DSA 8020 R Session 12: Time Series Analysis I

Whitney

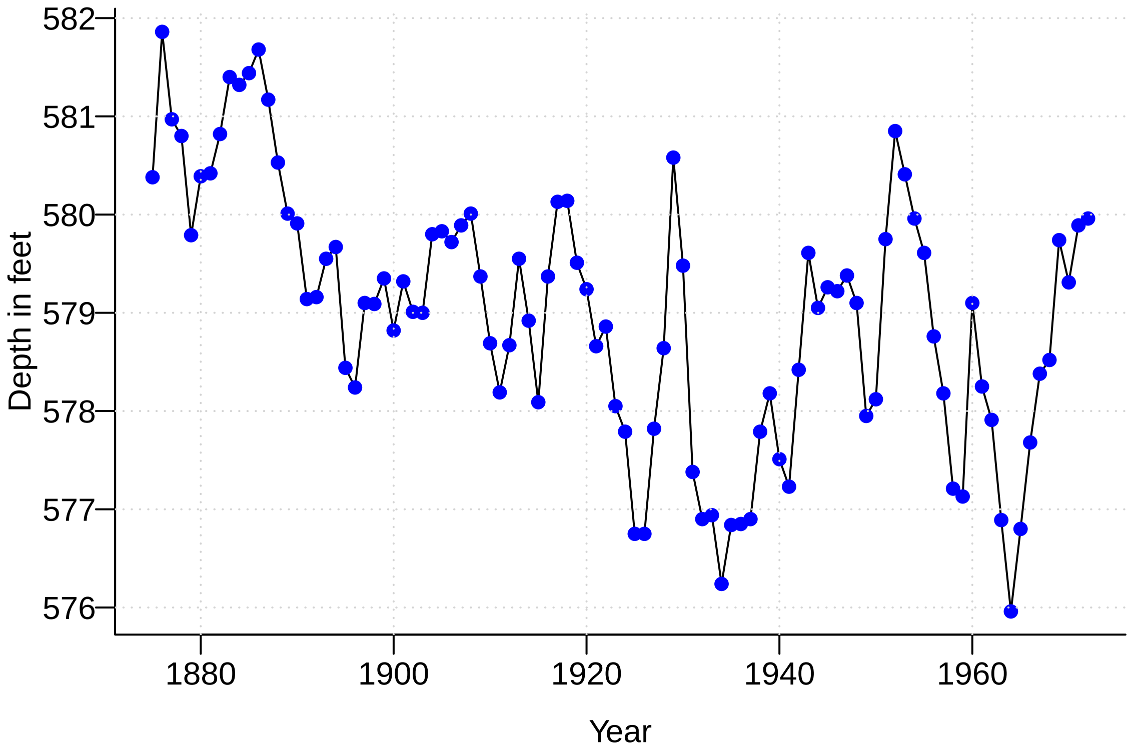
Contents

Time Series Data

Lake Huron Time Series

Annual measurements of the level of Lake Huron in feet

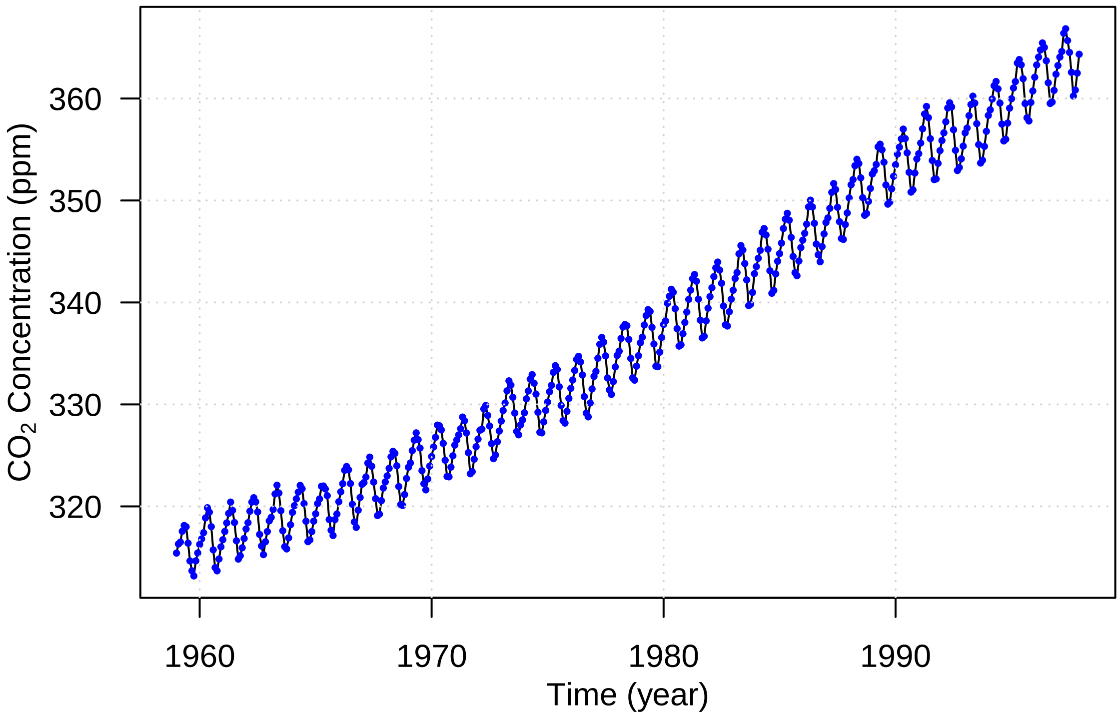
```r
par(mar = c(3.2, 3.2, 0.5, 0.5), mgp = c(2, 0.5, 0), bty = "L")
data(LakeHuron)
plot(LakeHuron, ylab = "Depth in feet", xlab = "Year", las = 1)
points(LakeHuron, pch = 16, col = "blue")
grid()
```



