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Subband design of fixed wideband beamformers based on the least squares approach

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ABSTRACT

Subband method is an effective way to reduce the computational complexity of a wideband system and in this paper we study the subband design problem for fixed wideband beamformers, with an emphasis on the design of frequency invariant beamformers (FIBs). We first express the equivalent fullband beam response as a function of the subband beamformer coefficients and then formulate the design problem based on the least squares approach. One direct least squares formulation is first proposed for the design of a general wideband beamformer, and then extended to the FIB design case, followed by three further variations.

1. Introduction

Wideband beamforming has found applications in various areas, ranging from radar, sonar and microphone arrays to wireless communications [1]. A fixed wideband beamformer maintains a response independent of the signal/interference scenarios, with a lower computational complexity compared to the adaptive one and can be implemented easily in practice. As one important class of fixed wideband beamformers, frequency invariant beamformer (FIB) can form beams pointing to the signal of interest with a constant beamwidth [2–9]. With the FIB technique, both adaptive wideband beamforming and wideband DOA (direction of arrival) estimation can be simplified significantly [10,11]. In [9], its design was achieved based on simple multi-dimensional inverse Fourier transforms. More recently, a direct optimization approach was adopted using the convex optimization method [12–14].

The least squares approach is a conventional and well-known method for the design of both FIR filters and wideband beamformers [15–17,5]. Compared with the convex optimization method, it can provide a closed-form solution and is more computationally efficient. Most recently, a series of least squares based methods to the design of FIBs was proposed in [18,19], where a frequency invariant controlling element was incorporated into the cost function, and then three formulations including the constrained least squares (CLS), the unconstrained least squares (ULS) and the constrained total least squares (CTLS) were studied.

To design a wideband beamformer with a high sidelobe attenuation and operating over a large bandwidth, we normally need a larger number of sensors and the attached tapped delay-line (TDL)/FIR filter coefficients, especially for the off-broadside main beam case [20,21]. In order to reduce the computational complexity and improve the performance of the system, subband methods have been introduced for adaptive wideband beamforming [22–31], where each received array signal is first split into subbands by a series of analysis filters, with a subband beamformer set up at each set of the subbands and the subbands beamformers' outputs are then combined together by a synthesis filter bank to form the final fullband beamformer

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output. For fixed wideband beamforming, a subband implementation for a FIB design based on multi-dimensional Fourier transforms was proposed in [21], where the subband array coefficients are obtained separately based on the spatio-temporal distribution of the subband signals at each of the subband arrays. One problem with this design is that its result is dependent on the perfect reconstruction (PR) property of the subband decomposition and any PR error in the system will cause similar errors in the transition areas of the resultant beam pattern between adjacent subbands.

In this paper, we will address the subband design problem for fixed wideband beamformers based on the least squares approach, with an emphasis on the design of FIBs. Instead of translating the desired fullband response into different subbands based on the PR property of subband decomposition, we express the equivalent fullband beam response of the entire subband beamforming system as a function of the subband beamformer coefficients directly, taking into account frequency responses of both the analysis and the synthesis filters. As a result, the traditional PR property is not required any more for subband decomposition. Based on this structure, a direct least squares formulation to the subband design of a general wideband beamformer is then proposed and extended to the FIB design case, followed by three further subband FIB formulations, i.e. CLS, ULS and CTLS.

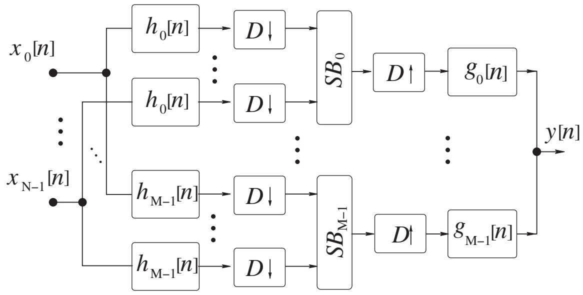

Fig. 1. The subband beamforming structure.

