STAT 8010 R Lab 6: Normal Distributions

Whitney Huang

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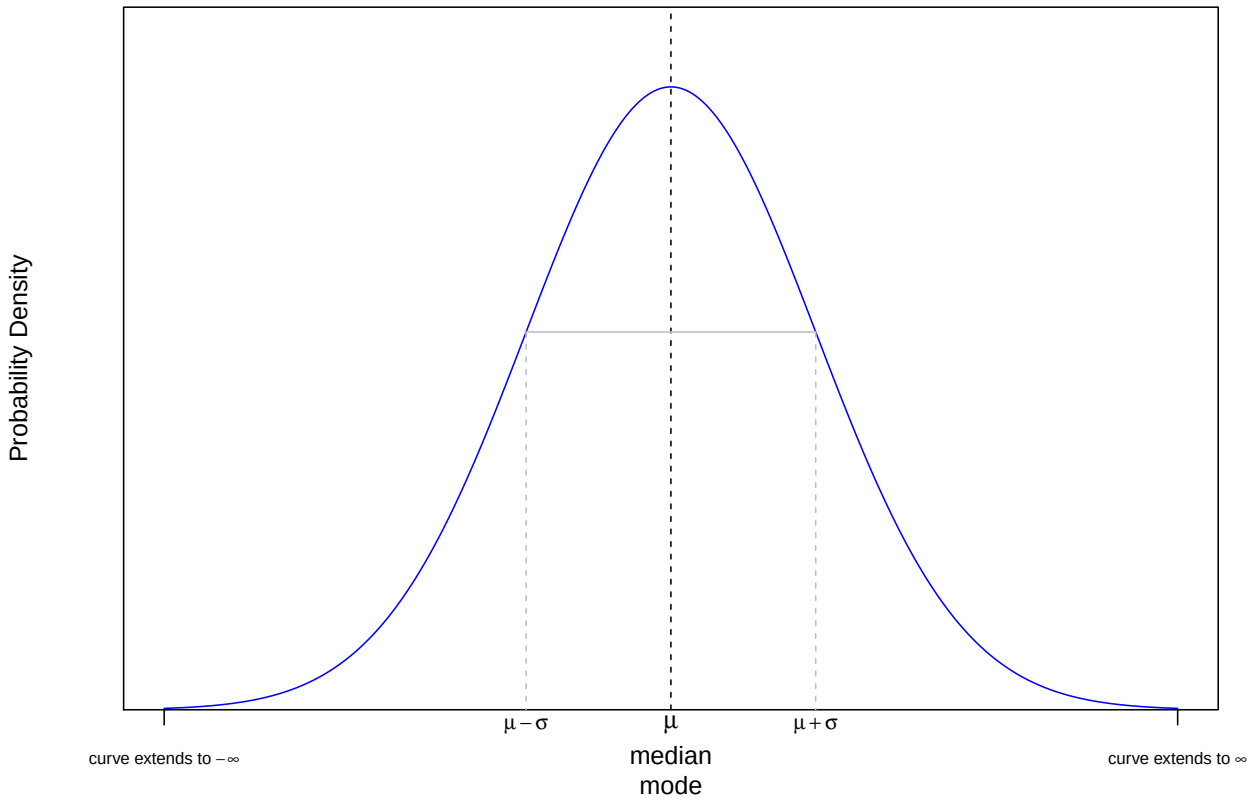
Contents

Normal distribution

Normal density curve

```
xg <- seq(-3.5, 3.5, 0.01)
yg <- dnorm(xg)

par(las = 1)
plot(xg, yg, type = "l", xlab = "",
     ylab = "Probability Density",
     col = "blue", xaxt = "n",
     yaxt = "n", yaxs = "i",
     ylim = c(0, 0.45))
abline(v = 0, lty = 2)
axis(1, at = 0, label = expression(mu),
     tick = F, line = -1)
axis(1, at = 0, label = "median", line = 0,
     tick = F)
axis(1, at = 0, label = "mode", line = 1,
     tick = F)
axis(1, at = -3.5, labels = expression(paste("curve extends to ", -infinity)), cex.axis = 0.7)
axis(1, at = 3.5, labels = expression(paste("curve extends to ", infinity)), cex.axis = 0.7)
segments(0, dnorm(1), -1, col = "gray")
segments(0, dnorm(1), 1, col = "gray")
segments(-1, dnorm(1), -1, 0, lty = 2,
         col = "gray")
segments(1, dnorm(1), 1, 0, lty = 2,
         col = "gray")
axis(1, at = -1, label = expression(mu - sigma), tick = F, cex.axis = 0.8, line = -1)
axis(1, at = 1, label = expression(mu + sigma), tick = F, cex.axis = 0.8, line = -1)
```



Standard Normal: Z $\mathrm{N}(0,1)$

We use $\Phi(\cdot)$ to denote the cdf of the standard normal distribution

1. $\Phi(0) = .50 \Rightarrow\}$ *Mean* and *Median* (50_{th} percentile) for standard normal are both 0
2. $\Phi(-z) = 1 - \Phi(z)\}$
3. $\mathbb{P}(Z > z) = 1 - \Phi(z) = \Phi(-z)\}$

```
pnorm(0)
```

```
## [1] 0.5
```

```
pnorm(-1)
```

```
## [1] 0.1586553
```

```
1 - pnorm(1)
```

```
## [1] 0.1586553
```

```
pnorm(1, lower.tail = F)
```

```
## [1] 0.1586553
```

```
pnorm(-1.75)
```

```
## [1] 0.04005916
```

```
pnorm(2) - pnorm(-2)
```

```
## [1] 0.9544997
```

```r
pnorm(0.5)
```

```
## [1] 0.6914625
```

