Fall 2019 Exam I

STAT 8020

September 27, 2019

Directions

- 1. Show your work on ALL questions (except those multiple choice questions). Unsupported work will NOT receive full credit.
- 2. Decimal answers should be exact, or to exactly 2 significant digits.
- 3. Please write legibly. If I cannot read your writing, NO credit will be given.
- 4. You are allowed the following aids:
 - (a) a one-page A4 handwritten cheat sheet
 - (b) A scientific Calculator
- 5. Turn off your cell phone before the exam begins.

Use your time wisely. Good Luck!!!

| Problem | Points Possible | Points Earned |
|---------|-----------------|---------------|
| 1 | 60 | |
| 2 | 20 | |
| 3 | 20 | |
| Total | 100 | |

Problem 1

A baseball fan would like to study the relationship between the annual salary Salary (in thousands of dollars) of major league players and the number of home runs during his career CHmRun. A simple linear regression is performed where Salary is the response. Use the R output below to answer the following questions: (12 points for each question.)

```
lm(formula = Salary ~ CHmRun)
Residuals:
   Min
        1Q Median 3Q
                              Max
-1427.7 -247.1 -109.3 169.2 1785.1
Coefficients:
         Estimate Std. Error t value Pr(>|t|)
0.2891 9.964 <2e-16 ***
CHmRun
          2.8809
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 384.7 on 261 degrees of freedom
 (59 observations deleted due to missingness)
Multiple R-squared: 0.2756, Adjusted R-squared: 0.2728
F-statistic: 99.27 on 1 and 261 DF, p-value: < 2.2e-16
```

1. Write down the least squares regression line and compute the fitted value with CHmRun = 100.

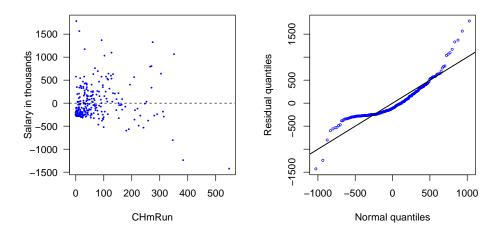
2. Construct the 95% confidence interval (using t(0.975, df = 261) = 1.97 and $\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2} = 1330.484$) for β_1 .

3. Test the following hypothesis: $H_0: \beta_1 = 0$ vs. $H_a: \beta_1 \neq 0$ with $\alpha = 0.05$. State your conclusion in plain language in the present context.

4. Fill in the missing values in the ANOVA table below and compute the \mathbb{R}^2 , the coefficient of determination.

| Source | df | SS | MS | F |
|--------|-----|----------------|---------|-----------|
| Model | ? | SSR = 14692193 | MSR = ? | $F^* = ?$ |
| Error | 261 | SSE = 38626920 | MSE = ? | |
| Total | ? | SST = ? | | |

5. Do the residual plot and the Normal Q-Q plot below suggest any regression assumptions may be violated? Explain your answer.



Problem 2

A researcher performs a multiple linear regression, using the Longley's macroeconomic data set, to study the relationship between Employed (number of people employed) and GNP.deflator, GNP, Unemployed, Armed.Forces, Population, and Year. Use the R outputs below to answer the following questions:

Full model Fit:

GNP.deflator

135.53244

Year 758.98060

1788.51348

```
lm(formula = Employed \sim ., data = longley)
Residuals:
     Min
              1Q
                   Median
                                3Q
                                        Max
-0.41011 -0.15767 -0.02816 0.10155 0.45539
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.482e+03 8.904e+02 -3.911 0.003560 **
GNP.deflator 1.506e-02 8.492e-02
                                    0.177 0.863141
GNP
            -3.582e-02 3.349e-02 -1.070 0.312681
            -2.020e-02 4.884e-03 -4.136 0.002535 **
Unemployed
Armed.Forces -1.033e-02 2.143e-03 -4.822 0.000944 ***
Population -5.110e-02 2.261e-01 -0.226 0.826212
             1.829e+00 4.555e-01 4.016 0.003037 **
Year
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.3049 on 9 degrees of freedom
Multiple R-squared: 0.9955, Adjusted R-squared: 0.9925
F-statistic: 330.3 on 6 and 9 DF, p-value: 4.984e-10
VIF:
```

Unemployed Armed. Forces

3.58893

33.61889

Population

399.15102

1. (10 points) Explain why the full model is highly significant (overall F-test P-value $< 5 \times 10^{-10}$ and with a very high R^2) but still have very high p-values on some of the regressor's t tests? (Hint: Check the VIF values.)

2. (10 points) Perform a general linear test using the R output below:

Problem 3

The dean of a college in a University would like to monitor salary differences between male and female faculty members and she performed a multiple linear regression where the response variable salary is regressed on sex (male, female) and yrs.service (years of service). Use the R output below to answer the following question:

```
lm(formula = salary ~ sex * yrs.service, data = Salaries)
Residuals:
  Min 1Q Median 3Q
-80381 -20258 -3727 16353 102536
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
                  82068.5 7568.7 10.843 < 2e-16 ***
(Intercept)
                  20128.6
                             7991.1 2.519 0.01217 *
sexMale
yrs.service
                   1637.3
                              523.0
                                     3.130 0.00188 **
sexMale:yrs.service -931.7
                               535.2 -1.741 0.08251 .
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
Residual standard error: 28420 on 393 degrees of freedom
Multiple R-squared: 0.1266, Adjusted R-squared: 0.1199
F-statistic: 18.98 on 3 and 393 DF, p-value: 1.622e-11
```

1. (20 points) Write down the regression equation for male and female faculty, respectively.