MATH 8090 Fall 2025 Exam I

September 25, 2025

Name:________________________________

Directions

1. Show your work on ALL questions (except for multiple-choice questions). Unsupported work will NOT receive full credit.
2. Please write legibly. If I cannot read your writing, NO credit will be given.
3. Decimal answers should be exact or rounded to exactly three decimal places.
4. Combine your work into a **single PDF file** and upload it to Canvas.

Use your time wisely. Good Luck!!!

Problem	Points Possible	Points Earned
1	20	
2	20	
3	20	
4	20	
Total	80	

Problem 1 (20 points total)

(a) (8 points) Consider the time series

$$\eta_t = 1.5 + 0.5\eta_{t-1} + Z_t,$$

where $\{Z_t\} \overset{i.i.d.}{\sim} \mathrm{N}(0, \sigma^2)$. Derive the mean function and the autocovariance function of this time series. Is this time series stationary? Justify your answer.

(b) (12 points) Consider the lag-d difference of the model

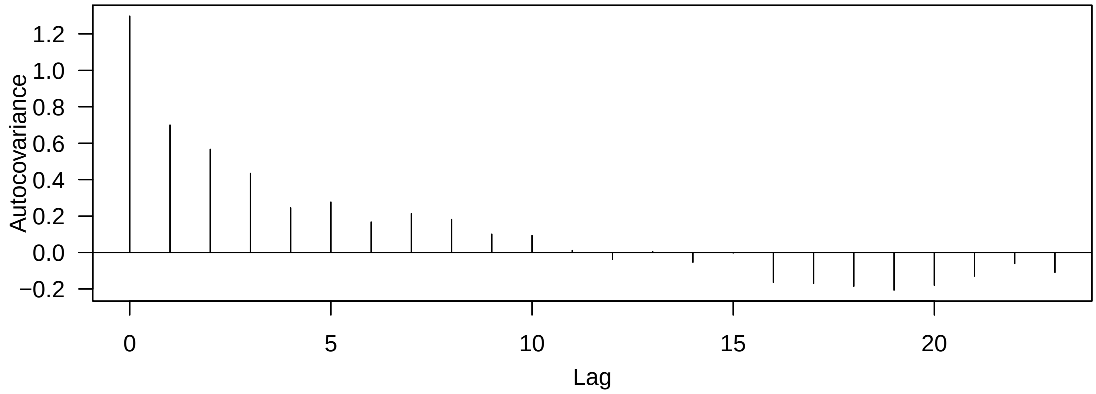
$$Y_t = \beta_0 + \beta_1 t + s_t + Z_t,$$

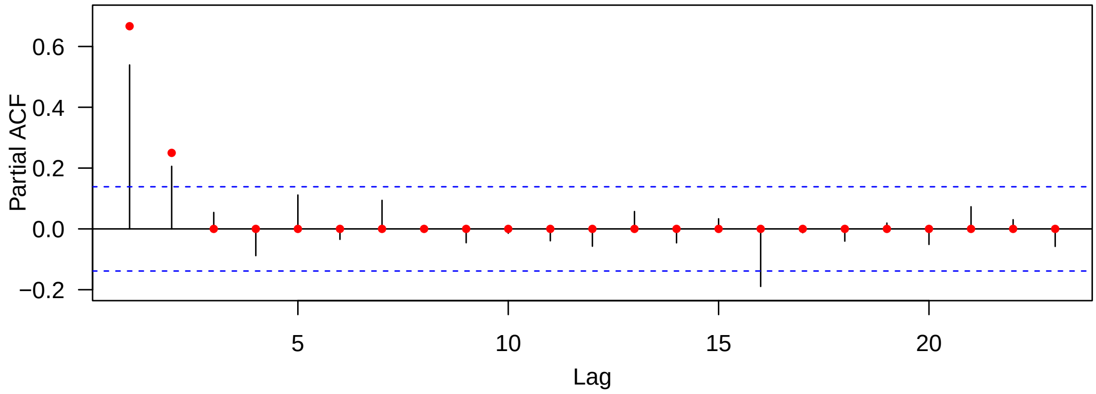
where $s_t = s_{t+jd}$ for all integers j and t, and $\{Z_t\} \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$. Let $X_t = Y_t - Y_{t-d}$ denote the differenced series.

1. Find the mean function of $\{X_t\}$.
2. Find the autocovariance function of $\{X_t\}$.
3. Determine whether $\{X_t\}$ is stationary and justify your answer.
4. Identify whether $\{X_t\}$ is an AR or MA process, and state its order.

Problem 2 (20 points total)

Let $\eta_1, \eta_2, \cdots, \eta_T$ be a realization of a stationary time series. Below are the sample autocovariance (ACVF) and the sample partial autocorrelation functions (along with their theoretical values).





(a) (10 points) We wish to model this time series. Would you suggest an AR(p) or an MA(q) model? What should its order be? Explain Why?