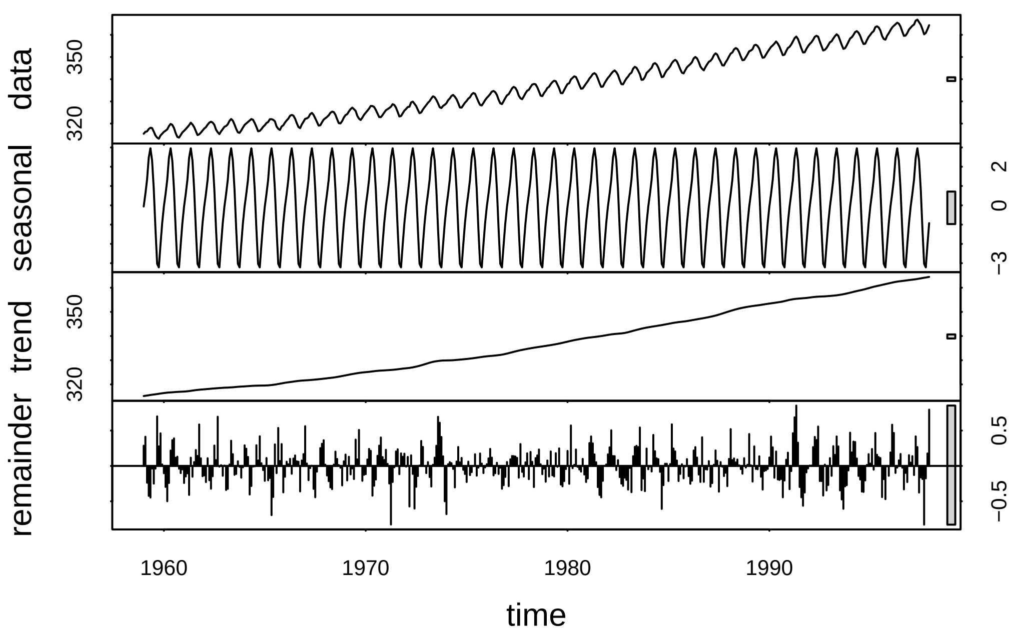
CO_2 **Concentration**

Atmospheric concentrations of CO2 are expressed in parts per million (ppm) and reported using the preliminary 1997 SIO manometric mole fraction scale.

```r
data(co2)
par(mar = c(3.8, 4, 0.8, 0.6))
plot(co2, las = 1, xlab = "", ylab = "")
points(co2, pch = 16, col = "blue", cex = 0.5)
mtext("Time (year)", side = 1, line = 2)
mtext(expression(paste("CO"[2], " Concentration (ppm)")), side = 2, line = 2.5)
grid()
```




```r
# Seasonal and Trend decomposition using Loess (STL)
par(mar = c(4, 3.6, 0.8, 0.6))
stl <- stl(co2, s.window = "periodic")
plot(stl, las = 1)
```



U.S. monthly unemployment rates

```r
library(quantmod)
getSymbols("UNRATE", src = "FRED")
```

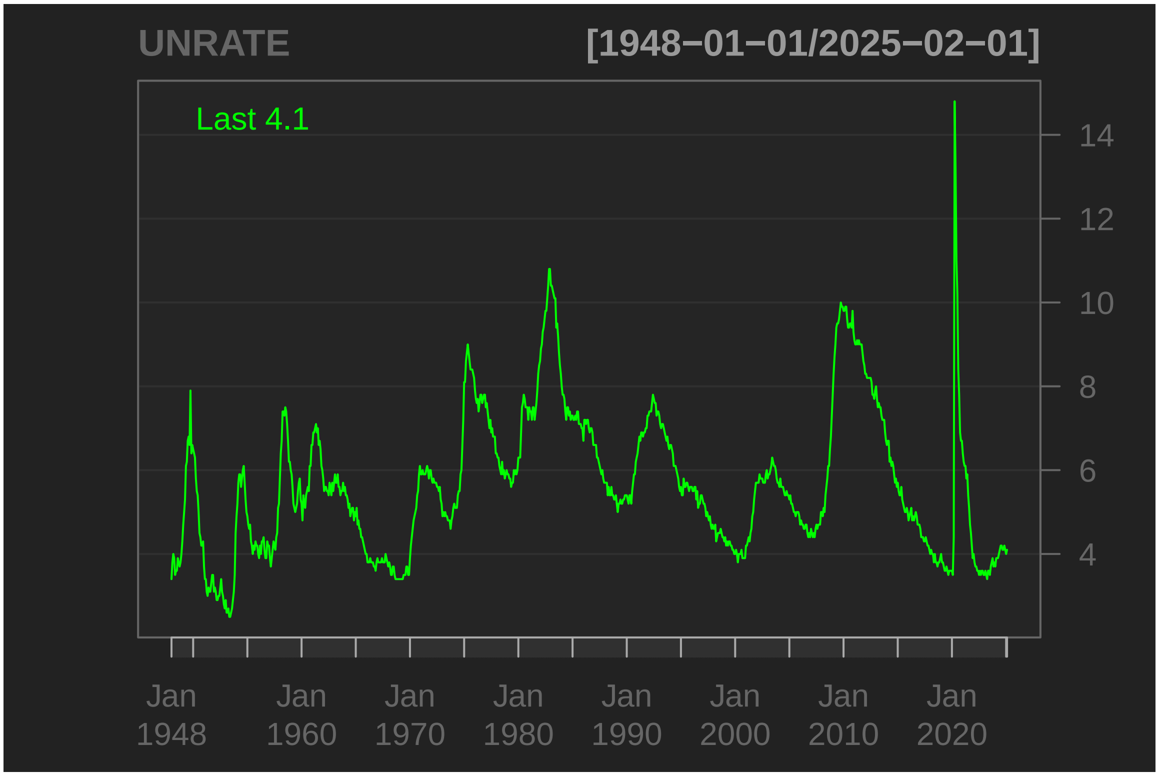
```
## [1] "UNRATE"
```

```r
head(UNRATE); tail(UNRATE)
```

```
##            UNRATE
## 1948-01-01    3.4
## 1948-02-01    3.8
## 1948-03-01    4.0
## 1948-04-01    3.9
## 1948-05-01    3.5
## 1948-06-01    3.6

##            UNRATE
## 2024-09-01    4.1
## 2024-10-01    4.1
## 2024-11-01    4.2
## 2024-12-01    4.1
## 2025-01-01    4.0
## 2025-02-01    4.1
```

