Lecture 3

Data Summary/Visualization II

Text: Chapter 3

STAT 8010 Statistical Methods I
January 16, 2020

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Clemson University

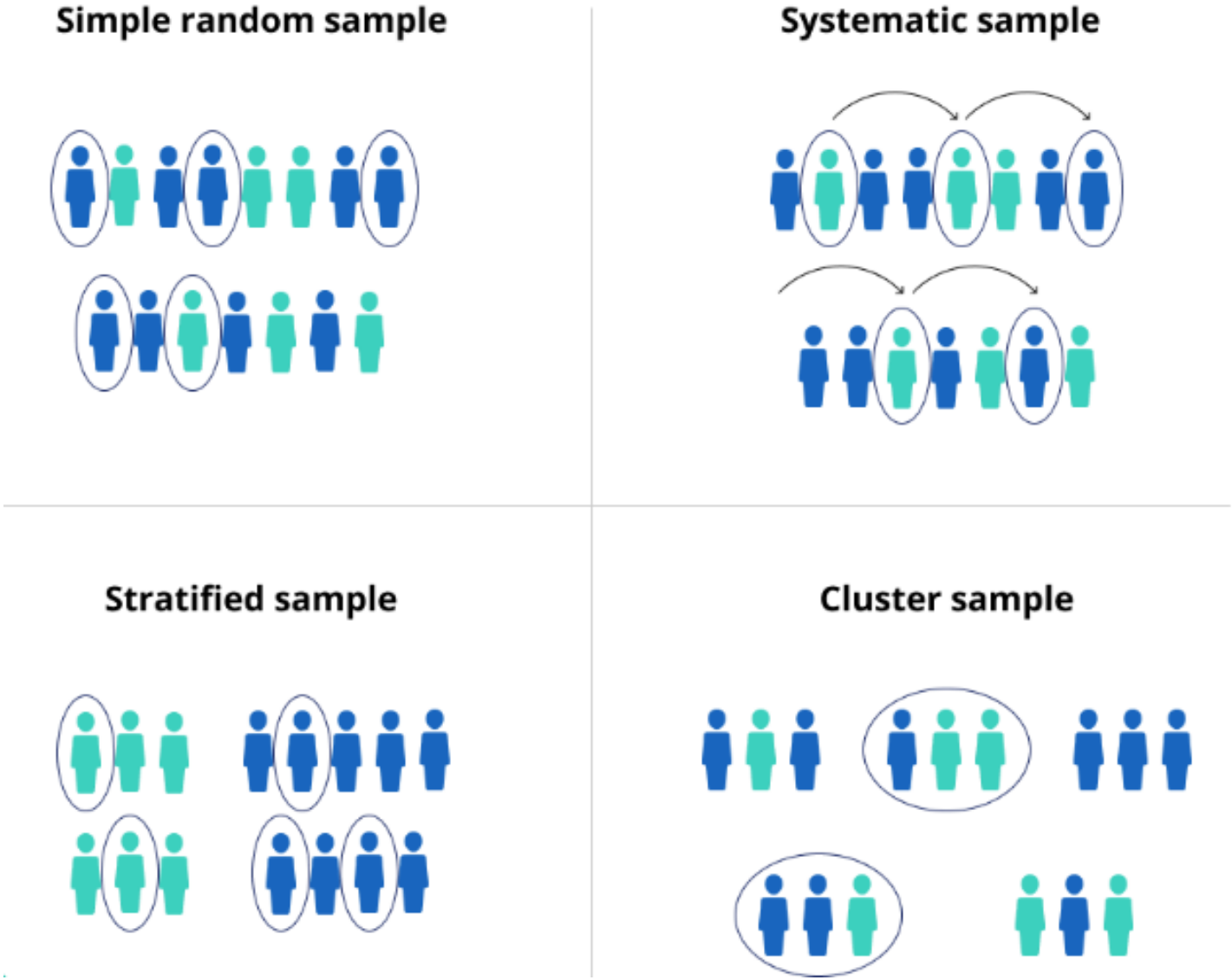
Agenda

1. Percentiles, Quartiles, and Boxplots
2. Visualizing numerical/categorical variables and two numerical variables
3. Visualizing Time Series, Cross-Sectional, and Spatio-Temporal Data sets

Last Lecture

- Sampling Techniques
- Numerical/Graphical Summaries of Categorical Variables
- Numerical/Graphical Summaries of Numerical Variables