STAT 8020 exam score example

```r
mu = 78; sigma2 = 36
1 - pnorm(84, mu, sqrt(sigma2))
```

```
## [1] 0.1586553
```

```r
(pnorm(84, mu, sqrt(sigma2)) - pnorm(75, mu, sqrt(sigma2))) / (1 - pnorm(75, mu, sqrt(sigma2)))
```

```
## [1] 0.7705512
```

Standard normal percentiles

```r
qnorm(c(.1, .55, .9))
```

```
## [1] -1.2815516  0.1256613  1.2815516
```

General normal percentiles

```r
qnorm(.8, 20, 7)
```

```
## [1] 25.89135
```

```r
20 + 7 * qnorm(.8)
```

```
## [1] 25.89135
```

Normal approximation of Binomial Distribution

```r
n = 400; p = 0.93
sum(dbinom(370:373, n, p))
```

```
## [1] 0.3009909
```

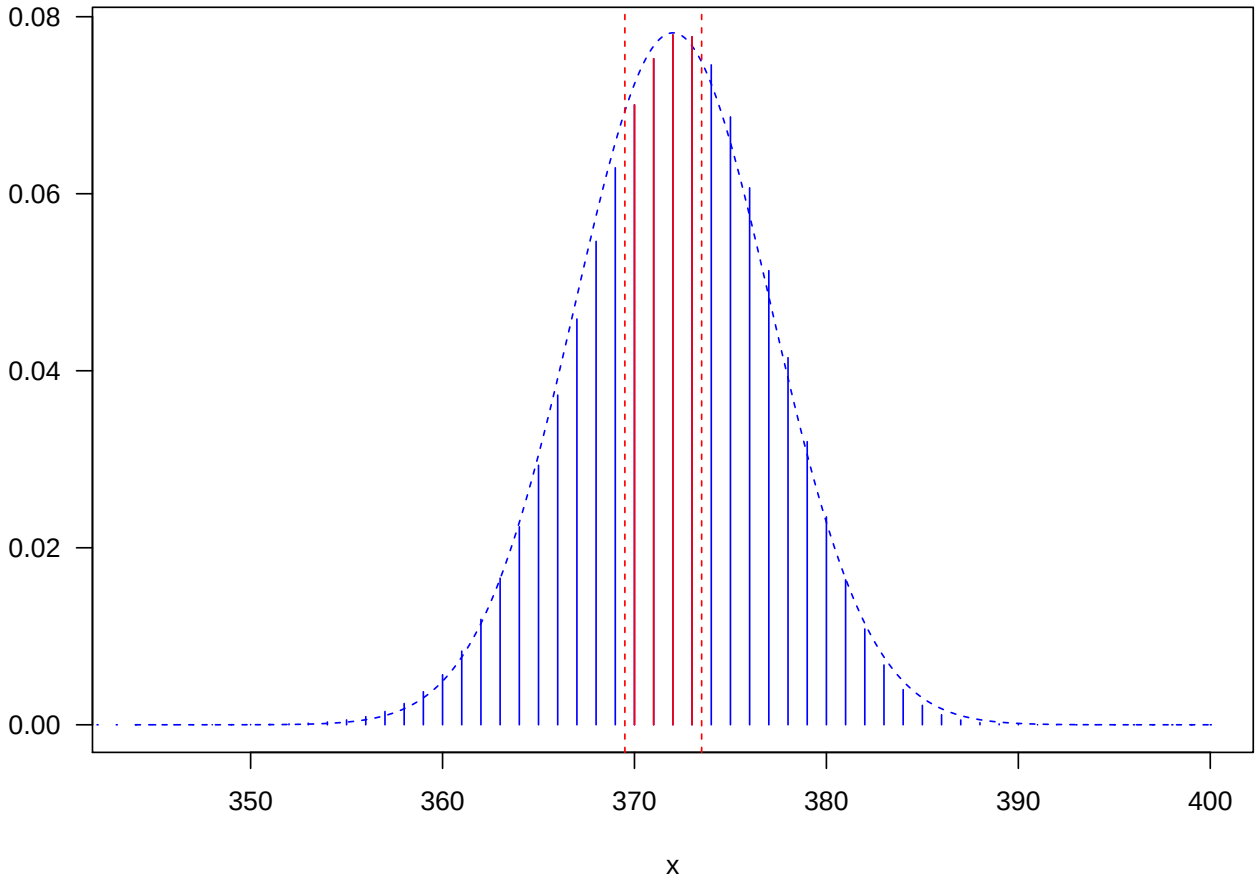
```r
pbinom(373, n, p) - pbinom(369, n, p)
```

```
## [1] 0.3009909
```

```r
mu = n * p; sigma = sqrt(n * p * (1 - p))
pnorm(373.5, mu, sigma) - pnorm(369.5, mu, sigma)
```

```
## [1] 0.3035037
```

```r
plot(0:400, dbinom(0:400, n, p), type = "h", las = 1,
     xlim = c(344, 400), xlab = "x", ylab = "", col = "blue")
xg <- seq(344, 400, 0.1); yg <- dnorm(xg, mu, sigma)
lines(xg, yg, col = "blue", lty = 2)
abline(v = c(369.5, 373.5), lty = 2, col = "red")
lines(370:373, dbinom(370:373, n, p), type = "h", col = "red")
```


0.08
0.06
0.04
0.02
0.00
350
360
370
380
390
400
x