Fall 2019 Exam I

STAT 8020

September 27, 2019

Name:____________________________

Directions

1. Show your work on ALL questions (except those multiple choice questions). Unsupported work will NOT receive full credit.
2. Decimal answers should be exact, or to exactly 2 significant digits.
3. Please write legibly. If I cannot read your writing, NO credit will be given.
4. You are allowed the following aids:
   (a) a one-page A4 handwritten cheat sheet
   (b) A scientific Calculator
5. Turn off your cell phone before the exam begins.

Use your time wisely. Good Luck!!!

Problem	Points Possible	Points Earned
1	60	
2	20	
3	20	
Total	100	

Problem 1

A baseball fan would like to study the relationship between the annual salary `Salary` (in thousands of dollars) of major league players and the number of home runs during his career `CHmRun`. A simple linear regression is performed where `Salary` is the response. Use the R output below to answer the following questions: **(12 points for each question.)**

```
lm(formula = Salary ~ CHmRun)

Residuals:
    Min      1Q  Median      3Q     Max
-1427.7  -247.1  -109.3   169.2  1785.1

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 336.4512    31.0408  10.839   <2e-16 ***
CHmRun        2.8809     0.2891   9.964   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 384.7 on 261 degrees of freedom
  (59 observations deleted due to missingness)
Multiple R-squared:  0.2756, Adjusted R-squared:  0.2728
F-statistic: 99.27 on 1 and 261 DF,  p-value: < 2.2e-16
```

1. Write down the least squares regression line and compute the fitted value with `CHmRun` $= 100$.

   Let's use Y to denote the response (`Salary`) and X to denote the predictor (`CHmRun`). The regression line equation is

   $$\boxed{\hat{Y} = 336.4512 + 2.8809X}.$$

   The fitted value of the response given $X = 100$ is

   $$336.4512 + 2.8809 * 100 = \boxed{624.54k}$$

2. Construct the 95% confidence interval (using $t(0.975, df = 261) = 1.97$ and $\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2} = 1330.484$) for β_1.

The 95% CI of β_1 is $\hat{\beta}_1 \pm t(0.975, 261) \times \hat{\sigma}_{\hat{\beta}_1}$ where $\hat{\beta}_1 = 2.8809$ and $\hat{\sigma}_{\hat{\beta}_1} = 0.2891$ (can be found from the R output above). Therefore the 95% CI of β_1 is $[2.8809 - 1.97 \times 0.2891, 2.8809 + 1.97 \times 0.2891] = \boxed{[2.31, 3.45]}$

3. Test the following hypothesis: $H_0 : \beta_1 = 0$ vs. $H_a : \beta_1 \neq 0$ with $\alpha = 0.05$. State your conclusion in plain language in the present context.

Method I: Because the 95% CI for β_1 DOES NOT contain 0, we reject $H_0 : \beta_1 = 0$ at 0.05 significance level.

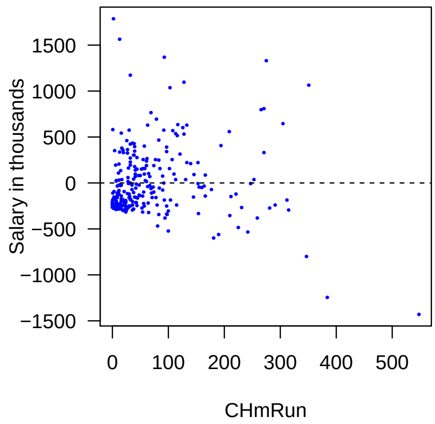
Method II: The P-value of the t-test for β_1 is less than 2×10^{-16} (again, can be found from the R output above), which is less than $= 0.05$. Therefore, we reject H_0, that is, we do have enough evidence to conclude that $\beta_1 \neq 0$ at 0.05 level.

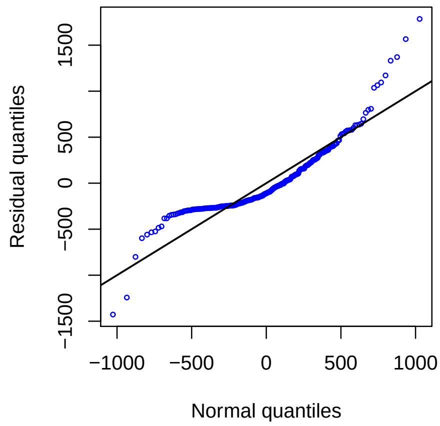
4. Fill in the missing values in the ANOVA table below and compute the R^2, the coefficient of determination.