Last Lecture: Sampling Techniques



Source:
https://www.scribbr.com/methodology/sampling-methods/

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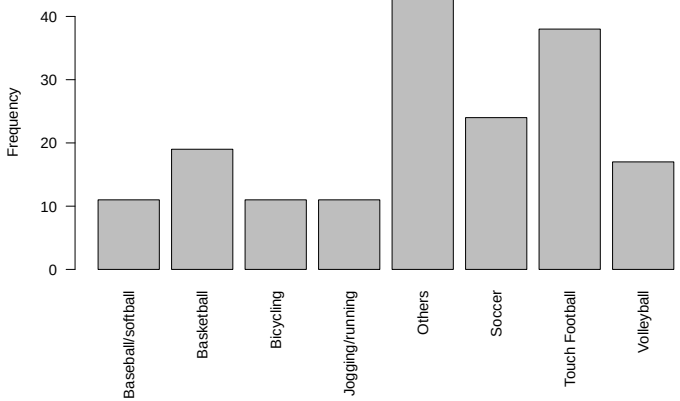
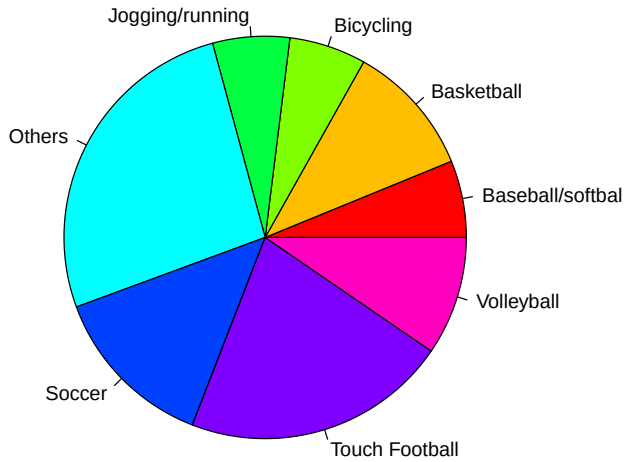
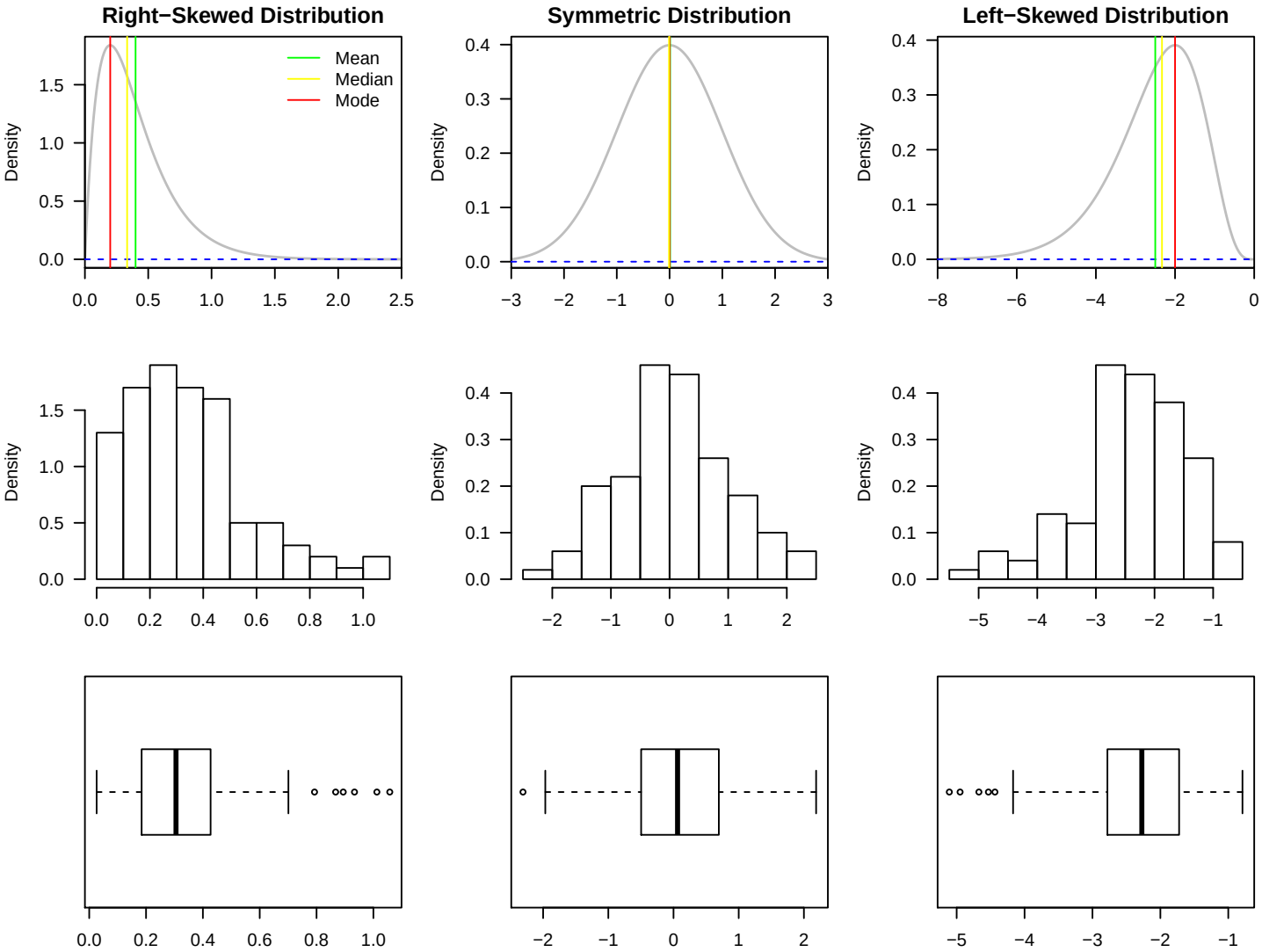
Last Lecture: Summarizing Categorical Variables

```
> table(sport)
sport
Baseball/softball        Basketball         Bicycling   Jogging/running
               11                19                11                11
           Others            Soccer    Touch Football        Volleyball
               47                24                38                17
> table(sport) / dim(sport)[1]
sport
Baseball/softball        Basketball         Bicycling   Jogging/running
       0.06179775        0.10674157        0.06179775        0.06179775
           Others            Soccer    Touch Football        Volleyball
       0.26404494        0.13483146        0.21348315        0.09550562
```





