Lecture 8

Introduction to Probability II

Text: Chapter 4

STAT 8010 Statistical Methods I
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Notes

Agenda

Notes

Intersection and Union

- Intersection: the intersection of two events A and B, denoted by $A \cap B$, is the event that contains all outcomes of A that also belong to $B \Rightarrow$ AND

  Example: Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 5\}$, then $A \cap B = \{1, 2\}$
- Union: the union of two events A and B, denoted by $A \cup B$, is the event of all outcomes that belong to either A or $B \Rightarrow$ OR

  Example: Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$

Notes

Example

Suppose we flipped 3 fair coins. Let A be the event of **exactly 2 tails**. Let B be the event that the **first 2 tosses are tails**. Let C be the event that **all 3 tosses are tails**. What are $A \cap B$, $A \cup C$, and $(A \cap B) \cup C$?

Solution.

$$A = \{(T, T, H), (T, H, T), (H, T, T)\}$$
$$B = \{(T, T, T), (T, T, H)\}$$
$$C = \{T, T, T\}$$

1. $A \cap B = \{(T, T, H)\}$
2. $A \cup C = \{(T, T, H), (T, H, T), (H, T, T), (T, T, T)\}$
3. $(A \cap B) \cup C =$
   $\{T, T, H\} \cup \{T, T, T\} = \{(T, T, H), (T, T, T)\}$

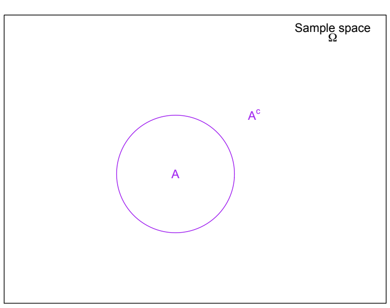
Notes

Logical Relationships among Events

- Mutually exclusive: refers to two (or more) events that cannot both occur when the random experiment is formed.
  $$A \cap B = \emptyset$$
- Exhaustive: refers to event(s) that comprise the sample space.
  $$A \cup B = \Omega$$
- Partition: events that are both mutually exclusive and exhaustive.
  $$A \cap B = \emptyset \quad \text{and} \quad A \cup B = \Omega$$

Notes

Complement



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Complement Rule

1. By the definition of complement
$$A \cup A^c = \Omega$$
2. Apply the probability operator
$$\mathbb{P}(A \cup A^c) = \mathbb{P}(\Omega) = 1$$
3. Since A and A^c are mutually exclusive
$$\mathbb{P}(A \cup A^c) = \mathbb{P}(A) + \mathbb{P}(A^c)$$
4. Hence we get $\boxed{\mathbb{P}(A) = 1 - \mathbb{P}(A^c)}$

Notes

Example

Suppose we rolled a fair, six–sided die 10 times. Let T be the event that we roll at least 1 three. If one were to calculate T you would need to find the probability of 1 three, 2 threes, $\cdots$, and 10 threes and add them all up. However, you can use the complement rule to calculate $\mathbb{P}(T)$

Solution.

Let X be the times that we rolled a 3, then
$\mathbb{P}(T) = \mathbb{P}(X \geq 1) =$
$\underbrace{\mathbb{P}(X = 1) + \mathbb{P}(X = 2) + \cdots + \mathbb{P}(X = 10)}_{\text{need to compute 10 probabilities}}$
If we apply the complement rule
$\mathbb{P}(T) = 1 - \mathbb{P}(T^c) = 1 - \mathbb{P}(X = 0)$

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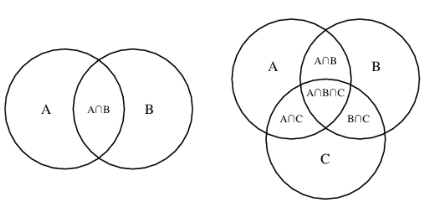
Venn Diagram

A Venn diagram is a diagram that shows all possible logical relations between a finite collection of events.



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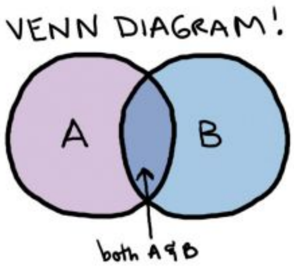
General Addition Rule

The general addition rule is a way of finding the probability of a union of 2 events. It is
$\boxed{\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)}$



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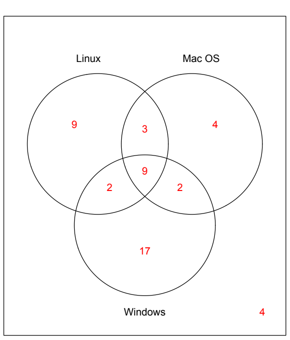
Example

Three of the major commercial computer operating systems are Windows, Mac OS, and Red Hat Linux Enterprise. A Computer Science professor selects 50 of her students and asks which of these three operating systems they use. The results for the 50 students are summarized below.

- 30 students use Windows
- 16 students use at least two of the operating systems
- 9 students use all three operating systems
- 18 students use Mac OS
- 46 students use at least one of the operating systems
- 11 students use both Windows and Linux
- 11 students use both Windows and Mac OS

Notes

Example cont'd



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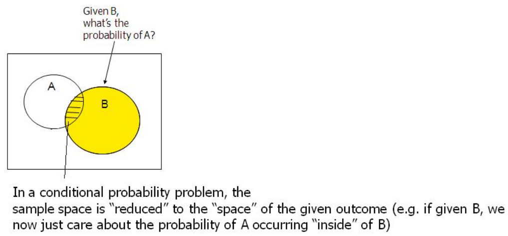
Conditional Probability

Let A and B be events. The probability that event A occurs given (knowing) that event B occurs is called a conditional probability and is denoted by $P(A|B)$. The formula of conditional probability is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



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Summary

In this lecture, we learned

- Union , Intersection
- **Logical Relationships among Events** Mutually Exclusive, Exhaustive, Partition
- Complement Rule and General Addition Rule
- Venn Diagram

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