Spatio-Temporal Data

Research Questions and Objectives

Spatio-Temporal Models and Methods

Lecture 15

Spatio-Temporal Models

MATH 8090 Time Series Analysis

Week 15

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Agenda

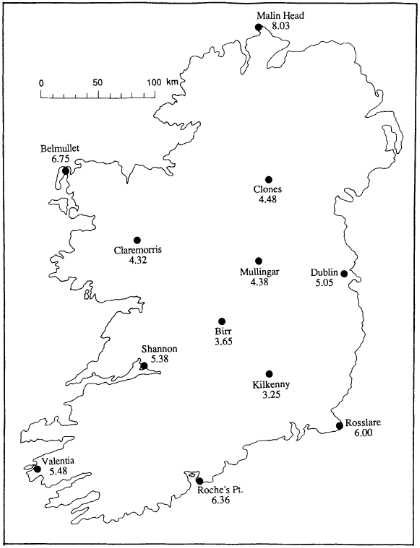
1 Spatio-Temporal Data

2 Research Questions and Objectives

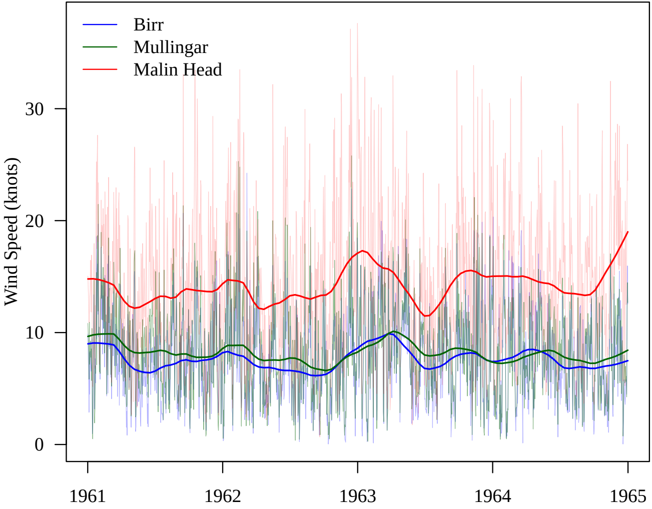
3 Spatio-Temporal Models and Methods

Ireland Wind Data

- Daily average wind speeds from 1961 to 1978
- Twelve synoptic meteorological stations in the Republic of Ireland
- Analyzed by Haslett and Raftery (1989), Gneiting (2002), Stein (2005), and many others

