Chebyshev-like Low Sidelobe Beampatterns with Adjustable Beamwidth and Steering-Invariance

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ABSTRACT

A modification of the classic Dolph-Chebyshev low sidelobe array pattern design is introduced, which allows the adjustment of the mainlobe's width relatively freely and independently from the sidelobe level. In order to illustrate the advantages of the tool provided by the new design method, its application to Uniform Linear Arrays (ULA's) and Uniform Circular Arrays (UCA's) is discussed. It is shown that by using the proposed design, low sidelobe beampatterns that are steering invariant within a certain range are achievable with a ULA. In addition the synthesis of beampatterns similar to those of sector antennas, but with electronically adjustable mainlobe, sidelobe level and steering direction are yielded with UCA's.

1. INTRODUCTION

Conventional Dolph-Chebyshev arrays were proposed by Dolph in 1946 [1] and are based on mapping the Chebyshev polynomial into the array's space factor. Dolph proved that for a desired SideLobe Ratio (SLR), the Chebyshev polynomial of order $L-1$ can be mapped into the space factor of a Uniform Linear Array (ULA) of L elements. This results in an equiripple low sidelobe beampattern with the narrowest possible mainlobe.

Originally, the design of Dolph's current distributions was restricted to linear arrays and was applicable to broadside steering only. Little has changed since the classic method came about, although some advances on how to compute its current distribution were made by Stegen [2], Davidson [3] and Jazi [4] and others.

Among modifications of a more fundamental quality, Jazi recently proposed [5] the use of a Chebyshev polynomial elevated to the n-th power, so as to increase the order of its zeros thus deepening the nulls and reducing the number of sidelobes in the pattern. This yields the same prescribed SLR, with higher directivity, at the expense of slight broadening of the mainlobe. Finally, the design of Chebyshev beampatterns for Uniform Circular Arrays (UCA's) was verified in [6], using a technique that transforms a UCA into a virtual ULA, presented long ago [7]. Using this last technique, given the fact that UCA's patterns are almost rotation-invariant, an angular span of 360° can be scanned with a near perfect invariant low sidelobe beampattern, by electronically rotating a Chebyshev beampattern.

However, since a UCA has a larger aperture than a ULA with the same number of elements and the same inter-element spacing, the UCA mainlobe beamwidth is always larger than that of a ULA. This is true for a certain angular range about broadside.

Equations (1) and (2) below, give the apertures of a ULA seen from an angle Θ to the end-fire, and of a UCA, for arrays with L elements and inter-element spacing Δe, respectively. It is easy to verify that for large arrays the ratio A_{ULA}/A_{UCA} tends to $\pi cos(\Theta)$, independently of Δe.

$$A_{ULA}(L,\Delta e) = (L-1)\Delta e\cos(\Theta) \tag{1}$$

$$A_{UCA}(L,\Delta e) = \frac{\Delta e}{2\sin\left(\frac{\pi}{L}\right)}\left(1-\cos\left(\frac{2\pi}{L}\left\lfloor\frac{L}{2}\right\rfloor\right)\right) \tag{2}$$

Therefore, in applications such radars for Intelligent Transport Systems (ITS), where it is desired to scan only a limited angular span ahead of the vehicle using a fixed-width, rotation-invariant and low-sidelobe pattern, we may have the following situation. While a ULA provides (at broadside) the narrowest beam for given SLR, L and Δe, it is unable to deliver rotation-invariance since the mainlobe's width enlarges as the beam is steered to angles closer to end-fire. A UCA provides rotation-invariance over the whole 360^0 span (which is rather unnecessary in the case) at the cost of delivering a much wider beamwidth. The dilemma is therefore of either sacrificing too much on the beamwidth with UCA beampattern or too much on the rotation invariance, with a ULA. Clearly, an algorithm that provides a balanced tradeoff between both features as design parameters is highly desirable.

Even when considering the fact that a whole 360° scan is desirable, there are many applications in communications such as Spatial Equalization and Space-Time Coding where it is desirable that the beamwidth of the rotation-invariant pattern be adjustable. In this light, the contribution in [6] does add rotation-invariance to the design of low-sidelobe patterns, but does not deliver the necessary flexibility, in terms of adaptability.

In this paper, we propose a new method, based on a modification of the Dolph-Chebyshev design that provides such flexibility. The paper is organized as follows. First, briefs on the Dolph design [1] and its application to UCA's [6] are given in sections 2 and 3. The proposed method is explained in section 4 followed by applications to UCA and ULA, demonstrated along with simulation results in sections 5 and 6. Finally, in section 7 conclusions and discussions regarding further research interests on the matter are given.

