

# DSA 8070 R Session 6: Multivariate Linear Regression

Whitney

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## Load the Motor Trend Car Road Tests data

```
data(mtcars)
head(mtcars)
```

```
##           mpg cyl  disp  hp  drat    wt  qsec vs am gear carb
## Mazda RX4      21.0   6  160 110 3.90 2.620 16.46 0  1    4    4
## Mazda RX4 Wag  21.0   6  160 110 3.90 2.875 17.02 0  1    4    4
## Datsun 710      22.8   4  108  93 3.85 2.320 18.61 1  1    4    1
## Hornet 4 Drive  21.4   6  258 110 3.08 3.215 19.44 1  0    3    1
## Hornet Sportabout 18.7   8  360 175 3.15 3.440 17.02 0  0    3    2
## Valiant         18.1   6  225 105 2.76 3.460 20.22 1  0    3    1
```

## Data Manipulation

```
mtcars$cyl <- factor(mtcars$cyl)
vars <- c("mpg", "disp", "hp", "wt")
Y <- as.matrix(mtcars[, vars])
```

## Summarizing the Responses

```
colMeans(Y)
```

```
##      mpg      disp      hp      wt  
## 20.09062 230.72188 146.68750  3.21725
```

```
apply(Y, 2, sd)
```

```
##      mpg      disp      hp      wt  
##  6.0269481 123.9386938 68.5628685  0.9784574
```

```
cov(Y); cor(Y)
```

```
##      mpg      disp      hp      wt  
## mpg    36.324103 -633.0972 -320.73206 -5.116685  
## disp -633.097208 15360.7998 6721.15867 107.684204  
## hp    -320.732056 6721.1587 4700.86694  44.192661  
## wt     -5.116685  107.6842  44.19266  0.957379
```

```
##      mpg      disp      hp      wt  
## mpg    1.0000000 -0.8475514 -0.7761684 -0.8676594  
## disp -0.8475514  1.0000000  0.7909486  0.8879799  
## hp    -0.7761684  0.7909486  1.0000000  0.6587479  
## wt    -0.8676594  0.8879799  0.6587479  1.0000000
```

```
library(GGally)
```

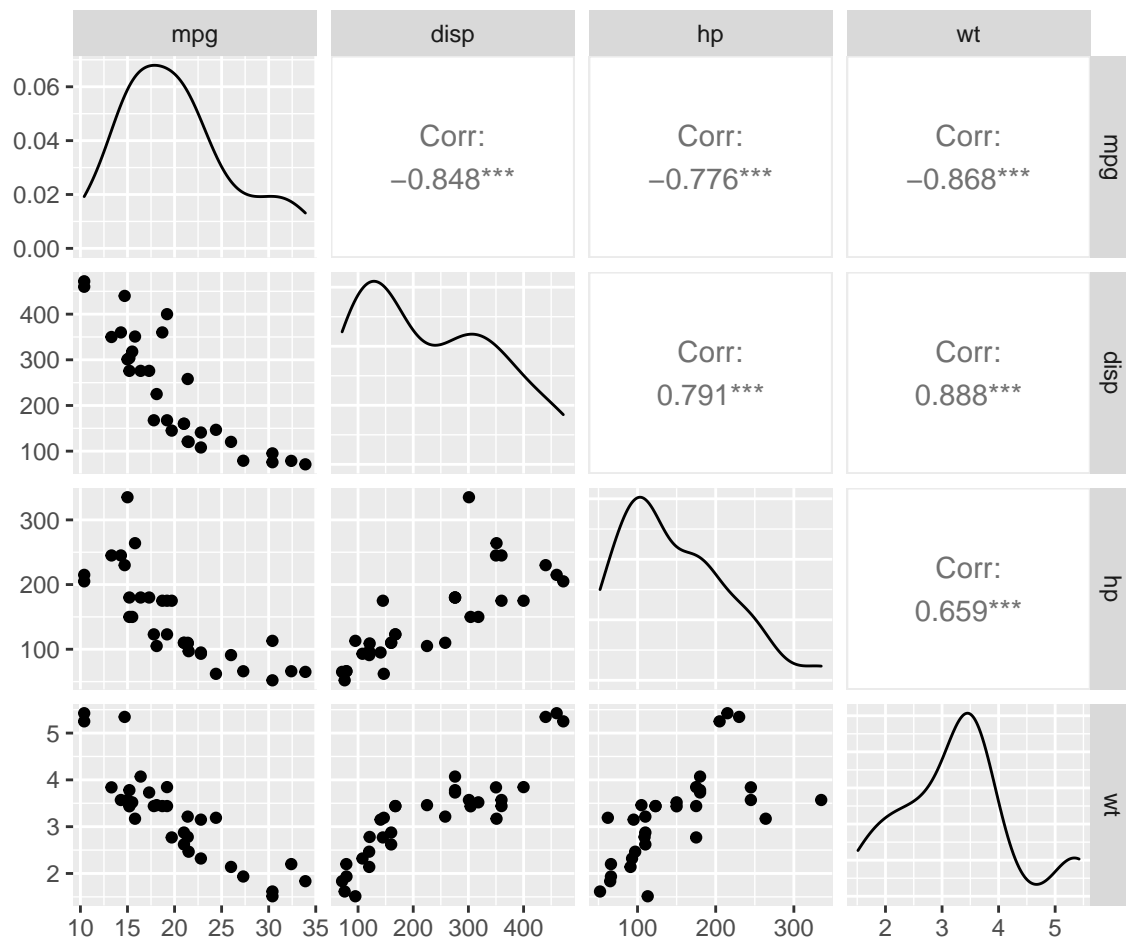
```
## Loading required package: ggplot2
```

```
## Warning: replacing previous import 'lifecycle::last_warnings' by  
## 'rlang::last_warnings' when loading 'tibble'
```

```
## Warning: replacing previous import 'lifecycle::last_warnings' by  
## 'rlang::last_warnings' when loading 'pillar'
```

```
## Registered S3 method overwritten by 'GGally':  
##   method from  
##   +.gg      ggplot2
```

```
ggpairs(as.data.frame(Y))
```



