MATH 8090: Spectral Analysis of Time Series I

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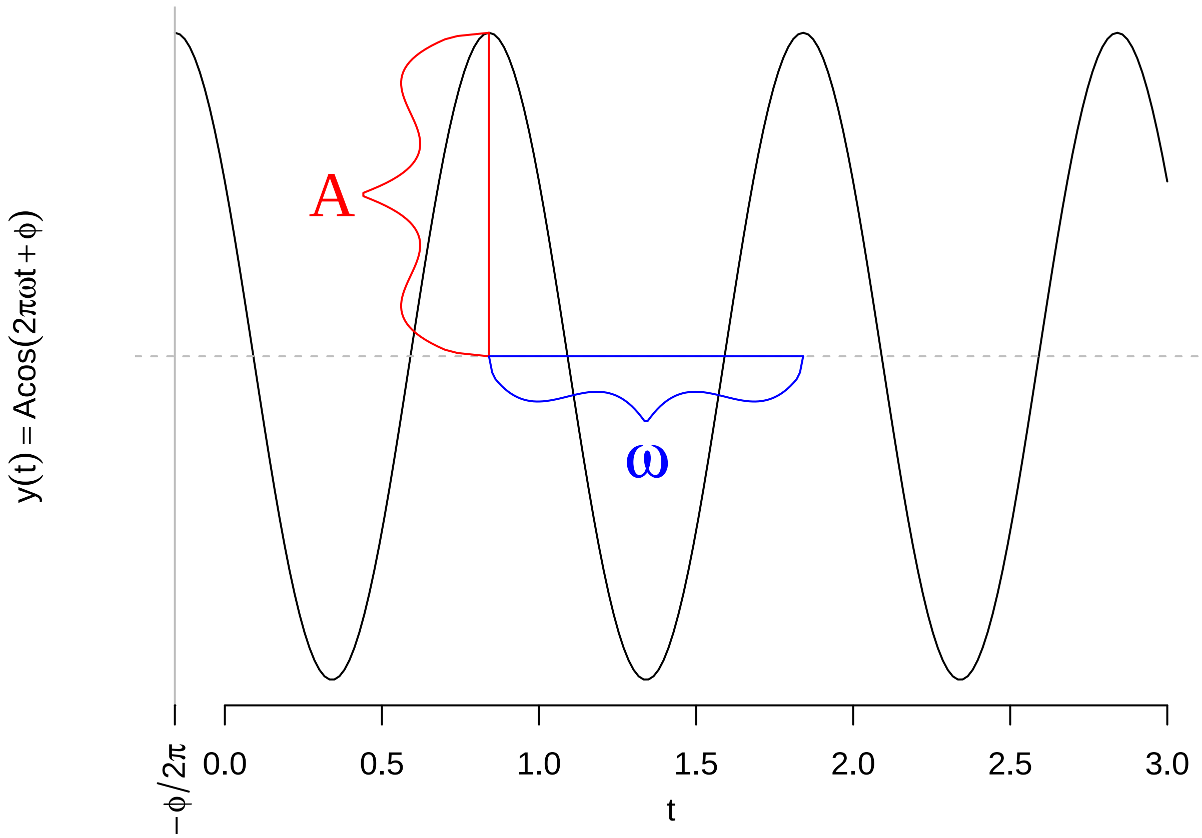
Contents

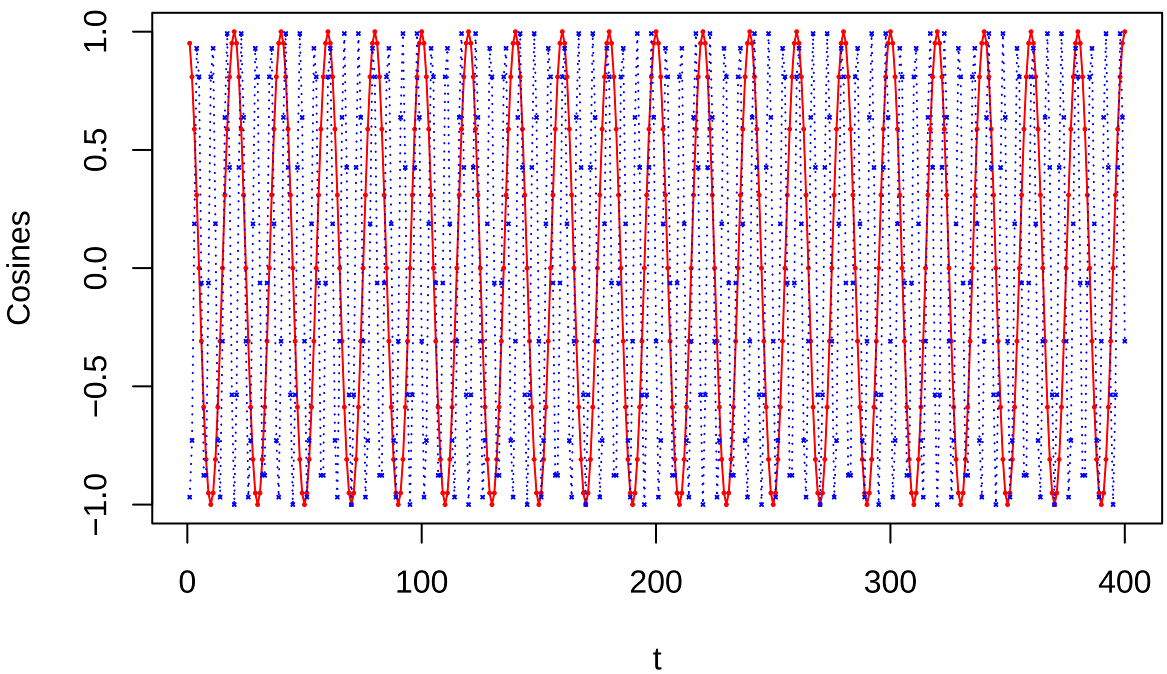
An example of periodic proces

```
source("curlyBraces.R")
A = 2
omega = 1
phi = 1
t <- seq(-phi / (2 * pi), 3, len = 200)
y <- A * cos (2 * pi * omega * t + phi)
par(mgp = c(2.2, 1, 0), mar = c(3.5, 4, 0.8, 0.6), las = 1)
fu <- expression(y(t) == paste("A", cos(2 * pi * omega * t + phi)))
plot(t, y, type = "l", yaxt = "n", bty = "n", ylab = fu)
abline(h = 0, lty = 2, col = "gray")
segments(1 - phi / (2 * pi), 0, 2 - phi / (2 * pi), col = "blue")
segments(1 - phi / (2 * pi), 0, y1 = A, col = "red")
CurlyBraces(1 - phi / (2 * pi), 0.5 * A, A, pos = 1, direction = 2, col = "red")
text(x = 1 - phi / (2 * pi) - 0.5, y = 0.5 * A, "A", family = "serif",
     col = "red", cex = 2)
CurlyBraces(0, 1.5 - phi / (2 * pi), omega, pos = 2, direction = 2, col = "blue")
text(x = 1.5 - phi / (2 * pi), y = -0.65, expression(omega), family = "serif",
     col = "blue", cex = 2)
abline(v = - phi / (2 * pi), col = "gray")
axis(1, at = - phi / (2 * pi), labels = expression(-phi/2*pi), cex = 0.5,
     las = 2)
```



An example from Cryer and Chan Cryer and Chan (2008), Chapter 13

```
t = 1:400
cos1 <- cos(2 * pi* t * 10 / 200)
cos2 <- cos(2 * pi * (t * 32 / 200 + .3))
plot(t, cos1, type = 'o', ylab = 'Cosines', col = "red", cex = 0.2)
lines(t, cos2, lty = 'dotted', type = 'o', pch = 4, col = "blue", cex = 0.2)
```




```
library(astsa)
y <- 3 * cos1 + 2 * cos2
par(mgp = c(2.7, 1, 0), mar = c(3.5, 4.5, 0.8, 0.6), las = 1, mfrow = c(3, 1))
tsplot(y, ylab = expression(y[t]))
acf(y)
library(TSA)
```

```
##
## Attaching package: 'TSA'

## The following objects are masked from 'package:stats':
##
##     acf, arima

## The following object is masked from 'package:utils':
##
##     tar
```

```
periodogram(y, ylab = ""); abline(h = 0)
axis(1, at = c(10 / 200, 32 / 200))
```

