Lecture 2

Data Summary/Visualization I

Text: Chapter 2 & Chapter 3

STAT 8010 Statistical Methods I
August 25, 2020

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Agenda

1. Sampling Techniques
2. Summarizing Categorical Data
3. Summarizing Numerical Data

Last Lecture

- Stating the problem, identifying the variable(s) of interest, and gathering data
  - Types of variables and datasets
  - Observational vs. Experimental Studies
  - Methods of sampling
- Summarizing the data
- Analyzing the data
- Reporting and interpreting the results

Today’s Lecture

- Stating the problem, identifying the variable(s) of interest, and gathering data
  - Types of variables and datasets
  - Observational vs. Experimental Studies
  - Sampling Techniques
- Summarizing the data
- Analyzing the data
- Reporting and interpreting the results

Collecting Data: Statistical Sampling

Statistical sampling is the procedure to select a subset from a statistical **population** that is representative of the population. There are several types of sampling:

- Simple random sampling (SRS): a sample selected such that each element in the population has the same probability of being selected

Simple random sample

Collecting Data: Statistical Sampling

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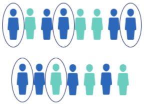
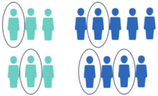
- Simple random sampling (SRS): a sample selected such that each element in the population has the same probability of being selected

Simple random sample



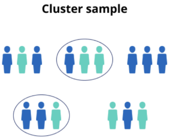
- Stratified sample: elements in the population are first divided into groups and a simple random sample is then taken from each group

Stratified sample



Sampling cont'd

- Cluster sampling: the elements in the population are first divided into separate groups called clusters and then a simple random sample of the clusters is taken that all elements in a selected cluster are part of a sample



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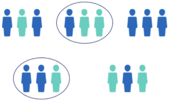
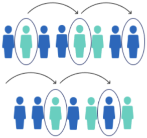
Cluster sample



- Systematic sampling: randomly select one of the first k elements from the population and then every k_{th} element thereafter is picked

Systematic sample



Sampling cont'd

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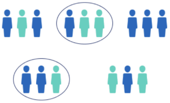
Cluster sample



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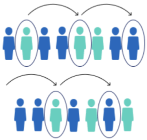
Systematic sample



- Convenience sampling: elements selected from the population on the basis of convenience

What type of sampling was used?

1. A researcher randomly chooses houses in a town. Once a particular house is chosen everyone living in the house is surveyed

2. A school principal decides to performs an exit interview with every 14^{th} name from a list of graduating seniors

3. A biologist knows that 40% of bats are male and that 60% are female so she randomly selects 20 males and randomly selects 30 females to be in her sample

4. A graduate student wants to do a study on why people like bluegrass music and uses the people she meets at the next show she attends as her sample

5. To get an idea of the average weight of his cattle, a rancher randomly chooses to weigh 25 from his list of the animals

Summarizing Categorical Variables

Example: Sport Injuries

The paper *"Profile of sport/leisure injuries treated at emergency rooms of urban hospitals."* by Pelletier et al. 1991 examined the nature and number of sport/leisure injuries treated in hospital emergency rooms in a large metropolitan city. They classified non-contact sports injuries by sport, resulting in the following data set:

Sport
Soccer
Basketball
Others
Basketball
Touch Football
Others
Touch Football
Volleyball
Baseball/softball
⋮

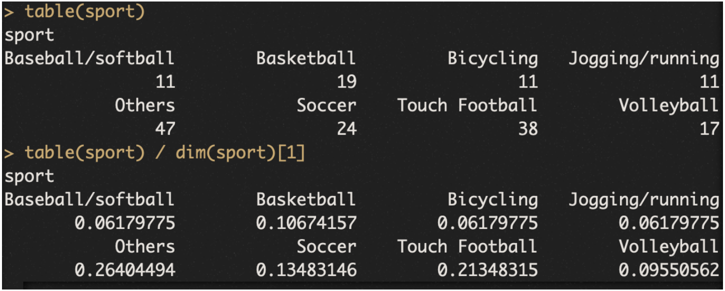
Question: How to summarize this data set?

