Lecture 6

Probability II

Text: Chapter 4

STAT 8010 Statistical Methods I
September 8, 2020

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Union, Intersection, and Logical Relationships among Events
Complement Rule and General Addition Rule
Independence and Conditional Probability
Law of Total Probability
Bayes' Rule

Agenda

1. **Union, Intersection, and Logical Relationships among Events**
2. **Complement Rule and General Addition Rule**
3. **Independence and Conditional Probability**
4. **Law of Total Probability**
5. **Bayes' Rule**

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Probability Rules

1. Any probability must be between 0 and 1 inclusively
2. The sum of the probabilities for all the experimental outcomes must equal 1

If a probability model satisfies the two rules above, it is said to be legitimate

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Example

An experiment with three outcomes has been repeated 50 times, and it was learned that outcome 1 occurred 20 times, outcome 2 occurred 13 times, and outcome 3 occurred 17 times. Assign probabilities to the outcomes. What method did you use?

Solution.

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Example

A decision maker subjectively assigned the following probabilities to the four possible outcomes of an experiment:

$$P(E_1) = 0.1 \; P(E_2) = 0.15 \; P(E_3) = 0.4 \; P(E_4) = 0.2$$

Are these probability assignments legitimate? Explain.

Solution.

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Intersection and Union

- Intersection: the intersection of two events A and B, denoted by $A \cap B$, is the event that contains all outcomes of A that also belong to $B \Rightarrow$ AND

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Example: Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 5\}$, then $A \cap B = \{1, 2\}$

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- Union: the union of two events A and B, denoted by $A \cup B$, is the event of all outcomes that belong to either A or $B \Rightarrow$ OR

Intersection and Union

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Example

Suppose we flipped 3 fair coins. Let A be the event of **exactly 2 tails**. Let B be the event that the **first 2 tosses are tails**. Let C be the event that **all 3 tosses are tails**. What are $A \cap B$, $A \cup C$, and $(A \cap B) \cup C$?

Solution.

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Solution.

$$A = \{(T,T,H),(T,H,T),(H,T,T)\}$$
$$B = \{(T,T,T),(T,T,H)\}$$
$$C = \{T,T,T\}$$

1. $A \cap B = \{(T,T,H)\}$

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Solution.

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\begin{aligned}
A &= \{(T,T,H),(T,H,T),(H,T,T)\} \\
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1. $A \cap B = \{(T,T,H)\}$
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1. $A \cap B = \{(T,T,H)\}$
2. $A \cup C = \{(T,T,H),(T,H,T),(H,T,T),(T,T,T)\}$
3. $(A \cap B) \cup C = \{T,T,H\} \cup \{T,T,T\} = \{(T,T,H),(T,T,T)\}$

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Logical Relationships among Events

- Mutually exclusive: refers to two (or more) events that cannot both occur when the random experiment is formed.

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$$A \cap B = \varnothing$$

- Exhaustive: refers to event(s) that comprise the sample space.

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- Partition: events that are both mutually exclusive and exhaustive.

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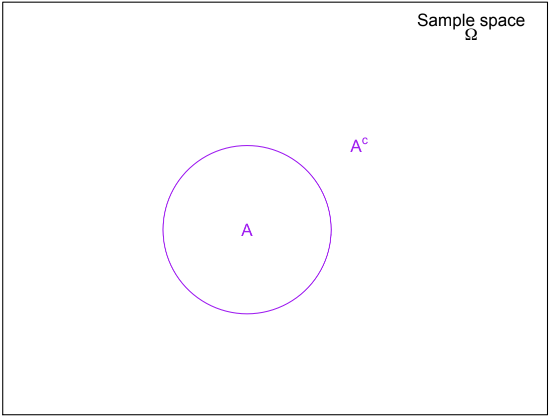
Complement Rule and General Addition Rule