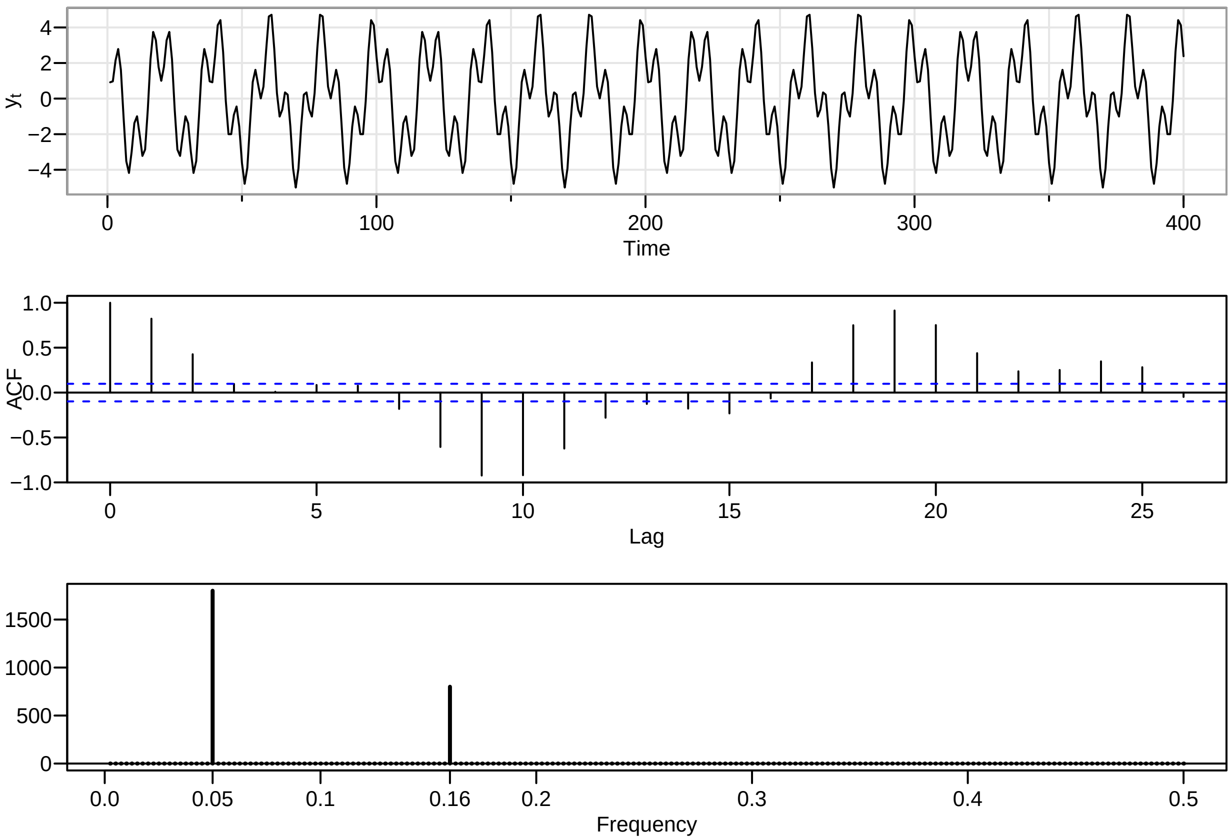
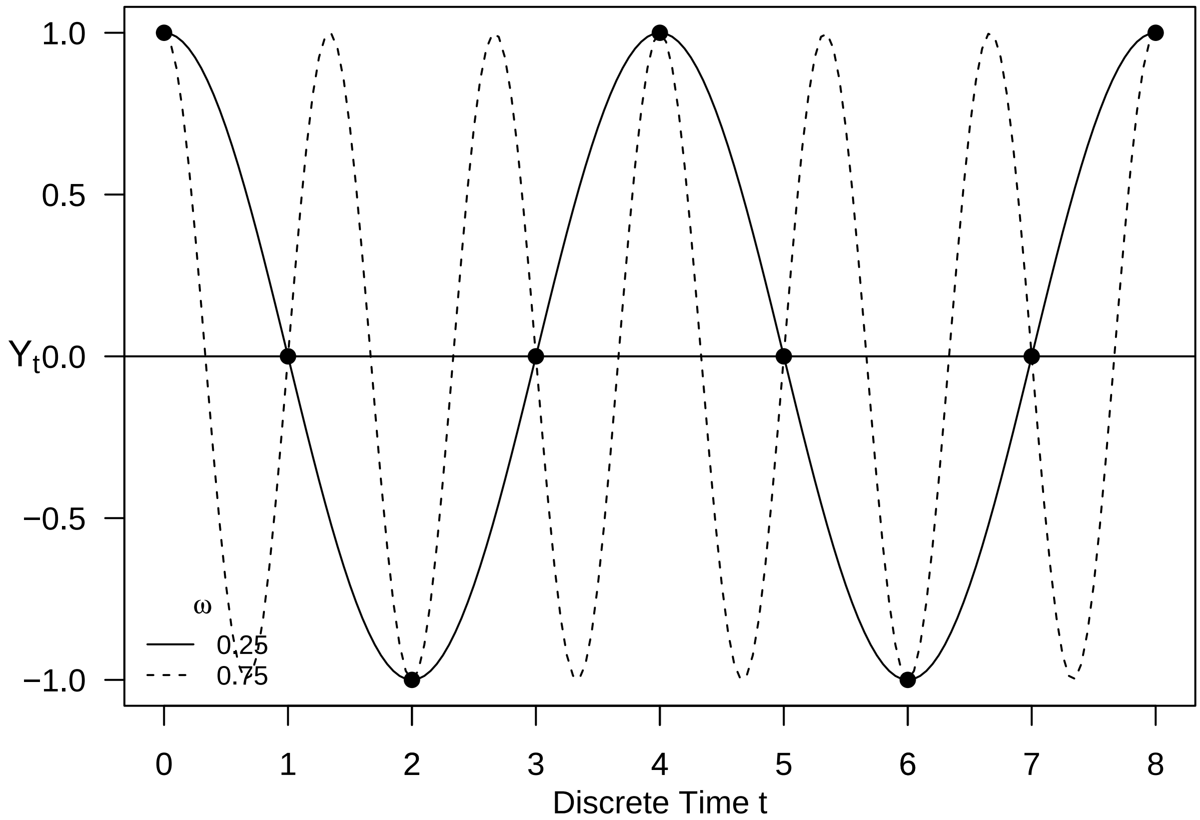


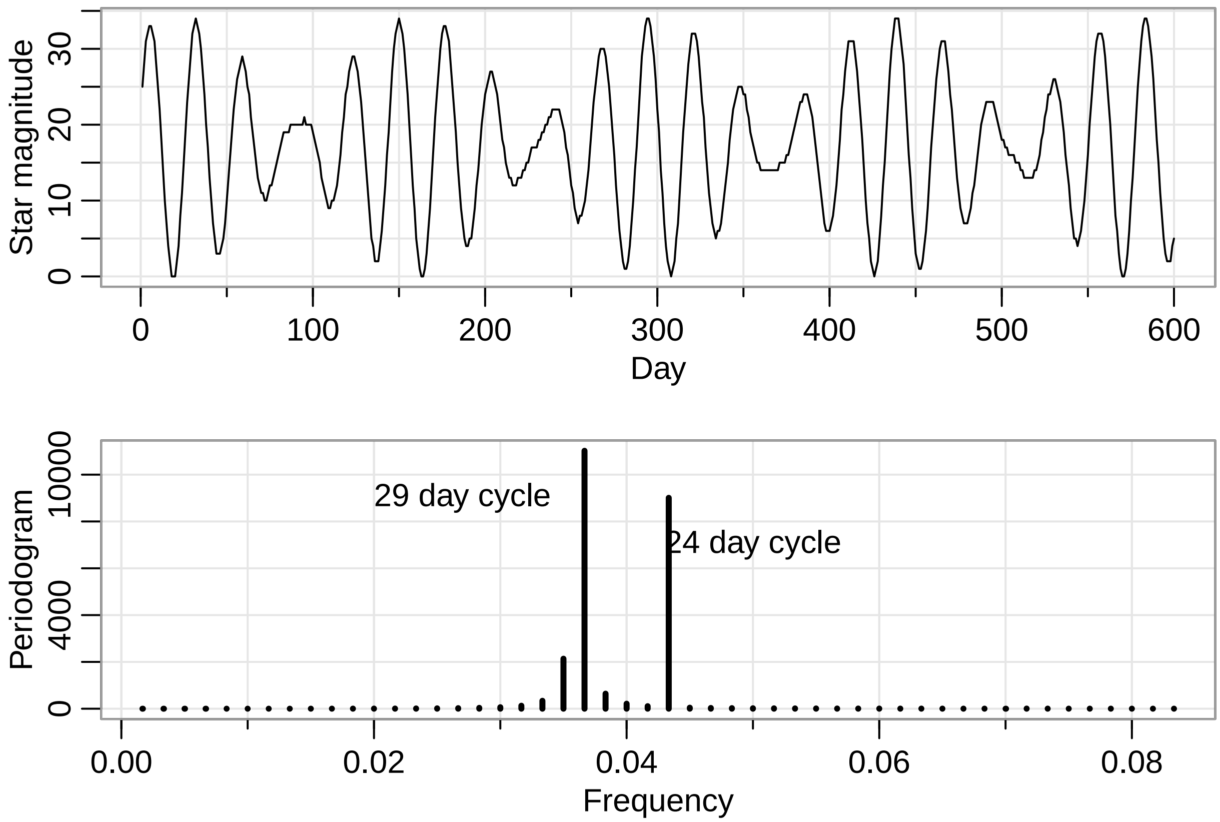
Illustration of aliasing

```
par(mgp = c(2, 1, 0), mar = c(3.5, 4, 0.8, 0.6), las = 1)
t <- seq(0, 8, by = .05)
plot(t, cos(2 * pi * t / 4), axes = F, type = 'l', ylab = "", xlab = 'Discrete Time t')
mtext(expression(Y[t]), 2, las = 1, line = 2.2, cex = 1.2)
axis(1, at = 1:7); axis(1); axis(2); box()
lines(t, cos(2 * pi * t * 3 / 4), lty = 'dashed', type = 'l'); abline(h = 0)
points(x= 0:8, y = cos(2 * pi * (0:8) / 4), pch = 19)
legend("bottomleft", legend = c(0.25, 0.75), title = expression(omega),
       lty = c(1, 2), bty = "n", cex = 0.8)
```



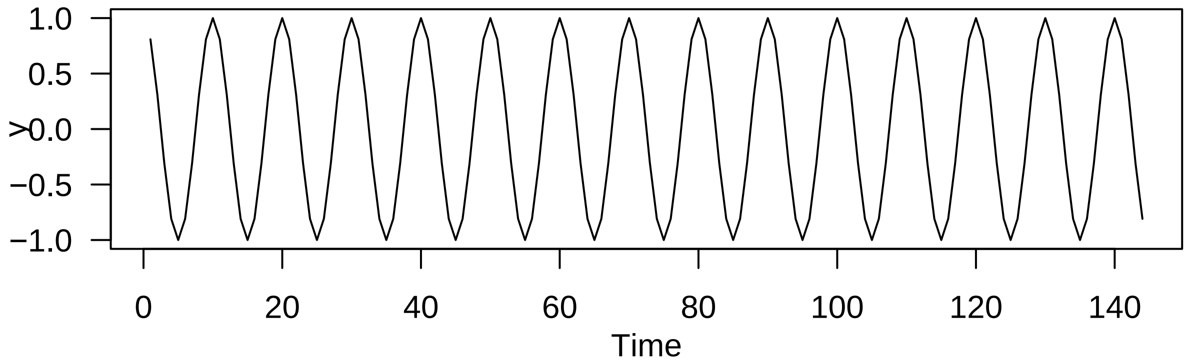
Example 4.3 from Shumway (n.d.)