Last Lecture: Shapes of Distributions



Last Lecture: Measures of Center & Spread

- Measures of Center: Mean, Median, Mode
- Measures of Spread: Range, Variance/Standard Deviation, Interquartile range (IQR)
- Resistant (Robust) Statistics

Percentiles, Quartiles, and Boxplots

Percentiles

- The p_{th} percentile is a value such that at least $p\%$ of the data set is less than or equal to this value [An Example]

- Calculation of percentiles using the indexing method:
  1. Sort the set of numbers in an increasing order

- Quartiles:

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  1. $Q1$: first quartile (25_{th} percentile)

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  4. Interquartile range or IQR: $Q3 - Q1$

Example

Find Q_1, M, Q_3 and IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13 using the indexing method

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Find Q_1, M, Q_3 and IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13 using the indexing method

1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21

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2. Find the sample size n and compute the indices for $p = 25, 50, 75$

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3. $n = 9 \Rightarrow$ the indices are $3, 5, 7 \Rightarrow Q_1 = 13$, $M = 14$, $Q_3 = 16$

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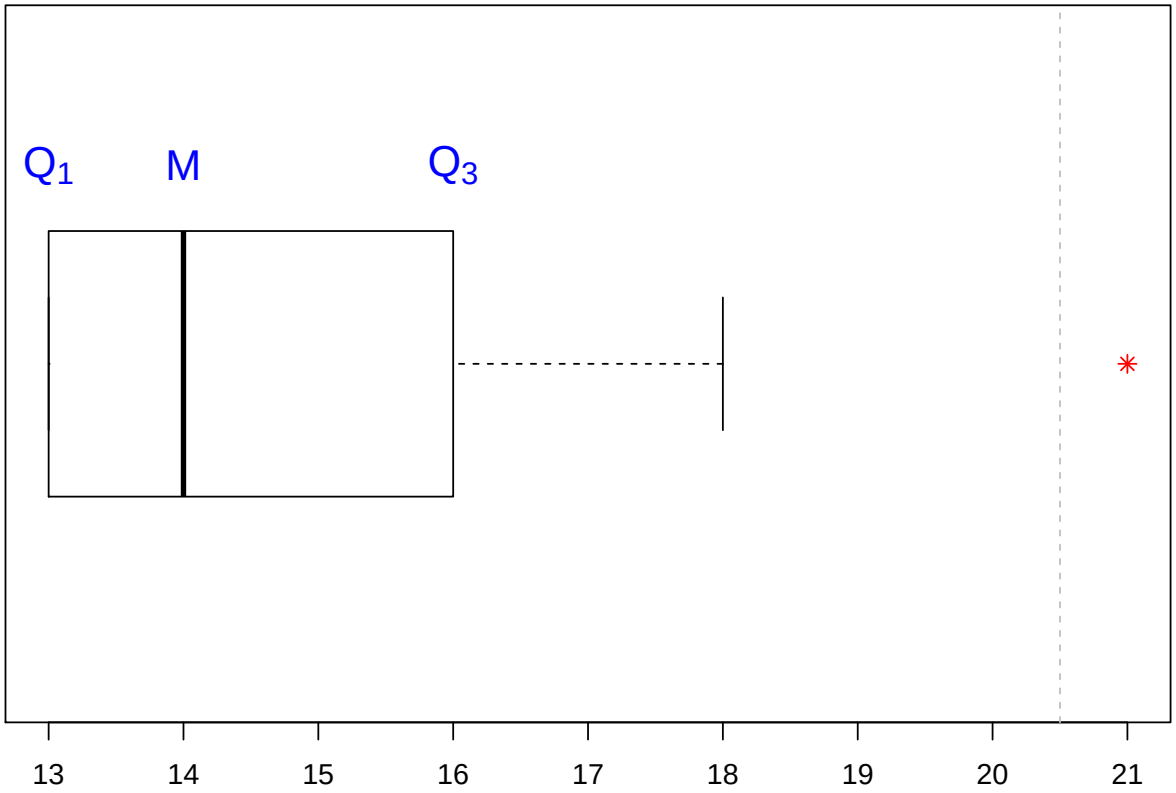
3. $n = 9 \Rightarrow$ the indices are $3, 5, 7 \Rightarrow Q_1 = 13$, $M = 14$, $Q_3 = 16$
4. IQR $= Q_3 - Q_1 = 16 - 13 = 3$

