Lecture 4

Data Summary/Visualization III

Text: Chapter 3

STAT 8010 Statistical Methods I
September 1, 2020

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Percentiles and Boxplots

Visualizing two variables simultaneously

Visualizing Time Series, Cross-Sectional, and Spatio-Temporal Data sets

Agenda

1. Percentiles and Boxplots
2. Visualizing two variables simultaneously
3. Visualizing Time Series, Cross-Sectional, and Spatio-Temporal Data sets

Interquartile range (IQR)

- IQR $= Q_3 - Q_1$, where Q_1 is the Lower Quartile (the median of the lower half of the data) and Q_3 is the Upper Quartile (the median of the upper half of the data)

- Compute the IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

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Question: Is IQR a robust statistic?

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Percentiles, Quartiles, and Boxplots

Percentiles

- The p_{th} percentile is a value such that at least $p\%$ of the data set is less than or equal to this value [An Example]

- Calculation of percentiles using the indexing method:
  1. Sort the set of numbers in an increasing order

- Quartiles:

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  1. $Q1$: first quartile (25_{th} percentile)

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  2. M $(Q2)$: median (second quartile, 50_{th} percentile)

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  3. $Q3$: third quartile (75_{th} percentile)

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  3. $Q3$: third quartile (75_{th} percentile)
  4. Interquartile range or IQR: $Q3 - Q1$

Example

Find Q_1, M, Q_3 and IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13 using the indexing method

1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21

Example

Find Q_1, M, Q_3 and IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13 using the indexing method

1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21
2. Find the sample size n and compute the indices for $p = 25, 50, 75$

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Find Q_1, M, Q_3 and IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13 using the indexing method

1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21
2. Find the sample size n and compute the indices for $p = 25, 50, 75$
3. $n = 9 \Rightarrow$ the indices are $3, 5, 7 \Rightarrow Q_1 = 13$, $M = 14$, $Q_3 = 16$

Example

Find Q_1, M, Q_3 and IQR of the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13 using the indexing method

1. Order the data first: 13, 13, 13, 13, 14, 14, 16, 18, 21
2. Find the sample size n and compute the indices for $p = 25, 50, 75$
3. $n = 9 \Rightarrow$ the indices are $3, 5, 7 \Rightarrow Q_1 = 13$, $M = 14$, $Q_3 = 16$
4. IQR $= Q_3 - Q_1 = 16 - 13 = 3$

