STAT 8010 Exam II

April 2, 2020

Name:______________________________

Directions

1. Show your work on ALL questions (except those multiple choice questions). Unsupported work will NOT receive full credit.
2. Decimal answers should be exact, or to exactly 3 significant digits.
3. You are allowed the following aids:
   (a) a one-page A4 cheat sheet
   (b) A scientific Calculator
4. Write up your work on blank sheets of paper and upload it to Canvas. Please write legibly. **If I cannot read your writing, No credit will be given.**

Use your time wisely. Good Luck!!!

Problem	Points Possible	Points Earned
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1

(4 points for each answer.)

(a) Which of the following corresponds to a Type I error in hypothesis testing?

True State	Decision	
	Reject H_0	Fail to reject H_0
H_0 is true	A	B
H_0 is false	C	D

(b) Which of the following is NOT a linear contrast of population means?

$A : \mu_1 - \mu_2$

$B : \mu_2 - \frac{1}{2}\mu_3 - \frac{1}{2}\mu_5$

$C : \mu_1 - \mu_2 + \frac{1}{3}\mu_3 - \frac{1}{4}\mu_4$

$D : \mu_1 + \mu_2 - \mu_3 - \mu_4$

(c) Suppose the standard deviation of random variable X is 6. What is the standard deviation of $\bar{X}_n = \frac{\sum_{i=1}^{n} X_i}{n}$ with sample size $n = 36$?

$A : 2$

$B : 5$

$C : 1$

$D :$ Can't be determined without sample mean

(d) What is the minimum sample size needed in order to estimate μ such that the 95% CI to be 4 in width if $\sigma = 10$

$A : 25$

$B : 97$

$C : 961$

(f) If the true means of the 4 populations are equal, then F = MSTr/MSE should be:

$A :$ more than 10.00

$B :$ close to -1.00

$C :$ close to 1.00

Problem 2

A graduate school administrator would like to know the average GRE Quantitative score for all the applicants. She take a random sample of 64 applicants. Use $\bar{x} = 148$, $s = 8$ to answer the following questions:

(a) Construct a 95% confidence interval (using $t_{0.025,df=63} = 1.998$) for the average GRE Quantitative score for all applicants. **(7 points)**

i Point estimate: $\bar{x}_n = 148$

ii Standard error (SE): $\frac{s}{\sqrt{n}} = \frac{8}{\sqrt{64}} = 1$

iii Margin of error (ME): $t_{\alpha/2,df=n-1} \times \text{SE} = 1.998 \times 1 = 1.998$

iv 95% confidence interval: Point estimate $\pm$ ME
$= 148 \pm 1.998 = (146.002, 149.998)$

(b) One of the graduate school administrators would like to know if the average GRE Quantitative score is greater than 145. Conduct a hypothesis test for this assessment (using $t_{0.05,df=63} = 1.669$) and $\alpha = 0.05$ for this test). **(7 points)**

i $H_0 : \mu = 145$ vs. $H_a : \mu > 145$

ii $t_{obs} = \frac{148-145}{\frac{8}{\sqrt{64}}} = \frac{3}{1} = 3$

iii Since $t_{obs} = 3 > t_{0.05,df=99} = 1.669$. Reject $H_0 : \mu = 145$.

iv We do have enough statistical evidence that the average GRE Quantitative score is greater than 145 at 0.05 level.

(c) Compute the minimum sample size needed such that the 95% CI for average GRE Quantitative score to be 2 in width if $\sigma = 7.5$ **(6 points)**

$$
\begin{aligned}
n &= \left(\frac{\sigma \times Z_{\alpha/2}}{\text{ME}}\right)^2 \\
&= \left(\frac{7.5 \times 1.96}{2/2}\right)^2 = 216.09 \\
&\Rightarrow n = 217.
\end{aligned}
$$

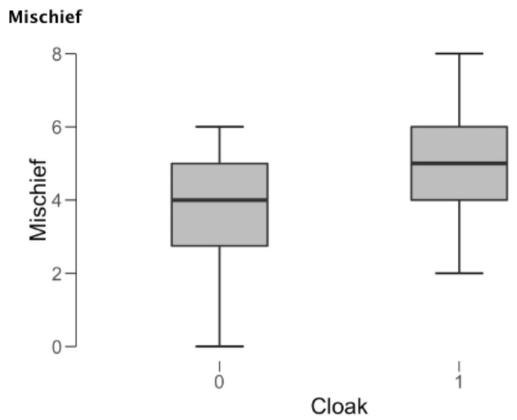
Problem 3

(5 points for each answer.) A researcher would like to examine if invisibility cloak affects the number mischievous acts committed. He conducted an experiment where he randomly assign the "treatment" (0 = without a cloak of invisibility, 1 = with a cloack of invisibility) to 24 study participants and he recorded the number of mischievous acts committed by each participant. Use the JASP output below to answer the following questions.