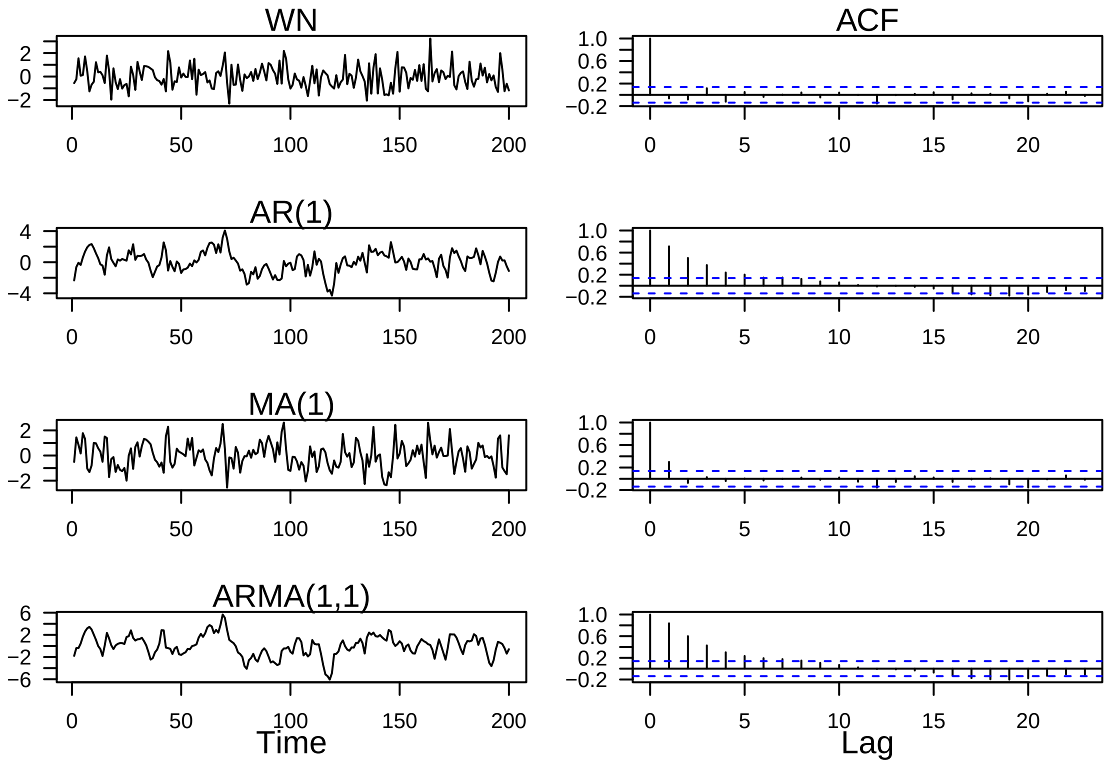
```r
chartSeries(UNRATE)
```



Time Series Models

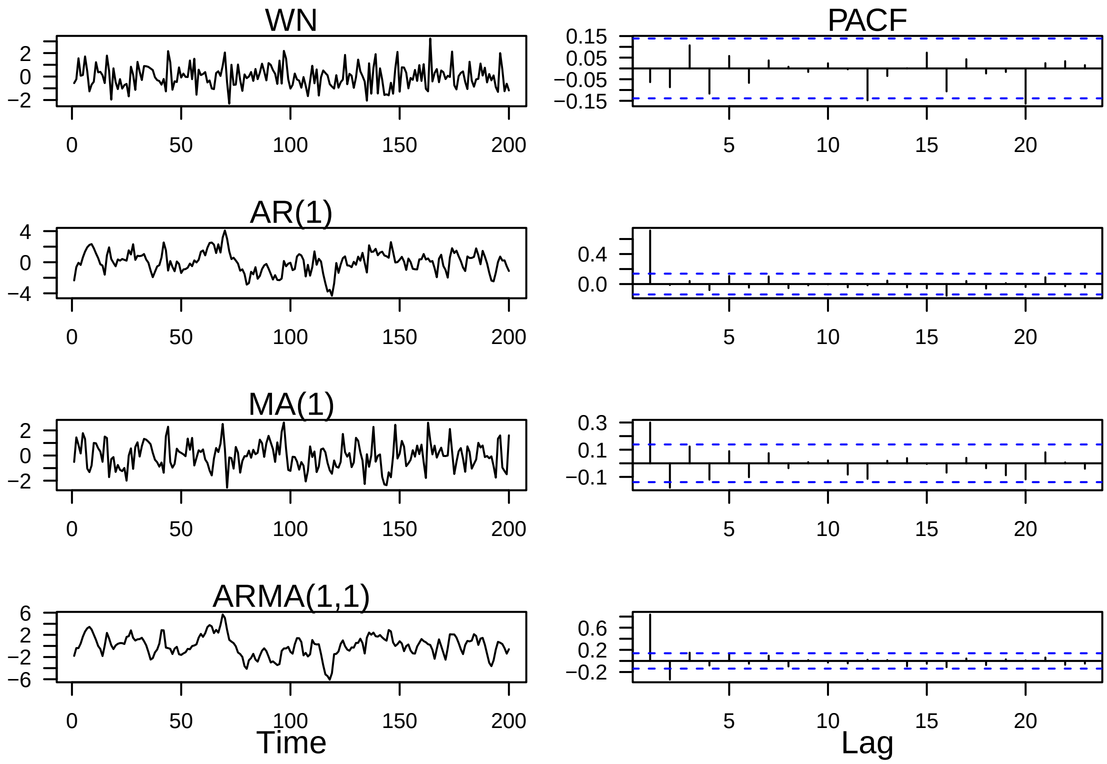
ARMA: Autocovariance Function (ACF)

```r
set.seed(123)
n = 200
WN <- rnorm(n)
par(mfrow = c(4, 2), mar = c(3.6, 3.6, 1.2, 0.6))
plot(1:n, WN, type = "l", las = 1, xlab = "", ylab = "")
mtext("WN")
acf(WN, xlab = "", ylab = "", main = "", las = 1)
mtext("ACF")
# AR(1) phi = 0.8
set.seed(123)
AR <- arima.sim(n = n, model = list(ar = 0.8))
plot(1:n, AR, type = "l", las = 1, xlab = "", ylab = "")
mtext("AR(1)")
acf(AR, xlab = "", ylab = "", main = "", las = 1)
# MA(1) theta = 0.5
set.seed(123)
MA <- arima.sim(n = n, model = list(ma = 0.5))
plot(1:n, MA, type = "l", las = 1, xlab = "", ylab = "")
mtext("MA(1)")
acf(MA, xlab = "", ylab = "", main = "", las = 1)
# ARMA(1, 1) phi = 0.8, theta = 0.5
set.seed(123)
ARMA <- arima.sim(n = n, model = list(ar = 0.8, ma = 0.5))
plot(1:n, ARMA, type = "l", las = 1, xlab = "", ylab = "")
mtext("ARMA(1,1)")
mtext("Time", side = 1, line = 2)
acf(ARMA, xlab = "", ylab = "", main = "", las = 1)
mtext("Lag", side = 1, line = 2)
```



ARMA: Partial Autocorrelation Function (PACF)

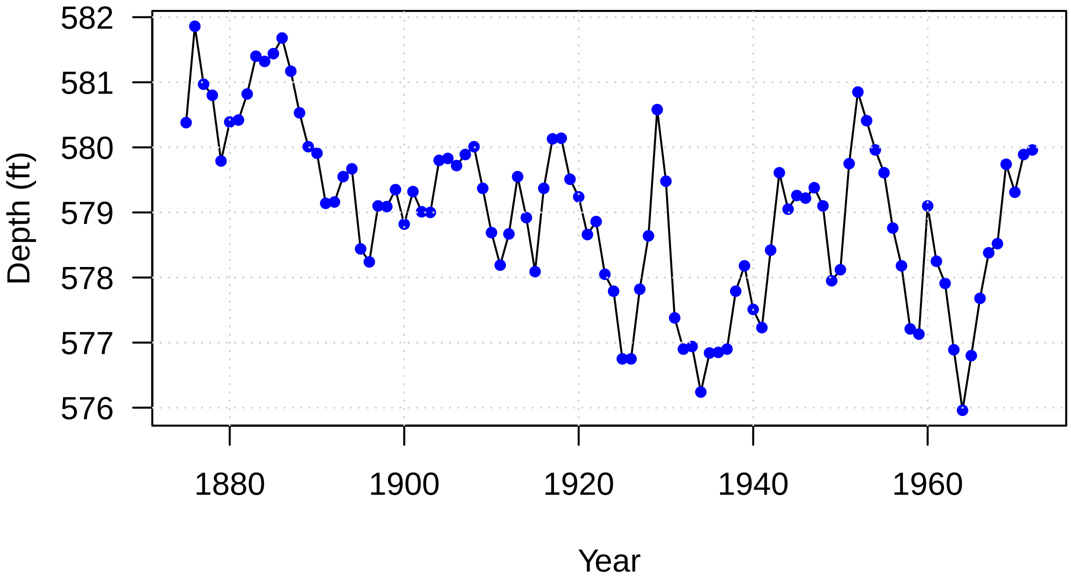
```
par(mfrow = c(4, 2), mar = c(3.6, 3.6, 1.2, 0.6))
plot(1:n, WN, type = "l", las = 1, xlab = "", ylab = "")
mtext("WN")
pacf(WN, xlab = "", ylab = "", main = "", las = 1)
mtext("PACF")
# AR(1) phi = 0.8
set.seed(123)
AR <- arima.sim(n = n, model = list(ar = 0.8))
plot(1:n, AR, type = "l", las = 1, xlab = "", ylab = "")
mtext("AR(1)")
pacf(AR, xlab = "", ylab = "", main = "", las = 1)
# MA(1) theta = 0.5
set.seed(123)
MA <- arima.sim(n = n, model = list(ma = 0.5))
plot(1:n, MA, type = "l", las = 1, xlab = "", ylab = "")
mtext("MA(1)")
pacf(MA, xlab = "", ylab = "", main = "", las = 1)
# ARMA(1, 1) phi = 0.8, theta = 0.5
set.seed(123)
ARMA <- arima.sim(n = n, model = list(ar = 0.8, ma = 0.5))
plot(1:n, ARMA, type = "l", las = 1, xlab = "", ylab = "")
mtext("ARMA(1,1)")
mtext("Time", side = 1, line = 2)
pacf(ARMA, xlab = "", ylab = "", main = "", las = 1)
mtext("Lag", side = 1, line = 2)
```