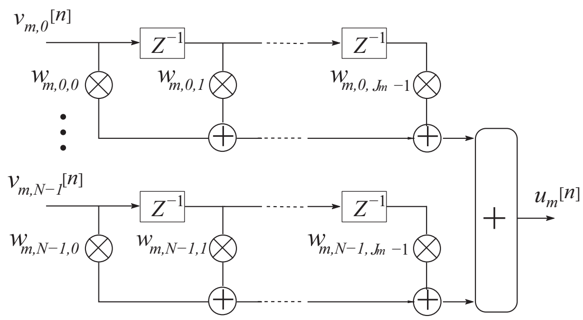

Fig. 2. The structure of SB_m in Fig. 1.

2. Wideband beamforming structure in subbands

Consider a wideband beamforming structure based on a uniform linear array with N sensors and J taps. Its response as a function of the signal frequency ω and arrival angle θ can be written as

$$\tilde{R}(\omega,\theta) = \sum_{l=0}^{N-1}\sum_{k=0}^{J-1} w_{l,k}^{*} e^{-jl\omega\Delta\tau} e^{-jk\omega T_s}, \tag{1}$$

where * denotes the conjugate operation and $\Delta\tau = (b/c)\cos\theta$, with b being the adjacent sensor spacing of the array, c the wave propagation speed, and T_s the delay between adjacent samples in the attached TDLs.

To avoid aliasing in both the spatial and the temporal domains, T_s is set to be half the period of the maximum signal frequency of interest and b is half of the corresponding wavelength λ_{min}. With normalized angular frequency $\Omega = \omega T_s$, we obtain the response as a function of Ω and θ

$$R(\Omega,\theta) = \sum_{l=0}^{N-1}\sum_{k=0}^{J-1} w_{l,k}^{*} e^{-jl\Omega\cos\theta} e^{-jk\Omega} = \mathbf{w}^H \mathbf{s}(\Omega,\theta), \tag{2}$$

where $\mathbf{w}$ is the coefficient vector and $\mathbf{s}(\Omega,\theta)$ is the corresponding $NJ \times 1$ steering vector.

To implement the wideband beamformer in subbands, an M-channel filter banks system is employed with a decimation factor of D, an analysis filter bank of $h_m[n]$ (m=0, 1,…,$M-1$) and a synthesis filter bank of $g_m[n]$ (m=0, 1,…,$M-1$). The resultant subband beamforming structure is shown in Fig. 1, where the received sensor signals are denoted by $x_l[n]$ (l=0, 1,…,$N-1$) with their Fourier transforms given by $X_l(e^{j\Omega})$ $(l=0,1,\ldots,N-1)$. Then we have

$$X_l(e^{j\Omega}) = X_0(e^{j\Omega})e^{-jl\Omega\cos\theta} \quad (l=0,1,\ldots,N-1). \tag{3}$$

Each $x_l[n]$ is split into M frequency bands by M analysis filters $h_m[n]$ with a frequency response $H_m(e^{j\Omega})$, $(m=0,1,\ldots,M-1)$ and decimated by a factor D. A series of subband wideband beamformers SB_m, m=0, 1, …, $M-1$, are then applied to the corresponding subbands. Each SB_m has the same structure as a conventional wideband beamformer and Fig. 2 shows its details, with its input given by $v_{m,l}[n]$ (l=0, 1, …, $N-1$), and its coefficients given by $w_{m,k,l}$ (k=0, 1, …, J_m-1; l=0, 1, …, $N-1$), where J_m is the length of the TDL of the m-th subband beamformer. $v_{m,l}[n]$ is the m-th decimated subband signal of $x_l[n]$ and in frequency domain is given by

$$V_{m,l}(e^{j\Omega}) = \frac{1}{D}\sum_{d=0}^{D-1} X_l(e^{j(\Omega-2\pi d)/D})H_m(e^{j(\Omega-2\pi d)/D}). \tag{4}$$

