Lecture 5

Analysis of Covariance, Polynomial Regression and Non-linear Regression

Reading: Faraway (2014) Chapters 9.4, 14.2-14.4; JWHT Chapter 3.3

DSA 8020 Statistical Methods II

Whitney Huang
Clemson University

Agenda

1. Analysis of Covariance
2. Polynomial Regression
3. Nonlinear Regression

Regression with Both Quantitative and Qualitative Predictors

Multiple Linear Regression

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_{p-1} x_{p-1} + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, \sigma^2)$$

$x_1, x_2, \cdots, x_{p-1}$ are the predictors.

Question: What if some of the predictors are qualitative (categorical) variables?

⇒ We will need to create **dummy (indicator) variables** for those categorical variables

Example: We can encode `Gender` into 1 (Female) and 0 (Male)

Salaries for Professors Data Set

The 2008-09 nine-month academic salary for Assistant Professors, Associate Professors and Professors in a college in the U.S. The data were collected as part of the on-going effort of the college's administration to monitor salary differences between male and female faculty members.

```
> head(Salaries)
       rank discipline yrs.since.phd yrs.service  sex salary
1      Prof          B            19          18 Male 139750
2      Prof          B            20          16 Male 173200
3  AsstProf          B             4           3 Male  79750
4      Prof          B            45          39 Male 115000
5      Prof          B            40          41 Male 141500
6 AssocProf          B             6           6 Male  97000
```

Predictors

```
> summary(Salaries)
       rank     discipline yrs.since.phd    yrs.service
 AsstProf : 67   A:181      Min.   : 1.00   Min.   : 0.00
 AssocProf: 64   B:216      1st Qu.:12.00   1st Qu.: 7.00
 Prof     :266              Median :21.00   Median :16.00
                            Mean   :22.31   Mean   :17.61
                            3rd Qu.:32.00   3rd Qu.:27.00
                            Max.   :56.00   Max.   :60.00
     sex          salary
 Female: 39   Min.   : 57800
 Male  :358   1st Qu.: 91000
              Median :107300
              Mean   :113706
              3rd Qu.:134185
              Max.   :231545
```

We have three categorical variables, namely, `rank`, `discipline`, and `sex`.

Dummy Variable

For binary categorical variables:

$$X_{\texttt{sex}} = \begin{cases} 1 & \text{if } \texttt{sex} = \text{male}, \\ 0 & \text{if } \texttt{sex} = \text{female}. \end{cases}$$

$$X_{\texttt{discip}} = \begin{cases} 0 & \text{if } \texttt{discip} = \text{A}, \\ 1 & \text{if } \texttt{discip} = \text{B}. \end{cases}$$

For categorical variable with more than two categories:

$$X_{\texttt{rank1}} = \begin{cases} 0 & \text{if } \texttt{rank} = \text{Assistant Prof}, \\ 1 & \text{if } \texttt{rank} = \text{Associated Prof}. \end{cases}$$

$$X_{\texttt{rank2}} = \begin{cases} 0 & \text{if } \texttt{rank} = \text{Associated Prof}, \\ 1 & \text{if } \texttt{rank} = \text{Full Prof}. \end{cases}$$

Design Matrix

```
> head(X)
  (Intercept) rankAssocProf rankProf disciplineB yrs.since.phd
1           1             0        1           1            19
2           1             0        1           1            20
3           1             0        0           1             4
4           1             0        1           1            45
5           1             0        1           1            40
6           1             1        0           1             6
  yrs.service sexMale
1          18       1
2          16       1
3           3       1
4          39       1
5          41       1
6           6       1
```

With the design matrix $\boldsymbol{X}$, we can now use method of least squares to fit the model $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$

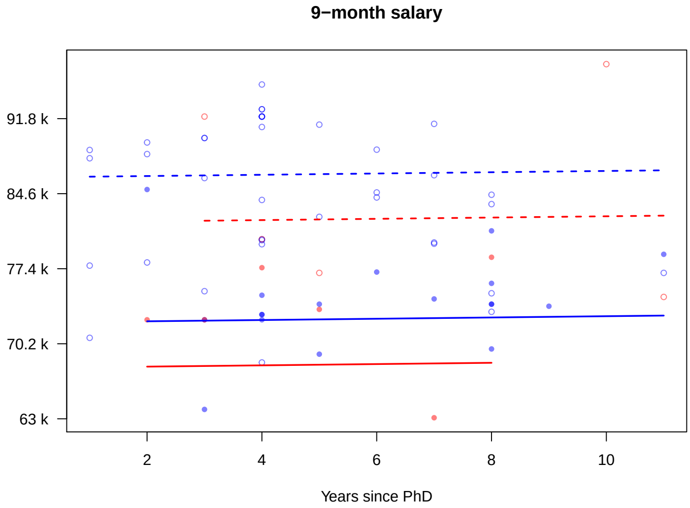
Model Fit: $\texttt{lm}(\texttt{salary} \sim \texttt{rank} + \texttt{sex} + \texttt{discipline} + \texttt{yrs.since.phd})$