Lake Huron Case Study

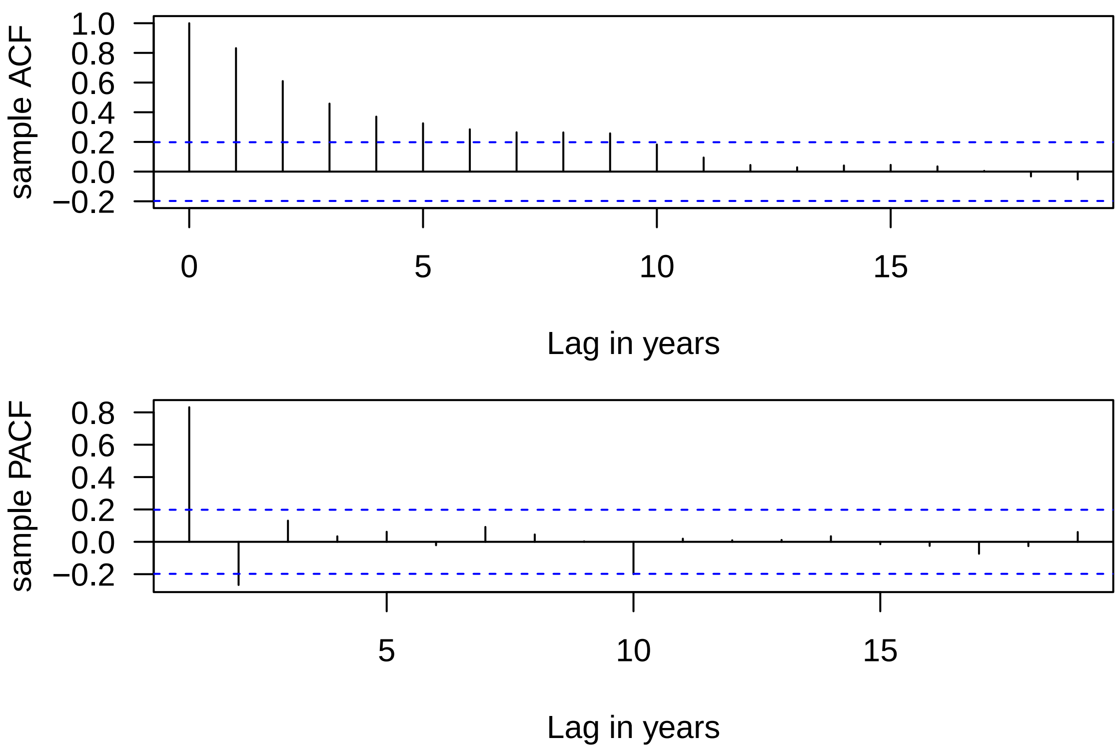
Plot the time series data

```r
## Let us create a 'years' variable.
years <- time(LakeHuron)
## Plot time series
plot(LakeHuron, ylab = "Depth (ft)", xlab = "Year", las = 1)
points(LakeHuron, col = "blue", pch = 16, cex = 0.8)
grid()
```



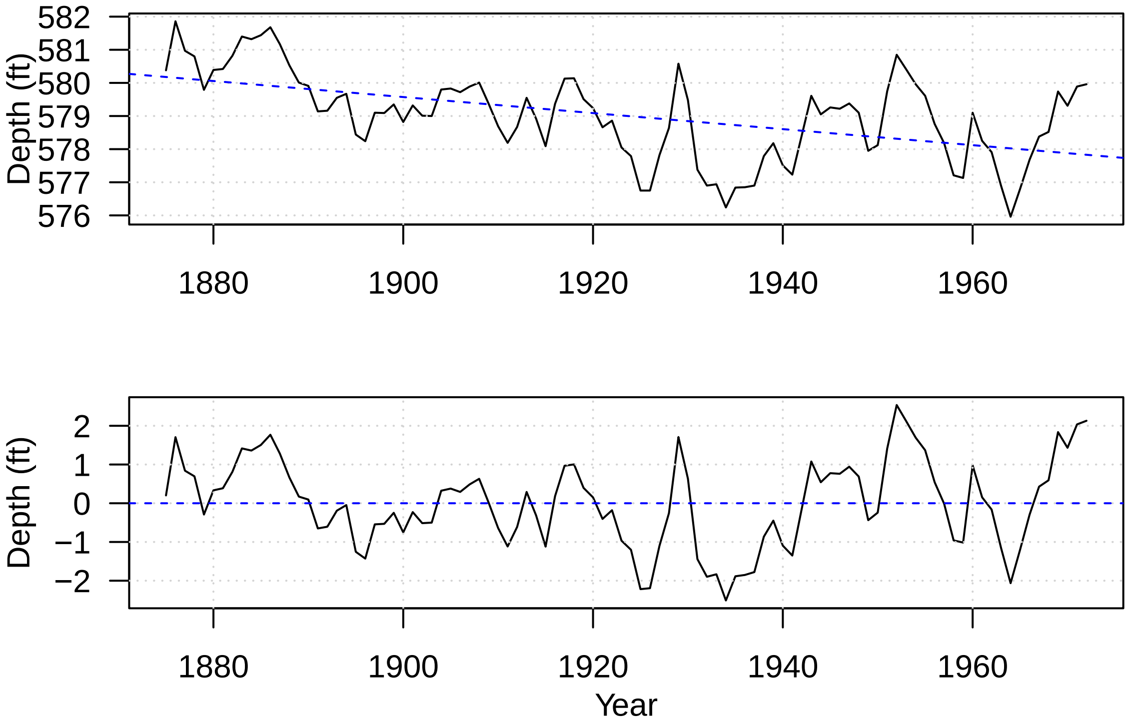
Calculate ACF and PACF

```r
par(mfrow = c(2, 1), mar = c(4, 4, 1, 1), las = 1)
acf(LakeHuron, xlab = "Lag in years", ylab = "sample ACF", main = "")
pacf(LakeHuron, xlab = "Lag in years", ylab = "sample PACF", main = "")
```