2. CLASSIC DOLPH-CHEBYSHEV ARRAYS

For the sake of convenience we briefly review the classic Dolph-Chebyshev design. We start with the Chebyshev polynomial, which can be written as [8]

$$T(N,x)=\begin{cases} cos(N\, arccos(x)) & if \quad |x|\le 1 \\ cosh(N\, arccos\, h(x)) & if \quad |x|\ge 1 \end{cases}. \quad (3)$$

Given a prescribed *SLR* first we compute the value of x that makes the $|T(N,x)|$ equal to *SLR* by

$$x_0 = cos\left(\frac{arccos\, h(SLR)}{N}\right). \quad (4)$$

Since the Chebyshev polynomial has only real coefficients and all roots in the interval $x \in [-1,1]$, $|T(N,x)|$ are monotonically increasing for $|x| > 1$. Therefore:

$$|T(N,y)| > |T(N,z)| > 1 \quad \forall \quad y > z > 1 \quad (5)$$

Within the interval $x \in [-1,1]$ the polynomial has its amplitude limited to 1. Dolph visualized that, if a ULA with L elements ($N = L - 1$) and interelement spacing Δe is used, the excitation of the n-th antenna element can be calculated by

$$A_n = \sum_{m=1}^{L} T\left(L-1, x_0\, cos\left(\frac{\pi m}{L}\right)\right) e^{-j(2n-L-1)\frac{\pi m}{L}} \quad (6)$$

where $n = 1,2,...,L$. If the phases of the signals at all elements are driven so as to steer the mainlobe towards an angle θ_s, the resulting beampattern will exhibit a space factor exactly given by the equation below.

$$\left|T(L-1, x_0\, cos(\pi \Delta e(cos(\theta) - cos(\theta_s))))\right| \quad (7)$$

While the amplitude limitation of the Chebyshev polynomial is responsible for the equiripple sidelobes, the monotonic behavior for $|x| > 1$ is responsible for its mainlobe's width inflexibility. The beamwidth of a pattern steered to θ_s can be computed once the criterion that defines the limits of the mainlobe is chosen. In the case of equiripple low sidelobe patterns, a reasonable choice is the point where the mainlobe crosses the sidelobe upper bound, i.e., the *sidelobe level beamwidth* ($\Delta\theta_{SL}$) is defined in terms of the distances between the steering angle (θ_s) and the angles to the right (θ_R) and to the left (θ_L) of the mainlobe's peak where the gain equals the sidelobe level. If a ULA is used, steering towards any direction other than broadside causes the mainlobe to enlarge, especially at angles close to the end-fire. There is therefore a limiting angle after which the beamwidth will enlarge to the extent that part of it falls outside the visible region. Making use of the symmetry of the array, this limit can be defined in terms of a minimum steering angle, where θ_L vanishes. Thus:

$$\theta_{s\,min} = arccos\left(1 - \frac{1}{\pi \Delta e} arccos\left(\frac{1}{x_{peak}}\right)\right); \quad (8)$$

$$\hat{\theta}_s = arcsin(sin(\theta_s)). \quad (9)$$

In the case of the Dolph-Chebyshev design, in equation (8) above, as well as in those that follow, x_{peak} assumes the values of x_0, given in (4). Then, if $\hat{\theta}_s \ge \hat{\theta}_{s\,min}$, θ_R and θ_L can be respectively calculated by

$$\theta_L = arccos\left(\frac{1}{\pi \Delta e} arccos\left(\frac{1}{x_{peak}}\right) + cos(\theta_s)\right); \quad (10)$$

$$\theta_R = \pi - arccos\left(\frac{1}{\pi \Delta e} arccos\left(\frac{1}{x_{peak}}\right) - cos(\theta_s)\right). \quad (11)$$

Accordingly, if $\hat{\theta}_s < \hat{\theta}_{s\,min}$, θ_R and θ_L can be calculated by

$$\theta_L = \pi - arccos\left(\frac{1}{\pi \Delta e} arccos\left(\frac{-1}{x_{peak}}\right) - cos(\theta_s)\right); \quad (12)$$

$$\theta_R = arccos\left(\frac{1}{\pi \Delta e} arccos\left(\frac{-1}{x_{peak}}\right) + cos(\theta_s)\right). \quad (13)$$

Using the above equations, $\Delta\theta_{SL}$ can be computed by

$$\Delta\theta_{SL} = \begin{cases} \theta_R - \theta_L & if \quad \hat{\theta}_s \ge \hat{\theta}_{s\,min} \\ \pi - \theta_R + \theta_L & if \quad \hat{\theta}_s < \hat{\theta}_{s\,min} \end{cases}. \quad (14)$$

Since in Chebyshev arrays x_0 is a function of *SLR* and *L*, as in (4), the above formulae establish a direct relationship between the steering direction and the beamwidth. In other words, in the classic design, the beamwidth is a function of the steering direction θ_s, the number of elements in the array L, its interelement spacing Δe and the desired sidelobe ratio *SLR*. Moreover, it was shown in [1] that for fixed θ_s, L, Δe and *SLR*, the beamwidth of the Dolph-Chebyshev array is the minimum possible at broadside. It is however known that the mainlobe's width and shape change (distort) as it is steered to angles closer to end-fire.

