MATH 8090: Univariate Volatility Modeling

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Contents

Introductory Example: Apple Stock Data

Load the Apple stock data via *quantmod*

```r
library(quantmod)
getSymbols("AAPL", src = "yahoo")
```

```
## [1] "AAPL"
```

```r
dim(AAPL)
```

```
## [1] 4728    6
```

```r
head(AAPL); tail(AAPL)
```

```
##            AAPL.Open AAPL.High AAPL.Low AAPL.Close AAPL.Volume AAPL.Adjusted
## 2007-01-03  3.081786  3.092143 2.925000   2.992857  1238319600      2.515686
## 2007-01-04  3.001786  3.069643 2.993571   3.059286   847260400      2.571523
## 2007-01-05  3.063214  3.078571 3.014286   3.037500   834741600      2.553212
## 2007-01-08  3.070000  3.090357 3.045714   3.052500   797106800      2.565819
## 2007-01-09  3.087500  3.320714 3.041071   3.306071  3349298400      2.778961
## 2007-01-10  3.383929  3.492857 3.337500   3.464286  2952880000      2.911952

##            AAPL.Open AAPL.High AAPL.Low AAPL.Close AAPL.Volume AAPL.Adjusted
## 2025-10-09    257.81    258.00   253.14     254.04    38322000        254.04
## 2025-10-10    254.94    256.38   244.00     245.27    61999100        245.27
## 2025-10-13    249.38    249.69   245.56     247.66    38142900        247.66
## 2025-10-14    246.60    248.85   244.70     247.77    35478000        247.77
## 2025-10-15    249.49    251.82   247.47     249.34    33893600        249.34
## 2025-10-16    248.25    249.04   245.13     247.45    39777000        247.45
```

```r
summary(AAPL)
```

```
##      Index              AAPL.Open         AAPL.High          AAPL.Low
##  Min.   :2007-01-03   Min.   :  2.835   Min.   :  2.929   Min.   :  2.793
##  1st Qu.:2011-09-11   1st Qu.: 13.659   1st Qu.: 13.768   1st Qu.: 13.510
##  Median :2016-05-23   Median : 29.379   Median : 29.571   Median : 29.128
##  Mean   :2016-05-23   Mean   : 65.714   Mean   : 66.424   Mean   : 65.045
##  3rd Qu.:2021-02-02   3rd Qu.:125.875   3rd Qu.:127.153   3rd Qu.:124.332
##  Max.   :2025-10-16   Max.   :258.190   Max.   :260.100   Max.   :257.630
##    AAPL.Close        AAPL.Volume         AAPL.Adjusted
##  Min.   :  2.793   Min.   :2.323e+07   Min.   :  2.348
##  1st Qu.: 13.616   1st Qu.:8.761e+07   1st Qu.: 11.445
##  Median : 29.378   Median :1.707e+08   Median : 26.738
##  Mean   : 65.766   Mean   :3.289e+08   Mean   : 63.706
##  3rd Qu.:125.642   3rd Qu.:4.473e+08   3rd Qu.:122.831
##  Max.   :259.020   Max.   :3.373e+09   Max.   :258.104
```

```r
chartSeries(AAPL)
```

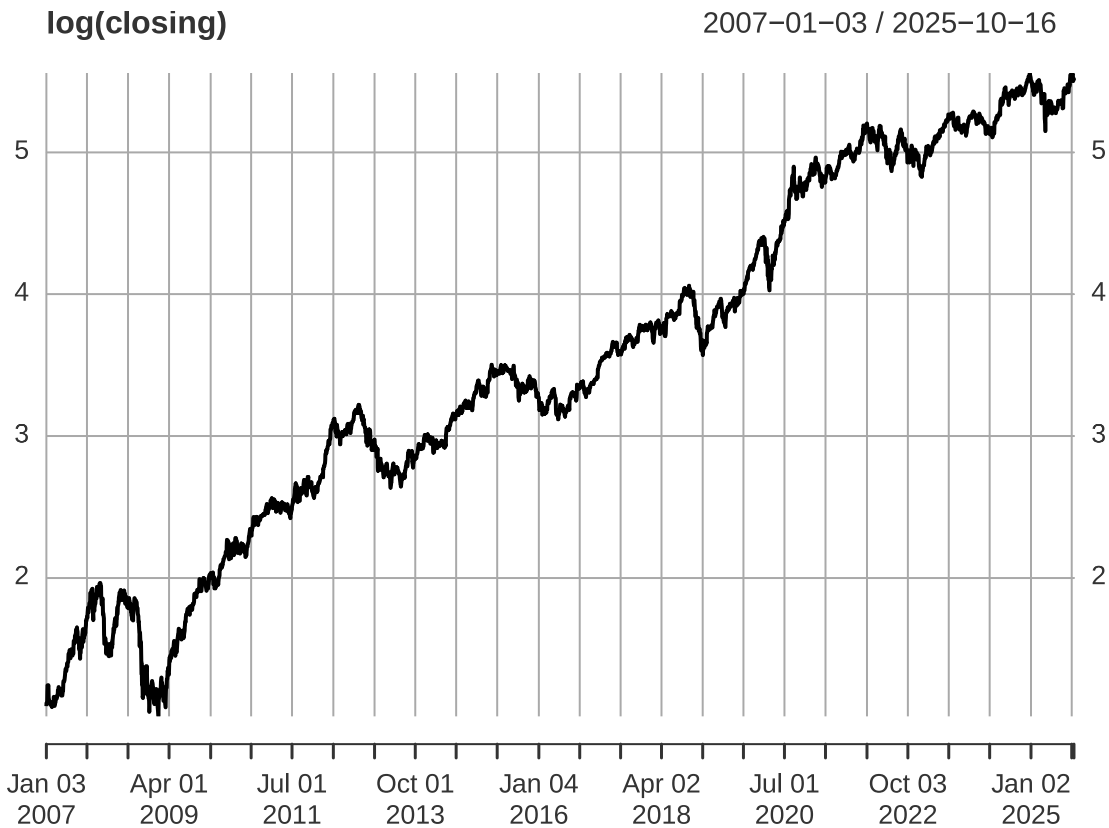
Visualize the Apple stock time series.

First, let's plot the daily closing values

```r
closing <- AAPL$AAPL.Close
plot(closing)
```

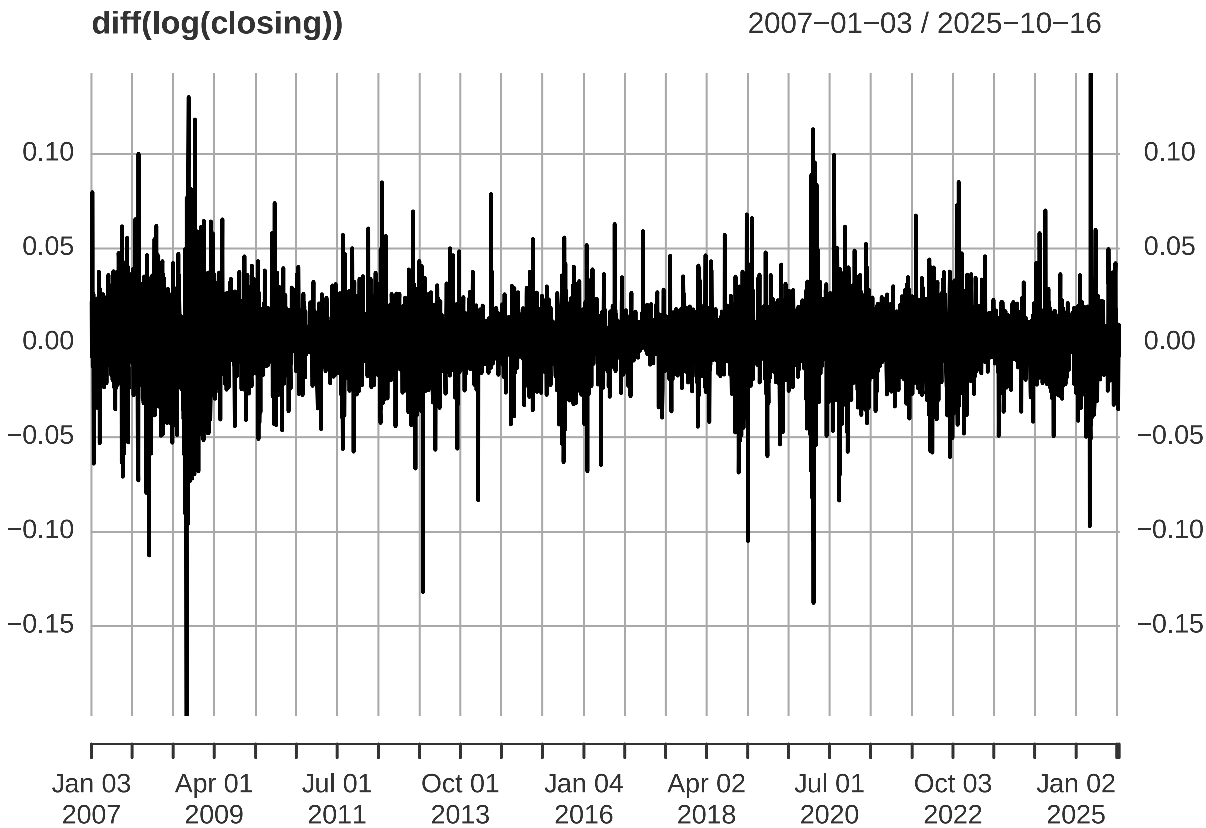
Next, apply a log transformation to stabilize the variance.

```
plot(log(closing))
```



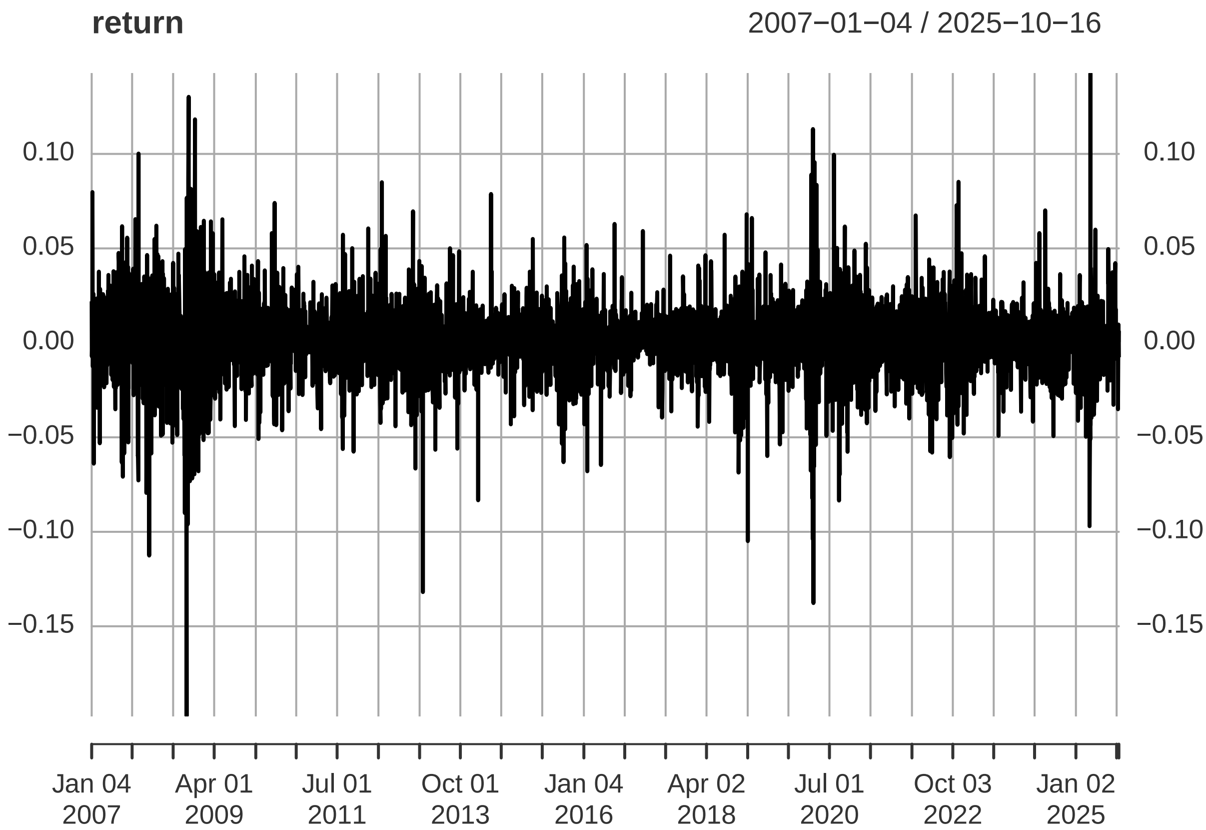
Perform first-order differencing to make the series approximately stationary

```
plot(diff(log(closing)))
```

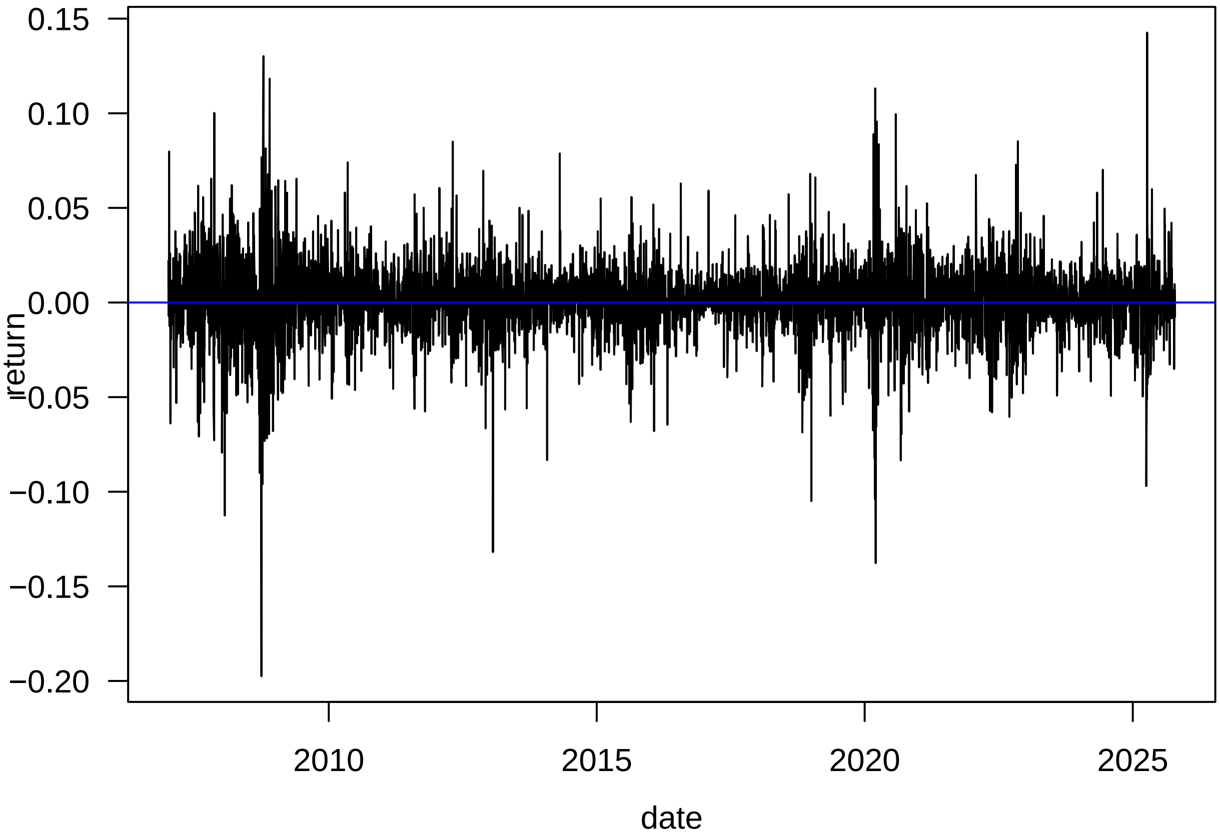


These series of operations lead to the log-return $r_t = \log(y_t) - \log(y_{t-1})$

```
temp <- diff(log(closing))
return <- temp[!is.na(temp)]

par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
plot(return)
```

