Lecture 18
Confidence Intervals & Hypothesis Testing

STAT 8010 Statistical Methods I
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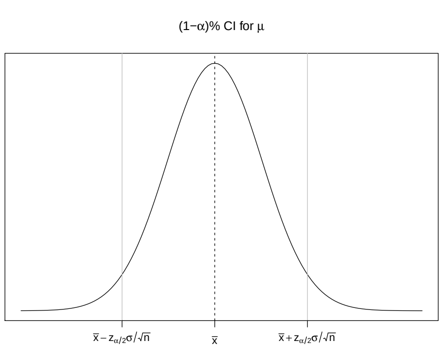
Notes

Agenda

1. Confidence Intervals
2. Hypothesis Testing

Notes

Last Lecture: Confidence Intervals for μ



Notes

Example: Average Height

We measure the heights of 40 randomly chosen men, and get a mean height of 5’9” (≈175cm). Suppose we know the standard deviation of men’s heights is 4” (≈10cm). Find the 95% confidence interval of the true mean height of ALL men.

Notes

Average Height Example Cont’d

1. Point estimate: $\bar{X}_{40} = \frac{\sum_{i=1}^{40} X_i}{40} = 69$ inches
2. Population standard deviation: $\sigma = 4$ inches
3. Standard error of $\bar{X}_{n=40} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{40}} = 0.63$ inches
4. 95%CI: Need to find $z_{0.05/2} = 1.96$ from the Z-table
5. 95% CI for μ_X is:

$$[69 - 1.96 \times 0.63, 69 + 1.96 \times 0.63]$$
$$= [67.77, 70.23]$$

Notes

Properties of Confidence Intervals

- In contrast with the point estimate, $\bar{X}_n$, a $(1-\alpha)$% CI is an interval estimate, where the **length of CI** reflects our estimation uncertainty
- Typical α values: $0.01, 0.05, 0.1 \Rightarrow 99\%, 95\%, 90\%$ confidence intervals. **Interpretation**: If we were to take random samples over and over again, then $(1-\alpha)$% of these confidence intervals will contain the true μ
- The length of a CI depends on
  - **Population Standard Deviation**: σ
  - **Confidence Level**: $1-\alpha$
  - **Sample Size**: n

Notes

Sample Size Calculation

- We may want to estimate μ with a confidence interval with a predetermined margin of error (i.e. $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$)
- For example, in estimating the true mean height of All men we may want our CI to be just 0.5 inches in width
- The question is then, "**how many observations do we need to take** so that we have the desired margin of error?"

Notes

Sample Size Calculation Cont'd

To compute the sample size needed to get a CI for μ with a specified margin of error, we use the formula below

$$n = \left(\frac{z_{\alpha/2} \times \sigma}{\text{Margin of error}}\right)^2$$

Exercise: Derive this formula using margin of error $= z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

Notes

Average Height Example Revisited

Compute the sample size needed in order to estimate the true mean height of All men such that the 95% CI to be 0.5 inches in width

1. Length of CI: $2 \times z_{0.025}\frac{\sigma}{\sqrt{n}} = 2\times$ margin of error
2. Want to find n s.t. $z_{0.025}\frac{\sigma}{\sqrt{n}} = 0.25$
3. We have $n = \left(\frac{1.96\times 4}{0.25}\right)^2 = 983.4496$

Therefore, the required sample size is 984

Notes

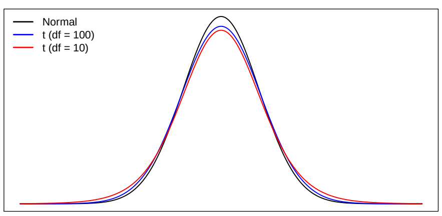
Confidence Intervals When σ Unknown

- In practice, it is unlikely that σ is available to us
- One reasonable option is to replace σ with s, the sample standard deviation
- We need to account for this added uncertainty with a (slightly) different sampling distribution that has fatter tails