3. DOLPH-CHEBYSHEV DESIGN FOR UCA

Consider the application of Dolph-Chebyshev to a UCA, as in [2]. For convenience, we briefly review the formulas and procedures here. Let the steering vector of a UCA of omnidirectional elements and minimum inter-element distance equal to Δe be:

$$\breve{\boldsymbol{a}}(\theta) = \left[e^{-j\frac{2\pi R}{\lambda} cos\left(\frac{2\pi(i-1)}{L} - \theta\right)} \right]_{i=1,\cdots,L-1} \quad (15)$$

where $R = \Delta e \lambda \left(2\, sin\left(\pi/L\right)\right)^{-1}$. If $\omega = exp\left(j 2\pi/L\right)$ and

$$h = max\left\{ h \le \frac{L-1}{2} \left| \frac{\left|J_{h-L}\left(2\pi R/\lambda\right)\right|}{\left|J_h\left(2\pi R/\lambda\right)\right|} \le \varepsilon << 1 \right. \right\}. \quad (16)$$

we can define the matrices $\boldsymbol{F}$ and $\boldsymbol{J}$ respectively by

$$\boldsymbol{F} = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & \omega^{-h} & \cdots & \omega^{-(L-1)h} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & \omega^{h} & \cdots & \omega^{(L-1)h} \end{bmatrix}, \quad (17)$$

$$J = diag\left\{ \left(j^m \sqrt{L} J_m \left(2\pi R / \lambda \right) \right)^{-1} \right\}, \qquad (18)$$

with $m = -h,...,0,...,h$ and $J_{-m}(x) = (-1)^m J_m(x)$. Then, if we right-multiply $\boldsymbol{JF}$ to $\breve{\boldsymbol{a}}(\theta)$, we have

$$\bar{\boldsymbol{a}}(\theta) = \boldsymbol{JF}\breve{\boldsymbol{a}}(\theta) = \begin{bmatrix} e^{-jh\theta} & \cdots & 1 & \cdots & e^{jh\theta} \end{bmatrix}. (19)$$

The resulting steering vector is similar to the steering vector of a linear array, except that the elements of $\bar{\boldsymbol{a}}(\theta)$ are no longer dependant on $cos(\theta)$, but on θ only. Using this transformation the design of Dolph-Chebyshev current distributions for a UCA is possible. The beamwidth of this pattern is not dependent on θ_s because in this case, equation (14) reduces to

$$\Delta\theta_{SL} = \theta_R - \theta_L = 4\, arccos\left(1 / x_{peak} \right). \qquad (20)$$

Once the virtual ULA has $2h+1$ elements [2], from (4) we see that (17) reduces to

$$\Delta\theta_{SL} = \theta_R - \theta_L = 4\, arccos\left(cosh\left(\frac{arccos\,h(SLR)}{2h} \right)^{-1} \right). (21)$$

Thus, the beamwidth of the pattern obtained with [2], although independent of the steering direction, is a function of L, Δe and SLR and therefore the mainlobe's width cannot be adjusted freely. Another drawback of this design is that since a UCA has an aperture narrower than that of a ULA with the same number of elements and the same interelement spacing. The beamwidth of this beamformer (22) is significantly larger than that of the Dolph-Chebyshev beampattern with a ULA (14) not only at broadside but also within a significantly large angular span around it

Figure 1 illustrates this fact showing curves of beamwidth against steering direction of ULA and UCA Dolph-Chebyshev beampatterns with L = 20 and Δe = 0.5λ for prescribed SLR = –20dB and –40dB. It is seen that the price for a rotation-invariant mainlobe obtained with a UCA is significant enlarging of the beamwidth.

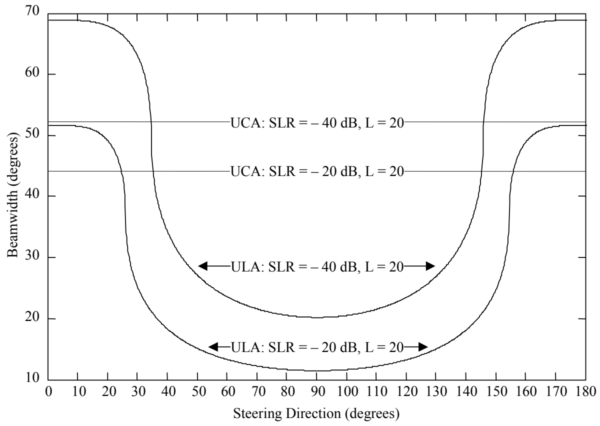

Figure 1 – Beamwidth of Conventional ULA and UCA Chebyshev against Steering Direction.