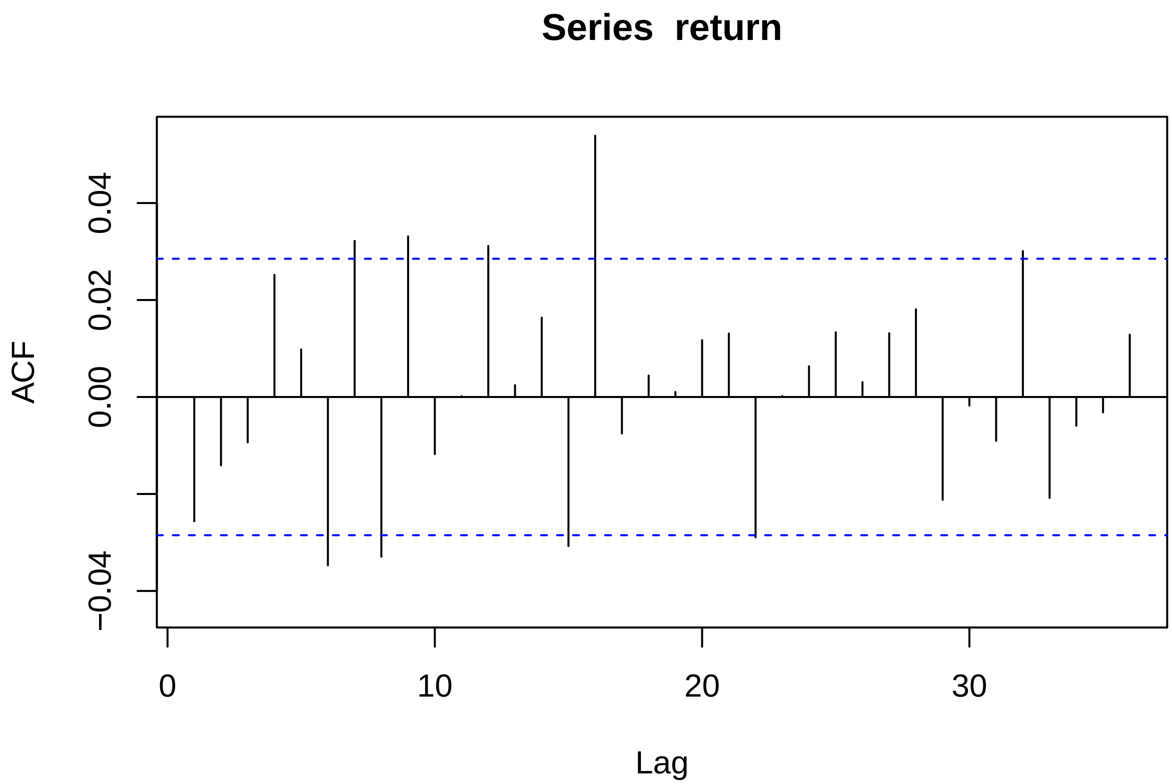



```r
date <- time(return)
par(las = 1, mgp = c(2.5, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
plot(date, return, type = "l")
abline(h = 0, col = "blue", lwd = 1)
```

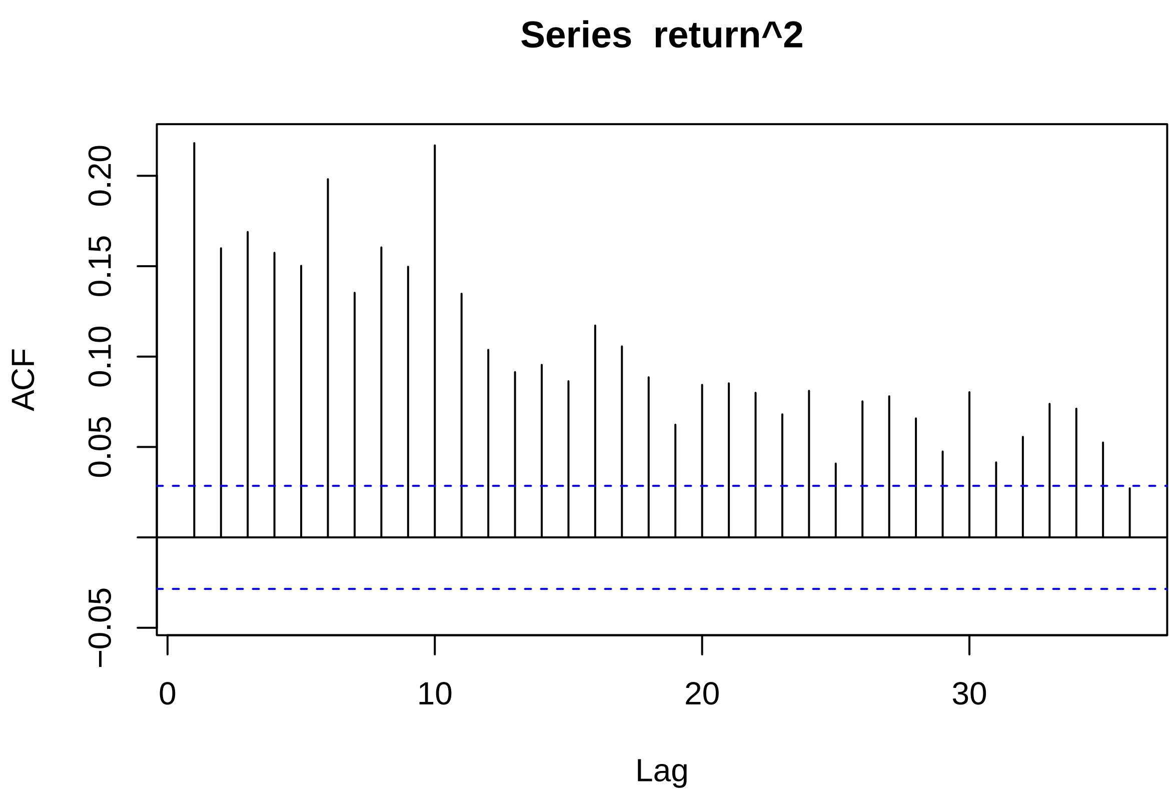


The resulting series is nearly uncorrelated but clearly dependent

```
library(forecast)
Acf(return)
```




```r
Acf(return^2)
```



ARCH Engle (1982)

An ARCH(m) model:

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2,$$

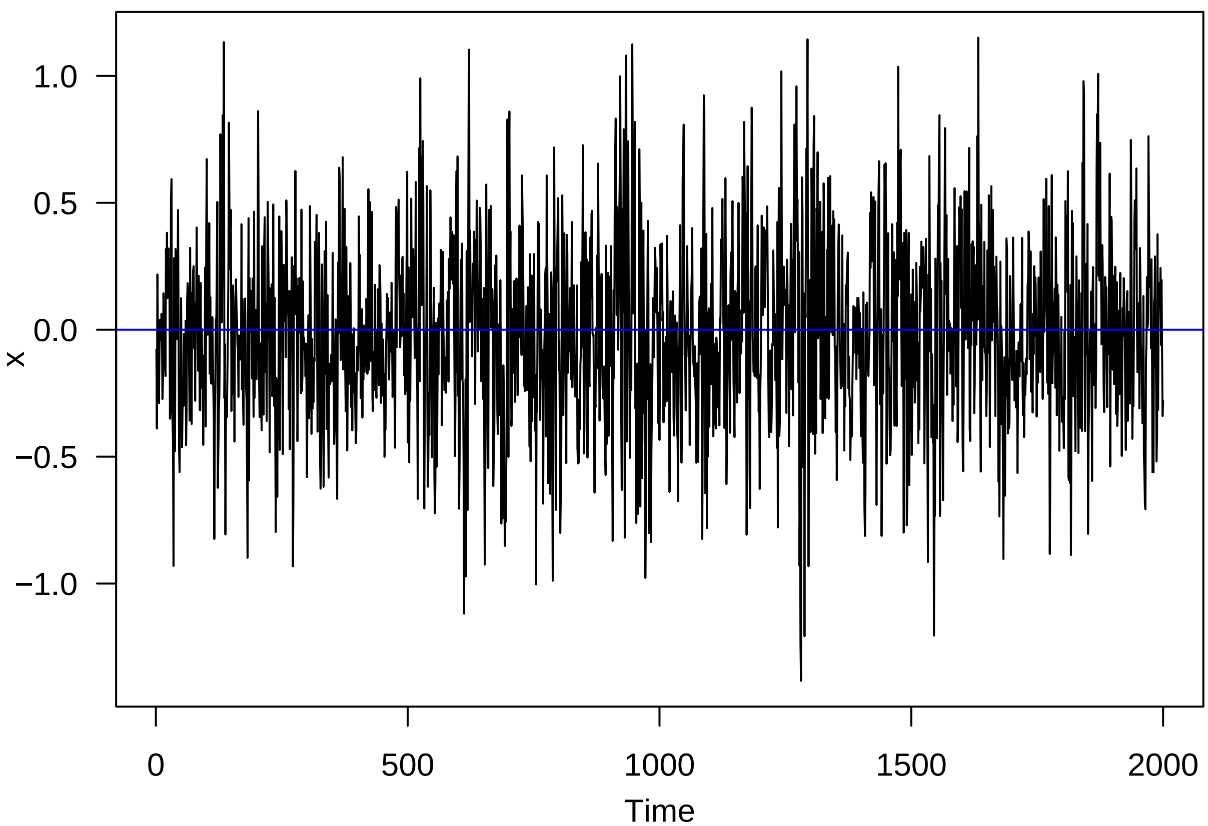
where $\{\epsilon_t\}$ is a sequence of i.i.d. r.v. with

- $\mathbb{E}(\epsilon_t) = 0$
- $\mathrm{Var}(\epsilon_t) = 1$
- $\alpha_i \geq 0$ for $1 \leq i \leq m$
- *Distribution*: standard normal, standardize Student-t, generalized error distribution, or their skewed counterparts

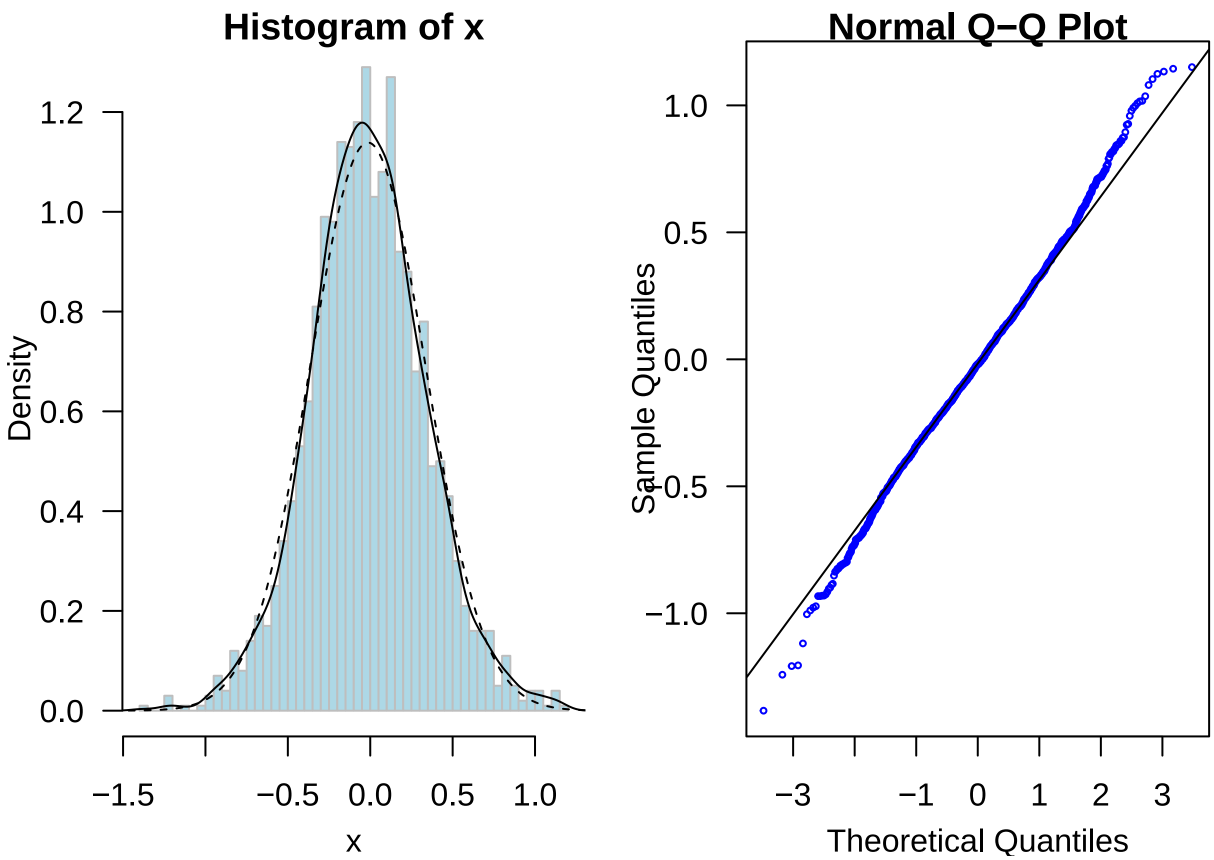
Simulation

We will use both the *fGarch* and *rugarch* packages for volatility modeling.