Frequency Table

- A frequency table for **categorical data** is a table that displays the possible categories along with the associated **frequencies** or **relative frequencies**
- The frequency for a particular category is the number of times the category appears in the data set
- The relative frequency for a particular category is the fraction or proportion of the time that the category appears in the data set. It is calculated as:

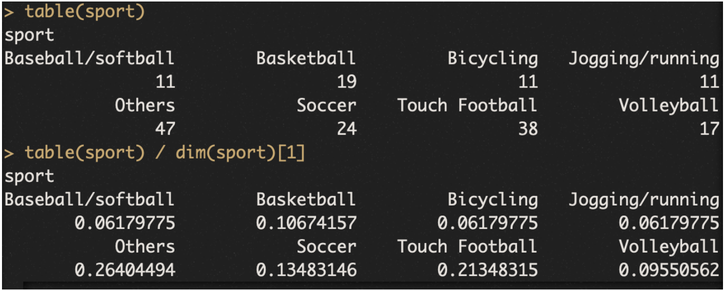
relative frequency =

Frequencies and Relative Frequencies



```
> table(sport)
sport
Baseball/softball        Basketball         Bicycling   Jogging/running
               11                19                11                11
           Others            Soccer    Touch Football        Volleyball
               47                24                38                17
> table(sport) / dim(sport)[1]
sport
Baseball/softball        Basketball         Bicycling   Jogging/running
       0.06179775        0.10674157        0.06179775        0.06179775
           Others            Soccer    Touch Football        Volleyball
       0.26404494        0.13483146        0.21348315        0.09550562
```

