# MATH 8090: Spectral Analysis of Time Series II

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## Contents



### Spetral ANOVA example

```
x <- c(1, 2, 3, 2, 1)
x <- x - mean(x)
c1 <- cos(2 * pi * (1:5) * (1 / 5)); s1 <- sin(2 * pi * (1:5) * (1 / 5))
c2 <- cos(2 * pi * (1:5) * (2 / 5)); s2 <- sin(2 * pi * (1:5) * (2 / 5))
omega1 <- cbind(c1, s1); omega2 <- cbind(c2, s2)
anova(lm(x ~ omega1 + omega2))
```

```
## Warning in anova.lm(lm(x ~ omega1 + omega2)): ANOVA F-tests on an essentially
## perfect fit are unreliable
```

```
## Analysis of Variance Table
##
## Response: x
##           Df  Sum Sq Mean Sq F value Pr(>F)
## omega1     2 2.74164 1.37082     NaN    NaN
## omega2     2 0.05836 0.02918     NaN    NaN
## Residuals  0 0.00000     NaN
```

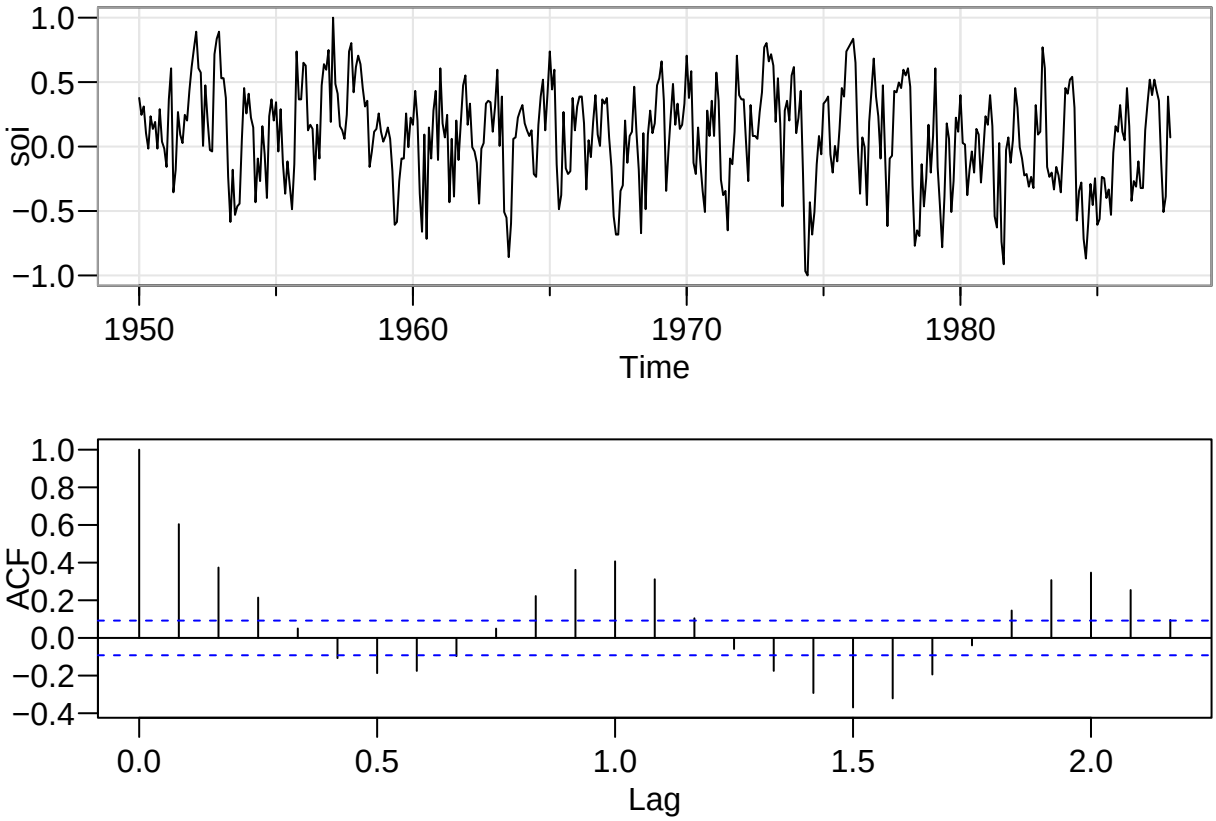
```r
Mod(fft(x))^2 / 5
```

```
## [1] 9.860761e-33 1.370820e+00 2.917961e-02 2.917961e-02 1.370820e+00
```

## SOI example

### Plot the time series and ACF

```r
library(astsa)
par(mgp = c(2.2, 1, 0), mar = c(3.5, 4, 0.8, 0.6), las = 1, mfrow = c(2, 1))
tsplot(soi)
acf(soi, main = "")
```

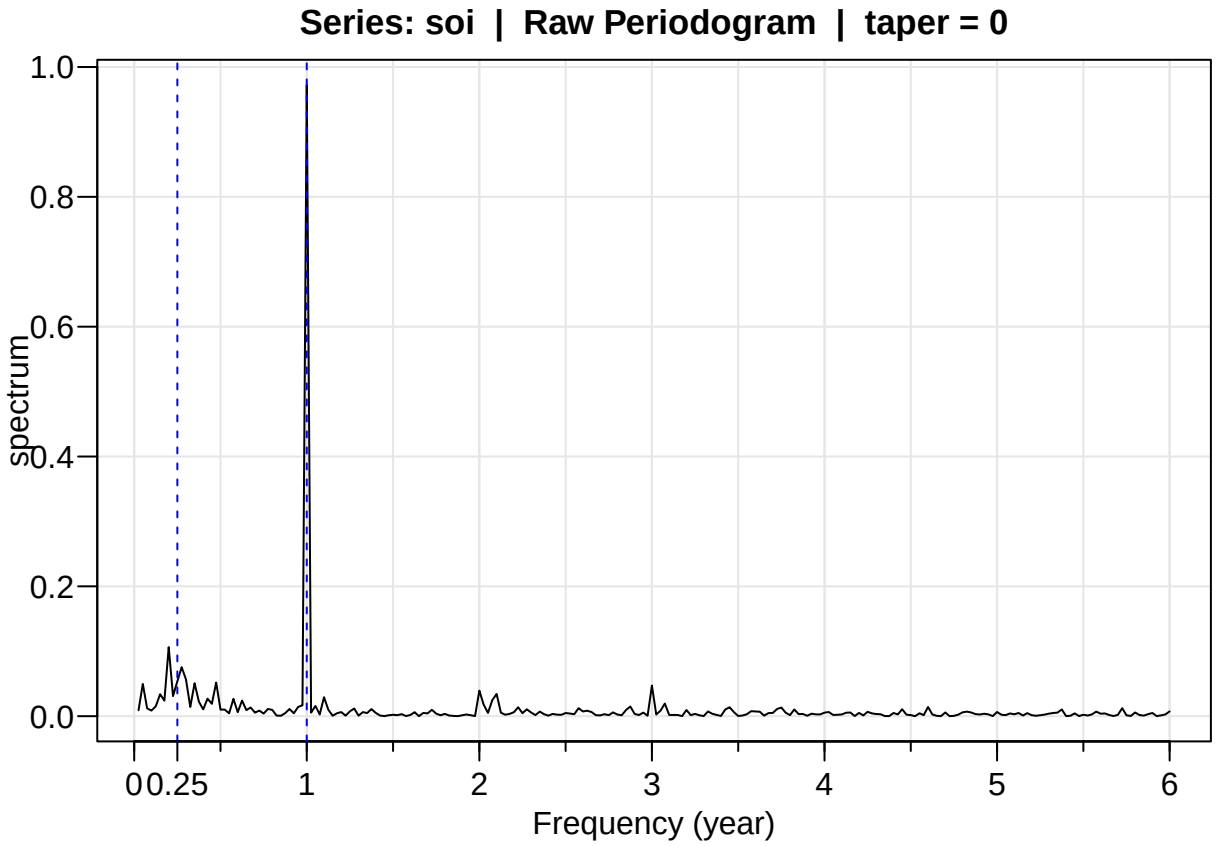



### Raw periodogram

An approximate $100(1-\alpha)\%$ confidence interval for $f(\omega)$

$$\frac{2I(\omega_j)}{\chi^2_2(1-\alpha/2)} \leq f(\omega) \leq \frac{2I(\omega_j)}{\chi^2_2(\alpha/2)}$$

```r
par(mgp = c(2.2, 1, 0), mar = c(3.5, 4, 1.4, 0.6), las = 1)
soi.per <- mvspec(soi, xlab = "Frequency (year)")
abline(v = c(1 / 4, 1), lty = 2, col = "blue")
axis(1, at = 0.25)
```




```
U = qchisq(.025, 2)
L = qchisq(.975, 2)
# 4-year period
soi.per$details[10,]
```

```
## frequency    period  spectrum
##    0.2500    4.0000    0.0537
```

```
c(2 * soi.per$spec[10] / L, 2 * soi.per$spec[10] / U)
```

```
## [1] 0.0145653 2.1222066
```

```
# 1-year period
soi.per$details[40,]
```

```
## frequency    period  spectrum
##    1.0000    1.0000    0.9722
```

```
c(2 * soi.per$spec[40] / L, 2 * soi.per$spec[40] / U)
```

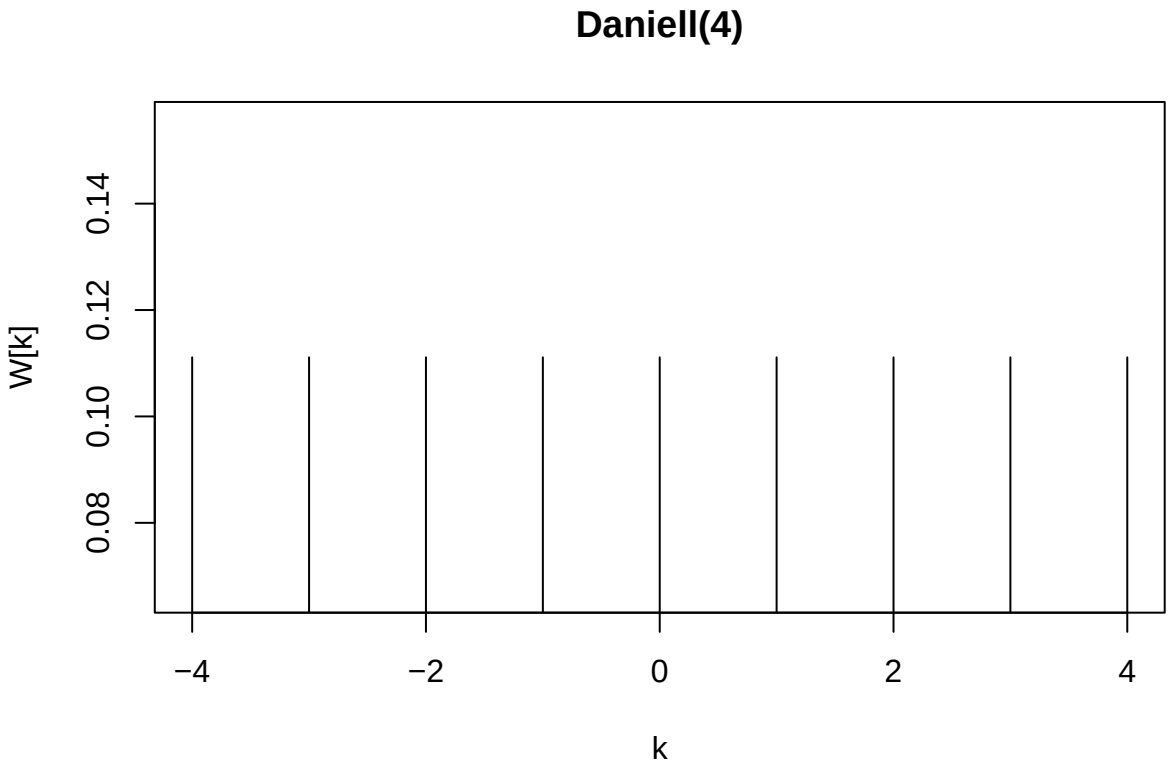
```
## [1]  0.2635573 38.4010800
```

### Averaged periodogram

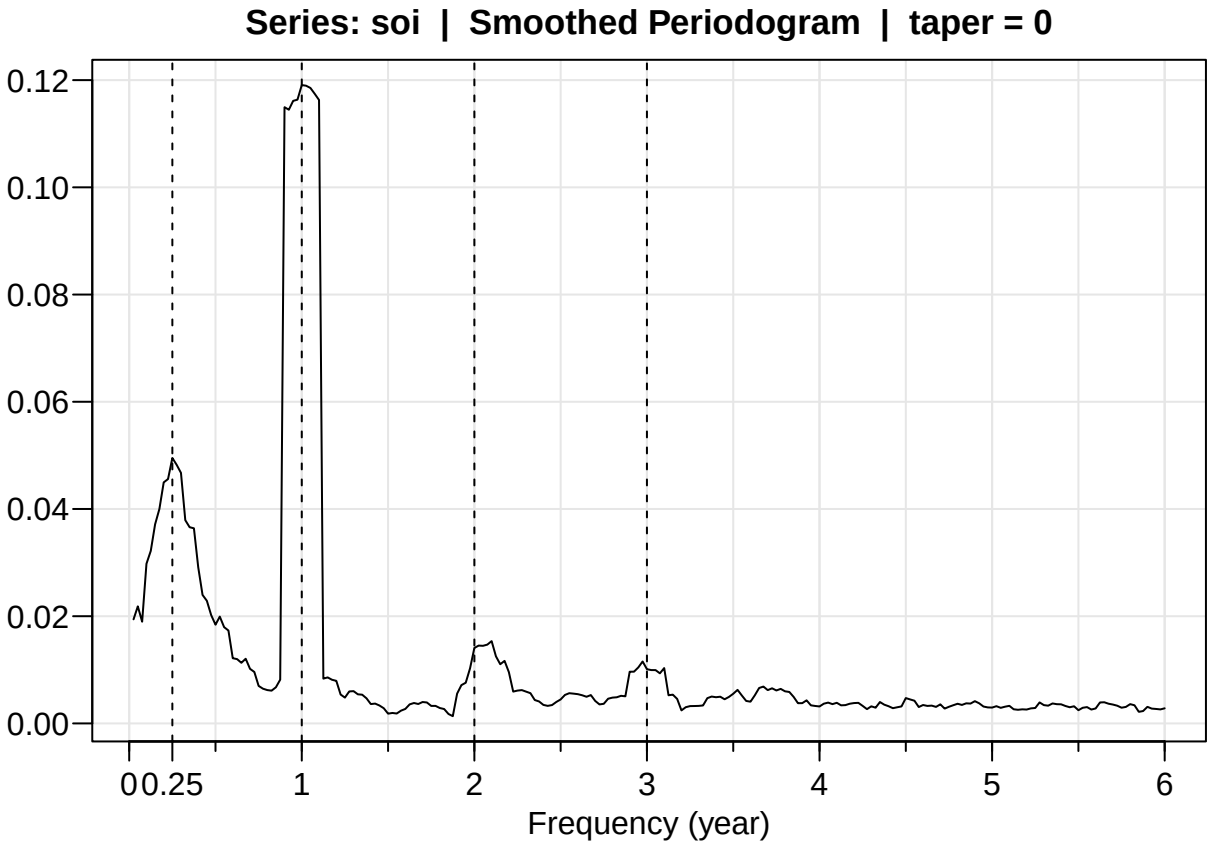
An approximate $100(1-\alpha)\%$ confidence interval for $f(\omega)$

$$\frac{2L\bar{f}(\omega)}{\chi^2_{2L}(1-\alpha/2)} \leq f(\omega) \leq \frac{2L\bar{f}(\omega)}{\chi^2_{2L}(\alpha/2)}$$

```
plot(kernel("daniell", 4))
```




```
par(mgp = c(3, 2, 0), mar = c(3.5, 4, 1.4, 0.6), las = 1)
soi.ave <- mvspec(soi, kernel('daniell', 4),
                  ylab = "", xlab = "Frequency (year)")
abline(v = c(.25, 1, 2, 3), lty = 2)
axis(1, at = 0.25)
```




```
soi.ave$bandwidth
```

```
## [1] 0.225
```

```
(df <- soi.ave$df)
```

```
## [1] 16.9875
```

```
(U <- qchisq(.025, df))
```

```
## [1] 7.555916
```

```
(L <- qchisq(.975, df))
```

```
## [1] 30.17425
```

```
soi.ave$spec[10]
```

```
## [1] 0.04952026
```

```
soi.ave$spec[40]
```

```
## [1] 0.11908
```

```
# intervals
c(df * soi.ave$spec[10] / L, df * soi.ave$spec[10] / U)
```

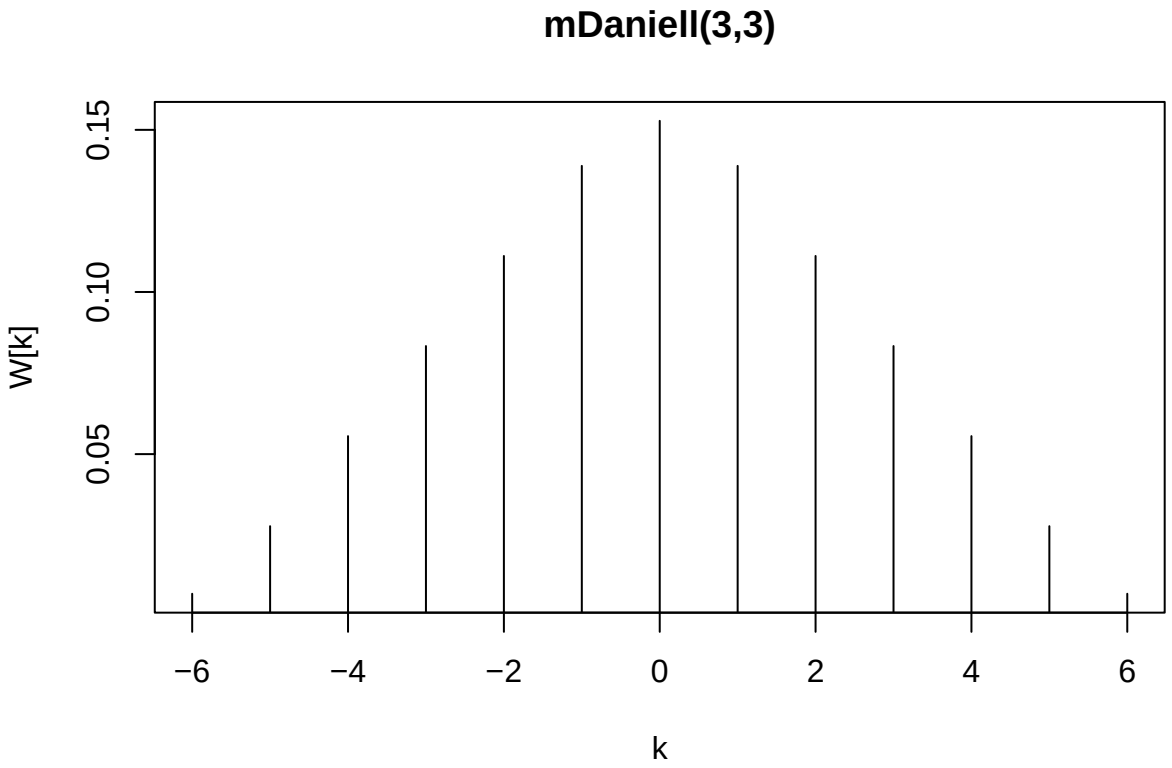
```
## [1] 0.02787891 0.11133335
```

```
c(df * soi.ave$spec[40] / L, df * soi.ave$spec[40] / U)
```

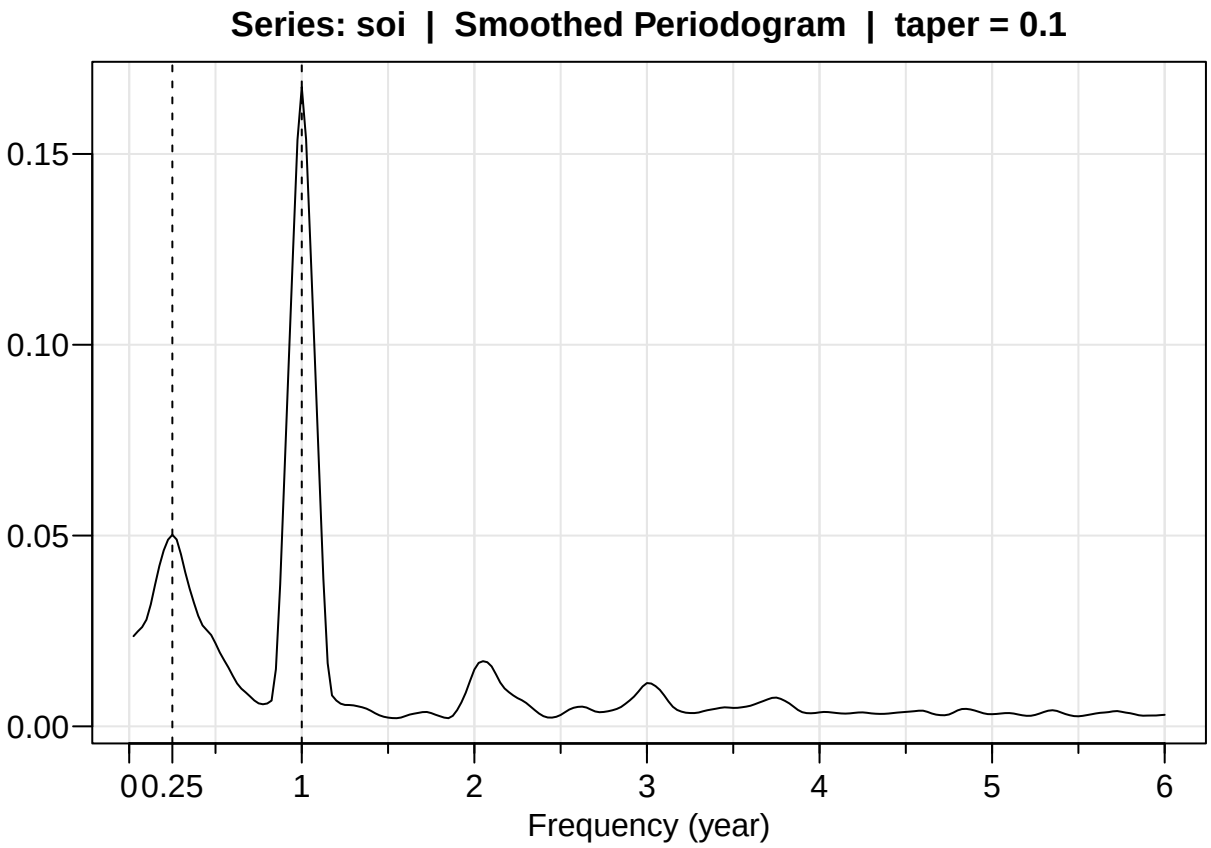
```
## [1] 0.06703963 0.26772011
```

**Smoothed periodogram**

```
plot(kernel("modified.daniell", c(3, 3)))
```




```
k <- kernel("modified.daniell", c(3, 3))

par(mgp = c(3, 2, 0), mar = c(3.5, 4, 1.4, 0.6), las = 1)
soi.smo <- mvspec(soi, kernel = k, taper = .1,
                  ylab = "", xlab = "Frequency (year)")
abline(v = c(.25, 1), lty = 2)
axis(1, at = 0.25)
```