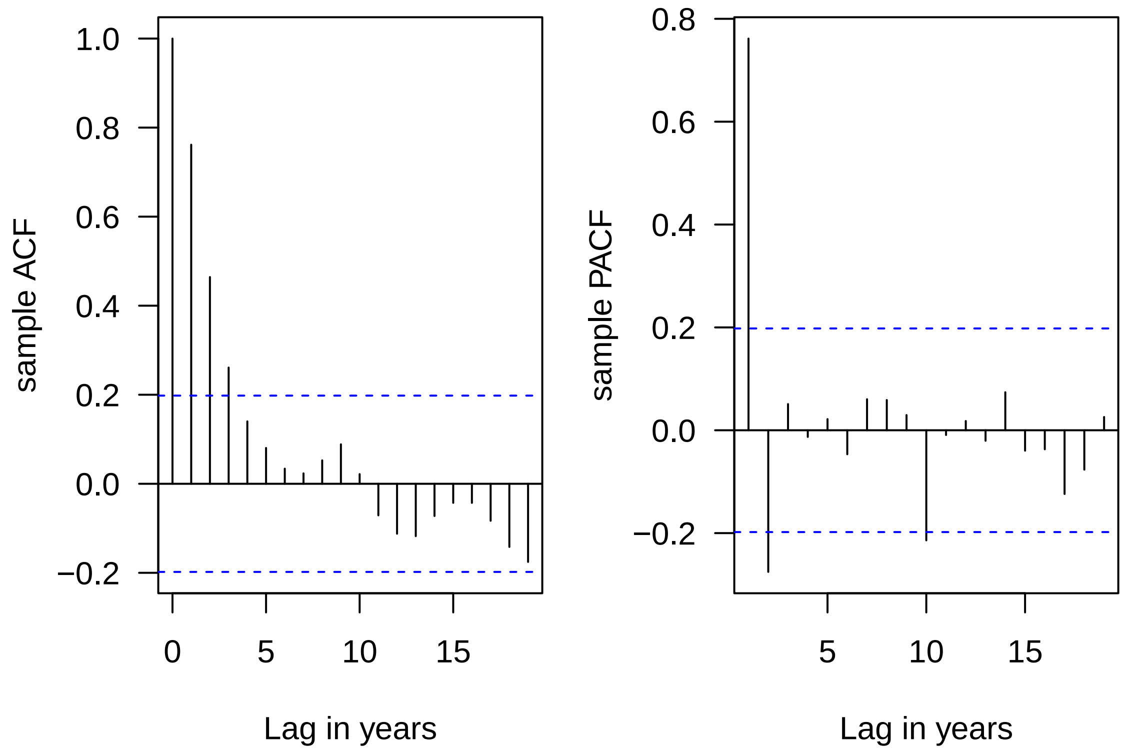
Estimate the linear trend

```r
lm <- lm(LakeHuron ~ years)
par(mfrow = c(2, 1), mar = c(3.5, 3.5, 1, 0.6))
plot(LakeHuron, ylab = "", xlab = "", las = 1); grid()
abline(lm, col = "blue", lty = 2)
mtext("Depth (ft)", 2, line = 2.4)
deTrend <- resid(lm)
plot(1875:1972, deTrend, type = "l", ylab = "", xlab = "", las = 1); grid()
abline(h = 0, col = "blue", lty = 2)
mtext("Year", 1, line = 2)
mtext("Depth (ft)", 2, line = 2.4)
```

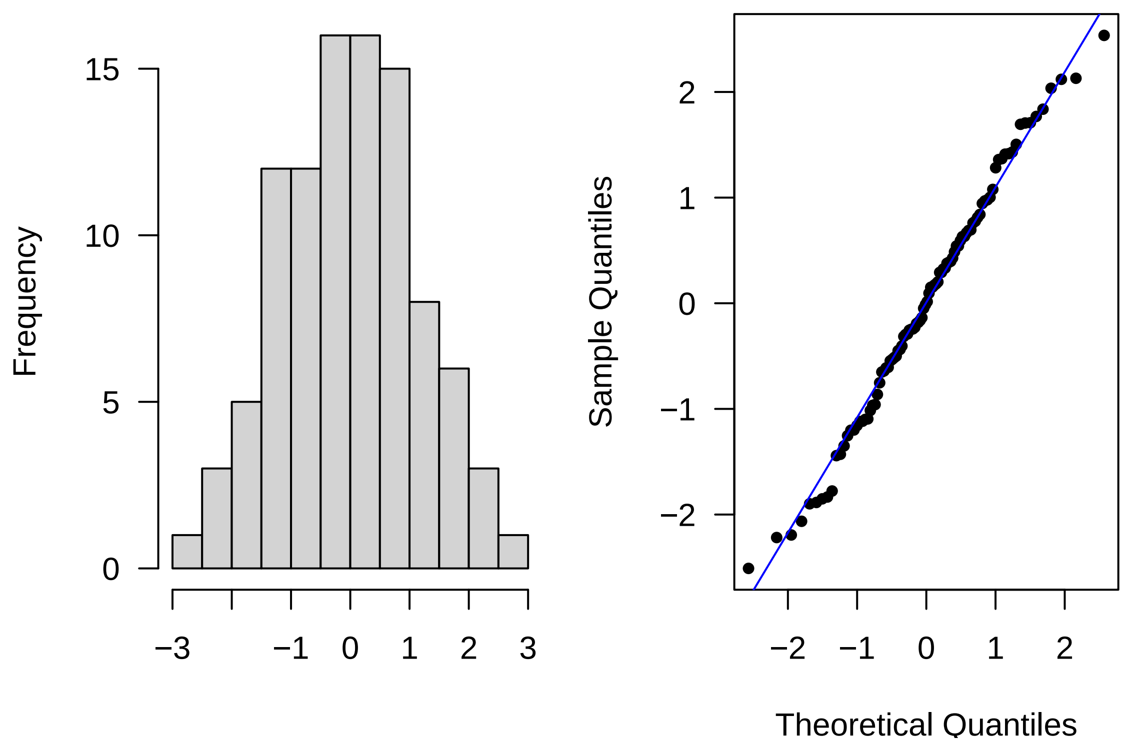


Recompute ACF and PACF for the detrended time series

```r
par(mfrow = c(1, 2), mar = c(4, 4, 1, 1), las = 1)
acf(deTrend, xlab = "Lag in years", ylab = "sample ACF", main = "")
pacf(deTrend, xlab = "Lag in years", ylab = "sample PACF", main = "")
```




```r
# Histogram and QQ plot
hist(deTrend, 15, main = "", xlab = "", las = 1)
qqnorm(deTrend, main = "", pch = 16, cex = 0.8); qqline(deTrend, col = "blue")
```



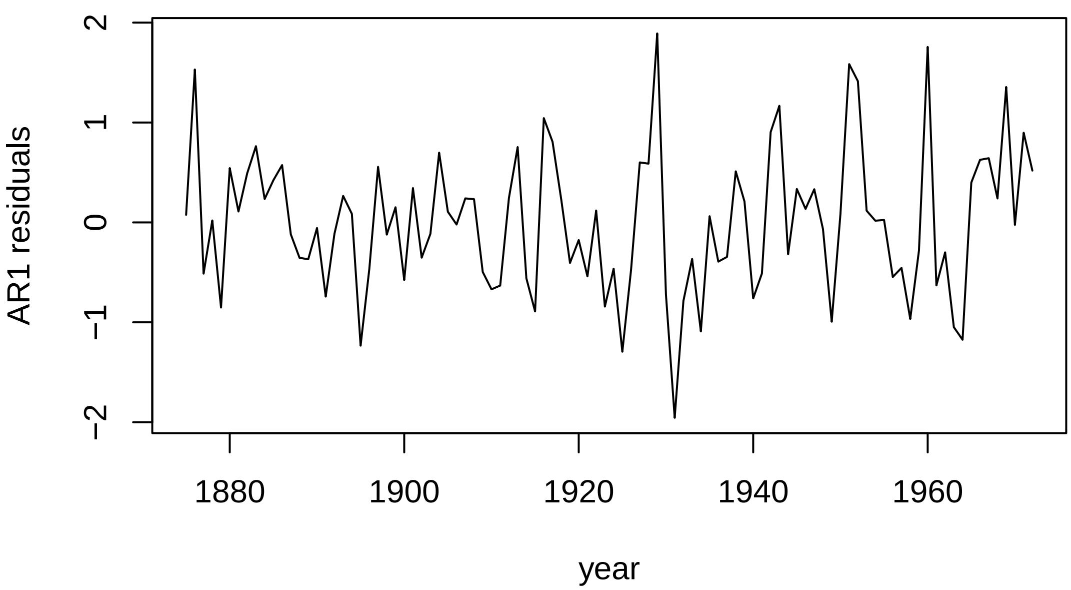
ARMA model fitting, selection, and diagnostics

```r
(ar1.model <- arima(deTrend, order = c(1, 0, 0), method = "ML"))
```

