Lecture 8

Introduction to Design of Experiments

Reading: Oehlert Chapters 1, 2; Dean-Voss-Draguljić Chapters 1, 2

DSA 8020 Statistical Methods II
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Definitions and Preliminaries

History of Experimental Design

Fundamental Principles of Experimental Design

Experimental Unit

Agenda

1. **Definitions and Preliminaries**
2. **History of Experimental Design**
3. **Fundamental Principles of Experimental Design**
4. **Experimental Unit**

Steps for Planning, Conducting and Analyzing an Experiment

- Statement of the problem
- Choice of factor
- Selection of the response variable(s)
- Choice of design
- Conducting the experiment
- Statistical analysis
- Drawing conclusions

Example: Battery Experiment [Dean-Voss-Draguljić p. 24]

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- **Specific question**: How do battery types vary with respect to life-per-unit cost?
- **Response**: Time (per unit cost) to exhaust battery under standard load
- **Comparative**: Difference between 4 battery types
- **Controlled**: All compared using the same device
- **Replication**: Four batteries of each type tested

Some Definitions

- **Factor:** variable whose influence upon a response variable is being studied in the experiment
- **Factor level:** numerical values or settings for a factor
- **Treatment:** set of values for all factors
- **Experimental unit:** object to which a treatment is applied
- **Randomization:** using a chance mechanism to assign treatments to experimental units

Main Elements of An Experiment

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An experiment applies treatments to experimental units and measures responses.

- Want to learn about **treatments** (e.g., dose of drug; nano-tech coating for a fabric)
- **Responses** tell us how the treatment worked (patient get better; stain resistance)
- Experimenter assigns treatments to **experimental units** (e.g., a patient; a bolt of fabric)

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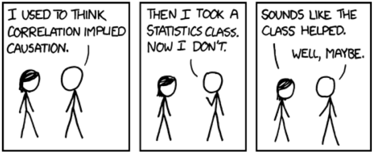
Observational vs. Experimental Studies

- An observational study has the same triple of treatment, unit, and response, but one **observes** the assignment of treatments to units (e.g., human health studies on cigarette smoke and adverse health effects)
- What makes an experimental study special is **control**. The experimenter gets to control the assignment of treatments to the experimental units
- Experiments can make causal inference while observational studies find association



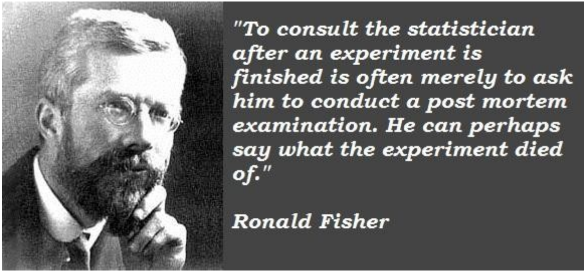
Source: Slide 5 at `http://users.stat.umn.edu/~gary/classes/5303/lectures/Introduction.pdf`

Why Designed Experiments?

- Design for direct comparison of treatments
- Design to reduce bias in comparisons
- Design to reduce and estimate the variability

A Brief History of Experimental Design

- 1. Agricultural Era:
  - R.A. Fisher, Rothamsted Agricultural Experimental Station (1930, England)
  - Introduced statistical experimental design and data analysis. Summarized the fundamental principles: **replication**, **randomization**, and **blocking**
  - An influential book, The Design of Experiments



A Brief History of Experimental Design Cont'd

- 2. Industrial Era:
  - Process modeling and optimization
  - G. E. P. Box and coworkers in chemical industries and other processing industries
  - Empirical modeling, response surface methodologies, central composite design
- 3. Quality Era:
  - Quality improvement and variation reduction
  - G. Taguchi and robust parameter design

A Brief History of Experimental Design Cont'd

- 4. Current State of Experimental Design:
  - Popular outside statistics, and an indispensable tool in many scientific/engineering endeavors
  - New challenges:
    - Large and complex experiments, e.g., screening design in pharmaceutical industry, experimental design in biotechnology
    - **Computer experiments:** efficient ways to model complex systems based on computer simulation
    - ...

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Fundamental Principles: Replication, Randomization, and Blocking

Replication

- Each treatment is applied to (experimental) units that are representative of the population
- Enable the estimation/quantification of **experimental error** using standard deviation
- Decrease variance of estimates and increase the power to detect significant differences: for independent $y_i's$,

$$\mathrm{Var}(\frac{1}{n}\sum_{i=1}^{n} y_i) = \frac{1}{n}\mathrm{Var}(y_1)$$

Randomization

Use of a chance mechanism such as random number generators to assign treatments to (experimental) units. It has the following advantages:

- Protect against latent variables or “lurking” variables
- Reduce influence of subjective bias in treatment assignments (e.g., clinical trials)
- Ensure validity of statistical inference

Blocking

A **block** refers to a collection of homogeneous units. Effective blocking : larger between-block variations than within-block variations.

Examples: hours, batches, lots, pairs of twins.

- Run and compare treatments within the same blocks to eliminate block-block variation and reduce variability of treatment effects estimates
- Block what you can and randomize what you cannot

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Experimental Unit

- Perhaps the most important concept in statistical design
- The experimental unit is the unit (subject, plant, pot, animal) which is randomly assigned to a treatment
- The experimental unit *defines the unit to be replicated to increase degrees of freedom*

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Experimental Units vs Measurement Units

If a group of "units" must have the same treatment, they are likely measurement units (MUs) rather than experimental units (EUs)

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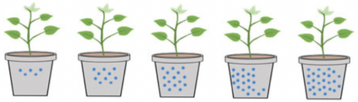
- Fertilizer is applied to the pots. Plants are not the EUs



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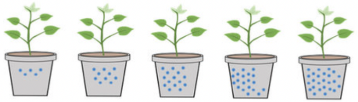

- Different food placed in tanks containing the fish. Fish are not the EUs

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Plans for the Next Three Weeks

- Completely Randomized Designs
- Randomized Complete Block Designs, Factorial Designs, and Split-Plot Designs
- Random and Mixed Effects Models, Computer Experiments

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