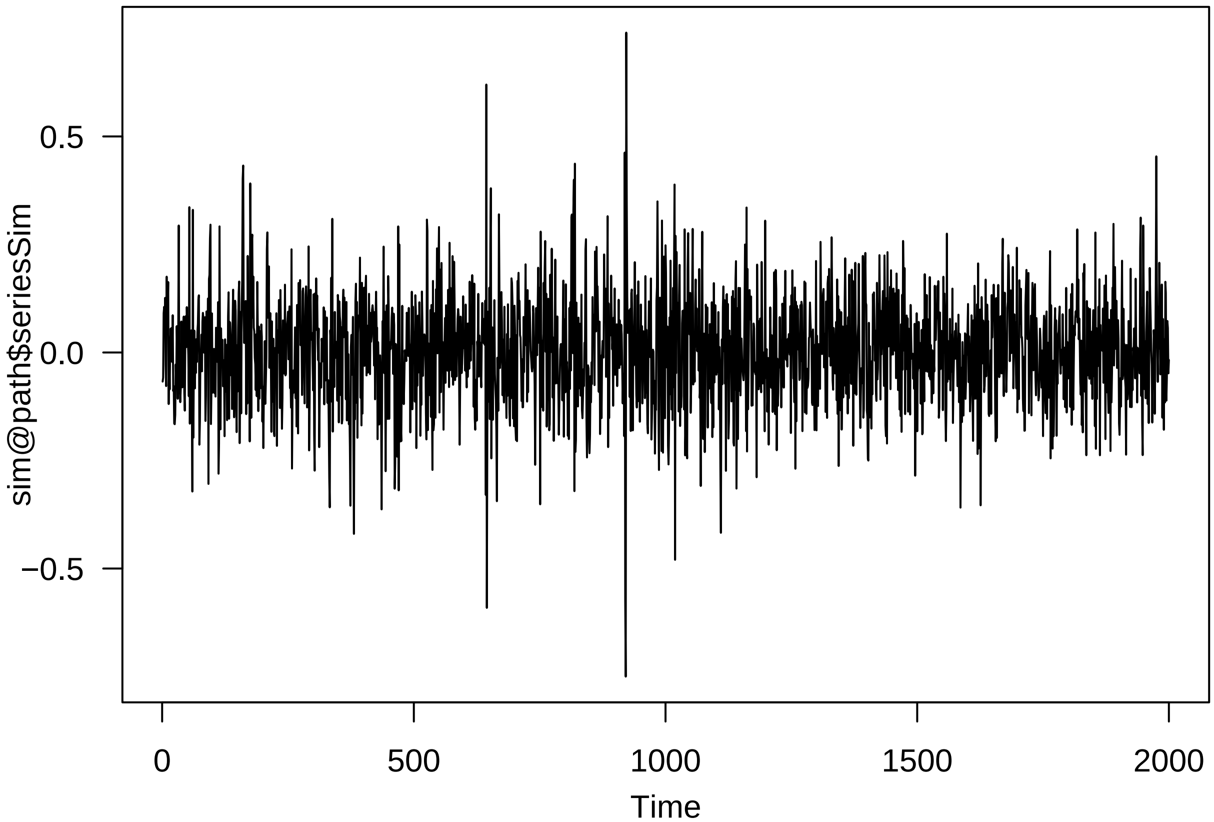
```r
library(fGarch)
mod_spec <- garchSpec(model = list(ar = c(.35), omega = 0.01))
set.seed(124)
x <- garchSim(spec = mod_spec, n = 2000)
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
ts.plot(x)
abline(h = 0, col = "blue")
```



```
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6), mfrow = c(1, 2))
hist(x, nclass = 40, col = "lightblue", border = "gray", prob = T)
den <- density(x)
rg <- 1.2 * range(x)
xg <- seq(rg[1], rg[2], .001)
yg <- dnorm(xg, mean(x), sd(x))
lines(den$x, den$y, xlab = "", ylab = "Density", type = "l")
lines(xg, yg, lty = 2)
qqnorm(x, col = "blue", cex = 0.4); qqline(x)
```




```
library(rugarch)
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
mod_spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 0)), mean.model = list
sim <- ugarchpath(mod_spec, n.sim = 2000)
ts.plot(sim@path$seriesSim)
```



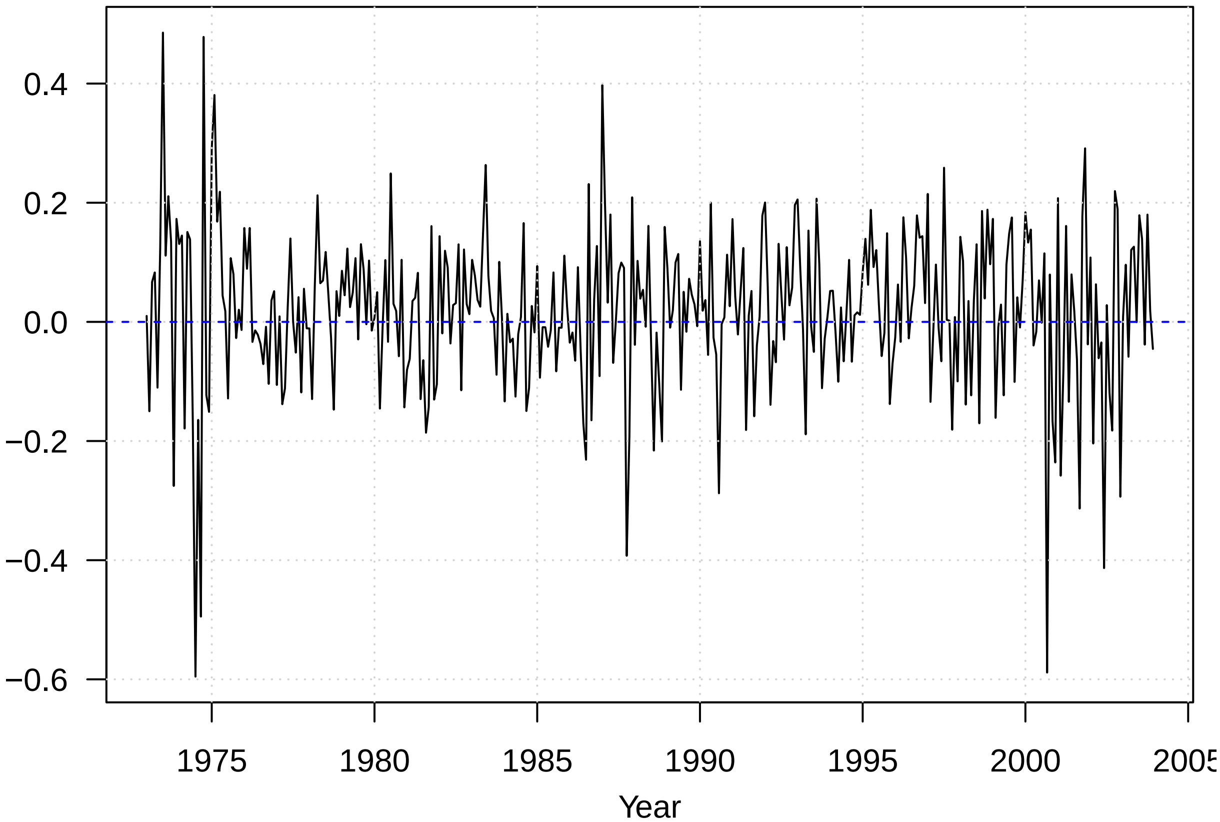
Intel Stock Example

Load and plot the monthly log returns of Intel stock

```
url <- "https://www.chicagobooth.edu/-/media/faculty/ruey-s-tsay/teaching/fts2//m-intc7303.txt"
dat1 <- read.table(url)
names(dat1) <- c("Date", "Return"); head(dat1)
```

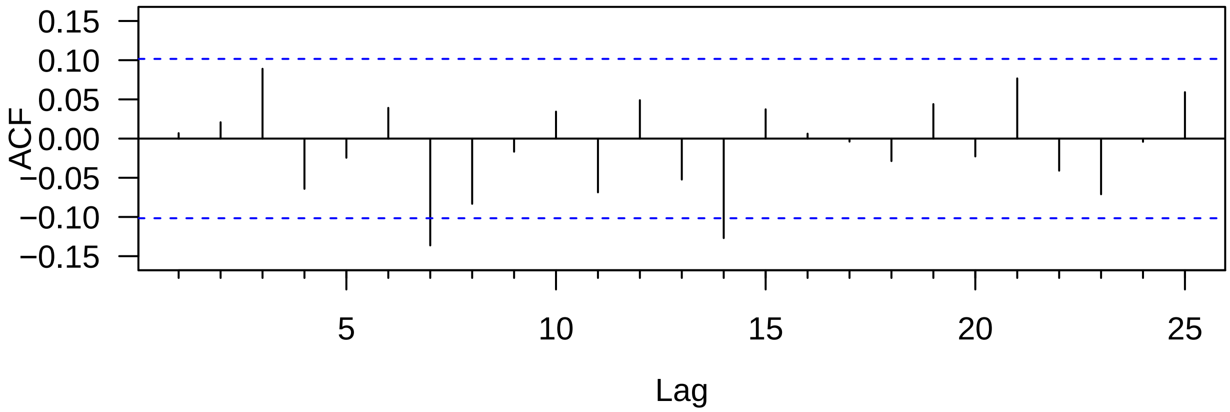
```
##       Date   Return
## 1 19730131  0.01005
## 2 19730228 -0.13930
## 3 19730330  0.06936
## 4 19730430  0.08649
## 5 19730531 -0.10448
## 6 19730629  0.13333
```

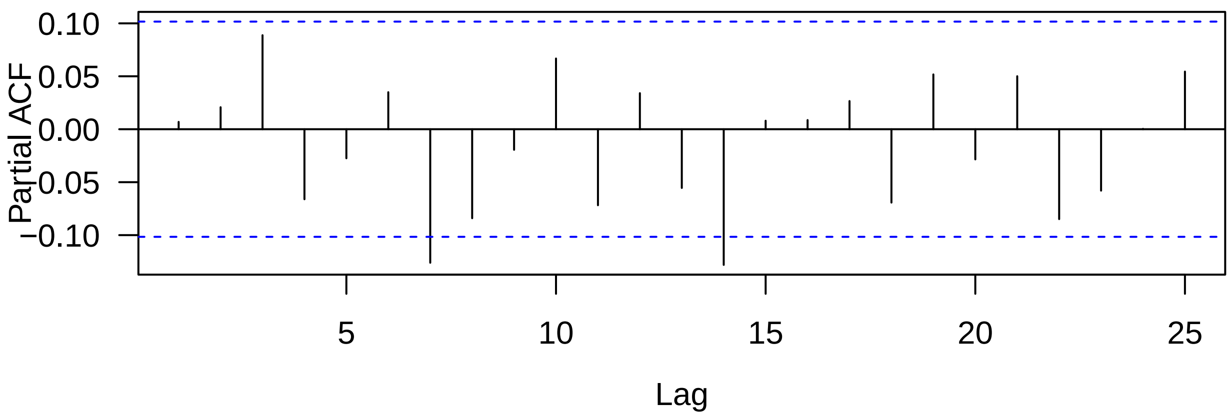
```
intc <- log(dat1$Return + 1)
return <- ts(intc, frequency = 12, start = c(1973, 1))
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
plot(return, type = "l", xlab = "Year", ylab = "")
grid()
abline(h = 0, lty = 2, col = "blue")
```



Examine the mean structure

```
par(las = 1, mgp = c(2.6, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6), mfrow = c(2, 1))
Acf(intc); pacf(intc)
```






```r
Box.test(intc, lag = 24, type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  intc
## X-squared = 32.764, df = 24, p-value = 0.1092
```

```r
t.test(intc)
```

```
##
##  One Sample t-test
##
## data:  intc
## t = 2.5944, df = 371, p-value = 0.00985
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.004354971 0.031624693
## sample estimates:
##  mean of x
## 0.01798983
```

```r
y <- intc - mean(intc)
```

Testing ARCH effect

```
Box.test(y^2, lag = 24, type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  y^2
## X-squared = 79.837, df = 24, p-value = 6.459e-08
```

```
# LM test for ARCH effects
source("archTest.R")
archTest(y, 12)
```

```
##
## Call:
## lm(formula = atsq ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.07368 -0.01295 -0.00729  0.00450  0.35621
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.007029   0.002752   2.554  0.01107 *
## x1           0.090001   0.052911   1.701  0.08984 .
## x2           0.155741   0.052830   2.948  0.00342 **
## x3           0.148341   0.053414   2.777  0.00578 **
## x4           0.020289   0.053994   0.376  0.70732
## x5           0.004670   0.053971   0.087  0.93110
## x6           0.007733   0.051753   0.149  0.88131
## x7           0.055361   0.051756   1.070  0.28552
## x8           0.009982   0.051805   0.193  0.84731
## x9           0.002042   0.051674   0.040  0.96850
## x10         -0.021888   0.051218  -0.427  0.66939
## x11         -0.057741   0.050622  -1.141  0.25481
## x12          0.162048   0.050563   3.205  0.00148 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03689 on 347 degrees of freedom
## Multiple R-squared:  0.1189, Adjusted R-squared:  0.0884
## F-statistic: 3.901 on 12 and 347 DF,  p-value: 1.236e-05
```

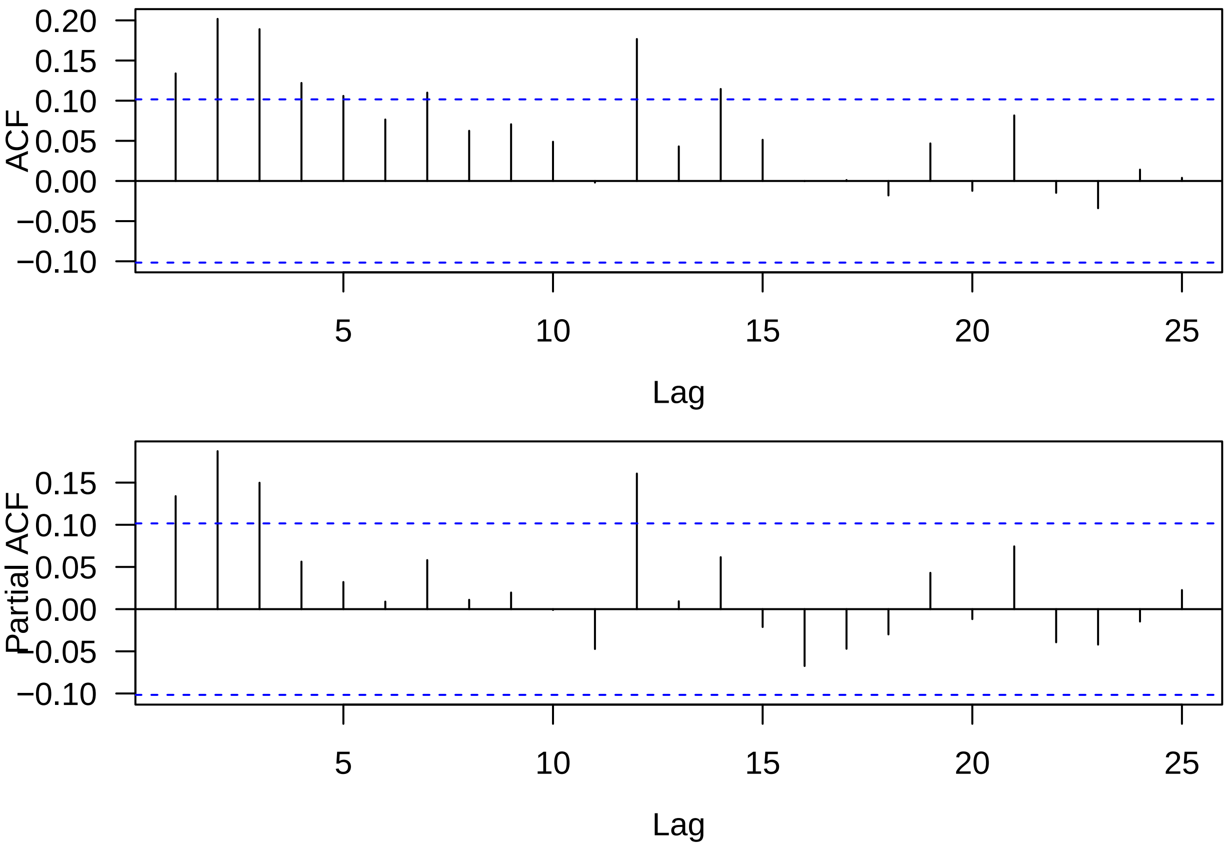
```
library(FinTS)
```

```
##
## Attaching package: 'FinTS'

## The following object is masked from 'package:forecast':
##
##     Acf
```

```r
ArchTest(y)
```

```
##
##  ARCH LM-test; Null hypothesis: no ARCH effects
##
## data:  y
## Chi-squared = 42.794, df = 12, p-value = 2.446e-05
```

```r
par(las = 1, mgp = c(2.6, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6), mfrow = c(2, 1))
Acf(y^2)
pacf(y^2)
```