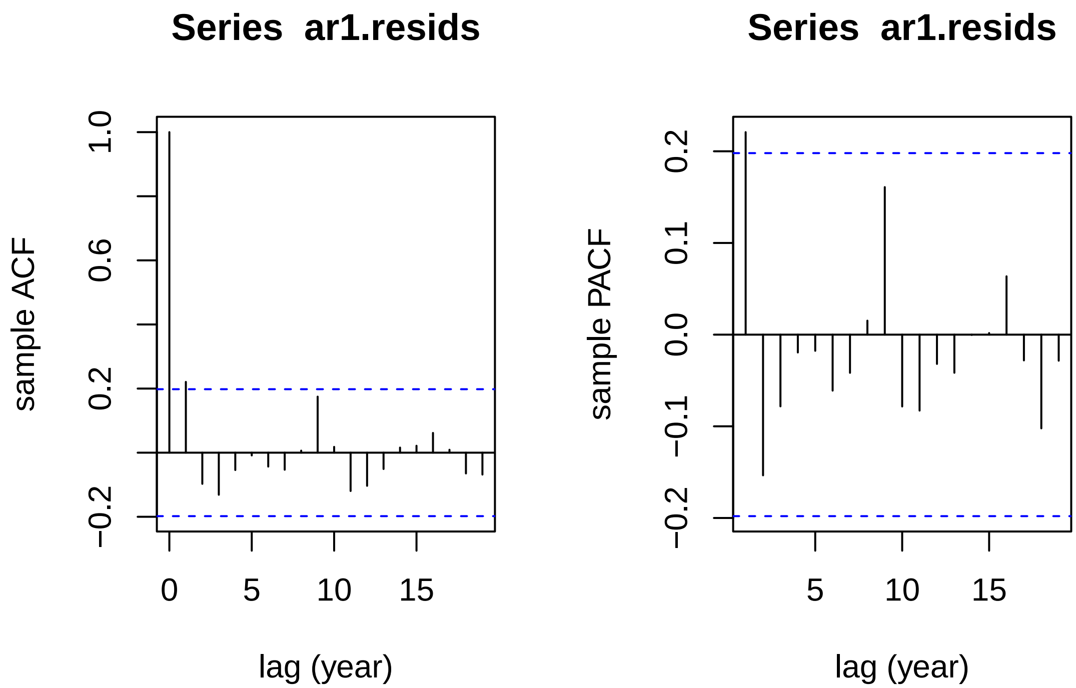
AR(1)

```
##
## Call:
## arima(x = deTrend, order = c(1, 0, 0), method = "ML")
##
## Coefficients:
##          ar1  intercept
##       0.7829     0.0799
## s.e.  0.0634     0.3179
##
## sigma^2 estimated as 0.4972:  log likelihood = -105.29,  aic = 216.58
```

```r
ar1.resids <- resid(ar1.model)
plot(1875:1972, ar1.resids, type = "l", xlab = "year", ylab = "AR1 residuals")
```

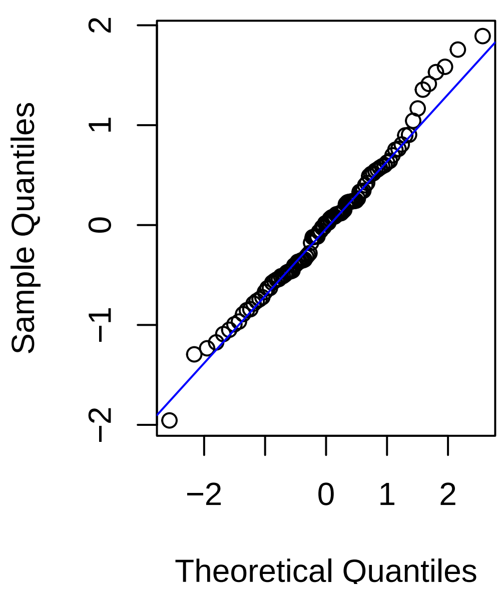



```r
## Sample ACF and PACF of the residuals
par(mfrow = c(1, 2))
acf(ar1.resids, ylab = "sample ACF", xlab = "lag (year)")
pacf(ar1.resids, ylab = "sample PACF", xlab = "lag (year)")
```




```r
## Normal Q-Q plot for the residuals
qqnorm(ar1.resids, main = ""); qqline(ar1.resids, col = "blue")
## Test for time dependence for the residuals
Box.test(ar1.resids, lag = 5, fitdf = 1, type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  ar1.resids
## X-squared = 7.9867, df = 4, p-value = 0.09207
```

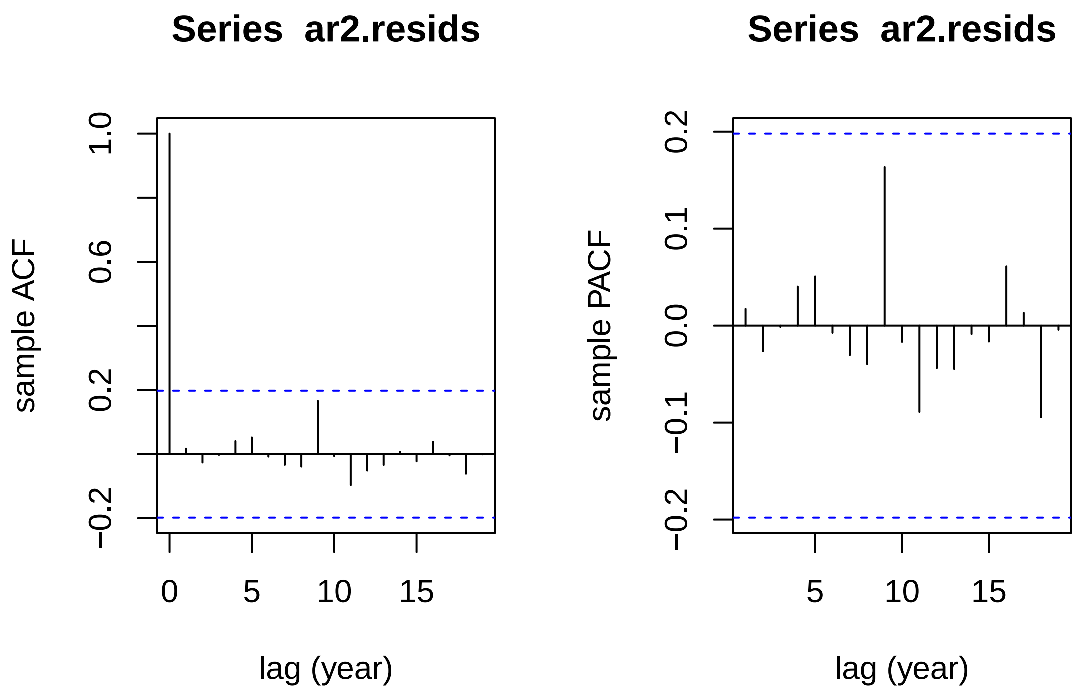


```r
(ar2.model <- arima(deTrend, order = c(2, 0, 0), method = "ML"))
```

AR(2)

```
##
## Call:
## arima(x = deTrend, order = c(2, 0, 0), method = "ML")
##
## Coefficients:
##          ar1      ar2  intercept
##       1.0047  -0.2919     0.0197
## s.e.  0.0977   0.1004     0.2350
##
## sigma^2 estimated as 0.4571:  log likelihood = -101.25,  aic = 210.5
```