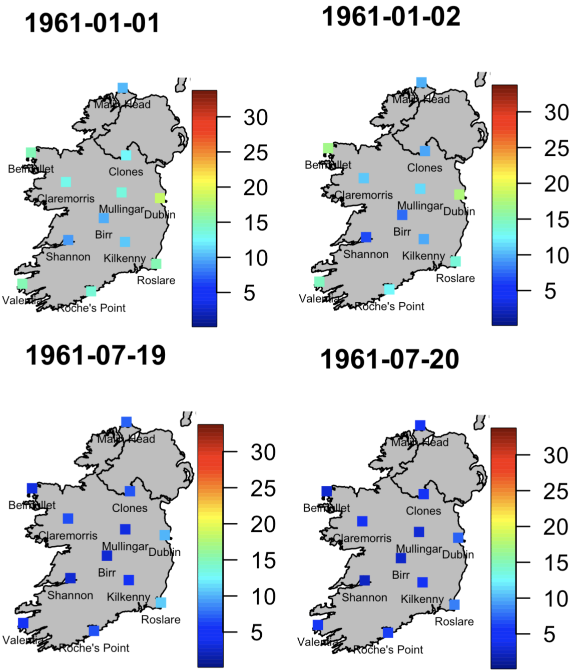


Spatially Structured Multivariate Time Series



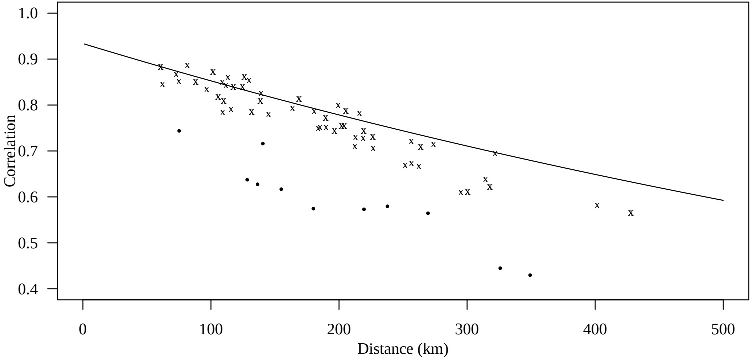
Temporal dynamics are similar for the nearby pair, while those of the far-away site are less alike

Spatial Snapshots Over Time

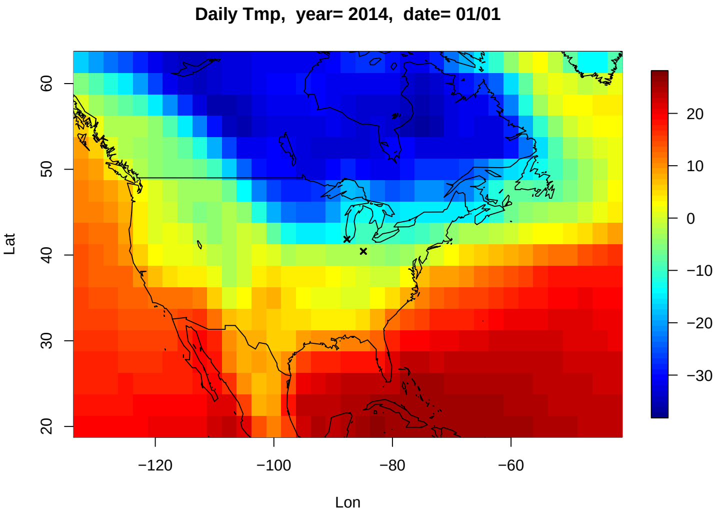


Spatial Correlation

- **Deseasonalize** wind speeds by removing the `LOWESS` seasonal cycle
- **Compute station distances and correlations**, treating ROS separately
- **Plot correlation vs. distance** with an exponential decay fit



Spatio-Temporal Data



Spatio-Temporal Data

- Say Y_i is observed at location $\boldsymbol{s}_i$ and time t_i.
  Example: Y_i is the air pollution level in Los Angeles $(\boldsymbol{s}_i)$ on August 1, 2020 (t_i)
- We may also have covariates, $\boldsymbol{X}_i = (X_{i1}, \cdots, X_{ip})^{\mathrm{T}}$
- An example of X_i:
  1. X_{i1} is the month of t_i
  2. X_{i2} is the elevation of $\boldsymbol{s}_i$
  3. X_{i3} is the wind speed at time t_i and location $\boldsymbol{s}_i$

Research Objectives

Spatio-Temporal Data
Research Questions and Objectives
Spatio-Temporal Models and Methods

- **Modeling**: Estimate covariate effects while properly accounting for underlying spatial and temporal correlation structures
- **Spatial Prediction**: Produce reliable predictions in unsampled regions using the fitted spatiotemporal model
- **Short-Term Forecasting**: Forecast near-term system behavior to support real-time or operational decisions
- **Temporal Dynamics**: Detect, quantify, and interpret changes over time, including trends and shifts in variability

General Model

We decompose the spatiotemporal process into a mean component, correlated residuals, and uncorrelated noise:

$$Y(\boldsymbol{s},t) = \mu(\boldsymbol{s},t) + \omega(\boldsymbol{s},t) + \varepsilon(\boldsymbol{s},t).$$