Fitting ARCH

First, let's fit an ARCH(3) model

```r
# fGarch
Intel_m1 <- garchFit(~ 1 + garch(3, 0), data = intc, trace = F)
summary(Intel_m1)
```

```
##
## Title:
##  GARCH Modelling
##
```

```
## Call:
##  garchFit(formula = ~1 + garch(3, 0), data = intc, trace = F)
##
## Mean and Variance Equation:
##  data ~ 1 + garch(3, 0)
## <environment: 0x12b4cf358>
##  [data = intc]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##       mu     omega    alpha1    alpha2    alpha3
## 0.016572  0.012043  0.208649  0.071837  0.049045
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.016572    0.006423    2.580  0.00988 **
## omega   0.012043    0.001579    7.627  2.4e-14 ***
## alpha1  0.208649    0.129177    1.615  0.10626
## alpha2  0.071837    0.048551    1.480  0.13897
## alpha3  0.049045    0.048847    1.004  0.31536
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  233.4286    normalized:  0.6274962
##
## Description:
##  Fri Oct 17 23:30:31 2025 by user:
##
##
## Standardised Residuals Tests:
##                                  Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  169.773032 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.960696 1.970626e-08
##  Ljung-Box Test     R    Q(10)   10.970251 3.598404e-01
##  Ljung-Box Test     R    Q(15)   19.590244 1.882211e-01
##  Ljung-Box Test     R    Q(20)   20.821922 4.076800e-01
##  Ljung-Box Test     R^2  Q(10)    5.376602 8.646439e-01
##  Ljung-Box Test     R^2  Q(15)   22.734596 8.993974e-02
##  Ljung-Box Test     R^2  Q(20)   23.705772 2.554810e-01
##  LM Arch Test       R    TR^2    20.485060 5.844884e-02
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -1.228111 -1.175437 -1.228466 -1.207193
```

```
# rugarch
M1 = ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(3, 0)),
                      mean.model = list(armaOrder = c(0, 0), include.mean = T),
```

```r
                      distribution.model = "norm", fixed.pars = list())
(Intel_m1 <- ugarchfit(M1, data = intc))
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(3,0)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.016560    0.006419  2.57981 0.009885
## omega   0.012050    0.001591  7.57490 0.000000
## alpha1  0.212954    0.131646  1.61763 0.105743
## alpha2  0.071933    0.048928  1.47016 0.141519
## alpha3  0.049129    0.049221  0.99813 0.318219
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.016560    0.006895   2.4016 0.016322
## omega   0.012050    0.002493   4.8345 0.000001
## alpha1  0.212954    0.193580   1.1001 0.271298
## alpha2  0.071933    0.036720   1.9590 0.050118
## alpha3  0.049129    0.031957   1.5373 0.124211
##
## LogLikelihood : 233.4331
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.2281
## Bayes        -1.1755
## Shibata      -1.2285
## Hannan-Quinn -1.2072
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.03327  0.8553
## Lag[2*(p+q)+(p+q)-1][2]   0.06686  0.9435
## Lag[4*(p+q)+(p+q)-1][5]   2.04550  0.6077
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
```

```
## Lag[1]                         0.5564  0.4557
## Lag[2*(p+q)+(p+q)-1][8]      1.4875  0.9287
## Lag[4*(p+q)+(p+q)-1][14]     6.7064  0.5442
## d.o.f=3
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[4]    0.5757 0.500 2.000  0.4480
## ARCH Lag[6]    0.8633 1.461 1.711  0.7873
## ARCH Lag[8]    1.7352 2.368 1.583  0.7945
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.8622
## Individual Statistics:
## mu     0.04824
## omega  0.31431
## alpha1 0.25825
## alpha2 0.57418
## alpha3 0.20981
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.28 1.47 1.88
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.22799 0.8198
## Negative Sign Bias 0.07266 0.9421
## Positive Sign Bias 0.27306 0.7850
## Joint Effect       0.10621 0.9911
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     32.95      0.02439
## 2    30     44.29      0.03444
## 3    40     47.78      0.15799
## 4    50     64.29      0.07027
##
##
## Elapsed time : 0.05422401
```

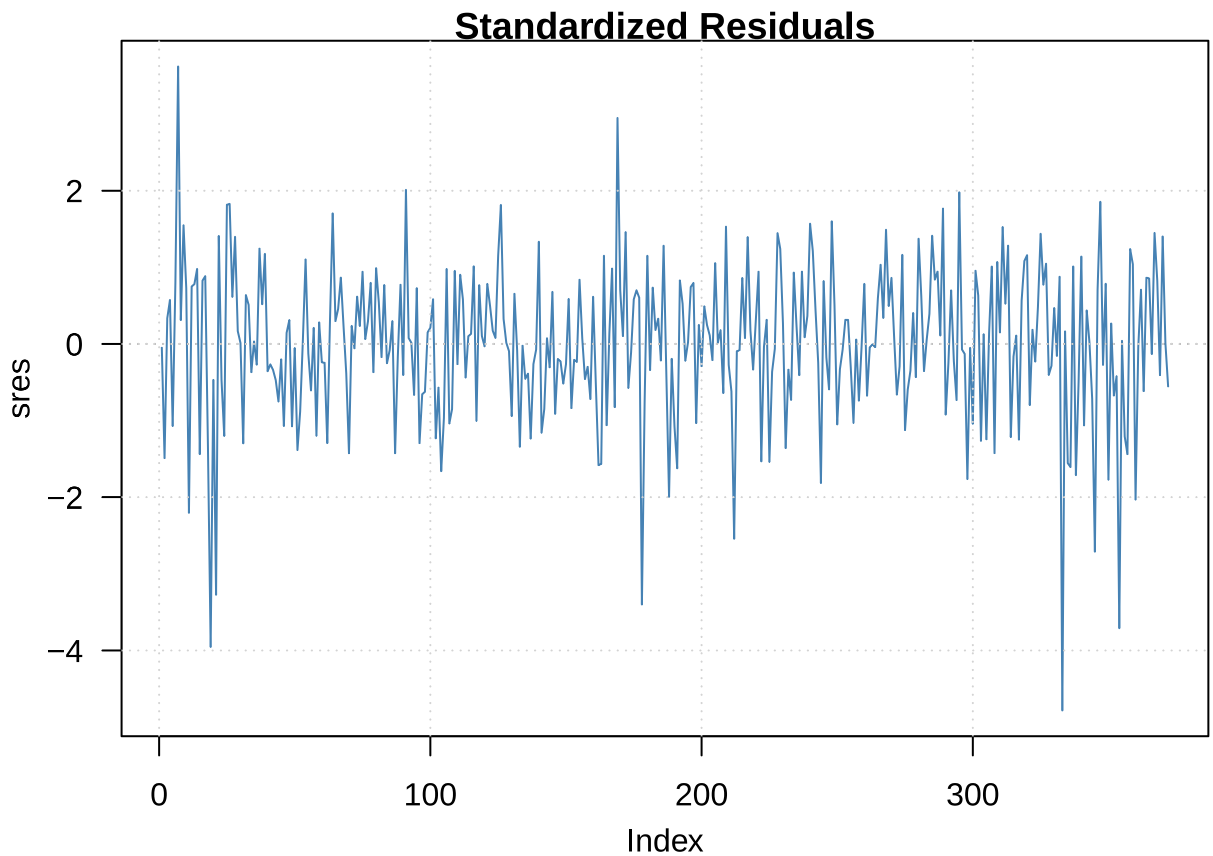
Let's fit an ARCH(1) model

```r
Intel_m2 <- garchFit(~ 1 + garch(1, 0), data = intc, trace = F)
summary(Intel_m2)
```

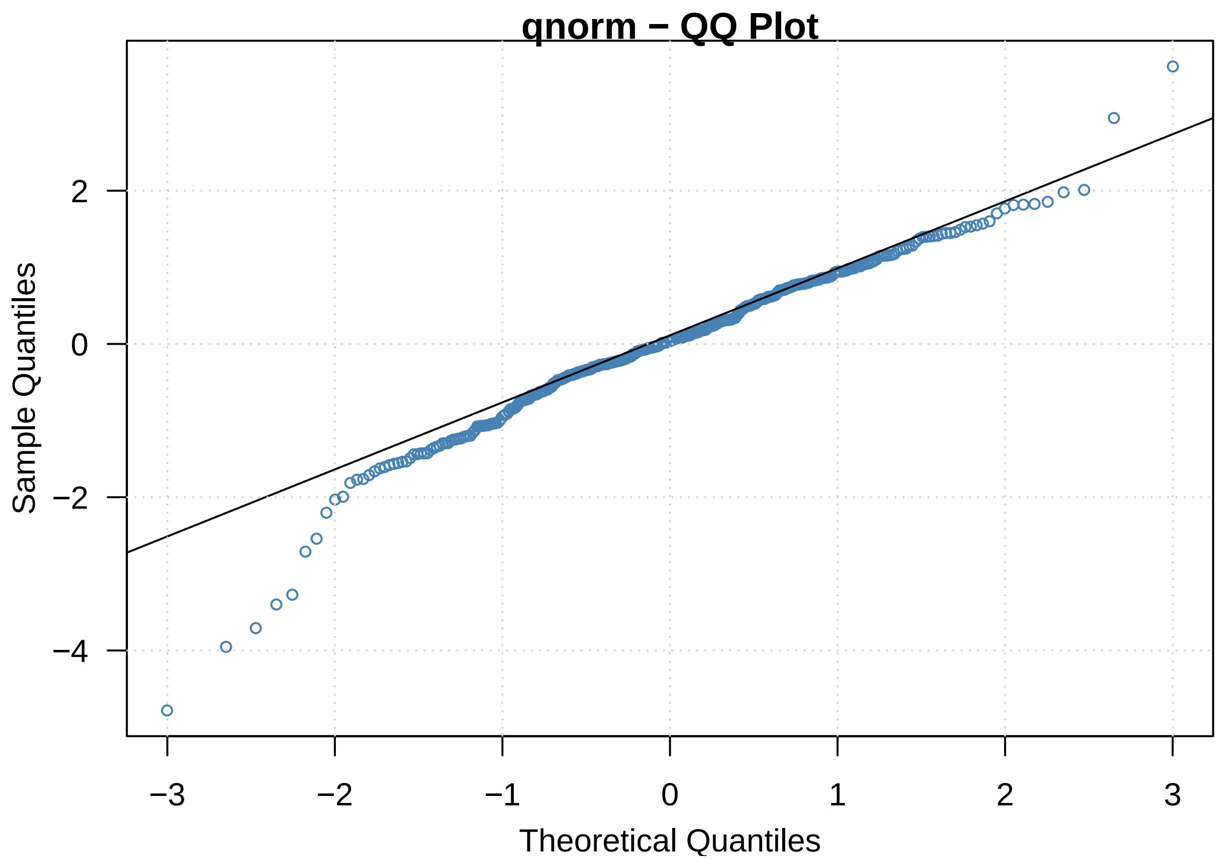
```
##
## Title:
```

```
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~1 + garch(1, 0), data = intc, trace = F)
##
## Mean and Variance Equation:
##  data ~ 1 + garch(1, 0)
## <environment: 0x11c3091c8>
##  [data = intc]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##      mu    omega   alpha1
## 0.01657  0.01249  0.36345
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.016570    0.006161    2.689  0.00716 **
## omega   0.012490    0.001549    8.061 6.66e-16 ***
## alpha1  0.363447    0.131598    2.762  0.00575 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  230.2423    normalized:  0.6189309
##
## Description:
##  Fri Oct 17 23:30:31 2025 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  122.4040147 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.9647625 8.273101e-08
##  Ljung-Box Test     R    Q(10)   13.7260350 1.858587e-01
##  Ljung-Box Test     R    Q(15)   22.3171422 9.975386e-02
##  Ljung-Box Test     R    Q(20)   23.8825692 2.475594e-01
##  Ljung-Box Test     R^2  Q(10)   12.5002502 2.529700e-01
##  Ljung-Box Test     R^2  Q(15)   30.1127648 1.152131e-02
##  Ljung-Box Test     R^2  Q(20)   31.4640428 4.935483e-02
##  LM Arch Test       R    TR^2    22.0360016 3.711830e-02
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -1.221733 -1.190129 -1.221861 -1.209182
```

```r
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
plot(Intel_m2, which = 9)
```




```r
plot(Intel_m2, which = 13)
```