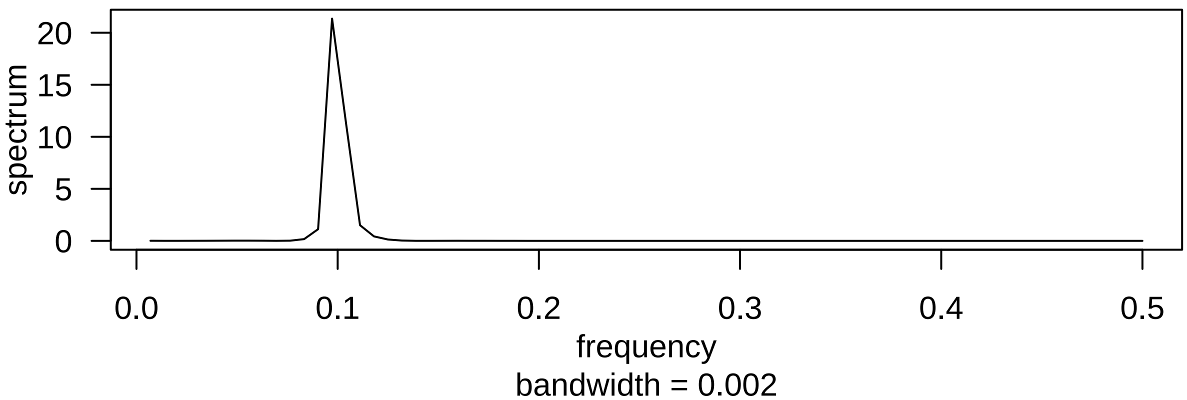
```
n = length(star)
par(mfrow = c(2, 1))
tsplot(star, ylab = "Star magnitude", xlab = "Day")
Per <- Mod(fft(star - mean(star)))^2 / n
Freq <- (1:(n - 1)) / n
tsplot(Freq[1:50], Per[1:50], type = 'h', lwd = 3, ylab = "Periodogram",
       xlab = "Frequency")
text(.05, 7000, "24 day cycle")
text(.027, 9000, "29 day cycle")
```



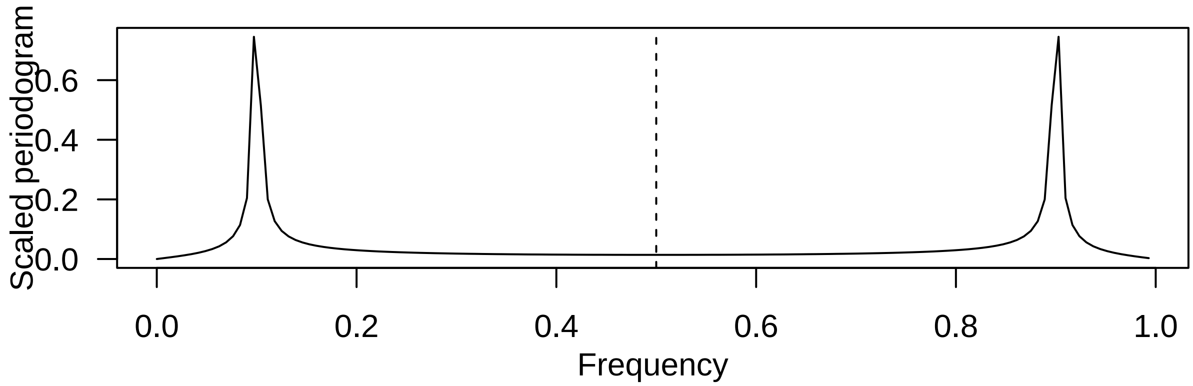
Periodogram

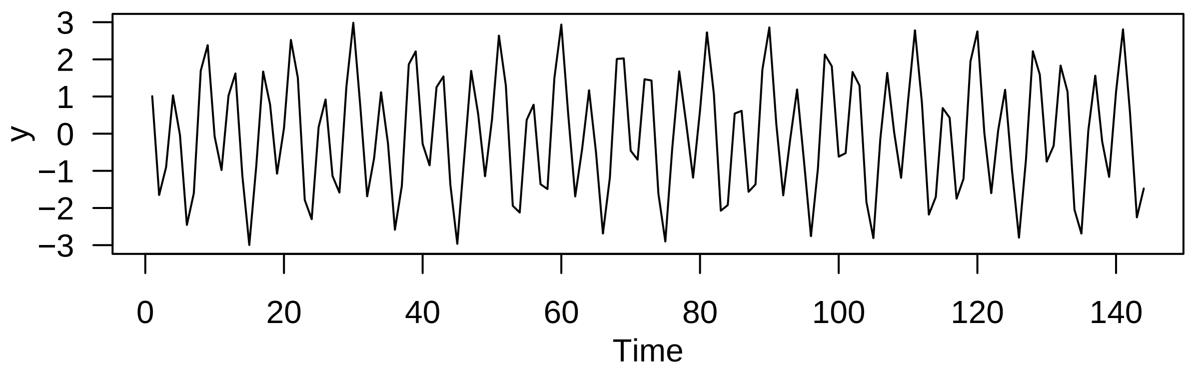
```
par(mfcol = c(2, 1), las = 1, mgp = c(2, 1, 0), mar = c(4, 4, 1, 0.6))
# one frequency:
y = cos(2 * pi * (0.1) * (1:144))
ts.plot(y); spectrum(y, log = "no", main = "")
```



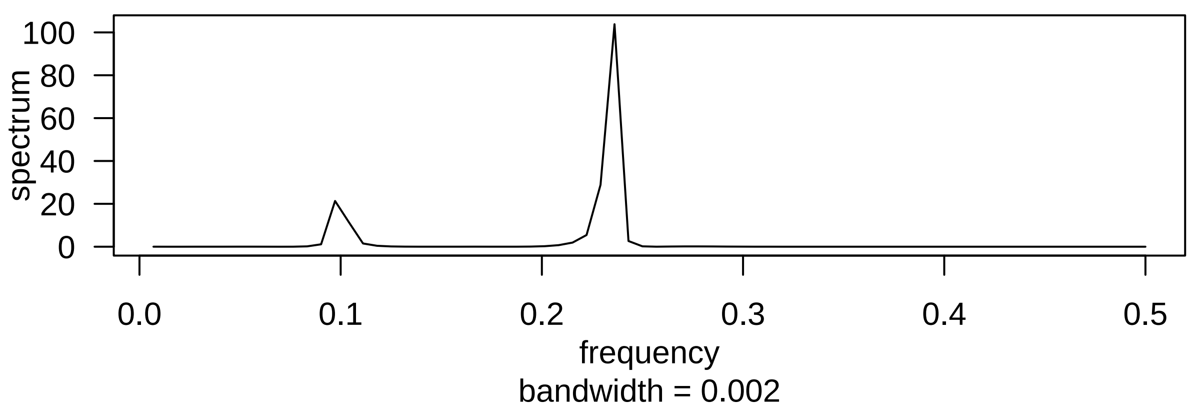


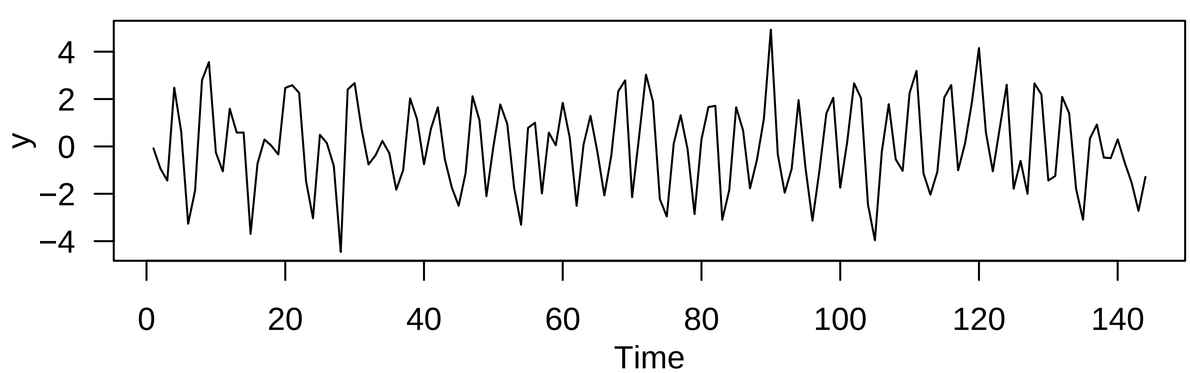

```
P <- Mod(2 * fft(y) / 144)
Fr <- 0:143 / 144
plot(Fr, P, type = "l", xlab = "Frequency", ylab = "Scaled periodogram")
abline(v = 0.5, lty = 2)
# and a second frequency:
y = y + 2 * cos(2 * pi * (0.234) * (1:144))
ts.plot(y); spectrum(y, log = "no", main = "")
```

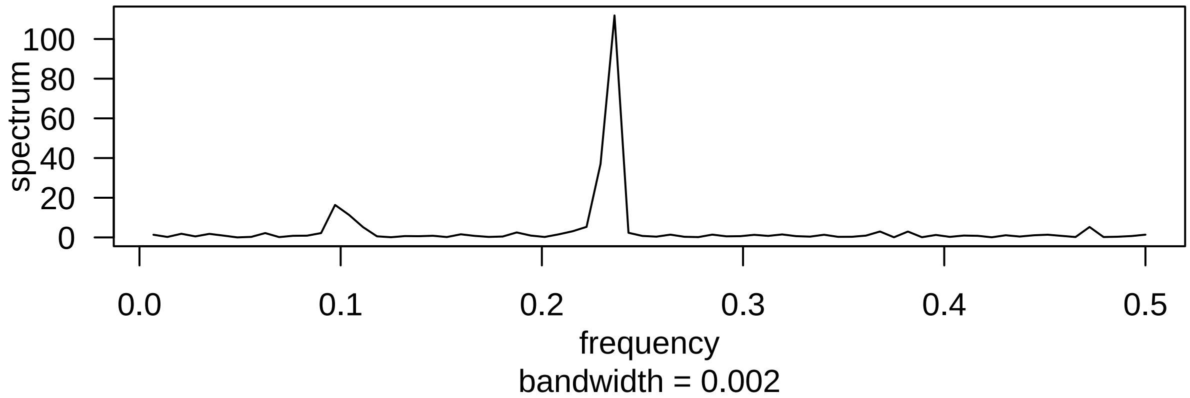





```r
# and added noise:
y = y + rnorm(144)
ts.plot(y); spectrum(y, log = "no", main = "")
```

