Review of Last Class

Analysis of Variance (ANOVA) Approach to Regression

Lecture 5

Simple Linear Regression III

Reading: Chapter 11

STAT 8020 Statistical Methods II

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Whitney Huang
Clemson University

Agenda

Review of Last Class

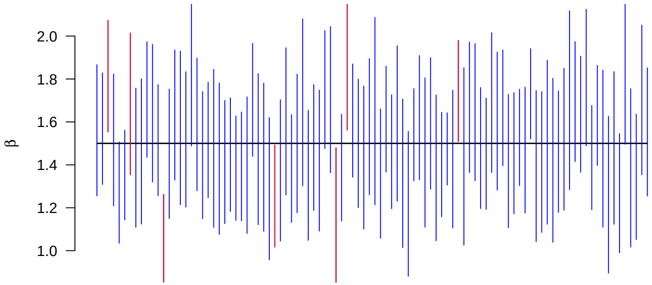
Analysis of Variance (ANOVA) Approach to Regression

1. Review of Last Class

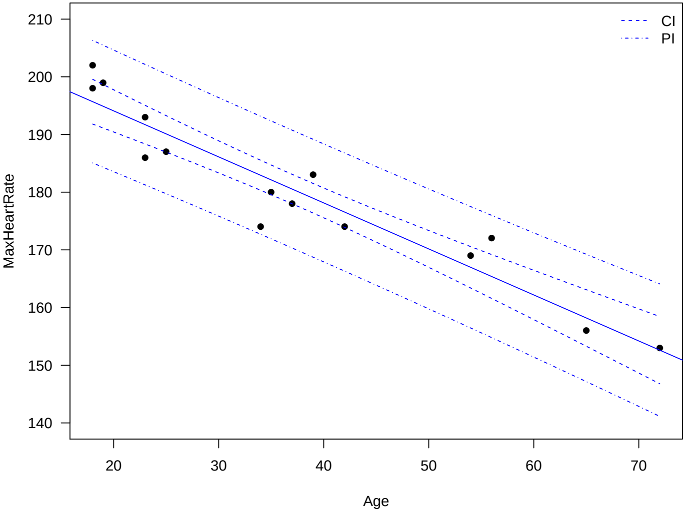
2. Analysis of Variance (ANOVA) Approach to Regression

Understanding Confidence Intervals

- Suppose $Y = \beta_0 + \beta_1 X + \varepsilon$, where $\beta_0 = 3$, $\beta_1 = 1.5$ and $\sigma^2 \sim \mathrm{N}(0, 1)$
- We take 100 random sample each with sample size 20
- We then construct the 95% CI for each random sample ($\Rightarrow$ 100 CIs)



Confidence Intervals vs. Prediction Intervals



Analysis of Variance (ANOVA) Approach to Regression

Review of Last Class

Analysis of Variance (ANOVA) Approach to Regression

Partitioning Sums of Squares

- Total sums of squares in response

$$\text{SST} = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

- We can rewrite SST as

$$\begin{aligned}\sum_{i=1}^{n}(Y_i - \bar{Y})^2 &= \sum_{i=1}^{n}(Y_i - \hat{Y}_i + \hat{Y}_i - \bar{Y})^2 \\ &= \underbrace{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}_{\text{Error}} + \underbrace{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}_{\text{Model}}\end{aligned}$$

Total Sum of Squares: SST

- If we ignored the predictor X, the $\bar{Y}$ would be the best (linear unbiased) predictor

$$Y_i = \beta_0 + \varepsilon_i \tag{1}$$

- SST is the sum of squared deviations for this predictor (i.e., $\bar{Y}$)

- The **total mean square** is SST$/(n-1)$ and represents an unbiased estimate of σ^2 under the model (1).

Regression Sum of Squares: SSR

- SSR: $\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$
- Degrees of freedom is 1 due to the inclusion of the slope, i.e.,

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \quad (2)$$

- "Large" MSR $=$ SSR$/1$ suggests a linear trend, because

$$\mathrm{E}[MSE] = \sigma^2 + \beta_1^2 \sum_{i=1}^{n}(X_i - \bar{X})^2$$

Error Sum of Squares: SSE

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Analysis of Variance (ANOVA) Approach to Regression

- SSE is simply the sum of squared residuals

$$\text{SSE} = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

- Degrees of freedom is $n - 2$ (Why?)
- SSE large when |residuals| are "large" $\Rightarrow$ Y_i's vary substantially around fitted regression line
- MSE $=$ SSE$/(n-2)$ and represents an unbiased estimate of σ^2 **when taking X into account**

ANOVA Table and F test

Source	df	SS	MS
Model	1	SSR $= \sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$	MSR = SSR/1
Error	$n-2$	SSE $= \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$	MSE = SSE/(n-2)
Total	$n-1$	SST $= \sum_{i=1}^{n}(Y_i - \bar{Y})^2$	

- **Goal:** To test $H_0 : \beta 1 = 0$
- Test statistics $F^* = \frac{\text{MSR}}{\text{MSE}}$
- If $\beta_1 = 0$ then F^* should be near one $\Rightarrow$ reject H_0 when F^* "large"
- We need sampling distribution of F^* under $H_0 \Rightarrow F_{1,n-2}$, where $F(d_1, d_2)$ denotes a F distribution with degrees of freedom d_1 and d_2

Correlation and Simple Linear Regression

- Pearson Correlation: $r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2 \sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$
- $-1 \leq r \leq 1$ measures the strength of the **linear relationship** between Y and X
- We can show $r = \hat{\beta}_{1,\text{LS}} \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$, this implies

$$\beta_1 = 0 \text{ in SLR } \Leftrightarrow \rho = 0$$

Coefficient of Determination R^2

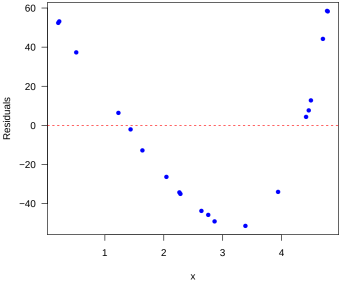
- Defined as the proportion of total variation explained by SLR

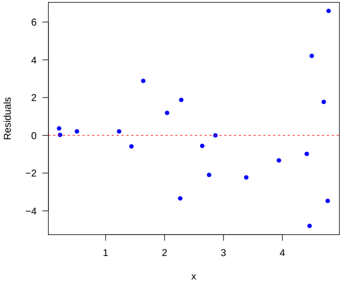
$$R^2 = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} = \frac{\text{SSR}}{\text{SST}} = 1 - \frac{\text{SSE}}{\text{SST}}$$

- We can show $r^2 = R^2$:

$$\begin{aligned} r^2 &= \left(\hat{\beta}_{1,\text{LS}} \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}}\right)^2 \\ &= \frac{\hat{\beta}_{1,\text{LS}}^2 \sum_{i=1}^n (X_i - \bar{X})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} \\ &= \frac{\text{SSR}}{\text{SST}} \\ &= R^2 \end{aligned}$$

Residual Plot Revisited





⇒ Nonlinear relationship

- Transform X
- Nonlinear regression

⇒ Non-constant variance

- Transform Y
- Weighted least squares

Summary

In this lecture, we learned ANOVA Approach to Regression

Next time: Multiple linear regression