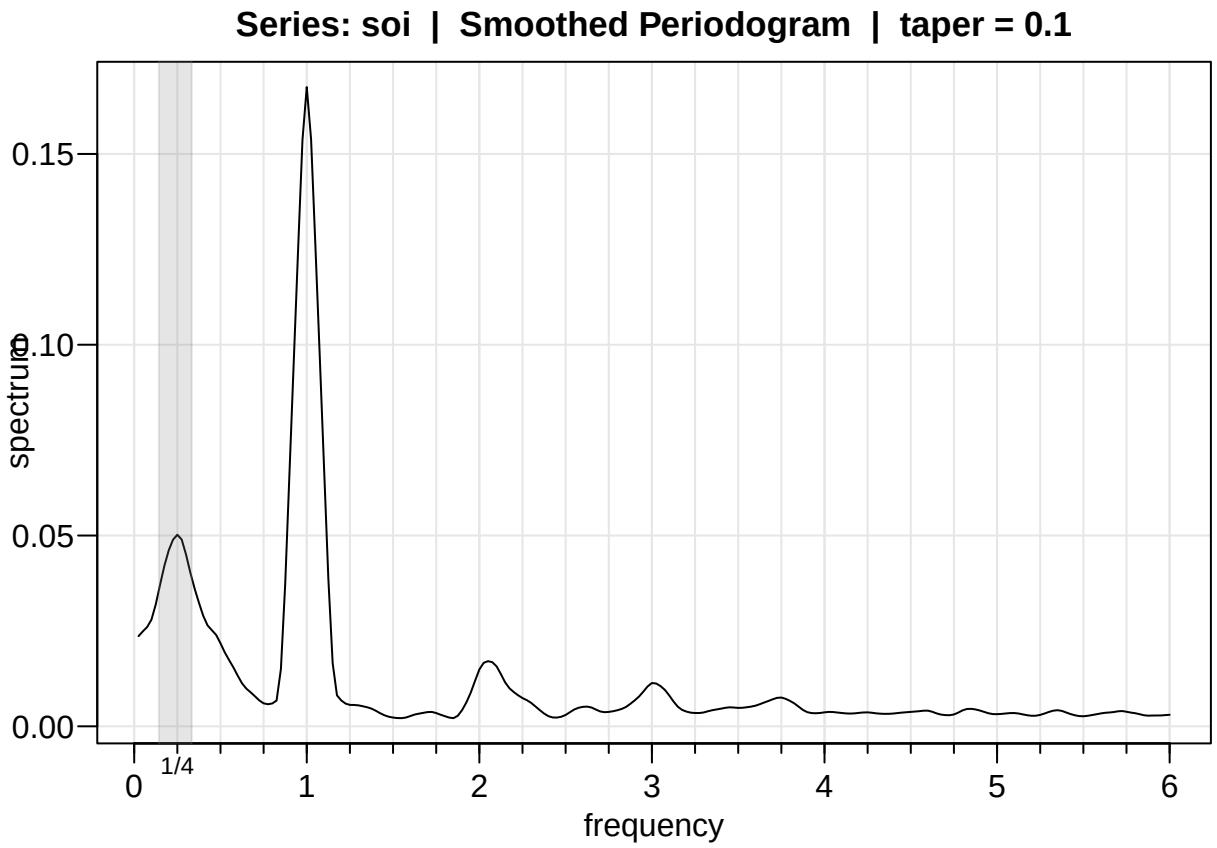

```
soi.smo$bandwidth
```

```
## [1] 0.2308103
```

```
(df <- soi.smo$df)
```

```
## [1] 17.42618
```

```
soi.smo <- mvspec(soi, spans = c(7, 7), taper = .1, nxm = 4)
rect(1/7, -1e5, 1/3, 1e5, density = NA, col = gray(.5, .2))
mtext("1/4", side = 1, line = 0, at = .25, cex = .75)
```




```
(U <- qchisq(.025, df))
```

```
## [1] 7.847084
```

```
(L <- qchisq(.975, df))
```

```
## [1] 30.76132
```

```
soi.smo$spec[10]
```

```
## [1] 0.05019866
```

```
soi.smo$spec[40]
```

```
## [1] 0.1675368
```

```
# intervals
c(df * soi.smo$spec[10] / L, df * soi.smo$spec[10] / U)
```

```
## [1] 0.02843736 0.11147718
```

```
c(df * soi.smo$spec[40] / L, df * soi.smo$spec[40] / U)
```

```
## [1] 0.09490899 0.37205231
```

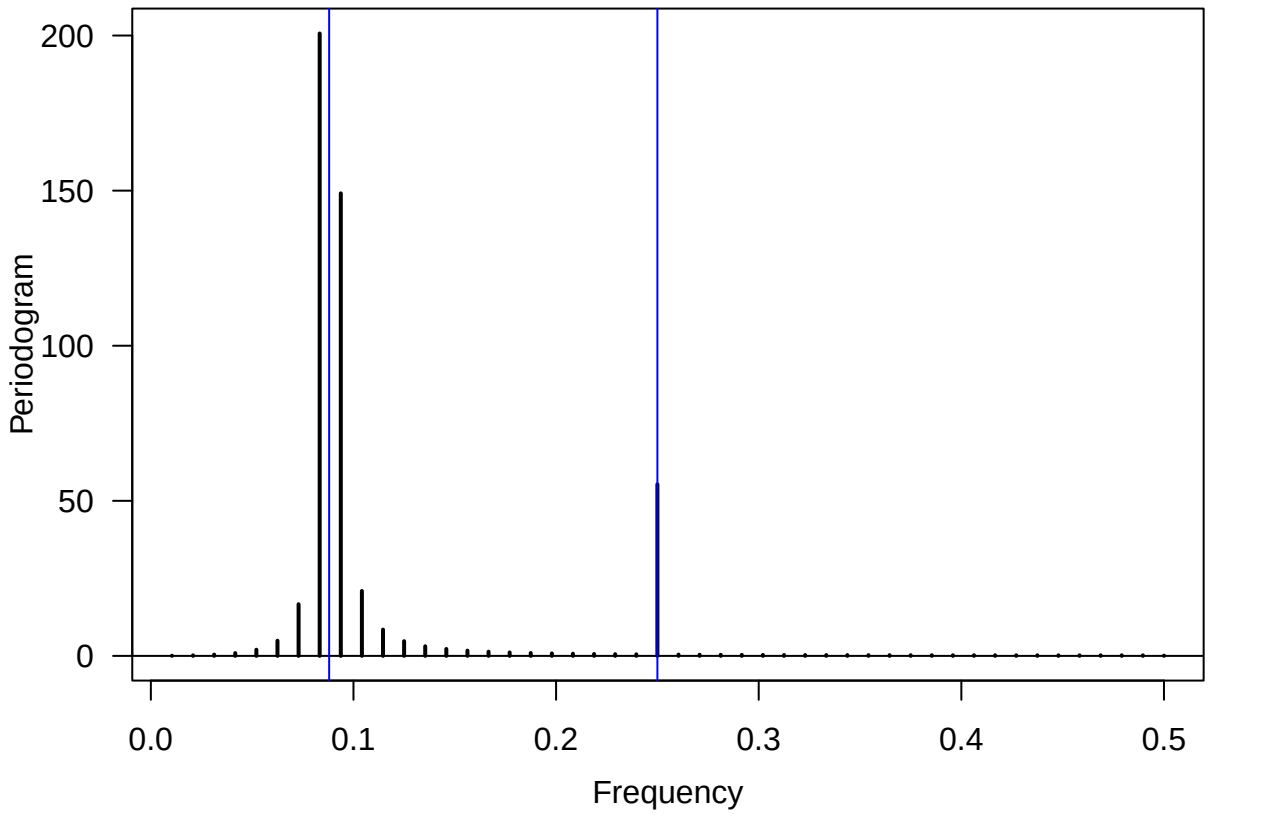
### Leakage and Tapering

```
library(TSA)
```

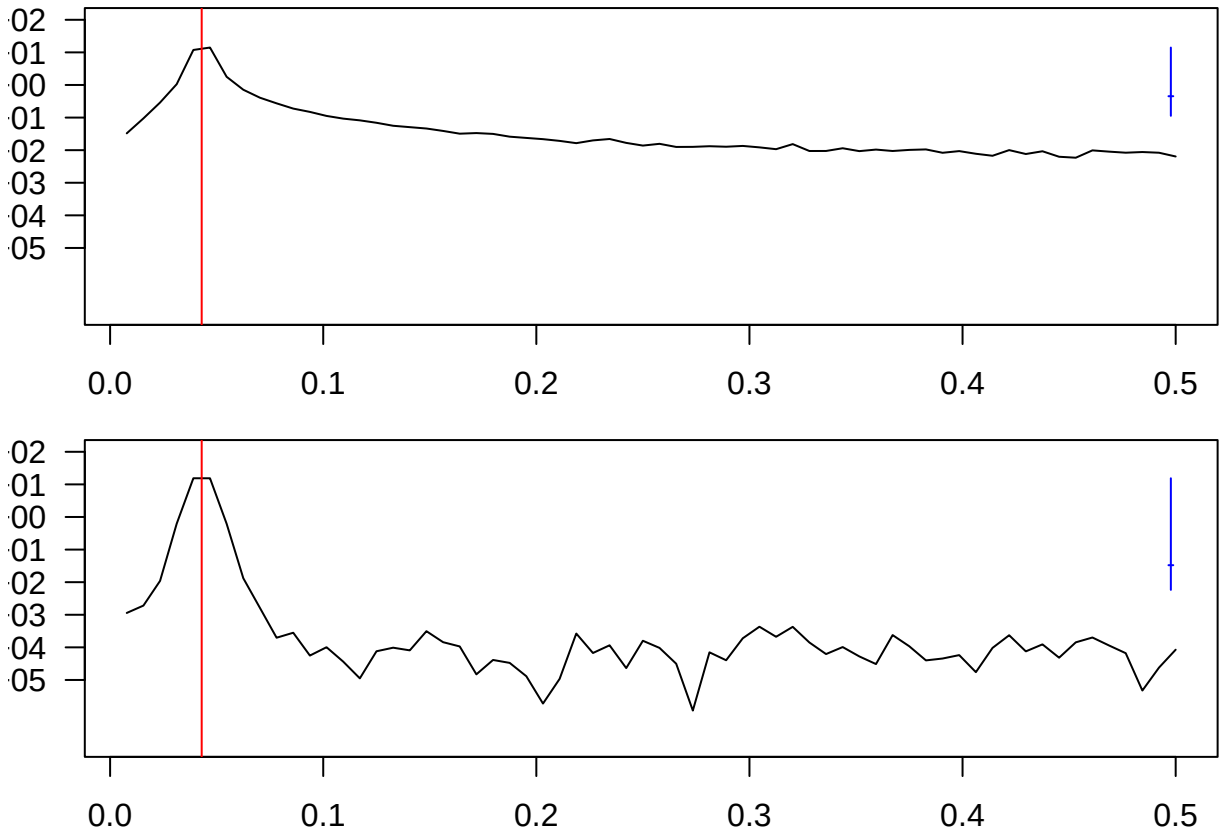
```
##
```

```
## Attaching package: 'TSA'

## The following objects are masked from 'package:stats':
##
##     acf, arima

## The following object is masked from 'package:utils':
##
##     tar
```

```
t = 1:96; f1 = 0.088; f2 = 24/96
y <- 3 * cos(f1 * 2 * pi * t) + sin(f2 * 2 * pi * t)

par(mgp = c(3, 2, 0), mar = c(3.5, 4, 1, 0.6), las = 1)
par(las = 1, mar = c(4, 4, 1, 0.6), mgp = c(2.4, 1, 0))
periodogram(y)
abline(h = 0)
abline(v = c(0.088, 24 / 96), col = "blue")
```



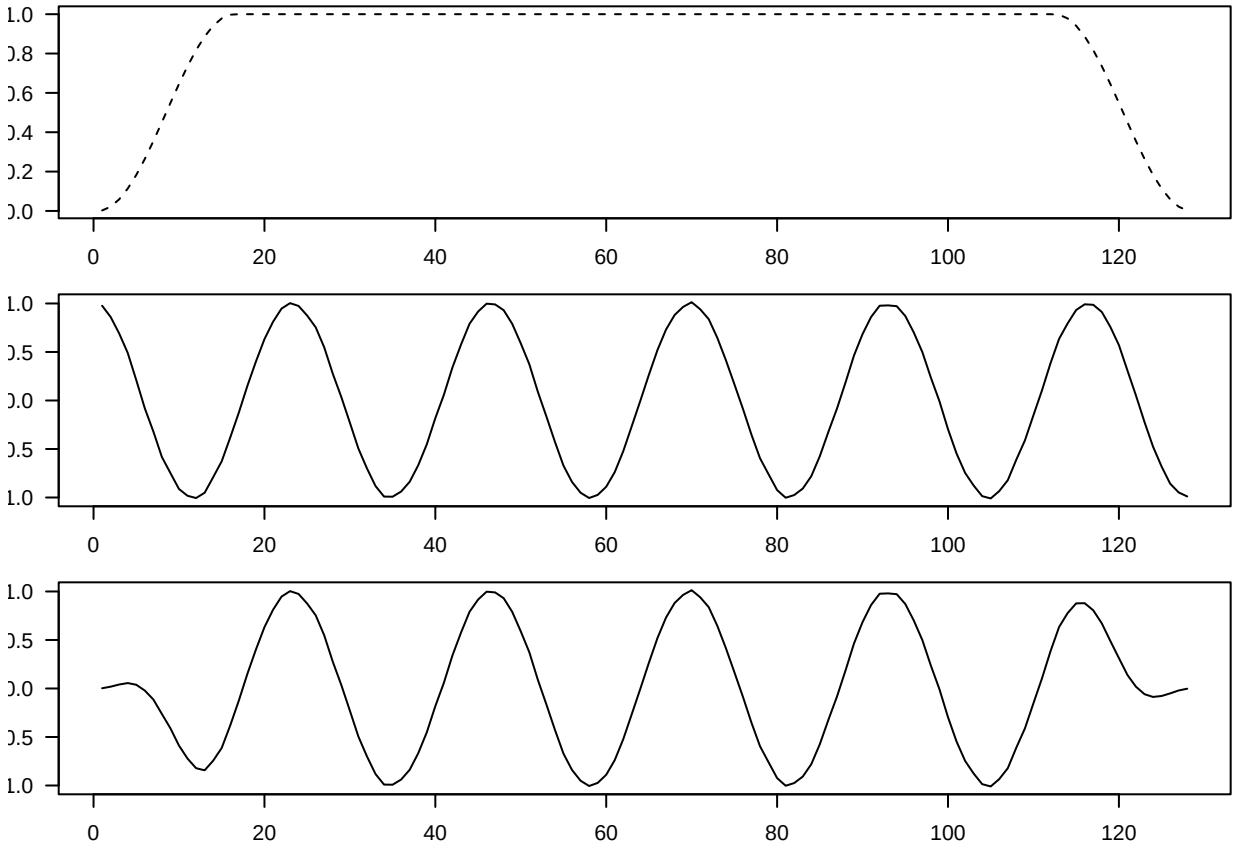

```
source("plotspectrum.R")

w <- rnorm(128, sd = 0.01)
par(mfcol = c(2, 1), mar = c(2, 2, 1, 1), las = 1)
#spectrum(x5, taper = 0, ylim = c(1e-7, 1e2), main = "")
x5h <- cos(2 * pi * (5.5 / 128) * (1:128)) + w
spectrum(x5h, taper = 0, ylim = c(1e-7, 1e2), main = "")
abline(v = 5.5 / 128, col = "red")
spectrum(x5h, taper = 0.5, ylim = c(1e-7, 1e2), main = "")
abline(v = 5.5 / 128, col = "red")
```





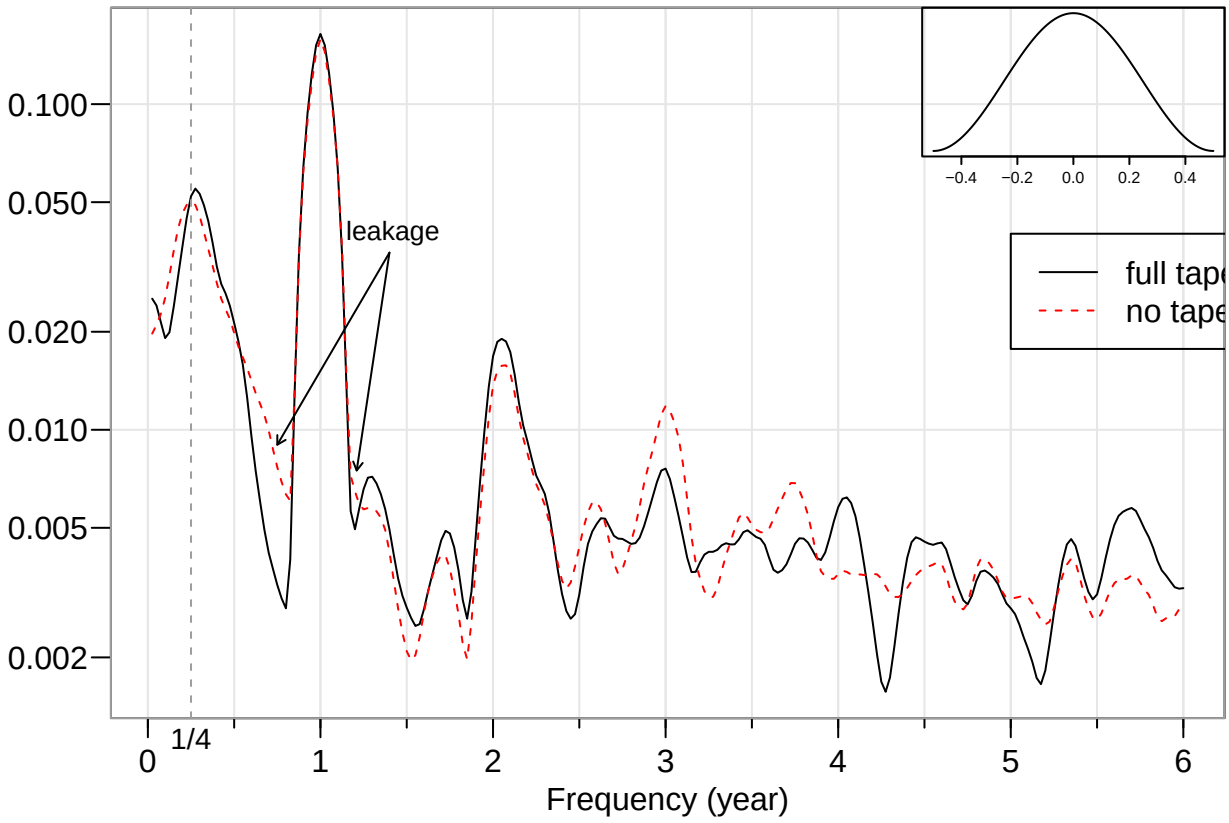


```
par(mfcol = c(3, 1), mar = c(2, 2, 1, 1))
plot(taper(rep(1, 128), 0.25), type = "l", lty = 2)
plot(x5h, type = "l")
plot(taper(x5h, 0.25), type = "l")
```