Steps to Making a Boxplot

1. Find Q_1, M, Q_3 and draw a box from Q_1 to Q_3. Add a vertical line inside the box at M
2. Compute the value of Lower Fence (LF) $= Q1 - 1.5$IQR and the Upper Fence (UF) $= Q3 + 1.5$IQR. Find the largest value $\leq$ UF and the smallest value $\geq$ LF. Draw whiskers go from Q_1, Q_3 to these two values
3. Plot the individual outlier(s) (i.e., the values either $>$ UF or $<$ LF)

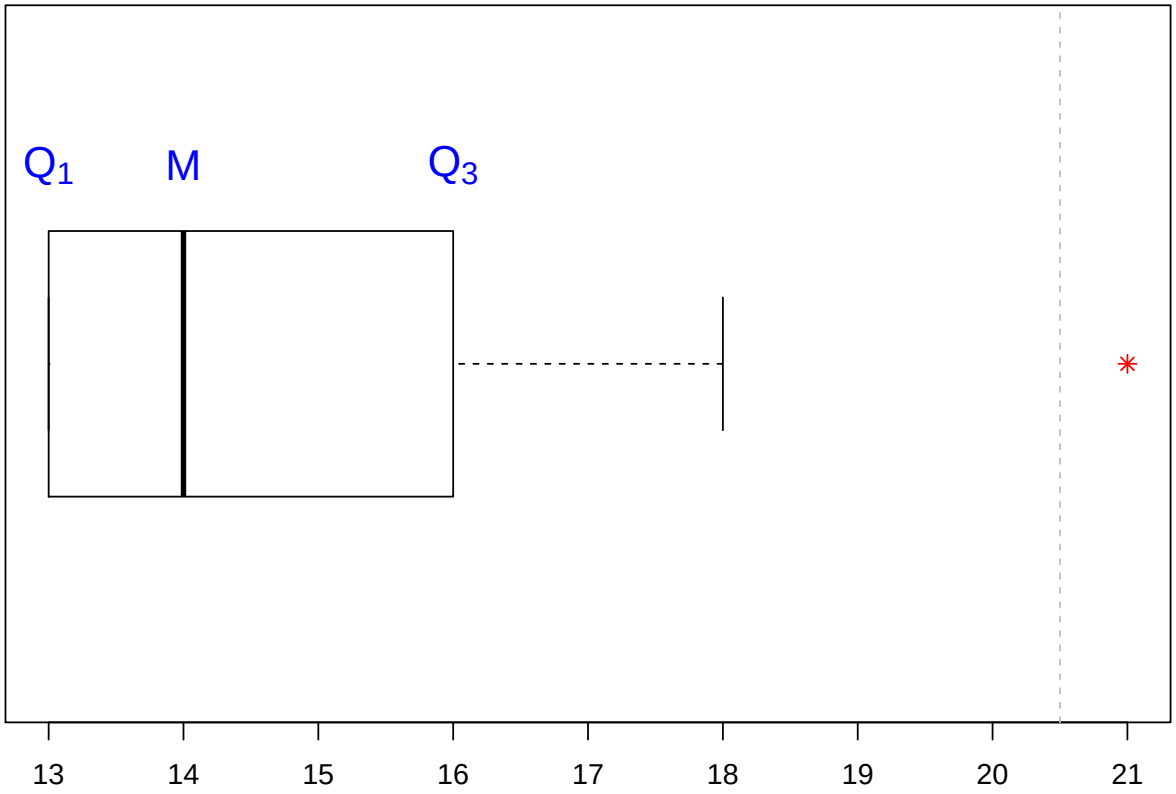
Bopxplot

- **Ordered data values:** 13, 13, 13, 13, 14, 14, 16, 18, 21



Bopxplot

- **Ordered data values:** 13, 13, 13, 13, 14, 14, 16, 18, 21
- **IQR** $16 - 13 = 3 \Rightarrow$ LF $= 13 - 1.5 \times 3 = 8.5$; UF $= 16 + 1.5 \times 3 = 20.5$



Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile

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- Find the 35th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$

Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

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Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$
  2. Compute the index value $i = \frac{35 \times 15}{100} = 5.25 \Rightarrow$ the 35th percentile is 13

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Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

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- Find the 65th percentile

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  2. Compute the index value $i = \frac{35 \times 15}{100} = 5.25 \Rightarrow$ the 35th percentile is 13
- Find the 65th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$
  2. Compute the index value $i = \frac{65 \times 15}{100} = 9.75 \Rightarrow$ the 65th percentile is 18