4. PROPOSED MODIFIED CHEBYSHEV DESIGN

In this section we introduce a modified Chebyshev-like design and demonstrate how the proposal yields the solution of the problems highlighted above.

We begin by noting that if the monotonic behavior of the Chebyshev polynomial at $|x| > 1$ is removed, while maintaining the limited amplitude in $x \in [-1,1]$, we obtain a pattern that is Chebyshev-like, with an adjustable beamwidth. Thus, we replace the Chebyshev polynomial with the function

$$G(N,x,\alpha,\beta) = \begin{cases} cos\left(N\left(\beta - e^{\alpha|x|} \right) arccos(x) \right) & |x| \le 1 \\ cosh\left(N\left(\beta - e^{\alpha|x|} \right) arccos\,h(x) \right) & |x| > 1 \end{cases}. \qquad (22)$$

If $\alpha = 0$ and $\beta = 2$, the proposed function reduces to the Chebyshev polynomial. However, the term $N(\beta - e^{\alpha|x|})arcos(x)$ vanishes if $\beta = e^{\alpha|x|}$. In this case $\exists\, x \mid G(N, x, \alpha, \beta) = 1$. Consequently, if appropriate α and β are chosen, an inflexion point can be added outside the interval $x \in [-1,1]$. On the other hand, within this interval, the term $\beta = e^{\alpha|x|}$ does nothing more than cause changes to the number and positions of zeros of $cos(N(\beta - e^{\alpha|x|})arcos(x))$, once this function is limited to $[-1,1]$. This is illustrated in Figure 2.

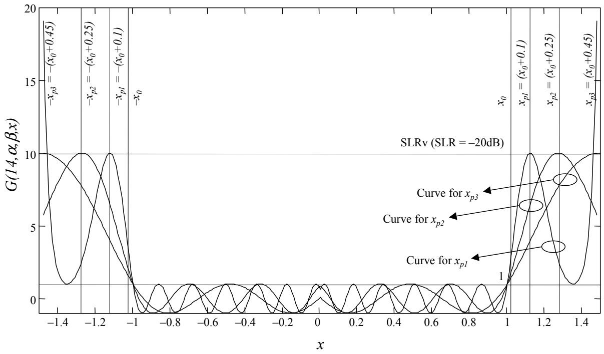

Figure 2 – Plots of Proposed Function for Different x_p.

Therefore, unlike the conventional Chebyshev, the proposed beampattern design involves the optimization of α and β so as to place the inflexion point at the value of $x_p \ge x_0$. This determines a beamwidth that is lower-bounded by Chebyshev's, while adjusting the peak value to the desired SLR and can be achieved by putting

$$G(N,x_p,\alpha,\beta) = cosh\left(N\left(\beta - e^{\alpha|x_p|} \right) arccos\,h(x_p) \right) = SLR\cdot \qquad (23)$$

which yields

$$\left(\beta - e^{\alpha|x_p|} \right) arccos\,h(x_p) = \frac{arccos\,h(SLR)}{N}. \qquad (24)$$

Next, we take

$$\left. \frac{\partial G(N,x,\alpha,\beta)}{\partial x} \right|_{x_p} = 0 \qquad (25)$$

which yields

$$\frac{\left(\beta - e^{\alpha|x_p|} \right)}{\sqrt{x_p^{\,2} - 1}} = \alpha e^{\alpha|x_p|}\, arccos\,h(x_p)\cdot \qquad (26)$$

Substituting (24) into (26) we get

$$\alpha e^{\alpha|x_p|} = \frac{arccos\,h(SLR)}{N\sqrt{x_p^{\,2} - 1}\, arccos\,h^2(x_p)}. \qquad (27)$$

The above equation allows for the optimization of α independent of β, and can be achieved by a simple linear regression. To this end we first rewrite (27) as

$$ln(\alpha) = ln(P) - \alpha|x_p| \rightarrow \alpha = e^{ln(P) - \alpha|x_p|} \qquad (28)$$

where $P = \dfrac{arccos\,h(SLR)}{N\sqrt{x_p^{\,2} - 1}\,arccos\,h^2(x_p)}$. The regression is done by repeatedly computing

$$\alpha_{k+1} = e^{ln(P) - \alpha_k|x_p|} \qquad (29)$$

where $\alpha_0 = 0$.
Finally, β can be calculate from:

$$\beta = e^{\alpha|x_p|} + \frac{arccos\,h(SLR)}{N\,arccos\,h(x_p)}. \qquad (30)$$