Frequencies and Relative Frequencies



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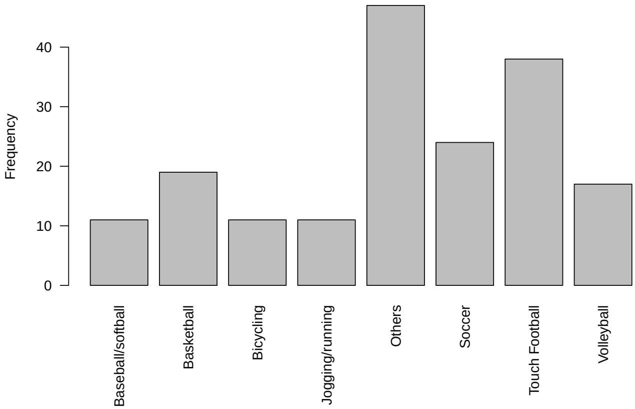
```

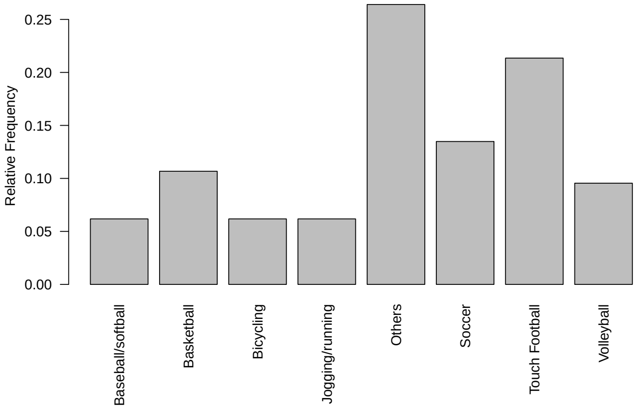
How could we visualize these information?
⇒ Making a bar chart and/or a pie chart

Bar Charts

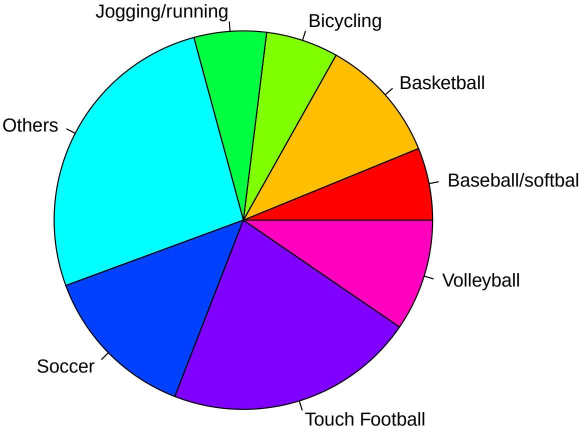
A bar chart draws a bar with a height proportional to the count in the table:



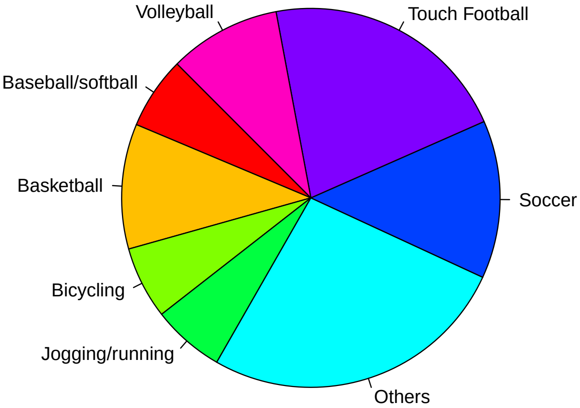
Bar Charts cont'd



Pie Charts


Jogging/running
Bicycling
Basketball
Baseball/softbal
Volleyball
Touch Football
Soccer
Others

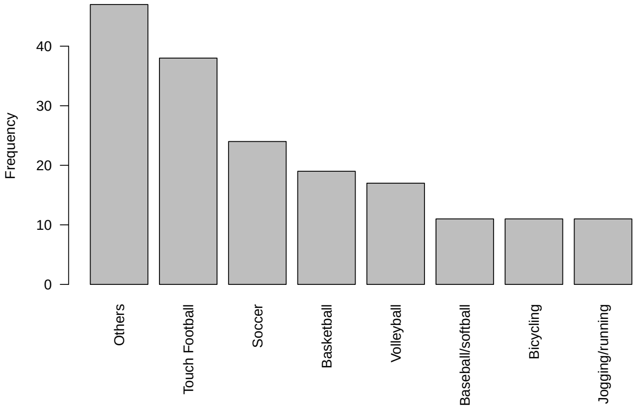
Pie Charts cont'd



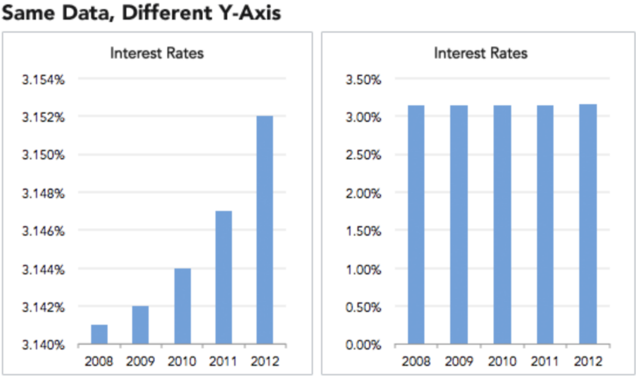
Bar Charts vs. Pie Charts

Discussion: Which one you prefer to visualize categorical variables. Why?

A Good Bar Chart



A (Potential) Misleading Bar Chart



Example: O'Hare Airport Flight Data

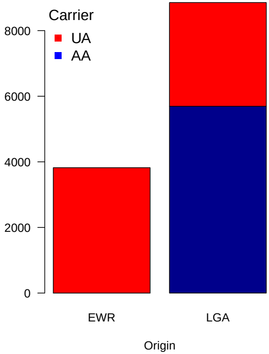
```
  carrier origin
1      UA    EWR
2      AA    LGA
3      AA    LGA
4      AA    LGA
5      UA    LGA
6      UA    EWR
```

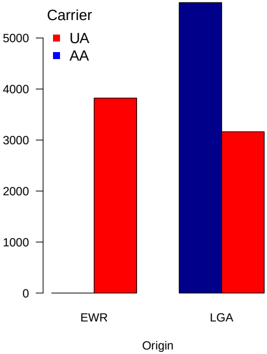
In this example, we have two categorical variables, `carrier` and `origin`, respectively. How to summarize/visualize this dataset?

ORD Flight Data Cont’d