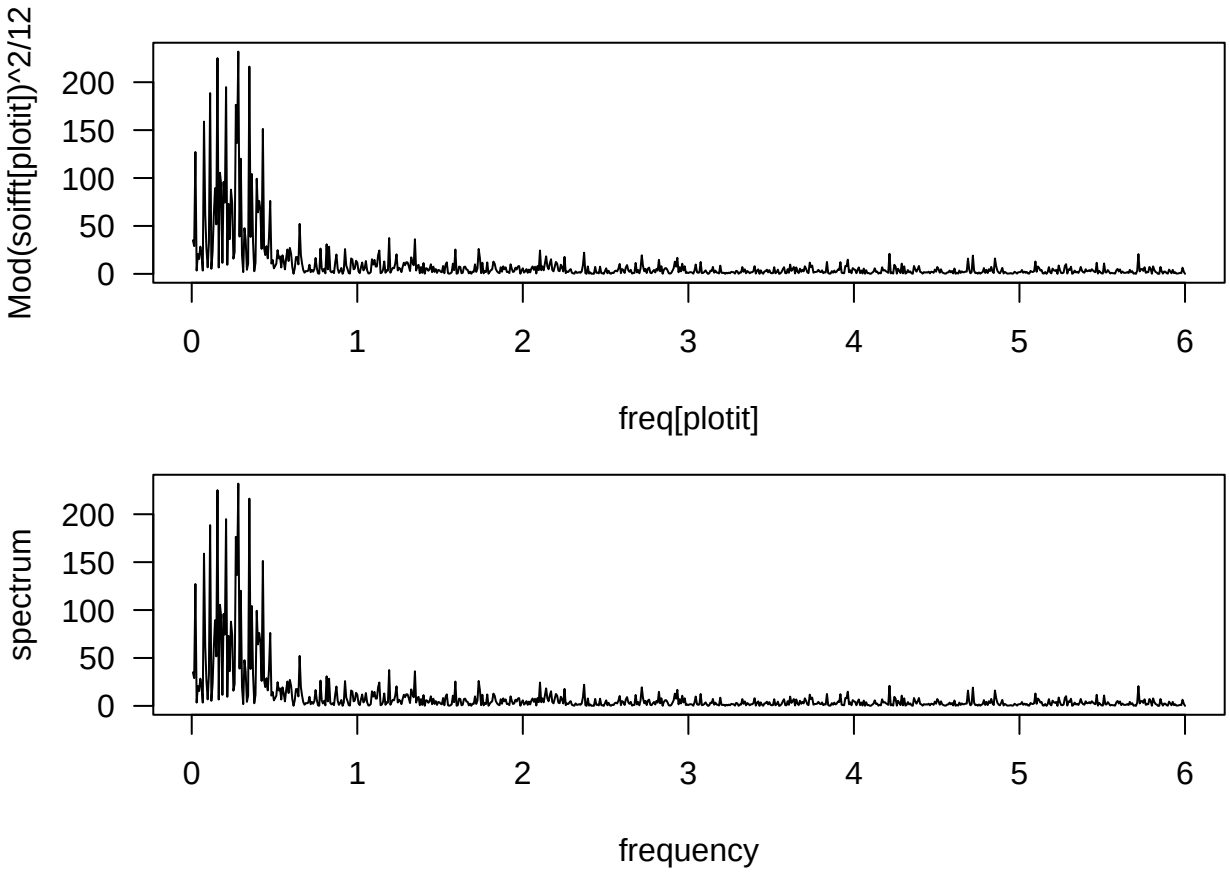

**SOI tapering**

```
par(mgp = c(3, 2, 0), mar = c(3.5, 4, 1.4, 0.6), las = 1)
s0 <- mvspec(soi, spans = c(7, 7), plot = FALSE)
s50 <- mvspec(soi, spans = c(7, 7), taper = .5, plot = FALSE)
tsplot(s50$freq, s50$spec, log = "y", type = "l",
       ylab = "", xlab = "Frequency (year)")
lines(s0$freq, s0$spec, lty = 2, col = "red")
abline(v = .25, lty = 2, col = 8)
mtext('1/4', side = 1, line = 0, at = .25, cex = .9)
legend(5, .04, legend = c('full taper', 'no taper'), lty = 1:2, col = c("black", "red"))

text(1.42, 0.04, 'leakage', cex = .8)
arrows(1.4, .035, .75, .009, length = 0.05, angle = 30)
arrows(1.4, .035, 1.21, .0075, length = 0.05, angle = 30)
par(fig = c(.65, 1, .65, 1),  new = TRUE, cex = .5,
    mgp = c(0, -.1, 0), tcl = -.2)
taper <- function(x) {.5 * (1 + cos(2 * pi * x))}
x <- seq(from = -.5, to = .5, by = 0.001)
plot(x, taper(x), type = "l",  lty = 1,  yaxt = 'n', ann = FALSE)
```



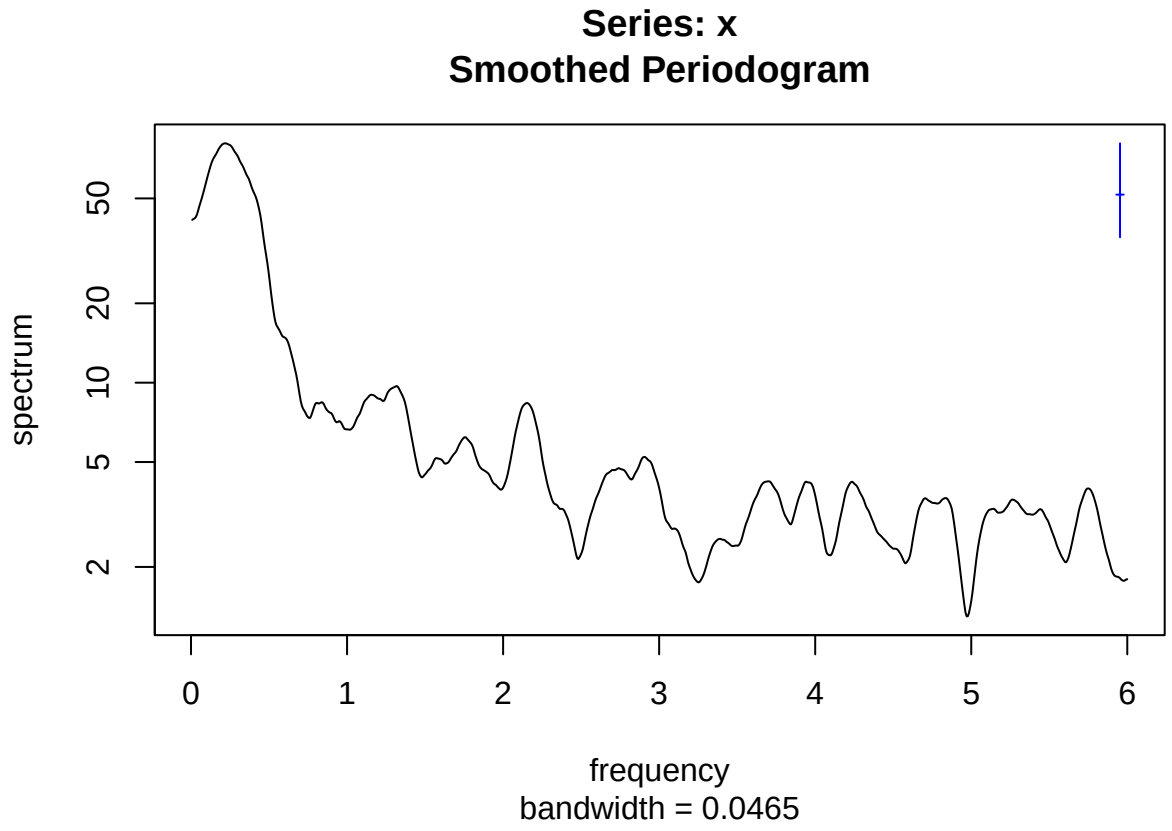

### Bloomfield's SOI Example

```
url <- "https://www.stat.ncsu.edu/people/bloomfield/courses/ST730/data/soi2010.txt"

soi_raw <- read.table(url, header = T)
SOI <- ts(c(t(soi_raw[, -1])), start = c(1876, 1),
          frequency = 12)

par(mfcol = c(2, 1), las = 1, mar = c(4, 4, 1, 0.6))
freq <- 12 * (0:(length(SOI) - 1)) / length(SOI)
plotit <- (freq > 0) & (freq <= 6)
soifft <- fft(SOI) / sqrt(length(SOI))
plot(freq[plotit], Mod(soifft[plotit])^2 / 12, type = "l")
# Use spectrum(); override some defaults to make it match:
spectrum(SOI, log = "no", fast = FALSE, taper = 0, detrend = FALSE, main = "")
```

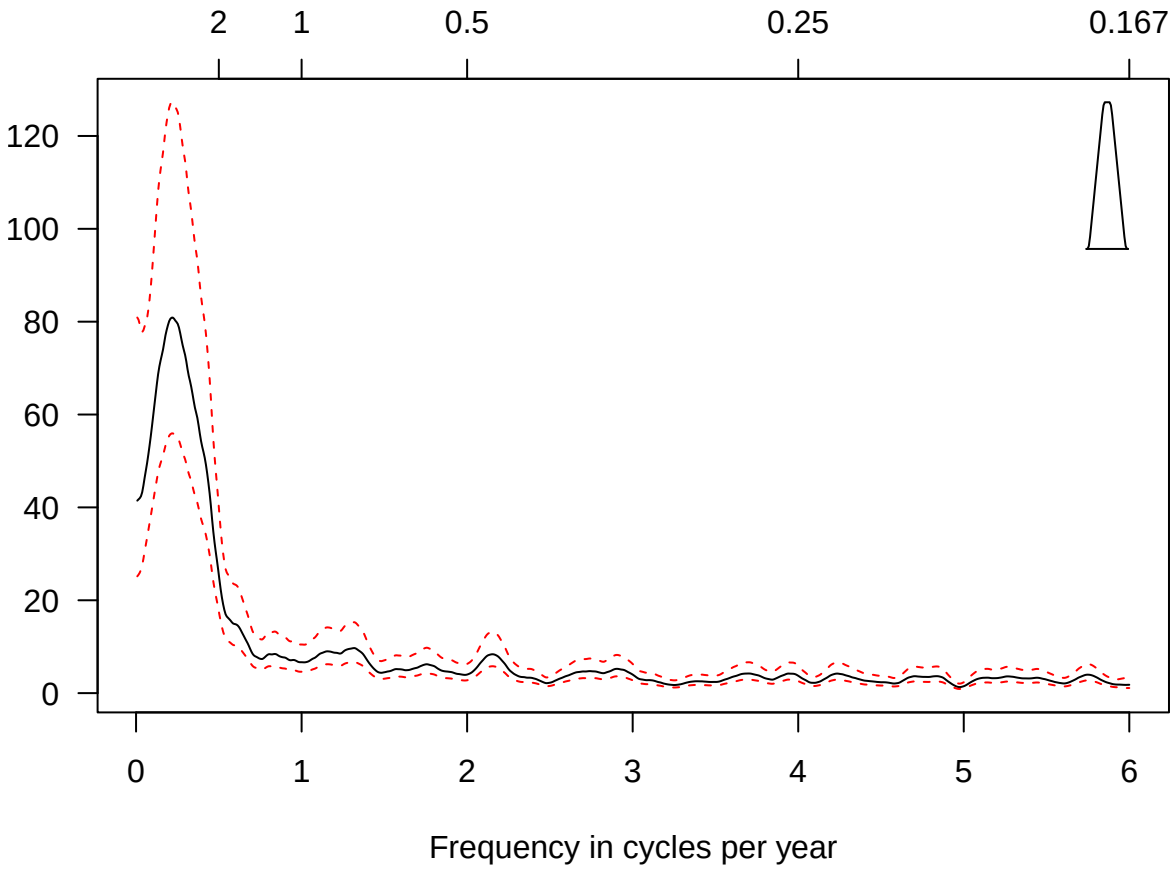



**Smoothed periodogram and pointwise CI**

```
source("plotspectrum.R")
soiAdj = residuals(lm(SOI ~ as.factor(cycle(SOI))))
soiAdj = ts(soiAdj, frequency = frequency(SOI), start = start(SOI))
k = kernel("modified.daniell", m = c(6, 9))
soiAdj.md = spectrum(soiAdj, k, fast = FALSE, detrend = FALSE, taper = 0)
```



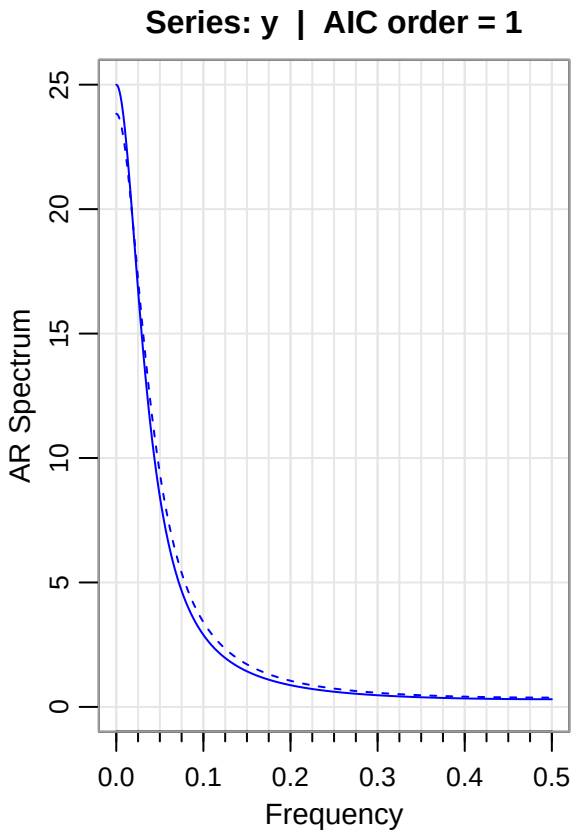

```
par(las = 1, mar = c(4, 4, 2, 0.6), mgp = c(3, 1, 0))
plotspectrum(soiAdj.md, log = "n", ci = 0.95, ci.type = "chisquare", ylab = "", unit.time = "year")
```

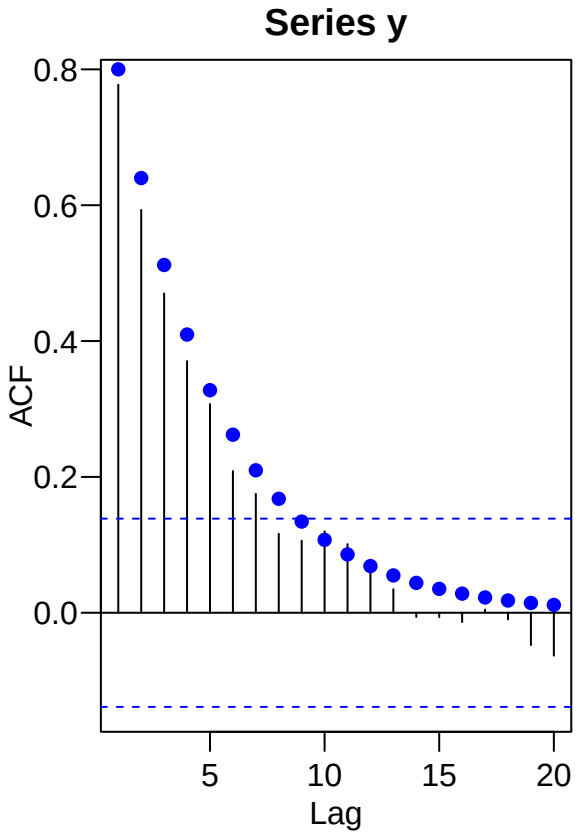



## Parametric Spectral Estimation

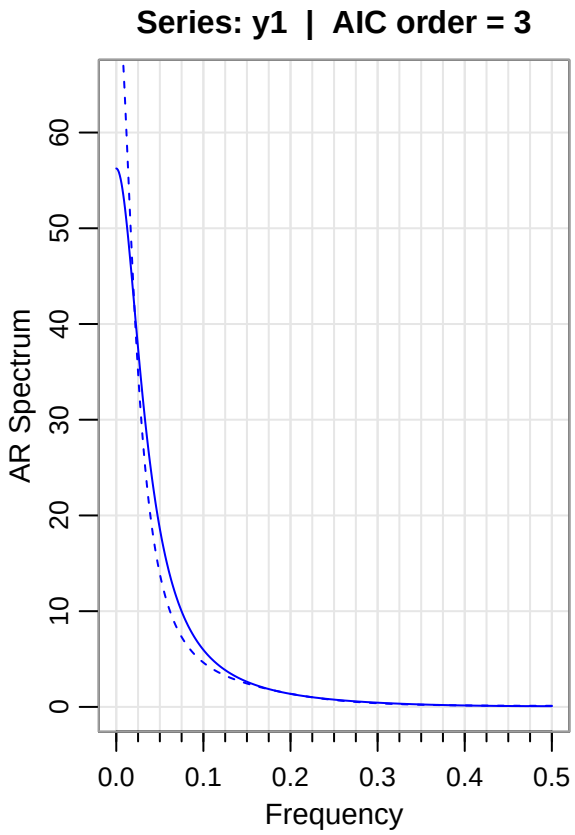
### Simulated examples

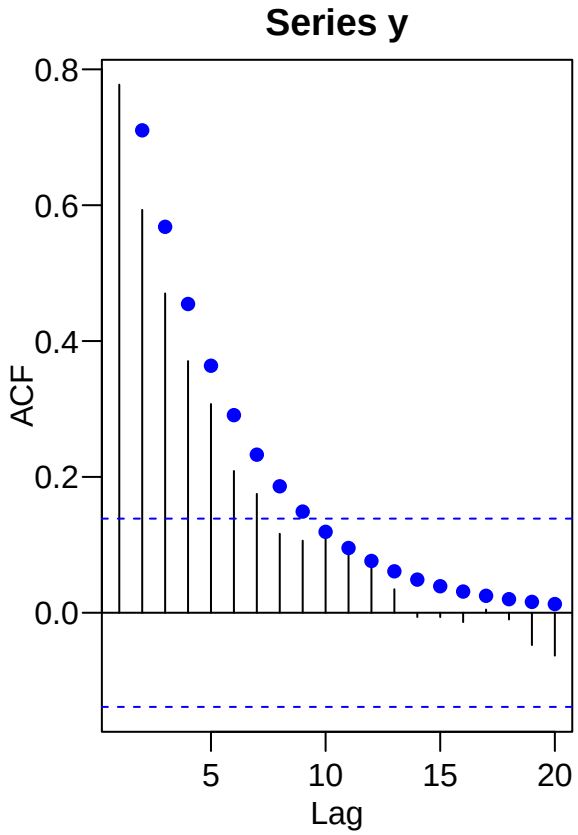
```
set.seed(12345)
n = 200; phi = 0.8; theta = 0.5
phi1 = 1.5; phi2 = -.95
y <- arima.sim(model = list(ar = phi), n = n)
y1 <- arima.sim(model = list(ar = phi, ma = theta), n = n)
y2 <- arima.sim(model = list(ar = c(phi1, phi2)), n = n)
##AR(1)
par(las = 1, mar = c(4, 4, 2, 0.6), mgp = c(3, 1, 0), mfrow = c(1, 2))
spec <- spec.ic(y, detrend = F, col = "blue", lwd = 1, nxm = 4,
                ylim = c(0, 25), lty = 2)
freq <- spec[[2]][, 1]
lines(freq, ARMAspec(model = list(ar = phi), freq = freq,
plot = F)$spec, col = "blue")
acf(y, lag = 20)
acf_true <- ARMAacf(ar = phi, lag.max = 20)
points(1:20, acf_true[2:21], col = "blue", pch = 16)
```



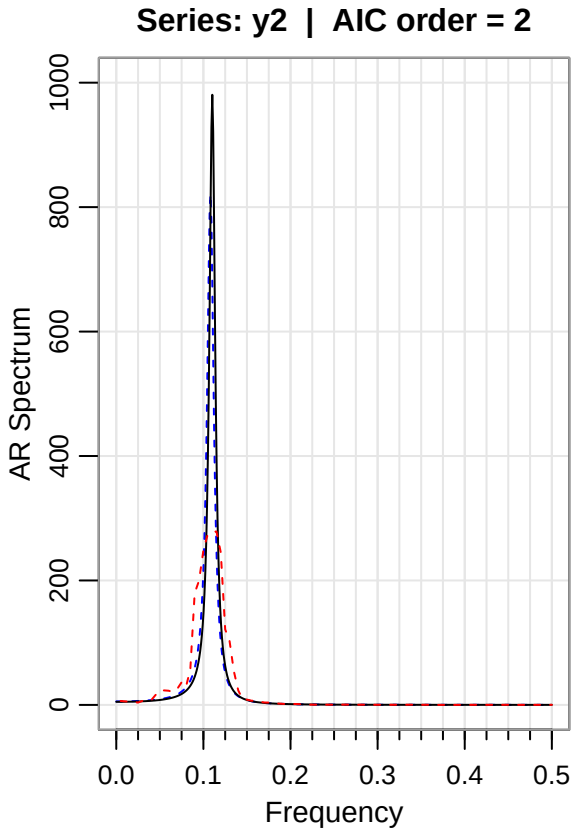




```
##ARMA(1,1)
par(las = 1, mar = c(4, 4, 2, 0.6), mgp = c(3, 1, 0), mfrow = c(1, 2))
spec <- spec.ic(y1, detrend = F, col = "blue", lwd = 1, nxm = 4, ylim = c(0, 65), lty = 2)
freq <- spec[[2]][, 1]
lines(freq, ARMAspec(model = list(ar = phi, ma = theta), freq = freq,
plot = F)$spec, col = "blue")
acf(y, lag = 20)
acf_true <- ARMAacf(ar = phi, ma = theta, lag.max = 20)
points(1:20, acf_true[2:21], col = "blue", pch = 16)
```



