Estimating Precipitation Extremes using Log-Histospline

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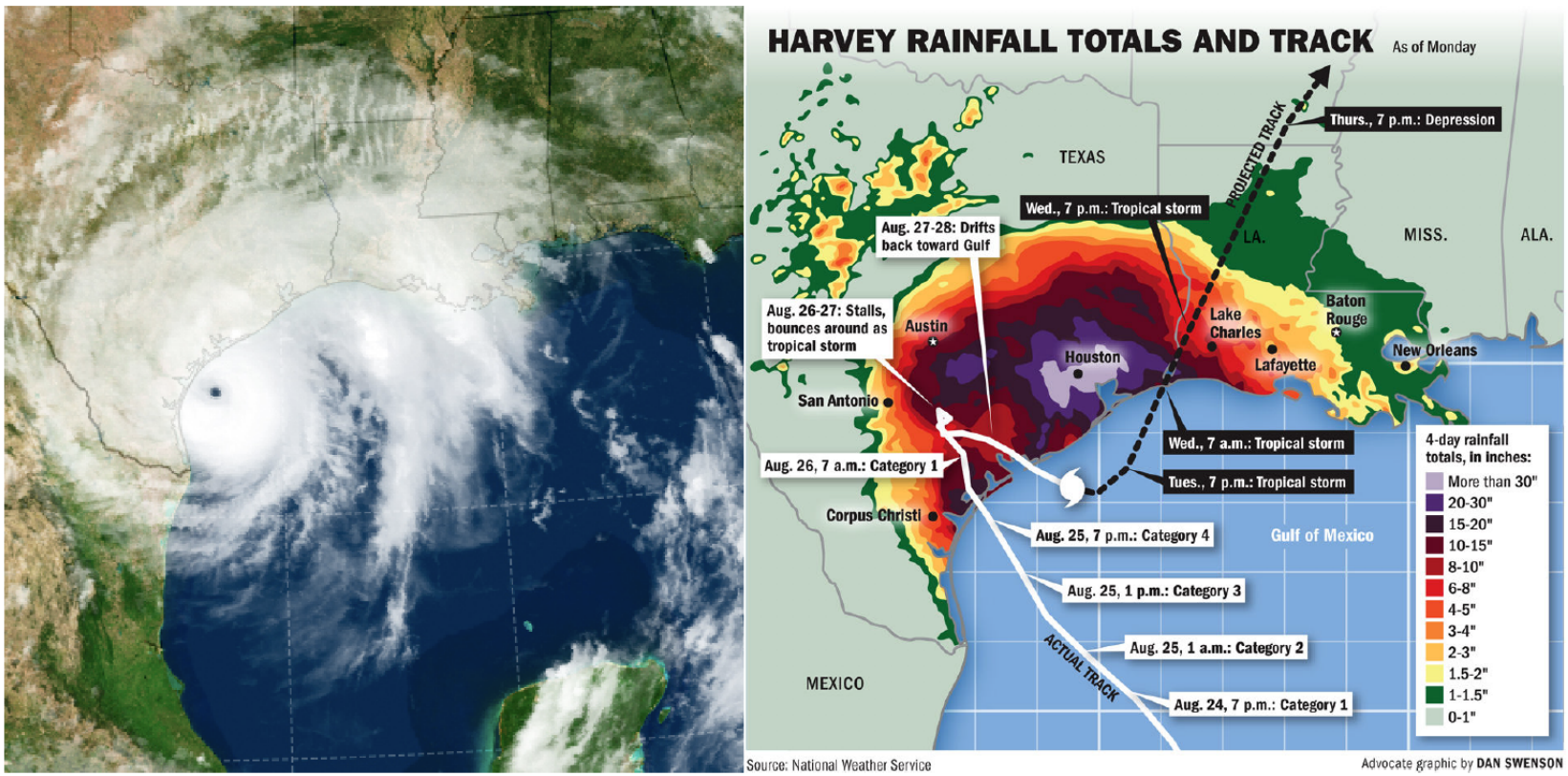
University of Georgia, August 29, 2019

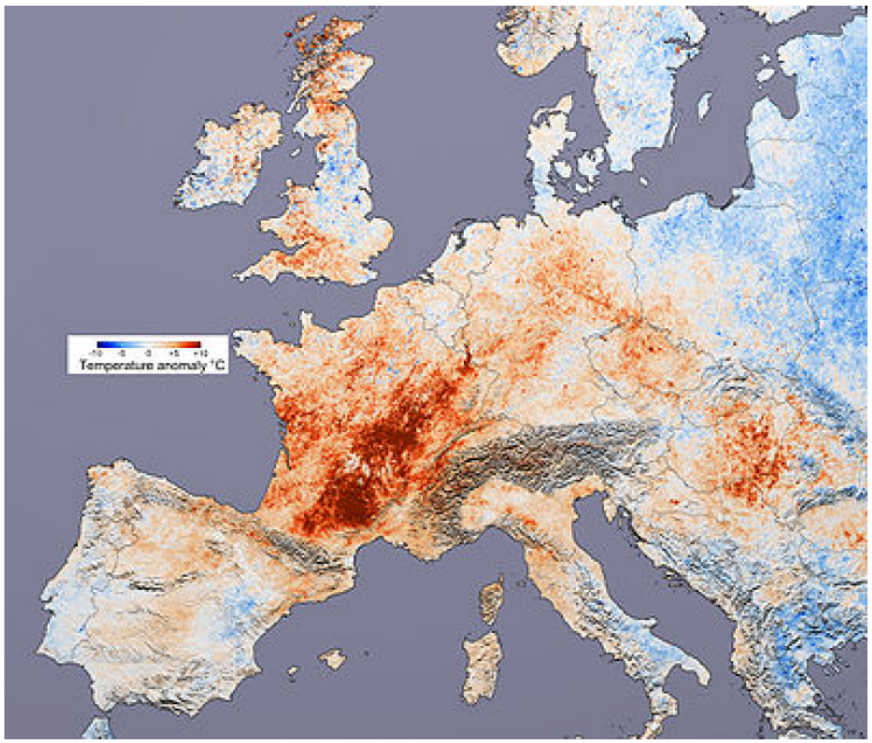
Extreme rainfall during Hurricane Harvey



Source: NASA (Left); National Weather Service (Right)

- *"A storm forces Houston, the limitless city, to consider its limits"* – The New York Times (8.31.17)

Environmental extremes: Heatwaves, storm surges, etc.