(a) What is the point estimate of the average difference (Without - With)?

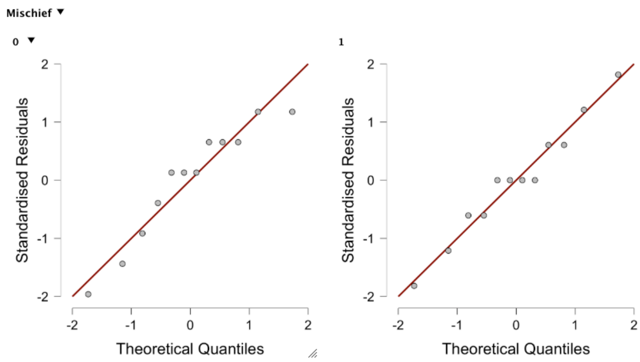
Descriptive Statistics ▼

	Mischief	
	0	1
Valid	12	12
Missing	0	0
Mean	3.750	5.000
Std. Error of Mean	0.552	0.477
Std. Deviation	1.913	1.651
Minimum	0.000	2.000
Maximum	6.000	8.000



$\hat{\mu}_{w/o} - \hat{\mu}_{w/} = 3.750 - 5.000 = -1.250$

(b) Assess the normality assumption using the QQ-plots below.



According to the QQ-plots, all the data points are relatively close to their 1-1 line. Therefore, the normality assumption is reasonable for both populations.

(c) State the null and alternative hypotheses

$H_0 : \mu_{w/o} - \mu_{w/} = 0$ vs. $H_a : \mu_{w/o} - \mu_{w/} \neq 0$

(d) Draw the conclusion at 0.05 level using the JASP output below:

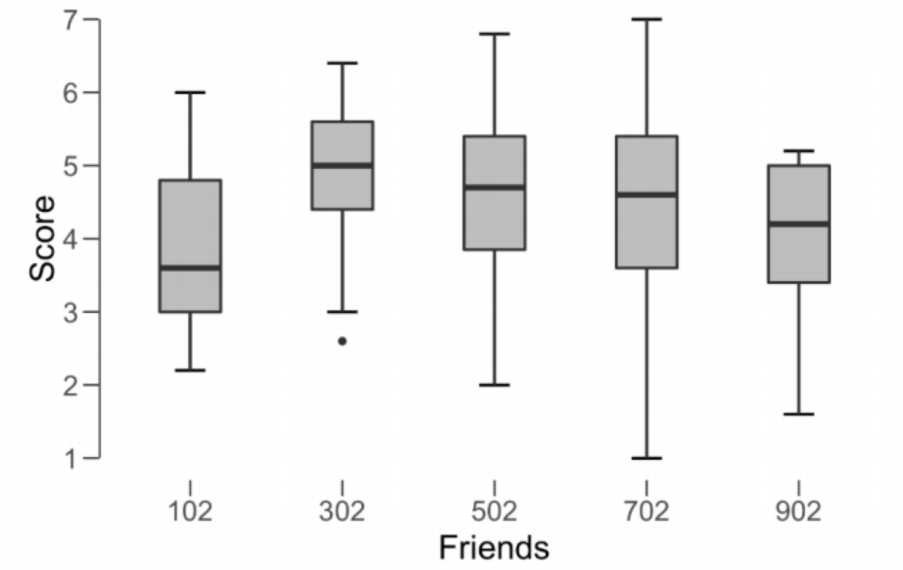
Independent Samples T-Test

	Test	Statistic	df	p	Mean Difference	SE Difference	95% CI for Mean Difference Lower	95% CI for Mean Difference Upper
Mischief	Student	−1.713	22.000	0.101	−1.250	0.730	−2.763	0.263
	Welch	−1.713	21.541	0.101	−1.250	0.730	−2.765	0.265

The P-value for both tests are greater than $\alpha = 0.05$. Therefore, we do not have enough statistical evidence that wearing invisibility cloak will affect the number mischievous acts committed at 0.05 level.

Problem 4

(5 points for each answer.) A researcher would like to investigate the relationship between Facebook social attractiveness and the number of Facebook friends. An experiment was conducted where five groups of participant judge the same Facebook profiles, except for the one aspect that was manipulated: the number of friends for that profile. Use the software output below to answer the following questions:



	Score				
	102	302	502	702	902
Valid	24	33	26	30	21
Missing	0	0	0	0	0
Mean	3.817	4.879	4.562	4.407	3.990
Std. Deviation	0.999	0.851	1.070	1.428	1.023
Minimum	2.200	2.600	2.000	1.000	1.600
Maximum	6.000	6.400	6.800	7.000	5.200

(a) State the null and alternative hypotheses of the overall F test.

$H_0 : \mu_{102} = \mu_{302} = \mu_{502} = \mu_{702} = \mu_{902}$ vs. H_a: At least one mean is different than others.