- The mean term $\mu(\boldsymbol{s},t)$ is modeled as $\boldsymbol{X}(\boldsymbol{s},t)^{\mathrm{T}}\boldsymbol{\beta}(\boldsymbol{s},t)$, capturing systematic effects
- The residual process $\omega(\boldsymbol{s},t)$ represents spatially and temporally correlated variation
- The “nugget” term $\varepsilon(\boldsymbol{s},t) \sim \mathrm{N}(0,\tau^2(\boldsymbol{s},t))$ captures measurement error or microscale variation and is assumed independent over space and time

Spatio-Temporal Data

Research Questions and Objectives

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Space-Time Domains

We consider two common frameworks for modeling spatio-temporal processes defined over a **continuous spatial domain** $\mathcal{S}$:

- **Continuous Space + Discrete Time**
  - Space is treated as continuous, but the process is observed or evolves at discrete time points (e.g., daily, monthly, yearly)
  - Suitable for dynamic space-time modeling, forecasting, and state-space formulations
- **Continuous Space + Continuous Time**
  - Both spatial location and time evolve continuously
  - Used in geostatistical space-time modeling with smooth temporal evolution

Spatio-Temporal Exploratory Analysis

Given a spatio-temporal data $\{y(\boldsymbol{s}_i, t_j)\}_{i=1,\cdots,n}^{j=1,\cdots,m}$

- Do a times series analysis at each site: check the mean structure $\mu(t)$ and ACF $\gamma(h)$
- Do a separate spatial analysis at each time point: check mean structure $\mu(\boldsymbol{s})$ and the variogram
- Plot the time series results over space and the spatial results over time, check for differences over space and time

Spatio-Temporal Random Effects Model

We can further decompose the spatio-temporal correlated term $\omega(\boldsymbol{s}, t)$ into additive spatial and temporal random effects, i.e.,

$$\omega(\boldsymbol{s}, t) = \alpha(\boldsymbol{s}) + \beta(t)$$

- The spatial terms have $\mathrm{Var}(\alpha(\boldsymbol{s})) = \sigma_{\boldsymbol{s}}^2$ and spatial correlation
- The temporal terms have $\mathrm{Var}(\beta(t)) = \sigma_t^2$ and autocorrelation
- The total variance is $\mathrm{Var}(Z_i) = \sigma_{\boldsymbol{s}}^2 + \sigma_t^2 + \tau^2$. The proportion of variance explained by spatial and temporal variation are

$$\frac{\sigma_{\boldsymbol{s}}^2}{\sigma_{\boldsymbol{s}}^2 + \sigma_t^2 + \tau^2} \text{ and } \frac{\sigma_t^2}{\sigma_{\boldsymbol{s}}^2 + \sigma_t^2 + \tau^2}$$

Note: This model cannot account for space-time interaction

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Dynamic Space-Time Models

Spatial Autoregressive Model

- The previous model has the same spatial trend for each time, and the same time trend for each location ⇒ A more flexible model may be needed

- A spatial AR1 model begins with the AR1 at each site,

$$\omega(\boldsymbol{s},t)|\omega(\boldsymbol{s},t-1) = \psi\omega(\boldsymbol{s},t-1) + e(\boldsymbol{s},t),$$

  where $e(\boldsymbol{s},t)$ are independent over time

- A spatial extension allows $e(\boldsymbol{s},t)$ to be spatially correlated

Dynamic Space-Time Regression Models

Here space is viewed as continuous, but time is assumed to be discrete

- Observation Equation:

$$y_t(s) = \boldsymbol{X}_t^{\mathrm{T}}(\boldsymbol{s})\boldsymbol{\beta}_t + \omega_t(s) + \epsilon_t(s), t = 1, \ldots, m,$$

  where $\varepsilon_t(s) \overset{ind.}{\sim} \mathrm{N}(0, \tau_t^2)$

- Evolution Equation:

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \overset{i.i.d.}{\sim} \mathrm{N}(\boldsymbol{0}, \Sigma_\eta)$$
$$\omega_t(\boldsymbol{s}) = \omega_{t-1}(\boldsymbol{s}) + e_t(s), \qquad e_t(s) \overset{ind.}{\sim} \mathrm{GP}(0, C_t(\cdot, \boldsymbol{\theta}_t))$$

- Initial State:

$$\boldsymbol{\beta}_0 \sim \mathrm{N}(\boldsymbol{\mu}_0, \Sigma_0)$$
$$\omega_0 = 0$$

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Geostatistical Space-Time Models

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Space–Time Covariance Functions

We now consider the setting where both **space and time are continuous**. Constructing a valid space-time covariance function is central to geostatistical space-time modeling

Key Question: What constitutes a sensible and scientifically meaningful space–time covariance function?

- Smoothness Controls how quickly correlations decay with spatial and temporal separation
- Space–Time Asymmetry Allows correlation from $(\boldsymbol{s}_1, t_1)$ to $(\boldsymbol{s}_2, t_2)$ to differ from the reverse ordering. Important for processes that propagate over time
- Markov Temporal Structure Many processes evolve over time with Markovian dynamics. Imposes temporal coherence and helps with computation

Properties of Space–Time Covariance Functions

- Fully Symmetric

$$\text{Cov}\{Y(\boldsymbol{s}_1, t_1), Y(\boldsymbol{s}_2, t_2)\} = \text{Cov}\{Y(\boldsymbol{s}_1, t_2), Y(\boldsymbol{s}_2, t_1)\}$$

  Rarely realistic for dynamic systems; symmetry rules out directional evolution or propagation

- Separable

$$\text{Cov}\{Y(\boldsymbol{s}_1, t_1), Y(\boldsymbol{s}_2, t_2)\} = C_S(\boldsymbol{s}_1, \boldsymbol{s}_2)\, C_T(t_1, t_2)$$

  Easy to estimate/compute, but overly restrictive-assumes spatial and temporal dependence operate independently

- Stationary

$$\text{Cov}\{Y(\boldsymbol{s}_1, t_1), Y(\boldsymbol{s}_2, t_2)\} = C(\boldsymbol{s}_1 - \boldsymbol{s}_2,\ t_1 - t_2)$$

  Dependence depends only on separations, not absolute location or time

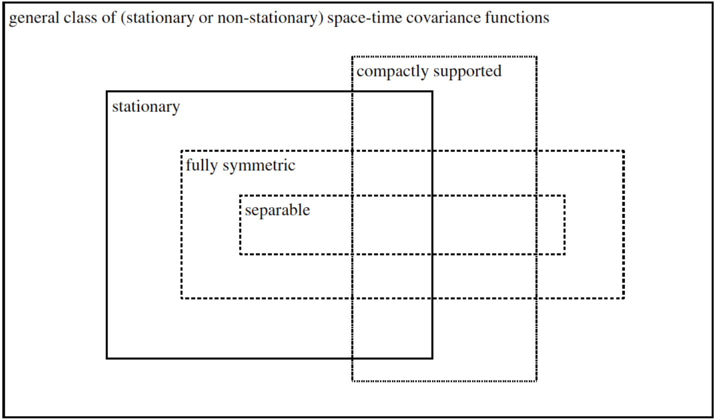
- Compactly Supported

$$\text{Cov}\{Y(\boldsymbol{s}_1, t_1), Y(\boldsymbol{s}_2, t_2)\} = 0 \quad \text{for large separations}$$

  Produces sparse covariance matrices → scalable for large datasets

Schematic Illustration

Gneiting *et al.* (07)