The output $u_m[n]$ of the beamformer SB_m in frequency domain is

$$U_m(e^{j\Omega}) = \sum_{k=0}^{J_m-1}\sum_{l=0}^{N-1} w_{m,k,l}^{*} V_{m,l}(e^{j\Omega})e^{-j\Omega k}. \tag{5}$$

At the synthesis side, the M outputs, $u_m[n]$ (m=0, 1, …, $M-1$), are upsampled to the original fullband data rate, by the same factor D, then processed by the synthesis filters $g_m[n]$ (m=0, 1, …, $M-1$) and summed up to give the final output $y[n]$. In frequency domain $y[n]$ is given by

$$Y(e^{j\Omega}) = \sum_{m=0}^{M-1} U_m(e^{jD\Omega})G_m(e^{j\Omega}). \tag{6}$$

where $G_m(e^{j\Omega})$ is the frequency response of $g_m[n]$. We can expand (6) as

$$Y(e^{j\Omega}) = \underbrace{\frac{1}{D}\sum_{m=0}^{M-1}\sum_{k=0}^{J_m-1}\sum_{l=0}^{N-1} w^*_{m,k,l} e^{-jl\Omega\cos\theta} e^{-jD\Omega k} X_0(e^{j\Omega}) H_m(e^{j\Omega}) G_m(e^{j\Omega})}_{Y_d(e^{j\Omega})} + \underbrace{\frac{1}{D}\sum_{m=0}^{M-1}\sum_{d=1}^{D-1}\sum_{k=0}^{J_m-1}\sum_{l=0}^{N-1} w^*_{m,k,l} e^{-jl\cos\theta(\Omega-2\pi d/D)} e^{-jD\Omega k} X_0(e^{j(\Omega-2\pi d/D)}) H_m(e^{j(\Omega-2\pi d/D)}) G_m(e^{j\Omega})}_{Y_a(e^{j\Omega})}. \tag{7}$$

$Y_d(e^{j\Omega})$ is the desired signal component and $Y_a(e^{j\Omega})$ is the aliasing part which can be effectively canceled by appropriate design of the filter banks system [32], i.e. we have

$$Y_a(e^{j\Omega}) \approx 0. \tag{8}$$

With the above condition, (7) simplifies to

$$Y(e^{j\Omega}) = \frac{1}{D}\sum_{m=0}^{M-1}\sum_{k=0}^{J_m-1}\sum_{l=0}^{N-1} w^*_{m,k,l} e^{-jl\Omega\cos\theta} e^{-jD\Omega k} X_0(e^{j\Omega}) H_m(e^{j\Omega}) G_m(e^{j\Omega}). \tag{9}$$

Then we obtain the response of the whole beamforming system as

$$R_{sub}(\Omega,\theta) = \frac{Y(e^{j\Omega})}{X_0(e^{j\Omega})} = \frac{1}{D}\sum_{m=0}^{M-1}\sum_{k=0}^{J_m-1}\sum_{l=0}^{N-1} w^*_{m,k,l} e^{-jl\Omega\cos\theta} e^{-jD\Omega k} H_m(e^{j\Omega}) G_m(e^{j\Omega}). \tag{10}$$

We can rewrite (10) as

$$R_{sub}(\Omega,\theta) = \sum_{m=0}^{M-1} H_m(e^{j\Omega}) G_m(e^{j\Omega}) \mathbf{w}^H_{sub_m} \mathbf{s}_{sub_m}(\Omega,\theta) \quad \text{with} \tag{11}$$

$$\begin{aligned}
\mathbf{w}_{sub_m} &= [w_{m,0,0} \dots w_{m,N-1,0} \dots w_{m,0,J_m-1} \dots w_{m,N-1,J_m-1}]^T \\
\mathbf{s}_{sub_m}(\Omega,\theta) &= [1, \dots, e^{-j\Omega(N-1)\cos\theta}, \dots, e^{-j\Omega D(J_m-1)}, \dots, \\
&\quad \times e^{-j\Omega(D(J_m-1)+(N-1)\cos\theta)}]^T.
\end{aligned} \tag{12}$$

