DSA 8070 R Session 13: Multidimensional Scaling

Whitney

November 14, 2021

Contents

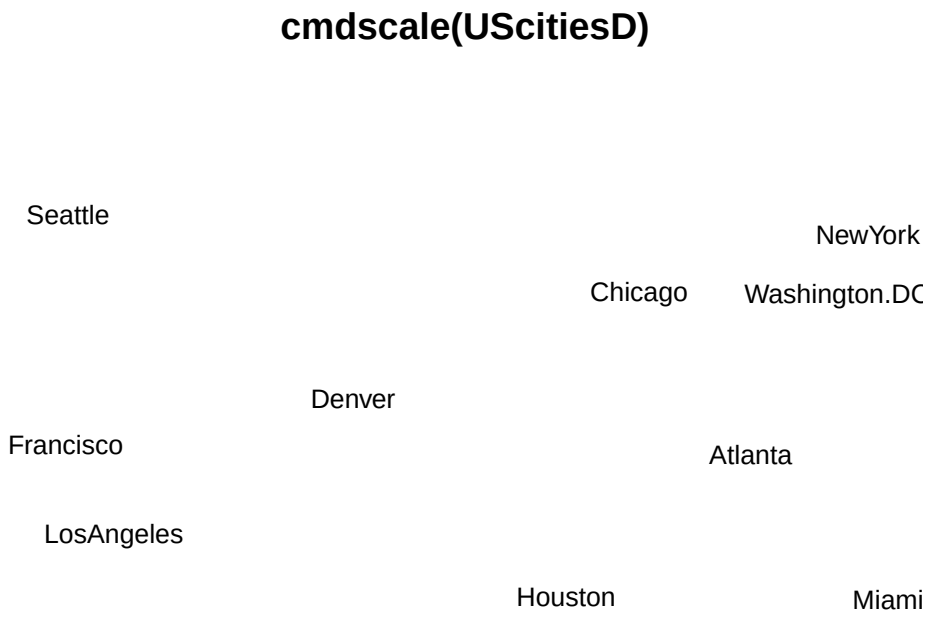
Classical Multidimensional Scaling

Distances Between US Cities

```
loc <- cmdscale(UScitiesD)
x <- loc[, 1]; y <- loc[, 2]
plot(x, y, type = "n", xlab = "", ylab = "", asp = 1,
     axes = FALSE, main = "cmdscale(UScitiesD)")
text(x, y, rownames(loc), cex = 0.8)
```


```
# Flip Axes
x1 <- -loc[, 1]; y1 <- -loc[, 2]
plot(x1, y1, type = "n", xlab = "", ylab = "", asp = 1,
```

```
     axes = FALSE, main = "cmdscale(UScitiesD)")
text(x1, y1, rownames(loc), cex = 0.8)
```