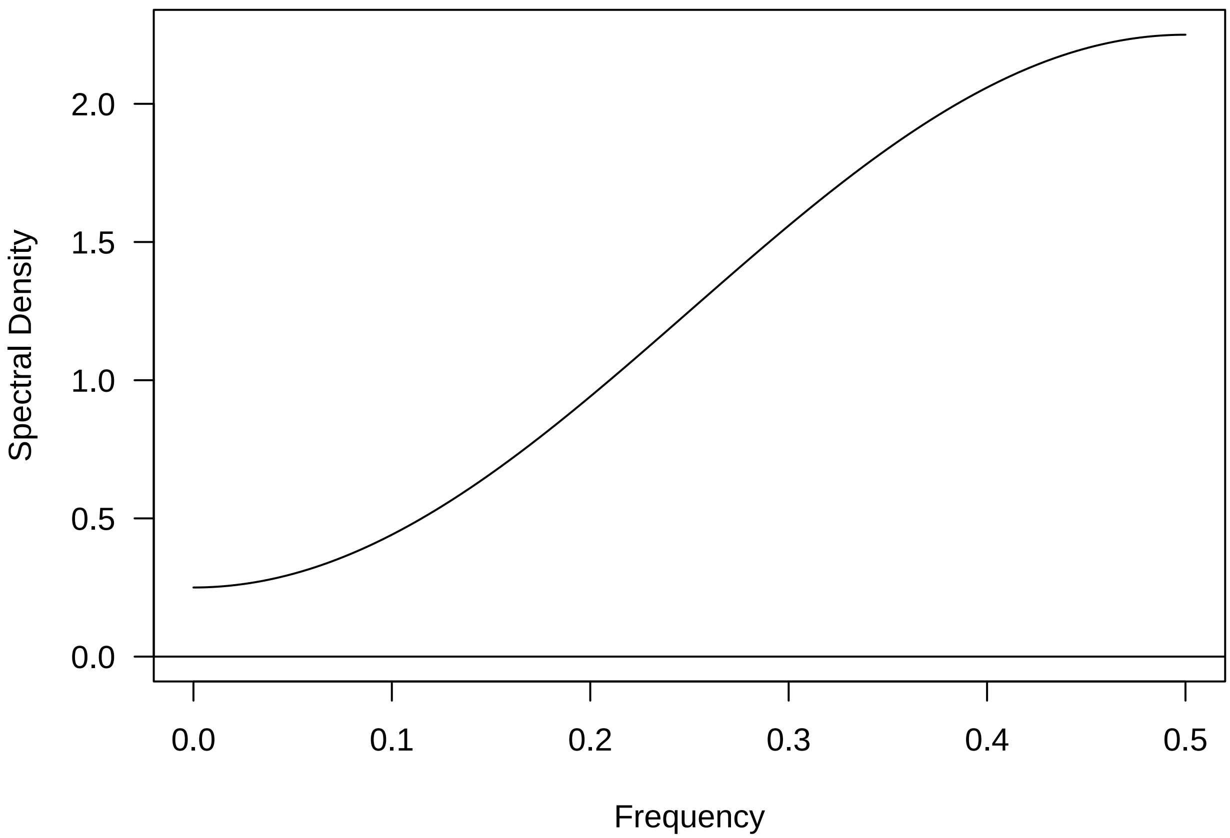






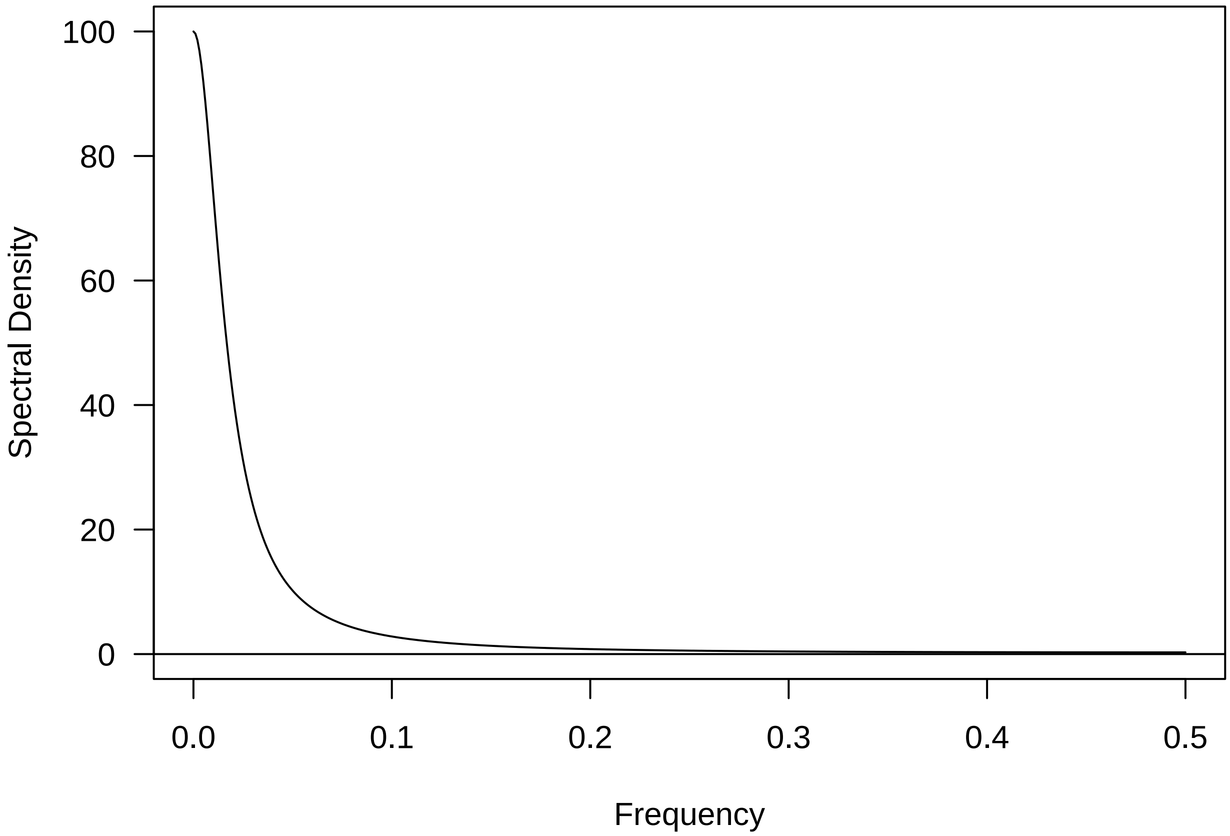
Spectral Density of MA(1) Process with $\theta = 0.5$

```
theta = .5
par(las = 1, mar = c(4, 4, 1, 0.6))
ARMAspec(model = list(ma = -theta))
```



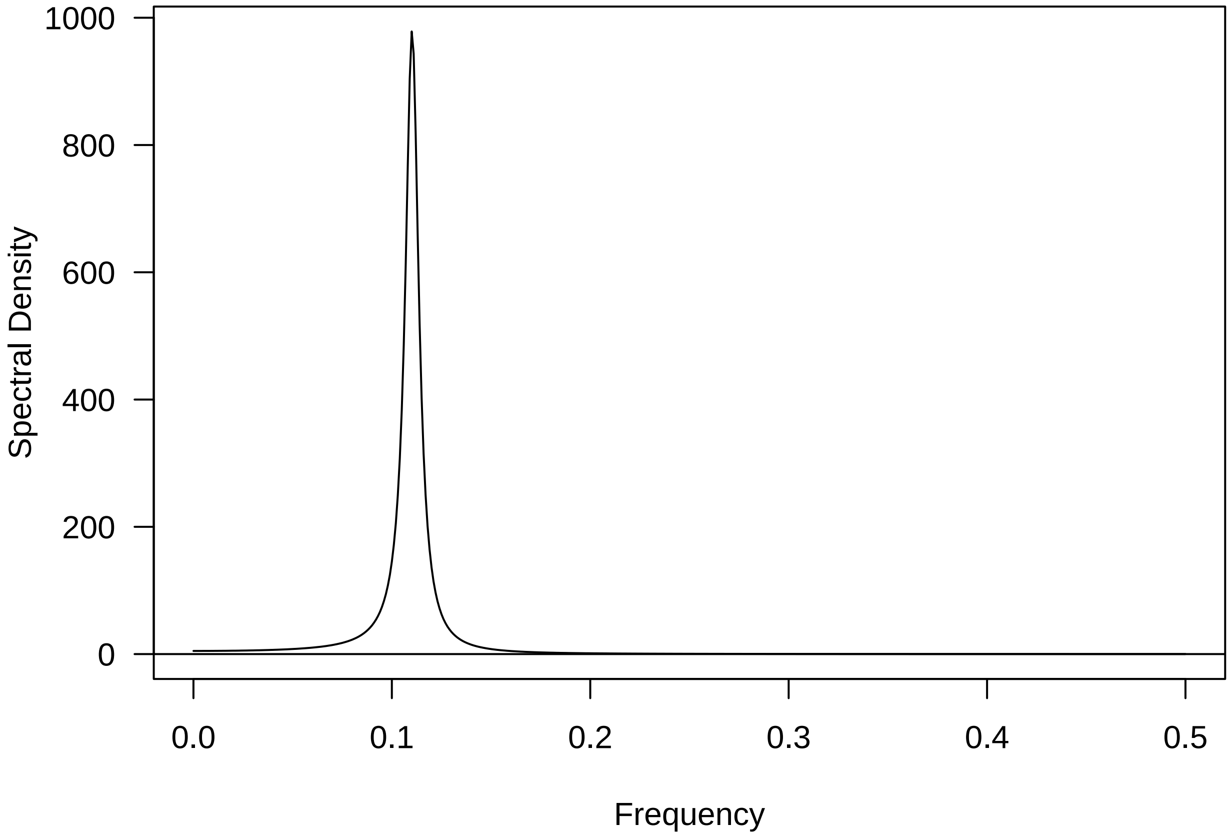
Spectral Density of AR(1) Process with $\phi = 0.9$

```
phi = .9
par(las = 1, mar = c(4, 4, 1, 0.6))
ARMAspec(model = list(ar = phi))
```



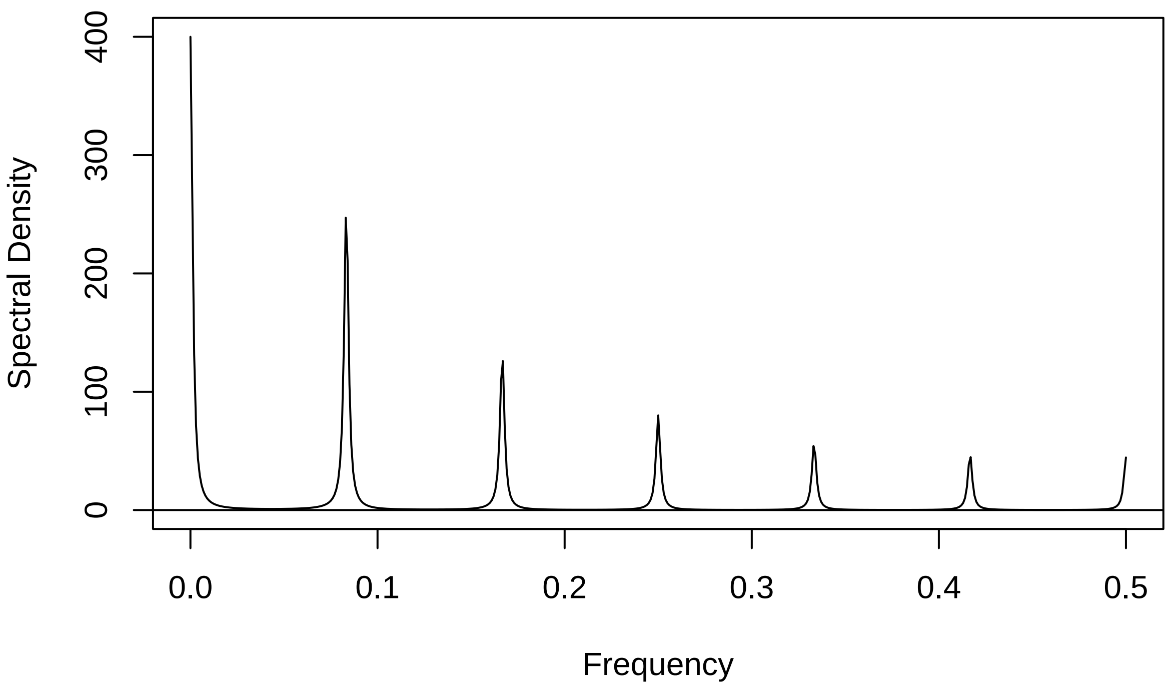
Spectral Density of AR(2) Process with $\phi_1 = 1.5, \phi_2 = -0.95$

```
phi = c(1.5, -0.95)
par(las = 1, mar = c(4, 4, 1, 0.6))
ARMAspec(model = list(ar = phi))
```