Complement



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Complement Rule

1. By the definition of complement

$$A \cup A^c = \Omega$$

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Complement Rule

1. By the definition of complement

$$A \cup A^c = \Omega$$

2. Apply the probability operator

$$\mathbb{P}(A \cup A^c) = \mathbb{P}(\Omega) = 1$$

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1. By the definition of complement

$$A \cup A^c = \Omega$$

2. Apply the probability operator

$$\mathbb{P}(A \cup A^c) = \mathbb{P}(\Omega) = 1$$

3. Since A and A^c are mutually exclusive

$$\mathbb{P}(A \cup A^c) = \mathbb{P}(A) + \mathbb{P}(A^c)$$

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3. Since A and A^c are mutually exclusive

$$\mathbb{P}(A \cup A^c) = \mathbb{P}(A) + \mathbb{P}(A^c)$$

4. Hence we get $\boxed{\mathbb{P}(A) = 1 - \mathbb{P}(A^c)}$

Example

Suppose we rolled a fair, six–sided die 10 times. Let T be the event that we roll at least 1 three. If one were to calculate T you would need to find the probability of 1 three, 2 threes, $\cdots$, and 10 threes and add them all up. However, you can use the complement rule to calculate $\mathbb{P}(T)$

Solution.

Let X be the times that we rolled a 3, then

$$\mathbb{P}(T) = \mathbb{P}(X \geq 1) = \underbrace{\mathbb{P}(X = 1) + \mathbb{P}(X = 2) + \cdots + \mathbb{P}(X = 10)}_{\text{need to compute 10 probabilities}}$$

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Solution.

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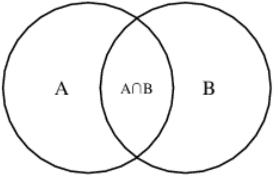
$$\mathbb{P}(T) = \mathbb{P}(X \geq 1) = \underbrace{\mathbb{P}(X = 1) + \mathbb{P}(X = 2) + \cdots + \mathbb{P}(X = 10)}_{\text{need to compute 10 probabilities}}$$

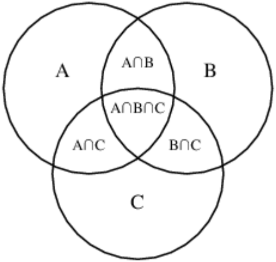
If we apply the complement rule

$$\mathbb{P}(T) = 1 - \mathbb{P}(T^c) = 1 - \mathbb{P}(X = 0)$$

Venn Diagram

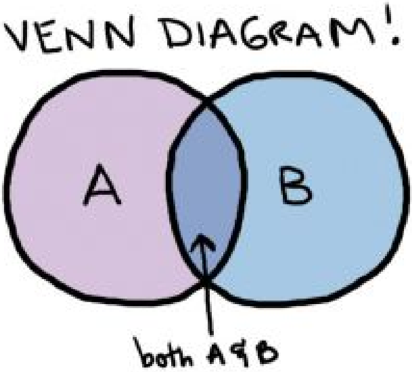
A Venn diagram is a diagram that shows all possible logical relations between a finite collection of events.





General Addition Rule

The general addition rule is a way of finding the probability of a union of 2 events. It is $\boxed{\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)}$



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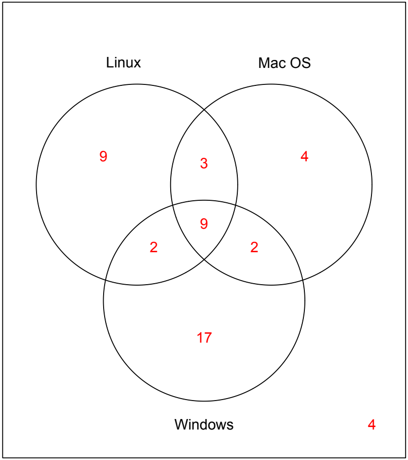
Example

Three of the major commercial computer operating systems are Windows, Mac OS, and Red Hat Linux Enterprise. A Computer Science professor selects 50 of her students and asks which of these three operating systems they use. The results for the 50 students are summarized below.

- 30 students use Windows
- 16 students use at least two of the operating systems
- 9 students use all three operating systems
- 18 students use Mac OS
- 46 students use at least one of the operating systems
- 11 students use both Windows and Linux
- 11 students use both Windows and Mac OS

Example cont'd



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Independence and Conditional Probability

Independence: A Motivating Example

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Example

You toss a fair coin and it comes up “Heads" three times. What is the chance that the next toss will also be a “Head"?