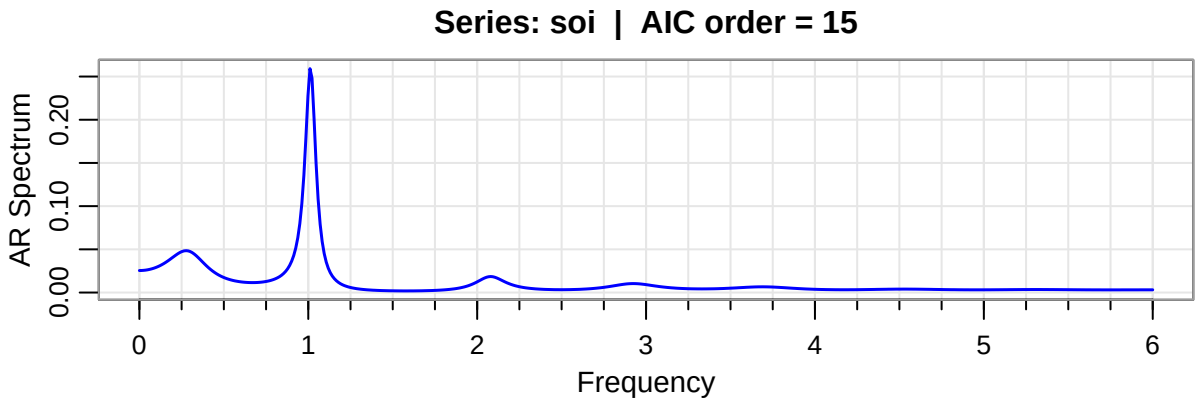



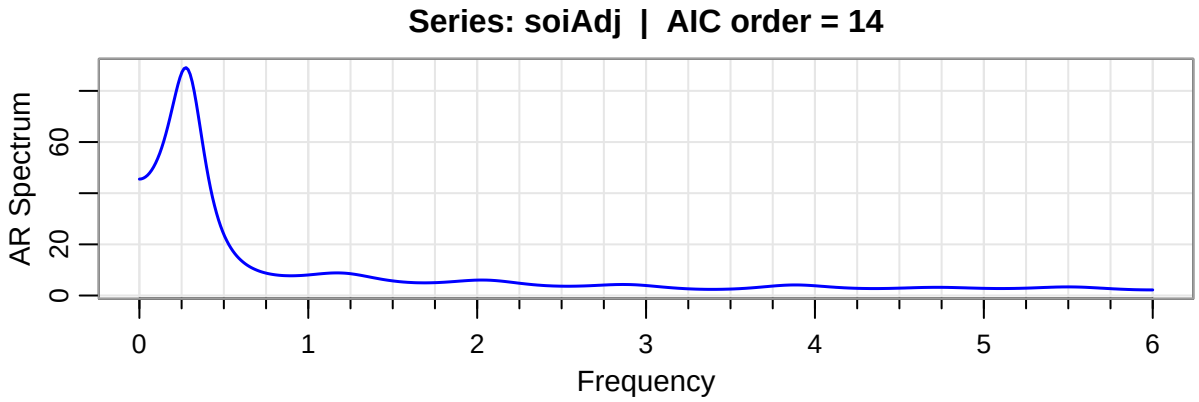

```
##AR(2)
par(las = 1, mar = c(4, 4, 2, 0.6), mgp = c(3, 1, 0))
spec <- spec.ic(y2, detrend = F, col = "blue", lwd = 1, nxm = 4, ylim = c(0, 1000), lty = 2)
freq <- spec[[2]][, 1]
lines(freq, ARMAspec(model = list(ar = c(phi1, phi2)), freq = freq,
plot = F)$spec)

AR2_nonpar <- mvspec(y2, kernel('daniell', 3), plot = F)
lines(AR2_nonpar$freq, AR2_nonpar$spec, col = "red", lty = 2)
```




**SOI Example**

```
par(las = 1, mar = c(4, 4, 2, 0.6), mgp = c(3, 1, 0), mfrow = c(2, 1))
u <- spec.ic(soi, detrend = TRUE, col = "blue", lwd = 1.5, nxm = 4)
u <- spec.ic(soiAdj, detrend = TRUE, col = "blue", lwd = 1.5, nxm = 4)
```




```
dev.off()
```

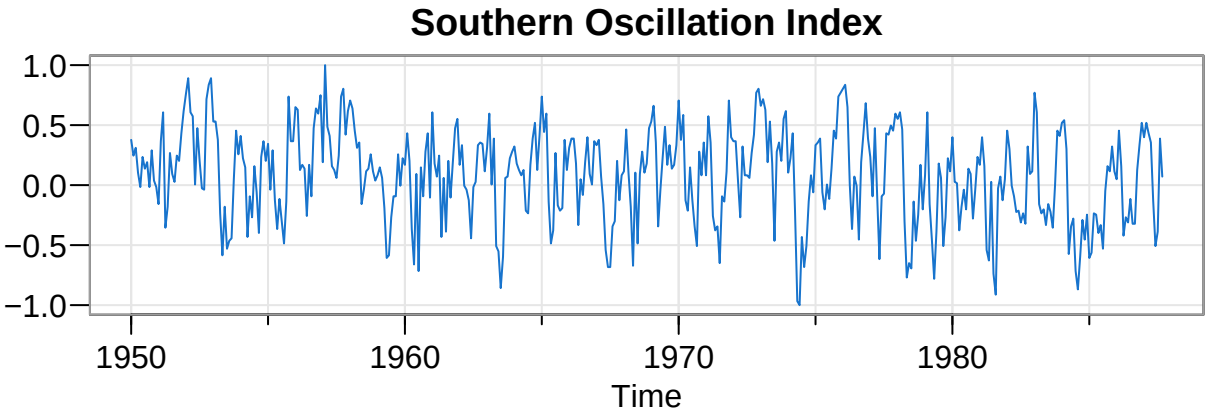
```
## null device
##           1
```

```
# plot AIC and BIC
tsplot(0:30, u[[1]][, 2:3], type = 'o', col = 2:3,
       xlab = 'ORDER', nxm = 5, lwd = 2, gg = TRUE)
```

## Lagged regression

### SOI and Recruitment time series

```
par(mfrow = c(2, 1), las = 1)
tsplot(soi, col = 4, ylab = "", main = "Southern Oscillation Index")
tsplot(rec, col = 4, ylab = "", main = "Recruitment")
```

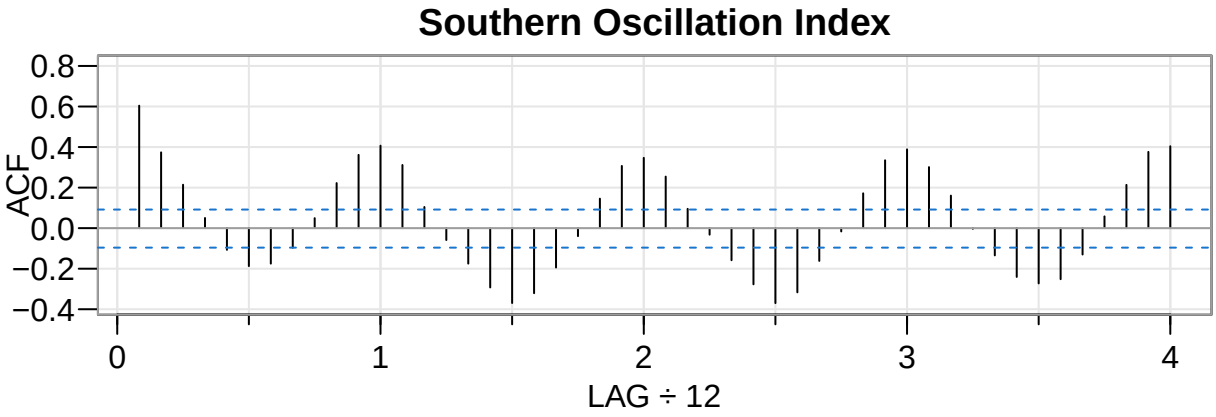


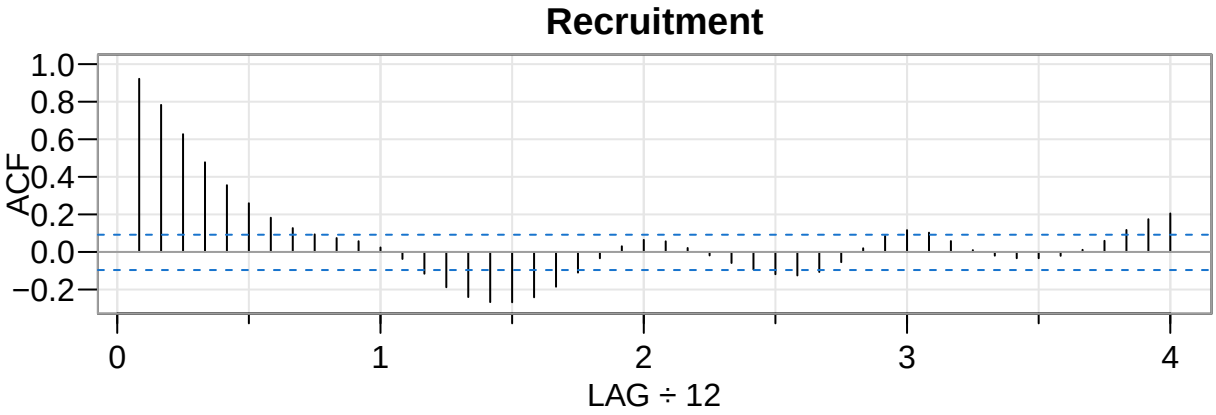


```
acf1(soi, main = "Southern Oscillation Index")
```

```
##  [1]  0.60  0.37  0.21  0.05 -0.11 -0.19 -0.18 -0.10  0.05  0.22  0.36  0.41
## [13]  0.31  0.10 -0.06 -0.17 -0.29 -0.37 -0.32 -0.19 -0.04  0.15  0.31  0.35
## [25]  0.25  0.10 -0.03 -0.16 -0.28 -0.37 -0.32 -0.16 -0.02  0.17  0.33  0.39
## [37]  0.30  0.16  0.00 -0.13 -0.24 -0.27 -0.25 -0.13  0.06  0.21  0.38  0.40
```

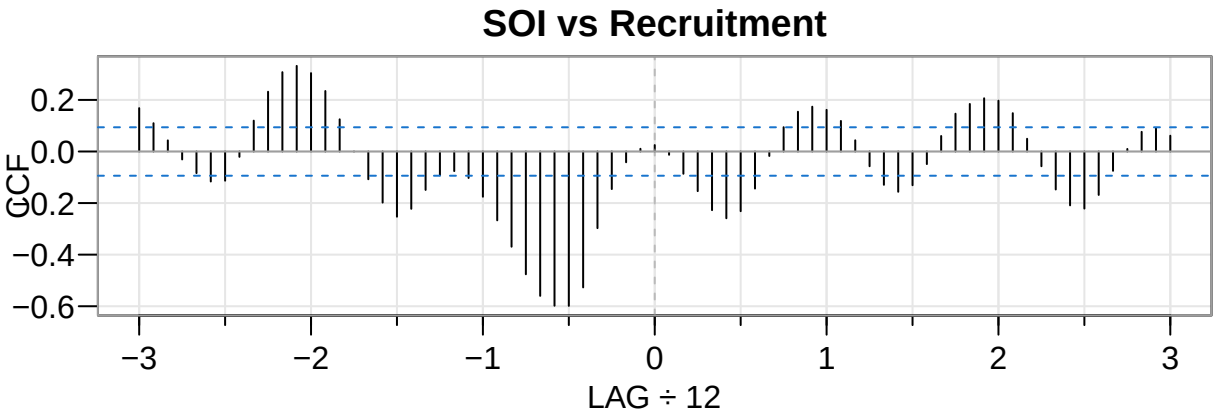
```
acf1(rec, main = "Recruitment")
```



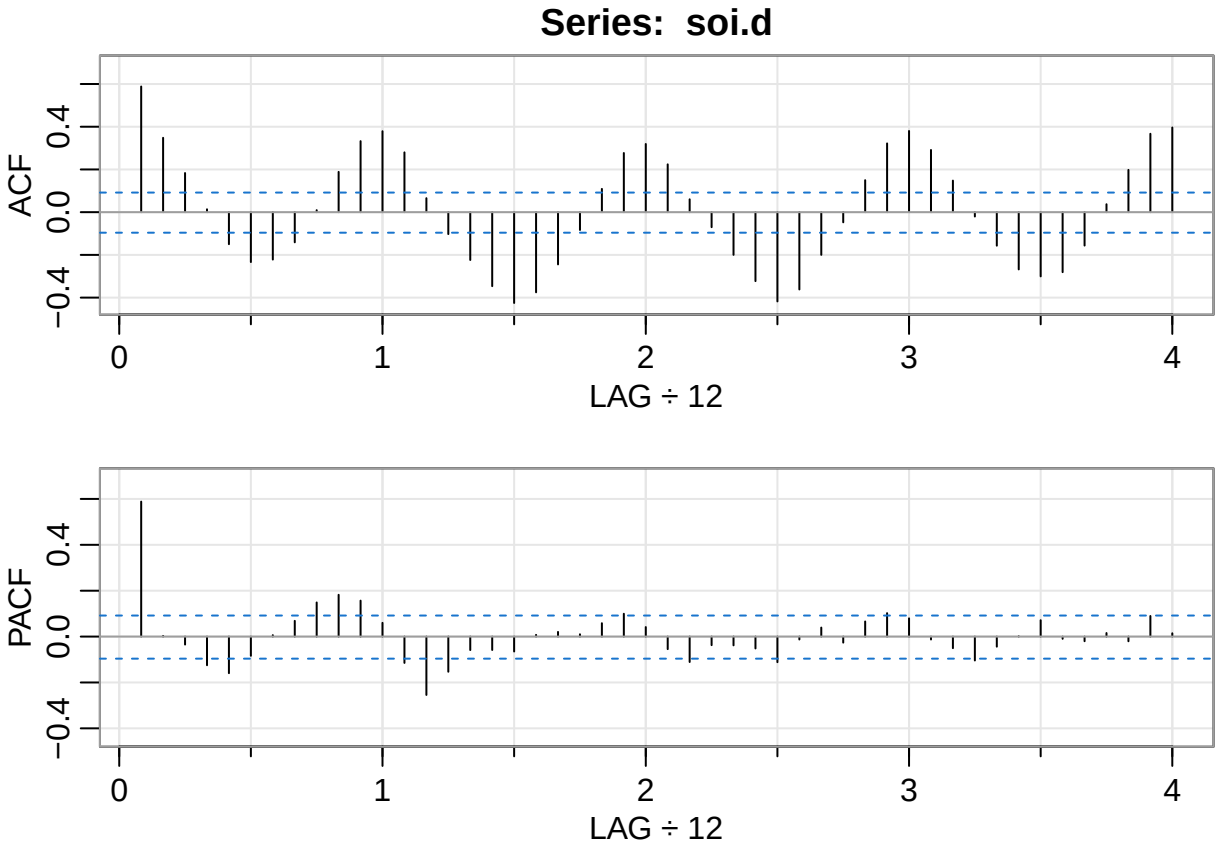

```
##  [1]  0.92  0.78  0.63  0.48  0.36  0.26  0.18  0.13  0.09  0.07  0.06  0.02
## [13] -0.04 -0.12 -0.19 -0.24 -0.27 -0.27 -0.24 -0.19 -0.11 -0.03  0.03  0.06
## [25]  0.06  0.02 -0.02 -0.06 -0.09 -0.12 -0.13 -0.11 -0.05  0.02  0.08  0.12
## [37]  0.10  0.06  0.01 -0.02 -0.03 -0.03 -0.02  0.01  0.06  0.12  0.17  0.20
```

```
ccf2(soi, rec, main = "SOI vs Recruitment", las = 1)
```



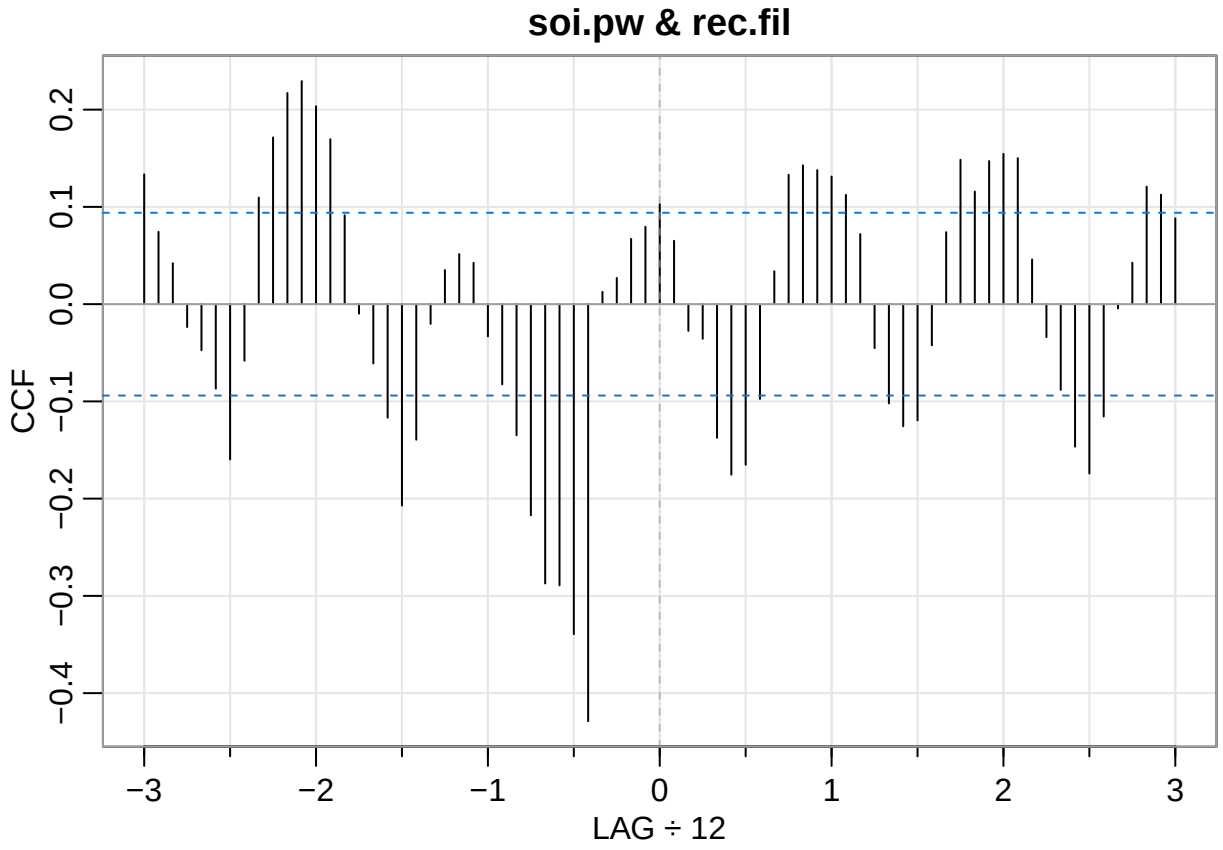

**Transfer function modeling**

```
soi.d <- resid(lm(soi ~ time(soi), na.action = NULL))
acf2(soi.d)
```




```
##      [,1] [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8] [,9] [,10] [,11] [,12] [,13]
## ACF  0.59 0.35  0.18  0.01 -0.15 -0.23 -0.22 -0.14 0.01  0.19  0.33  0.38  0.28
## PACF 0.59 0.00 -0.03 -0.12 -0.16 -0.08  0.01  0.07 0.15  0.18  0.16  0.06 -0.11
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.07 -0.10 -0.22 -0.35 -0.43 -0.38 -0.24 -0.08  0.11  0.28  0.32  0.22
## PACF -0.25 -0.15 -0.06 -0.06 -0.07  0.01  0.02  0.01  0.06  0.10  0.04 -0.05
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF   0.06 -0.07 -0.20 -0.32 -0.42 -0.36 -0.20 -0.05  0.15  0.32  0.38  0.29
## PACF -0.11 -0.04 -0.04 -0.05 -0.11 -0.01  0.04 -0.03  0.07  0.10  0.08 -0.01
##      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48]
## ACF   0.15 -0.02 -0.16 -0.27 -0.30 -0.28 -0.16  0.04  0.20  0.37  0.40
## PACF -0.05 -0.10 -0.04  0.00  0.07 -0.01 -0.02  0.02 -0.02  0.09  0.01
```

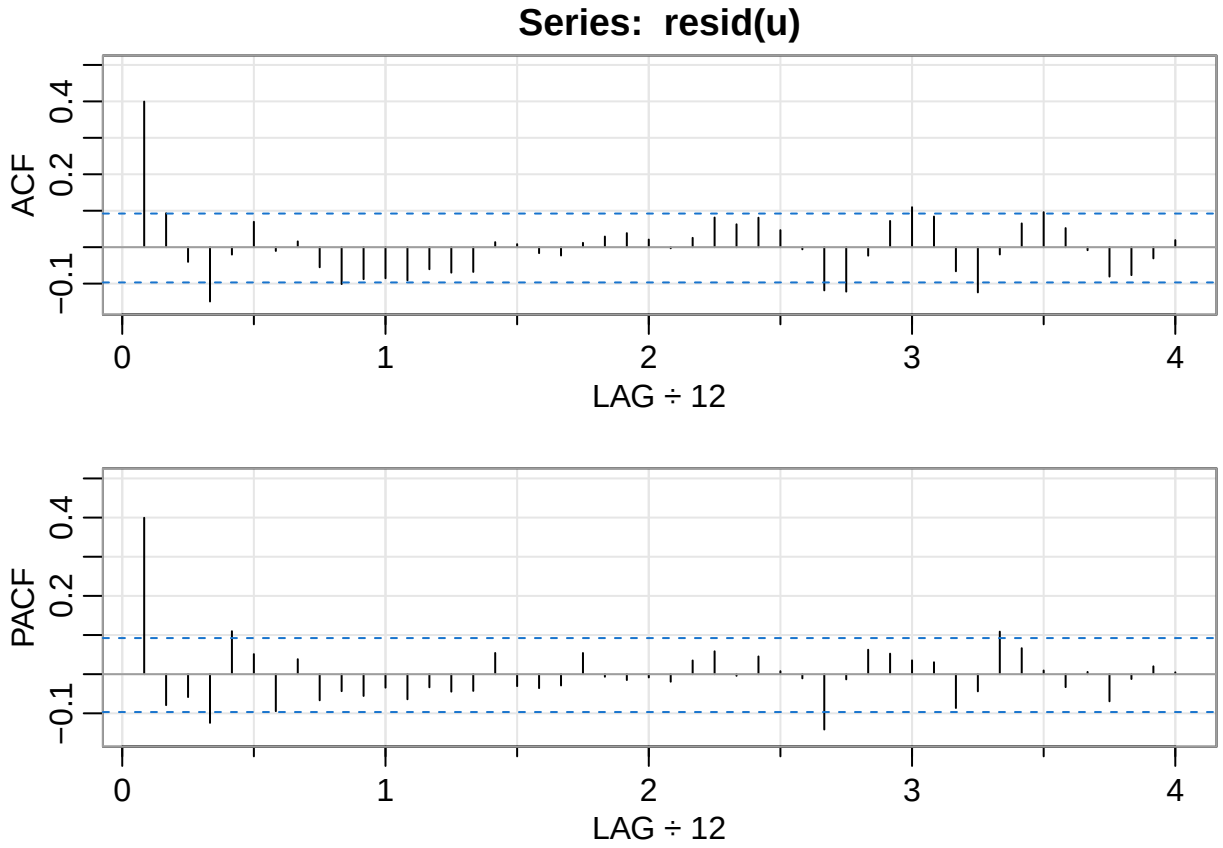
```
fit <- arima(soi.d, order = c(1, 0, 0))
ar1 <- as.numeric(coef(fit)[1])
soi.pw <- resid(fit)
rec.fil <- filter(rec, filter = c(1, -ar1), sides = 1)
ccf2(soi.pw, rec.fil)
```




```
fish <- ts.intersect(rec, RL1 = lag(rec, -1), SL5 = lag(soi.d, -5))
(u <- lm(fish[, 1] ~ fish[, 2:3], na.action = NULL))
```

```
##
## Call:
## lm(formula = fish[, 1] ~ fish[, 2:3], na.action = NULL)
##
## Coefficients:
##    (Intercept)  fish[, 2:3]RL1  fish[, 2:3]SL5
##         8.8971          0.8556        -20.3771
```

```
acf2(resid(u))
```