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   67884.32    4536.89  14.963  < 2e-16 ***
disciplineB   13937.47    2346.53   5.940 6.32e-09 ***
rankAssocProf 13104.15    4167.31   3.145  0.00179 **
rankProf      46032.55    4240.12  10.856  < 2e-16 ***
sexMale        4349.37    3875.39   1.122  0.26242
yrs.since.phd    61.01     127.01   0.480  0.63124
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 22660 on 391 degrees of freedom
Multiple R-squared:  0.4472,    Adjusted R-squared:  0.4401
F-statistic: 63.27 on 5 and 391 DF,  p-value: < 2.2e-16
```

Question: Interpretation of the slopes of these dummy variables (e.g. $\hat{\beta}_{\texttt{rankAssocProf}}$)? Interpretation of the intercept?

Model Fit for Assistant Professors



Model Fit for Associate Professors

Model Fit for Full Professors

Introducing Interaction Terms

```
lm(salary ~ sex * yrs.since.phd)
```

lm(salary ~ disp * yrs.since.phd)

Polynomial Regression

Suppose we would like to model the relationship between response Y and a predictor X as a p_{th} degree polynomial in x:

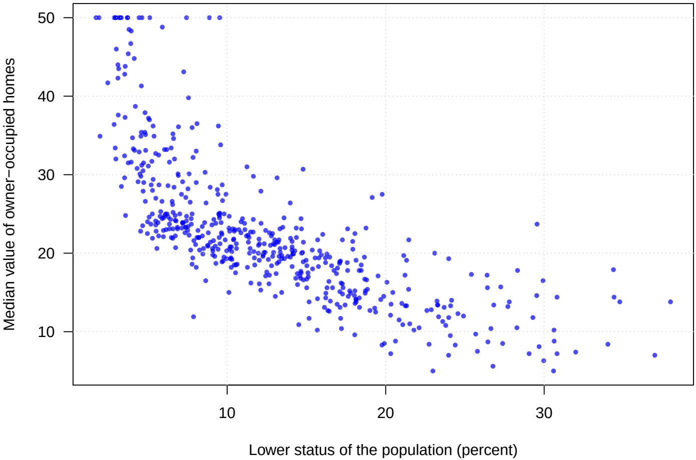
$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_p x^p + \varepsilon$$

We can treat polynomial regression as a special case of multiple linear regression. In specific, the design matrix takes the following form:

$$\boldsymbol{X} = \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^p \\ 1 & x_2 & x_2^2 & \cdots & x_2^p \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^p \end{pmatrix}$$

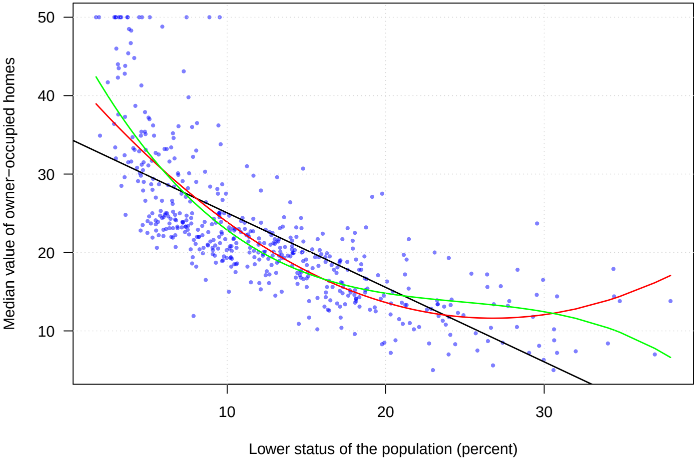