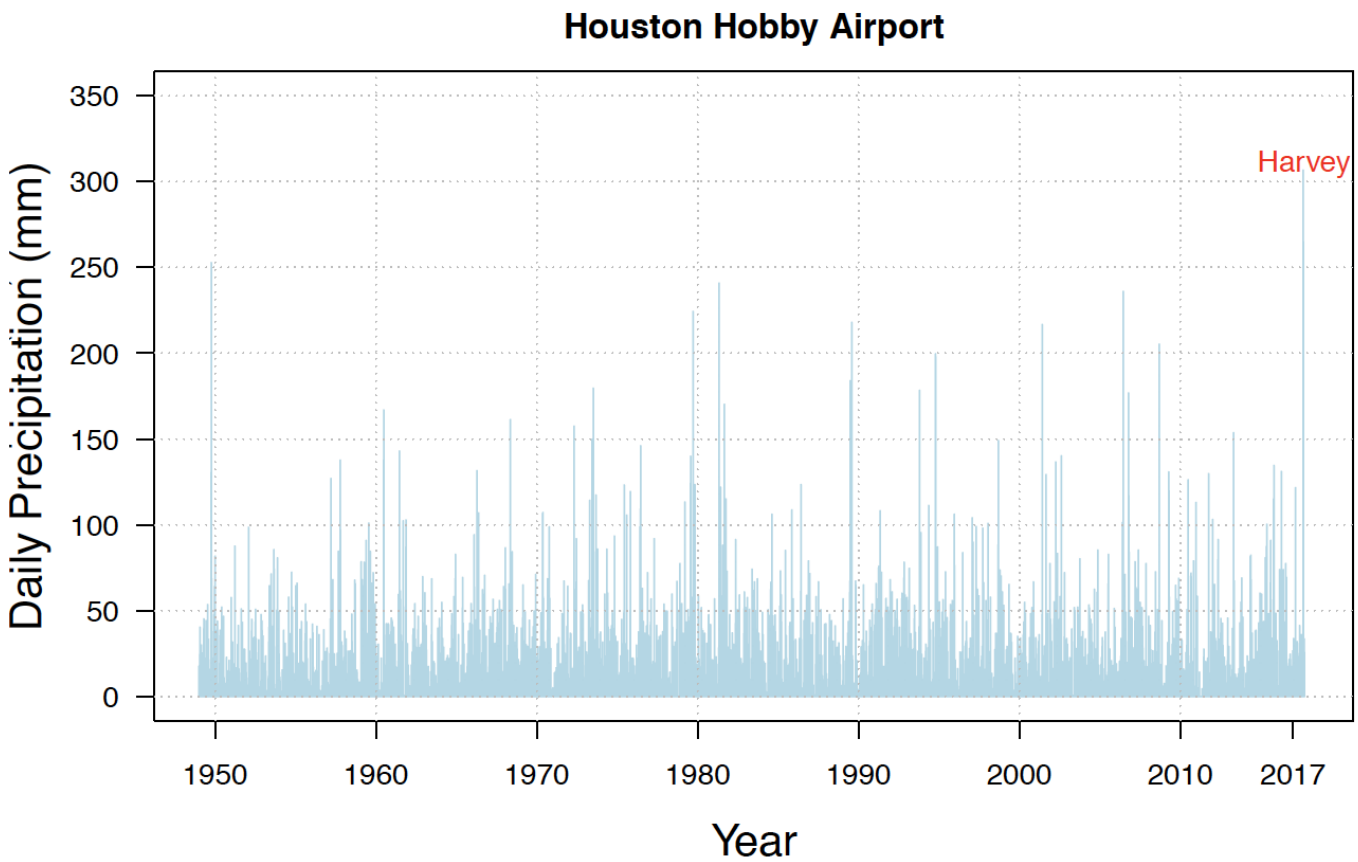
- **Heat wave**: The 2003 European heat wave led to the hottest summer on record in Europe since 1540 that resulted in at least 30,000 deaths

- **Storm Surge**: Hurricane Katrina produced the highest storm surge ever recorded (27.8 feet) on the U.S. coast

Big picture of the talk

- Scientific question: How to estimate the magnitude of extreme events (e.g. 100-year rainfall)
- Extreme value theory provides a very useful framework for estimating extremes but requires selecting a small fraction of the "large" observations
- Log-Histospline: combine **data transformation** and **spline smoothing** to estimate the precipitation extremes using the **full range** of the observations

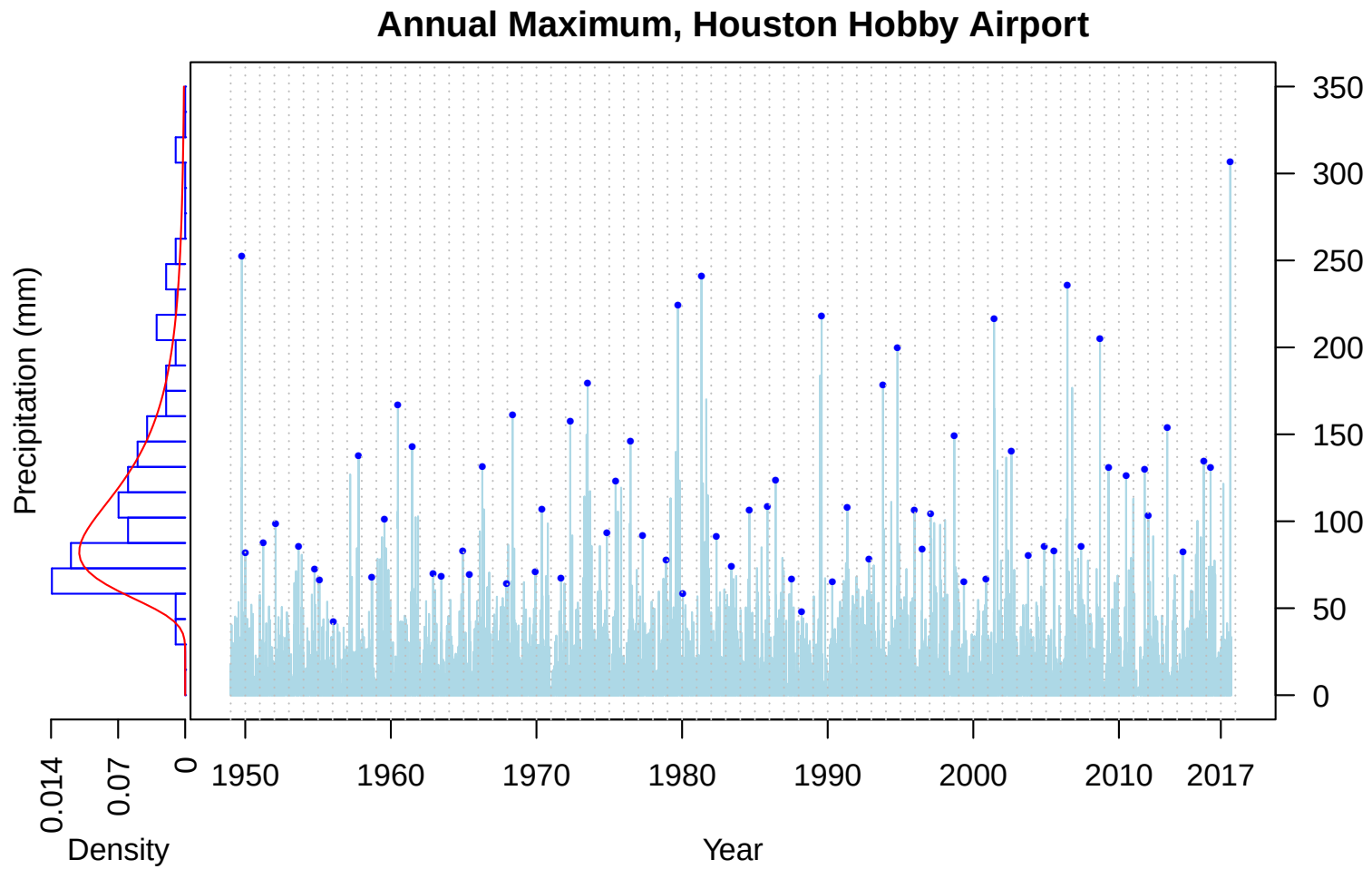
Part I: Extreme Value Analysis



Data source: Global Historical Climatology Network

Goal: To estimate the r-year return level (RL_r), the value whose probability of exceedance is $1/r$ in any given year.

Estimating extremes using block maxima (Gumbel 1958)



Which distribution to use for annual maxima?