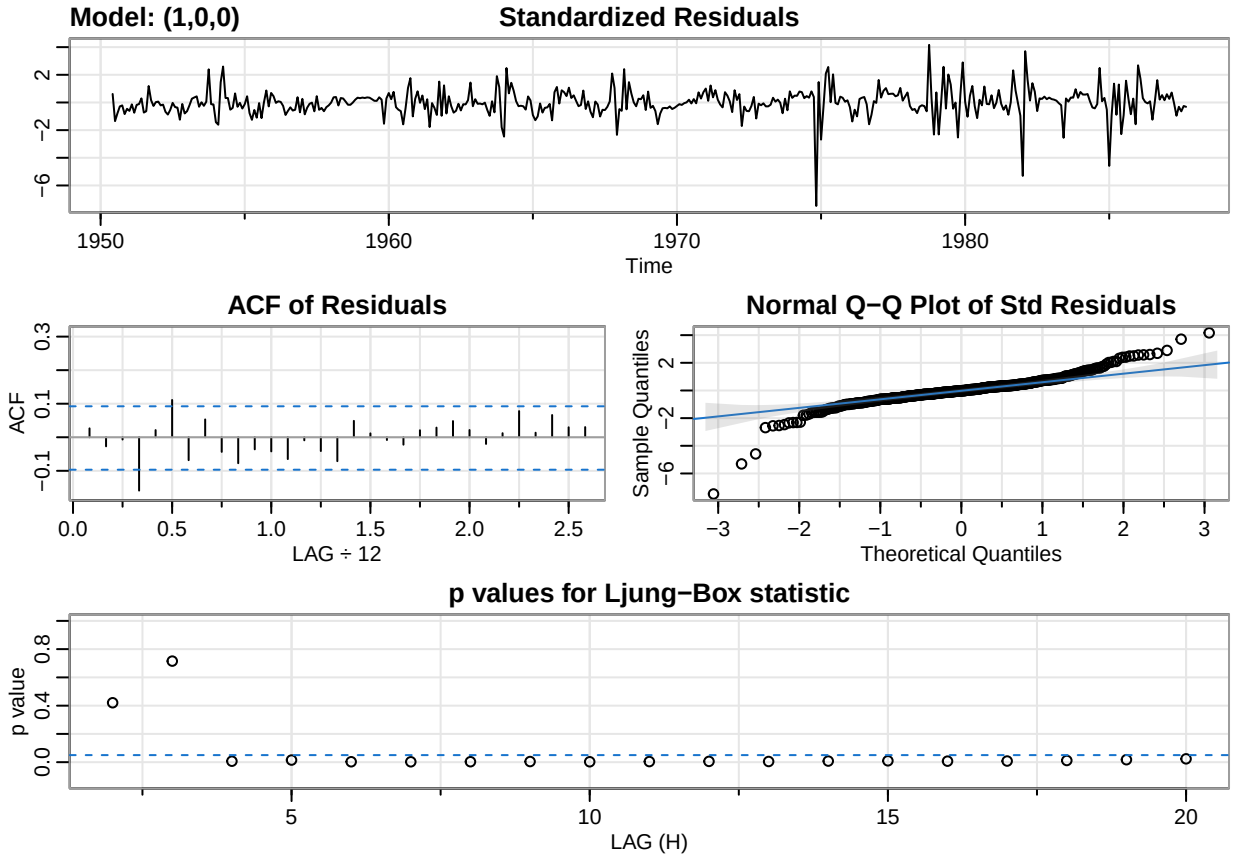



```
##      [,1]  [,2]  [,3]  [,4]  [,5] [,6]  [,7] [,8]  [,9] [,10] [,11] [,12] [,13]
## ACF   0.4  0.09 -0.04 -0.15 -0.02 0.07 -0.01 0.02 -0.06 -0.10 -0.09 -0.09 -0.09
## PACF  0.4 -0.08 -0.06 -0.12  0.11 0.05 -0.09 0.04 -0.07 -0.04 -0.06 -0.03 -0.06
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF  -0.06 -0.07 -0.07  0.01  0.01 -0.02 -0.02  0.01  0.03  0.04  0.02  0.00
## PACF -0.03 -0.04 -0.04  0.05 -0.03 -0.04 -0.03  0.05 -0.01 -0.01 -0.01 -0.02
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF   0.03  0.08  0.06  0.08  0.05 -0.01 -0.12 -0.12 -0.02  0.07  0.11  0.08
## PACF  0.03  0.06  0.00  0.05  0.01 -0.01 -0.14 -0.01  0.06  0.05  0.04  0.03
##      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48]
## ACF  -0.07 -0.12 -0.02  0.07  0.10  0.05 -0.01 -0.08 -0.08 -0.03  0.02
## PACF -0.09 -0.04  0.11  0.07  0.01 -0.03  0.01 -0.07 -0.01  0.02  0.00
```

```
(arx <- sarima(fish[, 1], 1, 0, 0, xreg = fish[, 2:3]))
```

```
## initial  value 2.050589
## iter   2 value 1.963560
## iter   3 value 1.962035
## iter   4 value 1.956727
## iter   5 value 1.956486
## iter   6 value 1.956230
## iter   7 value 1.956056
## iter   8 value 1.956027
## iter   9 value 1.956024
## iter  10 value 1.956024
## iter  10 value 1.956024
## final  value 1.956024
## converged
```

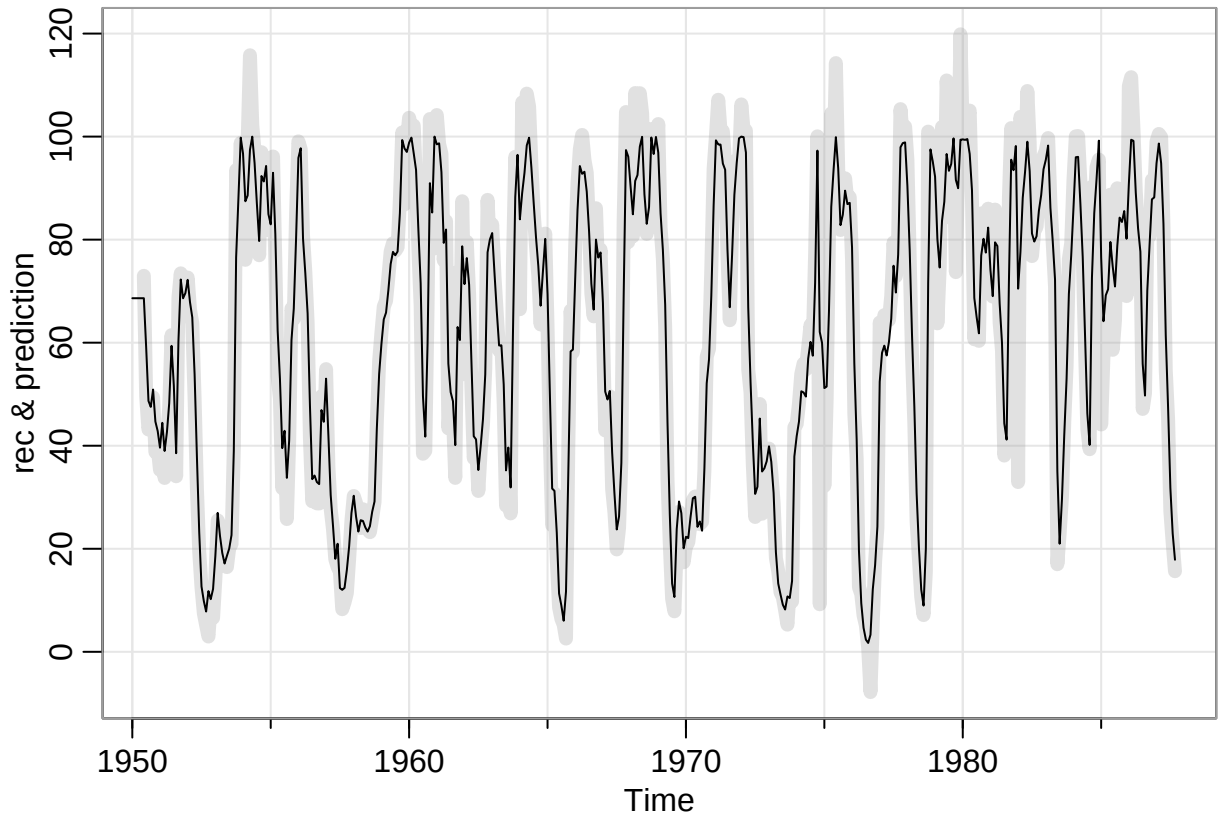
```
## initial  value 1.955587
## iter   2 value 1.955586
## iter   3 value 1.955585
## iter   4 value 1.955584
## iter   5 value 1.955584
## iter   6 value 1.955584
## iter   7 value 1.955584
## iter   8 value 1.955584
## iter   8 value 1.955584
## iter   8 value 1.955584
## final  value 1.955584
## converged
```




```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xreg, transform.pars = trans, fixed = fixed, optim.control = list(trace =
##     REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1  intercept     RL1       SL5
##       0.4487    12.3323  0.8005  -21.0307
## s.e.  0.0503     1.5746  0.0234    1.0915
##
## sigma^2 estimated as 49.93:  log likelihood = -1511.79,  aic = 3033.57
##
```

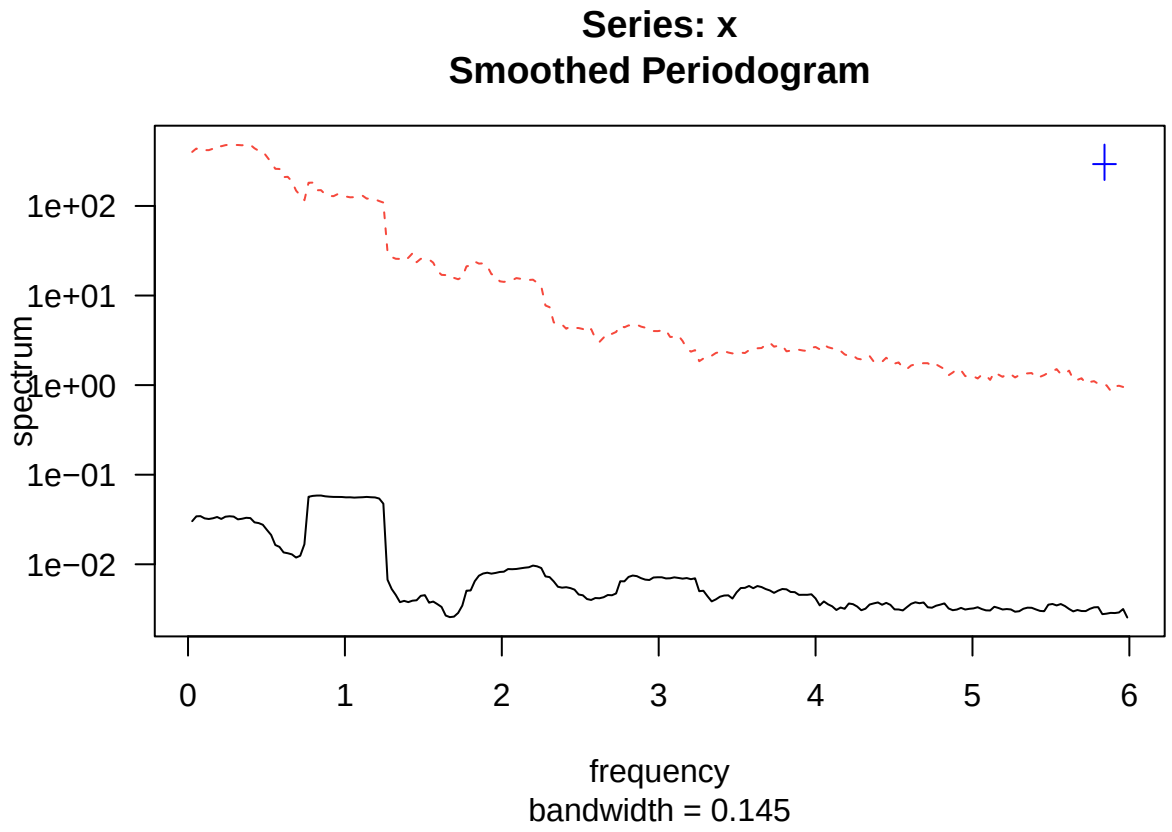
```
## $degrees_of_freedom
## [1] 444
##
## $ttable
##           Estimate     SE  t.value p.value
## ar1         0.4487 0.0503   8.9183       0
## intercept  12.3323 1.5746   7.8321       0
## RL1         0.8005 0.0234  34.2778       0
## SL5       -21.0307 1.0915 -19.2674       0
##
## $AIC
## [1] 6.771366
##
## $AICc
## [1] 6.771567
##
## $BIC
## [1] 6.817178
```

```
pred <- rec + resid(arx$fit)
tsplot(pred, col = astsa.col(8, .3), lwd = 7,
       ylab = 'rec & prediction')
lines(rec)
```

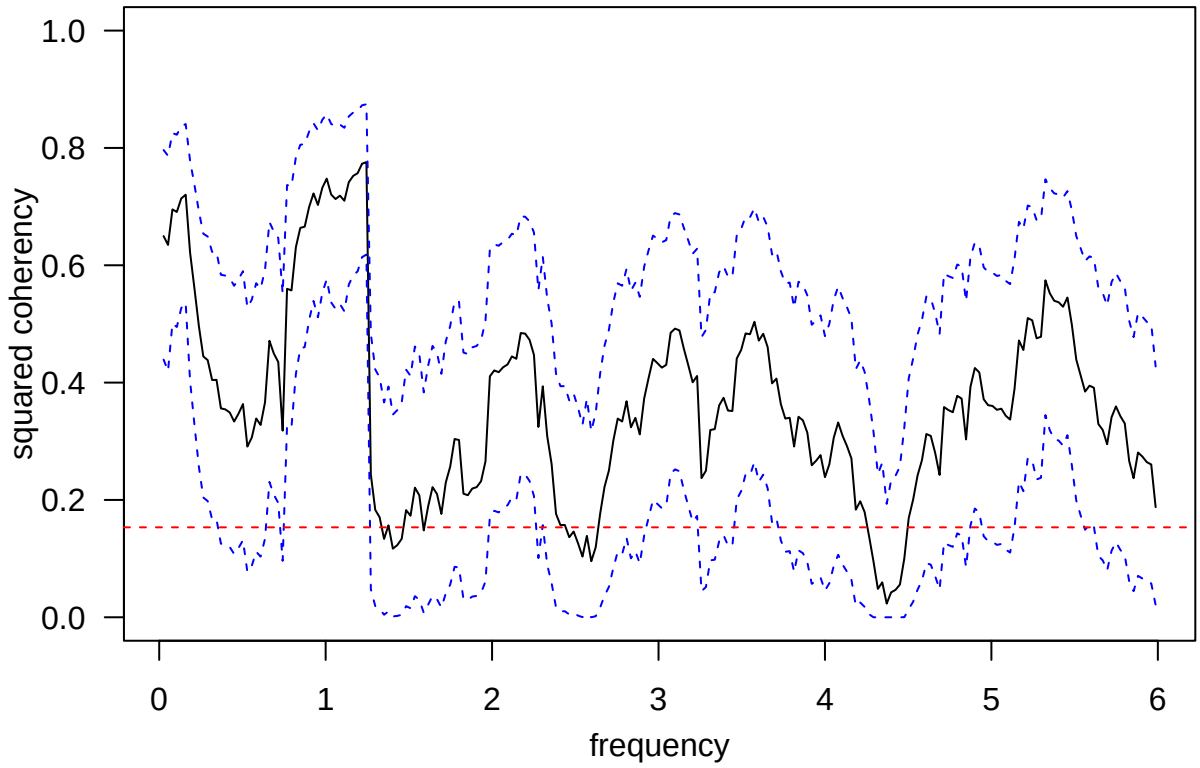


**Estimating cross-spectrum**

```
par(las = 1)
s = spectrum(cbind(soi, rec), kernel("daniell", 9),
             taper = 0, fast = FALSE)
```



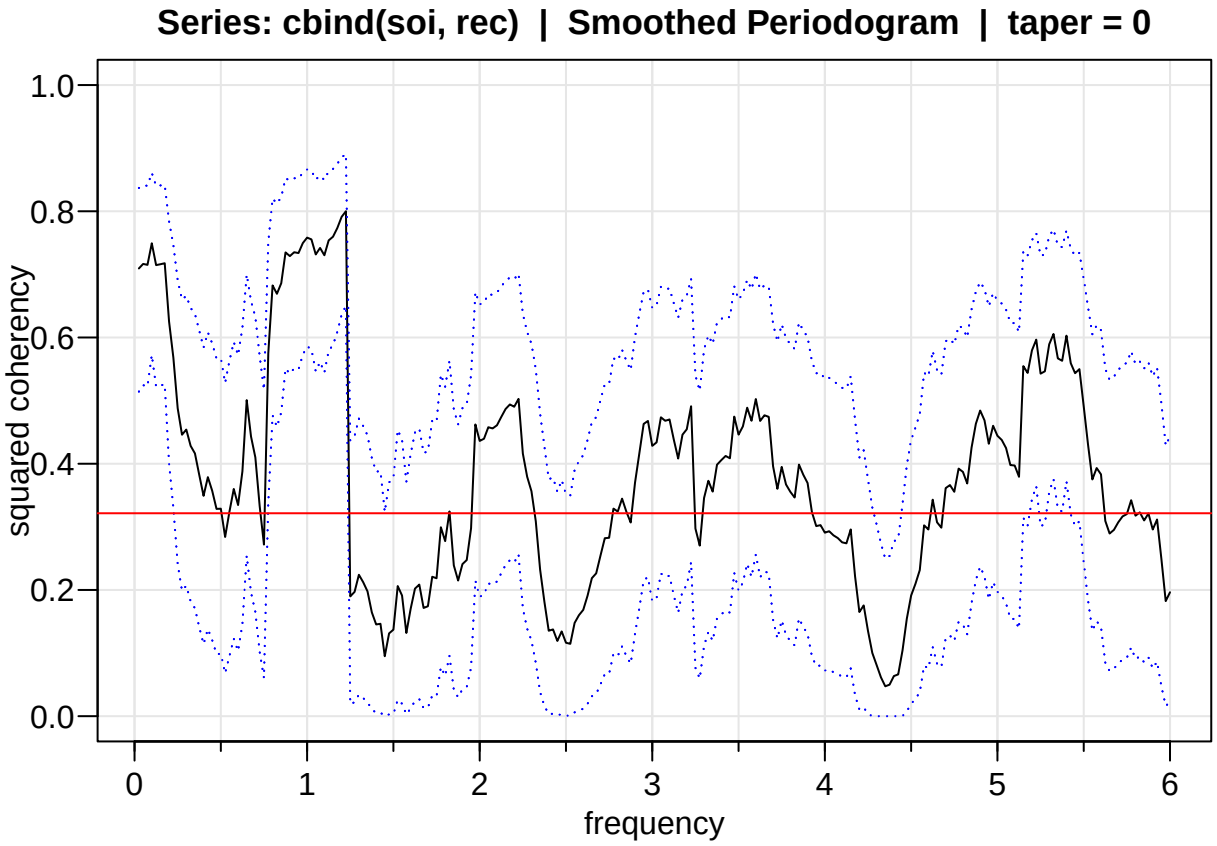

```
par(las = 1, mar = c(4, 4, 2, 0.6), mgp = c(2.2, 1, 0))
plot(s, plot.type = "coh", ci.lty = 2, main = "")
f = qf(.95, 2, s$df - 2);
abline(h = f / ((s$df - 2) / 2 + f), col = "red", lty = 2)
```