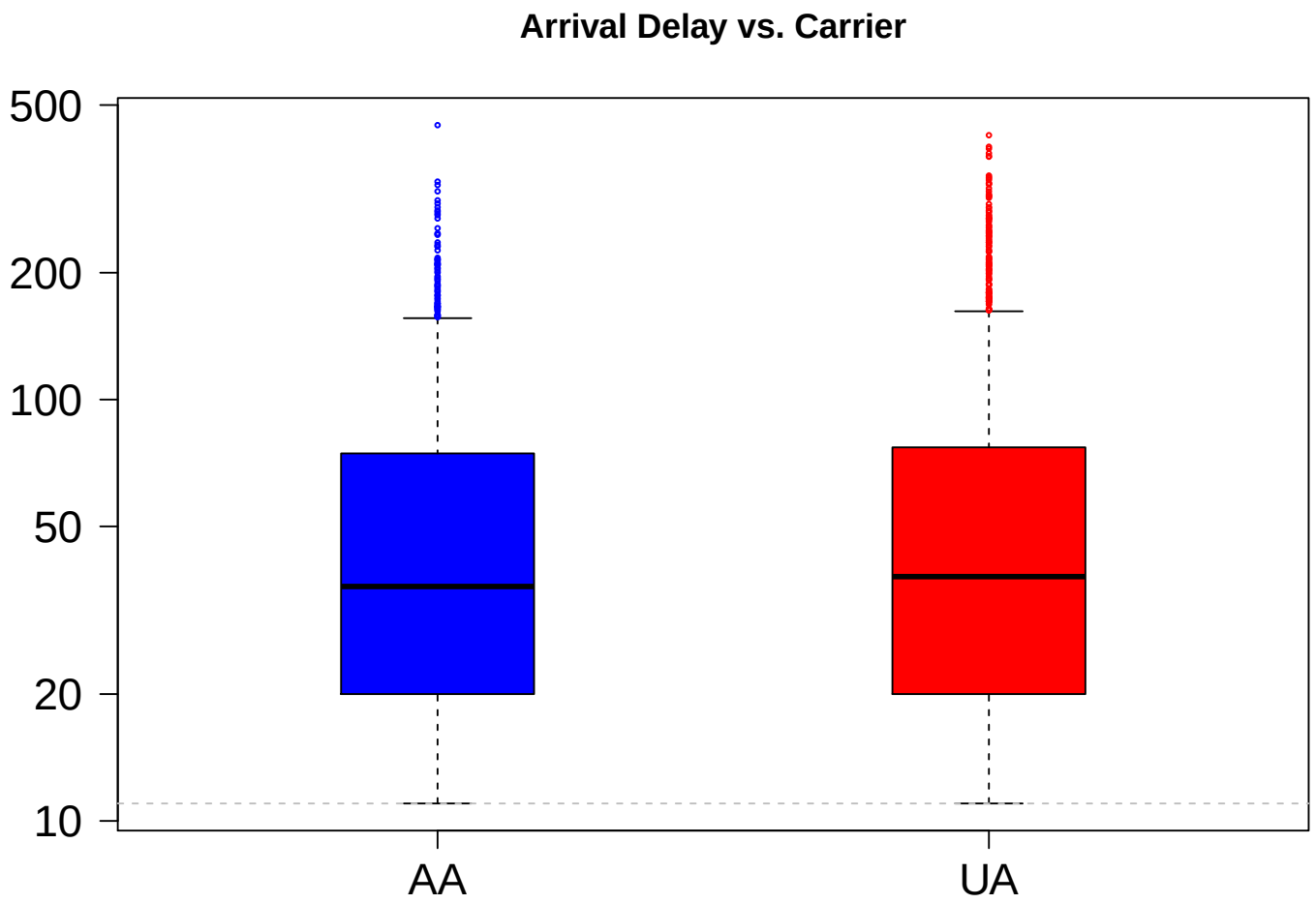
Visualizing numerical/categorical variables and two numerical variables

ORD Fligts Data Revisited

carrier	origin	arr_delay
UA	EWR	12
AA	LGA	8
AA	LGA	14
AA	LGA	4
UA	LGA	20
UA	EWR	21

In this example, we have two categorical variables, `carrier`, `origin` and a numerical variable `arr_delay`, respectively. How to visualize, for example, `arr_delay` vs. `carrier`?