$\Rightarrow$ generalized extreme value distribution (GEV(μ, σ, ξ))

Extremal types theorem (Fisher–Tippett 1928, Gnedenko 1943)

Define $M_n = \max\{X_1, \cdots, X_n\}$ where $X_1, \cdots, X_n \overset{\text{i.i.d.}}{\sim} F$. If $\exists\, a_n > 0$ and $b_n \in \mathbb{R}$ such that, as $n \to \infty$, if

$$\mathbb{P}\left(\frac{M_n - b_n}{a_n} \leq x\right) \xrightarrow{d} \mathsf{G}(x)$$

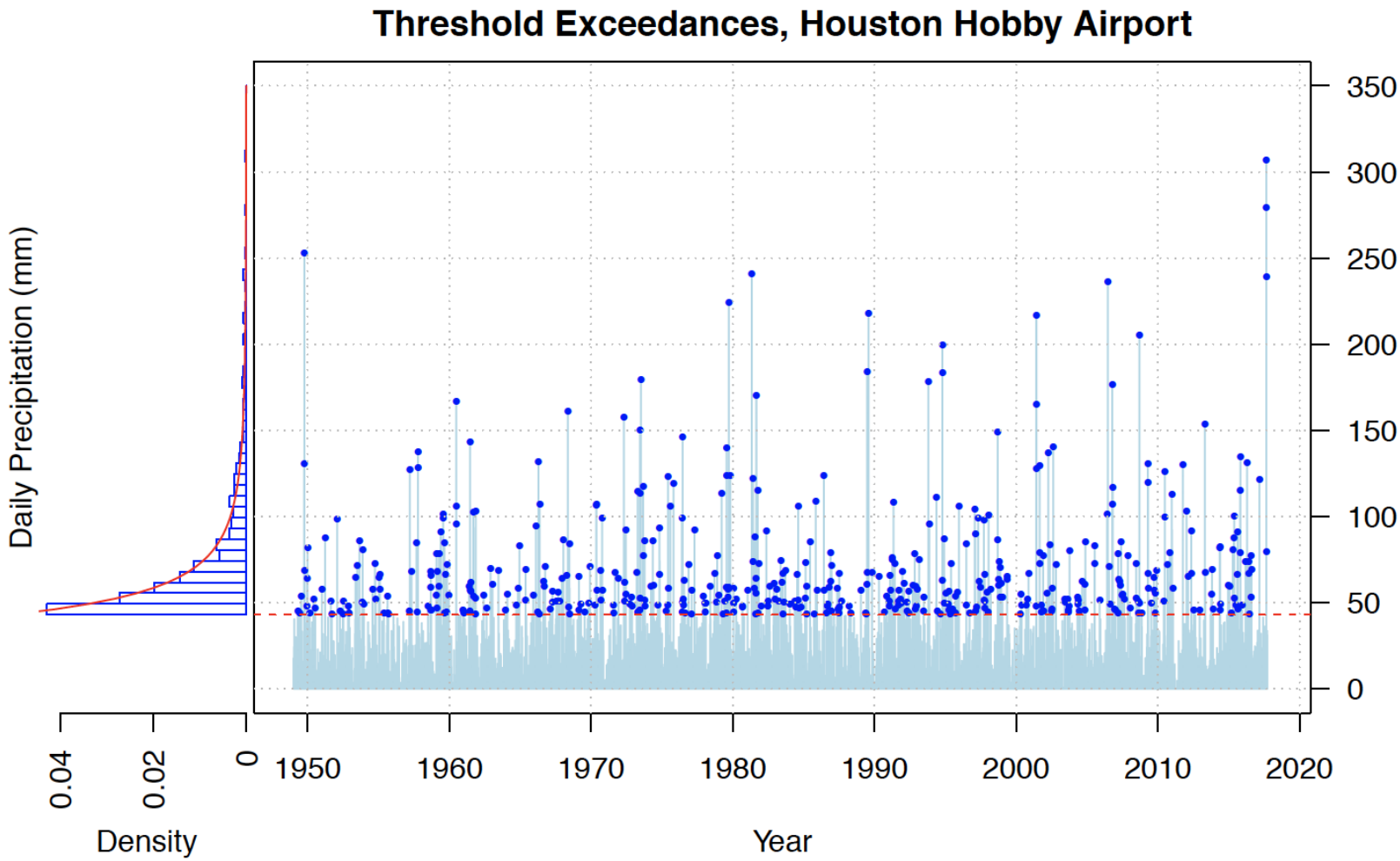
then G must be the same type of the following form:

$$\mathsf{G}(x; \mu, \sigma, \xi) = \exp\left\{ -\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]_+^{\frac{-1}{\xi}} \right\}$$

where $x_+ = \max(x, 0)$ and $G(x)$ is the distribution function of the generalized extreme value distribution (GEV)

- μ and σ are location and scale parameters
- ξ is a shape parameter determining the rate of tail decay, with
  - $\xi > 0$ giving the heavy-tailed case (Fréchet)
  - $\xi = 0$ giving the light-tailed case (Gumbel)
  - $\xi < 0$ giving the bounded-tailed case (reversed Weibull)

Peaks–over–threshold (POT) method [Davison & Smith 1990]



Which distribution to use for threshold exceedances?

$\Rightarrow$ generalized Pareto distribution ($\mathrm{GPD}_u(\tilde{\sigma}, \xi)$)

Pickands–Balkema–de Haan theorem (1974, 1975)

If $M_n = \max_{1\le i\le n}\{X_i\} \approx \text{GEV}(\mu,\sigma,\xi)$, then, for a "large" u (i.e., $u \to x_F = \sup\{x : F(x) < 1\}$), $F_u = \mathbb{P}(X - u < y | X > u)$ is well approximated by the generalized Pareto distribution (GPD). That is:

$$F_u(y) \xrightarrow{d} H_{\tilde{\sigma},\xi}(y) \qquad u \to x_F$$

where

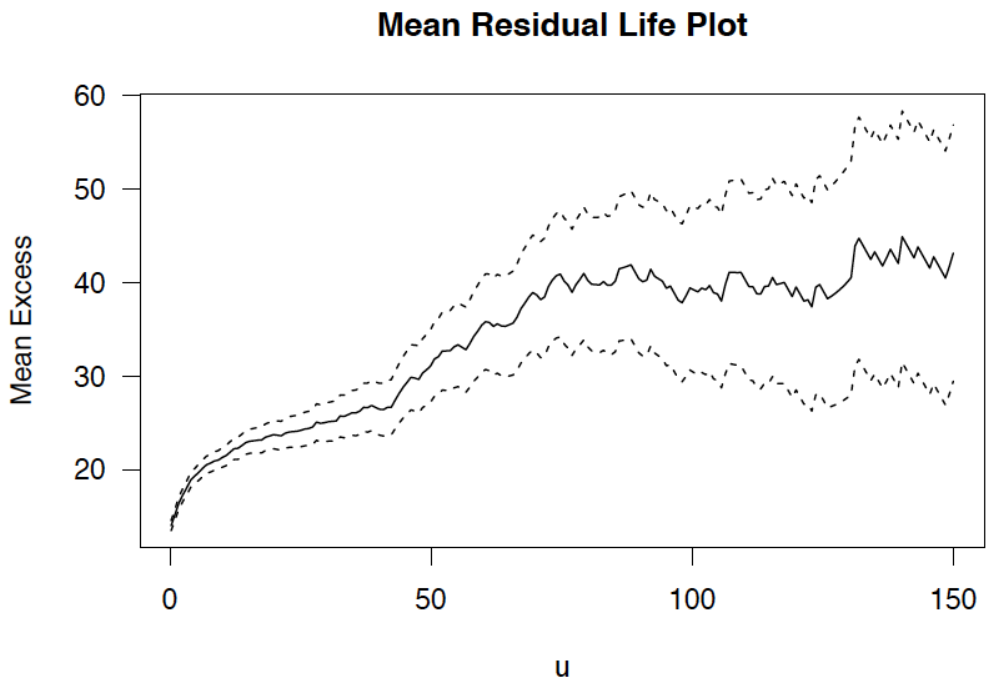
$$H_{\tilde{\sigma},\xi}(y) = \begin{cases} 1 - (1 + \xi y/\tilde{\sigma})^{-1/\xi} & \xi \neq 0; \\ \\ 1 - \exp(-y/\tilde{\sigma}) & \xi = 0. \end{cases}$$

and $\tilde{\sigma} = \sigma + \xi(u - \mu)$

Fit GPD to threshold exceedances: How to choose an "appropriate" u?