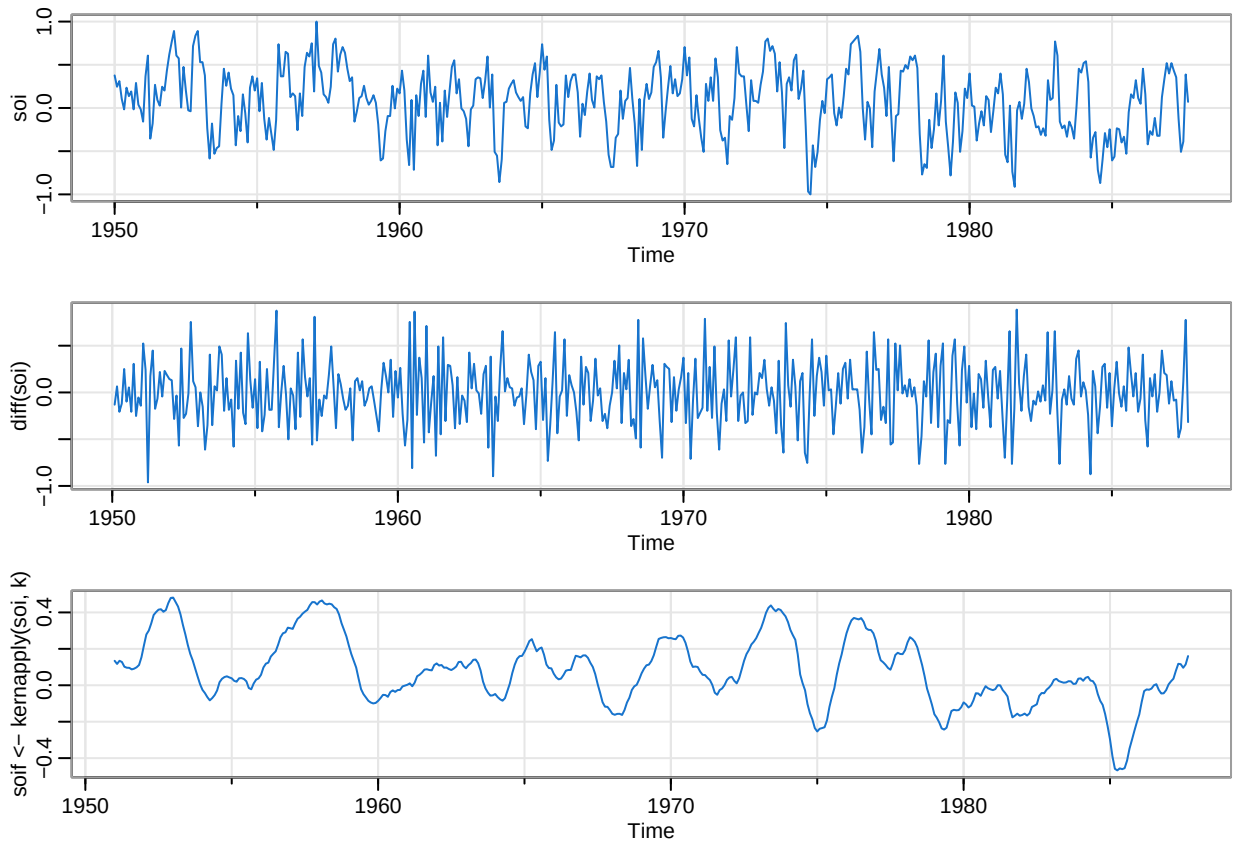

```
sr <- mvspec(cbind(soi, rec), kernel("daniell", 9),
             plot.type = "coh")
sr$df
```

```
## [1] 35.8625
```

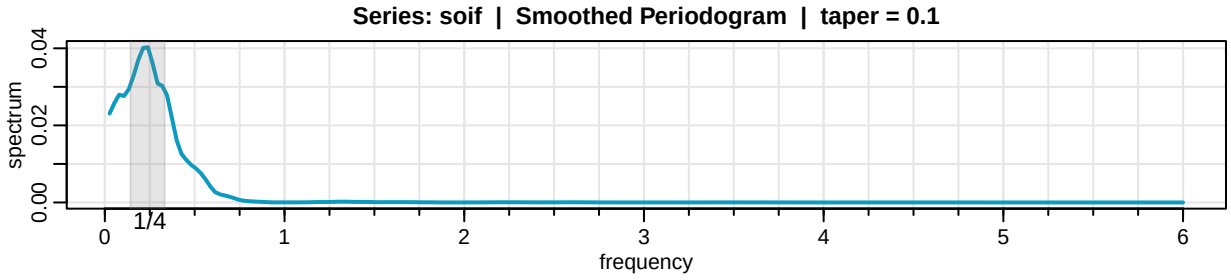
```
f = qf(.999, 2, sr$df - 2)
C = f / (18 + f)
abline(h = C, col = "red")
```



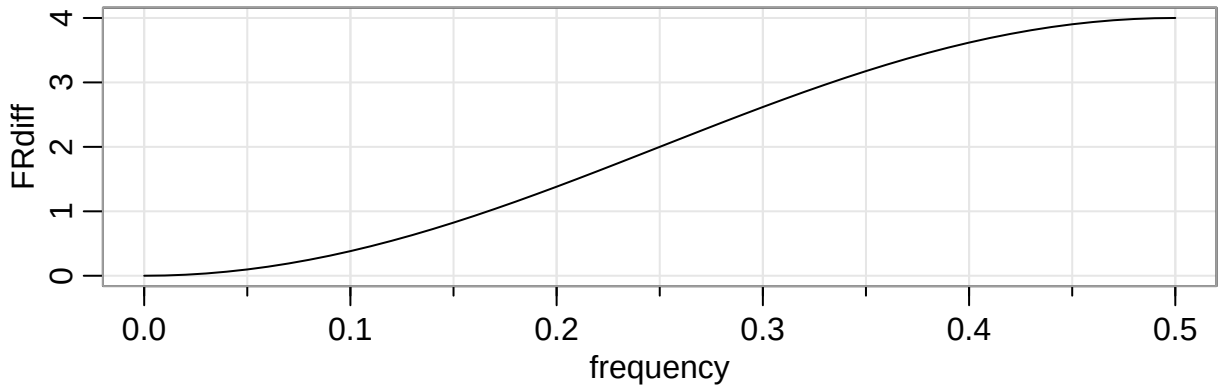

```
par(mfrow=c(3,1))
tsplot(soi, col=4)                          # plot data
tsplot(diff(soi), col=4)                    # plot first difference
k = kernel("modified.daniell", 6)           # filter weights
tsplot(soif <- kernapply(soi, k), col=4)    # plot 12 month filter
```

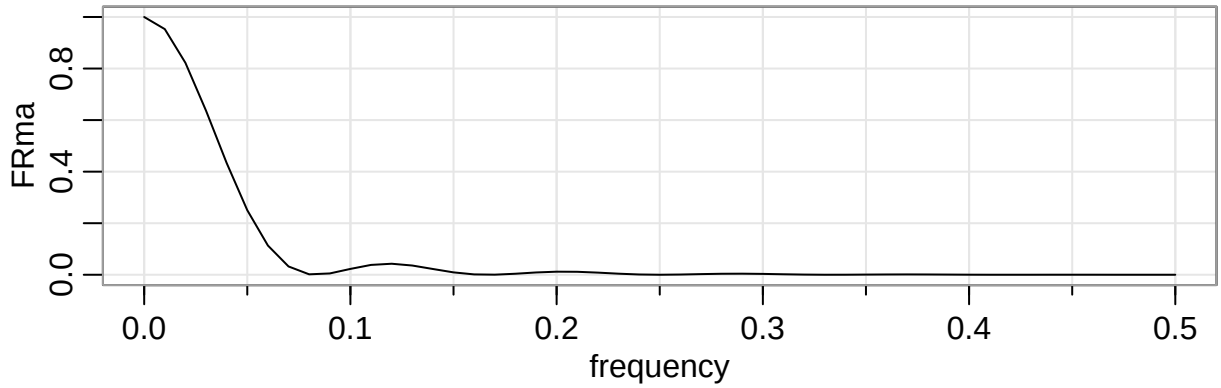



```
mvspec(soif, spans=9, lwd=2, col=5, nxm=4, taper=.1) # spectral analysis (not shown)
rect(1/7, -1e5, 1/3, 1e5, density=NA, col=gray(.5,.2))
mtext("1/4", side=1, line=0, at=.25, cex=.75)
##-- frequency responses --##
par(mfrow=c(2,1))
```




```
w = seq(0, .5, by=.01)
FRdiff = abs(1-exp(2i*pi*w))^2
tsplot(w, FRdiff, xlab='frequency')
u = cos(2*pi*w)+cos(4*pi*w)+cos(6*pi*w)+cos(8*pi*w)+cos(10*pi*w)
FRma = ((1 + cos(12*pi*w) + 2*u)/12)^2
tsplot(w, FRma, xlab='frequency')
```

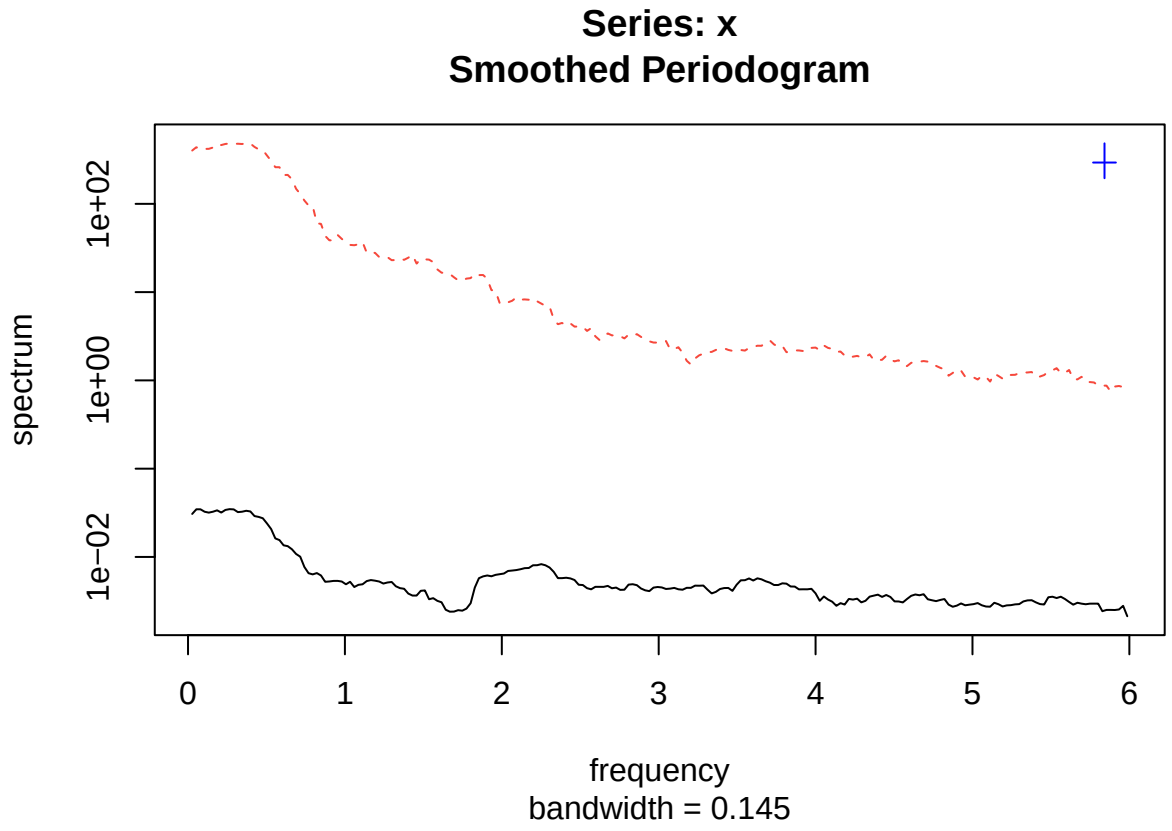


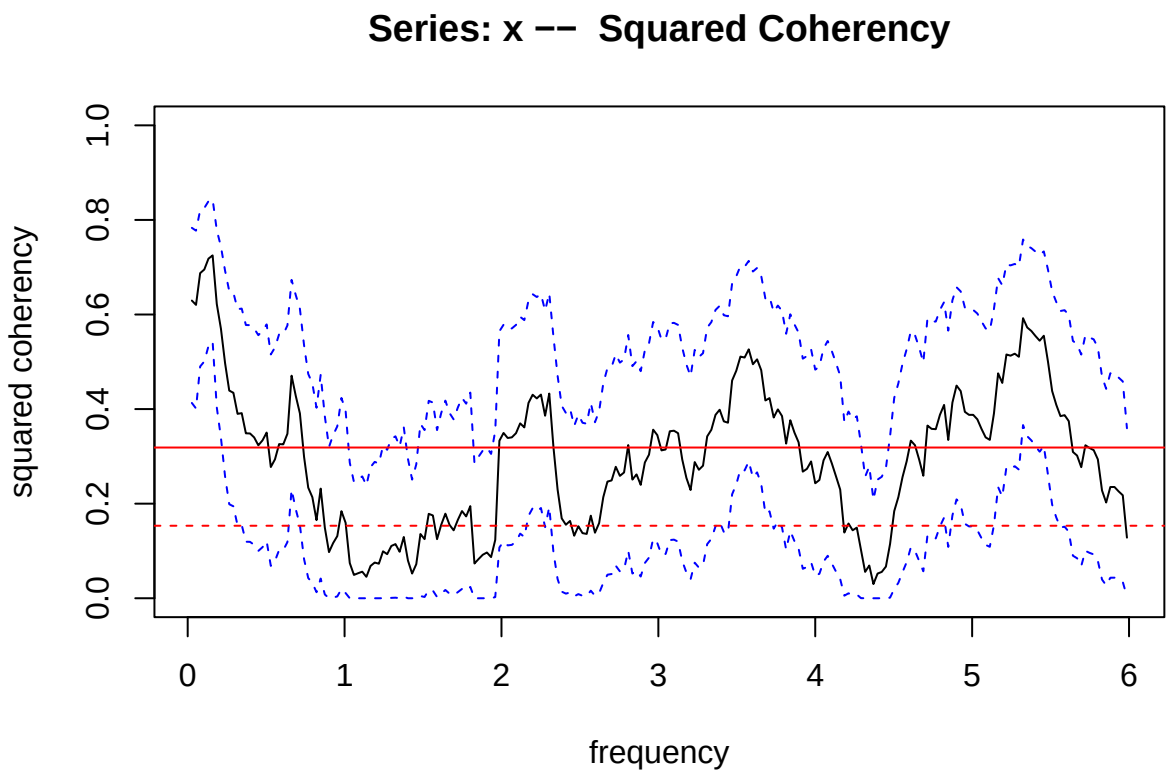

**Applying seasonal adjustment (code courtesy of**

Peter Bloomfield)

```
soiSA <- residuals(lm(soi ~ factor(cycle(soi))))
soiSA <- ts(soiSA, start = start(soi), frequency = frequency(soi))
recSA <- residuals(lm(rec ~ factor(cycle(rec))))
recSA <- ts(recSA, start = start(rec), frequency = frequency(rec))
sSA <- spectrum(cbind(soiSA, recSA), kernel("daniell", 9),
taper = 0, fast = FALSE)
```



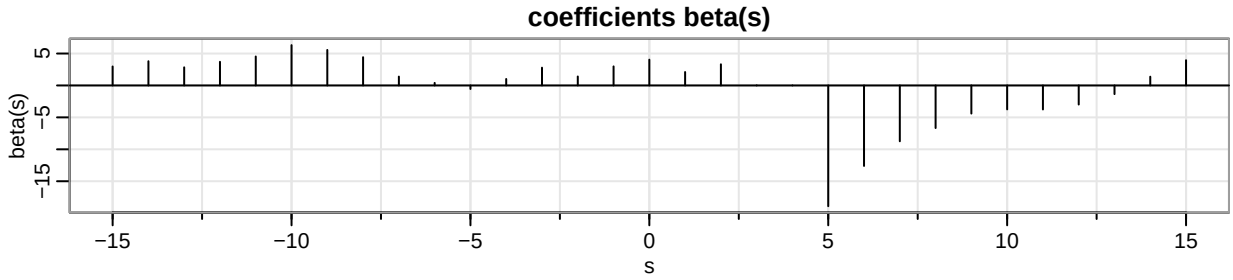

```
plot(sSA, plot.type = "coh", ci.lty = 2);
f <- qf(.999, 2, sSA$df - 2)
abline(h = f / ((sSA$df - 2) / 2 + f), col = "red");
f <- qf(.95, 2, sSA$df - 2)
abline(h = f / ((sSA$df - 2) / 2 + f), col = "red", lty = 2)
```