In a more compact form, we have

$$R_{sub}(\Omega,\theta) = \mathbf{w}^H_{sub} \mathbf{s}_{sub}(\Omega,\theta) \quad \text{with} \tag{13}$$

$$\begin{aligned}
\mathbf{w}_{sub} &= [\mathbf{w}^T_{sub_0}, \dots, \mathbf{w}^T_{sub_{M-1}}]^T \\
\mathbf{s}_{sub}(\Omega,\theta) &= [\mathbf{s}^T_{sub_0}(\Omega,\theta) H_0(e^{j\Omega}) G_0(e^{j\Omega}), \dots, \mathbf{s}^T_{sub_{M-1}}(\Omega,\theta) \\
&\quad \times H_{M-1}(e^{j\Omega}) G_{M-1}(e^{j\Omega})]^T.
\end{aligned} \tag{14}$$

3. The least squares approach in subbands

In this section, we will propose a series of least squares based methods for the design of wideband beamformers in subbands using the results derived in the last section.

3.1. Design of a general wideband beamformer in subbands

A direct least squares formulation to the design of a general subband wideband beamformer can be constructed as

$$\tilde{J}_{LS_{sub}} = \int_{\Omega_i}\int_{\Theta} F(\Omega,\theta) |R_{sub}(\Omega,\theta) - R_d(\Omega,\theta)|^2 \, d\Omega \, d\theta, \tag{15}$$

where Ω_i and Θ represent the frequency range of interest and the angle range, respectively, $F(\Omega,\theta)$ is a positive weighting function, and $R_d(\Omega,\theta)$ is the desired response. Without loss of generality, here we focus on a specific design case with $F(\Omega,\theta)=1$ and $R_d(\Omega,\theta)=1$ in the main lobe, and $F(\Omega,\theta)=\alpha$ and $R_d(\Omega,\theta)=0$ in the side lobe.

To calculate the integration approximately, we sample the frequency and angle ranges uniformly. Let $(*_k,\theta_n)$ denote one arbitrary sampling point. Then the formulation in (15) changes to

$$J_{LS_{sub}} = \sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_m} |R_{sub}(\theta_n,\Omega_k)-1|^2 + \alpha \sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_s} |R_{sub}(\theta_n,\Omega_k)|^2, \tag{16}$$

where Θ_s and Θ_m represent the sidelobe region and the mainlobe region, respectively.

Eq. (16) can be rewritten into a quadratic form

$$J_{LS_{sub}} = \mathbf{w}^H_{sub} \mathbf{Q}_{LS_{sub}} \mathbf{w}_{sub} - \mathbf{w}^H_{sub} \mathbf{a}_{sub} - \mathbf{a}^H_{sub} \mathbf{w}_{sub} + d_{LS_{sub}}, \tag{17}$$

with

$$\begin{aligned}
\mathbf{Q}_{LS_{sub}} &= \sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_m} \mathbf{S}_{sub}(\Omega_n,\theta_k) + \alpha \sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_s} \mathbf{S}_{sub}(\Omega_n,\theta_k), \\
\mathbf{a}_{sub} &= \sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_m} \mathbf{s}_{sub}(\Omega_n,\theta_k), d_{LS_{sub}} = \sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_m} 1,
\end{aligned} \tag{18}$$

where $\mathbf{S}_{sub}(\Omega_n,\theta_k) = \mathbf{s}_{sub}(\Omega_n,\theta_k)\mathbf{s}_{sub}(\Omega_n,\theta_k)^H$.

Minimizing the cost function in (17) leads to the following solution:

$$\mathbf{w}_{sub} = \mathbf{Q}^{-1}_{LS_{sub}} \mathbf{a}_{sub}. \tag{19}$$

This standard least squares design can be extended to some other variations, such as the total least squares design and eigenfilter design, etc. [16,17].