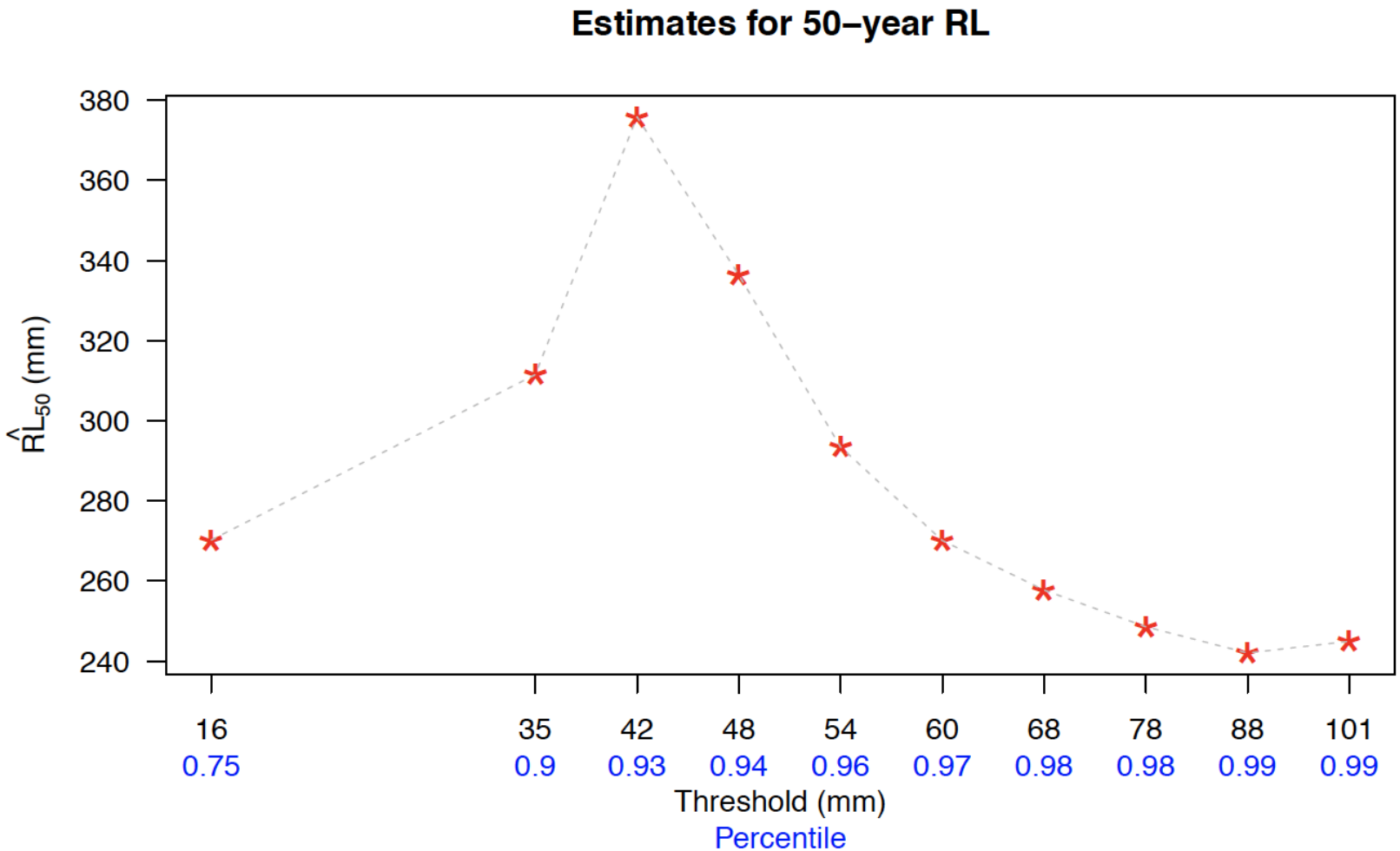
Bias–variance tradeoff:

- Threshold too low ⇒ bias because of the model asymptotics being invalid
- Threshold too high ⇒ variance is large due to few data points



Task: To choose a u_0 s.t. the Mean Residual Life curve behaves linearly $\forall u > u_0$

Return level estimate is sensitive to the chosen threshold ☹



Part II: Log-Histospline

Huang, W. K., Nychka, D. W., & Zhang, H.
Estimating precipitation extremes using the log–histospline.
Environmetrics, e2543, 2019

Motivation: To develop an alternative to POT method

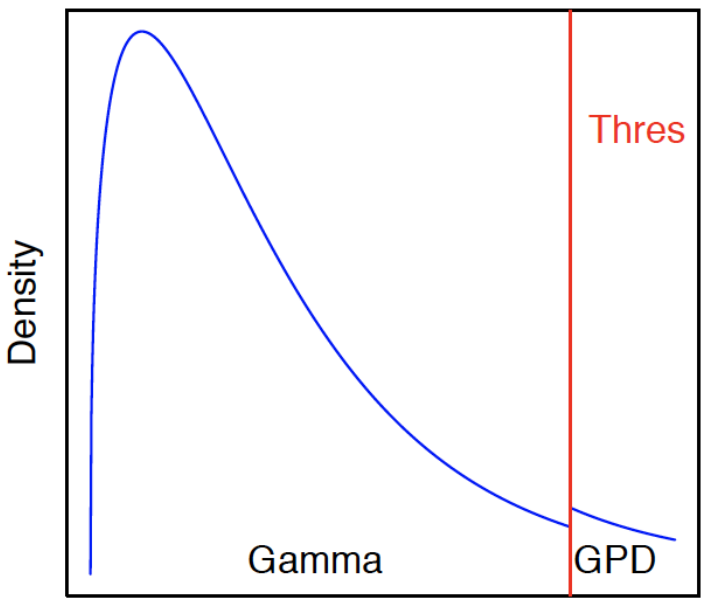
- Avoid the threshold selection
- Incorporate prior knowledge on the rainfall tail distributions
  - Polynomial upper tail behavior (i.e., $\xi > 0$)
  - Bounded lower tail

$\Rightarrow$ Model the full range of the distribution (i.e., $(0, \infty)$) while accounting for polynomial upper tail behavior $\xi > 0$ and bounded lower tail

Some existing methods for estimating extremes while modeling the full range of the distribution

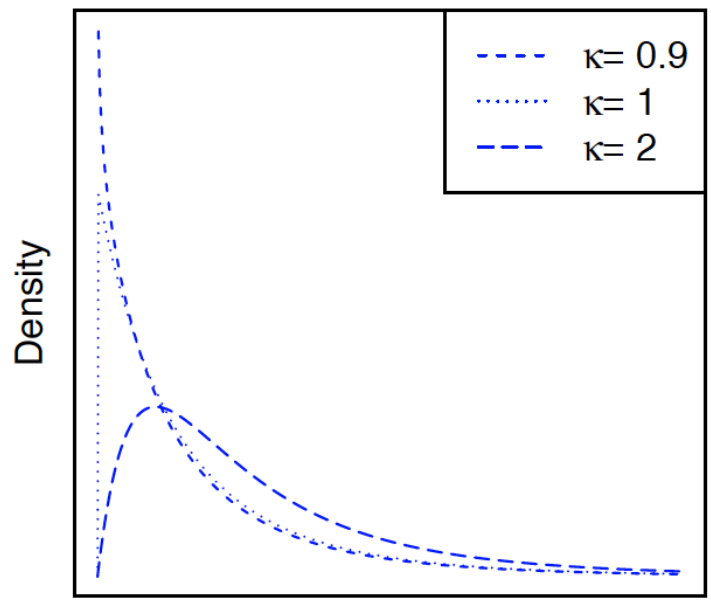
- Mixture models [e.g., Frigessi et al., 2002]

Parametric/Non-Parametric bulk+ GPD tail



- EGPD [e.g., Naveau et al., 2016]

$Y = H^{-1}_{\sigma,\xi}(G^{-1}(U))$ (e.g., $G(u) = u^{\kappa}, \quad \kappa > 0$)