```
    EWR  LGA
AA    0 5694
UA 3822 3162
```

```
    EWR  LGA
AA 0.00 0.45
UA 0.30 0.25
```





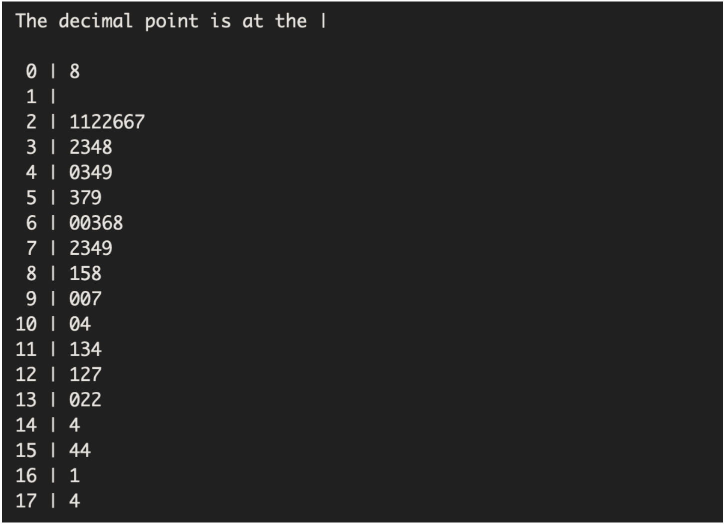
Summarizing Numerical Variables

Example: Murder arrests (per 100,000) in US States in 1973

Data: 13.2, 10.0, 8.1, 8.8, 9.0, 7.9, 3.3, 5.9, 15.4, 17.4, 5.3, 2.6, 10.4, 7.2, 2.2, 6.0, 9.7, 15.4, 2.1, 11.3, 4.4, 12.1, 2.7, 16.1, 9.0, 6.0, 4.3, 12.2, 2.1, 7.4, 11.4, 11.1, 13.0, 0.8, 7.3, 6.6, 4.9, 6.3, 3.4, 14.4, 3.8, 13.2, 12.7, 3.2, 2.2, 8.5, 4.0, 5.7, 2.6, 6.8.

Question: How to graphically summarize this data set?

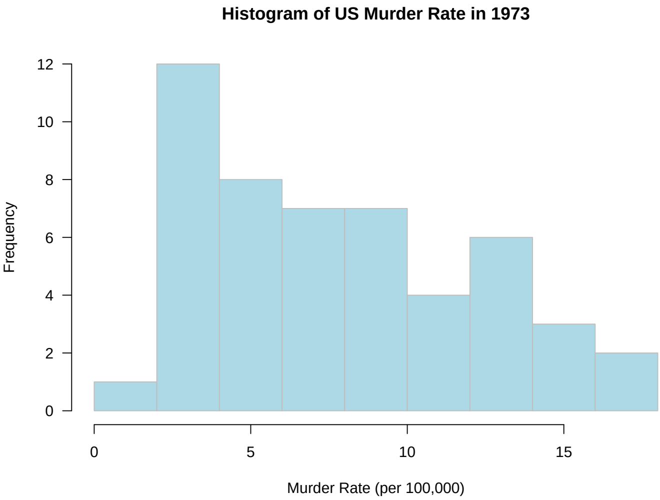
Stem-and-Leaf Plot



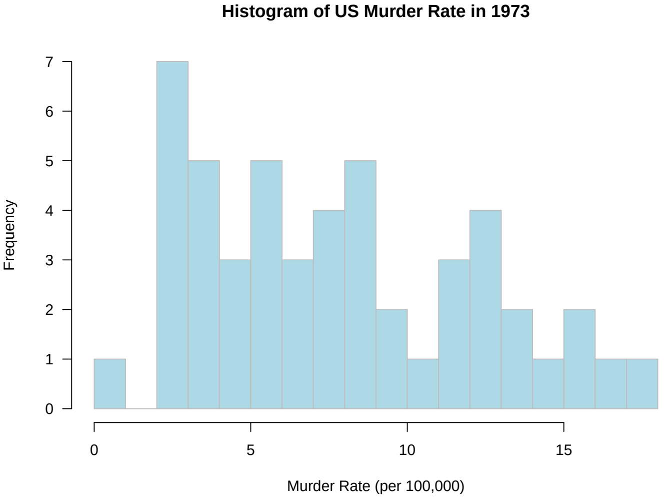
```
The decimal point is at the |