3.2. Design of FIBs in subbands

The above LS formulation as a general wideband beamforming design method has no mechanism to guarantee a frequency invariant property. To design a FIB in subbands, we introduce a new element to control the frequency invariant property, which is called response variation (RV), given by

$$RV = \sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_{FI}} |\mathbf{w}^H_{sub}\mathbf{s}_{sub}(\Omega_n,\theta_k) - \mathbf{w}^H_{sub}\mathbf{s}_{sub}(\Omega_r,\theta_k)|^2, \tag{20}$$

where Ω_r is a fixed reference frequency and Θ_{FI} represents the direction range in which frequency invariance is considered. It can be either the main beam direction area or the full angle range depending on the specific applications. Without loss of generality, we here only consider the full angle range. When the beamformer has a frequency invariant response, RV will be zero. Combining RV with (16), we obtain a formulation for the subband FIB design problem in subbands as

$$\tilde{J}_{FIB_{sub}} = J_{LS_{sub}} + \beta RV, \tag{21}$$

where β is a trade-off parameter between the frequency invariant property and the original least squares minimization.

Since RV is considered over the full angle range, the frequency invariant property will hold in the same range too. As a result, we only need to minimize the summations in $J_{LS_{sub}}$ at the reference frequency Ω_r over the mainlobe and the sidelobe regions, respectively, instead of over the entire frequency range. Thus we can simplify (21) to

$$J_{FIB_{sub}} = \sum_{\theta_k \in \Theta_m} |\mathbf{w}_{sub}^H \mathbf{s}_{sub}(\Omega_r,\theta_k) - 1|^2 + \alpha \sum_{\theta_k \in \Theta_s} |\mathbf{w}_{sub}^H \mathbf{s}_{sub}(\Omega_r,\theta_k)|^2 + \beta RV. \tag{22}$$

We can rewrite (22) as

$$J_{FIB_{sub}} = \mathbf{w}_{sub}^H \mathbf{Q}_{FIB_{sub}} \mathbf{w}_{sub} - \mathbf{w}_{sub}^H \mathbf{a}_{FIB_{sub}} - \mathbf{a}_{FIB_{sub}}^H \mathbf{w}_{sub} + d_{FIB_{sub}}, \tag{23}$$

with

$$\mathbf{Q}_{FIB_{sub}} = \sum_{\theta_k \in \Theta_m} \mathbf{S}_{sub}(\Omega_r,\theta_k) + \alpha \sum_{\theta_k \in \Theta_s} \mathbf{S}_{sub}(\Omega_r,\theta_k) + \beta \sum_{\Omega_n \in \Omega_i} \sum_{\theta_k \in \Theta_m} (\mathbf{s}_{sub}(\Omega_n,\theta_k) - \mathbf{s}_{sub}(\Omega_r,\theta_k))(\mathbf{s}_{sub}(\Omega_n,\theta_k) - \mathbf{s}_{sub}(\Omega_r,\theta_k))^H,$$

$$\mathbf{a}_{FIB_{sub}} = \sum_{\theta_k \in \Theta_m} \mathbf{s}_{sub}(\Omega_r,\theta_k), d_{FIB_{sub}} = \sum_{\theta_k \in \Theta_m} 1. \tag{24}$$

The optimum solution is given as

$$\mathbf{w}_{FIB_{sub}} = \mathbf{Q}_{FIB_{sub}}^{-1} \mathbf{a}_{FIB_{sub}}. \tag{25}$$

3.3. Variations of the FIB design in subbands

In (22), the two factors α and β provide the relative weighting for the sidelobe attenuation, the frequency invariant property and the gain over the mainlobe region. We can reduce the number of weighting factors by introducing the following constrained least squares (CLS) formulation:

$$J_{CLS_{sub}} = \beta RV + (1-\beta) \sum_{\theta_k \in \Theta_s} |\mathbf{w}_{sub}^H \mathbf{s}_{sub}(\Omega_r,\theta_k)|^2$$

$$\text{subject to } \mathbf{s}_{sub}(\Omega_r,\theta_k)^H \mathbf{w}_{sub} = 1, \theta_k \in \Theta_m. \tag{26}$$