ARCH(1) with Student-t Innovations for 5-step predictions

```r
Intel_m3 <- garchFit(~ 1 + garch(1, 0), data = intc, cond.dist = "std", trace = F)
summary(Intel_m3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~1 + garch(1, 0), data = intc, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ 1 + garch(1, 0)
## <environment: 0x118f7d8a8>
##  [data = intc]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu      omega     alpha1      shape
## 0.021571   0.013424   0.259867   5.985979
```

```
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.021571    0.006054    3.563 0.000366 ***
## omega   0.013424    0.001968    6.820 9.09e-12 ***
## alpha1  0.259867    0.119901    2.167 0.030209 *
## shape   5.985979    1.660029    3.606 0.000311 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  242.9678    normalized:  0.6531391
##
## Description:
##  Fri Oct 17 23:30:31 2025 by user:
##
##
## Standardised Residuals Tests:
##                                   Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  130.8930540 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.9637533 5.744995e-08
##  Ljung-Box Test     R    Q(10)   14.3128788 1.591926e-01
##  Ljung-Box Test     R    Q(15)   23.3404312 7.717449e-02
##  Ljung-Box Test     R    Q(20)   24.8728553 2.063387e-01
##  Ljung-Box Test     R^2  Q(10)   15.3591723 1.195054e-01
##  Ljung-Box Test     R^2  Q(15)   33.9631828 3.446127e-03
##  Ljung-Box Test     R^2  Q(20)   35.4682772 1.774746e-02
##  LM Arch Test       R    TR^2    24.1151683 1.961957e-02
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -1.284773 -1.242634 -1.285001 -1.268039
```

```r
predict(Intel_m3, 5)
```

```
##   meanForecast meanError standardDeviation
## 1     0.021571 0.1207911         0.1207911
## 2     0.021571 0.1312069         0.1312069
## 3     0.021571 0.1337810         0.1337810
## 4     0.021571 0.1344418         0.1344418
## 5     0.021571 0.1346130         0.1346130
```

```r
M3 = ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 0)),
                     mean.model = list(armaOrder = c(0, 0), include.mean = T),
                     distribution.model = "std", fixed.pars = list())
(Intel_m3 <- ugarchfit(M3, data = intc))
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
```

```
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,0)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : std
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.021565    0.006053   3.5627 0.000367
## omega   0.013476    0.001996   6.7508 0.000000
## alpha1  0.263911    0.121752   2.1676 0.030188
## shape   5.937754    1.655102   3.5875 0.000334
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.021565    0.006365   3.3880 0.000704
## omega   0.013476    0.002132   6.3203 0.000000
## alpha1  0.263911    0.153254   1.7220 0.085061
## shape   5.937754    1.475938   4.0230 0.000057
##
## LogLikelihood : 242.9753
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.2848
## Bayes        -1.2427
## Shibata      -1.2850
## Hannan-Quinn -1.2681
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                   0.003118  0.9555
## Lag[2*(p+q)+(p+q)-1][2]  0.193986  0.8579
## Lag[4*(p+q)+(p+q)-1][5]  3.649143  0.3012
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.5263 0.46815
## Lag[2*(p+q)+(p+q)-1][2]    3.5205 0.10150
## Lag[4*(p+q)+(p+q)-1][5]    6.6637 0.06253
## d.o.f=1
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[2]     5.924 0.500 2.000 0.01493
```

```
## ARCH Lag[4]     7.325 1.397 1.611 0.02349
## ARCH Lag[6]     8.323 2.222 1.500 0.03480
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.2181
## Individual Statistics:
## mu     0.05211
## omega  0.48258
## alpha1 0.37199
## shape  0.13634
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.07 1.24 1.6
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.18119 0.8563
## Negative Sign Bias 0.39211 0.6952
## Positive Sign Bias 0.06475 0.9484
## Joint Effect       0.16039 0.9837
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     32.95      0.02439
## 2    30     38.48      0.11195
## 3    40     42.62      0.31802
## 4    50     55.42      0.24547
##
##
## Elapsed time : 0.04685497
```

```
ugarchforecast(Intel_m3, n.ahead = 5)
```

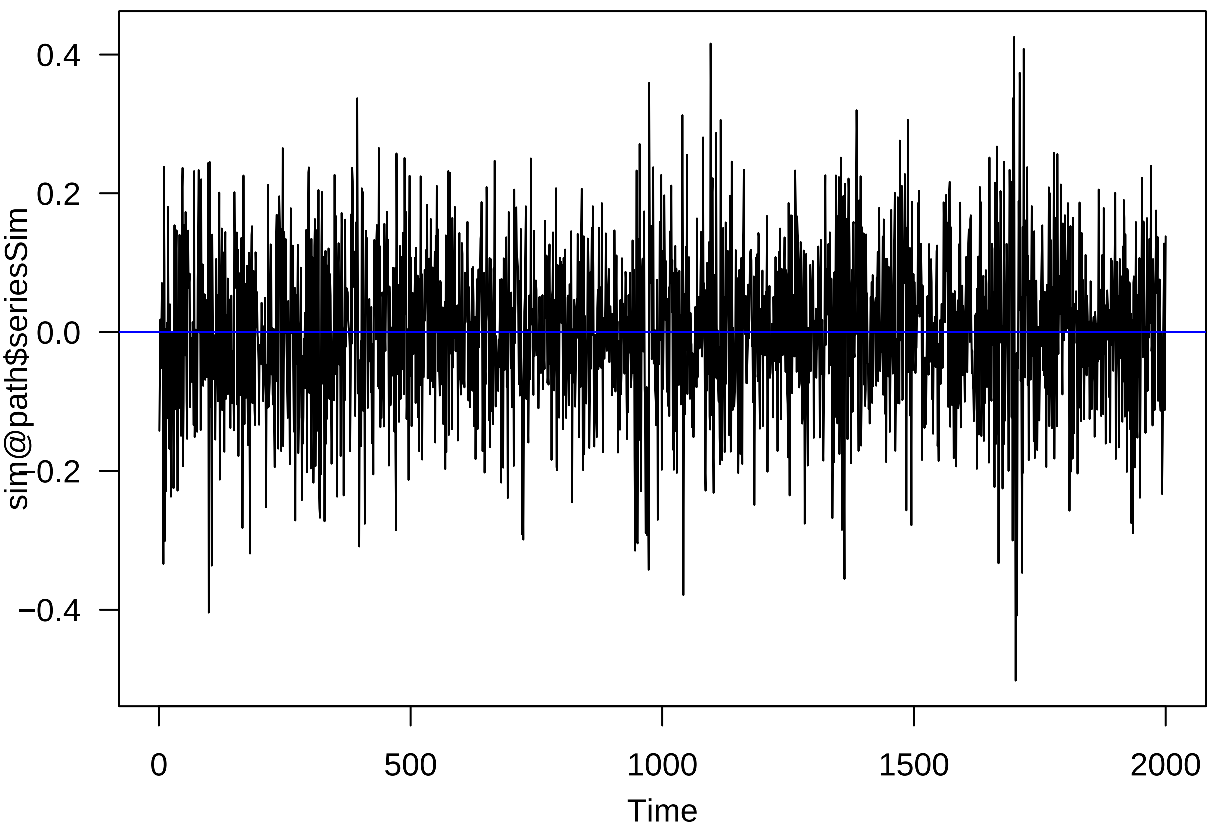
```
##
## *------------------------------------*
## *       GARCH Model Forecast         *
## *------------------------------------*
## Model: sGARCH
## Horizon: 5
## Roll Steps: 0
## Out of Sample: 0
##
## 0-roll forecast [T0=1971-01-08]:
##      Series  Sigma
## T+1 0.02156 0.1211
## T+2 0.02156 0.1317
## T+3 0.02156 0.1344
## T+4 0.02156 0.1351
## T+5 0.02156 0.1352
```

GARCH Bollerslev (1986)

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2.$$

Simulation

```
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
mod_spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                       mean.model = list(armaOrder = c(0, 0), include.mean = FALSE),
                       fixed.pars = list(omega = 0.001, alpha1 = 0.08, beta1 = 0.85))
sim <- ugarchpath(mod_spec, n.sim = 2000)
ts.plot(sim@path$seriesSim)
abline(h = 0, col = "blue")
```



Fit a GARCH(1,1)

```
Intel_m4 <- garchFit(~ 1 + garch(1, 1), data = intc, trace = F)
summary(Intel_m4)
```

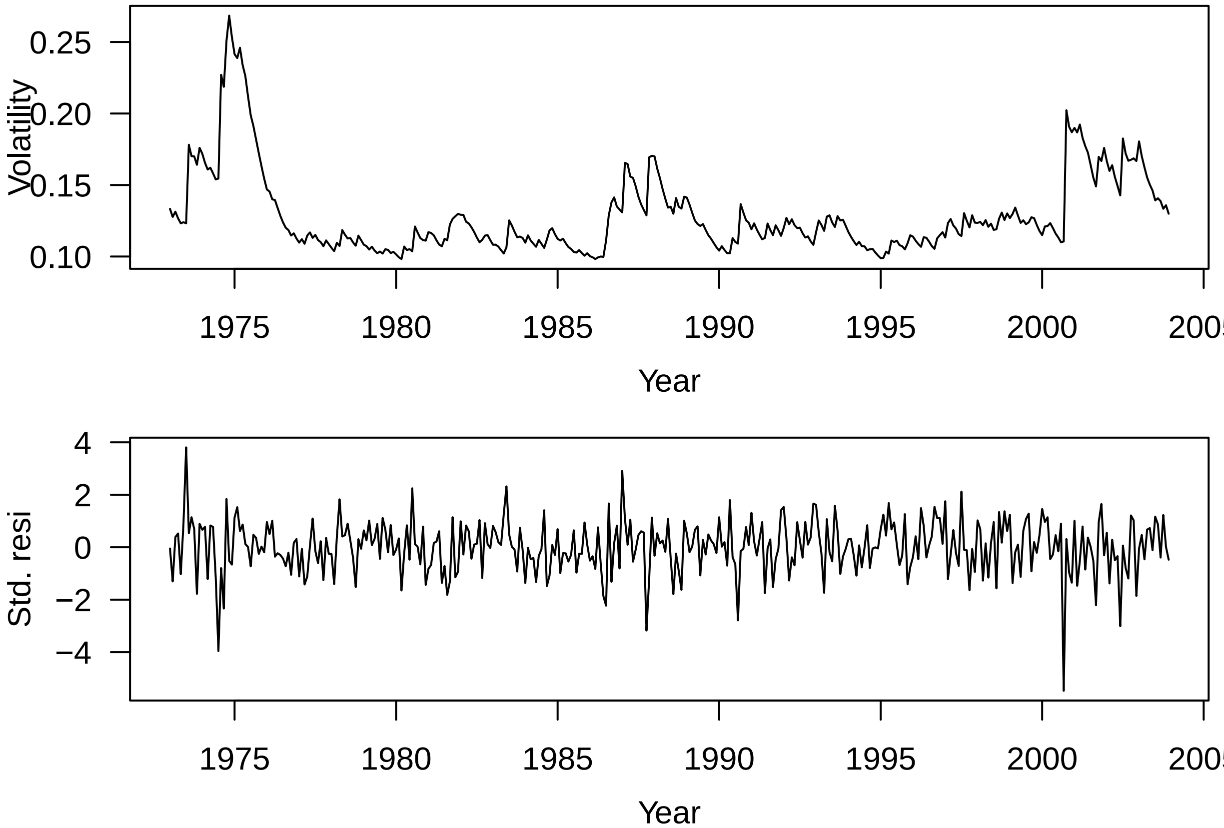
```
##
## Title:
##  GARCH Modelling
##
## Call:
```

```
##  garchFit(formula = ~1 + garch(1, 1), data = intc, trace = F)
##
## Mean and Variance Equation:
##  data ~ 1 + garch(1, 1)
## <environment: 0x11c7d02e8>
##  [data = intc]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu      omega     alpha1      beta1
## 0.0163276  0.0010918  0.0802716  0.8553014
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.0163276   0.0062624    2.607  0.00913 **
## omega   0.0010918   0.0005291    2.063  0.03907 *
## alpha1  0.0802716   0.0281162    2.855  0.00430 **
## beta1   0.8553014   0.0461374   18.538  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  239.5189    normalized:  0.6438681
##
## Description:
##  Fri Oct 17 23:30:32 2025 by user:
##
##
## Standardised Residuals Tests:
##                                  Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  156.5137854 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.9676933 2.471139e-07
##  Ljung-Box Test     R    Q(10)    9.8054846 4.577215e-01
##  Ljung-Box Test     R    Q(15)   16.5443535 3.468240e-01
##  Ljung-Box Test     R    Q(20)   17.8005009 6.005484e-01
##  Ljung-Box Test     R^2  Q(10)    0.5130171 9.999925e-01
##  Ljung-Box Test     R^2  Q(15)   10.2455702 8.040151e-01
##  Ljung-Box Test     R^2  Q(20)   11.7798769 9.234441e-01
##  LM Arch Test       R    TR^2     9.3344592 6.741288e-01
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -1.266231 -1.224092 -1.266459 -1.249496
```

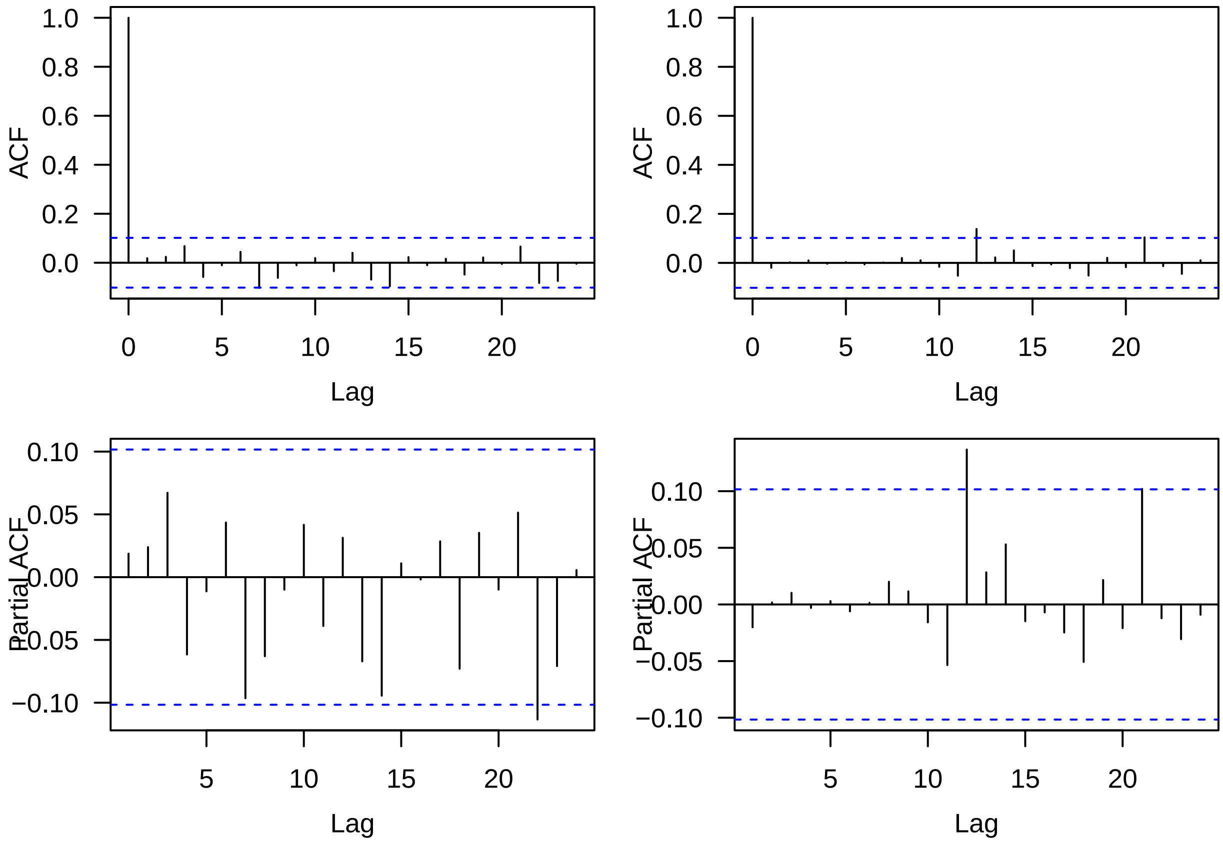
```r
mu <- Intel_m4@fit$par[1]

v1 <- volatility(Intel_m4)
resi <- residuals(Intel_m4, standardize = T)
vol <- ts(v1, frequency = 12, start = c(1973, 1))
```

```r
res <- ts(resi, frequency = 12, start = c(1973, 1))
```

