \hat{\mathbf{Q}}_1 = \begin{bmatrix} \mathbf{Q}_{1R} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}, \\ \hat{\mathbf{Q}}_2 &= \begin{bmatrix} \mathbf{Q}_{2R} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}, \hat{\mathbf{C}} = [\mathbf{C}^T, \mathbf{f}] \end{aligned} \tag{24}$$

The solution can be derived in a similar way as in [22].

A problem with this formulation is its high computational complexity and in the design examples, we will only use the formulation given in (21).

4. DESIGN EXAMPLES

To show the effectiveness of the proposed method, we provide several design examples based on a uniformly spaced linear array, a uniformly spaced circular array, and a uniformly spaced rectangular array, respectively.

4.1. Uniformly Spaced Linear Array

We apply the proposed method to a uniformly spaced linear array to design FIBs with a broadside main beam, i.e., a look direction $\theta_r = 90°$. The design is based on a linear array with 14 sensors and each followed by an 18-tap FIR filter. The array spacing is assumed to be half the wavelength corresponding to the maximum normalized signal frequency π so that $\mu = 1$. The frequency range of interest in the design is $[0.3\pi, 0.9\pi]$ and the mainlobe region is $[75°, 105°]$. The fixed reference frequency point is $\Omega_r = 0.7\pi$ and α is set to be 0.05.

The resultant beam pattern with the ULS formulation is shown in Fig. 4 which exhibits a good frequency invariant property over the frequency range $[0.3\pi, 0.9\pi]$ with a satisfactory sidelobe attenuation.

4.2. Uniformly Spaced Circular Array

The proposed least squares approach is general and can be applied to other array structures too. In the following we give an example based on a uniformly spaced circular array with

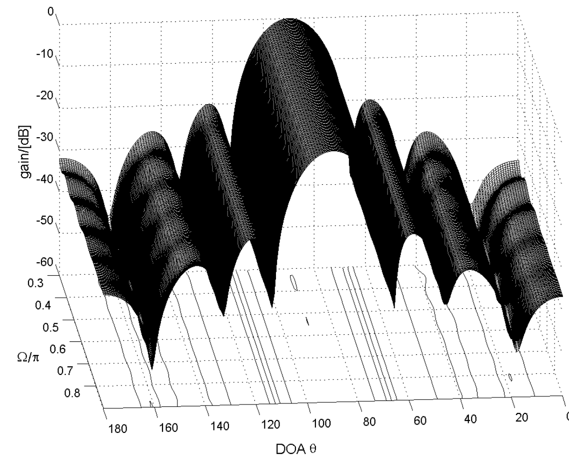

Fig. 4. The resultant beam pattern using the ULS formulation for a main beam direction at $\theta = 90°$.

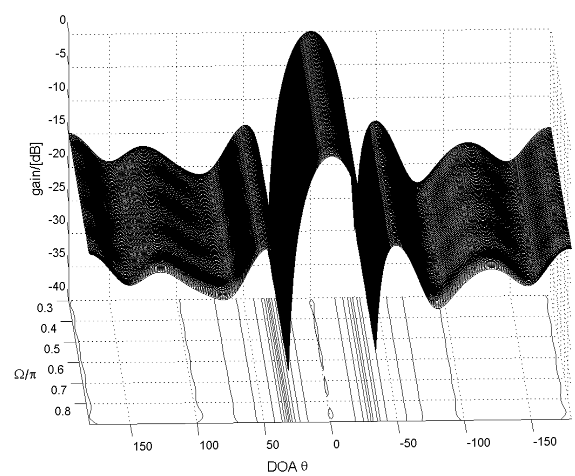

Fig. 5. The resultant beam pattern using the ULS formulation for the uniform circular array.

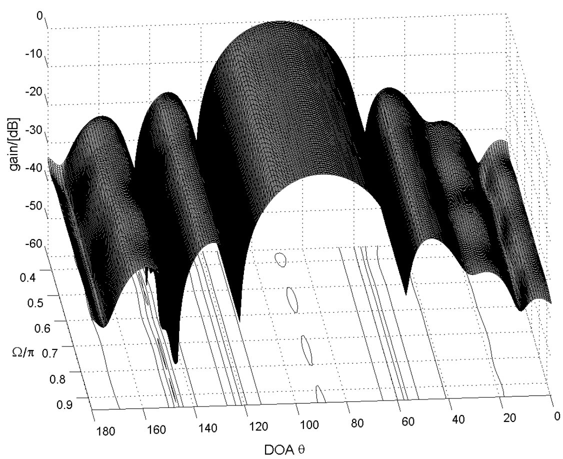

Fig. 6. The resultant beam pattern using the ULS formulation for the uniform rectangular array with a main beam direction at $\theta = 90^\circ$.

14 sensors and each followed by a $20-$tap FIR filter. The look direction is $\theta_r = 0^\circ$ and the mainlobe region is $[-30^\circ, 30^\circ]$. All of the other parameters are the same as in the linear array case except that α is set to be 0.1.

The resultant beam pattern is shown in Fig. 5. Again a good frequency invariant property is achieved over the frequency range of interest.

4.3. Uniformly Spaced Rectangular Array

Finally, we apply the proposed method to a uniformly spaced rectangular array without TDLs to design a broadside main beam. Except for the operating frequency range which changes to $[0.35\pi, 0.95\pi]$, all of the other parameters are the same as in the linear array case. The resultant beam pattern is shown in Fig. 6, with a good frequency invariance property and a satisfactory sidelobe performance.

5. CONCLUSION

A novel least squares formulation to the design of frequency invariant beamformers with an eigenfilter based solution has been proposed, which leads to satisfactory design results as demonstrated by the provided design examples. The proposed approach is general and can be applied to different array structures, such as linear arrays, circular arrays and rectangular arrays.

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