ORD Example: Arrival Delay vs. Air Carrier



Example: Max Heart Rate and Age

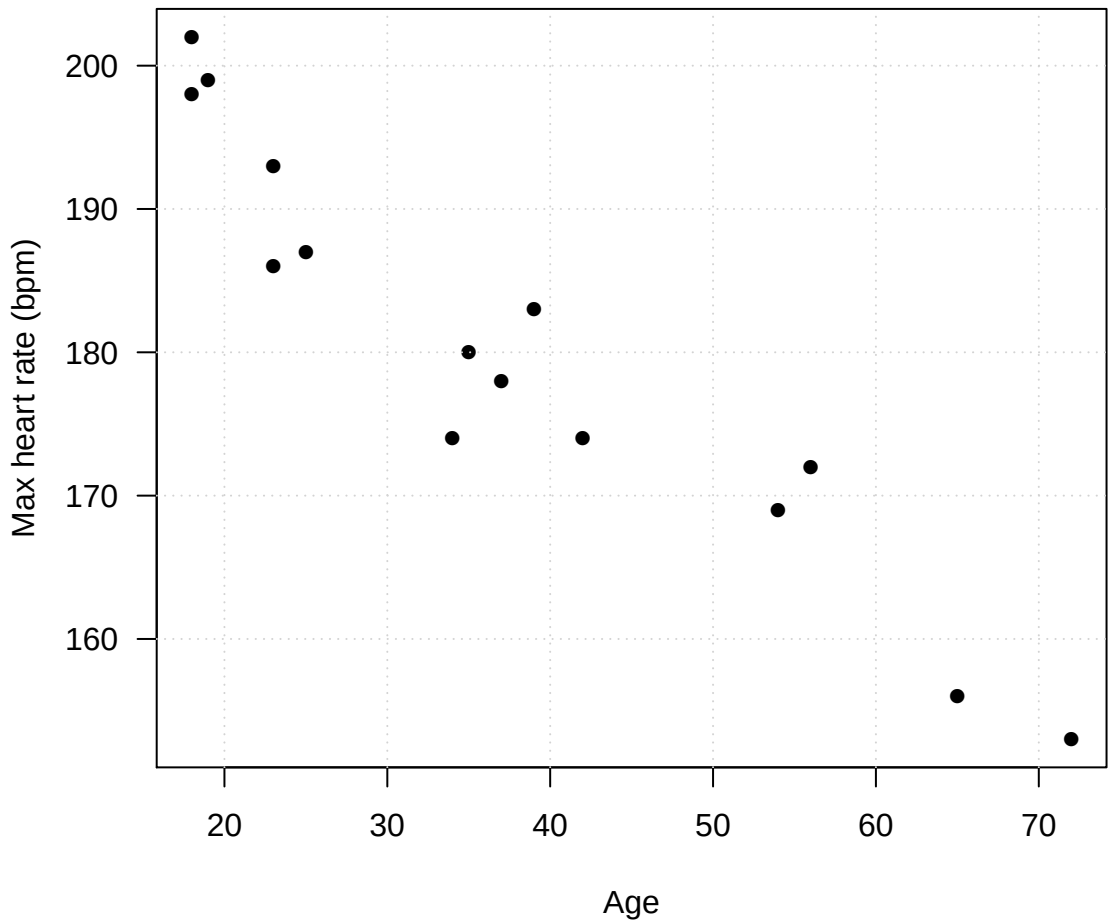
Suppose we have 15 people of varying ages are tested for their maximum heart rate (MHR)

Age	18	23	25	35	65	54	34	56	72	19	23	42	18	39	37
MHR	202	186	187	180	156	169	174	172	153	199	193	174	198	183	178

- How many variables do we have in this data set? What are the variable types?
- How to summarize these variables?