Once α and β are calculated, the function is mapped into the array factor as in the classic Dolph design, i.e., the current distribution is calculated using the Inverse Fourier Transform (IFT). For a ULA with L elements and interelement spacing Δe, the excitation of the n-th antenna element is given by

$$A_n = \sum_{m=1}^{L} G\left(L-1, x_p\,cos\left(\frac{\pi m}{L}\right), \alpha, \beta\right) e^{-j(2n-L-1)\frac{\pi m}{L}}. \qquad (31)$$

where $n = 1,2,...,L$. If the phases of the signals at all elements are driven so as to steer the mainlobe towards an angle θ_s, the resulting beampattern will exhibit a space factor approximately given by

$$\left|G\left(L-1, x_p\,cos\left(\pi\Delta e\left(cos(\theta) - cos(\theta_s)\right)\right), \alpha, \beta\right)\right|. \qquad (32)$$

The beampattern of the proposed design method is not exactly identical to that given by equation (32) because the function (22) is not a polynomial. For a L-element array, the true beampattern will therefore be given by the Fourier series of equation (32), truncated after L-terms. It is has however consistently been verified via simulations that the error incurred in using equation (32) is negligible. This is because: a) when L is large enough the truncation does not greatly affect the final value of the Fourier series; or b) when L is not that large, errors appear more often in the sidelobe region altering the number and location of nulls and sidelobes, rather than their level.

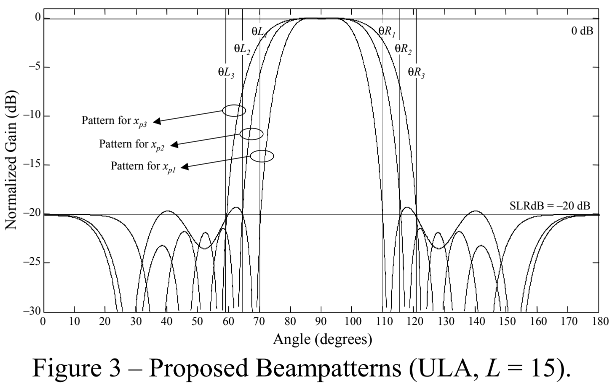

Figure 3 – Proposed Beampatterns (ULA, L = 15).

Figure 3 illustrates the effect of the choice of x_p in the proposed design method with the mainlobe steered to broadside (θ_s = 90°). It is seen that the enlarged mainlobe also exhibits (desirably) a flat top when x_p is large enough. This is expected and is due to the non-linear relationship between x and θ.

$$x = x_p\,cos\left(\pi\Delta e\left(cos(\theta)\right)\right). \qquad (33)$$

5. PROPOSED DESIGN APPLIED TO UCA

In this section we provide an example of application of the proposed beamformer illustrating its benefits compared to the one proposed in [2]. We start by observing that, like the Chebyshev polynomial, the proposed function returns 1 at $x = 1$, independently of the values of N, α and β. This means that the previous definitions of beamwidth ($\Delta\theta_{SL}$) given by (14) and (20) are equally applicable to the proposed beamformer. However, unlike the classic Dolph-Chebyshev, the value of $x_{peak} = x_p$ in (8), (10) ~ (14) and (20) can be freely chosen ($x_p \geq x_0$) in the proposed design and the beamwidth can therefore be adjusted (enlarged) as desired. For instance, if the proposed design method is applied to a UCA, equation (20) becomes:

$$\Delta\theta_{SL} = 4\,arccos\left(x_p^{-1}\right). \qquad (34)$$

In other words, for any desired $\Delta\theta_{SL}$ grater than that given by equation (20), equation (34) yields a value of $x_p > x_0$, i.e.:

$$x_p = cos\left(\frac{\Delta\theta_{SL}}{4}\right)^{-1}. \qquad (35)$$

Once the value of x_p that provides the desired beamwidth is known, it is can then be introduced as a parameter in the appropriate equations of the proposed beamformer (23) ~ (32).

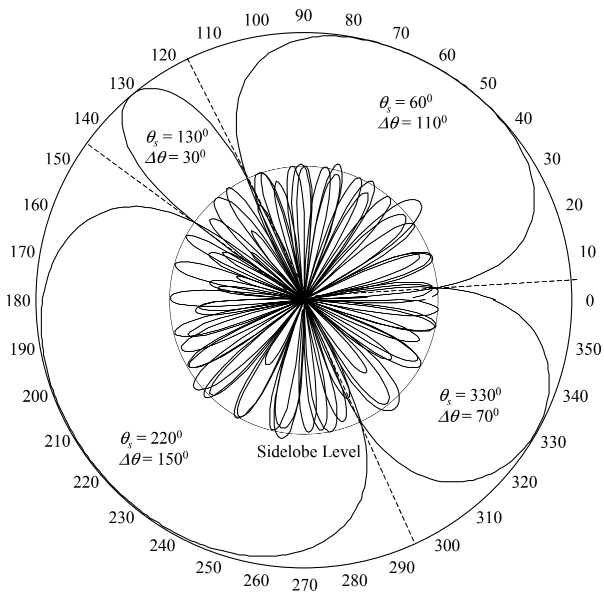

Figure 4 – Proposed Beampatterns (UCA, L = 35).