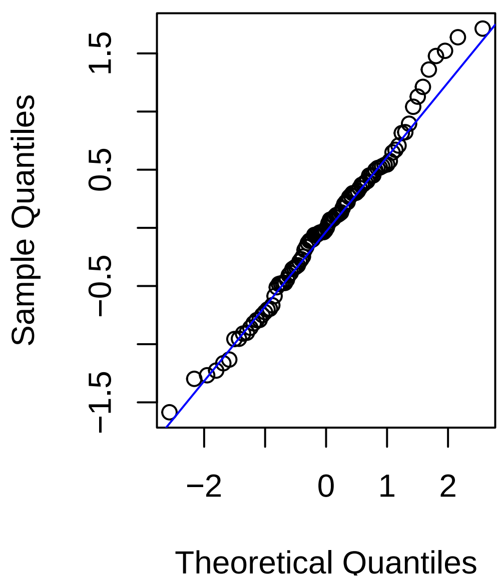
```r
## calculate the residuals
ar2.resids <- resid(ar2.model)
## Sample ACF and PACF of the residuals
par(mfrow = c(1, 2))
acf(ar2.resids,  ylab = "sample ACF", xlab = "lag (year)")
pacf(ar2.resids,  ylab = "sample PACF", xlab = "lag (year)")
```




```r
## Test for time dependence for the residuals
Box.test(ar2.resids, lag = 5, fitdf = 2, type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  ar2.resids
## X-squared = 0.55962, df = 3, p-value = 0.9056
```

```r
## Normal Q-Q plot for the residuals
qqnorm(ar2.resids, main = ""); qqline(ar2.resids, col = "blue")
```

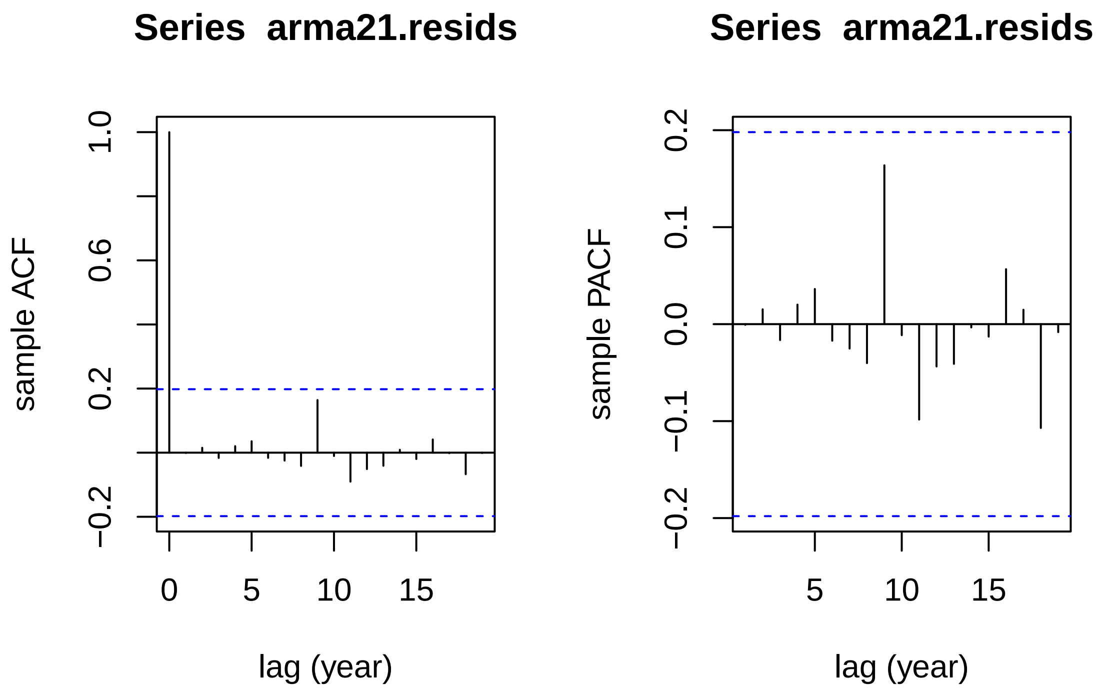



```r
(arma21.model <- arima(deTrend, order = c(2, 0, 1)))
```

ARMA(2, 1)

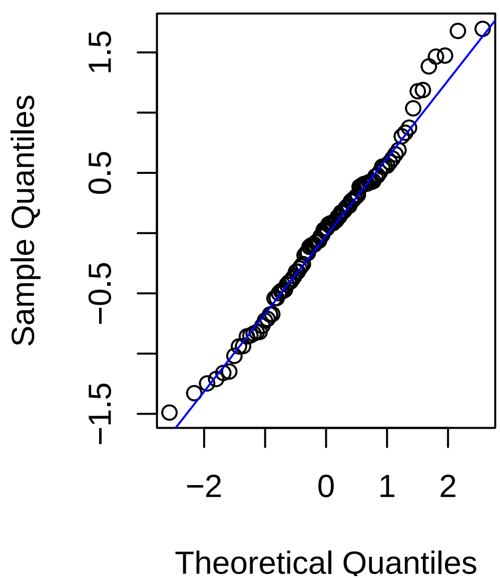
```
##
## Call:
## arima(x = deTrend, order = c(2, 0, 1))
##
## Coefficients:
##          ar1      ar2     ma1  intercept
##       0.8374  -0.1622  0.1846     0.0245
## s.e.  0.3180   0.2621  0.3180     0.2452
##
## sigma^2 estimated as 0.4556:  log likelihood = -101.09,  aic = 212.18
```

```r
## calculate the residuals
arma21.resids <- resid(arma21.model)
## Sample ACF and PACF of the residuals
par(mfrow=c(1,2))
acf(arma21.resids,  ylab = "sample ACF", xlab = "lag (year)")
pacf(arma21.resids,  ylab = "sample PACF", xlab = "lag (year)")
```




```r
## Normal Q-Q plot for the residuals
qqnorm(arma21.resids, main = ""); qqline(arma21.resids, col = "blue")
## Test
Box.test(arma21.resids, lag = 5, fitdf = 3, type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  arma21.resids
## X-squared = 0.2297, df = 2, p-value = 0.8915
```



```r
AIC(ar1.model); AIC(ar2.model); AIC(arma21.model)
```

Model selection using AIC

```
## [1] 216.5835
```

```
## [1] 210.5032
```

```
## [1] 212.1784
```

```r
library(forecast)
auto.arima(deTrend, trace = T)
```

```
##
##  ARIMA(2,0,2) with non-zero mean : 215.0455
##  ARIMA(0,0,0) with non-zero mean : 304.222
##  ARIMA(1,0,0) with non-zero mean : 216.8388
##  ARIMA(0,0,1) with non-zero mean : 235.4585
##  ARIMA(0,0,0) with zero mean     : 302.1373
##  ARIMA(1,0,2) with non-zero mean : 212.7747
##  ARIMA(0,0,2) with non-zero mean : 218.2478
##  ARIMA(1,0,1) with non-zero mean : 210.9477
##  ARIMA(2,0,1) with non-zero mean : 212.8306
##  ARIMA(2,0,0) with non-zero mean : 210.9333
##  ARIMA(3,0,0) with non-zero mean : 212.7787
##  ARIMA(3,0,1) with non-zero mean : Inf
##  ARIMA(2,0,0) with zero mean     : 208.7655
##  ARIMA(1,0,0) with zero mean     : 214.7735
##  ARIMA(3,0,0) with zero mean     : 210.569
##  ARIMA(2,0,1) with zero mean     : 210.6186
##  ARIMA(1,0,1) with zero mean     : 208.7891
##  ARIMA(3,0,1) with zero mean     : Inf
##
##  Best model: ARIMA(2,0,0) with zero mean
```