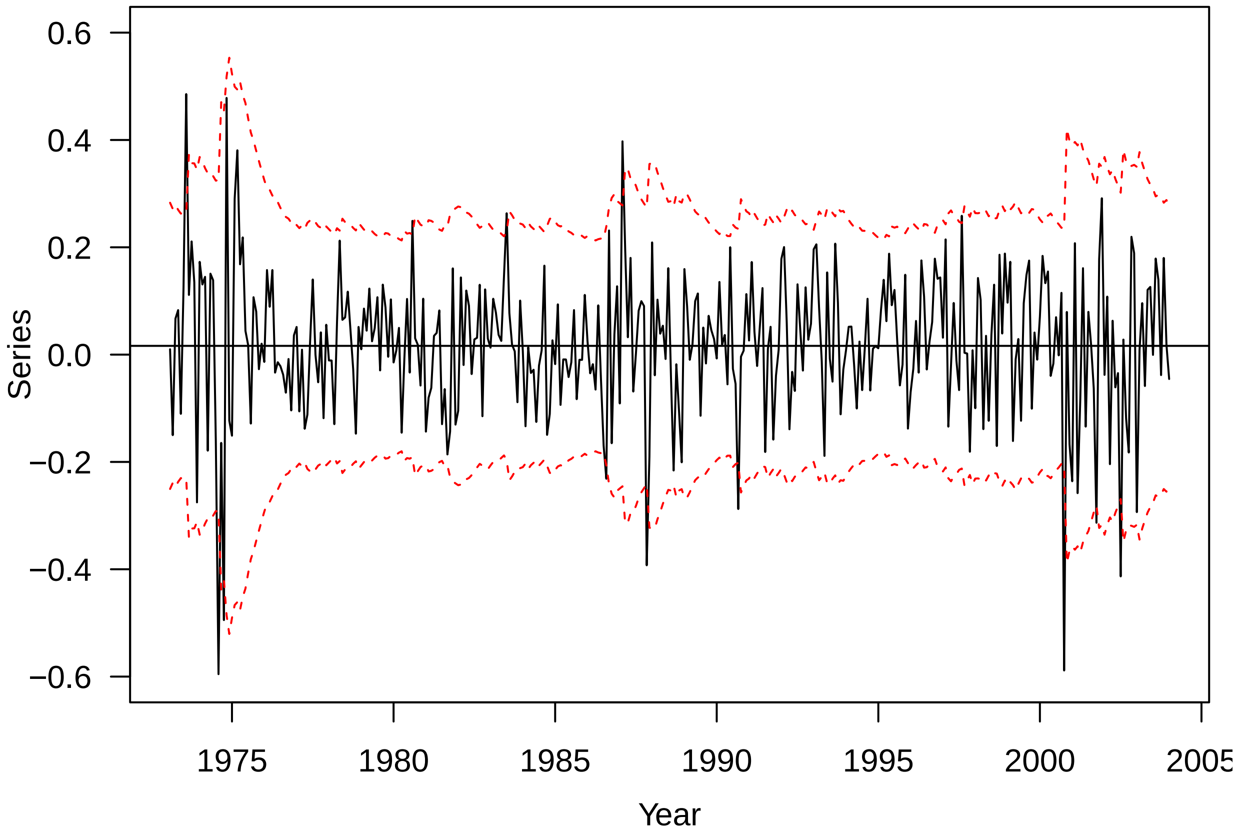
```r
par(las = 1, mgp = c(2.4, 1, 0), mar = c(3.6, 3.8, 0.8, 0.6), mfcol = c(2, 1))
plot(vol, xlab = 'Year', ylab = 'Volatility',type = 'l')
plot(res, xlab = 'Year', ylab = 'Std. resi', type = 'l')
```




```r
par(las = 1, mgp = c(2.4, 1, 0), mar = c(3.6, 3.8, 0.8, 0.6), mfcol = c(2, 2))
acf(resi, lag = 24)
pacf(resi, lag = 24)
acf(resi^2, lag = 24)
pacf(resi^2, lag = 24)
```




```r
par(las = 1, mgp = c(2.4, 1, 0), mar = c(3.6, 3.8, 0.8, 0.6))
upp = mu + 2 * v1; low = mu - 2 * v1
tdx <- (1:length(intc)) / 12 + 1973
plot(tdx, intc, xlab = 'Year', ylab = 'Series', type = 'l', ylim = c(-0.6, 0.6))
lines(tdx, upp, lty = 2, col = 'red'); lines(tdx, low, lty = 2, col = 'red')
abline(h = mu)
```




```r
M4 = ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                         mean.model = list(armaOrder = c(0, 0), include.mean = T),
                         distribution.model = "norm", fixed.pars = list())
Intel_m4 <- ugarchfit(M4, data = intc)
Intel_m4
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.016330    0.006262   2.6079 0.009111
## omega   0.001091    0.000529   2.0619 0.039214
## alpha1  0.079784    0.028023   2.8471 0.004412
## beta1   0.855460    0.046223  18.5074 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
```

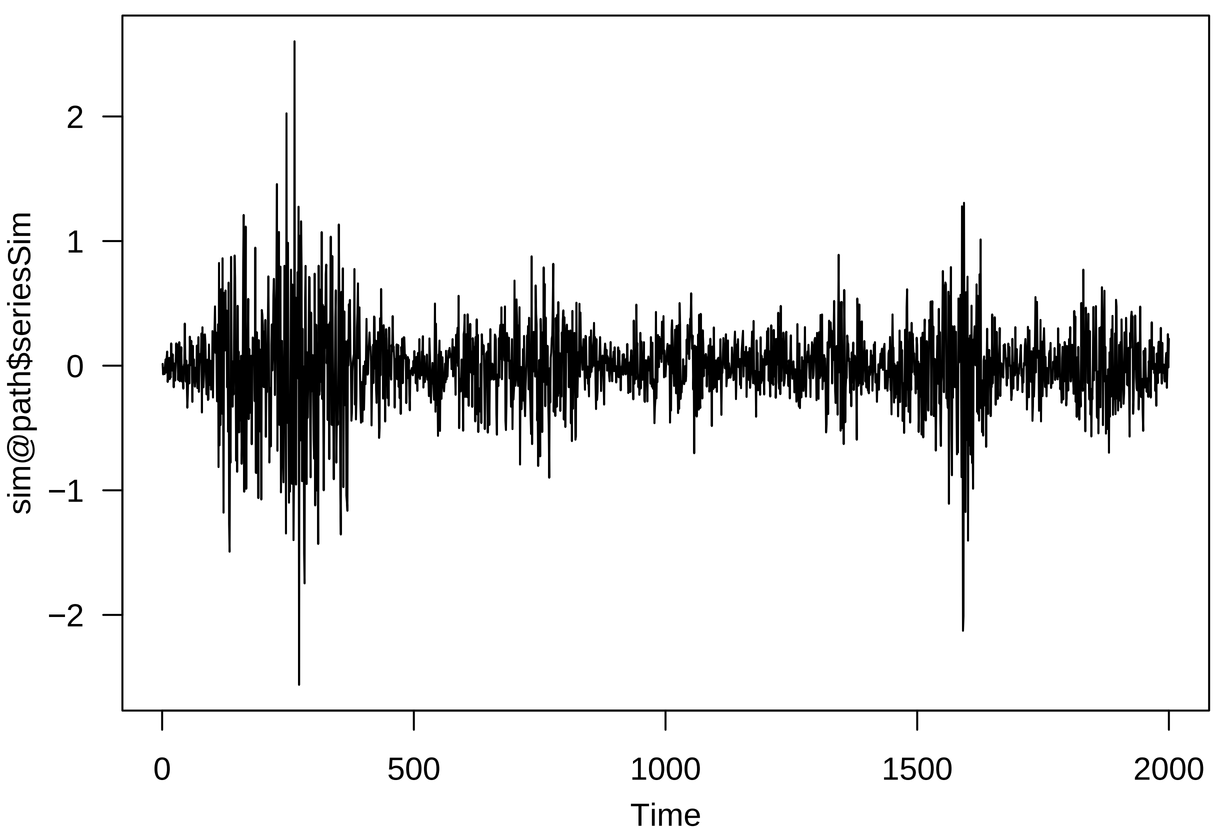
```
## mu      0.016330    0.007332   2.2272 0.025934
## omega   0.001091    0.000636   1.7159 0.086184
## alpha1  0.079784    0.032726   2.4379 0.014772
## beta1   0.855460    0.050233  17.0297 0.000000
##
## LogLikelihood : 239.5281
##
## Information Criteria
## ------------------------------------
##
## Akaike       -1.2663
## Bayes        -1.2241
## Shibata      -1.2665
## Hannan-Quinn -1.2495
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1307  0.7177
## Lag[2*(p+q)+(p+q)-1][2]    0.2414  0.8293
## Lag[4*(p+q)+(p+q)-1][5]    1.8780  0.6475
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1505  0.6981
## Lag[2*(p+q)+(p+q)-1][5]    0.1792  0.9940
## Lag[4*(p+q)+(p+q)-1][9]    0.2349  0.9999
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]   0.04014 0.500 2.000  0.8412
## ARCH Lag[5]   0.04586 1.440 1.667  0.9956
## ARCH Lag[7]   0.05466 2.315 1.543  0.9998
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.6271
## Individual Statistics:
## mu     0.04707
## omega  0.19694
## alpha1 0.10765
## beta1  0.20534
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.07 1.24 1.6
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
```

```
##                      t-value   prob sig
## Sign Bias           0.04526 0.9639
## Negative Sign Bias 0.43619 0.6630
## Positive Sign Bias 0.25615 0.7980
## Joint Effect       0.25672 0.9680
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     24.67       0.1718
## 2    30     36.06       0.1717
## 3    40     50.58       0.1013
## 4    50     51.39       0.3804
##
##
## Elapsed time : 0.02457714
```

IGARCH

Simulation

```r
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
mod_spec <- ugarchspec(variance.model = list(model = "iGARCH", garchOrder = c(1, 1)), mean.model = list
sim <- ugarchpath(mod_spec, n.sim = 2000)
ts.plot(sim@path$seriesSim)
```



```r
source("Igarch.R")
IGARCH_fit <- Igarch(intc, include.mean = T)
```

```
## Estimates:  0.01518916 0.930005
## Maximized log-likehood:  -231.7141
##
## Coefficient(s):
##        Estimate  Std. Error  t value Pr(>|t|)
## mu   0.01518916  0.00625338  2.42895 0.015143 *
## beta 0.93000501  0.01661928 55.95940  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
names(IGARCH_fit)
```

```
## [1] "par"        "volatility"
```

```r
Intel_m5 = ugarchspec(variance.model = list(model = "iGARCH", garchOrder = c(1, 1)), mean.model = list(a

(Intel_m5 <- ugarchfit(Intel_m5, data = intc))
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.015365    0.006210   2.4741 0.013357
## omega   0.000336    0.000206   1.6318 0.102719
## alpha1  0.113395    0.035860   3.1621 0.001566
## beta1   0.886605          NA       NA       NA
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.015365    0.007326   2.0973 0.035968
## omega   0.000336    0.000213   1.5765 0.114903
## alpha1  0.113395    0.037279   3.0418 0.002352
## beta1   0.886605          NA       NA       NA
##
## LogLikelihood : 236.014
##
## Information Criteria
## ------------------------------------
##
```

```
## Akaike       -1.2528
## Bayes        -1.2212
## Shibata      -1.2529
## Hannan-Quinn -1.2402
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1910  0.6621
## Lag[2*(p+q)+(p+q)-1][2]    0.3424  0.7726
## Lag[4*(p+q)+(p+q)-1][5]    1.7669  0.6744
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1677  0.6822
## Lag[2*(p+q)+(p+q)-1][5]    0.3334  0.9801
## Lag[4*(p+q)+(p+q)-1][9]    0.4189  0.9991
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]   0.01600 0.500 2.000  0.8993
## ARCH Lag[5]   0.07147 1.440 1.667  0.9918
## ARCH Lag[7]   0.12253 2.315 1.543  0.9990
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.5876
## Individual Statistics:
## mu     0.05671
## omega  0.13078
## alpha1 0.49311
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          0.846 1.01 1.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.00311 0.9975
## Negative Sign Bias 0.03265 0.9740
## Positive Sign Bias 0.02437 0.9806
## Joint Effect       0.00404 0.9999
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     29.08      0.06481
```

```
## 2    30     35.74      0.18120
## 3    40     46.28      0.19700
## 4    50     55.42      0.24547
##
##
## Elapsed time : 0.01326895
```

```r
ugarchforecast(Intel_m5, n.ahead = 10, data = intc)
```

```
## Warning in `setfixed<-`(`*tmp*`, value = as.list(pars)): Unrecognized Parameter
## in Fixed Values: beta1...Ignored
```

```
##
## *------------------------------------*
## *       GARCH Model Forecast         *
## *------------------------------------*
## Model: iGARCH
## Horizon: 10
## Roll Steps: 0
## Out of Sample: 0
##
## 0-roll forecast [T0=1971-01-08]:
##       Series  Sigma
## T+1  0.01536 0.1429
## T+2  0.01536 0.1441
## T+3  0.01536 0.1453
## T+4  0.01536 0.1464
## T+5  0.01536 0.1476
## T+6  0.01536 0.1487
## T+7  0.01536 0.1498
## T+8  0.01536 0.1510
## T+9  0.01536 0.1521
## T+10 0.01536 0.1532
```

EGARCH Nelson (1991)

IBM monthly returns

```r
source("Egarch.R")
IBM <- read.table("m-ibmsp6709.txt", header = T)
ibm <- log(IBM$ibm + 1)
Box.test(ibm, lag = 12, type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  ibm
## X-squared = 7.4042, df = 12, p-value = 0.8298
```