The constraints in (26) guarantee a desired response over the mainlobe region. It can be transformed to

$$\mathbf{S}_{sub}^H \mathbf{w}_{sub} = \mathbf{f} \quad \text{with} \tag{27}$$

$$\mathbf{S}_{sub} = [\mathbf{s}_{sub}(\Omega_r,\theta_1), \ldots, \mathbf{s}_{sub}(\Omega_r,\theta_L)],$$
$$\mathbf{f} = [1, \ldots, 1]^T \in \mathbf{C}^{L \times 1}, \tag{28}$$

where L is the number of angle samples chosen over the mainlobe region.

Substituting (27) into (26), we have

$$J_{CLS_{sub}} = \mathbf{w}_{sub}^H \mathbf{Q}_{CLS_{sub}} \mathbf{w}_{sub} \quad \text{subject to } \mathbf{S}_{sub}^H \mathbf{w}_{sub} = \mathbf{f}, \tag{29}$$

with

$$\mathbf{Q}_{CLS_{sub}} = \beta \sum_{\Omega_n \in \Omega_i} \sum_{\theta_k \in \Theta_{FI}} (\mathbf{s}_{sub}(\Omega_n,\theta_k) - \mathbf{s}_{sub}(\Omega_r,\theta_k))(\mathbf{s}_{sub}(\Omega_n,\theta_k) - \mathbf{s}_{sub}(\Omega_r,\theta_k))^H + (1-\beta) \sum_{\theta_k \in \Theta_s} \mathbf{s}_{sub}(\Omega_r,\theta_k) \mathbf{s}_{sub}(\Omega_r,\theta_k)^H. \tag{30}$$

Its solution can be obtained by the Lagrange multipliers method

$$\mathbf{w}_{CLS_{sub}} = \mathbf{Q}_{CLS_{sub}}^{-1} \mathbf{S}_{sub} (\mathbf{S}_{sub}^H \mathbf{Q}_{CLS_{sub}}^{-1} \mathbf{S}_{sub})^{-1} \mathbf{f}. \tag{31}$$

In a second variation—the ULS formulation, the cost function $J_{CLS_{sub}}$ in (26) will appear at the numerator and the energy at the reference frequency Ω_r over the mainlobe region will be considered at the denominator. The ULS cost function is simply a ratio between these two

$$J_{ULS_{sub}} = \frac{J_{CLS_{sub}}}{\sum_{\theta_k \in \Theta_m} |\mathbf{w}_{sub}^H \mathbf{s}_{sub}(\Omega_r,\theta_k)|^2}. \tag{32}$$

To control the response of the beamformer at the mainlobe region more effectively, we can use the CTLS formulation, which is given by

$$J_{CTLS} = \frac{J_{CLS_{sub}}}{\sum_{\theta_k \in \Theta_m} |\mathbf{w}_{sub}^H \mathbf{s}_{sub}(\Omega_r,\theta_k)|^2 + 1} \quad \text{subject to } \mathbf{S}_{sub}^H \mathbf{w}_{sub} = \mathbf{f}. \tag{33}$$

The optimum solutions $\mathbf{w}_{ULS_{sub}}$ and $\mathbf{w}_{CTLS_{sub}}$ to the above two formulations can be found in [19].

4. Design examples

We will first provide an example by a fullband FIB design method, and then give two examples by the proposed subband methods. Note that the fullband design can be obtained by replacing $R_{sub}(\Omega,\theta)$ by the fullband response $R(\Omega,\theta)$, and $\mathbf{w}_{sub}$ by $\mathbf{w}$. At the end, an example for a general subband wideband beamformer will be provided using the method in (19), to show its limitation in the FIB design.