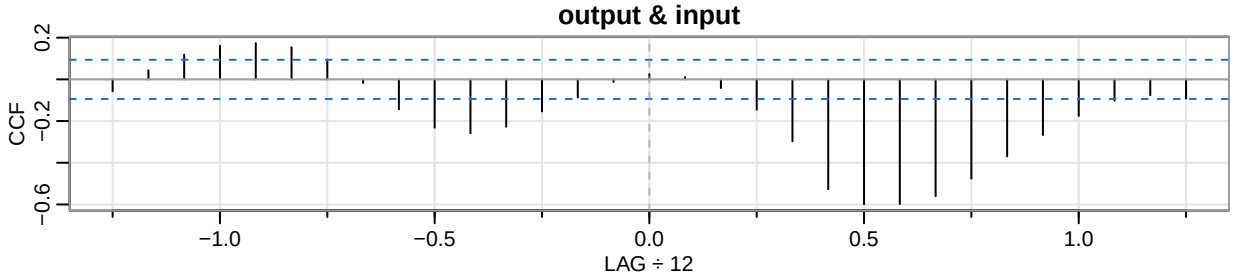


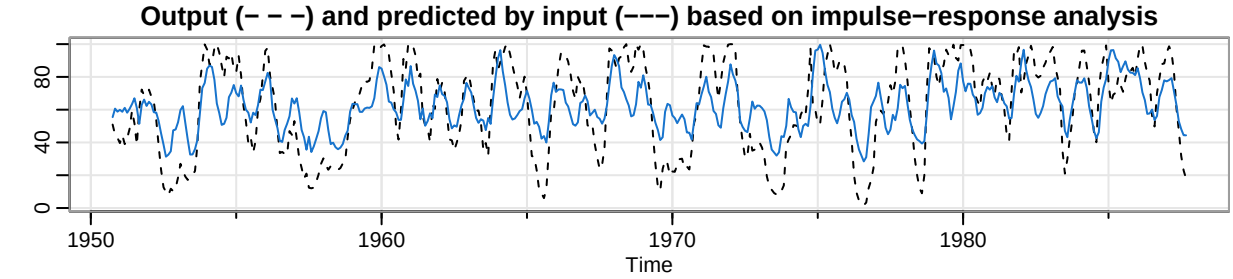

**Lagged Regression in frequency domain**

```
LagReg(soi, rec, L = 15, M = 32, threshold = 6)
```

```
## INPUT: soi OUTPUT: rec   L = 15    M = 32
##
## The coefficients beta(0), beta(1), beta(2) ... beta(M/2-1) are
##
## 4.03743 2.103372 3.31812 0.01247538 0.005194443 -18.90914 -12.60978 -8.746491
## -6.670373 -4.404543 -3.748336 -3.760936 -2.991477 -1.355261 1.375379 3.955252
##
##
##
## The coefficients beta(0), beta(-1), beta(-2) ... beta(-M/2+1) are
##
## 4.03743 2.987159 1.409949 2.788212 1.017324 -0.5528797 0.402843 1.389537
## 4.426287 5.563582 6.315986 4.540402 3.703423 2.840445 3.798354 2.974338

## The positive lags, at which the coefficients are large
## in absolute value, and the coefficients themselves, are:
##      lag s    beta(s)
## [1,]     5 -18.909140
## [2,]     6 -12.609781
## [3,]     7  -8.746491
## [4,]     8  -6.670373
```




```
##
## The prediction equation is
## rec(t) = alpha + sum_s[ beta(s)*soi(t-s) ], where alpha = 66.01941
## MSE = 411.5948
```

```
## $betas
##         S             b
##  [1,] -15   2.974338364
##  [2,] -14   3.798354349
##  [3,] -13   2.840445310
##  [4,] -12   3.703422986
##  [5,] -11   4.540402464
##  [6,] -10   6.315985671
##  [7,]  -9   5.563581967
##  [8,]  -8   4.426286804
##  [9,]  -7   1.389536617
## [10,]  -6   0.402842994
## [11,]  -5  -0.552879696
## [12,]  -4   1.017323713
## [13,]  -3   2.788212457
## [14,]  -2   1.409949212
## [15,]  -1   2.987158959
## [16,]   0   4.037429653
## [17,]   1   2.103372494
## [18,]   2   3.318120131
## [19,]   3   0.012475378
## [20,]   4   0.005194443
```

```
## [21,]   5 -18.909139762
## [22,]   6 -12.609781033
## [23,]   7  -8.746490655
## [24,]   8  -6.670373374
## [25,]   9  -4.404543150
## [26,]  10  -3.748336347
## [27,]  11  -3.760935651
## [28,]  12  -2.991477145
## [29,]  13  -1.355261381
## [30,]  14   1.375378996
## [31,]  15   3.955252475
##
## $fit
##              output      fit      resids
## Oct 1950  50.910000 55.46485  -4.5548459
## Nov 1950  44.700000 60.64946 -15.9494644
## Dec 1950  42.850000 58.79339 -15.9433933
## Jan 1951  39.620000 59.91178 -20.2917801
## Feb 1951  44.450000 58.73152 -14.2815208
## Mar 1951  38.980000 61.14768 -22.1676769
## Apr 1951  42.620000 58.15309 -15.5330909
## May 1951  48.270000 60.50992 -12.2399245
## Jun 1951  59.390000 63.41302  -4.0230240
## Jul 1951  51.660000 66.94203 -15.2820315
## Aug 1951  38.550000 60.97748 -22.4274757
## Sep 1951  60.330000 51.41505   8.9149504
## Oct 1951  72.270000 62.93072   9.3392833
## Nov 1951  68.620000 66.14905   2.4709533
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## Jan 1952  72.200000 64.82379   7.3762147
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## May 1952  37.960000 52.97174 -15.0117449
## Jun 1952  23.230000 45.49734 -22.2673382
## Jul 1952  12.680000 38.74233 -26.0623253
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## May 1953  17.140000 41.69523 -24.5552285
## Jun 1953  18.610000 32.62057 -14.0105722
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## Oct 1953  76.550000 57.53823  19.0117717
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## Jan 1954  96.690000 84.99525  11.6947548
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## Mar 1958  23.340000 47.59720 -24.2571980
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## Jul 1958  23.340000 35.86964 -12.5296450
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## Oct 1963  61.560000 51.33222  10.2277839
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## Jul 1967  23.750000 54.44341 -30.6934052
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## Jan 1972 100.000000 87.65982  12.3401757
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## Jan 1985  76.550000 89.91407 -13.3640659
## Feb 1985  64.170000 96.21493 -32.0449308
## Mar 1985  69.200000 96.37067 -27.1706743
## Apr 1985  70.370000 91.39517 -21.0251735
## May 1985  79.550000 89.27045  -9.7204546
## Jun 1985  74.790000 82.90792  -8.1179196
## Jul 1985  70.900000 86.50457 -15.6045748
```
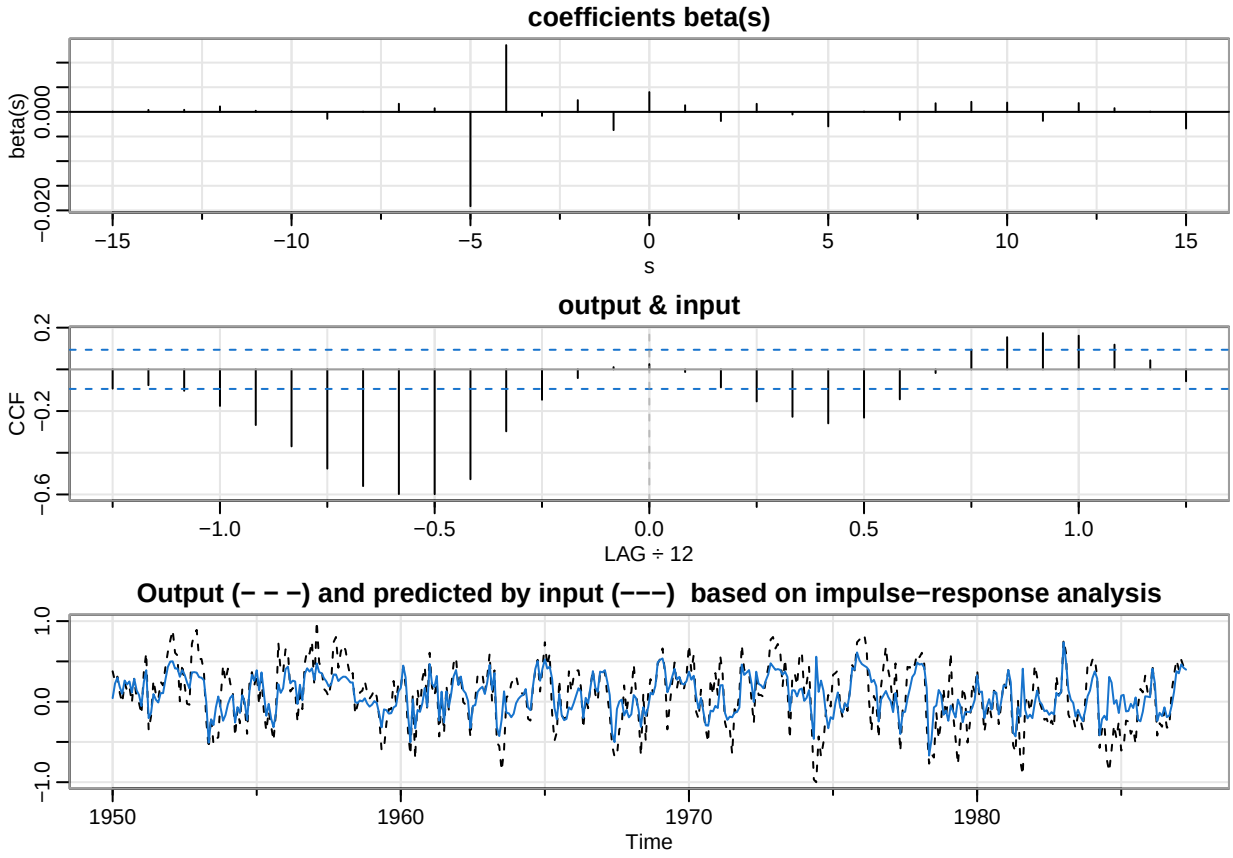

```
## Aug 1985  78.860000 89.49937 -10.6393748
## Sep 1985  84.280000 84.51239  -0.2323920
## Oct 1985  83.430000 82.60754   0.8224564
## Nov 1985  85.550000 82.47700   3.0729958
## Dec 1985  80.170000 81.06663  -0.8966264
## Jan 1986  90.820000 85.37107   5.4489314
## Feb 1986  99.390000 79.20320  20.1868020
## Mar 1986  99.180000 70.50652  28.6734846
## Apr 1986  89.100000 65.81639  23.2836147
## May 1986  82.180000 57.42544  24.7545593
## Jun 1986  77.640000 57.72474  19.9152602
## Jul 1986  55.930000 60.05927  -4.1292702
## Aug 1986  49.730000 53.66307  -3.9330704
## Sep 1986  70.120000 56.11125  14.0087499
## Oct 1986  79.200000 67.69005  11.5099468
## Nov 1986  87.830000 71.98548  15.8445220
## Dec 1986  88.200000 77.90792  10.2920768
## Jan 1987  94.830000 77.26789  17.5621145
## Feb 1987  98.660010 78.06609  20.5939177
## Mar 1987  94.839990 79.24883  15.5911592
## Apr 1987  83.060000 71.28067  11.7793334
## May 1987  61.420000 63.09806  -1.6780601
## Jun 1987  47.470000 53.05231  -5.5823076
## Jul 1987  31.810000 48.17713 -16.3671342
## Aug 1987  22.950000 44.41360 -21.4635966
## Sep 1987  17.870000 44.35441 -26.4844075
```

```
LagReg(rec, soi, L = 15, M = 32, inverse = TRUE, threshold = .01)
```

```
## INPUT: rec OUTPUT: soi   L = 15    M = 32
##
## The coefficients beta(0), beta(1), beta(2) ... beta(M/2-1) are
##
## 0.00401381 0.001342919 -0.001857819 0.001637061 -0.0005376305 -0.002942707
## 7.277553e-05 -0.001615343 0.001750733 0.002040213 0.001887672 -0.00182302
## 0.001794064 0.0007542296 5.312944e-05 -0.003376224
##
##
## The coefficients beta(0), beta(-1), beta(-2) ... beta(-M/2+1) are
##
## 0.00401381 -0.003704779 0.002380091 -0.0008144651 0.01355105 -0.01914634
## 0.0007317915 0.001648078 4.701496e-05 -0.001410605 0.0001172051 0.0002275781
## 0.001111257 0.0004196674 0.0003951922 5.630268e-05

## The negative lags, at which the coefficients are large
## in absolute value, and the coefficients themselves, are:
##      lag s     beta(s)
## [1,]     4  0.01355105
## [2,]     5 -0.01914634
```




```
##
## The prediction equation is
## soi(t) = alpha + sum_s[ beta(s)*rec(t+s) ], where alpha = 0.4284158
## MSE = 0.07168914
```

```
## $betas
##         S             b
##  [1,] -15  5.630268e-05
##  [2,] -14  3.951922e-04
##  [3,] -13  4.196674e-04
##  [4,] -12  1.111257e-03
##  [5,] -11  2.275781e-04
##  [6,] -10  1.172051e-04
##  [7,]  -9 -1.410605e-03
##  [8,]  -8  4.701496e-05
##  [9,]  -7  1.648078e-03
## [10,]  -6  7.317915e-04
## [11,]  -5 -1.914634e-02
## [12,]  -4  1.355105e-02
## [13,]  -3 -8.144651e-04
## [14,]  -2  2.380091e-03
## [15,]  -1 -3.704779e-03
## [16,]   0  4.013810e-03
## [17,]   1  1.342919e-03
## [18,]   2 -1.857819e-03
## [19,]   3  1.637061e-03
## [20,]   4 -5.376305e-04
```

```
## [21,]   5 -2.942707e-03
## [22,]   6  7.277553e-05
## [23,]   7 -1.615343e-03
## [24,]   8  1.750733e-03
## [25,]   9  2.040213e-03
## [26,]  10  1.887672e-03
## [27,]  11 -1.823020e-03
## [28,]  12  1.794064e-03
## [29,]  13  7.542296e-04
## [30,]  14  5.312944e-05
## [31,]  15 -3.376224e-03
##
## $fit
##              output          fit        resids
## Jan 1950  0.3770000  0.044411097  0.3325889035
## Feb 1950  0.2460000  0.225726913  0.0202730872
## Mar 1950  0.3110000  0.297669179  0.0133308213
## Apr 1950  0.1040000  0.178134968 -0.0741349682
## May 1950 -0.0160000  0.097892595 -0.1138925951
## Jun 1950  0.2350000  0.262458383 -0.0274583828
## Jul 1950  0.1370000  0.213727099 -0.0767270992
## Aug 1950  0.1910000  0.250500331 -0.0595003306
## Sep 1950 -0.0160000  0.114253635 -0.1302536353
## Oct 1950  0.2900000  0.284435663  0.0055643366
## Nov 1950  0.0380000  0.140618762 -0.1026187623
## Dec 1950 -0.0160000  0.081767770 -0.0977677700
## Jan 1951 -0.1580000 -0.054576082 -0.1034239180
## Feb 1951  0.3660000  0.244112761  0.1218872393
## Mar 1951  0.6070000  0.390371645  0.2166283554
## Apr 1951 -0.3550000 -0.204289826 -0.1507101742
## May 1951 -0.1800000 -0.137755270 -0.0422447296
## Jun 1951  0.2680000  0.093928375  0.1740716254
## Jul 1951  0.0930000  0.025129249  0.0678707515
## Aug 1951  0.0270000 -0.010390280  0.0373902804
## Sep 1951  0.2460000  0.107339554  0.1386604456
## Oct 1951  0.2020000  0.105336484  0.0966635158
## Nov 1951  0.4320000  0.276984202  0.1550157980
## Dec 1951  0.6170000  0.431344779  0.1856552212
## Jan 1952  0.7600000  0.498044178  0.2619558222
## Feb 1952  0.8910000  0.500431102  0.3905688984
## Mar 1952  0.6070000  0.411843144  0.1951568562
## Apr 1952  0.5740000  0.412033769  0.1619662307
## May 1952  0.0050000  0.308841156 -0.3038411558
## Jun 1952  0.4750000  0.392371592  0.0826284076
## Jul 1952  0.2020000  0.333513673 -0.1315136726
## Aug 1952 -0.0270000  0.237481214 -0.2644812136
## Sep 1952 -0.0380000  0.164088586 -0.2020885863
## Oct 1952  0.7160000  0.362712060  0.3532879400
## Nov 1952  0.8360000  0.366358664  0.4696413363
## Dec 1952  0.8910000  0.360156692  0.5308433081
## Jan 1953  0.5300000  0.304367438  0.2256325621
## Feb 1953  0.5300000  0.297291143  0.2327088575
## Mar 1953  0.3770000  0.266043253  0.1109567475
## Apr 1953 -0.2350000 -0.011168646 -0.2238313537
```

```
## May 1953 -0.5850000 -0.508880959 -0.0761190408
## Jun 1953 -0.1800000 -0.218937698  0.0389376977
## Jul 1953 -0.5300000 -0.290031955 -0.2399680453
## Aug 1953 -0.4640000 -0.070448968 -0.3935510318
## Sep 1953 -0.4430000  0.064319431 -0.5073194312
## Oct 1953  0.0490000 -0.082336150  0.1313361503
## Nov 1953  0.4540000 -0.236795527  0.6907955269
## Dec 1953  0.2570000 -0.165747866  0.4227478657
## Jan 1954  0.4100000 -0.033219398  0.4432193982
## Feb 1954  0.2240000  0.048599350  0.1754006504
## Mar 1954  0.1480000  0.080683800  0.0673162002
## Apr 1954 -0.4320000 -0.259323261 -0.1726767389
## May 1954 -0.0930000 -0.068014053 -0.0249859468
## Jun 1954 -0.2680000 -0.140200103 -0.1277998967
## Jul 1954  0.1580000  0.079933781  0.0780662192
## Aug 1954 -0.0600000 -0.008994280 -0.0510057198
## Sep 1954 -0.3990000 -0.227480385 -0.1715196148
## Oct 1954  0.2350000  0.136390275  0.0986097246
## Nov 1954  0.3660000  0.332706696  0.0332933040
## Dec 1954  0.2020000  0.259030255 -0.0570302549
## Jan 1955  0.3440000  0.389631128 -0.0456311276
## Feb 1955 -0.0380000  0.142710864 -0.1807108642
## Mar 1955  0.2900000  0.363375421 -0.0733754208
## Apr 1955 -0.1260000  0.101473749 -0.2274737486
## May 1955 -0.3660000 -0.174751167 -0.1912488330
## Jun 1955 -0.1150000 -0.027083322 -0.0879166781
## Jul 1955 -0.3010000 -0.207931486 -0.0930685137
## Aug 1955 -0.4860000 -0.317718860 -0.1682811399
## Sep 1955 -0.1370000 -0.143943752  0.0069437517
## Oct 1955  0.7380000  0.216593125  0.5214068749
## Nov 1955  0.3660000  0.109644880  0.2563551197
## Dec 1955  0.3660000  0.166264796  0.1997352036
## Jan 1956  0.6500000  0.402926742  0.2470732584
## Feb 1956  0.6280000  0.434697651  0.1933023491
## Mar 1956  0.1260000  0.227323765 -0.1013237649
## Apr 1956  0.1690000  0.261260858 -0.0922608576
## May 1956  0.1370000  0.251709562 -0.1147095617
## Jun 1956 -0.2570000 -0.028843728 -0.2281562723
## Jul 1956  0.1690000  0.209538482 -0.0405384816
## Aug 1956 -0.0930000  0.018195784 -0.1111957838
## Sep 1956  0.4750000  0.343129132  0.1318708678
## Oct 1956  0.6390000  0.414665761  0.2243342394
## Nov 1956  0.5960000  0.373029181  0.2229708186
## Dec 1956  0.7490000  0.413876527  0.3351234732
## Jan 1957  0.1910000  0.271322072 -0.0803220716
## Feb 1957  1.0000000  0.475876608  0.5241233917
## Mar 1957  0.4860000  0.365711840  0.1202881602
## Apr 1957  0.4100000  0.353828875  0.0561711248
## May 1957  0.1580000  0.292349019 -0.1343490189
## Jun 1957  0.1260000  0.252007903 -0.1260079034
## Jul 1957  0.0600000  0.189676462 -0.1296764623
## Aug 1957  0.2460000  0.213673996  0.0323260035
## Sep 1957  0.7380000  0.337051554  0.4009484461
## Oct 1957  0.8030000  0.336713267  0.4662867333
```

```
## Nov 1957  0.4210000  0.255508353  0.1654916469
## Dec 1957  0.6170000  0.288328120  0.3286718802
## Jan 1958  0.7050000  0.310036921  0.3949630789
## Feb 1958  0.6390000  0.308933618  0.3300663817
## Mar 1958  0.4540000  0.277909568  0.1760904321
## Apr 1958  0.3110000  0.238009986  0.0729900139
## May 1958  0.3550000  0.238314193  0.1166858069
## Jun 1958 -0.1580000 -0.005201018 -0.1527989824
## Jul 1958 -0.0380000 -0.017194324 -0.0208056757
## Aug 1958  0.1150000  0.014903194  0.1000968057
## Sep 1958  0.1370000  0.002904748  0.1340952520
## Oct 1958  0.2570000  0.042134571  0.2148654291
## Nov 1958  0.1150000 -0.023264614  0.1382646139
## Dec 1958  0.0380000 -0.061580950  0.0995809497
## Jan 1959  0.0820000 -0.037926982  0.1199269824
## Feb 1959  0.1480000  0.006881537  0.1411184630
## Mar 1959  0.0600000 -0.016365955  0.0763659550
## Apr 1959 -0.1910000 -0.148595467 -0.0424045328
## May 1959 -0.6070000 -0.319980445 -0.2870195548
## Jun 1959 -0.5850000 -0.096020387 -0.4889796133
## Jul 1959 -0.2680000 -0.105224269 -0.1627757311
## Aug 1959 -0.0930000 -0.149094179  0.0560941794
## Sep 1959 -0.0930000 -0.143138594  0.0501385944
## Oct 1959  0.2570000 -0.071406285  0.3284062851
## Nov 1959 -0.0050000 -0.051604212  0.0466042119
## Dec 1959  0.2240000  0.124449462  0.0995505375
## Jan 1960  0.1690000  0.171531240 -0.0025312400
## Feb 1960  0.4320000  0.447601454 -0.0156014543
## Mar 1960  0.2020000  0.301132253 -0.0991322527
## Apr 1960 -0.3660000 -0.150260476 -0.2157395239
## May 1960 -0.6610000 -0.503244858 -0.1577551416
## Jun 1960  0.0930000  0.028737916  0.0642620842
## Jul 1960 -0.7160000 -0.330855583 -0.3851444168
## Aug 1960  0.1480000 -0.102393629  0.2503936295
## Sep 1960 -0.0930000 -0.126549469  0.0335494690
## Oct 1960  0.2790000 -0.016619769  0.2956197693
## Nov 1960  0.4320000  0.170373576  0.2616264239
## Dec 1960 -0.1040000 -0.065462790 -0.0385372095
## Jan 1961  0.6070000  0.468149100  0.1388509001
## Feb 1961  0.1800000  0.221677494 -0.0416774938
## Mar 1961  0.0710000  0.179975268 -0.1089752681
## Apr 1961  0.2460000  0.319387730 -0.0733877297
## May 1961 -0.4320000 -0.234901255 -0.1970987449
## Jun 1961  0.0600000  0.124128999 -0.0641289989
## Jul 1961 -0.3880000 -0.258809959 -0.1291900412
## Aug 1961  0.2020000  0.128680206  0.0733197940
## Sep 1961 -0.1040000 -0.067800465 -0.0361995354
## Oct 1961  0.1910000  0.099945867  0.0910541334
## Nov 1961  0.4750000  0.312925770  0.1620742301
## Dec 1961  0.5520000  0.392425225  0.1595747755
## Jan 1962  0.1690000  0.205796705 -0.0367967053
## Feb 1962  0.3330000  0.311778248  0.0212217519
## Mar 1962 -0.0050000  0.143323776 -0.1483237762
## Apr 1962 -0.0380000  0.104486377 -0.1424863774
```

```
## May 1962 -0.1260000  0.009901654 -0.1359016540
## Jun 1962 -0.4430000 -0.327144813 -0.1158551875
## Jul 1962 -0.0160000 -0.052925850  0.0369258502
## Aug 1962  0.0270000 -0.043443636  0.0704436364
## Sep 1962  0.3330000  0.121057483  0.2119425170
## Oct 1962  0.3550000  0.160694872  0.1943051285
## Nov 1962  0.3440000  0.185156818  0.1588431818
## Dec 1962  0.1150000  0.095145511  0.0198544887
## Jan 1963  0.3110000  0.240872745  0.0701272555
## Feb 1963  0.5960000  0.457572663  0.1384273366
## Mar 1963  0.0050000  0.146395934 -0.1413959338
## Apr 1963  0.3880000  0.354946703  0.0330532974
## May 1963 -0.5080001 -0.317954290 -0.1900458099
## Jun 1963 -0.5520000 -0.427045955 -0.1249540451
## Jul 1963 -0.8580000 -0.222559915 -0.6354400851
## Aug 1963 -0.5960000  0.128406335 -0.7244063349
## Sep 1963  0.0600000 -0.139090571  0.1990905708
## Oct 1963  0.0710000 -0.144898359  0.2158983586
## Nov 1963  0.2240000 -0.193636766  0.4176367660
## Dec 1963  0.2790000 -0.150264697  0.4292646970
## Jan 1964  0.3220000 -0.019272294  0.3412722941
## Feb 1964  0.1800000  0.035469694  0.1445303058
## Mar 1964  0.1260000  0.067655738  0.0583442617
## Apr 1964  0.0820000  0.085438312 -0.0034383116
## May 1964  0.1260000  0.156596315 -0.0305963149
## Jun 1964 -0.2130000 -0.075181805 -0.1378181951
## Jul 1964 -0.2350000 -0.105418241 -0.1295817589
## Aug 1964  0.1690000  0.184819173 -0.0158191727
## Sep 1964  0.3880000  0.404752152 -0.0167521522
## Oct 1964  0.5190000  0.504697152  0.0143028476
## Nov 1964  0.1260000  0.259389969 -0.1333899695
## Dec 1964  0.4100000  0.409414206  0.0005857938
## Jan 1965  0.7380000  0.524843995  0.2131560051
## Feb 1965  0.4430000  0.410262496  0.0327375044
## Mar 1965  0.5960000  0.434380791  0.1616192088
## Apr 1965 -0.1370000  0.286116487 -0.4231164869
## May 1965 -0.4860000 -0.076074590 -0.4099254097
## Jun 1965 -0.3770000 -0.218734332 -0.1582656678
## Jul 1965  0.2680000  0.095453266  0.1725467337
## Aug 1965 -0.1690000 -0.167086739 -0.0019132613
## Sep 1965 -0.2130000 -0.229496984  0.0164969836
## Oct 1965 -0.1910000 -0.214754472  0.0237544720
## Nov 1965  0.3770000 -0.070061408  0.4470614081
## Dec 1965  0.1260000 -0.098509326  0.2245093264
## Jan 1966  0.3110000 -0.019048398  0.3300483984
## Feb 1966  0.3880000  0.075879892  0.3121201082
## Mar 1966  0.3880000  0.166773512  0.2212264879
## Apr 1966  0.1800000  0.124802947  0.0551970527
## May 1966 -0.3330000 -0.203613504 -0.1293864961
## Jun 1966  0.0490000  0.047692930  0.0013070698
## Jul 1966 -0.0820000 -0.018690612 -0.0633093881
## Aug 1966  0.1910000  0.181975915  0.0090240850
## Sep 1966  0.3990000  0.378955461  0.0200445387
## Oct 1966  0.0930000  0.175418613 -0.0824186129
```

```
## Nov 1966  0.0050000  0.122440105 -0.1174401049
## Dec 1966  0.3660000  0.373103235 -0.0071032349
## Jan 1967  0.3330000  0.363732173 -0.0307321731
## Feb 1967  0.3770000  0.390927068 -0.0139270682
## Mar 1967  0.0710000  0.247087456 -0.1760874563
## Apr 1967 -0.1580000  0.082441161 -0.2404411610
## May 1967 -0.5410000 -0.394041563 -0.1469584366
## Jun 1967 -0.6830000 -0.502443142 -0.1805568585
## Jul 1967 -0.6830000 -0.091810655 -0.5911893453
## Aug 1967 -0.3440000  0.001757251 -0.3457572510
## Sep 1967 -0.3010000  0.026168468 -0.3271684684
## Oct 1967  0.2020000 -0.170996092  0.3729960917
## Nov 1967 -0.1260000 -0.104684825 -0.0213151753
## Dec 1967  0.0820000 -0.194677132  0.2766771323
## Jan 1968  0.1150000 -0.156907336  0.2719073363
## Feb 1968  0.4640000  0.082209412  0.3817905876
## Mar 1968  0.1040000  0.041669152  0.0623308477
## Apr 1968 -0.1800000 -0.098610481 -0.0813895189
## May 1968 -0.6720000 -0.313236595 -0.3587634050
## Jun 1968  0.1040000 -0.068702193  0.1727021931
## Jul 1968 -0.4860000 -0.175766898 -0.3102331023
## Aug 1968  0.0930000 -0.072381089  0.1653810890
## Sep 1968  0.2790000  0.111640607  0.1673593928
## Oct 1968  0.1040000  0.089041086  0.0149589144
## Nov 1968  0.1800000  0.194910807 -0.0149108067
## Dec 1968  0.4750000  0.489473411 -0.0144734108
## Jan 1969  0.5300000  0.519847314  0.0101526862
## Feb 1969  0.6610000  0.536810847  0.1241891533
## Mar 1969  0.3550000  0.404250171 -0.0492501712
## Apr 1969 -0.3440000  0.116341744 -0.4603417440
## May 1969 -0.0380000  0.192647161 -0.2306471614
## Jun 1969  0.2350000  0.308607454 -0.0736074543
## Jul 1969  0.4860000  0.408424597  0.0775754028
## Aug 1969  0.1690000  0.273118654 -0.1041186539
## Sep 1969  0.3330000  0.308451049  0.0245489509
## Oct 1969  0.1370000  0.225853403 -0.0888534027
## Nov 1969  0.1690000  0.212700952 -0.0437009525
## Dec 1969  0.3330000  0.256067798  0.0769322021
## Jan 1970  0.7050000  0.372003676  0.3329963237
## Feb 1970  0.3770000  0.272626391  0.1043736089
## Mar 1970  0.5850000  0.321318399  0.2636816010
## Apr 1970 -0.1260000  0.064872902 -0.1908729019
## May 1970 -0.2130000 -0.086611502 -0.1263884980
## Jun 1970  0.1480000  0.047240420  0.1007595802
## Jul 1970 -0.1260000 -0.124840627 -0.0011593730
## Aug 1970 -0.3440000 -0.294180960 -0.0498190399
## Sep 1970 -0.5080001 -0.298369774 -0.2096303260
## Oct 1970  0.2790000 -0.111767649  0.3907676488
## Nov 1970  0.0820000 -0.122033770  0.2040337701
## Dec 1970  0.3550000 -0.051981723  0.4069817235
## Jan 1971  0.0820000 -0.079065765  0.1610657654
## Feb 1971  0.5740000  0.201768318  0.3722316821
## Mar 1971  0.3550000  0.205839080  0.1491609197
## Apr 1971 -0.2570000 -0.140323936 -0.1166760640
```

```
## May 1971 -0.3770000 -0.226274511 -0.1507254894
## Jun 1971 -0.3440000 -0.185939709 -0.1580602909
## Jul 1971 -0.6500000 -0.194176210 -0.4558237899
## Aug 1971 -0.0930000 -0.135584982  0.0425849816
## Sep 1971 -0.1370000 -0.129198502 -0.0078014980
## Oct 1971  0.1150000 -0.073306249  0.1883062491
## Nov 1971  0.7050000  0.461140960  0.2438590400
## Dec 1971  0.3990000  0.331633284  0.0673667162
## Jan 1972  0.3660000  0.360822454  0.0051775457
## Feb 1972  0.3660000  0.391888631 -0.0258886306
## Mar 1972  0.0380000  0.230306783 -0.1923067834
## Apr 1972 -0.2680000 -0.004354769 -0.2636452314
## May 1972  0.3220000  0.371885457 -0.0498854565
## Jun 1972  0.0820000  0.220709954 -0.1387099539
## Jul 1972  0.0820000  0.203072585 -0.1210725846
## Aug 1972  0.0600000  0.165786168 -0.1057861677
## Sep 1972  0.2680000  0.262850193  0.0051498067
## Oct 1972  0.4210000  0.337513954  0.0834860459
## Nov 1972  0.7700000  0.476366943  0.2936330571
## Dec 1972  0.8030000  0.436477139  0.3665228613
## Jan 1973  0.6610000  0.395195440  0.2658045604
## Feb 1973  0.7160000  0.404105926  0.3118940739
## Mar 1973  0.6280000  0.395080978  0.2329190224
## Apr 1973  0.1910000  0.333655329 -0.1426553288
## May 1973  0.5300000  0.374528793  0.1554712075
## Jun 1973  0.1800000  0.306491106 -0.1264911061
## Jul 1973 -0.4640000 -0.111094929 -0.3529050711
## Aug 1973  0.2790000  0.140861821  0.1381381792
## Sep 1973  0.3550000  0.140260278  0.2147397219
## Oct 1973  0.2020000  0.065510842  0.1364891579
## Nov 1973  0.5520000  0.149865683  0.4021343171
## Dec 1973  0.6170000  0.162066012  0.4549339881
## Jan 1974  0.1040000  0.009733739  0.0942662608
## Feb 1974  0.2240000  0.048033314  0.1759666862
## Mar 1974  0.4320000  0.143306847  0.2886931533
## Apr 1974 -0.2130000 -0.165214929 -0.0477850715
## May 1974 -0.9670000 -0.462800777 -0.5041992230
## Jun 1974 -1.0000000  0.557865669 -1.5578656690
## Jul 1974 -0.4320000  0.121594526 -0.5535945255
## Aug 1974 -0.6830000  0.261162611 -0.9441626106
## Sep 1974 -0.5080001  0.136304997 -0.6443050974
## Oct 1974 -0.1370000 -0.139166216  0.0021662159
## Nov 1974  0.0820000 -0.330183030  0.4121830302
## Dec 1974 -0.0600000 -0.192623412  0.1326234121
## Jan 1975  0.3330000 -0.216196230  0.5491962301
## Feb 1975  0.3550000 -0.014909728  0.3699097282
## Mar 1975  0.3880000  0.114617508  0.2733824919
## Apr 1975 -0.0600000 -0.073661472  0.0136614721
## May 1975 -0.2020000 -0.135702078 -0.0662979217
## Jun 1975  0.0050000 -0.022255144  0.0272551444
## Jul 1975 -0.1150000 -0.062736938 -0.0522630621
## Aug 1975  0.1370000  0.106976544  0.0300234558
## Sep 1975  0.4540000  0.420911895  0.0330881047
## Oct 1975  0.3880000  0.396156581 -0.0081565815
```

```
## Nov 1975  0.7380000  0.611250572  0.1267494284
## Dec 1975  0.7700000  0.514088013  0.2559119870
## Jan 1976  0.8030000  0.467115763  0.3358842369
## Feb 1976  0.8360000  0.446378344  0.3896216565
## Mar 1976  0.6500000  0.427464589  0.2225354109
## Apr 1976  0.0050000  0.388157778 -0.3831577784
## May 1976 -0.3660000  0.241160222 -0.6071602217
## Jun 1976  0.0710000  0.270940095 -0.1999400949
## Jul 1976 -0.0050000  0.190952932 -0.1959529315
## Aug 1976 -0.4540000 -0.245944730 -0.2080552700
## Sep 1976  0.1910000  0.027316859  0.1636831405
## Oct 1976  0.4320000  0.077378777  0.3546212227
## Nov 1976  0.6830000  0.132321754  0.5506782457
## Dec 1976  0.3880000  0.056602822  0.3313971778
## Jan 1977  0.2350000  0.004855222  0.2301447782
## Feb 1977 -0.0930000 -0.129928933  0.0369289328
## Mar 1977  0.4750000  0.108857047  0.3661429533
## Apr 1977 -0.0820000 -0.103043557  0.0210435573
## May 1977 -0.6170000 -0.401663706 -0.2153362943
## Jun 1977 -0.0930000 -0.135039420  0.0420394198
## Jul 1977 -0.0710000 -0.126743558  0.0557435582
## Aug 1977  0.4320000  0.037322876  0.3946771242
## Sep 1977  0.4210000  0.162832362  0.2581676379
## Oct 1977  0.4970000  0.306383583  0.1906164172
## Nov 1977  0.4540000  0.349019801  0.1049801989
## Dec 1977  0.5960000  0.485700349  0.1102996514
## Jan 1978  0.5520000  0.459942184  0.0920578157
## Feb 1978  0.6070000  0.469639634  0.1373603656
## Mar 1978  0.4640000  0.420369829  0.0436301713
## Apr 1978 -0.3010000  0.166467576 -0.4674675764
## May 1978 -0.7700000 -0.669885678 -0.1001143217
## Jun 1978 -0.6500000 -0.469292853 -0.1807071468
## Jul 1978 -0.6940000 -0.071668670 -0.6223313300
## Aug 1978 -0.1370000 -0.048012723 -0.0889872769
## Sep 1978 -0.4640000  0.144663291 -0.6086632908
## Oct 1978 -0.2460000  0.085593935 -0.3315939350
## Nov 1978  0.1690000 -0.162594302  0.3315943018
## Dec 1978 -0.2020000 -0.110527418 -0.0914725817
## Jan 1979  0.0820000 -0.237875225  0.3198752250
## Feb 1979  0.6070000 -0.049648486  0.6566484864
## Mar 1979 -0.1580000 -0.119616506 -0.0383834944
## Apr 1979 -0.4540000 -0.196424127 -0.2575758731
## May 1979 -0.7810000  0.025108879 -0.8061088786
## Jun 1979 -0.3880000 -0.053095828 -0.3349041715
## Jul 1979  0.1800000 -0.255215235  0.4352152353
## Aug 1979  0.0600000 -0.129040253  0.1890402528
## Sep 1979 -0.5080001 -0.126368509 -0.3816315910
## Oct 1979 -0.2680000 -0.130459863 -0.1375401375
## Nov 1979  0.2240000 -0.073286080  0.2972860803
## Dec 1979  0.1150000  0.023434570  0.0915654300
## Jan 1980  0.3990000  0.327133167  0.0718668329
## Feb 1980  0.0270000  0.114071417 -0.0870714169
## Mar 1980  0.0160000  0.125878372 -0.1098783718
## Apr 1980 -0.3770000 -0.206594678 -0.1704053217
```