Air Pollution in US Cities

```
library(HSAUR3)
```

```
## Loading required package: tools
```

```
data(USairpollution)
dat <- USairpollution
head(dat)
```

```
##             SO2 temp manu popul wind precip predays
## Albany       46 47.6   44   116  8.8  33.36     135
## Albuquerque  11 56.8   46   244  8.9   7.77      58
## Atlanta      24 61.5  368   497  9.1  48.34     115
## Baltimore    47 55.0  625   905  9.6  41.31     111
## Buffalo      11 47.1  391   463 12.4  36.11     166
## Charleston   31 55.2   35    71  6.5  40.75     148
```

```
summary(dat)
```

```
##       SO2             temp           manu           popul
##  Min.   :  8.00   Min.   :43.50   Min.   :  35.0   Min.   :  71.0
##  1st Qu.: 13.00   1st Qu.:50.60   1st Qu.: 181.0   1st Qu.: 299.0
##  Median : 26.00   Median :54.60   Median : 347.0   Median : 515.0
##  Mean   : 30.05   Mean   :55.76   Mean   : 463.1   Mean   : 608.6
##  3rd Qu.: 35.00   3rd Qu.:59.30   3rd Qu.: 462.0   3rd Qu.: 717.0
##  Max.   :110.00   Max.   :75.50   Max.   :3344.0   Max.   :3369.0
##       wind           precip         predays
##  Min.   : 6.000   Min.   : 7.05   Min.   : 36.0
##  1st Qu.: 8.700   1st Qu.:30.96   1st Qu.:103.0
##  Median : 9.300   Median :38.74   Median :115.0
```

```
##  Mean   : 9.444   Mean   :36.77   Mean   :113.9
##  3rd Qu.:10.600   3rd Qu.:43.11   3rd Qu.:128.0
##  Max.   :12.700   Max.   :59.80   Max.   :166.0
```

```
xs <- apply(dat, 2, function(x) (x - min(x)) / (diff(range(x))))
summary(xs)
```

```
##       SO2               temp             manu             popul
##  Min.   :0.00000   Min.   :0.0000   Min.   :0.00000   Min.   :0.00000
##  1st Qu.:0.04902   1st Qu.:0.2219   1st Qu.:0.04412   1st Qu.:0.06913
##  Median :0.17647   Median :0.3469   Median :0.09429   Median :0.13463
##  Mean   :0.21616   Mean   :0.3832   Mean   :0.12937   Mean   :0.16301
##  3rd Qu.:0.26471   3rd Qu.:0.4938   3rd Qu.:0.12904   3rd Qu.:0.19588
##  Max.   :1.00000   Max.   :1.0000   Max.   :1.00000   Max.   :1.00000
##       wind            precip          predays
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.4030   1st Qu.:0.4533   1st Qu.:0.5154
##  Median :0.4925   Median :0.6008   Median :0.6077
##  Mean   :0.5140   Mean   :0.5634   Mean   :0.5992
##  3rd Qu.:0.6866   3rd Qu.:0.6836   3rd Qu.:0.7077
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
```

```
## compute distance matrix
poldist <- dist(xs)
## reduce to 2 dimensions
(pol.mds <- cmdscale(poldist, k = 2, eig = TRUE))
```

```
## $points
##                       [,1]         [,2]
## Albany          0.140558172 -0.046859954
## Albuquerque    -0.364824787 -0.636602091
## Atlanta        -0.155922591  0.244511276
## Baltimore       0.153189990  0.067519907
## Buffalo         0.256244063  0.003022604
## Charleston     -0.128730958  0.215783429
## Chicago         1.197000315  0.009638168
## Cincinnati     -0.084166097  0.106828800
## Cleveland       0.531787447  0.056305378
## Columbus        0.025412911  0.033574934
## Dallas         -0.258008194 -0.062640448
## Denver         -0.110682033 -0.510378502
## Des Moines     -0.007603614 -0.244344703
## Detroit         0.341537781 -0.105917971
## Hartford        0.206766531  0.105259858
## Houston        -0.188167760  0.243707765
## Indianapolis    0.069589745  0.010565926
## Jacksonville   -0.349520267  0.412490203
## Kansas City    -0.106424371 -0.085118726
## Little Rock    -0.355970056  0.194004542
## Louisville     -0.046780470  0.144850917
## Memphis        -0.249259311  0.208737990
## Miami          -0.449823739  0.604996816
## Milwaukee       0.217298744 -0.249612250
## Minneapolis     0.326439578 -0.242858309
## Nashville      -0.215002650  0.211835269
```