$\Rightarrow$ Student t Distribution (William Gosset, 1908)

Notes

Student t Distribution



- Recall the standardize sampling distribution $\frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \mathrm{N}(0, 1)$
- Similarly , the studentized sampling distribution $\frac{\bar{X}_n - \mu}{\frac{s}{\sqrt{n}}} \sim \mathrm{t}(df = n - 1)$

Notes

Confidence Intervals (CIs) for μ When σ is Unknown

- $(1 - \alpha) \times 100\%$ CI for μ:

$$\left[\bar{X}_n - t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}, \bar{X}_n + t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}\right],$$

 where $t_{\frac{\alpha}{2}, n-1}$ is the $1 - \frac{\alpha}{2}$ percentile of a student t distribution with the degrees of freedom $= n - 1$

- $\frac{s}{\sqrt{n}}$ is an estimate of the standard error of $\bar{X}_n$

Notes

Average Height Example Revisited

We measure the heights of 40 randomly chosen men, and get a mean height of 5'9" (≈175cm), and a standard deviation of 4.5" (≈11.4cm). Find the 95% confidence interval of the true mean height of ALL men.

Notes

Average Height Example Cont'd

1. Point estimate: $\bar{X}_{40} = \frac{\sum_{i=1}^{40} X_i}{40} = 69$ inches
2. Sample standard deviation: $s = 4.5$ inches
3. (Estimated) standard error of $\bar{X}_{n=40} = \frac{s}{\sqrt{n}} = \frac{4.5}{\sqrt{40}} = 0.71$ inches
4. 95%CI: Need to find $t_{0.05/2,39} = 2.02$ from a t-table (or using a statistical software)
5. 95% CI for μ_X is:

$$[69 - 2.02 \times 0.71, 69 + 2.02 \times 0.71]$$
$$= [67.57, 70.43]$$

Notes

Hypothesis Testing

- **Hypothesis Testing**: A method for using sample data to decide between two competing claims (hypotheses) about a population characteristic (a parameter. e.g. μ)
- **Examples**:
  - The true mean starting salary for graduates of four-year business schools is $4,500 per month $\Rightarrow \mu = 4,500$
  - The true mean monthly income for systems analysts is at least $6,000 $\Rightarrow \mu \geq 6,000$

Notes

Hypotheses

- **Null Hypothesis**: A claim about a population characteristic that is initially assumed to be true. We use H_0 to denote a null hypothesis
- **Alternative Hypothesis**: The competing claim, denoted by H_a
- In carrying out a test of H_0 versus H_a , the hypothesis H_0 will be rejected in favor of H_a only if sample evidence strongly suggests that H_0 is false. If the sample data does not contain such evidence, H_0 will not be rejected
- Therefore, the two possible decisions in a hypothesis test are:
  - Reject H_0 (and go with H_a)
  - Fail to Reject H_0

Notes

Courtroom Analogy

- In a criminal trial, we use the rule "innocent until proven guilty"
- Therefore, our hypotheses are:
  - H_0: Innocent
  - H_a: Guilty
- If we have strong evidence that the accused is not innocent, we reject H_0 (innocent) and conclude H_a (guilty)
- If we do not have enough evidence to say that the accused is guilty, we do not say that the accused is "innocent". Instead, we say that the accused is "not guilty"

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Hypotheses

Be careful setting up hypotheses:

- A statistical hypothesis test is only capable of demonstrating strong support for the alternative hypothesis H_a (by rejecting the null hypothesis H_0)
- Failing to reject H_0 does not show strong support for the null hypothesis – **only a lack of strong evidence against H_0, the null hypothesis**

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The 2×2 Decision Paradigm for Hypothesis Testing

	Decision	
True State	Reject H_0	Fail to reject H_0
H_0 is true	Incorrect: Type I error	Correct
H_0 is false	Correct	Incorrect: Type II error

Errors in Hypothesis Testing

- The probability of a type I error is denoted by α
- The probability of a type II error is denoted by β

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