(b) Fill in the missing elements in the ANOVA table below:

Source	df	SS	MS	F statistic
Friends	4	SSTr = 19.89	MSTr = 4.9726	F_{obs} = 4.142
Error	129	SSE = 154.87	MSE = 1.2005	
Total	133	SSTo = 174.76		

(c) Perform an overall F-test for part (a) at 0.05 level
(using $F_{0.95,df_1=4,df_2=129} = 2.442$).

Since $F_{obs} = 4.142 > F_{0.95,df_1=4,df_2=129} = 2.442$. We do have enough evidence that the number of Face-book friends affects Facebook social attractiveness at 0.05 level.

(e) Construct a 95% confidence interval (using $t_{0.025,129} = 1.9785$) for $L = \mu_{302} - \frac{1}{3}\mu_{502} - \frac{1}{3}\mu_{702} - \frac{1}{3}\mu_{902}$

i Point estimate: $\hat{L} = \hat{\mu}_{302} - \frac{1}{3}\hat{\mu}_{502} - \frac{1}{3}\hat{\mu}_{702} - \frac{1}{3}\hat{\mu}_{902} = 4.879 - \frac{1}{3} \times (4.562 + 4.407 + 3.990) = 0.5593$

ii ME: $t_{\alpha/2,df} \times \sqrt{MSE \times \left(\frac{c_2^2}{n_2} + \frac{c_3^2}{n_3} + \frac{c_4^2}{n_4} + \frac{c_5^2}{n_5}\right)} = 1.9785 \times \sqrt{1.2005 \times \left(\frac{1}{33} + \frac{1/9}{26} +\right.}$
$= 0.4525$

iii 95% confidence interval: Point estimate ± ME = $0.5593 \pm 0.4525 = (0.1068, 1.0118)$

Problem 5

Suppose concrete cylinders for bridge supports. There are three ways of drying green concrete (A, B, and C), and you want to find the one that gives you the best compressive strength. The concrete is mixed in batches that are large enough to produce exactly three cylinders, and your production engineer believes that there is substantial variation in the quality of the concrete from batch to batch. You have data from 4 batches on each of the 3 drying processes. Use the R output below to answer the following questions:

```
Analysis of Variance Table

Response: x
          Df Sum Sq Mean Sq F value    Pr(>F)
trt        2  74.00   37.00   9.250 0.0146878
blk        3 362.25  120.75  30.188 0.0005124
Residuals  6  24.00    4.00
```

(a) Test, at the 5% level of significance, whether these data provide sufficient evidence that at least one of the three treatments (A, B, and C) affects the average compressive strength. **(7 points)**

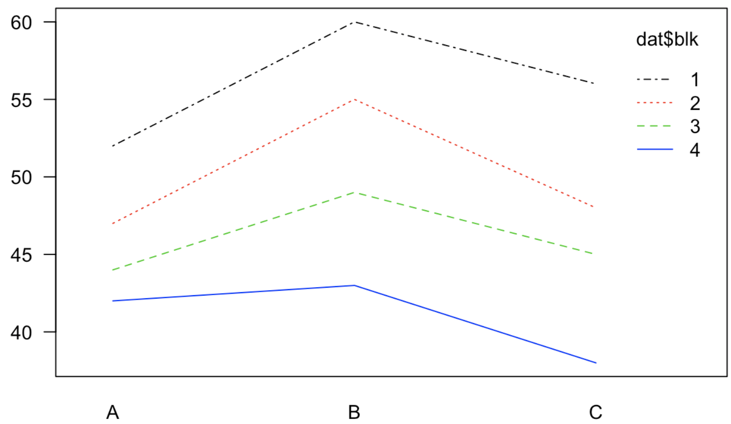
i $H_0 : \mu_A = \mu_B = \mu_C$ vs. H_a : at least one mean is different than the others.

ii $F_{obs} = 9.250$, P-value $= 0.0146878 < \alpha = 0.05$

iii Reject H_0.

iv We do have enough statistical evidence that at least one way of drying green concrete gives different average compressive strength than others at 0.05 level.

(b) Use the interaction plot below to assess the appropriateness of the assumption that treatments have the same effect across blocks. **(7 points)**



Based on the interaction plot above, it is reasonable to assume that the treatment effects are consistent across blocks.

(c) Use the R output of Fisher's LSD with Bonferroni correction (Left) and Tukey's HSD (Right) below to determine which pairs are significantly different than each other at 0.05 level after accounting for multiple comparisons. **(6 points)**

	x	groups
B	51.75	a
C	46.75	b
A	46.25	b

	diff	lwr	upr	p adj
B-A	5.5	1.160805	9.8391953	0.01895344
C-A	0.5	-3.839195	4.8391953	0.93415461
C-B	-5.0	-9.339195	-0.6608047	0.02848377

B-A and B-C pairs.