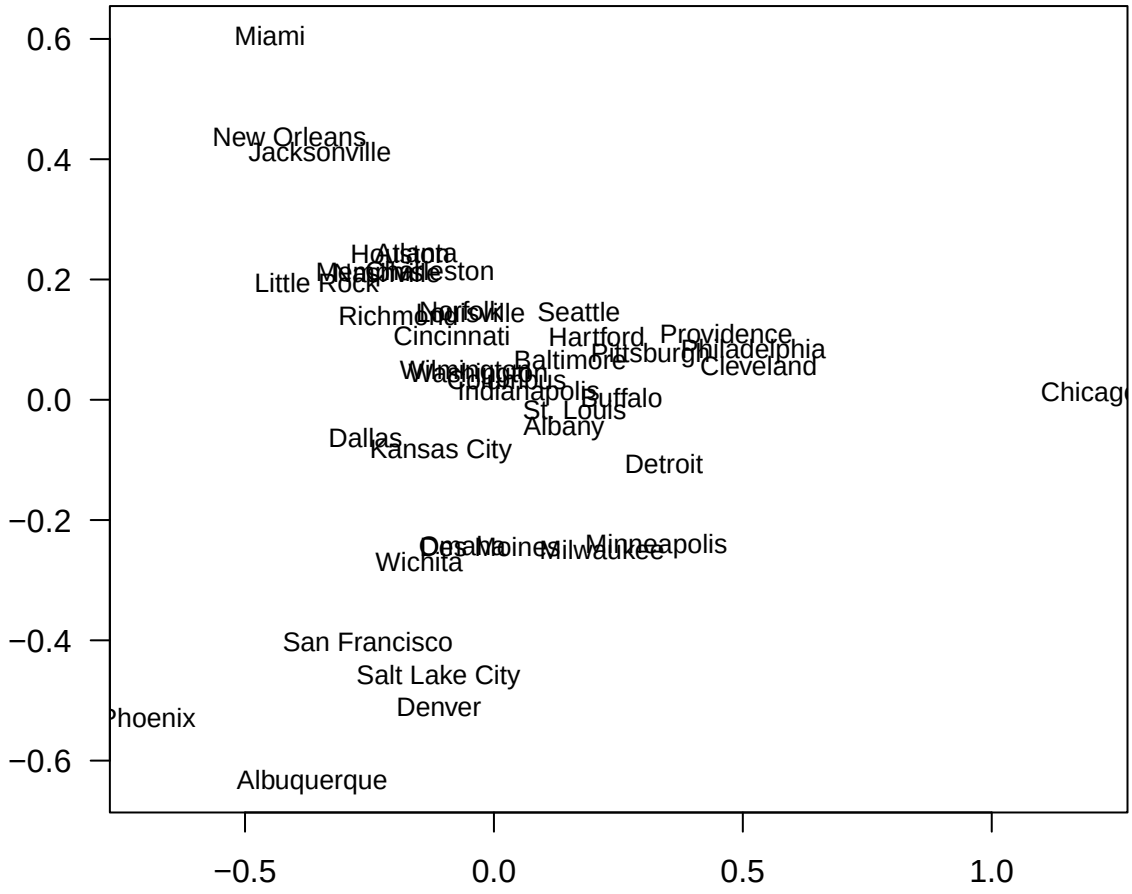
```
## New Orleans    -0.410715158  0.438263300
## Norfolk        -0.066285208  0.149134571
## Omaha          -0.063335982 -0.241936316
## Philadelphia    0.521031706  0.081089446
## Phoenix        -0.695773353 -0.527859295
## Pittsburgh      0.314965899  0.074640031
## Providence      0.466505620  0.110503750
## Richmond       -0.191967563  0.140461889
## Salt Lake City -0.111111665 -0.461383196
## San Francisco  -0.253430076 -0.401897024
## Seattle         0.170829143  0.147411289
## St. Louis       0.162208664 -0.016576959
## Washington     -0.031338057  0.041417952
## Wichita        -0.149744969 -0.268806546
## Wilmington     -0.056777379  0.046236280
##
## $eig
##  [1]  4.456648e+00  2.819944e+00  2.256196e+00  1.651762e+00  6.199354e-01
##  [6]  1.904906e-01  3.068220e-02  1.558353e-15  9.406328e-16  2.494225e-16
## [11]  1.736021e-16  1.471280e-16  1.356518e-16  8.017147e-17  7.511957e-17
## [16]  6.686099e-17  5.684599e-17  5.034791e-17  4.025565e-17  3.312471e-17
## [21]  2.974204e-17  1.555983e-17  1.132251e-17  3.668800e-18 -5.206488e-18
## [26] -8.948794e-18 -9.519928e-18 -1.506805e-17 -1.853275e-17 -2.314710e-17
## [31] -2.858271e-17 -3.093804e-17 -3.151435e-17 -3.396470e-17 -7.209856e-17
## [36] -7.714641e-17 -1.524915e-16 -2.390840e-16 -2.833661e-16 -3.238640e-16
## [41] -1.263609e-15
##
## $x
## NULL
##
## $ac
## [1] 0
##
## $GOF
## [1] 0.6050889 0.6050889
```

```r
## reduce to 3 dimensions
(pol.mds3 <- cmdscale(poldist, k = 3, eig = TRUE))
```