Figure 4 demonstrates the flexibility of the proposed design method applied to a UCA. Four different beampatterns are displayed, all with the SLR = –20dB and designed with a UCA of 35 and $\Delta e = 0.5\lambda$, but with different θ_s and $\Delta\theta$. It can be seen that the adjustability of the mainlobe's width comes at the expense of increasing the possibility of obtaining patterns with non-equiripple sidelobes, but the prescribed sidelobe level is rarely violated, and only slightly if so.

The proposed design method applied to a UCA can be summarized as follows (given L, Δe, SLR, $\Delta\theta$ and θ_s).
1) Use (17) to calculate h.
2) Use (4) to compute the value of x_o associated to the narrowest beamwidth.
3) Use (20) to calculate the narrowest possible beamwidth.
4) For a desired beamwidth larger than the one calculated in the step above, use (35) to compute the value of x_p associated to it.
5) Use (29) to calculate the optimum value of α.
6) Use (30) to compute the optimum value of β.
7) Use (31) to compute the current distribution.
8) Multiply every element of the transformed steering vector (19) towards θ_s with the correspondent current distribution obtained above.

6. PROPOSED DESIGN APPLIED TO ULA

The application of the proposed beamforming method to ULA's, however, is another issue. Obviously, it is impossible to avoid the deformation caused by steering the beam anywhere other than broadside. The extended Chebyshev design proposed here, however, can be used to allow almost perfect rotation-invariant low sidelobe scanning over a limited range with a ULA. Note that unlike the UCA case, a value of x_p associated to a desired beamwidth cannot be calculated directly. In order to derive a method to calculate x_p, we revisit the expressions for the beamwidth of the steered extended Chebyshev. Let us put

$$A = \frac{1}{\pi\Delta e} arccos\left(\frac{1}{x_p}\right) - cos(\theta_s); \tag{36}$$

$$B = \frac{1}{\pi\Delta e} arccos\left(\frac{1}{x_p}\right) + cos(\theta_s) \cdot \tag{37}$$

Then, within the limit $\hat{\theta}_s \geq \hat{\theta}_{s\,min}$, from (10), (11) and (14) we have

$$\Delta\theta_{SL} = \pi - arccos(A) - arccos(B). \tag{38}$$

Substituting (41) and (42) we get to (from now we drop the subscript SL of $\Delta\theta$):

$$B = -A\,cos(\Delta\theta) + sin(\Delta\theta)\,sin(arccos(A)). \tag{39}$$

Since $B = A + 2\,cos(\theta_s)$ we get

$$\begin{aligned} A_{k+1} &= -2\,cos(\hat{\theta}_s) - A_k\,cos(\Delta\theta) + \\ &+ sin(\Delta\theta)\,sin(arccos(A_k)) \end{aligned}. \tag{40}$$

Proceeding in a similar way, we derive

$$\begin{aligned} B_{k+1} &= 2\,cos(\hat{\theta}_s) - B_k\,cos(\Delta\theta) + \\ &+ sin(\Delta\theta)\,sin(arccos(B_k)) \end{aligned}. \tag{41}$$

Given A_k and B_k, (36) and (37) respectively yield

$$x_p = \frac{1}{cos\left(\pi\Delta e\left(A_k + cos(\hat{\theta}_s)\right)\right)}; \tag{42}$$

$$x_p = \frac{1}{cos\left(\pi\Delta e\left(B_k - cos(\hat{\theta}_s)\right)\right)} \cdot \tag{43}$$

The above equations allow for a simple recursive procedure to compute the x_p necessary to obtain a mainlobe of width $\Delta\theta$ steered towards θ_s as follows:
1) Start with $x_p = x_o$.
2) Use (36) to compute A_o.
3) Use (40) to update A.
4) Use (42) to recalculate x_p.
5) Use (37) to compute B.
6) Use (41) to update B.
7) Use (43) to recalculate x_p.
8) Go back to step 3).