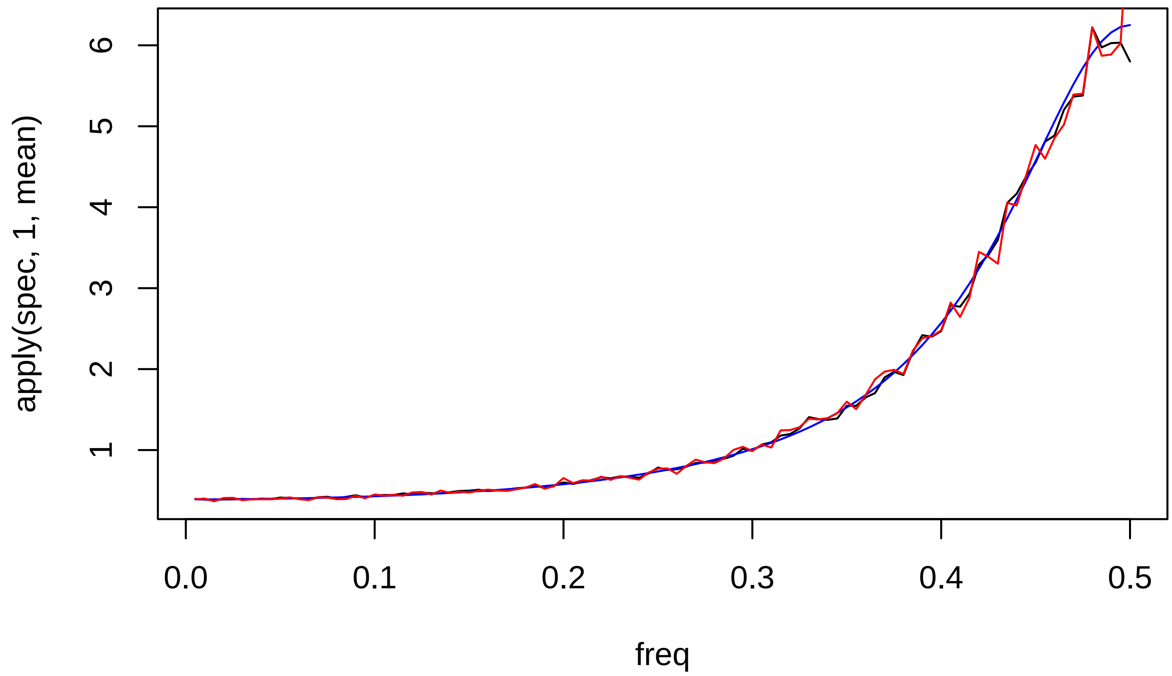
Spectral Density of SARMA$

```
phi = 0.5; PHI = .9
ARMAspec(model = list(ar = phi, seasonal = list(sar = PHI, period = 12)))
```

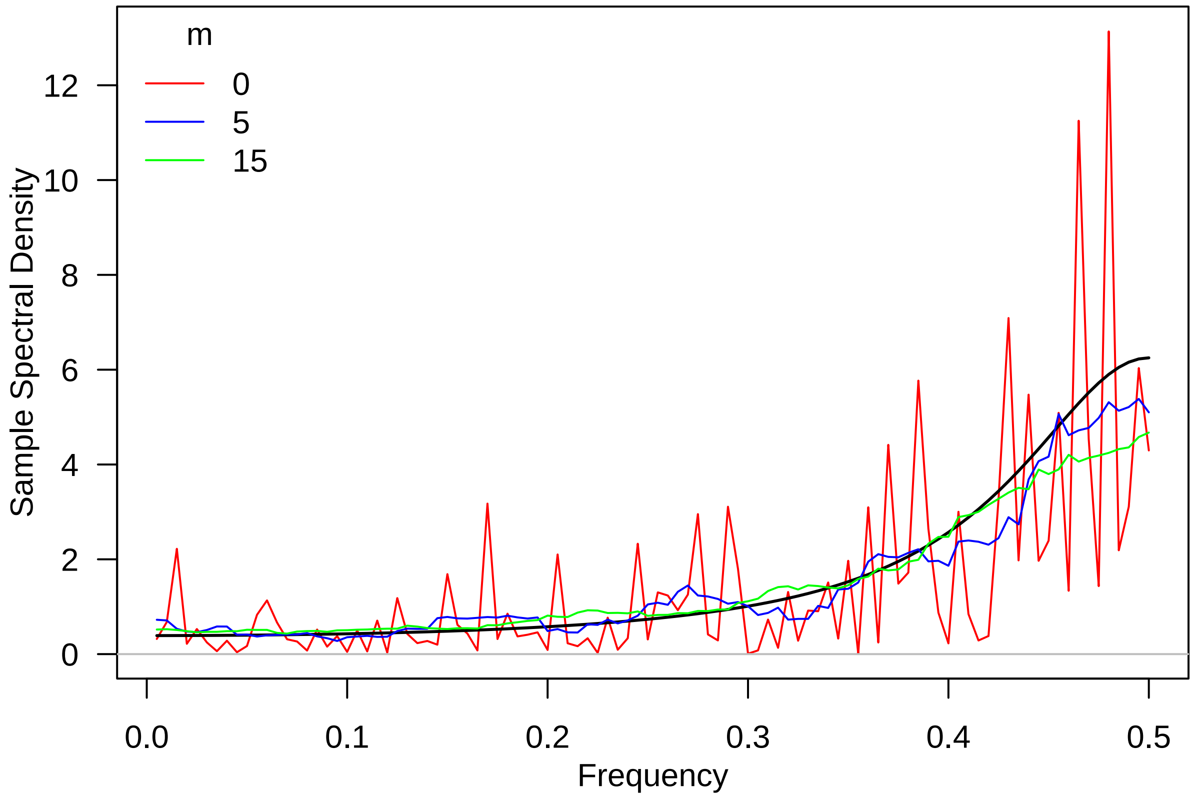


The periodogram is not a consistent estimator!

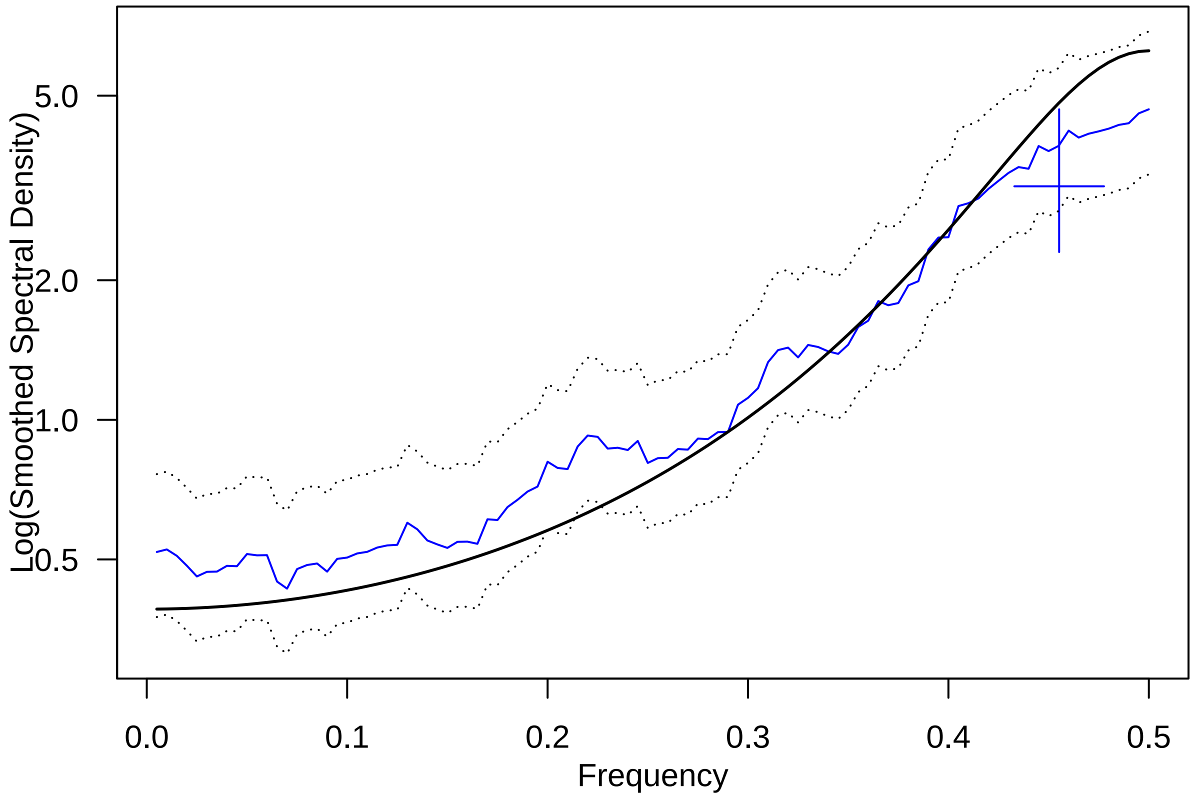
```
n = 200; phi= -0.6; N = 1000
y = replicate(N, arima.sim(model = list(ar = phi), n = n))
spec <- apply(y, 2, function(x) spec(x, log = "no", plot = F)$spec)

freq <- 1:(0.5 * n) / n
plot(freq, apply(spec, 1, mean), type = "l")
lines(freq, ARMAspec(model = list(ar = phi), freq = freq, plot = F)$spec, col = "blue")
lines(freq, apply(spec, 1, sd), col = "red")
```



