# Lecture 35
## Correlation & Regression

*STAT 8010 Statistical Methods I*
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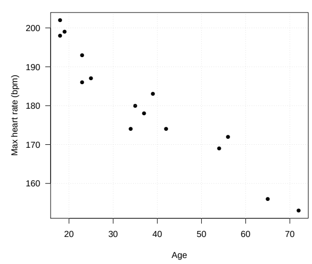
Notes

## Motivated Example: Maximum Heart Rate vs. Age

Suppose we have 15 people of varying ages are tested for their maximum heart rate (bpm):

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Age | 18 | 23 | 25 | 35 | 65 | 54 | 34 | 56 | 72 | 19 | 23 | 42 | 18 | 39 | 37 |
| MaxHeartRate | 202 | 186 | 187 | 180 | 156 | 169 | 174 | 172 | 153 | 199 | 193 | 174 | 198 | 183 | 178 |




**Question:** How to describe the relationship between maximum heart rate and age?

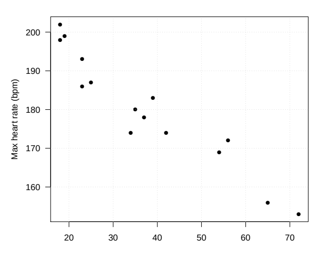




Notes

## Scatterplot

A scatterplot is a useful tool to graphically display the relationship between two numerical variables. Each dot on the scatterplot represents one observation from the data








Notes

## Scatterplot Cont'd



Typical questions we want to ask for a scatterplot:

- the form of relationship between two variables e.g. linear, quadratic, ···
- the strength of the relationship between two variables e.g. weak, moderate, strong
- the direction of the relationship between two variables e.g positive, negative

In the next few slides we will learn how to quantify the strength and direction of the linear relationship between two variables



Notes

## Variance, Covariance, and Correlation



- **Recall:** Variance is a measure of the variability of one quantitative variable
- Covariance is a measure of how much two quantitative random variables change together
- The sign of the covariance shows the direction in the linear relationship between the variables
- The normalized version of the covariance, the correlation shows both the the direction and the strength of the linear relation



Notes

## Correlation: Pearson Correlation Coefficient $(\rho)$



- We use $\rho$ to denote the population correlation and $r$ to denote the sample correlation
- The value of the correlation is between $-1$ and $1$
- The strength of the linear relation:
  - If $\rho = 1$ $(-1)$: the two variables have a perfect positive (negative) linear relationship
  - If $0.7 < |\rho| < 1$: we say the two variables have a strong linear relationship
  - If $0.3 < |\rho| < 0.7$: we say the two variables have a moderate linear relationship
  - If $0 < |\rho| < 0.3$: we say the two variables have a weak linear relationship
  - If $\rho = 0$: we say the two variables have no linear relationship



Notes

## Scatterplot & Pearson Correlation Coefficient



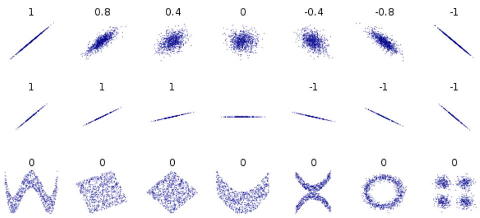



**Figure:** Image courtesy of Wikipedia at `https://en.wikipedia.org/wiki/Correlation_and_dependence`



Notes

## Formulas of Covariance and Correlation



- **Recall:** Variance
  - Sample variance: $s_X^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$
  - Population variance: $\sigma_X^2 = \mathrm{E}[(X - \mu_X)^2]$
- Covariance
  - Sample covariance: $s_{X,Y} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$
  - Population covariance: $\sigma_{X,Y} = \mathrm{E}[(X - \mu_X)(Y - \mu_Y)]$
- Correlation
  - Sample correlation: $r_{X,Y} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2}\sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}}$ or $\frac{s_{X,Y}}{s_X s_Y}$
  - Population correlation: $\rho_{X,Y} = \frac{\mathrm{E}[(X-\mu_X)(Y-\mu_Y)]}{\sqrt{\mathrm{E}[(X-\mu_X)^2]}\sqrt{\mathrm{E}[(Y-\mu_Y)^2]}}$ or $\frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$



Notes

## A Toy Example



You wonder how sleep affects productivity. You take a sample of 4 of your friends and measure last night's sleep and today's productivity in hours. Here are the results:

| Sleep ($X$) | Productivity ($Y$) |
|---|---|
| 2 | 4 |
| 4 | 12 |
| 6 | 14 |
| 10 | 10 |

