Lecture 1

Overview

DSA 8070 Multivariate Analysis
August 22-26, 2022

Whitney Huang
Clemson University

Agenda

1. **Introduction**
2. **Objectives of Multivariate Analysis**
3. **Useful Tools for Multivariate Analysis**

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Introduction

- In many observational or experimental studies, measurements are collected simultaneously on more than one variable on each unit

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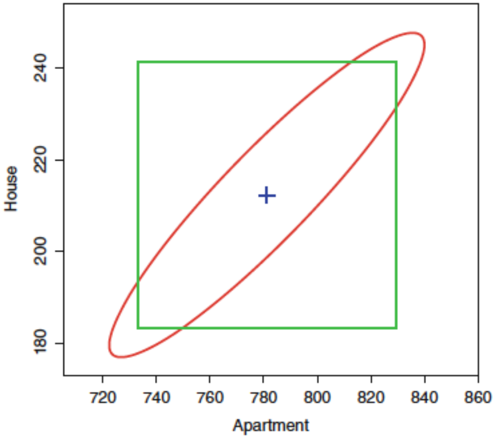
1 0.00632 18  2.31    0 0.538 6.575 65.2 4.0900   1 296    15.3 396.90  4.98 24.0
2 0.02731  0  7.07    0 0.469 6.421 78.9 4.9671   2 242    17.8 396.90  9.14 21.6
3 0.02729  0  7.07    0 0.469 7.185 61.1 4.9671   2 242    17.8 392.83  4.03 34.7
4 0.03237  0  2.18    0 0.458 6.998 45.8 6.0622   3 222    18.7 394.63  2.94 33.4
5 0.06905  0  2.18    0 0.458 7.147 54.2 6.0622   3 222    18.7 396.90  5.33 36.2
6 0.02985  0  2.18    0 0.458 6.430 58.7 6.0622   3 222    18.7 394.12  5.21 28.7
```

- Multivariate analysis is the collection of statistical methods that can be used to (jointly) analyze these multiple measurements

  $\Rightarrow$ *some are extensions of familiar methods (t-test, ANOVA, linear regression,...) while others are unique to multivariate analysis*

- Idea is to exploit potential "correlations" among the multiple measurements to improve inference (see an example in the next slide)

Using Multivariate Methods Could Lead to Sharper Inference



Source: Fig. 1.1 of Applied Multivariate Statistics with R by Zelterman

Objectives of Multivariate Analysis

Dimensionality Reduction or Structural Simplification

- Goal: to reduce the “dimensionality” by considering a small number of (linear) combinations of a large number of measurements without losing important information

- Examples:
  - A single index of patient reaction to radiotherapy can be constructed from measurements on several response variables
  - Wildlife ecologists can construct a few indices of habitat preference from measurements of dozens of features of nesting sites selected by a certain bird species

- Techniques:
  - **Principal Component Analysis** (Week 9)
  - **Factor Analysis** (Week 10)
  - **Multidimensional Scaling** (Week 14)

Grouping or Classification

- Goal: to **identify** groups of “similar” units or to **classify** units into previously defined groups

- Examples:
  - Using the concentration of elements (copper, silver, tin, antimony) in the lead alloy used in bullets, the FBI **identifies** ‘similar’ bullets that may be used to infer whether bullets were produced from the same batch of lead
  - The US IRS uses data collected from tax returns (income, amount withheld, deductions, ...) to **classify** taxpayers into two groups: those who will be audited and those who will not

- Techniques:
  - **Classification Analysis** (Week 12)
  - **Cluster Analysis** (Week 13)

Dependence among Variables and Prediction

- Goal: to estimate the relationship among variables and to predict the value of some of them given information on the others

- Examples:
  - The associations between measures of risk-taking propensity and measures of socioeconomic characteristics for top-level business executives were used to assess the relation between risk-taking behavior and performance
