Lecture 7

Binomial, Hypergeometric R.V.s & Continuous Random Variables

Text: Chapter 4

STAT 8010 Statistical Methods I
January 30, 2020

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Agenda

1. Binomial Random Variables

2. Hypergeometric Random Variable

3. Continuous Random Variables

4. Normal Distributions

Binomial Random Variable

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$$Var(X) = np(1-p)$$

Example

To test for Extrasensory perception (ESP), we have 4 cards. They will be shuffled and one randomly selected each time, and you are to guess which card is selected. This is repeated 10 times. Suppose you do not have ESP. Let R be the number of times you guess a card correctly. What are the distribution and parameter(s) of R? What is the expected value of R? Furthermore, suppose that you get certified as having ESP if you score at least an 8 on the test. What is the probability that you get certified as having ESP?

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Solution.

$R \sim Binomial(n = 10, p = \frac{1}{4} = .25)$
$\mathbb{E}[R] = n \times p = 2.5$
$\mathbb{P}(X \geq 8) = .000416$

Example

Suppose that 95% of consumers can recognize Coke in a blind taste test. Assume consumers are independent of one another. The company randomly selects 4 consumers for a taste test. Let X be the number of consumers who recognize Coke.

1. What is the probability that X is at least 1?
2. What is the probability that X is at most 3?

Binomial and Hypergeometric r.v.s

The binomial distribution describes the probability of k successes in n trials with replacement.

We want a distribution to describe the probability of k successes in n trials without replacement from a finite population of size N containing exactly K successes.

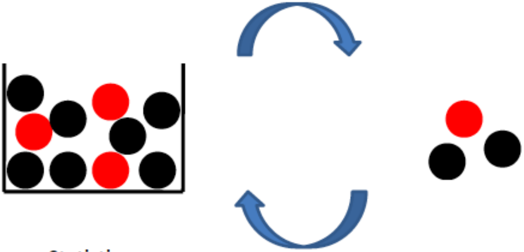
$\Rightarrow$ Hypergeometric Distribution

Important applications are **quality control** and statistical **estimation of population proportions**. The hypergeometric r.v. is the equivalent of a Binomial r.v. except that sampling is done without replacement.

An Example of Hypergeometric r.v.

Probability:

What is the probability to get 1 red and 2 black balls?



Statistics:

What percentage of balls in the box are red?

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- The expected value: $\mathbb{E}[X] = n\frac{K}{N}$
- The variance: $Var(X) = n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}$

Example

There are 100 identical looking 52" TVs at Best Buy in Anderson, SC. Let 10 of them be defective. Suppose we want to buy 8 of the aforementioned TVs (at random). What is the probability that we don't get any defective TVs?

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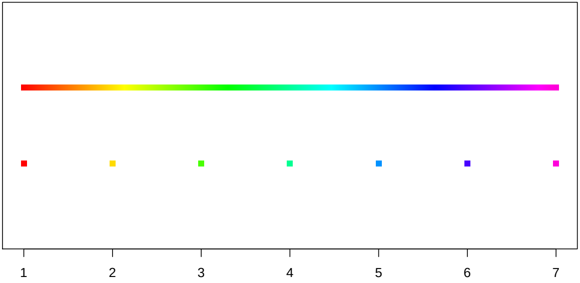
Let D be the number of defective TVs in the sample.

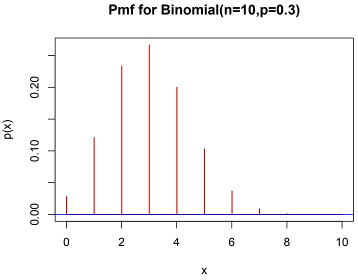
$D \sim Hyp(N = 100, n = 8, K = 10)$

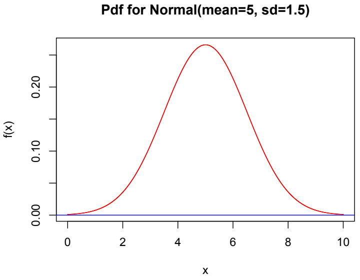
$\mathbb{P}(D = 0) = \frac{\binom{10}{0}\binom{90}{8}}{\binom{100}{8}} = 0.4166$

From Discrete to Continuous Random Variables



Probability Mass Functions vs. Probability Density Functions





Remarks:

- pmf assigns probabilities to each possible values of a discrete random variable
- pdf describes the relative likelihood for a continuous random variable to take on a given **interval**

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Cumulative Distribution Functions (cdfs) for Continuous Distribution

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- The cdf $F_X(x)$ is defined as $F_X(x) = \mathbb{P}(X \le x) = \int_{-\infty}^{x} f_X(x)\,dx$
- we use cdf to calculate probabilities of a continuous random variable within an interval, i.e. $\mathbb{P}(a \le X \le b) = \int_a^b f_X(x)\,dx = \int_{-\infty}^{b} f_X(x)\,dx - \int_{-\infty}^{a} f_X(x)\,dx = \boxed{F_X(b) - F_X(a)}$