Main idea: Model the log-density as a natural cubic spline

Let Y be a non-negative random variable and $X = \log(Y)$ with density $f(x), x \in \mathbb{R}$. We model the log-density $g(x) = \log f(x)$

- Working on log-density makes life easier

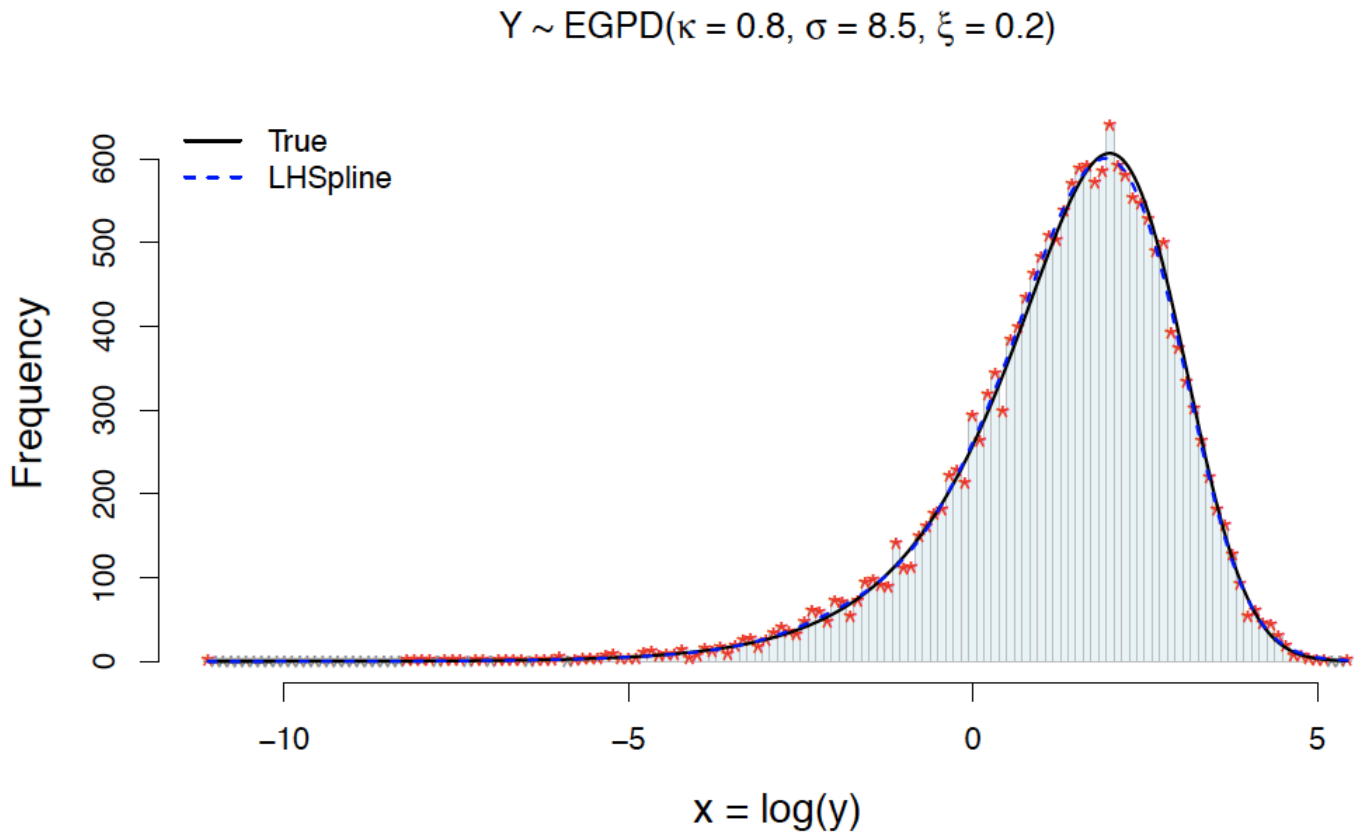
  $\Rightarrow$ remove the positivity constraint

- Estimate $g(x)$ as a flexible (cubic) spline function
  - $g(x)$ is a cubic polynomial for $(x_k, x_{k+1}]$, $k = 1, \cdots, K$
  - $g(x)$ extrapolates linearly

- Unlogged variable Y will have a polynomial tail i.e. $\xi > 0$

  linear tail in $g \rightarrow$ exponential tail in $X \rightarrow$ **polynomial tail** in Y ☺

Log-Histospline (LHSpline): Smooth the histogram by fitting a (penalized) Possion regression [Efron & Tibshirani 1996]



Assumption: $Z(x_j) \overset{ind}{\sim} \text{Poi}(\tilde{f}(x_j)), j = 1, \cdots, N$, where $Z(x_j)$ is the j$^{\text{th}}$ bin count. We use a **penalized Possion regression** to estimate $g(x), x \in (-\infty, \infty)$, the log-density

Estimating (log)-density by maximizing penalized likelihood

[Good and Gaskins, 1971, R. Tapia, 1978, Silverman, 1982]

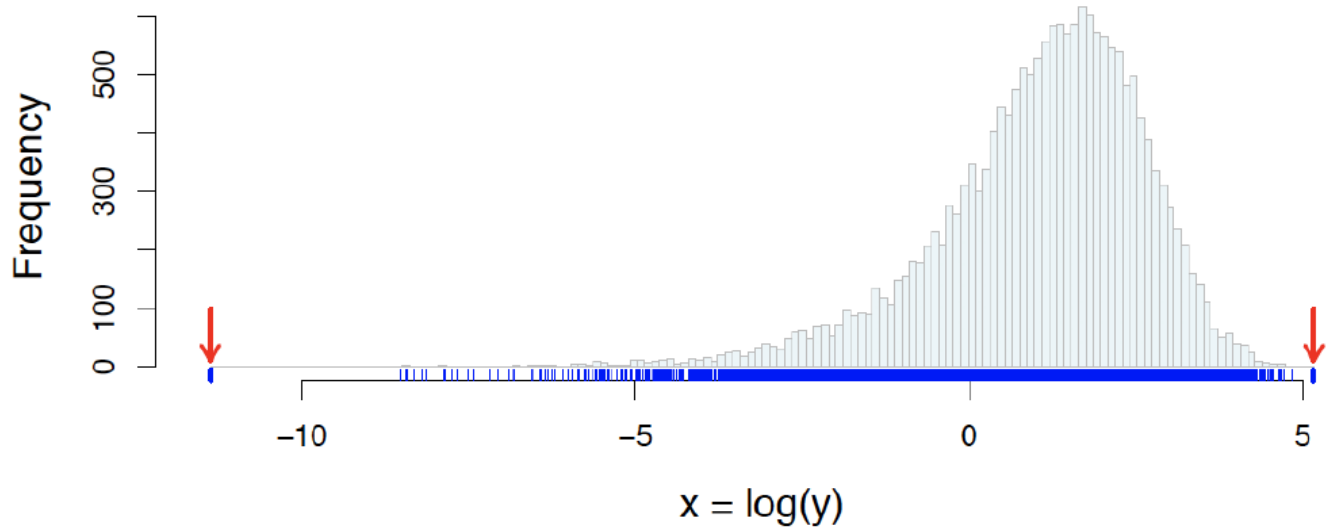
Penalized negative log likelihood:

$$-\sum_{j=1}^{N}\left\{\tilde{g}_j z_j - e^{\tilde{g}_j} - \log(z_j!)\right\} + \lambda\left(\int_{x\in\mathbb{R}}\left(\tilde{g}''(x)\right)^2\,dx\right)$$

where λ is the smoothing parameter

- The minimizer $\hat{g}_\lambda(x)$ is a natural cubic spline
- λ is chosen by cross validation (CV) (more on this in next slide)
- Bayesian interpretation: $\hat{g}_\lambda$ is the posterior mode with the prior proportional to $\lambda \int {g''}^2$