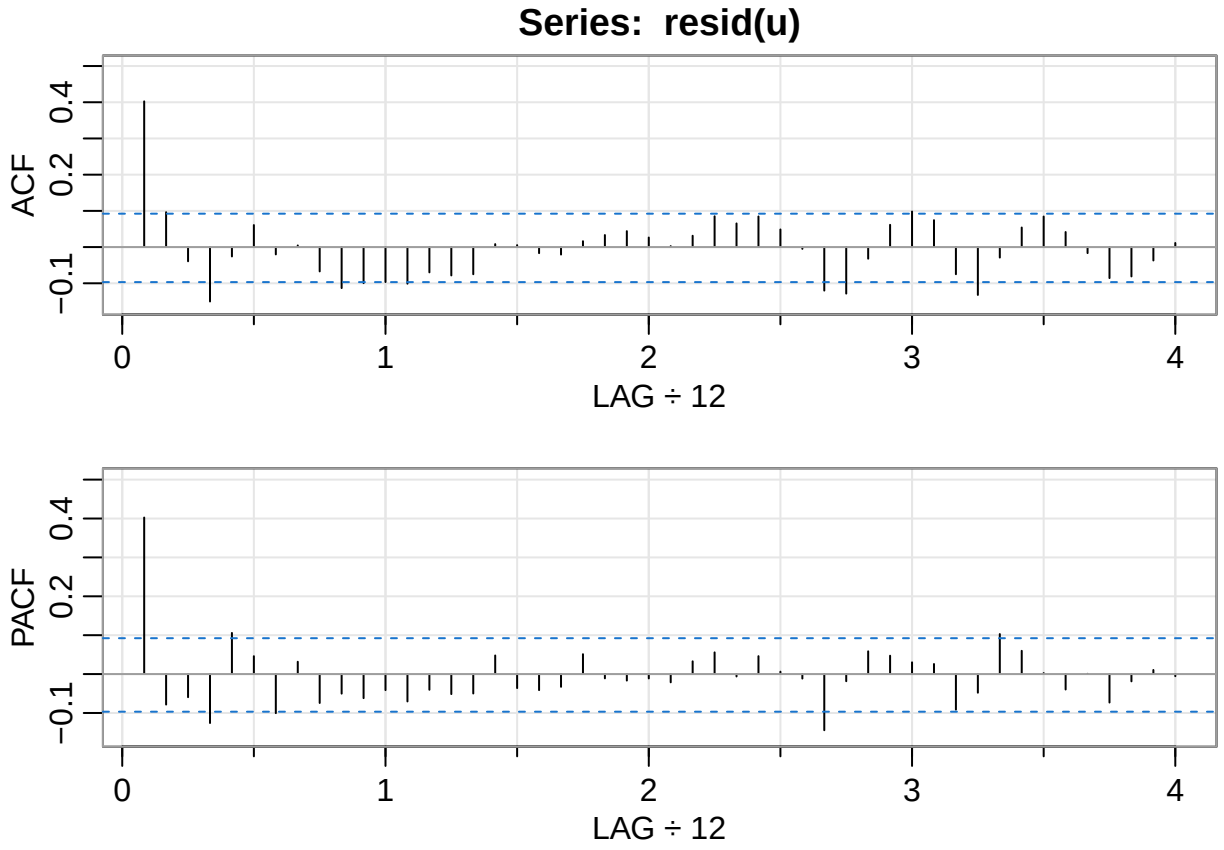
```
## May 1980 -0.1800000 -0.064199451 -0.1158005495
## Jun 1980 -0.0380000  0.031344912 -0.0693449119
## Jul 1980 -0.2020000 -0.098158281 -0.1038417194
## Aug 1980  0.1370000  0.125274035  0.0117259650
## Sep 1980  0.0930000  0.111012305 -0.0180123045
## Oct 1980 -0.2790000 -0.157906438 -0.1210935622
## Nov 1980 -0.0380000 -0.002512296 -0.0354877037
## Dec 1980  0.2350000  0.202361257  0.0326387428
## Jan 1981  0.1690000  0.195391784 -0.0263917842
## Feb 1981  0.3990000  0.392067606  0.0069323937
## Mar 1981  0.1580000  0.240958616 -0.0829586160
## Apr 1981 -0.5410000 -0.383089550 -0.1579104502
## May 1981 -0.6280000 -0.431135689 -0.1968643113
## Jun 1981  0.0270000 -0.066988037  0.0939880371
## Jul 1981 -0.7380000 -0.184619499 -0.5533805011
## Aug 1981 -0.9130000  0.409423649 -1.3224236489
## Sep 1981 -0.0270000 -0.102836367  0.0758363668
## Oct 1981  0.0710000 -0.206600126  0.2776001264
## Nov 1981 -0.1260000 -0.161082684  0.0350826844
## Dec 1981  0.0490000 -0.204982940  0.2539829404
## Jan 1982  0.4540000 -0.016439675  0.4704396753
## Feb 1982  0.3010000  0.137990035  0.1630099649
## Mar 1982 -0.0050000  0.004273568 -0.0092735676
## Apr 1982 -0.0930000 -0.037049279 -0.0559507208
## May 1982 -0.2240000 -0.117922023 -0.1060779766
## Jun 1982 -0.2130000 -0.108722419 -0.1042775805
## Jul 1982 -0.3110000 -0.163202641 -0.1477973587
## Aug 1982 -0.2350000 -0.130812308 -0.1041876917
## Sep 1982 -0.3220000 -0.159105007 -0.1628949931
## Oct 1982  0.3220000  0.109854337  0.2121456632
## Nov 1982  0.0930000  0.070609006  0.0223909936
## Dec 1982  0.1150000  0.124784482 -0.0097844822
## Jan 1983  0.7700000  0.736343951  0.0336560488
## Feb 1983  0.6070000  0.501825398  0.1051746020
## Mar 1983 -0.1580000  0.144644969 -0.3026449692
## Apr 1983 -0.2350000  0.024606064 -0.2596060643
## May 1983 -0.2020000 -0.015210608 -0.1867893916
## Jun 1983 -0.3330000 -0.183633822 -0.1493661777
## Jul 1983 -0.1580000 -0.102051402 -0.0559485981
## Aug 1983 -0.2240000 -0.180273465 -0.0437265347
## Sep 1983 -0.3550000 -0.241667759 -0.1133322415
## Oct 1983  0.0050000 -0.110072221  0.1150722213
## Nov 1983  0.4540000  0.067405584  0.3865944160
## Dec 1983  0.4100000  0.137565909  0.2724340907
## Jan 1984  0.5190000  0.290313924  0.2286860759
## Feb 1984  0.5410000  0.375689163  0.1653108369
## Mar 1984  0.3010000  0.286561845  0.0144381546
## Apr 1984 -0.5740000 -0.417342247 -0.1566577530
## May 1984 -0.3440000 -0.218607176 -0.1253928241
## Jun 1984 -0.2790000 -0.173926430 -0.1050735703
## Jul 1984 -0.7160000 -0.228110649 -0.4878893510
## Aug 1984 -0.8690000  0.307298662 -1.1762986616
## Sep 1984 -0.5960000  0.237128062 -0.8331280620
## Oct 1984 -0.2900000 -0.026939990 -0.2630600101
```

```
## Nov 1984 -0.4540000  0.018820567 -0.4728205666
## Dec 1984 -0.2460000 -0.141088086 -0.1049119143
## Jan 1985 -0.6070000  0.074447102 -0.6814471015
## Feb 1985 -0.5630000  0.084423366 -0.6474233660
## Mar 1985 -0.2350000 -0.120695057 -0.1143049427
## Apr 1985 -0.2460000 -0.116601865 -0.1293981354
## May 1985 -0.3990000 -0.026880798 -0.3721192024
## Jun 1985 -0.3330000 -0.078989424 -0.2540105761
## Jul 1985 -0.5300000  0.052746094 -0.5827460937
## Aug 1985 -0.0490000 -0.224067039  0.1750670391
## Sep 1985  0.1580000 -0.243832490  0.4018324905
## Oct 1985  0.1150000 -0.123679278  0.2386792783
## Nov 1985  0.3220000  0.066470084  0.2555299162
## Dec 1985  0.1150000  0.062368176  0.0526318244
## Jan 1986  0.0490000  0.055519296 -0.0065192958
## Feb 1986  0.4540000  0.409664525  0.0443354746
## Mar 1986  0.1580000  0.234178566 -0.0761785661
## Apr 1986 -0.4210000 -0.240231753 -0.1807682465
## May 1986 -0.2680000 -0.137774630 -0.1302253705
## Jun 1986 -0.3110000 -0.179964006 -0.1310359935
## Jul 1986 -0.1150000 -0.070102607 -0.0448973928
## Aug 1986 -0.3220000 -0.192028937 -0.1299710628
## Sep 1986 -0.3220000 -0.175516153 -0.1464838469
## Oct 1986  0.1260000 -0.050476112  0.1764761115
## Nov 1986  0.3330000  0.123302279  0.2096977214
## Dec 1986  0.5190000  0.377997813  0.1410021874
## Jan 1987  0.3990000  0.351844542  0.0471554576
## Feb 1987  0.5190000  0.462639068  0.0563609317
## Mar 1987  0.4320000  0.420066207  0.0119337929
## Apr 1987  0.3550000  0.397267316 -0.0422673165
```

```
fish = ts.intersect(R = rec, RL1 = lag(rec, -1), SL5 = lag(soi, -5))
(u = lm(fish[, 1] ~ fish[, 2:3], na.action = NULL))
```

```
##
## Call:
## lm(formula = fish[, 1] ~ fish[, 2:3], na.action = NULL)
##
## Coefficients:
##    (Intercept)  fish[, 2:3]RL1  fish[, 2:3]SL5
##        11.3136          0.8434        -20.3004
```

```
acf2(resid(u))
```




```
##      [,1]  [,2]  [,3]  [,4]  [,5] [,6]  [,7] [,8]  [,9] [,10] [,11] [,12] [,13]
## ACF   0.4  0.10 -0.04 -0.15 -0.03 0.06 -0.02 0.00 -0.07 -0.11 -0.10 -0.10 -0.10
## PACF  0.4 -0.08 -0.06 -0.13  0.11 0.05 -0.10 0.03 -0.07 -0.05 -0.06 -0.04 -0.07
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF  -0.07 -0.08 -0.08  0.01  0.01 -0.02 -0.02  0.02  0.03  0.04  0.03  0.00
## PACF -0.04 -0.05 -0.05  0.05 -0.04 -0.04 -0.03  0.05 -0.01 -0.02 -0.01 -0.02
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF   0.03  0.09  0.07  0.08  0.05  0.00 -0.12 -0.13 -0.03  0.06  0.10  0.07
## PACF  0.03  0.06 -0.01  0.05  0.01 -0.01 -0.14 -0.02  0.06  0.05  0.03  0.03
##      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48]
## ACF  -0.07 -0.13 -0.03  0.05  0.08  0.04 -0.02 -0.09 -0.08 -0.04  0.01
## PACF -0.09 -0.05  0.10  0.06  0.00 -0.04  0.00 -0.07 -0.02  0.01 -0.01
```

```
sarima(fish[, 1], 1, 0, 0, xreg = fish[, 2:3], details = FALSE)
```

```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xreg, transform.pars = trans, fixed = fixed, optim.control = list(trace =
##     REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1  intercept     RL1       SL5
##       0.4489    14.6838  0.7902  -20.9988
## s.e.  0.0495     1.5605  0.0229    1.0812
##
## sigma^2 estimated as 49.57:  log likelihood = -1510.14,  aic = 3030.28
```

```
##
## $degrees_of_freedom
## [1] 444
##
## $ttable
##           Estimate     SE  t.value p.value
## ar1         0.4489 0.0495   9.0591       0
## intercept  14.6838 1.5605   9.4098       0
## RL1         0.7902 0.0229  34.4532       0
## SL5       -20.9988 1.0812 -19.4218       0
##
## $AIC
## [1] 6.764027
##
## $AICc
## [1] 6.764229
##
## $BIC
## [1] 6.80984
```

## References