STAT 8010 R Lab 5: Binomial, Hypergeometric, and Normal Distributions

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September 17, 2020

Contents

Binomial distribution

The probability mass function of a binomial random variable with parameters n, the number of trials, and p, the success probability of every trial is

$$p_X(x) = \binom{n}{x}(p)^x(1-p)^{n-x},\ x = 0, 1, \cdots, n$$

ESP example

To test for Extrasensory perception (ESP), we have 4 cards. They will be shuffled and one randomly selected each time, and you are to guess which card is selected. This is repeated 10 times. Suppose you do not have ESP. Let R be the number of times you guess a card correctly. What are the distribution and parameter(s) of R? What is the expected value of R? Furthermore, suppose that you get certified as having ESP if you score at least an 8 on the test. What is the probability that you get certified as having ESP?

```
n = 10; p = 1 / 4
choose(10, 8)
```

```
## [1] 45
```

```
#P(R=8)
choose(n, 8) * p^8 * (1 - p)^(n - 8)
```

```
## [1] 0.0003862381
```

```
dbinom(8, n, p)
```

```
## [1] 0.0003862381
```

```
sum(dbinom(8:10, n, p)) # P(R=8) + P(R=9) + P(R=10)
```

```
## [1] 0.000415802
```

```
1 - pbinom(7, n, p) # 1 - (P(R=0) + P(R=1) + ... +P(R=7))
```

```
## [1] 0.000415802
```

```
pbinom(7, n, p, lower.tail = F) # P(R=8) + P(R=9) + P(R=10)
```

```
## [1] 0.000415802
```

Coke blind taste test

What is the probability that X is at least 1?

```
n = 4; p = .95
1 - 0.05^4
```

```
## [1] 0.9999938
```

```
1 - dbinom(0, n, p)
```

```
## [1] 0.9999938
```

What is the probability that X is at most 3?

```
sum(dbinom(0:3, n, p))
```

```
## [1] 0.1854938
```

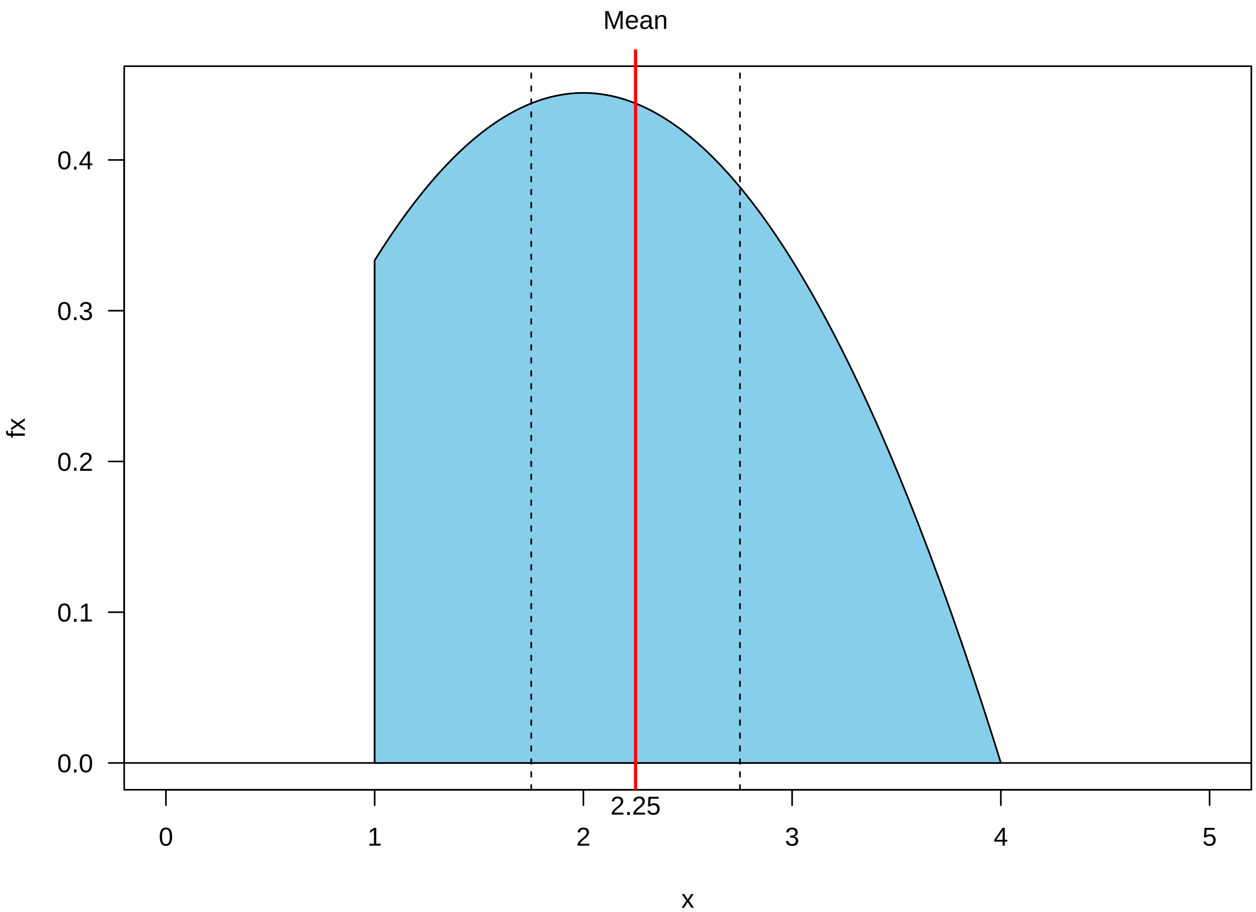
```
pbinom(3, n, p)
```

```
## [1] 0.1854938
```

General continuous random variable

```
x <- seq(1, 4, 0.01)
fx <- (4 * x - x^2) / 9

plot(x, fx, type = "l", las = 1, xlim = c(0, 5))
segments(1, 0, 1, 1 / 3)
abline(h = 0)
polygon(c(x, rev(x)), c(fx, rep(0, length(fx))), col = "skyblue")
abline(v = 2.25, lwd = 2, col = "red")
axis(3, at = 2.25, labels = "Mean", line = 0, col = "red",
     lwd = 2)
axis(1, at = 2.25, line = -1, col = "red")
abline(v = 1.75, lty = 2); abline(v = 2.75, lty = 2)
```



Normal distribution

1. $\Phi(0) = .50 \Rightarrow \}$ Mean and Median (50_{th} percentile) for standard normal are both 0
2. $\Phi(-z) = 1 - \Phi(z)\}$
3. $\mathbb{P}(Z > z) = 1 - \Phi(z) = \Phi(-z)\}$

```r
pnorm(0)
```

```
## [1] 0.5
```

```r
pnorm(-1)
```

```
## [1] 0.1586553
```

```r
1 - pnorm(1)
```

```
## [1] 0.1586553
```

```r
pnorm(1, lower.tail = F)
```

```
## [1] 0.1586553
```

```r
pnorm(-1.75)
```

```
## [1] 0.04005916
```

```r
pnorm(2) - pnorm(-2)
```

```
## [1] 0.9544997
```