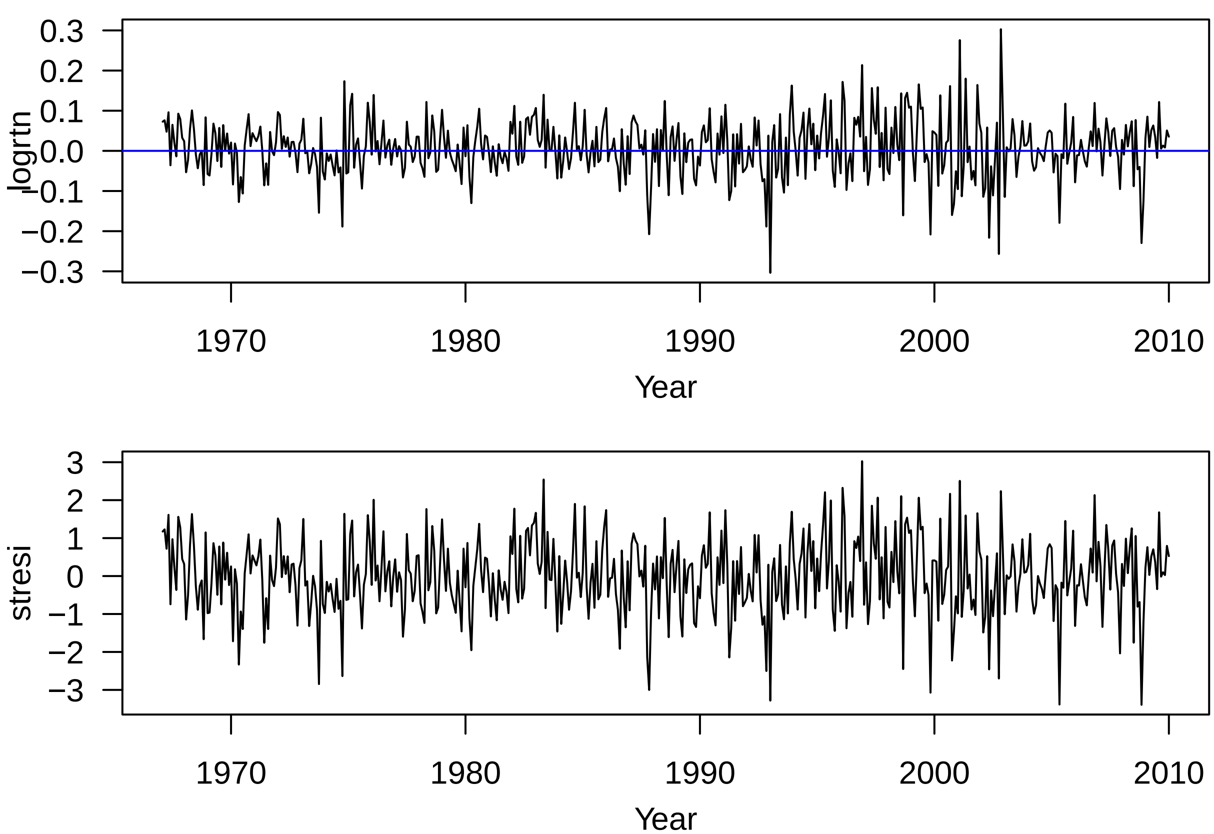
```r
EGARCH_fit <- Egarch(ibm)
```

```
##
## Estimation results of EGARCH(1,1) model:
## estimates:  0.006732418 -0.5983265 0.2176024 -0.4243194 0.9201499
## std.errors:  0.002877668 0.2349184 0.05916505 0.1683056 0.03886579
## t-ratio:  2.339539 -2.546954 3.677888 -2.521125 23.67506
```

```r
names(EGARCH_fit)
```

```
## [1] "residuals"  "volatility"
```

```r
stresi <- EGARCH_fit$residuals / EGARCH_fit$volatility
tdx = (1:length(ibm)) / 12 + 1967
par(las = 1, mgp = c(2.2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6), mfcol = c(2, 1))
plot(tdx, ibm, xlab = 'Year', ylab = 'logrtn', type = 'l')
abline(h = 0, col = "blue")
plot(tdx,stresi, xlab = 'Year', ylab = 'stresi',type = 'l')
```




```r
Box.test(stresi, lag = 10, type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  stresi
## X-squared = 5.2866, df = 10, p-value = 0.8712
```

```r
Box.test(stresi, lag = 20, type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  stresi
## X-squared = 20.983, df = 20, p-value = 0.3981
```

```r
Box.test(stresi^2, lag = 10, type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  stresi^2
## X-squared = 5.0469, df = 10, p-value = 0.888
```

```r
Box.test(stresi^2, lag = 20, type = 'Ljung')
```

```
##
##  Box-Ljung test
##
## data:  stresi^2
## X-squared = 14.261, df = 20, p-value = 0.817
```

Fit EGARCH using *ugarch*

```r
IBM_egarch <- ugarchspec(variance.model = list(model = "eGARCH", garchOrder = c(1, 1)), mean.model = lis

(IBM_egarch <- ugarchfit(IBM_egarch, data = ibm))
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(1,1)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.006649    0.002963   2.2442 0.024820
## omega  -0.423213    0.223675  -1.8921 0.058480
## alpha1 -0.094813    0.039373  -2.4081 0.016037
## beta1   0.920484    0.041730  22.0583 0.000000
## gamma1  0.218711    0.060802   3.5971 0.000322
```

```
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.006649    0.003073   2.1635 0.030502
## omega  -0.423213    0.308233  -1.3730 0.169743
## alpha1 -0.094813    0.051835  -1.8291 0.067382
## beta1   0.920484    0.057270  16.0726 0.000000
## gamma1  0.218711    0.061769   3.5408 0.000399
##
## LogLikelihood : 651.634
##
## Information Criteria
## ------------------------------------
##
## Akaike       -2.5063
## Bayes        -2.4652
## Shibata      -2.5065
## Hannan-Quinn -2.4902
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.237  0.2661
## Lag[2*(p+q)+(p+q)-1][2]     1.344  0.3989
## Lag[4*(p+q)+(p+q)-1][5]     1.867  0.6501
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                   0.009632  0.9218
## Lag[2*(p+q)+(p+q)-1][5]  1.087450  0.8396
## Lag[4*(p+q)+(p+q)-1][9]  2.511477  0.8360
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]   0.09128 0.500 2.000  0.7626
## ARCH Lag[5]   1.10480 1.440 1.667  0.7021
## ARCH Lag[7]   2.20198 2.315 1.543  0.6746
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.1719
## Individual Statistics:
## mu     0.21948
## omega  0.61756
## alpha1 0.15868
## beta1  0.61824
## gamma1 0.06386
##
## Asymptotic Critical Values (10% 5% 1%)
```

```
## Joint Statistic:          1.28 1.47 1.88
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.1014 0.9192
## Negative Sign Bias  0.2560 0.7980
## Positive Sign Bias  0.1888 0.8503
## Joint Effect        0.3726 0.9458
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     13.07       0.8350
## 2    30     22.26       0.8094
## 3    40     28.03       0.9040
## 4    50     42.53       0.7314
##
##
## Elapsed time : 0.03070617
```

Stochastic Volatility (SV) Model Melino and Turnbull (1990); Harvey, Ruiz, and Shephard (1994); Jacquier, Polson, and Rossi (2002)

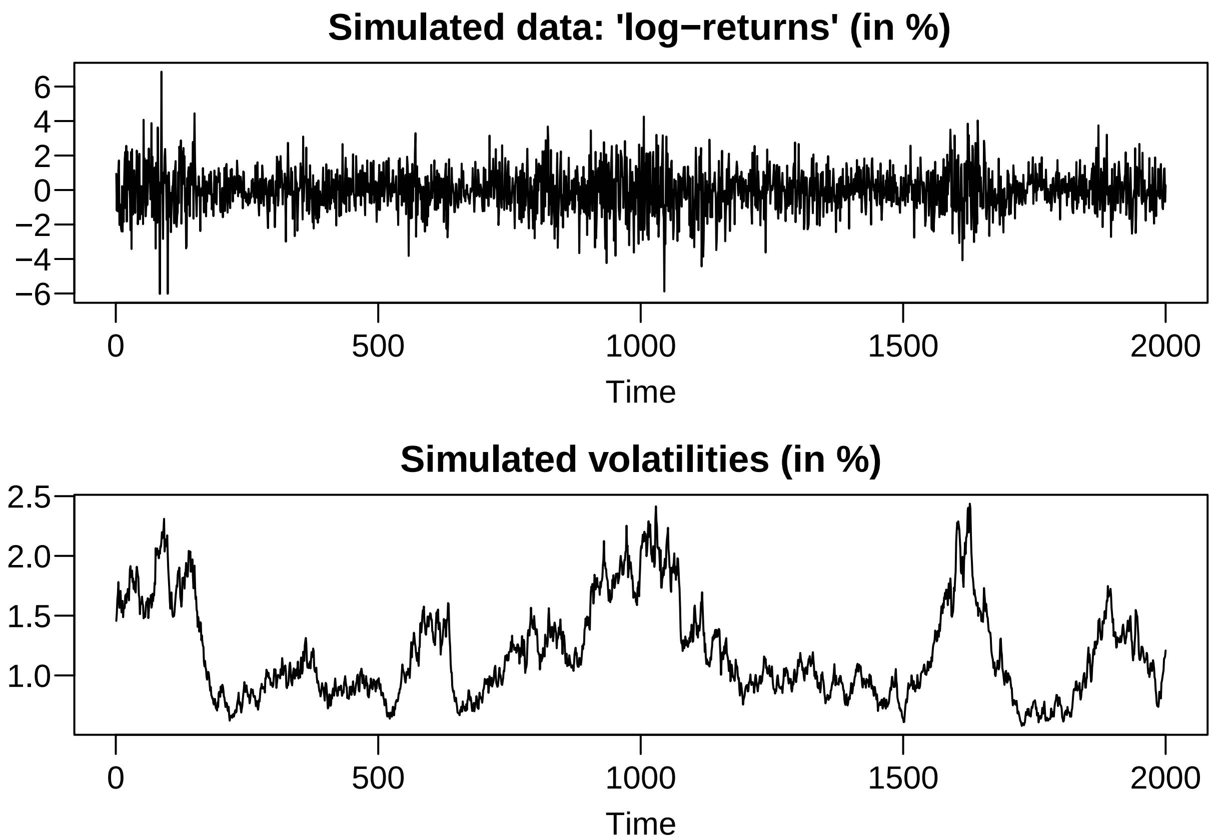
Simulation

Let's simulate realization from a stochastic volatility model where $\log(\sigma_t)$ follows an AR(1) process. That is

$$(1 - \phi B)\log(\sigma_t^2) = \mu + \nu_t,$$

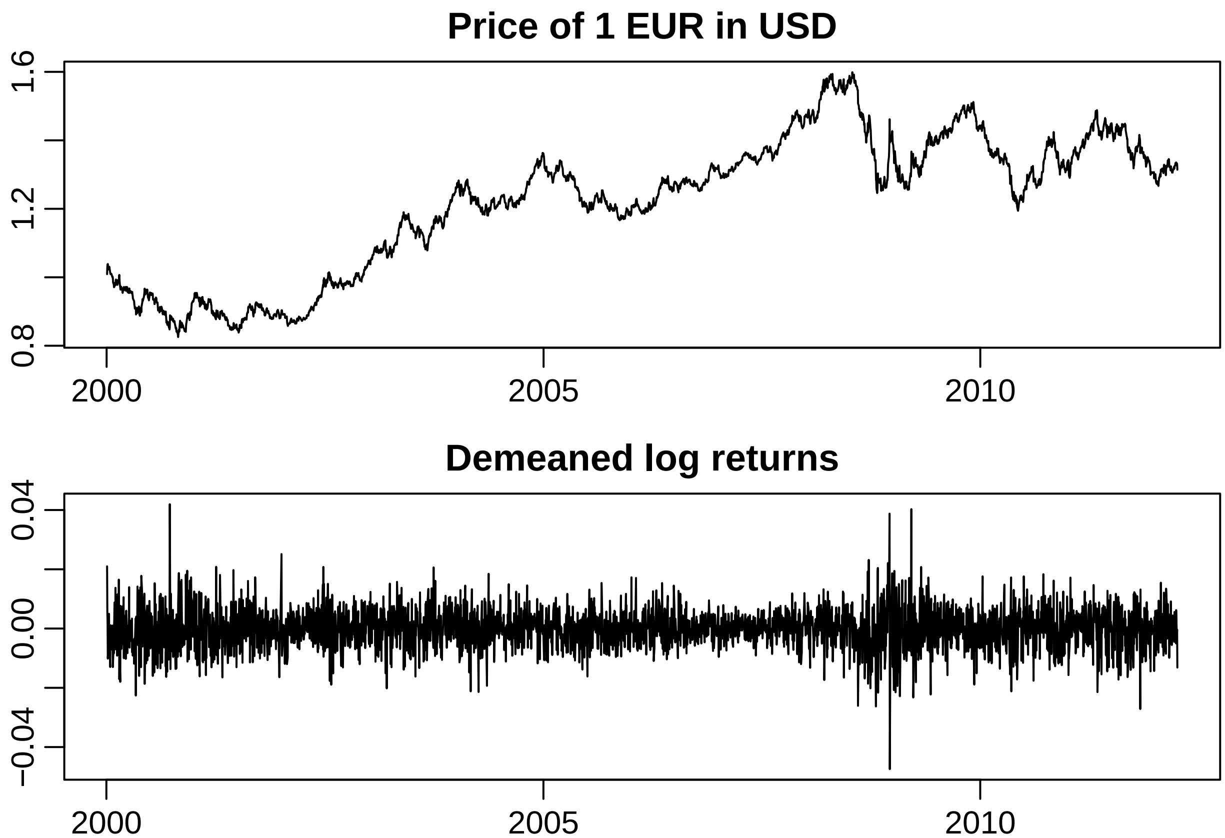
where $\nu_t \sim \mathrm{N}(0, \sigma_\nu^2)$.

```r
library(stochvol)
sim <- svsim(2000, mu = -9, phi = 0.99, sigma = 0.1)
par(mfrow = c(2, 1), las = 1)
plot(sim)
```



Euro exchange rate example

```
data(exrates)
# Computes the Log Returns
ret <- logret(exrates$USD, demean = TRUE)
par(mfrow = c(2, 1), mar = c(1.9, 1.9, 1.9, 0.5), mgp = c(2, 0.6, 0))
plot(exrates$date, exrates$USD, type = "l", main = "Price of 1 EUR in USD")
plot(exrates$date[-1], ret, type = "l", main = "Demeaned log returns")
```



Perform Markov Chain Monte Carlo (MCMC) sampling for the Stochastic Volatility (SV) Model

Prior

- $\pi(\mu) \sim N(-10, 1)$
- $\pi(\phi) \sim \text{Beta}(20, 1.1)$

```
res <- svsample(ret, priormu = c(-10, 1), priorphi = c(20, 1.1), priorsigma = 0.1)
```

```
## Done!
```