The above calculations are verified to converge extremely quickly and stably to the desired value of x_p. In fact, the convergence can be made even faster and more secure if we observe that x_p is bounded by the values associated to broadside and to end-fire. Making $\theta_s = 90^o$ in equation (38), we can derive:

$$x_{p_{max}} = \frac{1}{cos\left(\frac{1}{\pi\Delta e} cos\left(\frac{\pi - \Delta\theta}{2}\right)\right)} \cdot \tag{44}$$

For the sake of consistency, we also derive the formulas to compute the values of x_p to obtain a beamwidth of $\Delta\theta$ when the array is steered to angles below θ_{smin}. To this end, we proceed analogously to the above. Let A be as in (36) and B now be given by

$$B = \frac{1}{\pi\Delta e} arccos\left(\frac{-1}{x_0}\right) - cos(\hat{\theta}_s) \cdot \tag{45}$$

Since $arccos(-x) = \pi - arccos(-x)$, we have

$$A = -B + \frac{1}{\Delta e} - 2\,cos(\hat{\theta}_s) \cdot \tag{46}$$

Then, from (12) to (14) we have, for $\hat{\theta}_s < \hat{\theta}_{s\,min}$,

$$\begin{aligned} A_{k+1} &= \frac{1}{\Delta e} - 2\,cos(\hat{\theta}_s) + A_k\,cos(\Delta\theta) + \\ &- sin(\Delta\theta)\,sin(arccos(A_k)) \end{aligned}; \tag{47}$$

$$\begin{aligned} B_{k+1} &= \frac{1}{\Delta e} - 2\,cos(\hat{\theta}_s) + B_k\,cos(\Delta\theta) + \\ &+ sin(\Delta\theta)\,sin(arccos(B_k)) \end{aligned}. \tag{48}$$

The recursive procedure to compute the value of x_p now becomes:
1) Start with $x_p = x_o$.
2) Use (36) to compute A_o.
3) Use (47) to update A.
4) Use (42) to recalculate x_p.
5) Use (45) to compute B.
6) Use (48) to update B.
7) Use (43) to recalculate x_p.
8) Go back to step 3).

Similarly to before, the lower bound on x_p is

$$x_{p_{min}} = \frac{1}{cos\left(1 + \frac{1}{\pi\Delta e} cos\left(\pi - \frac{\Delta\theta}{2}\right)\right)} \cdot \tag{49}$$

Figures 5 to 8 illustrate the possibilities of beamforming with the proposed algorithm and a ULA, with respect to performing uniform scanning of a limited angular range with a rotation invariant low sidelobe beampattern.

In figure 5, L = 20 and SLR = –20dB. The beamwidth curve of the proposed beamformer is contrasted with those of the conventional ULA Dolph-Chebyshev and of the method proposed in [2]. The interval of interest is 35°~145° and the objective is to scan it with a beamwidth-invariant pattern. It is seen that the proposed algorithm delivers the best possible trade-off between the beamwidth and invariance, since with the proposed method a 21° wide mainlobe is achieved, against an almost 45° wide mainlobe obtained with the technique in [2].

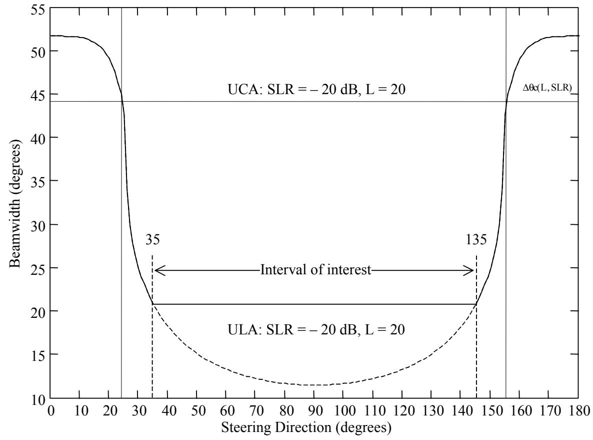

Figure 5 – Beamwidths of Proposed Beamformer with ULA and of Chebyshev with ULA and UCA.

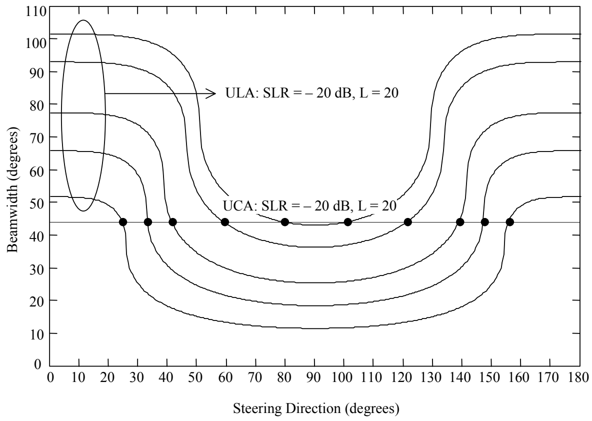

Figure 6 – Beamwidths of Proposed Beamformer with ULA for Different x_p and of Chebyshev with UCA.