## Fitting Linear Regression

```
mvlm <- lm(Y ~ cyl + am + carb, data = mtcars)
summary(mvlm)
```

```
## Response mpg :
##
## Call:
## lm(formula = mpg ~ cyl + am + carb, data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -5.9074 -1.1723  0.2538  1.4851  5.4728
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  25.3203     1.2238  20.690 < 2e-16 ***
## cyl6         -3.5494     1.7296  -2.052  0.049959 *
## cyl8         -6.9046     1.8078  -3.819  0.000712 ***
## am           4.2268     1.3499   3.131  0.004156 **
## carb        -1.1199     0.4354  -2.572  0.015923 *
```

```

## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.805 on 27 degrees of freedom
## Multiple R-squared:  0.8113, Adjusted R-squared:  0.7834
## F-statistic: 29.03 on 4 and 27 DF,  p-value: 1.991e-09
##
##
## Response disp :
##
## Call:
## lm(formula = disp ~ cyl + am + carb, data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -82.694 -21.442   0.254  26.500 111.779
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   134.325     21.836   6.152 1.42e-06 ***
## cyl6           61.843     30.860   2.004  0.0552 .
## cyl8          218.991     32.256   6.789 2.72e-07 ***
## am            -43.803     24.086  -1.819  0.0801 .
## carb           1.726       7.768   0.222  0.8258
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 50.05 on 27 degrees of freedom
## Multiple R-squared:  0.858, Adjusted R-squared:  0.8369
## F-statistic: 40.78 on 4 and 27 DF,  p-value: 4.537e-11
##
##
## Response hp :
##
## Call:
## lm(formula = hp ~ cyl + am + carb, data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -41.520 -17.941  -4.378  19.799  41.292
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   46.5201     10.4825   4.438 0.000138 ***
## cyl6           0.9116     14.8146   0.062 0.951386
## cyl8          87.5911     15.4851   5.656 5.25e-06 ***
## am             4.4473     11.5629   0.385 0.703536
## carb          21.2765       3.7291   5.706 4.61e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.03 on 27 degrees of freedom
## Multiple R-squared:  0.893, Adjusted R-squared:  0.8772
## F-statistic: 56.36 on 4 and 27 DF,  p-value: 1.023e-12

```

```
##
##
## Response wt :
##
## Call:
## lm(formula = wt ~ cyl + am + carb, data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.66317 -0.34384 -0.03802  0.12334  1.19083
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.76121     0.22133   12.476 1.01e-12 ***
## cyl6          0.19572     0.31280    0.626  0.53675
## cyl8          0.77231     0.32695    2.362  0.02564 *
## am           -1.02547     0.24414   -4.200  0.00026 ***
## carb          0.17491     0.07874    2.222  0.03489 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5073 on 27 degrees of freedom
## Multiple R-squared:  0.7659, Adjusted R-squared:  0.7312
## F-statistic: 22.08 on 4 and 27 DF,  p-value: 3.484e-08
```