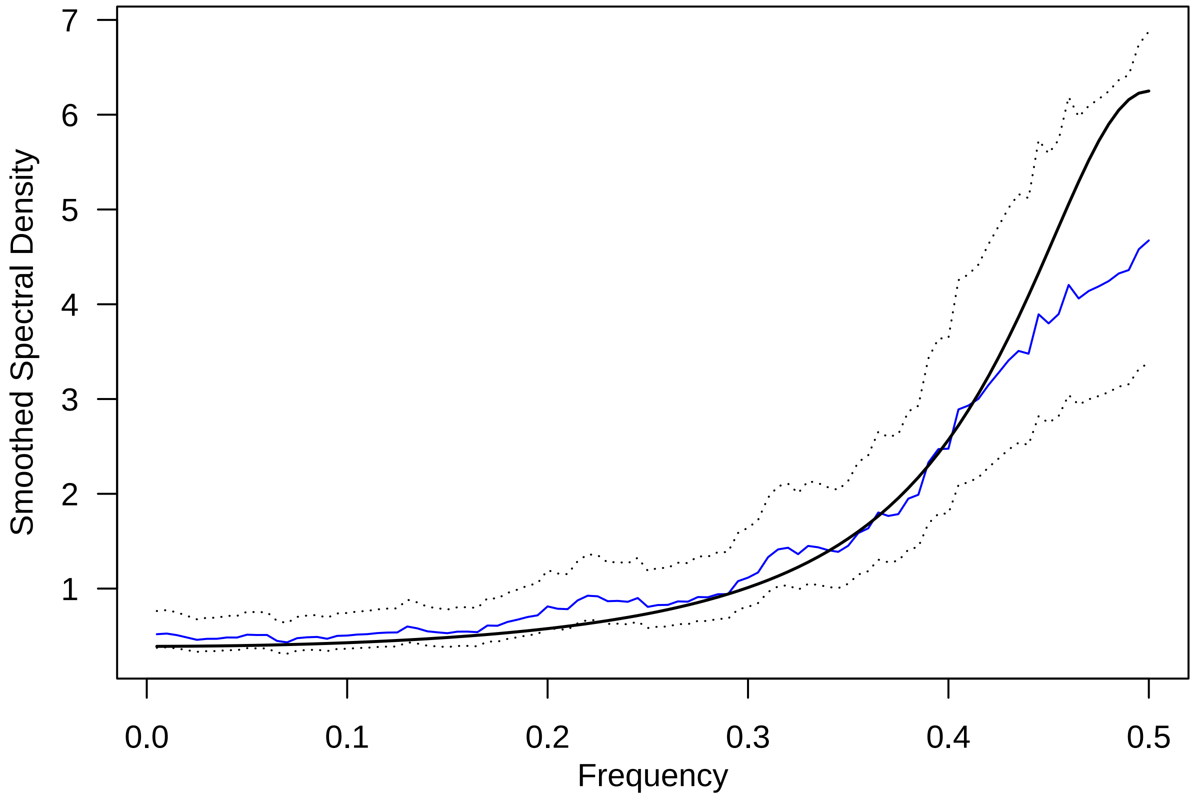

```
par(las = 1, mar = c(4, 4, 1, 0.6), mgp = c(2, 1, 0))
sp <- spec(y[, 1], log = 'no', xlab = 'Frequency',
           ylab = 'Sample Spectral Density', sub = '', main = "",
           col = "red")
lines(sp$freq, ARMAspec(model = list(ar = phi), freq = sp$freq,
                        plot = F)$spec, lwd = 1.5)
abline(h = 0, col = "gray")
k = kernel("daniell", m = 5)
sp1 <- spec(y[, 1], kernel = k, log = 'no', plot = F)
lines(sp1$freq, sp1$spec, col = "blue")
k = kernel("daniell", m = 15)
sp2 <- spec(y[, 1], kernel = k, log = 'no', plot = F)
lines(sp2$freq, sp2$spec, col = "green")
legend("topleft", legend = c(0, 5, 15),
       col = c("red", "blue", "green"), title = "m",
       lty = 1, bty = "n")
```




```r
par(las = 1, mar = c(4, 4, 1, 0.6), mgp = c(2, 1, 0))
k <- kernel('daniell', m = 15)
sp <- spec(y[, 1], kernel = k, sub = '', xlab = 'Frequency',
           ylab = 'Log(Smoothed Spectral Density)', ci.plot = T, ci.col = "blue",
           col = "blue", main = "")
lines(sp$freq, ARMAspec(model = list(ar = phi), sp$freq, plot = F)$spec, lwd = 1.5)
```




```
par(las = 1, mar = c(4, 4, 1, 0.6), mgp = c(2, 1, 0))
k <- kernel('daniell', m = 15)
sp <- spec(y[, 1], kernel = k, sub = '', xlab = 'Frequency',
           ylab = 'Smoothed Spectral Density', ci.plot = T,
           col = "blue", main = "", log = "no")
lines(sp$freq, ARMAspec(model = list(ar = phi), sp$freq, plot = F)$spec, lwd = 1.5)
```



Spetral ANOVA example

```r
x <- c(1, 2, 3, 2, 1)
c1 <- cos(2 * pi * (1:5) * (1 / 5)); s1 <- sin(2 * pi * (1:5) * (1 / 5))
c2 <- cos(2 * pi * (1:5) * (2 / 5)); s2 <- sin(2 * pi * (1:5) * (2 / 5))
omega1 <- cbind(c1, s1); omega2 <- cbind(c2, s2)
anova(lm(x ~ omega1 + omega2))
```

```
## Warning in anova.lm(lm(x ~ omega1 + omega2)): ANOVA F-tests on an essentially
## perfect fit are unreliable
```

```
## Analysis of Variance Table
##
## Response: x
##           Df  Sum Sq Mean Sq F value Pr(>F)
## omega1     2 2.74164 1.37082     NaN    NaN
## omega2     2 0.05836 0.02918     NaN    NaN
## Residuals  0 0.00000     NaN
```

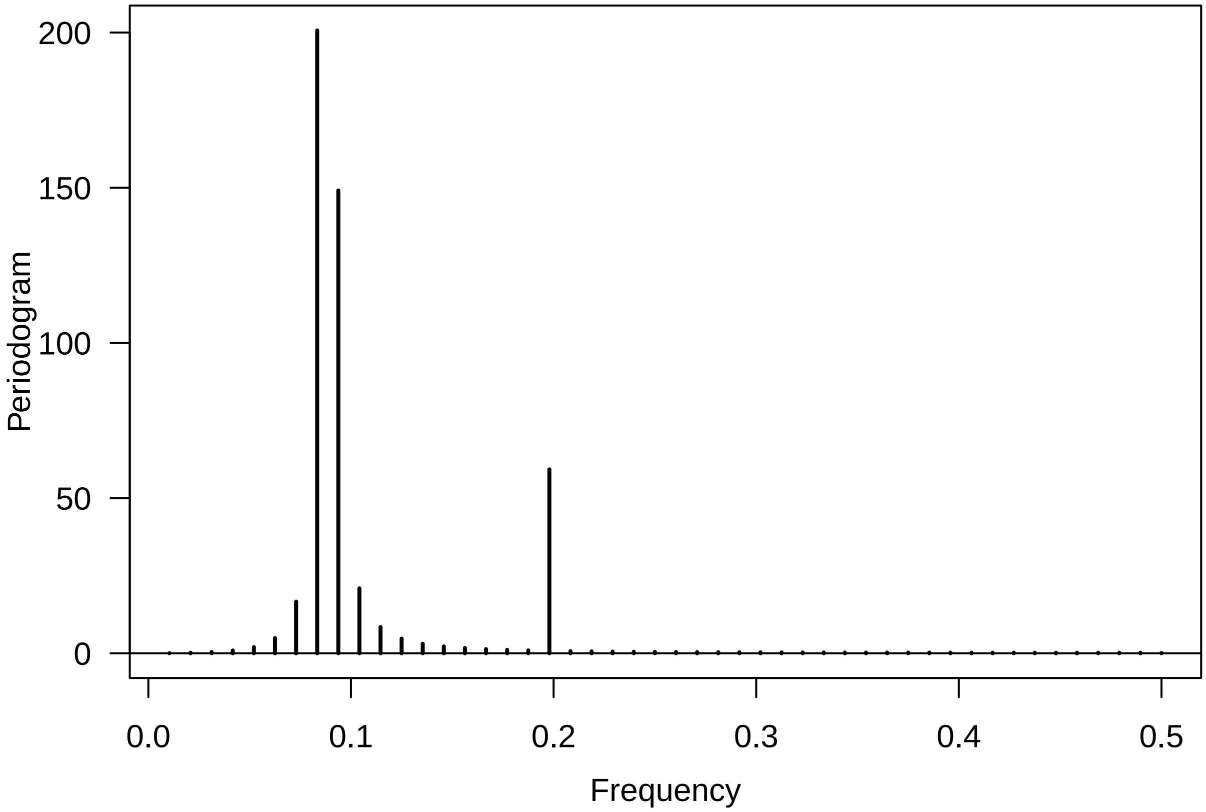
```r
Mod(fft(x))^2 / 5
```

```
## [1] 16.20000000  1.37082039  0.02917961  0.02917961  1.37082039
```