In order to provide better understanding of the above result, the beamwidth curve of the steered –20dB conventional Chebyshev pattern of a 20-element UCA is compared to those of the extended ULA Chebyshev patterns of the same size, with various values of $x_p \geq x_o$ in figure 6. It is seen that the extended design encompasses a family of curves covering the whole region above that of the conventional ULA Chebyshev, which forms the lower bound. Any desired beamwidth curve can then be obtained, including a straight line (invariant beam scanning) as a special case.

Figure 7 shows some of the beams in the interval of interest so as to demonstrate how the mainlobe's shape in the proposed beampattern is approximately preserved while that of the Chebyshev beamformer varies greatly. This will be true whenever the interval of interest is well within the limits determined by equations (9) and (10).

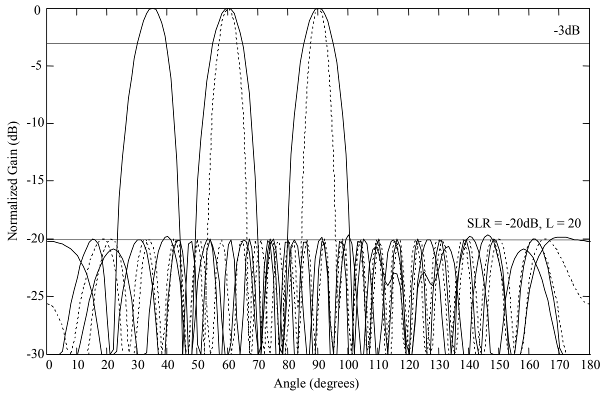

Figure 7 – Proposed and Chebyshev Patterns with ULA.

Finally, figure 8 compares the proposed steered beampatterns obtained with a 20-element ULA to those of a 41-element conventional Dolph-Chebyshev beampattern obtained with a UCA as proposed in [2]. Both arrays have half wavelength interelement spacing and are set to deliver a sidelobe ratio of –20dB. It can be seen that with the proposed algorithm a ULA with only 20 elements can deliver, within the interval 35°~145°, the same result as the one yield by a UCA with 41 elements using the Dolph-Chebyshev design. Of course, for even narrower angles of interest the economy in terms of number of antenna elements is even greater.

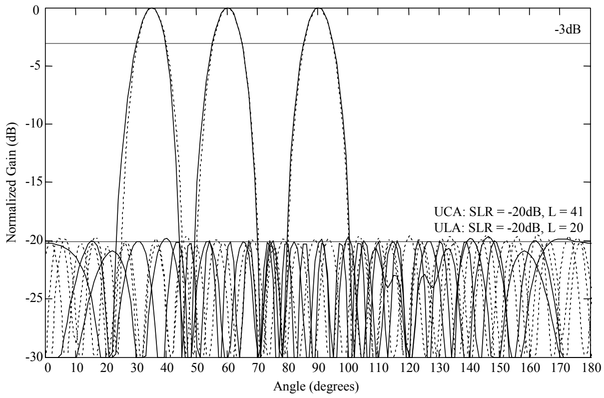

Figure 8 – Proposed Beampatterns with a ULA and Chebyshev Beampatterns with a UCA.

7. CONCLUSIONS

In this paper we have proposed a technique to design Chebyshev-like low sidelobe beampatterns that offer the possibility of prescribing both the sidelobe level and the beamwidth. The new design method represents an extension of the classic Dolph-Chebyshev design that has remained almost unchanged since its proposal in 1946. The proposed method offers new possibilities for applications in many communication and radar systems where low sidelobe level and adjustable beamwidth mainlobes are desired.

We have also demonstrated 2 examples of applications of the proposed design. The first consists of a fully adjustable sector-antenna-like beampattern obtained with a UCA, and will find direct application in Space-Domain Multiple Access (SDMA), Beam Space-Time Coding systems etc. The second consists of a low sidelobe beampattern that is rotation-invariant over a wide range around broadside, obtained with a ULA containing much less elements than what would be necessary if UCA were used. This design example will find straightforward application in radar systems in which the angular spatial span to be scanned is limited and in which uniform precision is desired, such as that required in ITS systems.

An expected improvement that might be obtained over these findings is the substitution of the proposed function given by equation (22) by a polynomial family that would completely remove the possibility of sidelobes surpassing the prescribed level, thus leading to an exact extension of the Chebyshev polynomial. Even in this case, however, the basic idea of the modification proposed here would be the same.

In addition, many other applications to the algorithm proposed here are imaginable if the relationship between array antenna theory and digital filters is considered. Even though the context in which the developments above are explained relate to array antennas (spatial frequency filtering), the method described is easily extended to the time frequency domain as well.

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