(b) (10 points) Which of the following values of T are plausible? Show your work to receive credit.
(i) 10 (ii) 50 (iii) 200 (iv) 1000

Problem 3 (20 points total; 5 points each)

Consider the following ARMA model:

$$(1-0.2B)(1-0.4B)(1+0.6B)\eta_t = (1+0.6B)(1+0.5B)(1+0.4B)Z_t,$$

where $Z_t \overset{i.i.d.}{\sim} \mathrm{N}(0, \sigma^2)$

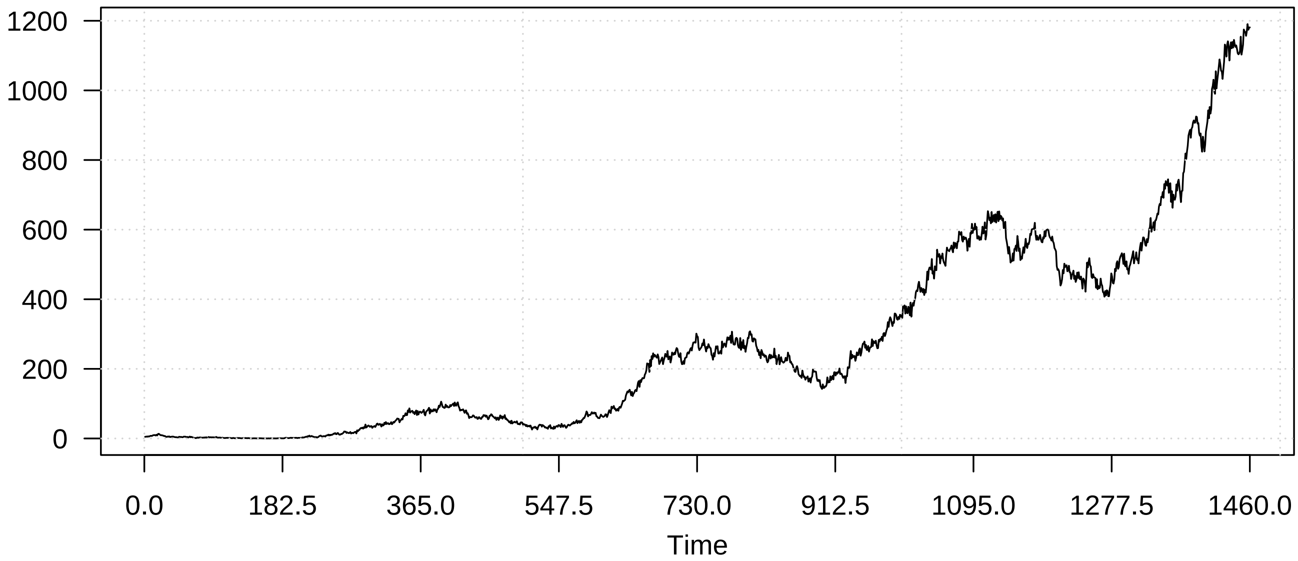
(a) What are the orders p and q for this ARMA model?

(b) Write down the AR and MA characteristic polynomials, respectively.

(c) Is this model causal? Why or why not?

(d) Is this model invertible? Why or why not?

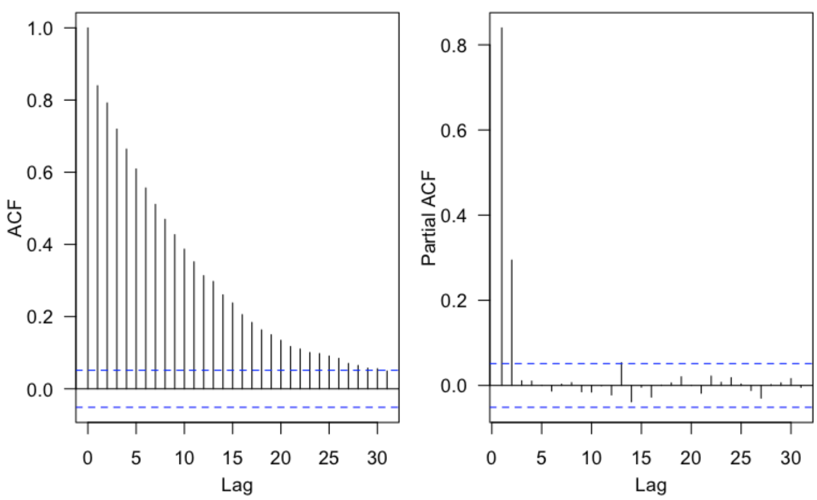
Problem 4 (20 points total; 5 points each)



1. Describe the steps for modeling the time series above. Do you need a transformation first? Why or why not?

2. Describe a possible trend model, suggest a seasonal model, and visually estimate the period d.

3. Use the sample ACF and PACF plots to identify an ARMA model for the noise term.



4. Ljung–Box test has been performed on the fitted residuals of the identified ARMA model. Based on the output of the Box test, explain whether the chosen model has adequately accounted for the temporal dependence.

```
        Box-Ljung test

data:  ts_fit$residuals
X-squared = 20.464, df = 36, p-value = 0.9825
```