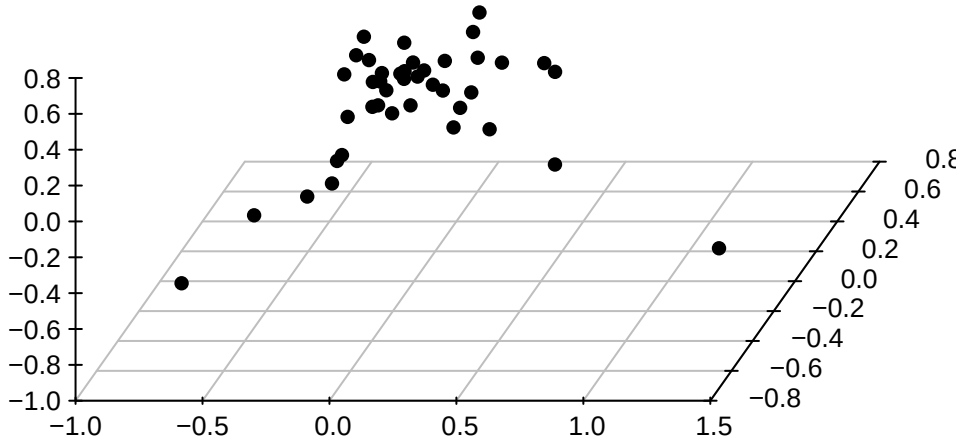
```
## $points
##                       [,1]         [,2]         [,3]
## Albany          0.140558172 -0.046859954  0.267632311
## Albuquerque    -0.364824787 -0.636602091 -0.102087912
## Atlanta        -0.155922591  0.244511276 -0.046494117
## Baltimore       0.153189990  0.067519907 -0.089541686
## Buffalo         0.256244063  0.003022604  0.495974986
## Charleston     -0.128730958  0.215783429  0.150107702
## Chicago         1.197000315  0.009638168 -0.824422653
## Cincinnati     -0.084166097  0.106828800  0.039366111
## Cleveland       0.531787447  0.056305378  0.120608309
## Columbus        0.025412911  0.033574934  0.147295326
## Dallas         -0.258008194 -0.062640448 -0.244388882
## Denver         -0.110682033 -0.510378502 -0.029698665
## Des Moines     -0.007603614 -0.244344703  0.268379415
## Detroit         0.341537781 -0.105917971 -0.064362771
```

```
## Hartford        0.206766531  0.105259858  0.158528863
## Houston        -0.188167760  0.243707765 -0.266857401
## Indianapolis    0.069589745  0.010565926  0.086925283
## Jacksonville   -0.349520267  0.412490203 -0.110633886
## Kansas City    -0.106424371 -0.085118726  0.051579961
## Little Rock    -0.355970056  0.194004542 -0.008211894
## Louisville     -0.046780470  0.144850917  0.020360112
## Memphis        -0.249259311  0.208737990 -0.062727649
## Miami          -0.449823739  0.604996816 -0.140722158
## Milwaukee       0.217298744 -0.249612250  0.271313047
## Minneapolis     0.326439578 -0.242858309  0.255203375
## Nashville      -0.215002650  0.211835269 -0.015978258
## New Orleans    -0.410715158  0.438263300 -0.105127995
## Norfolk        -0.066285208  0.149134571  0.095300814
## Omaha          -0.063335982 -0.241936316  0.174099908
## Philadelphia    0.521031706  0.081089446 -0.416784375
## Phoenix        -0.695773353 -0.527859295 -0.571707666
## Pittsburgh      0.314965899  0.074640031  0.156669771
## Providence      0.466505620  0.110503750  0.124107741
## Richmond       -0.191967563  0.140461889 -0.002992969
## Salt Lake City -0.111111665 -0.461383196  0.055129138
## San Francisco  -0.253430076 -0.401897024 -0.193036023
## Seattle         0.170829143  0.147411289  0.266888111
## St. Louis       0.162208664 -0.016576959 -0.128585601
## Washington     -0.031338057  0.041417952 -0.053942965
## Wichita        -0.149744969 -0.268806546  0.140296711
## Wilmington     -0.056777379  0.046236280  0.132538529
##
## $eig
##  [1]  4.456648e+00  2.819944e+00  2.256196e+00  1.651762e+00  6.199354e-01
##  [6]  1.904906e-01  3.068220e-02  1.558353e-15  9.406328e-16  2.494225e-16
## [11]  1.736021e-16  1.471280e-16  1.356518e-16  8.017147e-17  7.511957e-17
## [16]  6.686099e-17  5.684599e-17  5.034791e-17  4.025565e-17  3.312471e-17
## [21]  2.974204e-17  1.555983e-17  1.132251e-17  3.668800e-18 -5.206488e-18
## [26] -8.948794e-18 -9.519928e-18 -1.506805e-17 -1.853275e-17 -2.314710e-17
## [31] -2.858271e-17 -3.093804e-17 -3.151435e-17 -3.396470e-17 -7.209856e-17
## [36] -7.714641e-17 -1.524915e-16 -2.390840e-16 -2.833661e-16 -3.238640e-16
## [41] -1.263609e-15
##
## $x
## NULL
##
## $ac
## [1] 0
##
## $GOF
## [1] 0.792704 0.792704
```