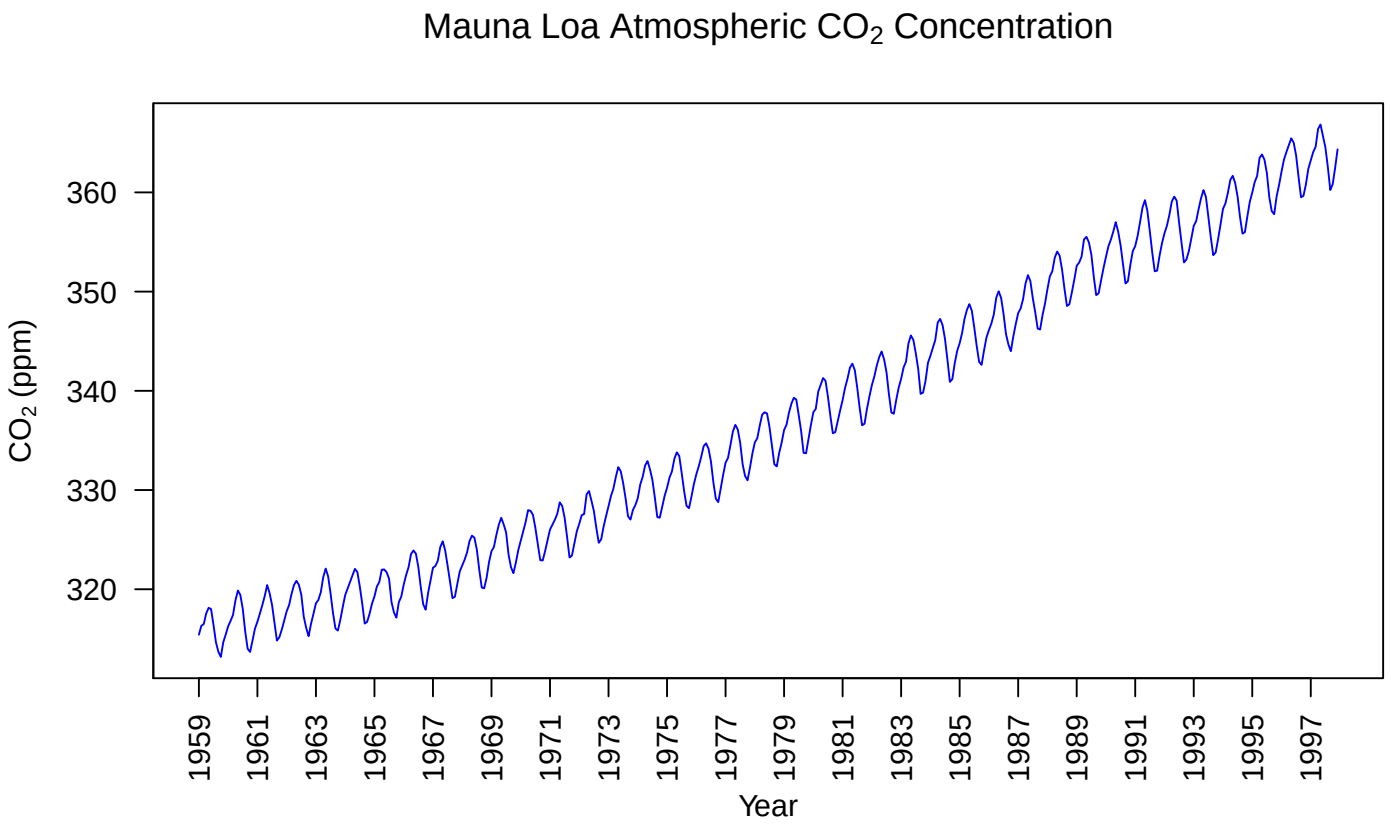
Scatterplot

A scatterplot is a useful tool to graphically display the relationship between two numerical variables. Each dot on the scatterplot represents one observation from the data