For the fullband FIB design, it is based on a uniform linear array with the following parameters: $N=10$, $J=140$, $\Omega_i = [0.15\pi, \pi], \Theta_s = [0^\circ, 40^\circ] \cup [80^\circ, 180^\circ]$. The mainlobe area is a single angle point $\Theta_m = 60^\circ$, with the weighting factor $\beta = 0.95$. Fig. 3 shows the resultant beam pattern based on the fullband CLS formulation. Although the design result is satisfactory, it requires a large number of array

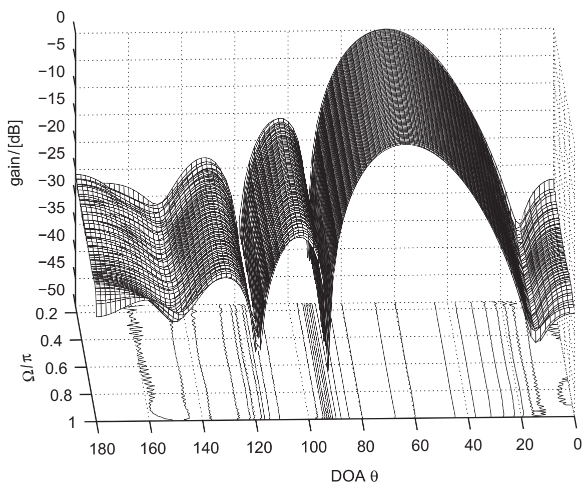

Fig. 3. The resultant beam pattern using a fullband FIB design method.

coefficients and for this case, it is $10\times 140=1400$, which leads to a high computational complexity.

To reduce the computational complexity, we implement the wideband beamformer in subbands using our proposed subband design methods. we employ a set of GDFT filter banks with M=10, D=8 and l_p=110 [21], where l_p is the length of the prototype filter of the GDFT filter banks. All of the subband beamformers have the same TDL length $J_m=(J+l_p)/D=31$, according to [21]. For the subband formulation in (22), α is set to be 0.01. All the other specifications are the same as the fullband one.

Figs. 4 and 5 show the resultant beam patterns by the proposed subband method in (25) and the subband CLS method, respectively. Both have a satisfactory performance in terms of sidelobe attenuation and frequency invariant property over the entire angle-frequency range of interest. (To save space, the design examples for CLS and CTLS are omitted.) To compare the result of the subband design with the fullband one, we define the following performance indexes:

$$PI_1=Mean\left\{\sum_{\Omega_n\in\Omega_i}\sum_{\theta_k\in\Theta_{FI}}|\hat{R}(\Omega_n,\theta_k)-\hat{R}(\Omega_r,\theta_k)|^2\right\},$$

$$PI_2=Mean\left\{\sum_{\theta_k\in\Theta_s}|\hat{R}(\Omega_r,\theta_k)|^2\right\}, \tag{34}$$

where $\hat{R}(\Omega,\theta)$ is the resultant beampattern of the corresponding designs and $Mean\{\bullet\}$ denotes the averaging operation. PI_1 is used to measure the minimization of the RV element and PI_2 is for the sidelobe attenuation. As shown in Table 1, for all the provided design examples, the subband ones have achieved a better performance than the fullband one.

In addition, compared to the results in [21], there are no distortions around the subband transition areas caused by PR error of the filter banks system. This can be explained by the fact that PR error has been compensated by the subband coefficients during the design process since our approach is general and has taken into consideration the overall response of the filter banks system, which effectively eliminates the traditional PR requirement.

For the computational complexity, for a real-valued input signal, it is 1164 real multiplications for the subband implementation and 1400 for the fullband one; for a complex-valued input, we have 2145 and 2800, respectively. In both cases, a much lower computational complexity has been achieved by the subband-based beamformer.

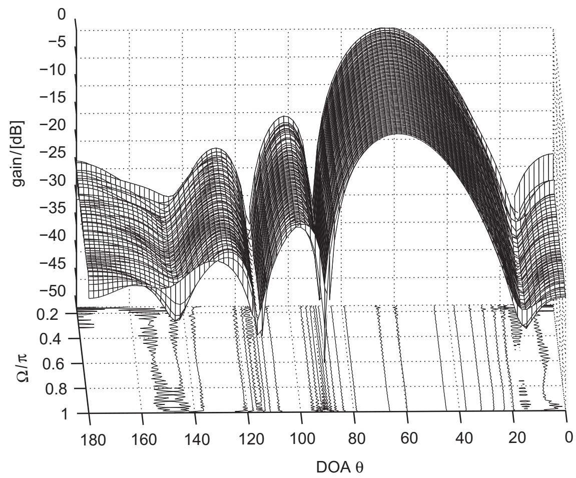

Fig. 4. The resultant beam pattern using the method in (25).

