MATH 8090 Fall 2023 Exam I

October 05, 2023

Name:______________________________

Directions

1. Show your work on ALL questions (except those multiple choice questions). Unsupported work will NOT receive full credit.
2. Please write legibly. If I cannot read your writing, NO credit will be given.
3. Put your work into a **single file** and upload it to Canvas.

Use your time wisely. Good Luck!!!

Problem	Points Possible	Points Earned
1	40	
2	30	
3	30	
4 (bonus question)	5	
Total	105	

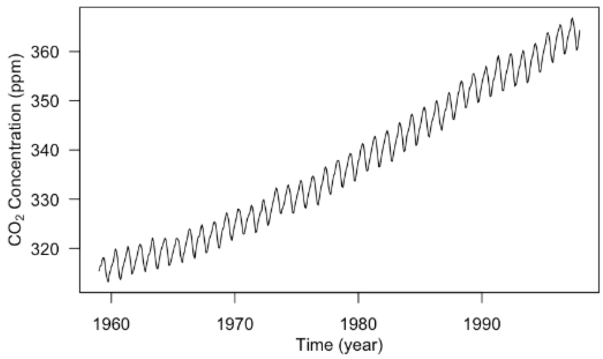
Problem 1 (40 points, 10 points for each question.)

(a) Plot the theoretical autocorrelation and partial autocorrelation functions for the following ARMA process

$$(1 - 0.8B)X_t = Z_t, \quad \{Z_t\} \sim \mathrm{WN}(0, \sigma^2),$$

up to lag 5.

(b) Describe the main features of the monthly atmospheric concentrations of CO_2 at the Mauna Loa Observatory



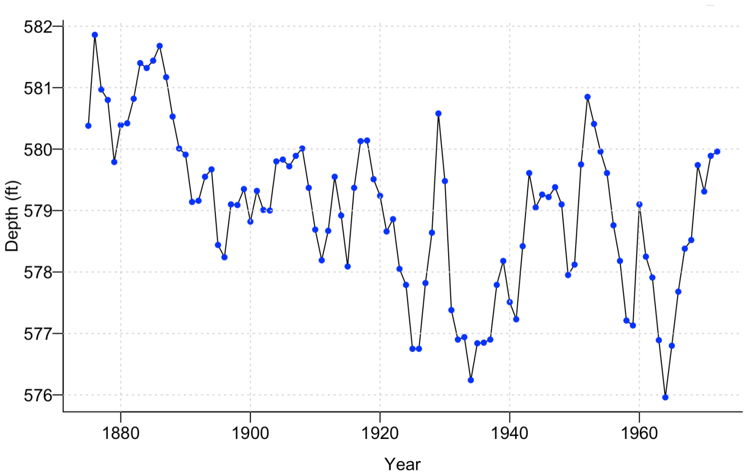
(c) Show that for an MA(1) process, the lag 1 autocorrelation $\rho(1)$ satisfies $-\frac{1}{2} \leq \rho(1) \leq \frac{1}{2}$

(d) Based on the `R` outputs below, which time series model was fitted to the Lake Huron data set? Are these parameters statistically significant at 0.05 level? Is the fitted model adequately account for the temporal dependence structure?

```
> (MLE_est3 <- arima(LakeHuron, order = c(2, 0, 0), xreg = yr))

Call:
arima(x = LakeHuron, order = c(2, 0, 0), xreg = yr)

Coefficients:
         ar1      ar2  intercept       yr
      1.0048  -0.2913   620.5115  -0.0216
s.e.  0.0976   0.1004    15.5771   0.0081

sigma^2 estimated as 0.4566:  log likelihood = -101.2,  aic = 212.4
> Box.test(MLE_est3$residuals, type = "Ljung-Box")

        Box-Ljung test

data:  MLE_est3$residuals
X-squared = 0.03358, df = 1, p-value = 0.8546
```

Problem 2 (30 points)

Describe concisely the typical steps for modeling a time series like the one below including exploratory analysis, model building, estimation, diagnostic, and forecasting



Problem 3 (30 points, 10 points for each question.)

(a) Consider a causal and stationary AR(1) process $X_t = \phi X_{t-1} + Z_t$, $\{Z_t\} \sim \mathrm{WN}(0, \sigma^2)$. Which of the following is/are false?

A : $\mathrm{Var}(X_t) = \frac{\sigma^2}{(1-\phi)^2}$

B : $\mathrm{Corr}(X_t, X_{t-2}) = \phi^2 = \mathrm{Corr}(X_t, X_{t+2})$

C : The best linear predictor of X_{t+1} is ϕX_t

D : ϕ can take the value -1.01

(b) Which of the following is/are true?

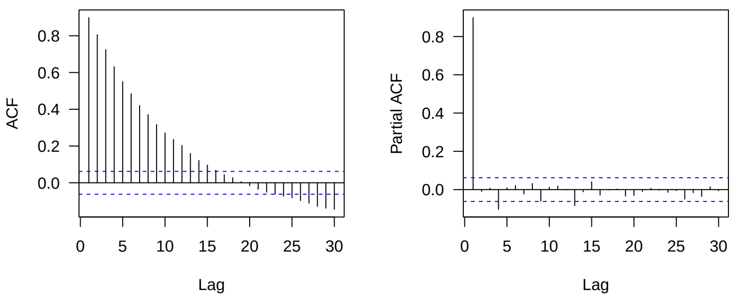
$A : X_t = 0.75X_{t-1} + Z_t + 0.5Z_{t-1}$ is a casual, stationary, and invertiable ARMA process

B : The second order difference of a random walk is a stationary process

C : The autocovariance function of a stationary process can be negative definite

D : $\text{Corr}(X_t, X_{t-2}) = 0.4$ for the MA(1) process with $\theta = 0.5$

(c) The sample ACF and PACF plots below is most likely obtained from which of the following stationary process?



A : White Noise

B : AR(1)

C : MA(2)

D : ARMA$(5, 4)$

Problem 4 (5 bonus points)

Derive the best linear predictor of X_2 given X_1 and X_3 if $\{X_t\}$ is a causal AR(1) process. Also, derive its MSPE.