Source	df	SS	MS	F
Model	1	SSR = 14692193	MSR = 14692193	F* = 99.27
Error	261	SSE = 38626920	MSE = 147995.86	
Total	262	SST = 53319113		

$R^2 = 14692193/53319113 = 0.28$

5. Do the residual plot and the Normal Q-Q plot below suggest any regression assumptions may be violated? Explain your answer.





Yes. The residual plot suggests the constant variance assumption for error may be violated. Moreover, the Normal QQ plot suggests normality assumption on the error distribution is probably not true.

Problem 2

A researcher performs a multiple linear regression, using the Longley's macroeconomic data set, to study the relationship between `Employed` (number of people employed) and `GNP.deflator`, `GNP`, `Unemployed`, `Armed.Forces`, `Population`, and `Year`. Use the R outputs below to answer the following questions:

Full model Fit:

```
lm(formula = Employed ~ ., data = longley)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.41011 -0.15767 -0.02816  0.10155  0.45539 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -3.482e+03  8.904e+02  -3.911 0.003560 ** 
GNP.deflator  1.506e-02  8.492e-02   0.177 0.863141    
GNP          -3.582e-02  3.349e-02  -1.070 0.312681    
Unemployed   -2.020e-02  4.884e-03  -4.136 0.002535 ** 
Armed.Forces -1.033e-02  2.143e-03  -4.822 0.000944 ***
Population   -5.110e-02  2.261e-01  -0.226 0.826212    
Year          1.829e+00  4.555e-01   4.016 0.003037 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3049 on 9 degrees of freedom
Multiple R-squared:  0.9955, Adjusted R-squared:  0.9925 
F-statistic: 330.3 on 6 and 9 DF,  p-value: 4.984e-10
```

VIF:

```
GNP.deflator          GNP   Unemployed Armed.Forces   Population 
   135.53244   1788.51348     33.61889      3.58893    399.15102 
        Year 
   758.98060 
```

1. **(10 points)** Explain why the full model is highly significant (overall F-test P-value $< 5 \times 10^{-10}$ and with a very high R^2) but still have very high p-values on some of the regressor's t tests? (Hint: Check the VIF values.)

Most of the predictors have fairly high VIF values indicates there is a high multicollearity and this leads to poorly estimated βs and inflated standard error.

2. **(10 points)** Perform a general linear test using the R output below:

```
## Analysis of Variance Table
##
## Model 1: Employed ~ GNP + Unemployed + Armed.Forces + Year
## Model 2: Employed ~ GNP.deflator + GNP + Unemployed + Armed.Forces + Populat
ion +
##     Year
##   Res.Df     RSS Df Sum of Sq      F Pr(>F)
## 1     11 0.85868
## 2      9 0.83642  2  0.022256 0.1197 0.8885
```

Full model: $\texttt{Employed} = \beta_0 + \beta_1\texttt{GNP.deflator} + \beta_2\texttt{GNP} + \beta_3\texttt{Unemployed} + \beta_4\texttt{Armed.Forces} + \beta_5\texttt{Population} + \beta_6\texttt{Year}$

Reduced model: $\texttt{Employed} = \beta_0 + \beta_2\texttt{GNP} + \beta_3\texttt{Unemployed} + \beta_4\texttt{Armed.Forces} + \beta_6\texttt{Year}$

The P-value of the general linear is 0.8885, which is greater than any reasonable α level. Therefore, we DO NOT have enough evidence to reject $H_0 : \beta_1 = \beta_5 = 0$.

Problem 3

The dean of a college in a University would like to monitor salary differences between male and female faculty members and she performed a multiple linear regression where the response variable `salary` is regressed on `sex` (male, female) and `yrs.service` (years of service). Use the R output below to answer the following question:

```
Call:
lm(formula = salary ~ sex * yrs.service, data = Salaries)

Residuals:
   Min      1Q Median     3Q    Max 
-80381 -20258  -3727  16353 102536 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)          82068.5     7568.7  10.843  < 2e-16 ***
sexMale              20128.6     7991.1   2.519  0.01217 *  
yrs.service           1637.3      523.0   3.130  0.00188 ** 
sexMale:yrs.service   -931.7      535.2  -1.741  0.08251 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 28420 on 393 degrees of freedom
Multiple R-squared:  0.1266,	Adjusted R-squared:  0.1199 
F-statistic: 18.98 on 3 and 393 DF,  p-value: 1.622e-11
```

1. **(20 points)** Write down the regression equation for male and female faculty, respectively.

Female: $\texttt{salary} = 82068.5 + 1637.3\texttt{yrs} + \varepsilon$

Male: $\texttt{salary} = (82068.5 + 20128.6) + (1637.3 - 931.7)\texttt{yrs}$

$$= 102197.1 + 705.6\texttt{yrs} + \varepsilon$$