  0 | 8
  1 |
  2 | 1122667
  3 | 2348
  4 | 0349
  5 | 379
  6 | 00368
  7 | 2349
  8 | 158
  9 | 007
 10 | 04
 11 | 134
 12 | 127
 13 | 022
 14 | 4
 15 | 44
 16 | 1
 17 | 4
```

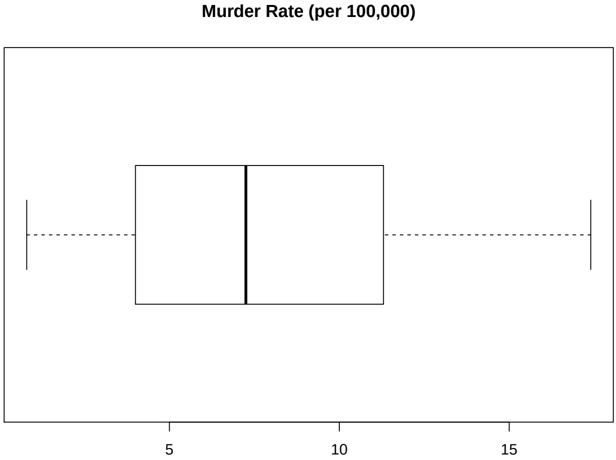
Histogram


Histogram of US Murder Rate in 1973
Frequency
0
2
4
6
8
10
12
0
5
10
15
Murder Rate (per 100,000)

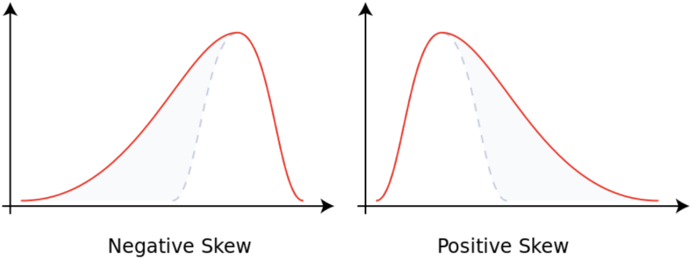
Histogram


Histogram of US Murder Rate in 1973
Frequency
0
1
2
3
4
5
6
7
0
5
10
15
Murder Rate (per 100,000)

Box-and-Whisker Plot



Shape of Distributions



Source: Skewness - Wikipedia

In the rest of the class, we will talk about how to summarize a numerical variable in terms of its center and spread

Measures of Center

- A **measure of center** attempts to report a “typical" value for the variable
- When a measure of center is calculated with **sample data** it is a **statistic**
- When a measure of center is calculated with popular (e.g., census data) it is a **parameter**
- **Measures:** Mean, Median, Mode

Mean

- The population mean, denoted by μ_X, is the sum of all the population values ($\{X_i, \cdots, X_N\}$) divided by the size of the population (N). That is,

$$\mu_X = \frac{\sum_{i=1}^{N} X_i}{N}$$

- The sample mean, denoted by $\bar{X}$ is the sum of all the sample values ($\{X_1, \cdots, X_n\}$) divided by the sample size (n). That is,

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Median

The median is the value separating the higher half from the lower half of a data sample

How to compute the median: Order the n observations in a data set from smallest to largest, then

$$\text{Median} = \begin{cases} \text{the single middle value,} & \text{n odd} \\ \text{the average of the middle two values,} & \text{n even} \end{cases}$$

Mode

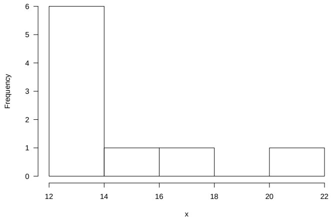
The mode is the value of the observation that appears most frequently

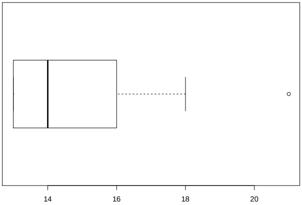
How to compute the mode(s): Order the observations in a data set from smallest to largest, then find the number that is repeated more often than any other

Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

- Plot this “data set" and describe the shape of the distribution





Example cont'd

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

- Find the sample mean

$$\bar{X} = \sum_{i=1}^{9} \frac{13+18+13+14+13+16+14+21+13}{9} = 15$$

- Find the sample median
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21

Example cont'd

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

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- Find the sample median
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21
  2. Compute the sample size n and identify (or compute) the median value

Example cont'd

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

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- Find the sample median
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21
  2. Compute the sample size n and identify (or compute) the median value
  3. $n = 9 \Rightarrow$ the median is the 5th number, which is 14

Example cont'd

- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21

Example cont'd

- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21

Example cont'd

- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21
  2. We have 3 13 and 2 14 ⇒ 13 is the mode

Example: Resistant (Robust) Statistics

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

- Find the sample mean

$$\bar{X} = \sum_{i=1}^{9} \frac{13 + 18 + 13 + 14 + 13 + 16 + 14 + 210 + 13}{9} = 36$$

- Find the sample median
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210

Example: Resistant (Robust) Statistics

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

- Find the sample mean

$$\bar{X} = \sum_{i=1}^{9} \frac{13 + 18 + 13 + 14 + 13 + 16 + 14 + 210 + 13}{9} = 36$$