## SSCP

```
ybar <- colMeans(Y)
n <- nrow(Y); d <- ncol(Y)
Ybar <- matrix(ybar, n, d, byrow = TRUE)
SSCP.Tot <- crossprod(Y - Ybar)
SSCP.Reg <- crossprod(mvml$fitted.values - Ybar)
SSCP.Err <- crossprod(Y - mvml$fitted.values)
SSCP.Tot
```

```
##           mpg      disp      hp      wt
## mpg    1126.0472 -19626.01 -9942.694 -158.61723
## disp -19626.0134 476184.79 208355.919 3338.21032
## hp    -9942.6938 208355.92 145726.875 1369.97250
## wt    -158.6172  3338.21  1369.972   29.67875
```

```
SSCP.Reg + SSCP.Err
```

```
##           mpg      disp      hp      wt
## mpg    1126.0472 -19626.01 -9942.694 -158.61723
## disp -19626.0134 476184.79 208355.919 3338.21033
## hp    -9942.6938 208355.92 145726.875 1369.97250
## wt    -158.6172  3338.21  1369.973   29.67875
```

## Estimated Error Covariance Matrix in R

```
p <- nrow(coef(mvlm)) - 1
SigmaHat <- SSCP.Err / (n - p - 1)
SigmaHat
```

```
##           mpg      disp          hp      wt
## mpg      7.8680094 -53.27166 -19.7015979 -0.6575443
## disp -53.2716607 2504.87095 425.1328988 18.1065416
## hp    -19.7015979 425.13290 577.2703337 0.4662491
## wt     -0.6575443 18.10654 0.4662491 0.2573503
```

## Testing if we need “cyl”

```
mvlm0 <- lm(Y ~ am + carb, data = mtcars)
anova(mvlm, mvlm0, test = "Wilks")
```

```
## Analysis of Variance Table
##
## Model 1: Y ~ cyl + am + carb
## Model 2: Y ~ am + carb
##   Res.Df Df Gen.var.   Wilks approx F num Df den Df    Pr(>F)
## 1      27      29.862
## 2      29  2  43.692 0.16395   8.8181      8    48 2.525e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(mvlm, mvlm0, test = "Pillai")
```

```
## Analysis of Variance Table
##
## Model 1: Y ~ cyl + am + carb
## Model 2: Y ~ am + carb
##   Res.Df Df Gen.var. Pillai approx F num Df den Df    Pr(>F)
## 1      27      29.862
## 2      29  2  43.692 1.0323   6.6672      8    50 6.593e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Confidence/Prediction Intervals

```
newdata <- data.frame(cyl = factor(6, levels = c(4, 6, 8)),
                      am = 1, carb = 4)
# confidence interval
predict(mvlm, newdata, interval = "confidence")
```

```
##           mpg      disp      hp      wt
## 1 21.51824 159.2707 136.985 2.631108
```

```
# prediction interval
predict(mvmlm, newdata, interval = "prediction")
```

```
##          mpg      disp      hp      wt
## 1 21.51824 159.2707 136.985 2.631108
```

R does not yet have the capability to produce CIs/PIs with multivariate responses!!

Below is R function to calculate the Multivariate Regression CIs and PIs (taken from Prof. Helwig at U of Minnesota)

```
pred.mlm <- function(object, newdata, level = 0.95,
                      interval = c("confidence", "prediction")){
  form <- as.formula(paste("~", as.character(formula(object))[3]))
  xnew <- model.matrix(form, newdata)
  fit <- predict(object, newdata)
  Y <- model.frame(object)[, 1]; X <- model.matrix(object)
  n <- nrow(Y); d <- ncol(Y); p <- ncol(X) - 1
  sigmas <- colSums((Y - object$fitted.values)^2) / (n - p - 1)
  fit.var <- diag(xnew %*% tcrossprod(solve(crossprod(X)), xnew))
  if(interval[1] == "prediction") fit.var <- fit.var + 1
  const <- qf(level, df1 = d, df2 = n - p - d) * d * (n - p - 1) / (n - p - d)
  vmat <- (n / (n - p - 1)) * outer(fit.var, sigmas)
  lwr <- fit - sqrt(const) * sqrt(vmat)
  upr <- fit + sqrt(const) * sqrt(vmat)
  if(nrow(xnew) == 1L){
    ci <- rbind(fit, lwr, upr)
    rownames(ci) <- c("fit", "lwr", "upr")
  } else {
    ci <- array(0, dim = c(nrow(xnew), d, 3))
    dimnames(ci) <- list(1:nrow(xnew), colnames(Y), c("fit", "lwr", "upr"))
    ci[, , 1] <- fit; ci[, , 2] <- lwr; ci[, , 3] <- upr
  }
  ci
}
```

```
# confidence interval
pred.mlm(mvmlm, newdata)
```

```
##          mpg      disp      hp      wt
## fit 21.51824 159.2707 136.98500 2.631108
## lwr 16.65593  72.5141  95.33649 1.751736
## upr 26.38055 246.0273 178.63351 3.510479
```

```
# prediction interval
pred.mlm(mvmlm, newdata, interval = "prediction")
```

```
##          mpg      disp      hp      wt
## fit 21.518240 159.27070 136.98500 2.6311076
## lwr  9.680053 -51.95435  35.58397 0.4901152
## upr 33.356426 370.49576 238.38603 4.7720999
```