Bias correction for removing edge effects



These "bumps" causes overestimation in extreme upper quantiles

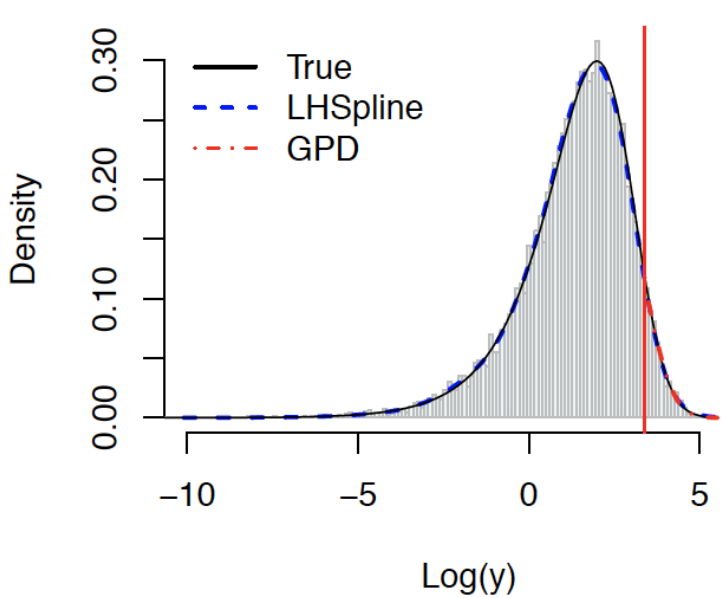
- Extend the histogram beyond the data range by exploiting the equivalence of density and intensity estimation for Poisson processes, and
  1. **Bootstrap for bias correction**: Use the "parametric" bootstrap to estimate/correct the bias
  2. **Adjust the smoothing parameter** λ: The λ chosen by CV maybe be too "large" to explore the tail features

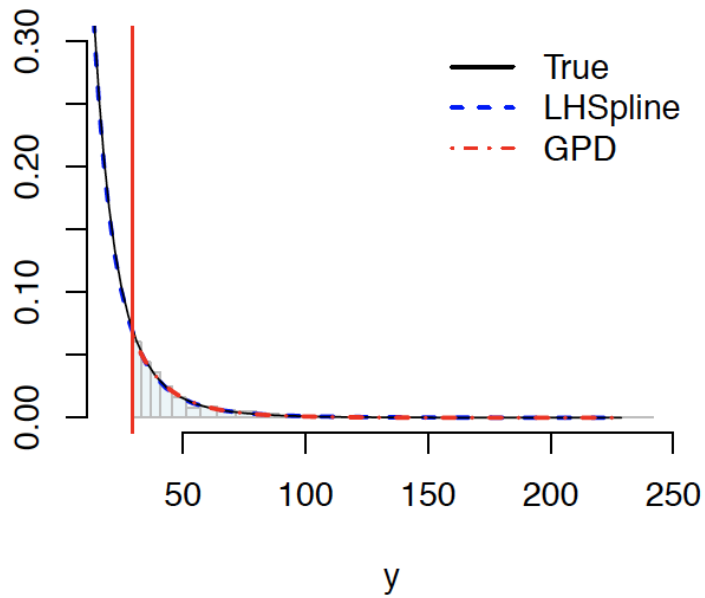
Simulation Study

Set-up:

- Generate 100 data sets each with 50 years ($n = 18{,}250$) of observations that follows EGPD($\kappa = 0.8, \sigma = 8.5, \xi = 0.2$)

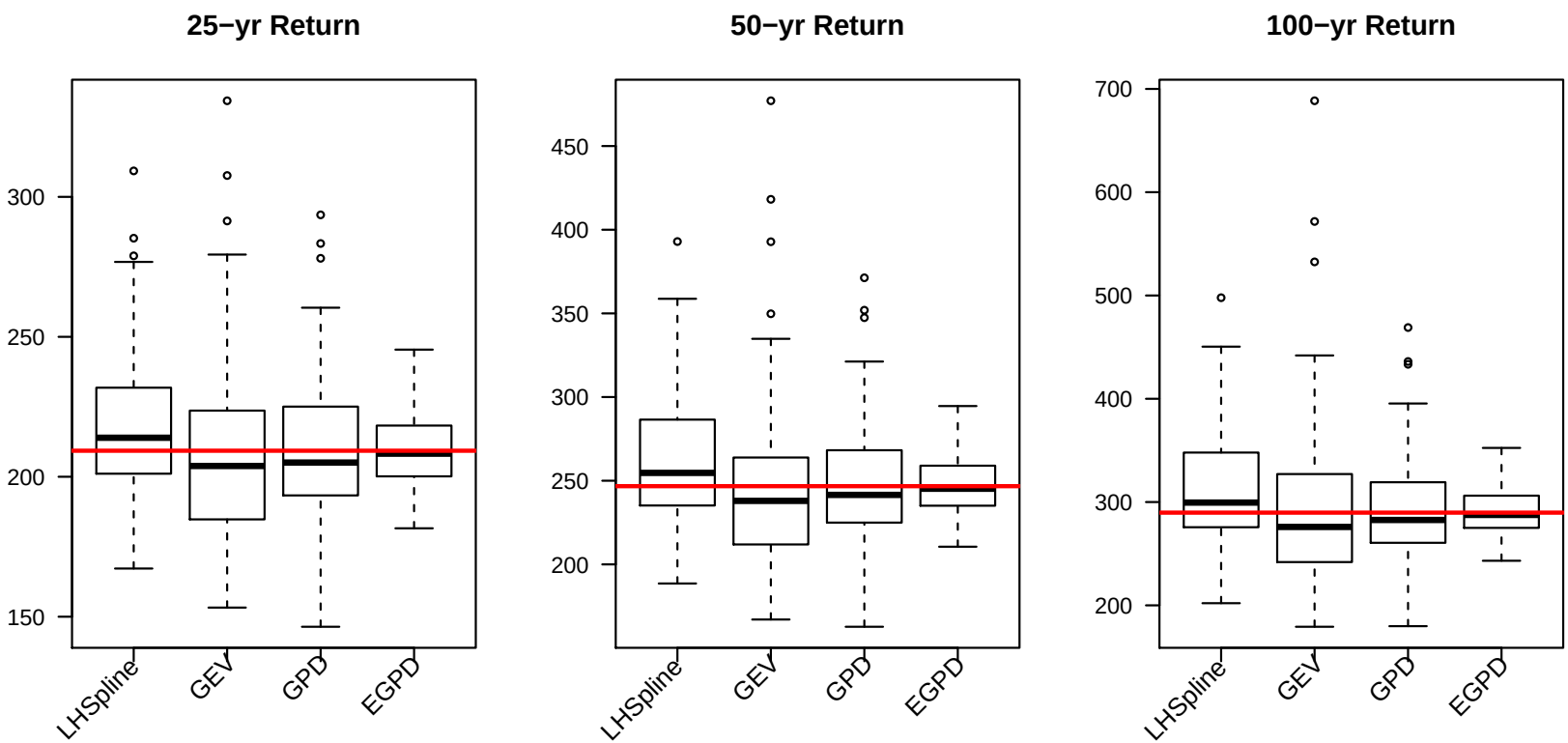
- Estimate return levels using LHSpline and compare the estimator performance with the GEV/GPD





