STAT 8010 Statistical Methods I
Homework 4

Instructor: Whitney Huang (wkhuang@clemson.edu)

Due Date: April 16, 9:30am

Problem 1

The following contingency table contains enrollment data for a **random sample** of students from the college of Liberal Arts and Engineering at a University during the 2016-2017 academic year. The table lists the number of male and female students enrolled in each college.

(Observed)	Female	Male	Total
Liberal Arts	155	75	230
Engineering	80	320	400
Total	235	395	630

a. Let $p = \mathbb{P}(Female|Liberal\ Arts)$ in that university. Construct a 95% confidence interval for p.

i Point estimate: $\hat{p} = \frac{155}{230} = 0.6739$

ii Standard error (SE): $\sqrt{\frac{\hat{p}\times(1-\hat{p})}{n}} = \sqrt{\frac{0.6739\times 0.3261}{230}} = 0.0309$

iii Margin of error (ME): $z_{0.05/2} \times \text{SE} = 1.96 \times 0.0309 = 0.0606$

iv 95% confidence interval: Point estimate $\pm$ ME
$= 0.6739 \pm 0.0606 = (0.6133, 0.7345)$

b. Let $p_1 = \mathbb{P}(Female|Liberal\ Arts)$ and $p_2 = \mathbb{P}(Female|Engineering)$. Perform a hypothesis test with $H_0 : p_1 = p_2$ vs. $H_a : p_1 \neq p_2$.

i $\hat{p}_1 = \frac{155}{230} = 0.6739$; $\hat{p}_2 = \frac{80}{400} = 0.2$

ii $\bar{p} = \frac{155+80}{230+400} = 0.3730$

iii $z_{obs} = \frac{0.6739-0.2}{\sqrt{\frac{0.3730\times0.6270}{230}+\frac{0.3730\times0.6270}{400}}} = 11.8421$

iv Since $z_{obs} = 11.8421 > Z_{0.025} = 1.96 \Rightarrow$ Reject H_0 and conclude $p_1 \neq p_2$ at 0.05 level.

c. Use the contingency table to conduct a χ^2 test for independence from beginning to end. Use $\alpha = .01$.

First, calculate the row sums, column sums, and the grand total:

(Observed)	Female	Male	Total
Liberal Arts	155	75	230
Engineering	80	320	400
Total	235	395	630

Second, calculate the expected cell counts:

(Observed)	Female	Male
Liberal Arts	$\frac{230\times235}{630} = 85.7937$	$\frac{230\times395}{630} = 144.2063$
Engineering	$\frac{400\times235}{630} = 149.2064$	$\frac{400\times395}{630} = 250.7936$

Third, compute the partial χ^2:

(Observed)	Female	Male
Liberal Arts	$\frac{(155-85.7937)^2}{85.7937} = 55.8260$	$\frac{(75-144.2063)^2}{144.2063} = 33.2130$
Engineering	$\frac{(80-149.2064)^2}{149.2064} = 32.1000$	$\frac{(320-250.7936)^2}{250.7936} = 19.0975$

Fourth, compute the χ^2 and compare with the critical value: $\chi^2_{obs} = 55.8260 + 33.2130 + 32.1000 + 19.0975 = 140.2364 > \chi^2_{0.01,df=1} = 6.635 \Rightarrow$ Reject $H_0 \Rightarrow$ There is a relationship between gender and college at 0.01 level.

Problem 2

The following information represents data gathered during an observational study of Clemson residents. The table depicts the number of people in categories based on marital status and level of happiness.

	Happy	So-So	Unhappy	Totals
Married	60	80	140	
Single	100	80	160	
Totals				

a. Calculate and fill in the row and column totals as well as the overall total.

	Happy	So-So	Unhappy	Totals
Married	60	80	140	280
Single	100	80	160	340
Totals	160	160	300	620

b. Use the information above to create a table of expected counts.

	Happy	So-So	Unhappy
Married	72.25806	72.25806	135.4839
Single	87.74194	87.74194	164.5161

c. Construct a table of partial χ^2 values (a χ^2 value for each individual cell).

	Happy	So-So	Unhappy
Married	2.079493	.829493	.150538
Single	1.712524	.683112	.123972

d. What is the χ^2 value?
5.579132

e. What are the degrees of freedom (df)?
df = (2-1)(3-1) = 2

f. At the $\alpha = .01$ level, what is the χ^2 critical value?
9.210

g. What is your conclusion?
Do not reject the null hypothesis that there is no relationship between marital status and state of happiness.