Lecture 1

Introduction

STAT 8020 Statistical Methods II
August 20, 2020

Whitney Huang
Clemson University

Who is the instructor?

Who am I?

- **Second year** Assistant Professor of Applied Statistics and Data Science
- Born in Laramie, Wyoming, grew up in Taiwan

- With a B.S. in Mechanical Engineering, switched to Statistics in graduate school
- Got a Ph.D. (Statistics) in 2017 at Purdue University.

How to reach me?

- **Email:** wkhuang@clemson.edu
- **Office**: O-221 Martin Hall
- **Office Hours**: TR 11:00am – 12:00pm and by appointment

Class Policies / Schedule

Logistics

- We will meet TR 12:30pm – 1:45pm via Zoom
- There will be three online exams and a (comprehensive) online final. The (tentative) dates for the three exams are:
  - **Exam I**: Sept. 24, Thursday
  - **Exam II**: Oct. 20, Tuesday
  - **Exam II**: Nov. 12, Tuesday
  - The **Final Exam** will be given on Wednesday, Dec. 7, 3:00 pm -5:30 pm.
- No classes on Nov. 3 (Fall Break) & 26 (Thanksgiving)

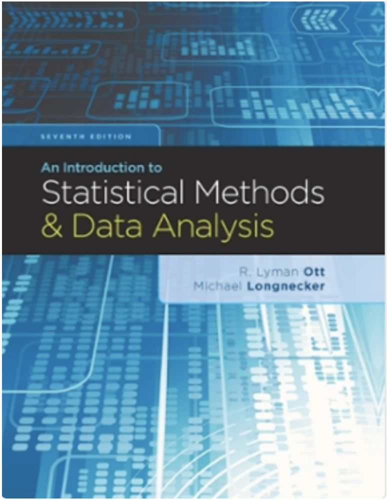
Class Website

CANVAS and my teaching website (link: https://whitneyhuang83.github.io/STAT8020/Fall2020/stat8020_2020Fall.html)

- Course syllabus [Link] / Announcements
- Lecture slides/notes
- Homework assignments
- Exam and homework schedule
- Data sets for lectures and homework
- R code

Recommended Textbook

An Introduction to Statistical Methods and Data Analysis, 6th Edition. **Lyman Ott and Micheal T. Longnecker, Duxbury, 2010**; **ISBN-13:** 978-1305269477



Evaluation

- Grade Distribution:

Exam I:	25%
Exam II	25%
Exam III	25%
Final Exam	25%

- Letter Grade:

>= 90.00	A
$88.00 \sim 89.99$	A-
$85.00 \sim 87.99$	B+
$80.00 \sim 84.99$	B
$78.00 \sim 79.99$	B-
$75.00 \sim 77.99$	C+
$70.00 \sim 74.99$	C
$68.00 \sim 69.99$	C-
<= 67.99	F

Tentative Topics and Dates

Part I: Regression Analysis (August 20 – September 24)

- Review of Simple Linear Regression
- Multiple Linear Regression: Statistical Inference; Model Selection and Diagnostics
- Regression Models with Quantitative and Qualitative Predictors
- Nonlinear and Non-parametric Regression

Part II: Categorical Data Analysis (September 29 – October 20)

- Review of Inference for Proportions and Contingency Tables
- Relative Risk and Odds Ratio
- Logistic Regression and Poisson Regression

Tentative Topics and Dates cont'd

Part III: Experimental Design (October 22 – November 12)

- Introduction to Experimental Design: Principles and Techniques
- Completely randomized Designs, Block Designs, Latin Square Designs, Nested and Split-Plot Designs
- Computer experiments

Part IV: Multivariate, Spatial and Time Series Analysis (November 17 – December 3)

- Discriminate Analysis, Principle Components Analysis, and Cluster Analysis
- Basic of time series and spatial data analysis

Computing

We will use software to perform statistical analyses. The recommended software for this course are `JASP` and `R/Rstudio`