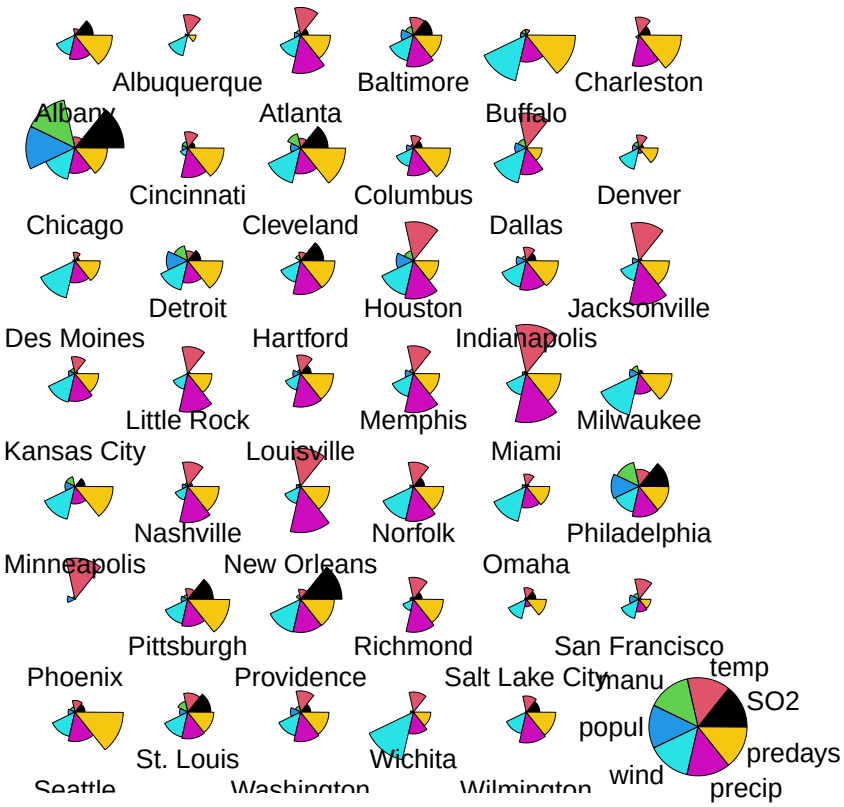
```r
## plot
par(las = 1, mgp = c(2, 1, 0), mar = c(3, 3, 1, 0.5))
x <- pol.mds$points
plot(x[, 1], x[, 2], type = "n", xlab = "", ylab = "")
text(x[, 1], x[, 2], labels = rownames(x), cex = 0.8)
```




```
library(scatterplot3d)
par(las = 1)
scatterplot3d(pol.mds3$points, pch = 16, angle = 75,
              box = F, xlab = "", ylab = "",
              zlab = "")
```




```
stars(xs, draw.segments = TRUE, key.loc = c(15, 2))
```