Table 1
Performance comparison between subband and fullband designs.

Performance index	PI_1	PI_2/(dB)
Fullband	2.3113e−005	−17.6278
Subband method in (25)	2.3012e−005	−17.6321
Subband CLS	2.2130e−005	−18.0125
Subband ULS	2.1986e−005	−18.1207
Subband CTLS	2.2040e−005	−18.0106

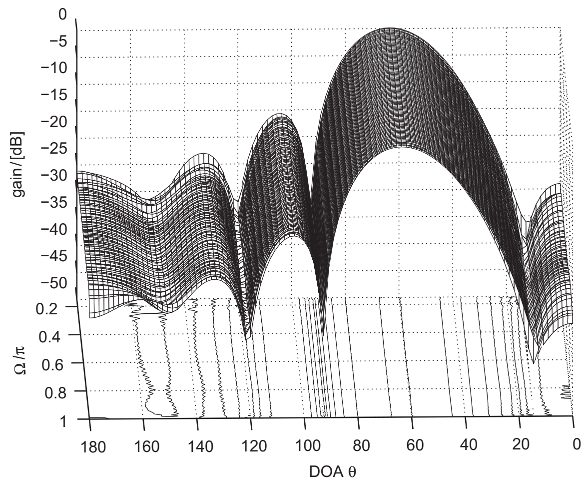

Fig. 5. The resultant beam pattern using the CLS method in subbands.

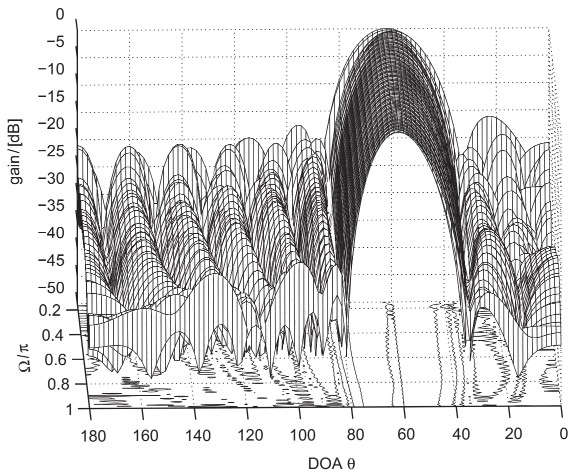

Fig. 6. The resultant beam pattern using the method in (19).

Finally we give an example using the method in (19) for the design of a general wideband beamformer in subbands. Its specifications are the same as the subband FIB design examples except that α is set to be 0.1. Fig. 6 shows the resultant beam pattern with a satisfactory sidelobe attenuation and unity gain at the look direction. However, as we discussed, such a general design approach cannot provide a frequency invariant response, especially for the transition area since there is no constraint imposed on that part.

5. Conclusion

The design of subband fixed wideband beamformers has been studied based on a series of least squares formulations, with an emphasis on the design of FIBs. Instead of translating the fullband desired response into subbands and then designing the subband beamformers separately in each set of subbands, we express the equivalent fullband beam response of the subband beamformer as a function of the subband beamformer coefficients directly taking into consideration frequency responses of both the analysis and the synthesis filters, therefore eliminating the traditional PR requirement for subband decomposition. One direct least squares formulation has been proposed for the design of a general wideband beamformer, and then extended to the FIB case, which is thereafter followed by three further variations. Design examples have been provided with an improved performance and reduced computational complexity. Although we have used a uniform linear array in our derivation, the proposed method can be extended to other array geometries easily.

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