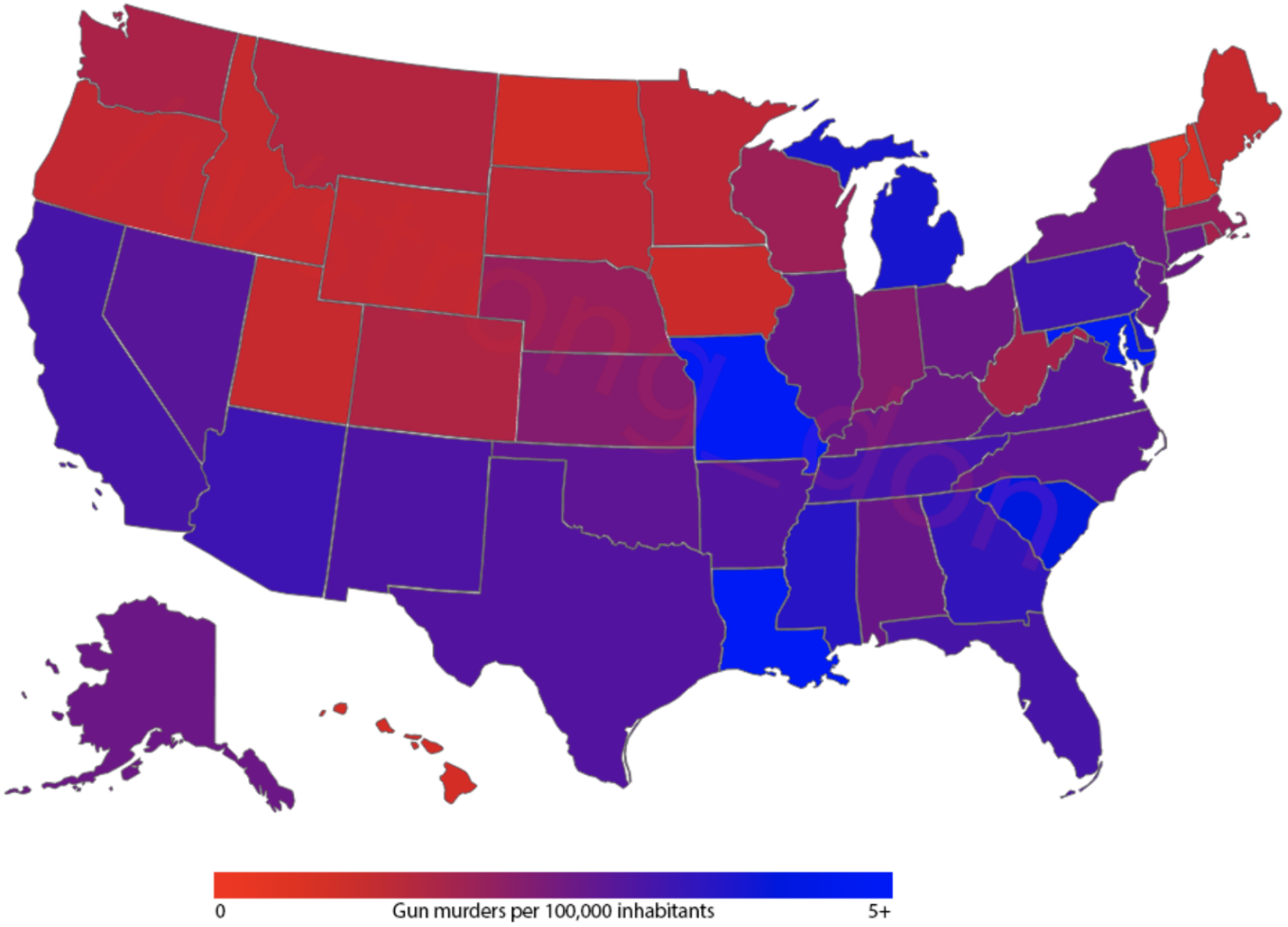


Visualizing Time Series, Cross-Sectional, and Spatio-Temporal Data sets

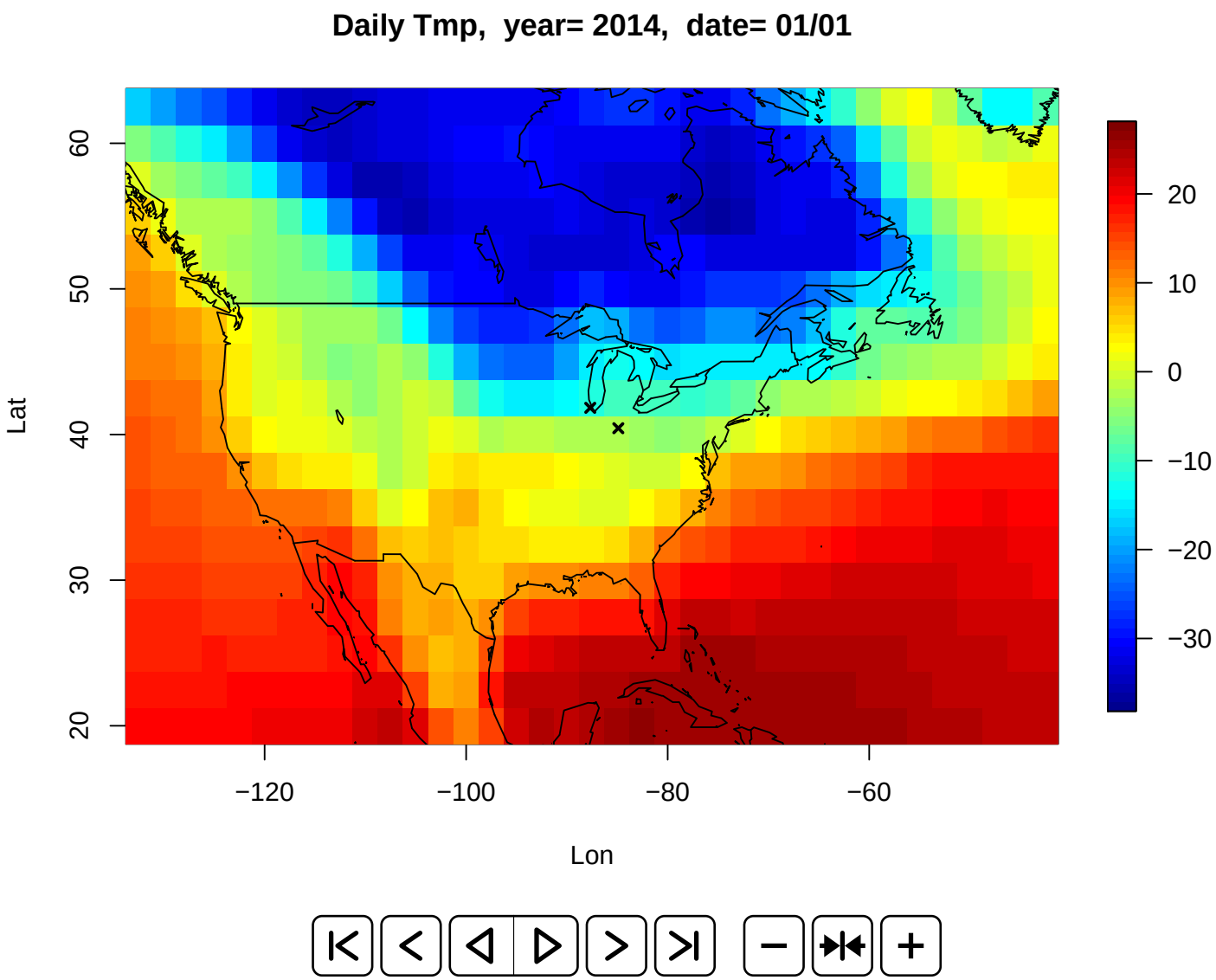
Visualizing Time Series Data



Visualizing Cross-Sectional Data



Visualizing Spatio-Temporal Data



Summary

In this lecture, we learned

- Percentiles and Quartiles
- How to construct a Boxplot
- How to visualize numerical/categorical and two numerical Variables
- How to visualize time series, cross-sectional, spatio-temporal data sets

We will talk about Probability in the next few weeks