Calculate the means, variances, and standard deviations of each variable and the correlation coefficient of these two variables



Notes

## Toy Example Cont'd



**Solution.**
Let $X$ denote last night's sleep in hours and $Y$ denote today's productivity in hours

- $\bar{X} = \frac{2+4+6+10}{4} = 5.5, \quad \bar{Y} = \frac{4+12+14+10}{4} = 10$
- $s_X^2 = \frac{(2-5.5)^2+(4-5.5)^2+(6-5.5)^2+(10-5.5)^2}{4-1} = \frac{35}{3}$
  $s_Y^2 = \frac{(4-10)^2+(12-10)^2+(14-10)^2+(10-10)^2}{4-1} = \frac{56}{3}$
- $s_X = \sqrt{s_X^2} = \sqrt{\frac{35}{3}}, \quad s_Y = \sqrt{s_Y^2} = \sqrt{\frac{56}{3}}$
- $r_{X,Y} = \frac{s_{X,Y}}{s_X s_Y}$
  $s_{X,Y} = \frac{(2-5.5)(4-10)+(4-5.5)(12-10)+(6-5.5)(14-10)+(10-5.5)(10-10)}{3}$
  $= \frac{20}{3} \Rightarrow r_{X,Y} = \frac{\frac{20}{3}}{\sqrt{\frac{35}{3}}\sqrt{\frac{56}{3}}} = \frac{20}{\sqrt{35\times 56}} = 0.4518$



Notes

## Inference/Hypothesis Test on $\rho$



1. $H_0 : \rho = 0$ vs. $H_a : \rho \neq 0$
2. Test statistic: $t^* = r\sqrt{\frac{n-2}{1-r^2}}$
3. Under $H_0$: $t^* \sim t_{df=n-2}$

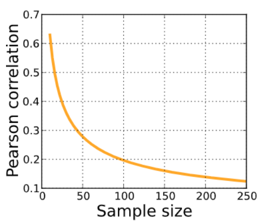


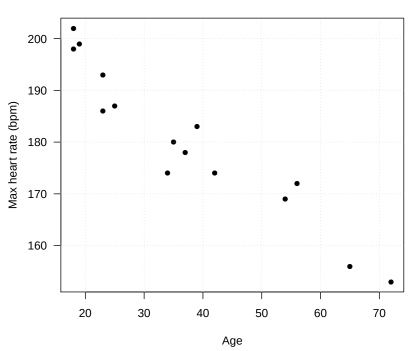

**Figure:** Image courtesy of Wikipedia



Notes

## Maximum Heart Rate Example Revisited





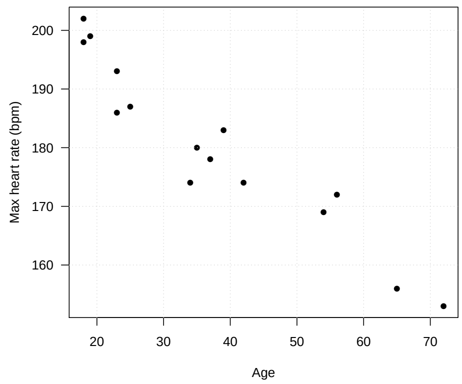

We may want to predict maximum heart rate for an individual based on his/her age ⇒ Regression Analysis



Notes

## What is Regression Analysis?

**Regression analysis**: A set of statistical procedures for estimating the relationship between response variable and predictor variable(s)



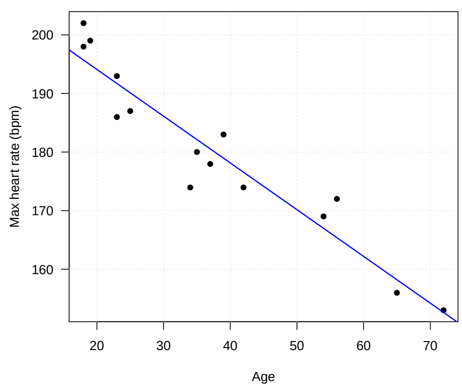







Notes

## Scatterplot: Is Linear Trend Reasonable?










Notes

## Simple Linear Regression (SLR)

$Y$: dependent (response) variable; $X$: independent (predictor) variable

- In SLR we **assume** there is a **linear relationship** between $X$ and $Y$:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

- We will need to estimate $\beta_0$ (intercept) and $\beta_1$ (slope)
- Then we can use the estimated regression equation to
  - make predictions
  - study the relationship between response and predictor
  - control the response

Next lecture we will learn how to estimate the regression parameters $\beta_0, \beta_1$ and how to quantify the estimation uncertainty







Notes