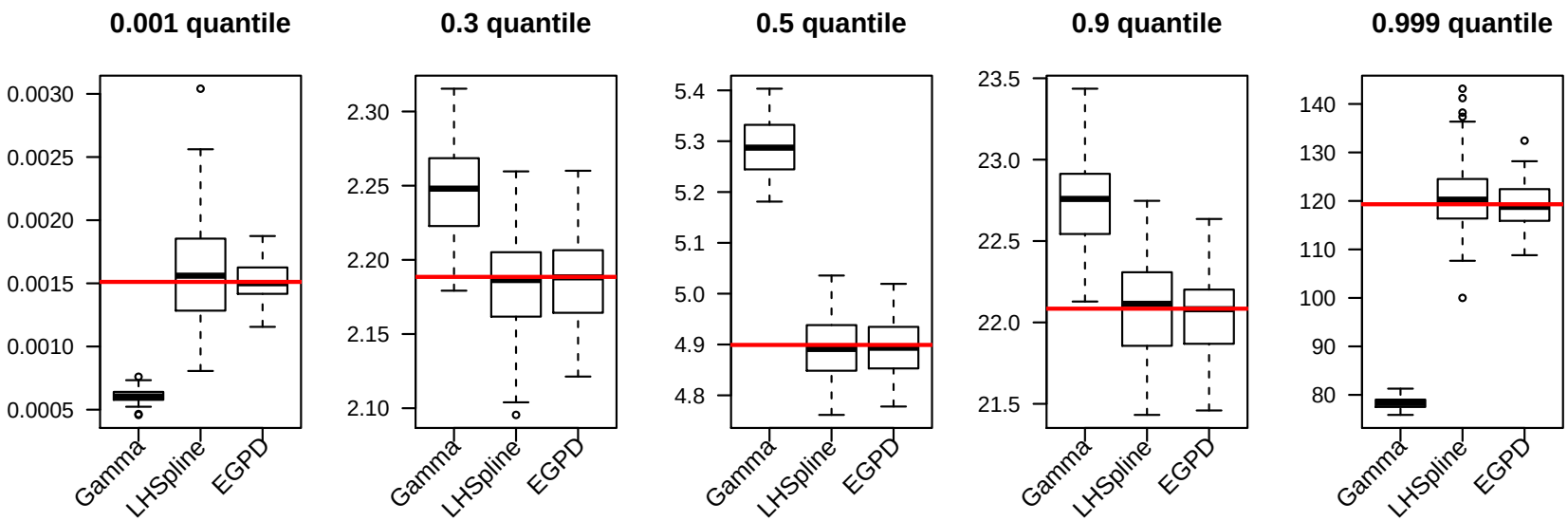
Return level estimates

- LHSpline Fit Log-Histospline to the full range of the log-transformed data
- GEV: Fit GEV to "annual maxima"
- GPD: Fit GPD to threshold exceedances
- EGPD: Fit EGPD to the full range of the data ("Oracle")



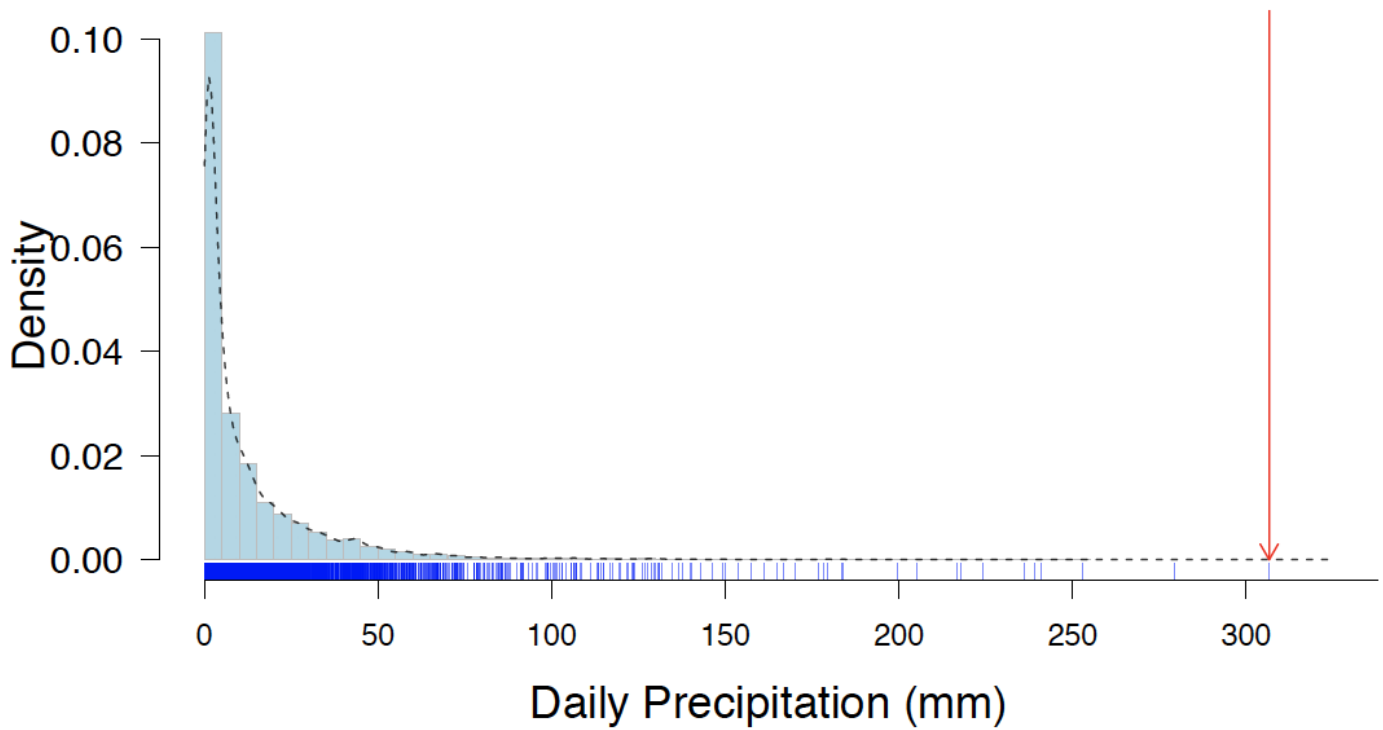
Quantile estimates

- LHSpline Fit Log-Histospline to the full range of the log-transformed data
- Gamma: Fit Gamma to the full range of the data [Wilks, 2011]
- EGPD: Fit EGPD to the full range of the data ("Oracle")