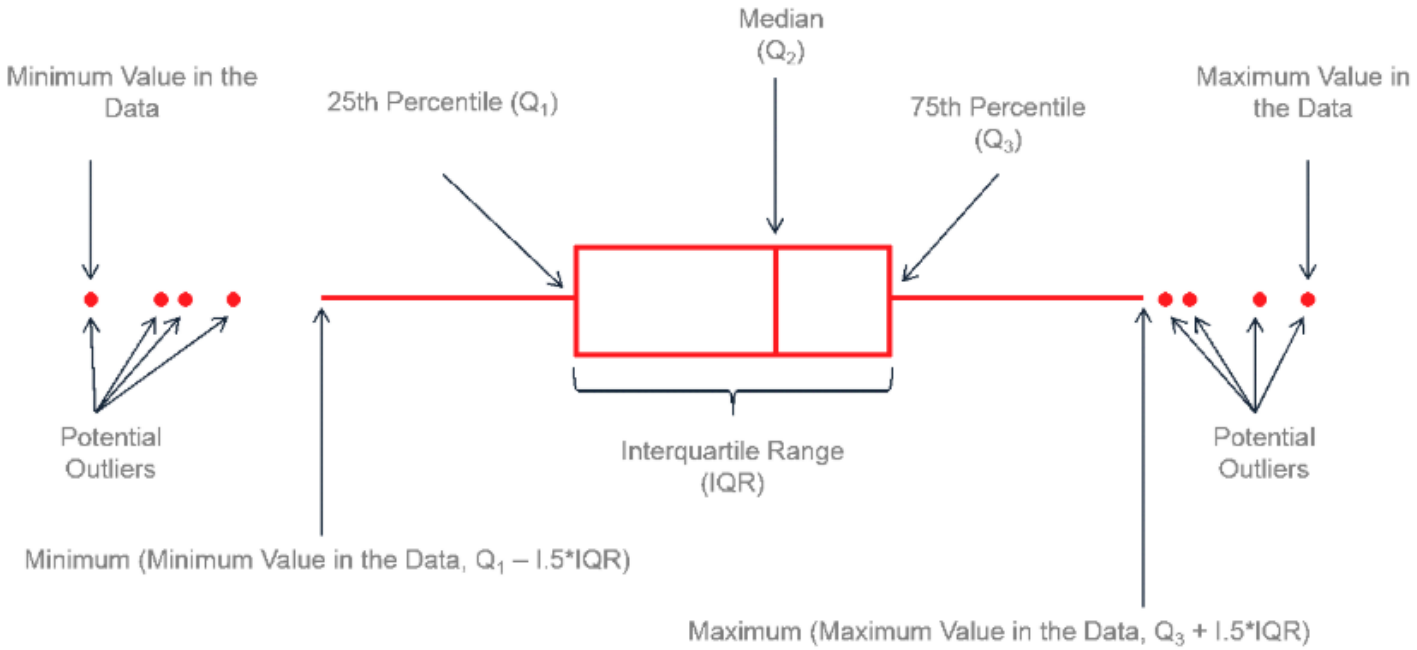
Steps to Making a Boxplot

1. Find Q_1, M, Q_3 and draw a box from Q_1 to Q_3. Add a vertical line inside the box at M

2. Compute the value of Lower Fence (LF) $= Q1 - 1.5$IQR and the Upper Fence (UF) $= Q3 + 1.5$IQR. Find the largest value $\leq$ UF and the smallest value $\geq$ LF. Draw whiskers go from Q_1, Q_3 to these two values

3. Plot the individual outlier(s) (i.e., the values either $>$ UF or $<$ LF)

Boxplot Anatomy



Source: `https://https://www.leansigmacorporation.com/box-plot-with-minitab/`

Example

- **Ordered data values:** 13, 13, 13, 13, 14, 14, 16, 18, 21

Percentiles and Boxplots

Visualizing two variables simultaneously

Visualizing Time Series, Cross-Sectional, and Spatio-Temporal Data sets

Example

- **Ordered data values:** 13, 13, 13, 13, 14, 14, 16, 18, 21
- **IQR** $16 - 13 = 3 \Rightarrow$ LF $= 13 - 1.5 \times 3 = 8.5$; UF $= 16 + 1.5 \times 3 = 20.5$

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- **Ordered data values:** 13, 13, 13, 13, 14, 14, 16, 18, 21
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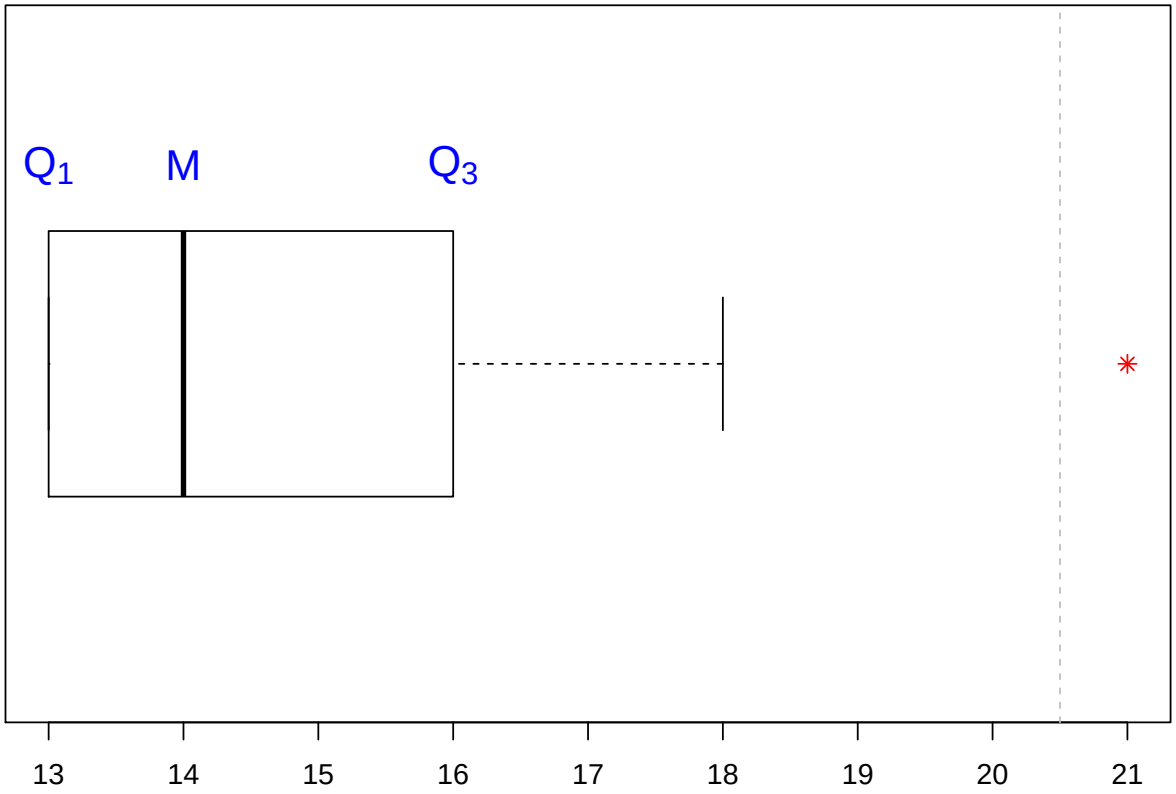

