Lecture 13
Model Diagnostics

STAT 8020 Statistical Methods II
September 18, 2019

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Notes

Agenda

1. Leverage
2. Studentized & Jackknife Residuals
3. DFFITS
4. Non-Constant Variance & Transformation

Notes

Leverage

Recall in MLR that $\hat{Y} = X(X^TX)^{-1}X^TY = HY$ where H is the hat-matrix

- The leverage score for the i_{th} observation is defined as:

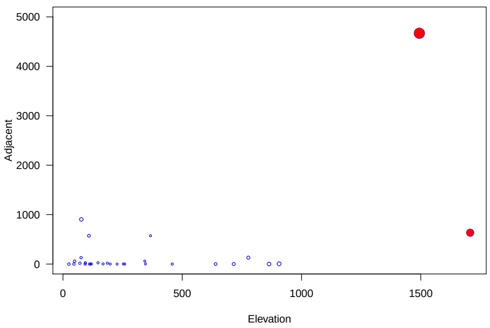
$$h_i = H_{ii}$$

- Can show that Var$(e_i) = \sigma^2(1 - h_i)$, where $e_i = Y_i - \hat{Y}_i$ is the residual for the i_{th} observation

- $\frac{1}{n} \leq h_i \leq 1, \quad 1 \leq i \leq n$ and $\bar{h}_i = \frac{p}{n} \Rightarrow$ a "rule of thumb" is that leverages of more than $\frac{2p}{n}$ should be looked at more closely

Notes

Leverage Scores of `Species` $\sim$ `Elev` + `Adj`



Notes

Studentized Residuals

As we have seen Var$(e_i) = \sigma^2(1 - h_i)$, this suggests the use of $r_i = \frac{e_i}{\hat{\sigma}\sqrt{(1-h_i)}}$

- r_i's are called **studentized residuals**. r_i's are sometimes preferred in residual plots as they have been standardized to have equal variance.
- If the model assumptions are correct then Var$(r_i) = 1$ and Corr(e_i, e_j) tends to be small

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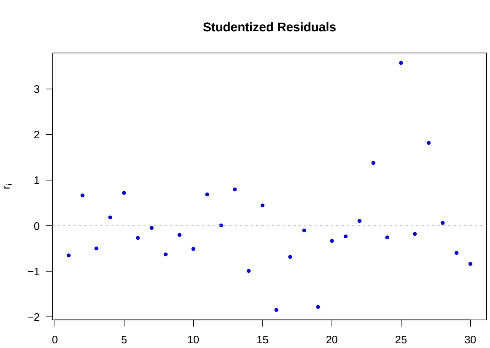
Studentized Residuals of `Species` $\sim$ `Elev` + `Adj`



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Jackknife Residuals

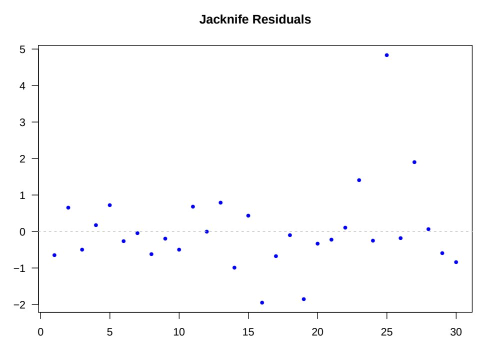
- For a given model, exclude the observation i and recompute $\hat{\beta}_{(i)}$, $\hat{\sigma}_{(i)}$ to obtain $\hat{Y}_{i(i)}$
- The observation i is an outlier if $\hat{Y}_{i(i)} - Y_i$ is "large"
- Can show $\mathrm{Var}(\hat{Y}_{i(i)} - Y_i) = \sigma^2 \left(1 + x_i^T (X_{(i)}^T X_i)^{-1} x_i\right)$
- Define the **jackknife residuals** as

$$t_i = \frac{\hat{Y}_{i(i)} - Y_i}{\sqrt{\hat{\sigma}^2 \left(1 + x_i^T (X_{(i)}^T X_i)^{-1} x_i\right)}}$$

which are distributed as a t_{n-p} if the model is correct and $\varepsilon \sim \mathrm{N}(0, \sigma^2 I)$

Notes

Jackknife Residuals of `Species` $\sim$ `Elev` $+$ `Adj`



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Influential Observations

DFFITS

- Difference between the fitted values $\hat{Y}_i$ and the predicted values $\hat{Y}_{i(i)}$
- $\mathrm{DFFITS}_i = \frac{\hat{Y}_i - \hat{Y}_{i(i)}}{\sqrt{\mathrm{MSE}_{(i)} h_i}}$
- Concern if absolute value greater than 1 for small data sets, or greater than $2\sqrt{p/n}$ for large data sets

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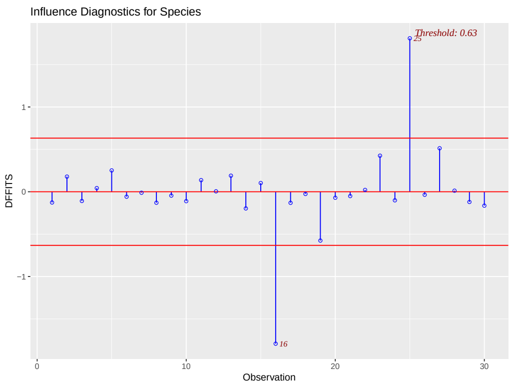
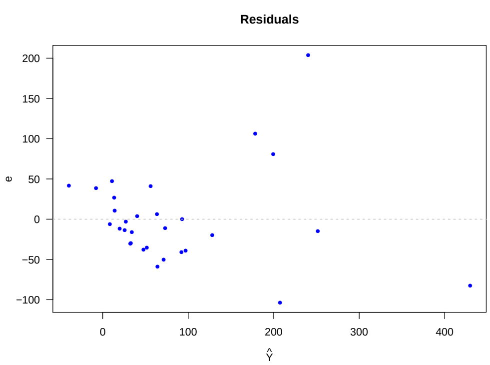
DFFITS of `Species` ∼ `Elev` + `Adj`



Notes

Residual Plot of `Species` ∼ `Elev` + `Adj`



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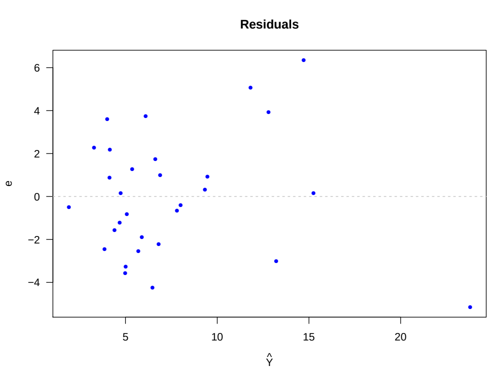
Residual Plot After Square Root Transformation



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