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Conditional Probability

Let A and B be events. The probability that event B occurs given (knowing) that event A occurs is called a conditional probability and is denoted by $P(B|A)$. The formula of conditional probability is

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Independent events

Suppose $P(A) > 0,\ P(B) > 0$. We say that event B is independent of event A if the occurrence of event A does not affect the probability that event B occurs.

$$\boxed{P(B|A) = P(B) \Rightarrow P(B \cap A) = P(B)P(A)}$$

Law of Partitions & Multiplication Rule

Law of partitions

Let $A_1, A_2, \cdots, A_k$ form a partition of Ω. Then, for all events B,

$$\mathbb{P}(B) = \sum_{i=1}^{k} \mathbb{P}(A_i \cap B)$$

Multiplication rule

- 2 events:

$$\mathbb{P}(B \cap A) = \mathbb{P}(A) \times \mathbb{P}(B|A) = \mathbb{P}(B) \times \mathbb{P}(A|B)$$

- More than 2 events:

$$\mathbb{P}(\cap_{i=1}^{n} A_i) = \mathbb{P}(A_1) \times \mathbb{P}(A_2|A_1) \times \mathbb{P}(A_3|A_1 \cap A_2) \times \cdots \times \mathbb{P}(A_n|A_{n-1} \cap \cdots \cap A_1)$$

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Let $A_1, A_2, \cdots, A_k$ form a partition of Ω. Then, for all events B,

$$\begin{aligned}\mathbb{P}(B) &= \underbrace{\sum_{i=1}^{k} \mathbb{P}(A_i \cap B)}_{\text{Law of partitions}} \\ &= \underbrace{\sum_{i=1}^{k} \mathbb{P}(B|A_i) \times \mathbb{P}(A_i)}_{\text{Multiplication rule}}\end{aligned}$$

Example

Suppose that two factories supply light bulbs to the market. Factory X's bulbs work for over 5000 hours in 99% of cases, whereas factory Y's bulbs work for over 5000 hours in 95% of cases. It is known that factory X supplies 60% of the total bulbs available and Y supplies 40% of the total bulbs available. What is the chance that a purchased bulb will work for longer than 5000 hours?

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Bayes' Rule: Motivating example

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The Monty Hall Problem

There was an old television show called Let's Make a Deal, whose original host was named Monty Hall. The set–up is as follows. You are on a game show and you are given the choice of three doors. Behind one door is a car, behind the others are goats. You pick a door, and the host, who knows what is behind the doors, opens another door (not your pick) which has a goat behind it. Then he asks you if you want to change your original pick. The question we ask you is, "Is it to your advantage to switch your choice?"

The Monty Hall Problem

The Monty Hall Problem Solution

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General form

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

Let $A_1, A_2, \cdots, A_k$ form a partition of the sample space. Then for every event B in the sample space,

$$\boxed{\mathbb{P}(A_j|B) = \frac{\mathbb{P}(B|A_j) \times \mathbb{P}(A_j)}{\sum_{i=1}^{k} \mathbb{P}(B|A_i) \times \mathbb{P}(A_i)}}, \; j = 1, 2, \cdots, k$$

Example

Let us assume that a specific disease is only present in 5 out of every 1,000 people. Suppose that the test for the disease is accurate 99% of the time a person has the disease and 95% of the time that a person lacks the disease. What is the probability that the person has the disease given that they tested positive?

Solution.

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Solution.

$$\mathbb{P}(D|+) = \frac{\mathbb{P}(D\cap+)}{\mathbb{P}(+)} = \frac{.005\times.99}{.005\times.99+.995\times.05} = \frac{.00495}{.0547} = .0905$$

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Solution.

$$\mathbb{P}(D|+) = \frac{\mathbb{P}(D \cap +)}{\mathbb{P}(+)} = \frac{.005 \times .99}{.005 \times .99 + .995 \times .05} = \frac{.00495}{.0547} = .0905$$

The reason we get such a surprising result is because the disease is so rare that the number of false positives greatly outnumbers the people who truly have the disease.

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Review of Probability (we learned so far)

Basic Concepts:

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Review: Probability Rules

- $0 \leq \mathbb{P}(A) \leq 1$ for any event A, $\mathbb{P}(\varnothing) = 0$, $\mathbb{P}(\Omega) = 1$

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- Multiplication rule:
  $\mathbb{P}(A \cap B) = \mathbb{P}(A|B) \times \mathbb{P}(B) = \mathbb{P}(B|A) \times \mathbb{P}(A)$

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Union, Intersection, and Logical Relationships among Events

Complement Rule and General Addition Rule

Independence and Conditional Probability

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- Independence: if A and B are independent, then
  $\mathbb{P}(A|B) = \mathbb{P}(A)$, $\mathbb{P}(B|A) = \mathbb{P}(B)$, and $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$

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