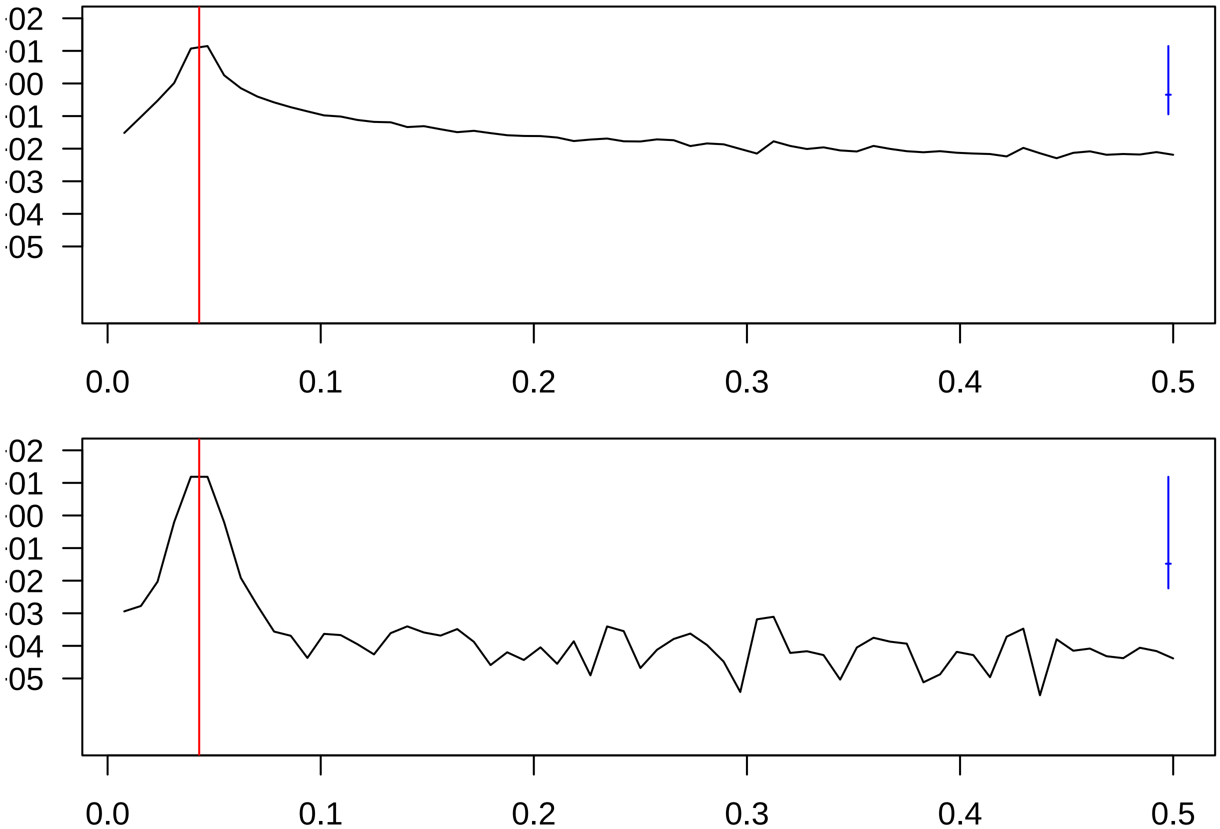
Leakage and Tapering

```r
t = 1:96; f1 = 0.088; f2 = 19/96
y <- 3 * cos(f1 * 2 * pi * t) + sin(f2 * 2 * pi * t)
```

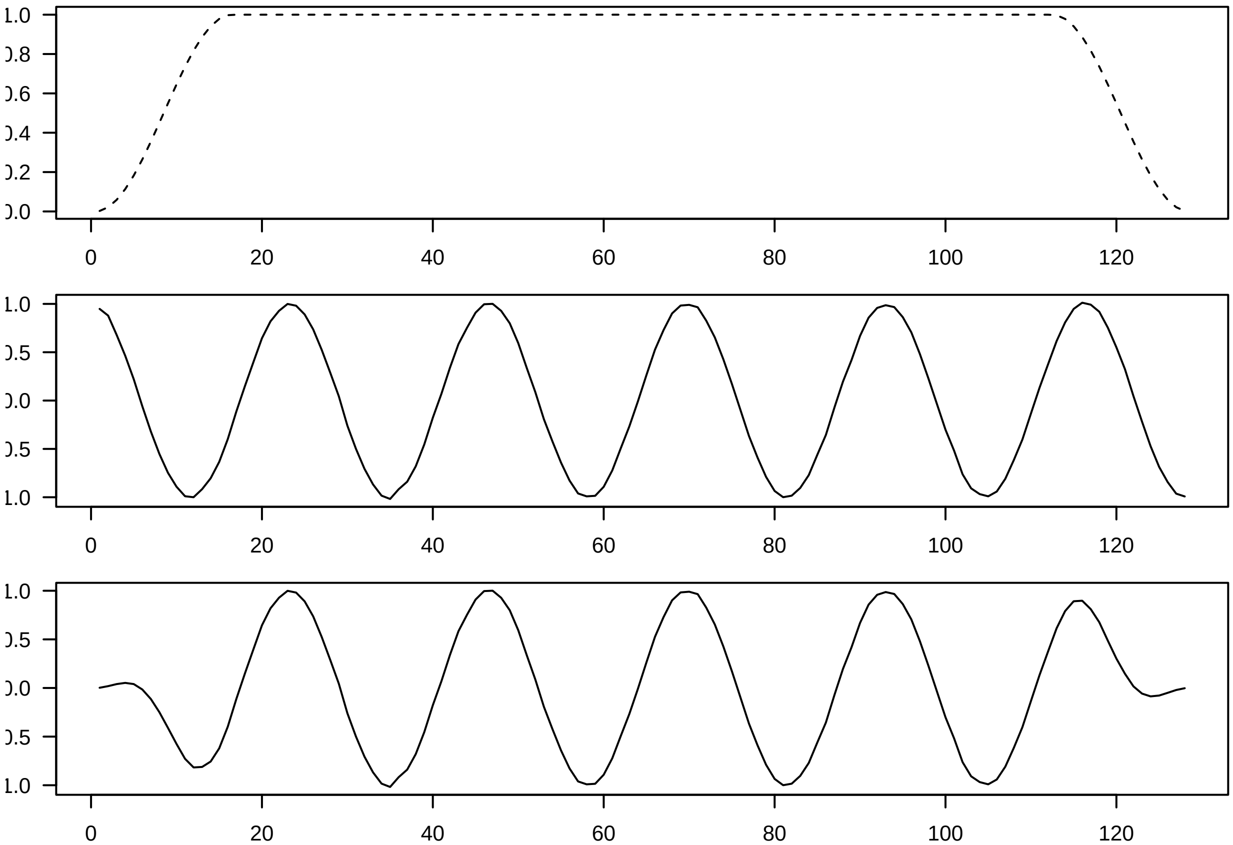
```r
par(las = 1, mar = c(4, 4, 1, 0.6), mgp = c(2.4, 1, 0))
periodogram(y)
abline(h = 0)
```




```r
source("plotspectrum.R")

w <- rnorm(128, sd = 0.01)
x5 <- cos(2 * pi * (5 / 128) * (1:128)) + w

par(mfcol = c(2, 1), mar = c(2, 2, 1, 1), las = 1)
#spectrum(x5, taper = 0, ylim = c(1e-7, 1e2), main = "")
x5h <- cos(2 * pi * (5.5 / 128) * (1:128)) + w
spectrum(x5h, taper = 0, ylim = c(1e-7, 1e2), main = "")
abline(v = 5.5 / 128, col = "red")
spectrum(x5h, taper = 0.5, ylim = c(1e-7, 1e2), main = "")
abline(v = 5.5 / 128, col = "red")
```

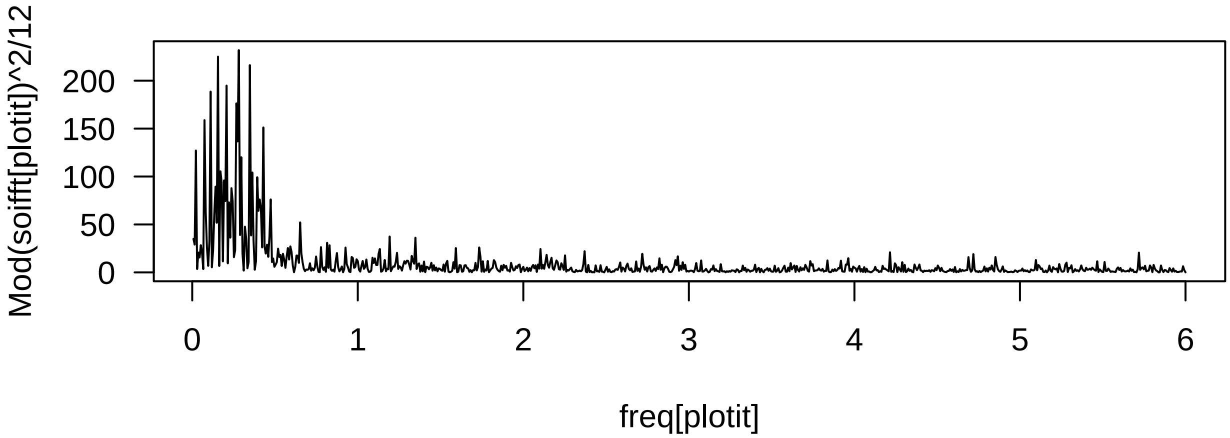


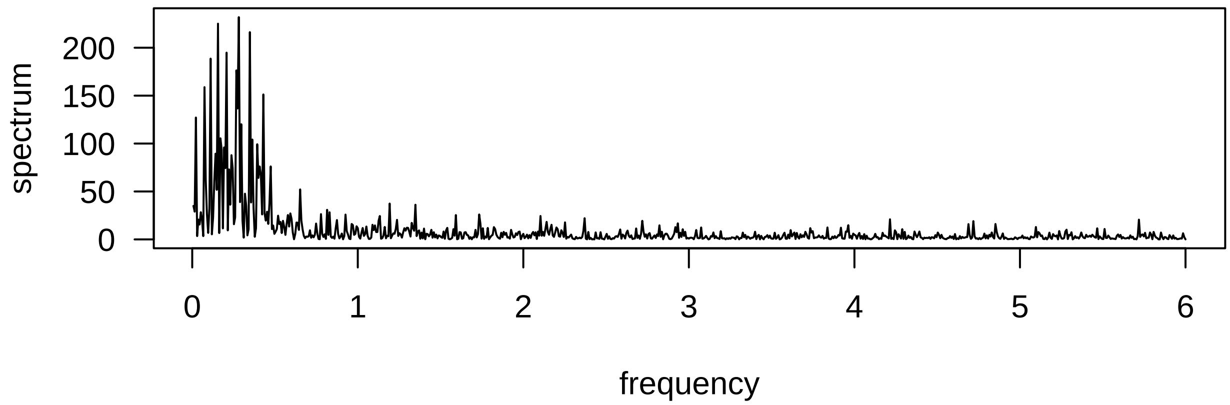

```r
par(mfcol = c(3, 1), mar = c(2, 2, 1, 1))
plot(taper(rep(1, 128), 0.25), type = "l", lty = 2)
plot(x5h, type = "l")
plot(taper(x5h, 0.25), type = "l")
```