  - The association between test scores, and several college performance variables were used to develop predictors of success in college

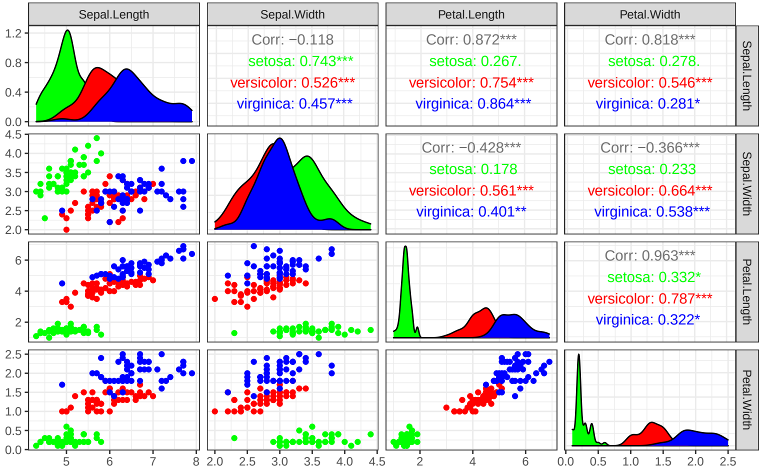
- Techniques:
  - **Multivariate Regression** (Week 7)
  - **Repeated Measures Analysis** (Week 8)
  - **Canonical Correlation Analysis** (Week 11)

Hypothesis Testing

- Goal: to test if differences in sets of response mean vectors for two or more groups large enough to be distinguished from sampling variation
- Examples:
  - A transportation company wants to know if means for gasoline mileage, repair costs, downtime due to repairs differ for different truck models
  - An insurance company wants to know if changing case management practices leads to changes in mean length of hospital stay, mean infection rates, and mean costs
- Techniques:
  - **Hotelling's T^2 and MAVONA** (Week 5 and Week 6)

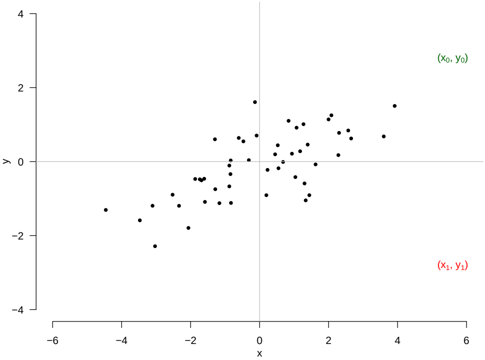
Useful Concepts/Tools for Multivariate Analysis

Exploratory Data Analysis [EDA, Tukey 1977]



Statistical Distance

Multivariate methods rely on "distances" between data points: clustering (group units that are "close"); classification (allocate each unit to the "closest" group)



Question: which one ((x_0, y_0) or (x_1, y_1)) is closer the center of the observations? ⇒ We will learn **Mahalanobis distance** to formally answer this question

Matrix Algebra (Week 3)

The study of multivariate methods is greatly facilitated by the use of matrix algebra

- Many operations performed on multivariate data are presented using vector/matrix notation, e.g., $\boldsymbol{X}_{n\times p}$ (Data matrix); $\hat{\boldsymbol{\mu}}_{p\times 1}$ (estimated mean vector); $\hat{\boldsymbol{\Sigma}}_{p\times p}$ (estimated covaraince matrix)
- The computation of eigenvalues and eigenvectors (i.e., the **spectral decomposition**) plays an important role in multivariate analysis
- We will use `R` to perform the needed matrix operations

Multivariate Normal Distribution (Week 4)

- We will often assume the joint distribution of $\boldsymbol{X} = (X_1, X_2, \cdots, X_p)^{\mathrm{T}}$ follows a multivariate normal distribution with the probability density function:

$$f(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{\frac{d}{2}} \det(\boldsymbol{\Sigma})^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu})\right]$$

- The multivariate normal assumption is often appropriate:
  - Variables can sometimes be assumed to be multivariate normal (perhaps after transformation)
  - Central limit theorem tells us that distribution of many **multivariate sample statistics** is approximately normal, regardless of the form of the population distribution

Data Mining, Machine Learning, and Multivariate Analysis

- Data Mining is the process of extracting and discovering patterns (e.g., unexpected structures or relationships, trends, clusters, and outliers) in **massive data sets**
- Supervised learning and unsupervised learning are two most common problems in machine learning
- Data mining/machine learning applications usually involve many variables, often related in complex ways, hence techniques from multivariate analysis play an important role