- **JASP**
  - a **free/open-source** graphical program for statistical analysis
  - available at `https://jasp-stats.org/`
- R / R Studio
  - a **free/open-source** programming language for statistical analysis
  - available at `https://www.r-project.org/` (`R`); `https://rstudio.com/` (`Rstudio`)

You are welcome to use a different package (e.g. SAS, JMP, SPSS, Minitab) if you prefer

Tell us about yourself

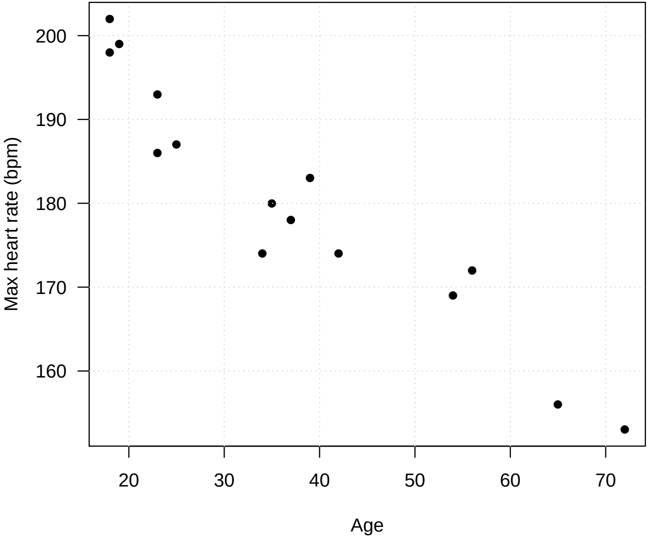
Tell us about yourself

- Your name
- Degree program
- Your background in Statistics/Computing

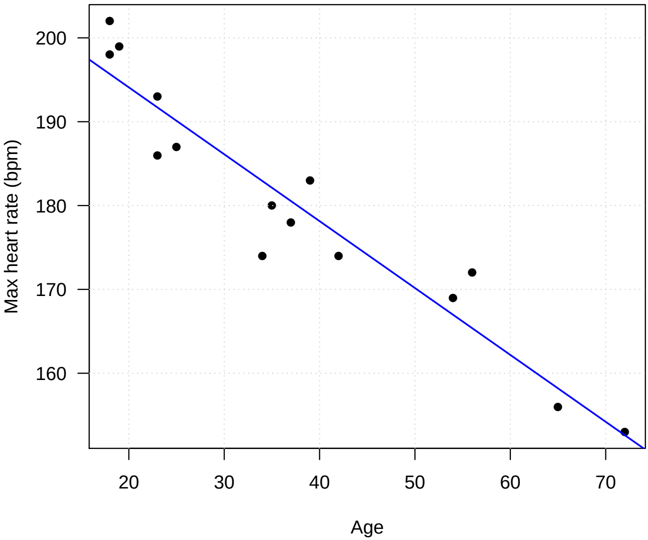
Review of Simple Linear Regression

What is Regression Analysis?

Regression analysis: A set of statistical procedures for estimating the relationship between response variable and predictor variable(s)



Scatterplot: Is Linear Trend Reasonable?



Simple Linear Regression (SLR)

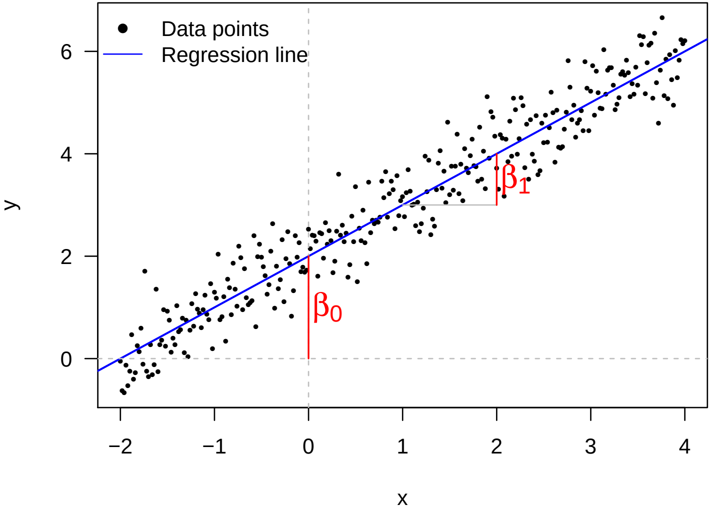
Y: dependent (response) variable; X: independent (predictor) variable