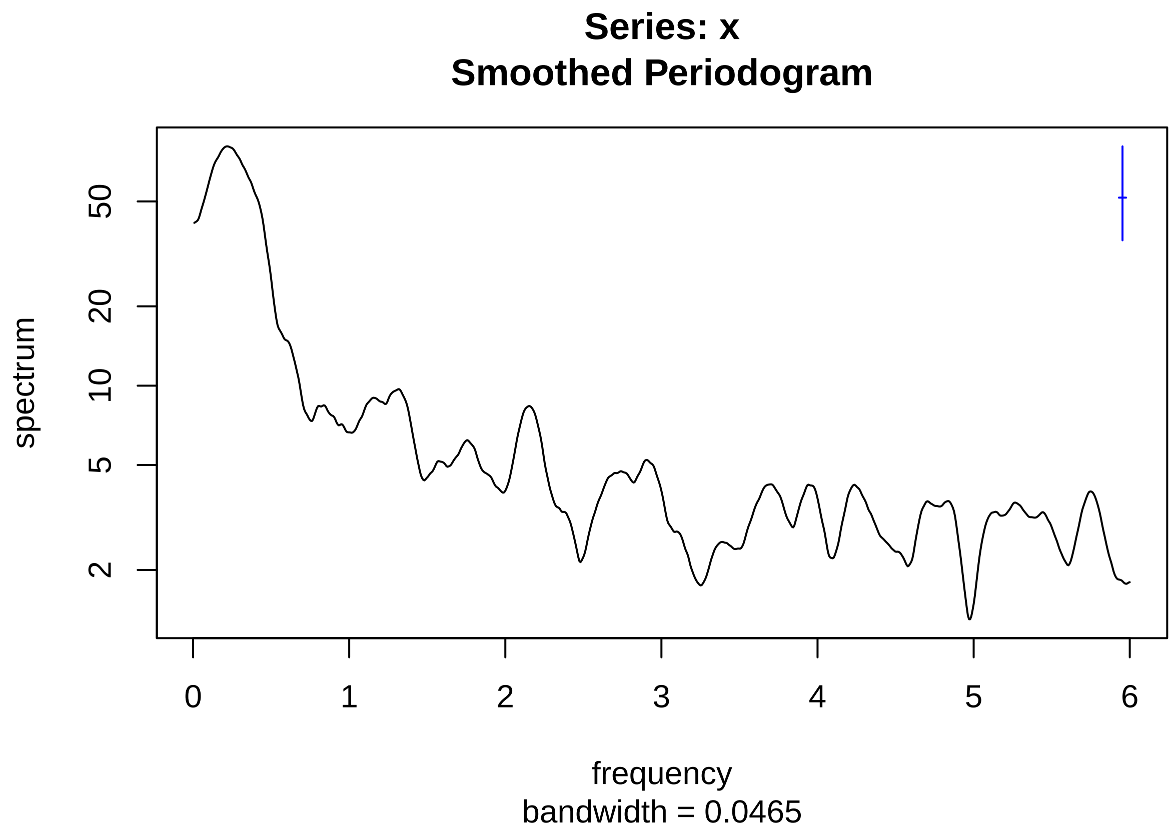
SOI example

```
url <- "https://www.stat.ncsu.edu/people/bloomfield/courses/ST730/data/soi2010.txt"

soi_raw <- read.table(url, header = T)
soi <- ts(c(t(soi_raw[, -1])), start = c(1876, 1),
          frequency = 12)
par(mfcol = c(2, 1), las = 1, mar = c(4, 4, 1, 0.6))
# Use fft() to calculate the periodogram directly; note that
# frequencies are expressed in cycles per year, and the
# periodogram values are similarly scaled by 12:
freq <- 12 * (0:(length(soi) - 1)) / length(soi)
plotit <- (freq > 0) & (freq <= 6)
soifft <- fft(soi) / sqrt(length(soi))
plot(freq[plotit], Mod(soifft[plotit])^2 / 12, type = "l")
# Use spectrum(); override some defaults to make it match:
spectrum(soi, log = "no", fast = FALSE, taper = 0, detrend = FALSE,
         main = "")
```

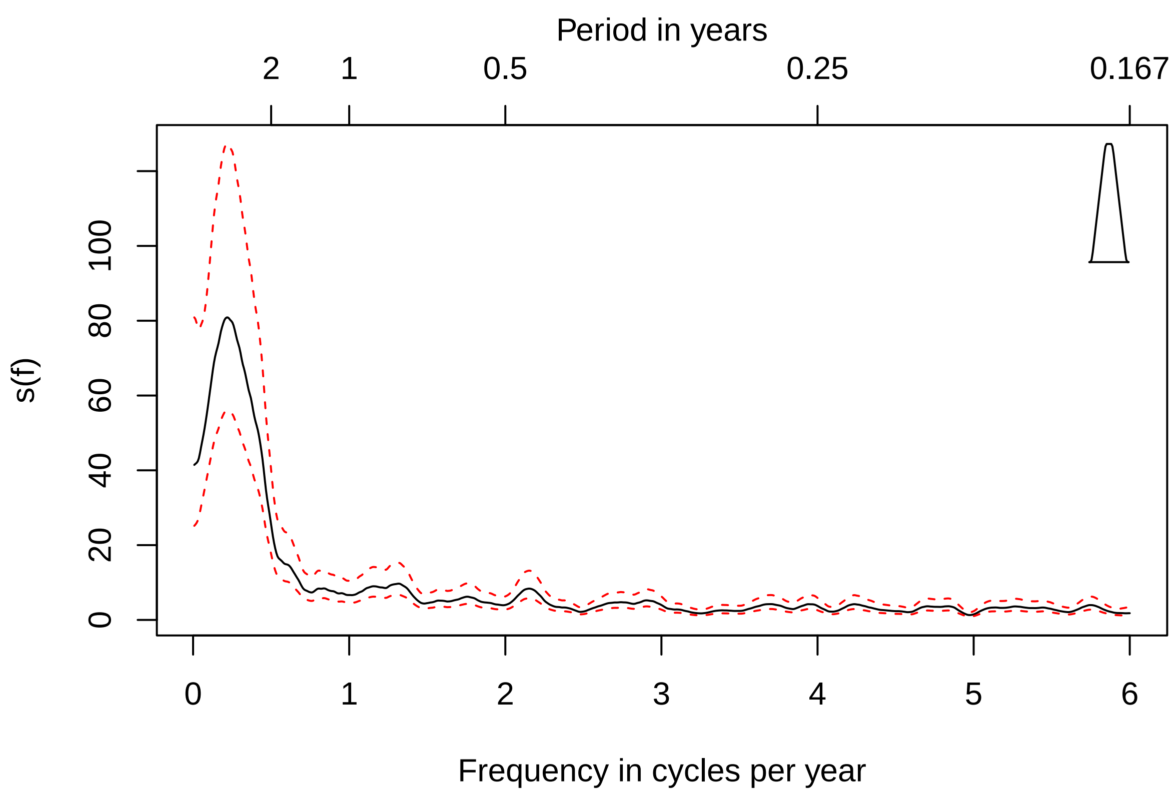




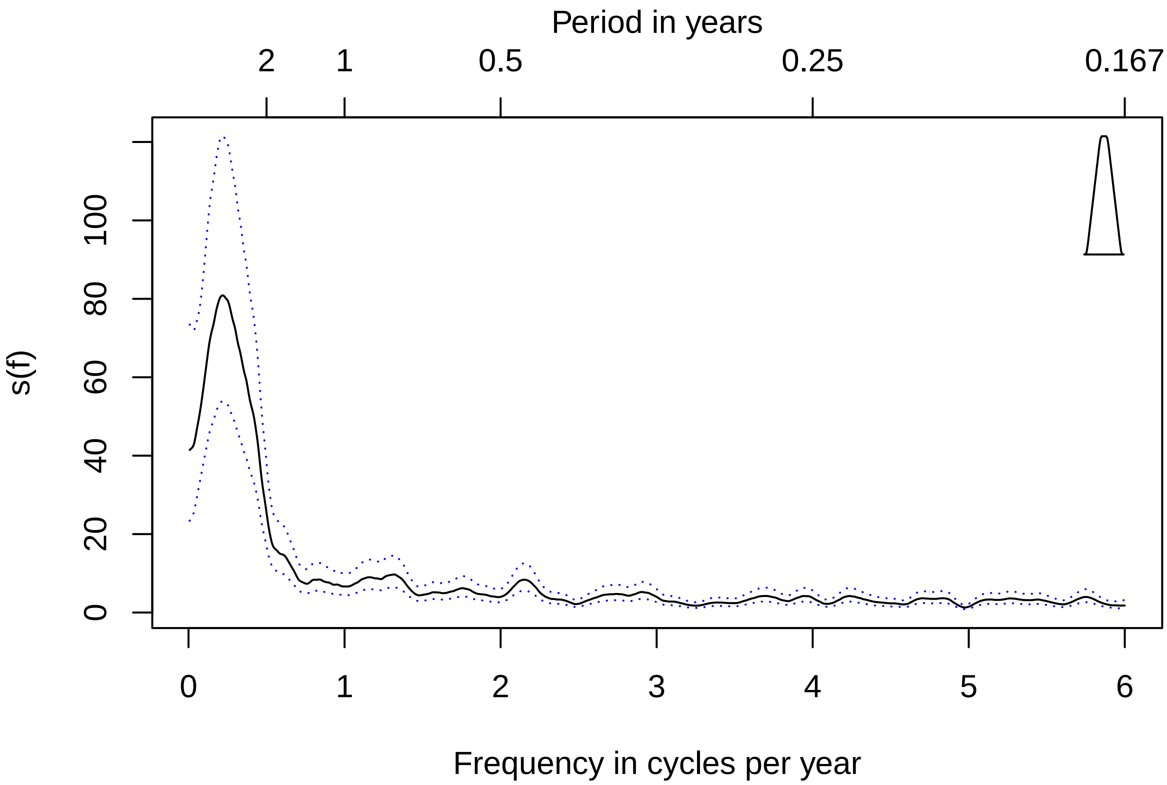

```
source("plotspectrum.R")
soiAdj = residuals(lm(soi ~ as.factor(cycle(soi))))
soiAdj = ts(soiAdj, frequency = frequency(soi), start = start(soi))
k = kernel("modified.daniell", m = c(6, 9))
soiAdj.md = spectrum(soiAdj, k, fast = FALSE, detrend = FALSE, taper = 0)
```




```
plotspectrum(soiAdj.md, log = "n", ci = 0.95, ci.type = "chisquare",
unit.time = "year")
```



```
plotspectrum(soiAdj.md, log = "n", ci = 0.95, ci.type = "lognormal",
unit.time = "year")
```



References

Cryer, Jonathan D, and Kung-Sik Chan. 2008. *Time Series Analysis: With Applications in r*. Vol. 2. Springer.

Shumway, Robert H. n.d. *Time Series Analysis and Its Applications*. Vol. 3. Springer.