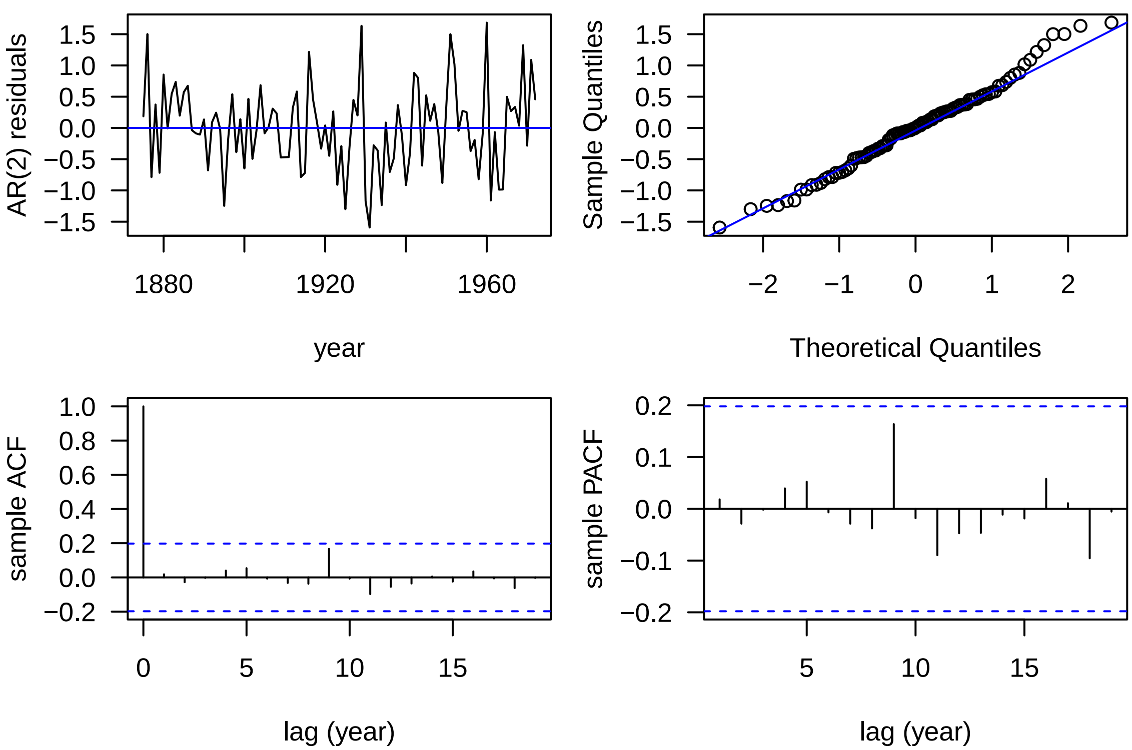
```
## Series: deTrend
## ARIMA(2,0,0) with zero mean
##
## Coefficients:
##          ar1      ar2
##       1.0050  -0.2925
## s.e.  0.0976   0.1002
##
## sigma^2 = 0.4667:  log likelihood = -101.26
## AIC=208.51   AICc=208.77   BIC=216.27
```

AR(2) Fitting and Forecasting

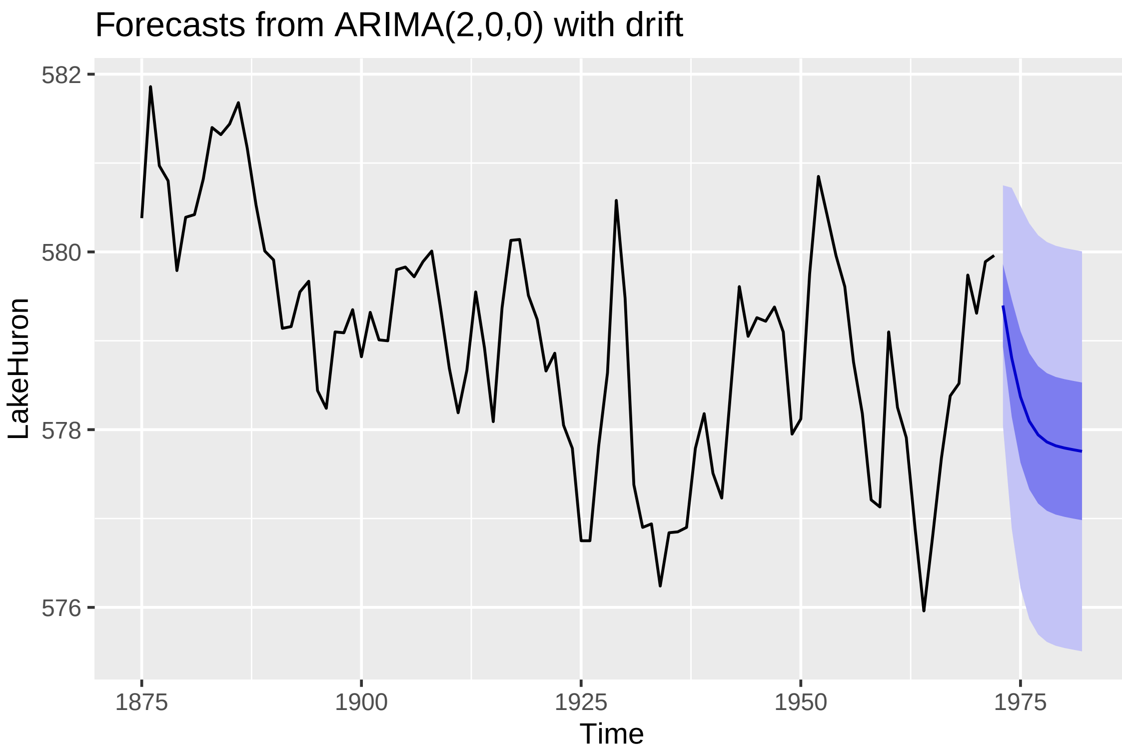
```r
(fit <- Arima(LakeHuron, order = c(2, 0, 0), include.drift = T))
```

```
## Series: LakeHuron
## ARIMA(2,0,0) with drift
##
## Coefficients:
##          ar1      ar2  intercept    drift
##       1.0048  -0.2913   580.0915  -0.0216
## s.e.  0.0976   0.1004     0.4636   0.0081
##
## sigma^2 = 0.476:  log likelihood = -101.2
## AIC=212.4   AICc=213.05   BIC=225.32
```

```r
par(mfrow = c(2, 2), mar = c(4.1, 4, 1, 0.8), las = 1)
res <- fit$residuals
plot(res, type = "l", xlab = "year", ylab = "AR(2) residuals", las = 1)
abline(h = 0, col = "blue")
qqnorm(res, main = ""); qqline(res, col = "blue")
acf(res,  ylab = "sample ACF", xlab = "lag (year)")
pacf(res,  ylab = "sample PACF", xlab = "lag (year)")
```




```r
# 10-year-ahead forecasts
autoplot(forecast(fit, h = 10, level = c(50, 95)))
```


Forecasts from ARIMA(2,0,0) with drift
582
580
578
576
LakeHuron
1875
1900
1925
1950
1975
Time