Remark: $\mathbb{P}(X = x) = \int_x^x f_X(x)\,dx = 0$ for all possible values of x

Expected Value and Variance

Recall the expected value formula for the discrete random variable: $\mathbb{E}[X] = \sum_x x p_X(x)$
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Let a, b, and c are constant real numbers

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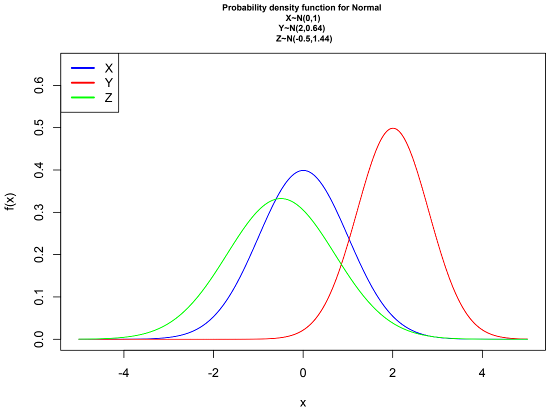
Normal Density Curves



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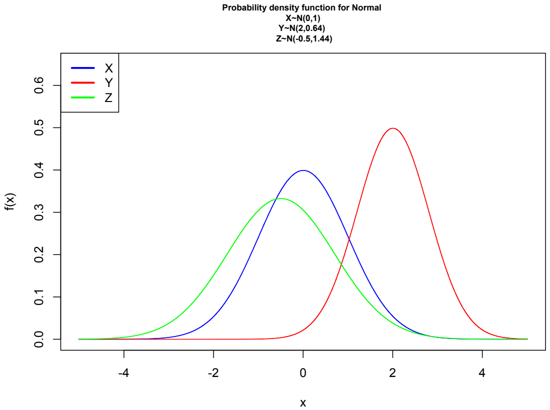

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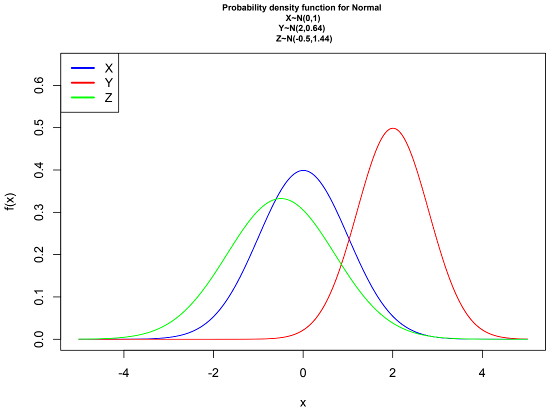

- The parameter μ determines the center of the distribution
- The parameter σ^2 determines the spread of the distribution
- Also called bell-shaped distribution

Standard Normal $Z \sim N(\mu = 0, \sigma^2 = 1)$

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- The probability $\mathbb{P}(a \leq X \leq b)$ where $X \sim N(\mu, \sigma^2)$ can be compute

$$\mathbb{P}(a \leq X \leq b) = \mathbb{P}\left(\frac{a - \mu}{\sigma} \leq Z \leq \frac{b - \mu}{\sigma}\right)$$
$$= \Phi\left(\frac{b - \mu}{\sigma}\right) - \Phi\left(\frac{a - \mu}{\sigma}\right)$$

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- $\mathbb{P}(Z > z) = 1 - \Phi(z) = \Phi(-z)$

Example

Let us examine Z. Find the following probabilities with respect to Z:

1. Z is at most -1.75
2. Z is between -2 and 2 inclusive
3. Z is less than .5

Example Cont'd

Solution.

1. $\mathbb{P}(Z \leq -1.75) = \Phi(-1.75) = .0401$

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Example Cont'd

Solution.

1. $\mathbb{P}(Z \le -1.75) = \Phi(-1.75) = .0401$
2. $\mathbb{P}(-2 \le Z \le 2) = \Phi(2) - \Phi(-2) = .9772 - .0228 = .9544$
3. $\mathbb{P}(Z < .5) = \Phi(.5) = .6915$

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- This can be applied for any integer n

Example

Let X_1, X_2, and X_3 be mutually independent, Normal random variables. Let their means and standard deviations be $3k$ and k for $k = 1, 2,$ and 3 respectively. Find the following distributions:

1. $\sum_{i=1}^{3} X_i$
2. $X_1 + 2X_2 - 3X_3$
3. $X_1 + 5X_3$

Example Cont'd

Solution.

1. $\sum_{i=1}^{3} X_i \sim N(\mu = 3 + 6 + 9 = 15, \sigma^2 = 1^2 + 2^2 + 3^2 = 14)$
2. $X_1 + 2X_2 - 3X_3 \sim N(\mu = 3 + 12 - 27 = -12, \sigma^2 = 1^2 + 4 \times 2^2 + 9 \times 3^2 = 98)$
3. $X_1 + 5X_3 \sim N(\mu = 3 + 45 = 48, \sigma^2 = 1^2 + 25 \times 3^2 = 226)$