- In SLR we **assume** there is a **linear relationship** between X and Y:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

- We will need to estimate β_0 (intercept) and β_1 (slope)
- Then we can use the estimated regression equation to
  - make predictions
  - study the relationship between response and predictor
  - control the response
- Yet we need to quantify our uncertainty regarding the linear relationship

Regression equation: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$



Assumptions about ε

In order to estimate β_0 and β_1, we make the following assumptions about ε

- $\mathrm{E}[\varepsilon_i] = 0$
- $\mathrm{Var}[\varepsilon_i] = \sigma^2$
- $\mathrm{Cov}[\varepsilon_i, \varepsilon_j] = 0, \quad i \neq j$

Therefore, we have

$$\mathrm{E}[Y_i] = \beta_0 + \beta_1 X_i, \text{ and}$$
$$\mathrm{Var}[Y_i] = \sigma^2$$

The regression line $\beta_0 + \beta_1 x$ represents the **conditional expectation curve** whereas σ^2 measures the magnitude of the **variation** around the regression curve

Estimation: Method of Least Square

For the given observations $(x_i, y_i)_{i=1}^n$, choose β_0 and β_1 to minimize the *sum of squared errors*:

$$\mathrm{L}(\beta_0, \beta_1) = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

Solving the above minimization problem requires some knowledge from Calculus....

- $\hat{\beta}_1 = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{(X_i - \bar{X})^2}$
- $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$

We also need to **estimate** σ^2

- $\hat{\sigma}^2 = \frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-2}$, where $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

Properties of Least Squares Estimates

- **Gauss-Markov** theorem states that in a linear regression these least squares estimators

  1. **Are unbiased**, i.e.,
     - $\mathrm{E}[\hat{\beta}_1] = \beta_1; \mathrm{E}[\hat{\beta}_0] = \beta_0$
     - $\mathrm{E}[\hat{\sigma}^2] = \sigma^2$
  2. Have **minimum variance** among all unbiased linear estimators

Note that we do not make any distributional assumption on ε_i

Example: Maximum Heart Rate vs. Age

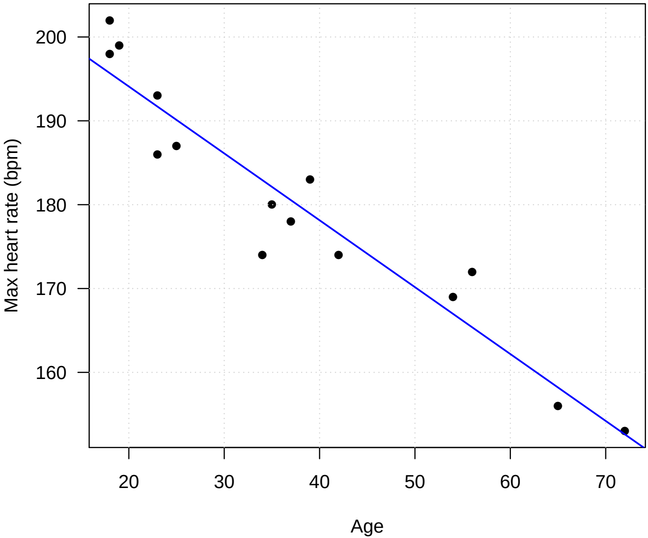
The maximum heart rate `MaxHeartRate` of a person is often said to be related to age `Age` by the equation:

$$\text{MaxHeartRate} = 220 - \text{Age}.$$

Suppose we have 15 people of varying ages are tested for their maximum heart rate (bpm) (link to the “dataset": http://whitneyhuang83.github.io/STAT8010/Data/maxHeartRate.csv)