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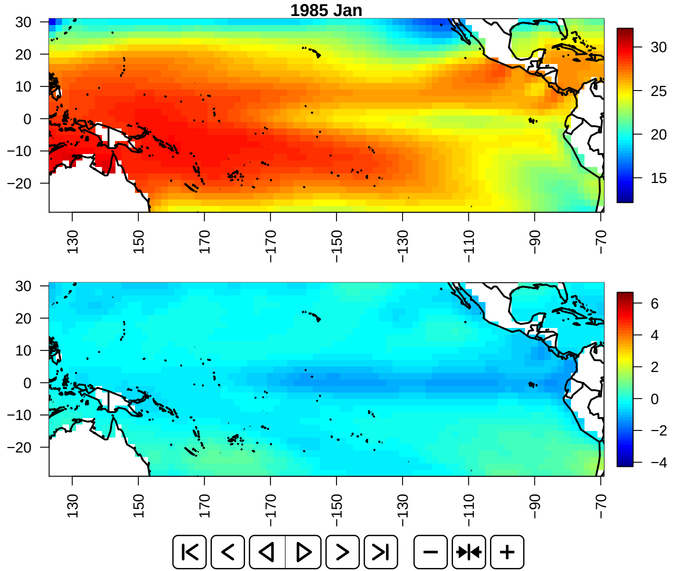
Spatio-Temporal Models and Methods

Summarizing Space-Time Fields Using Empirical Orthogonal Functions

Example: Monthly Sea Surface Temperatures



Sea Surface Temperatures and Anomalies

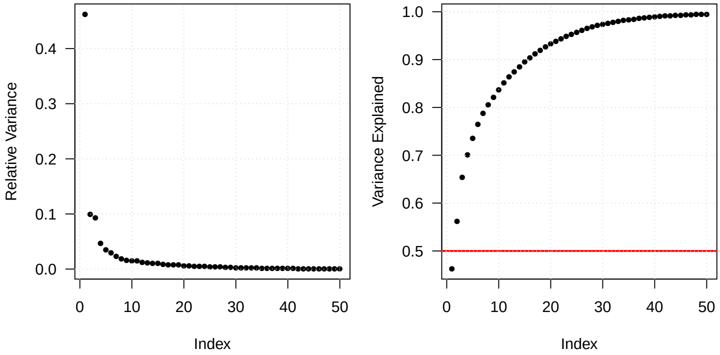
- The "data" are gridded at a $2°$ by $2°$ resolution from $124°E – 70°W$ and $30°S – 30°N$. The dimension of this SST data set is 2303 (number of grid points in space) $\times$ 552 (monthly time series from 1970 Jan. to 2015 Dec.)
- Sea-surface temperature anomalies are the temperature differences from the climatology (i.e. long-term monthly mean temperatures)
- We will demonstrate the use of Empirical Orthogonal Function (EOF) analysis to uncover the low-dimensional structure of this spatio-temporal data set

The Emipirical Orthogonal Function (EOF) Decomposition

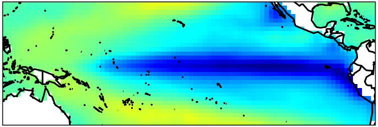
Empirical orthogonal functions (EOFs) are the geophysicist's terminology for the eigenvectors in the eigen-decomposition of an empirical covariance matrix. In its discrete formulation, EOF analysis is simply Principal Component Analysis (PCA). EOFs are usually used

- To find principal spatial structures
- To reduce the dimension (spatially or temporally) in large spatio-temporal datasets

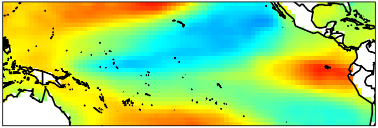
Screen Plot for EOFs



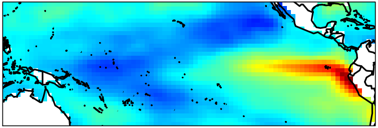
Perform EOF Decomposition and Plot the First Three Modes



EOF1: The classic ENSO pattern



EOF2: A modulation of the center



EOF3: Messing with the coast of SA and the Northern Pacific.

1998 Jan El Niño Event