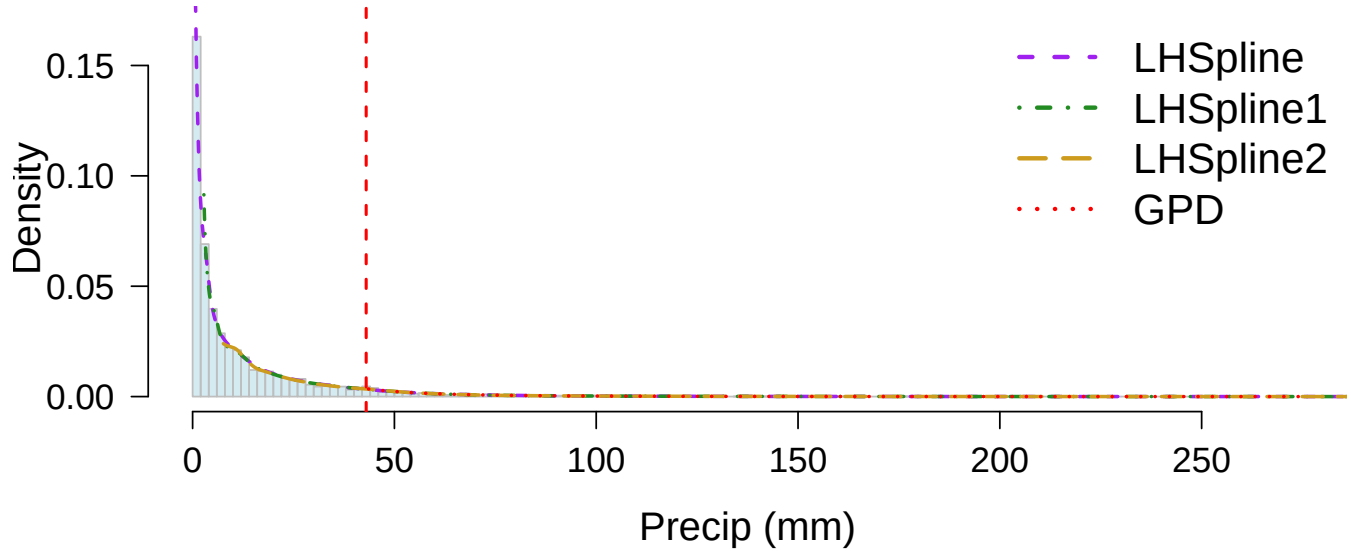


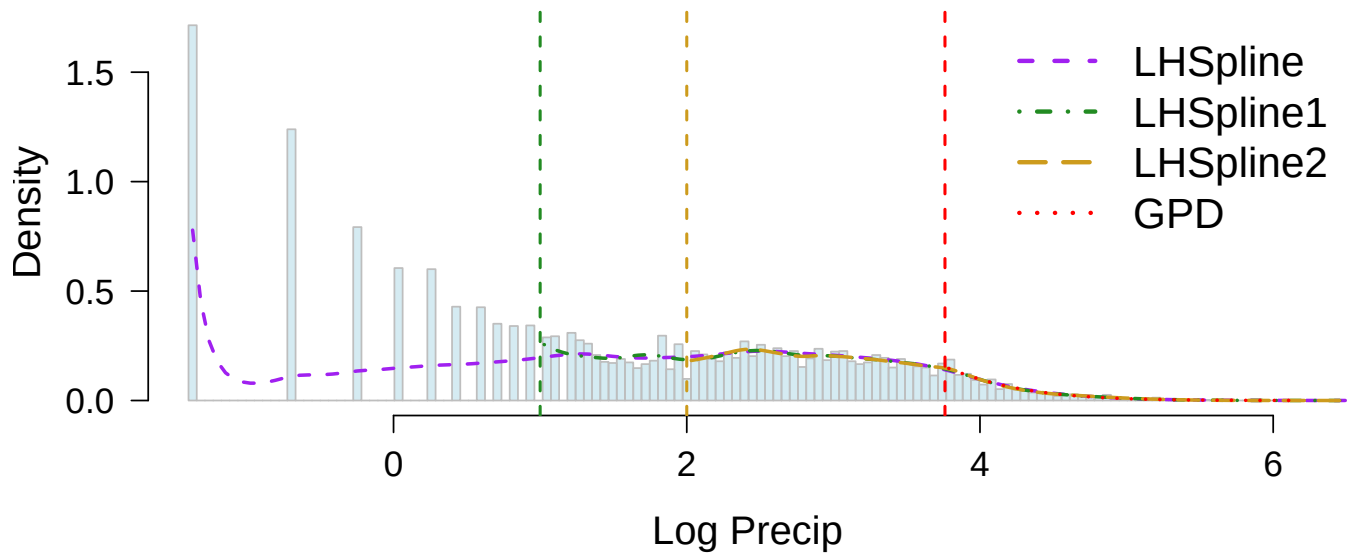
Part III: Houston Precipitation Extremes Revisited

- Use the daily rainfall data (wet day only $\sim 28\%$) from 1949 - 2016 for the model fitting
- **Question**: *How unusual was the event of 306.58 mm on 26th August, 2017?*



LHSpline fits to Houston Hobby Airport rainfall data





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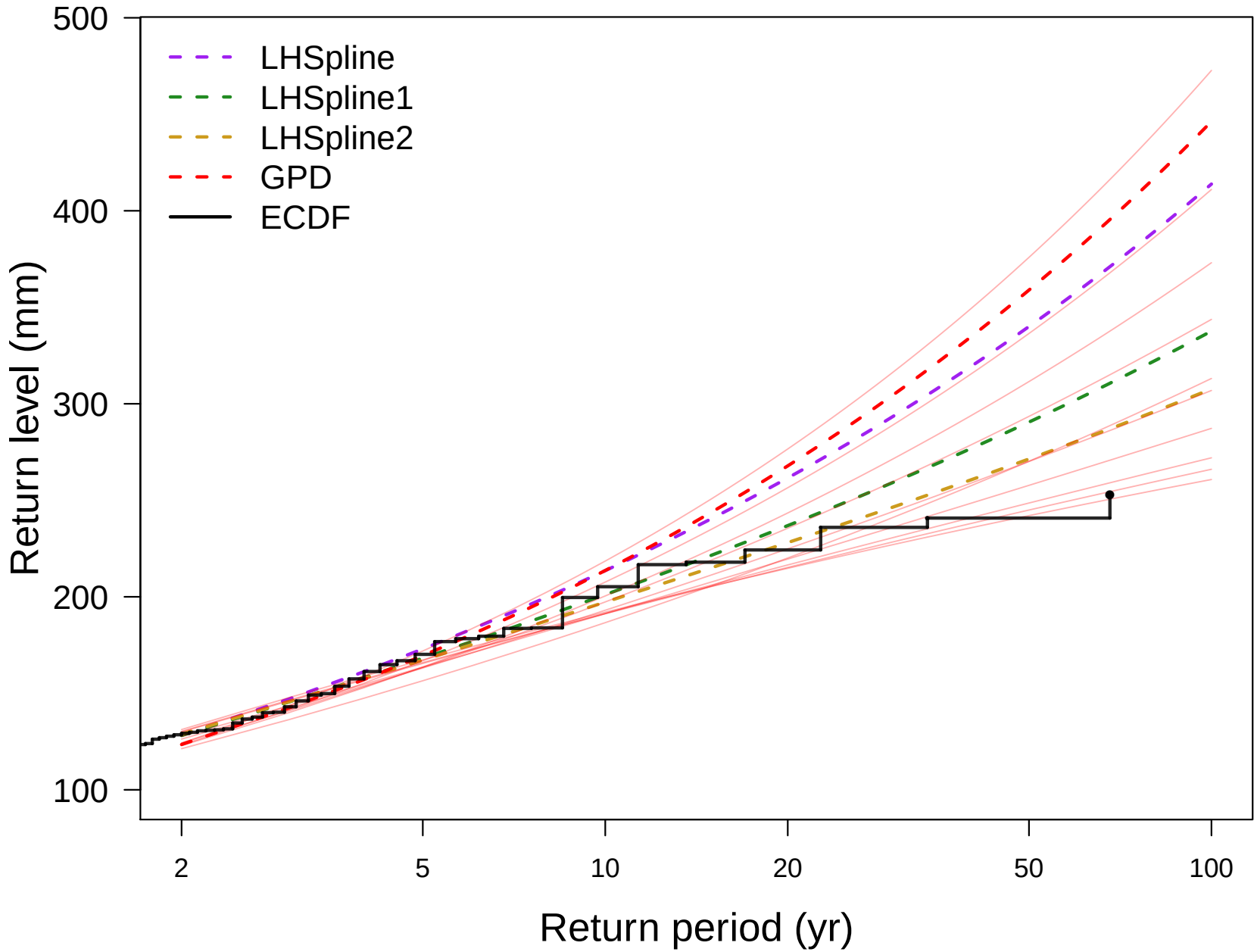

How unusual was the event of 306.58 mm on 26th August, 2017?

Method	GPD	LHSpline1	LHSpline2
Estimate (years)	30.5	64.0	98.0
90% CI Lower limit	17.0	19.1	22.0
90% CI Upper limit	73.3	172.4	345.7

Estimation Uncertainty

- LHSpline: Via conditional simulation to obtain Bayesian "confidence interval" [Wahba, 1990, Green & Silverman 1994]

- GPD: Via profile likelihood [Coles, 2001]

Summary and discussion

LHSpline

+: **Does not require the threshold selection** when modeling the tail distribution

+: Applicable to **heavy-tailed distribution**: streamflow, financial return, ...

-: Only applicable to heavy-tailed distribution

-: LHSpline requires some tunings

Ongoing work

- **Nonstationary extension** to account for both seasonality and long term trend
- **Spatial extension** by using the methods from functional data analysis to facilitate the spatial smoothing of the log-densities in a spatial region to compute return level maps

Thank you for your attention!

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