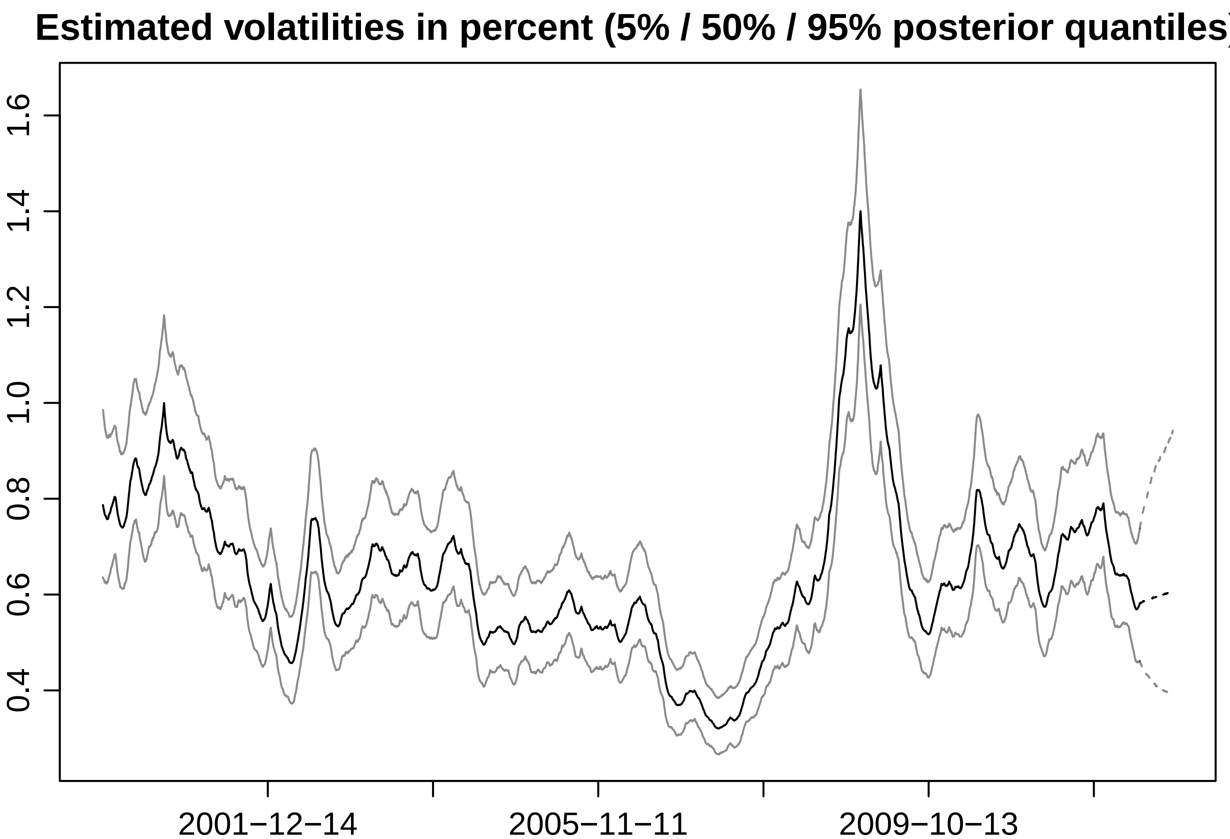
```
## Summarizing posterior draws...
```

```
summary(res, showlatent = FALSE)
```

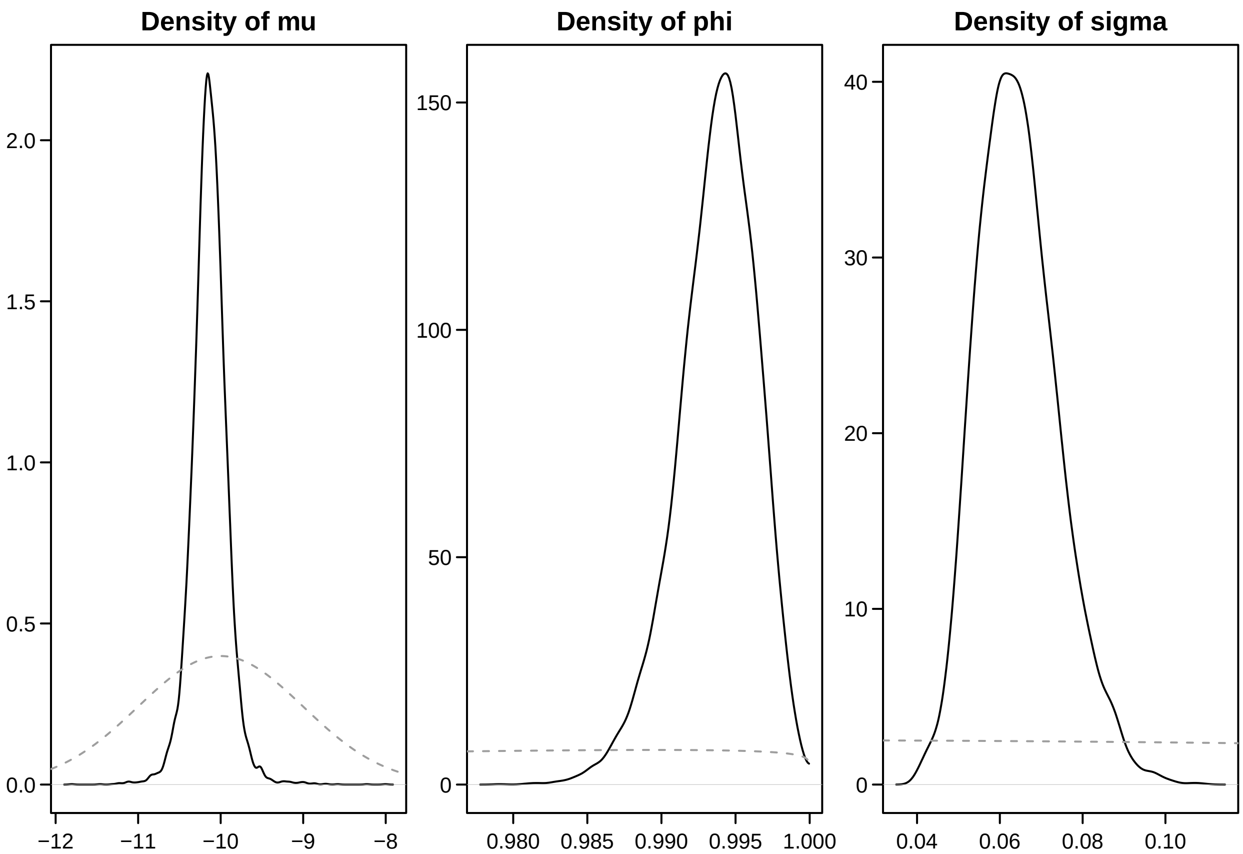
```
##
## Summary of 'svdraws' object
##
## Prior distributions:
## mu         ~ Normal(mean = -10, sd = 1)
## (phi+1)/2  ~ Beta(a = 20, b = 1.1)
## sigma^2    ~ Gamma(shape = 0.5, rate = 5)
## nu         ~ Infinity
```

```
## rho          ~ Constant(value = 0)
##
## Stored 10000 MCMC draws after a burn-in of 1000.
## No thinning.
##
## Posterior draws of SV parameters (thinning = 1):
##                  mean      sd        5%       50%      95%  ESS
## mu          -10.1343 0.21782 -10.4657 -10.1377 -9.8059 4836
## phi           0.9937 0.00268   0.9889   0.9939  0.9977  330
## sigma         0.0645 0.00981   0.0503   0.0637  0.0821  151
## exp(mu/2)     0.0063 0.00072   0.0053   0.0063  0.0074 4836
## sigma^2       0.0043 0.00132   0.0025   0.0041  0.0067  151
```

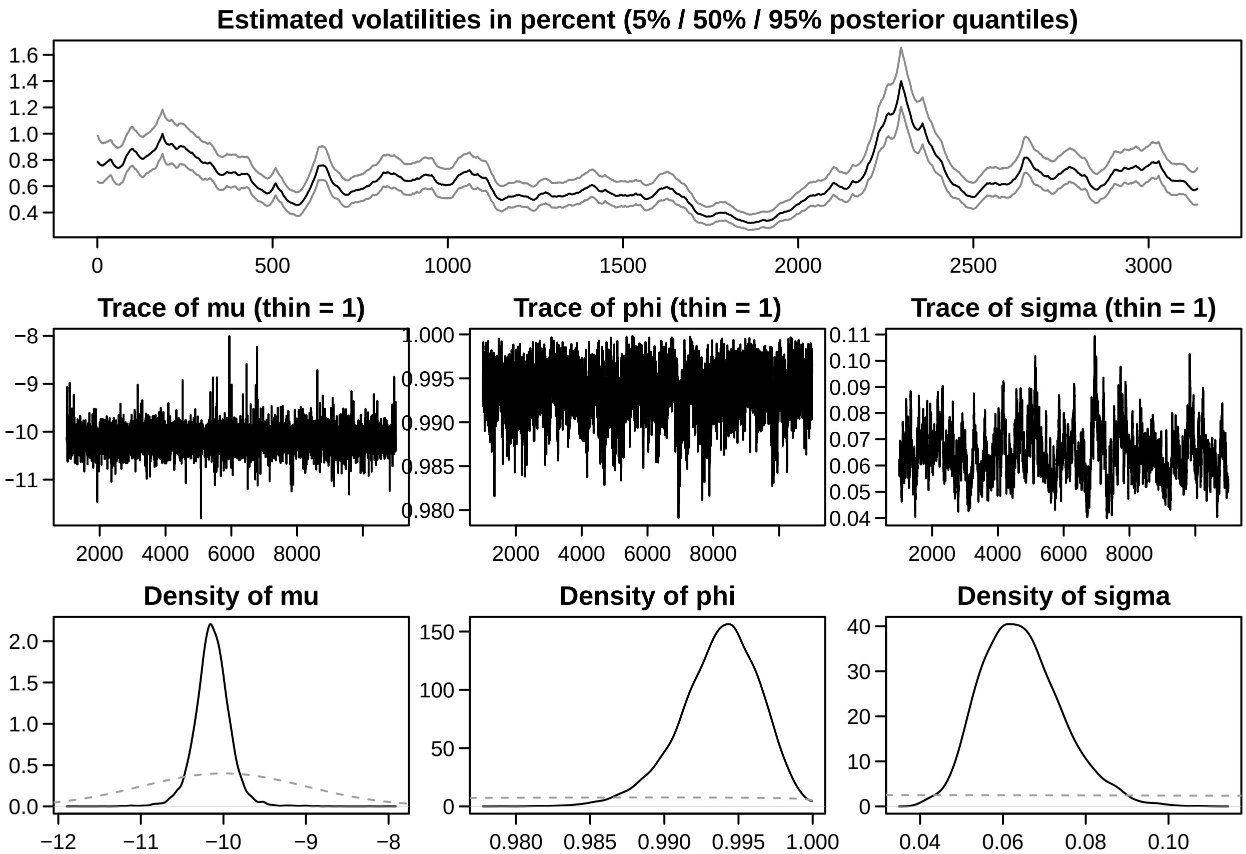
```
volplot(res, forecast = 100, dates = exrates$date[-1])
```



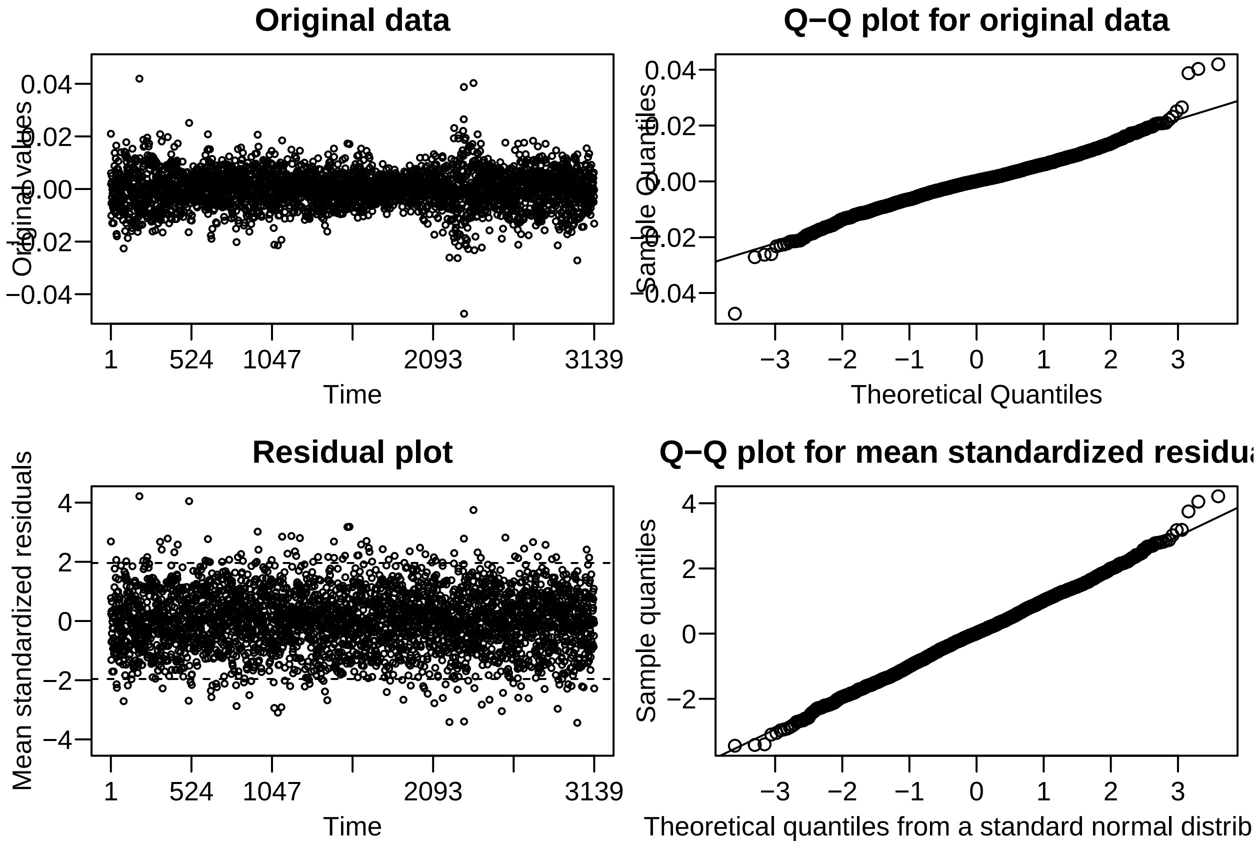

```
par(mfrow = c(1, 3), las = 1, mar = c(3.5, 4, 1, 0.5))
paradensplot(res, showobs = FALSE, cex = 0.5)
```




```
plot(res, showobs = FALSE, cex = 0.5)
```




```r
myresid <- resid(res)
plot(myresid, ret, cex = 0.5)
```



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