Non-Metric Multidimensional Scaling

House of Representatives Voting Data

```
data("voting", package = "HSAUR2")
voting
```

```
##                     Hunt(R) Sandman(R) Howard(D) Thompson(D) Freylinghuysen(R)
## Hunt(R)                   0          8        15           15                 10
## Sandman(R)                8          0        17           12                 13
## Howard(D)                15         17         0            9                 16
## Thompson(D)              15         12         9            0                 14
## Freylinghuysen(R)        10         13        16           14                  0
## Forsythe(R)               9         13        12           12                  8
## Widnall(R)                7         12        15           13                  9
## Roe(D)                   15         16         5           10                 13
## Heltoski(D)              16         17         5            8                 14
## Rodino(D)                14         15         6            8                 12
## Minish(D)                15         16         5            8                 12
## Rinaldo(R)               16         17         4            6                 12
## Maraziti(R)               7         13        11           15                 10
## Daniels(D)               11         12        10           10                 11
## Patten(D)                13         16         7            7                 11
##                     Forsythe(R) Widnall(R) Roe(D) Heltoski(D) Rodino(D) Minish(D)
## Hunt(R)                       9          7     15           16        14        15
## Sandman(R)                   13         12     16           17        15        16
## Howard(D)                    12         15      5            5         6         5
```

```
## Thompson(D)                12         13     10          8         8         8
## Freylinghuysen(R)           8          9     13         14        12        12
## Forsythe(R)                 0          7     12         11        10         9
## Widnall(R)                  7          0     17         16        15        14
## Roe(D)                     12         17      0          4         5         5
## Heltoski(D)                11         16      4          0         3         2
## Rodino(D)                  10         15      5          3         0         1
## Minish(D)                   9         14      5          2         1         0
## Rinaldo(R)                 10         15      3          1         2         1
## Maraziti(R)                 6         10     12         13        11        12
## Daniels(D)                  6         11      7          7         4         5
## Patten(D)                  10         13      6          5         6         5
##                   Rinaldo(R) Maraziti(R) Daniels(D) Patten(D)
## Hunt(R)                   16          7         11        13
## Sandman(R)                17         13         12        16
## Howard(D)                  4         11         10         7
## Thompson(D)                6         15         10         7
## Freylinghuysen(R)         12         10         11        11
## Forsythe(R)               10          6          6        10
## Widnall(R)                15         10         11        13
## Roe(D)                     3         12          7         6
## Heltoski(D)                1         13          7         5
## Rodino(D)                  2         11          4         6
## Minish(D)                  1         12          5         5
## Rinaldo(R)                 0         12          6         4
## Maraziti(R)               12          0          9        13
## Daniels(D)                 6          9          0         9
## Patten(D)                  4         13          9         0
```