Housing Values in Suburbs of Boston Data Set

- y: the median value of owner-occupied homes (in thousands of dollars)
- x: percent of lower status of the population



Polynomial Regression Fits

1st, 2nd, and 3rd polynomial regression fits

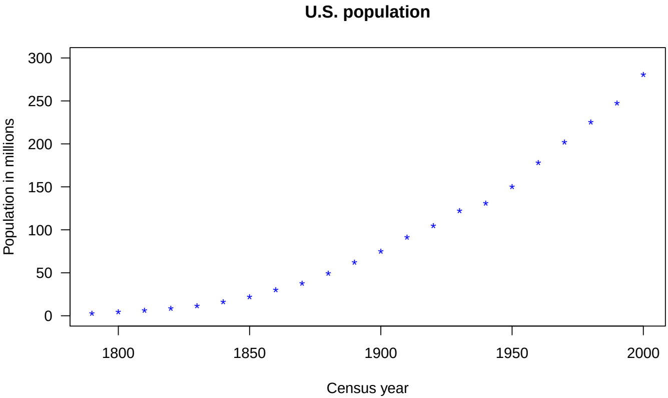


Moving Away From Linear Regression

- We have mainly focused on linear regression so far
- The class of polynomial regression can be thought as a starting point for relaxing the linear assumption
- In the next few slides we are going to discuss non-linear regression modeling

Population of the United States

Let's look at the `USPop` data set, a bulit-in data set in R. This is a decennial time-series from 1790 to 2000.

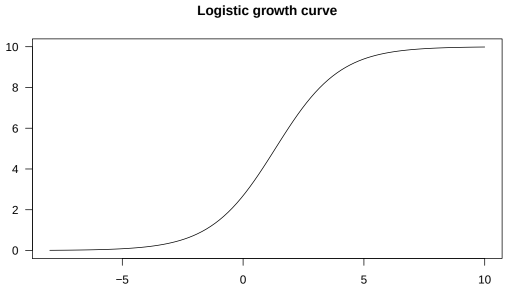


Logistic Growth Curve

A simple model for population growth is the logistic growth model,

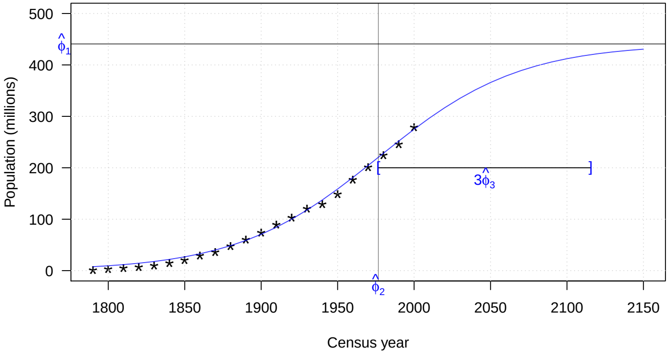
$$Y = \frac{\phi_1}{1 + \exp\left[-(x - \phi_2)/\phi_3\right]} + \varepsilon,$$

where ϕ_1 is the curve's maximum value; ϕ_2 is the curve's midpoint in x; and ϕ_3 is the "range" (or the inverse growth rate) of the curve.

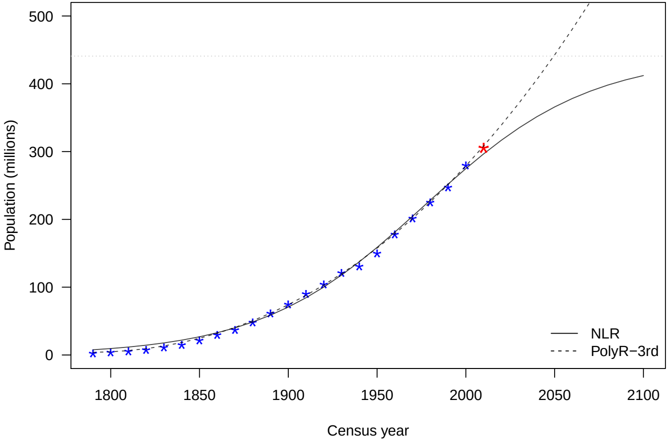


Fitting logistic growth curve to the U.S. population

$$\hat{\phi}_1 = 440.83,\ \hat{\phi}_2 = 1976.63,\ \hat{\phi}_3 = 46.29$$



Comparing the Logistic Growth Curve Fit and Cubic Polynomial Fit



Summary

This slides cover:

- Analysis of Covariance to handle the situations where there both some of the predictors are categorical variables
- Polynomial Regression, where polynomial terms are added to increase the model flexibility
- Nonlinear Regression