- Find the sample median
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210

Example: Resistant (Robust) Statistics

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

- Find the sample mean

$$\bar{X} = \sum_{i=1}^{9} \frac{13+18+13+14+13+16+14+210+13}{9} = 36$$

- Find the sample median
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210
  2. Compute the sample size n and identify (or compute) the median value

Example: Resistant (Robust) Statistics

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

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$$\bar{X} = \sum_{i=1}^{9} \frac{13 + 18 + 13 + 14 + 13 + 16 + 14 + 210 + 13}{9} = 36$$

- Find the sample median
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Example: Resistant (Robust) Statistics

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

- Find the sample mean

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- Find the sample median
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210
  2. Compute the sample size n and identify (or compute) the median value
  3. $n = 9 \Rightarrow$ the median is the 5th number, which is (still) 14

Example cont'd

- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210

Example cont'd

- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210

Example cont'd

- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210
  2. We have 3 13 and 2 14 ⇒ 13 is (still) the mode

Example cont'd

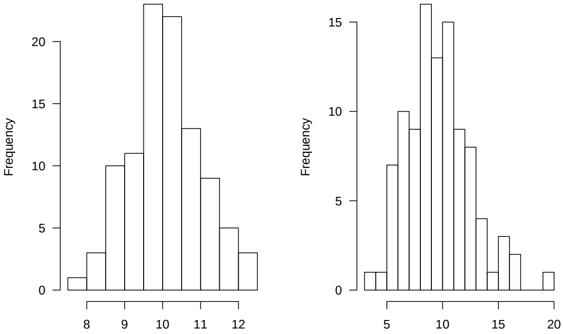
- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210
  2. We have 3 13 and 2 14 ⇒ 13 is (still) the mode

Example cont'd

- Find the mode
  1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 210
  2. We have 3 13 and 2 14 ⇒ 13 is (still) the mode

What is the take-home message?

Measures of Spread



- **Measures:** Range, Variance/Standard Deviation, Interquartile range (IQR)

Range

The range of a dataset is the difference between the largest and smallest values

$$\boxed{\text{Range} = \text{Largest Value} - \text{Smallest Value}}$$

- Compute the range of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13
- Compute the range of the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

Question: Is Range a robust statistic?

Standard Deviation/Variance

- The sample standard deviation (variance), denoted by s (s^2), is a measure of the amount of variation of data. s (s^2) can be used as the estimate of the population standard deviation (varaince), denoted by σ (σ^2)

- s is calculated in the following way:

  1. Calculate the sample mean $\bar{X}$
  2. Calculate the deviation (from the sample mean) for each observation (i.e., $X_i - \bar{X}, \quad i = 1, , \cdots, n$)
  3. Square each deviation and add them (i.e., $\sum_{i=1}^{n}(X_i - \bar{X})^2$)
  4. Divide by $n - 1$ and take the square root, that is,

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

Example

- Compute s of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

- Compute s of the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

Question: Is standard deviation a robust statistic?

Interquartile range (IQR)

- IQR $= Q_3 - Q_1$, where Q_1 is the Lower Quartile (the median of the lower half of the data) and Q_3 is the Upper Quartile (the median of the upper half of the data)
- Compute the IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13
- Compute the IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 210, 13

Question: Is IQR a robust statistic?

Summary

In this lecture, we learned

- Sampling Techniques
- Summarizing Categorical Data
- Summarizing Numerical Data

In next lecture we will learn

- How to construct a boxplot
- How to visualize numerical + categorical variables and numerical + numerical variables
- How to visualize time series, cross-sectional, and spatio-temporal Data sets