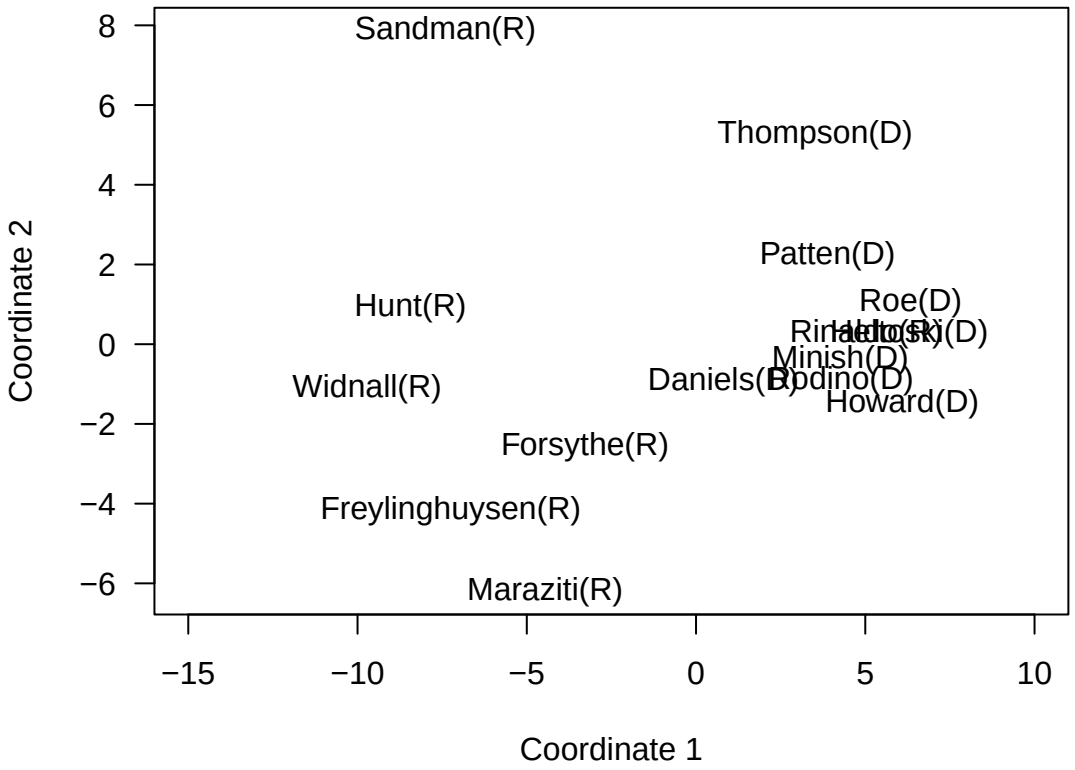
```
library(MASS)
voting_mds <- isoMDS(voting, k = 2)
```

```
## initial  value 15.268246
## iter   5 value 10.264075
## final  value 9.879047
## converged
```

```
str(voting_mds)
```

```
## List of 2
##  $ points: num [1:15, 1:2] -8.44 -7.41 6.09 3.52 -7.25 ...
##   ..- attr(*, "dimnames")=List of 2
##   .. ..$ : chr [1:15] "Hunt(R)" "Sandman(R)" "Howard(D)" "Thompson(D)" ...
##   .. ..$ : NULL
##  $ stress: num 9.88
```

```
par(las = 1)
plot(voting_mds$points, type = "n", xlim = c(-15, 10),
     xlab = "Coordinate 1", ylab = "Coordinate 2")
text(voting_mds$points, labels = rownames(voting_mds$points))
```


Sandman(R)
Thompson(D)
Patten(D)
Roe(D)
Hunt(R)
Widnall(R)
Daniels(D)
Howard(D)
Forsythe(R)
Freylinghuysen(R)
Maraziti(R)
Coordinate 1
Coordinate 2
8
6
4
2
0
−2
−4
−6
−15
−10
−5
0
5
10