1. Compute the estimates for the regression coefficients
2. Compute the fitted values
3. Compute the estimate for σ

Linear Regression Fit



Question: Is linear relationship between max heart rate and age reasonable? ⇒ Residual Analysis

Residuals

- The residuals are the differences between the observed and fitted values:

$$e_i = Y_i - \hat{Y}_i,$$

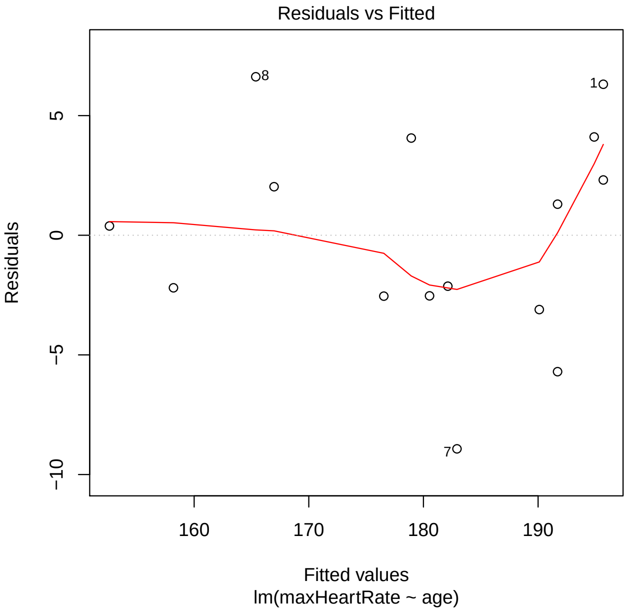
where $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

- e_i is NOT the error term $\varepsilon_i = Y_i - \mathrm{E}[Y_i]$

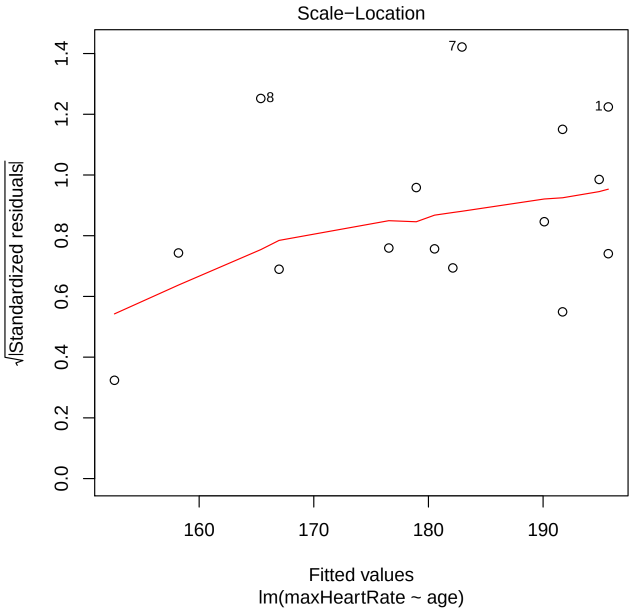
- Residuals are very useful in assessing the appropriateness of the assumptions on ε_i. Recall

  - $\mathrm{E}[\varepsilon_i] = 0$
  - $\mathrm{Var}[\varepsilon_i] = \sigma^2$
  - $\mathrm{Cov}[\varepsilon_i, \varepsilon_j] = 0, \quad i \neq j$

Residual Analysis



Residual Analysis



Summary

In this lecture, we reviewed

- Simple Linear Regression: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$
- Method of Least Square for parameter estimation
- Residual analysis to check model assumptions

Next time we will talk about

1. More on residual analysis
2. Normal Error Regression Model and statistical inference for β_0, β_1, and σ^2
3. Prediction