Another Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile

Another Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$

Another Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$
  2. Compute the index value $i = \frac{35 \times 15}{100} = 5.25 \Rightarrow$ the 35th percentile is 13

Another Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$
  2. Compute the index value $i = \frac{35 \times 15}{100} = 5.25 \Rightarrow$ the 35th percentile is 13
- Find the 65th percentile

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Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$
  2. Compute the index value $i = \frac{35 \times 15}{100} = 5.25 \Rightarrow$ the 35th percentile is 13
- Find the 65th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$

Another Example

Suppose we have the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13, 9, 27, 18, 25, 20, 6

- Find the 35th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$
  2. Compute the index value $i = \frac{35 \times 15}{100} = 5.25 \Rightarrow$ the 35th percentile is 13
- Find the 65th percentile
  1. Sort the data: $6, 9, 13, 13, 13, 13, 14, 14, 16, 18, 18, 20, 21, 25, 27$
  2. Compute the index value $i = \frac{65 \times 15}{100} = 9.75 \Rightarrow$ the 65th percentile is 18

Percentiles and Boxplots

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Visualizing two variables simultaneously

Example: O'Hare Airport Flight Data

	carrier	origin
1	UA	EWR
2	AA	LGA
3	AA	LGA
4	AA	LGA
5	UA	LGA
6	UA	EWR

In this example, we have two categorical variables, `carrier` and `origin`, respectively. How to summarize/visualize this dataset?

Percentiles and Boxplots

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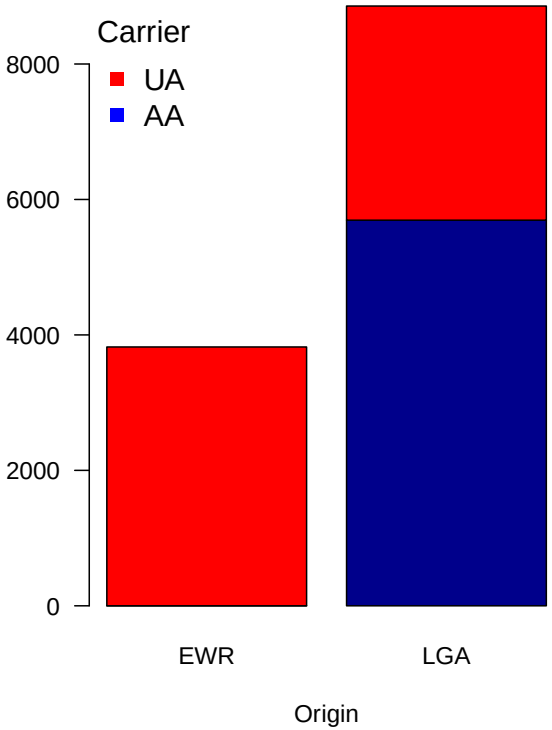
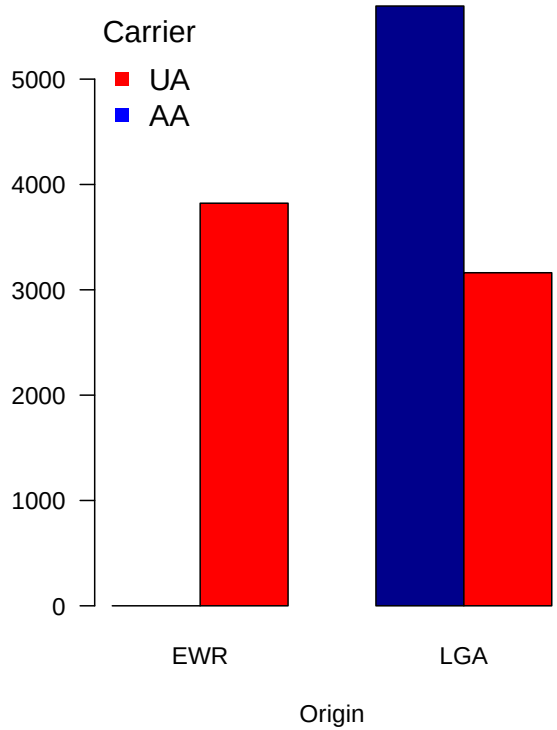
ORD Flight Data Cont'd

	EWR	LGA
AA	0	5694
UA	3822	3162

	EWR	LGA
AA	0.00	0.45
UA	0.30	0.25





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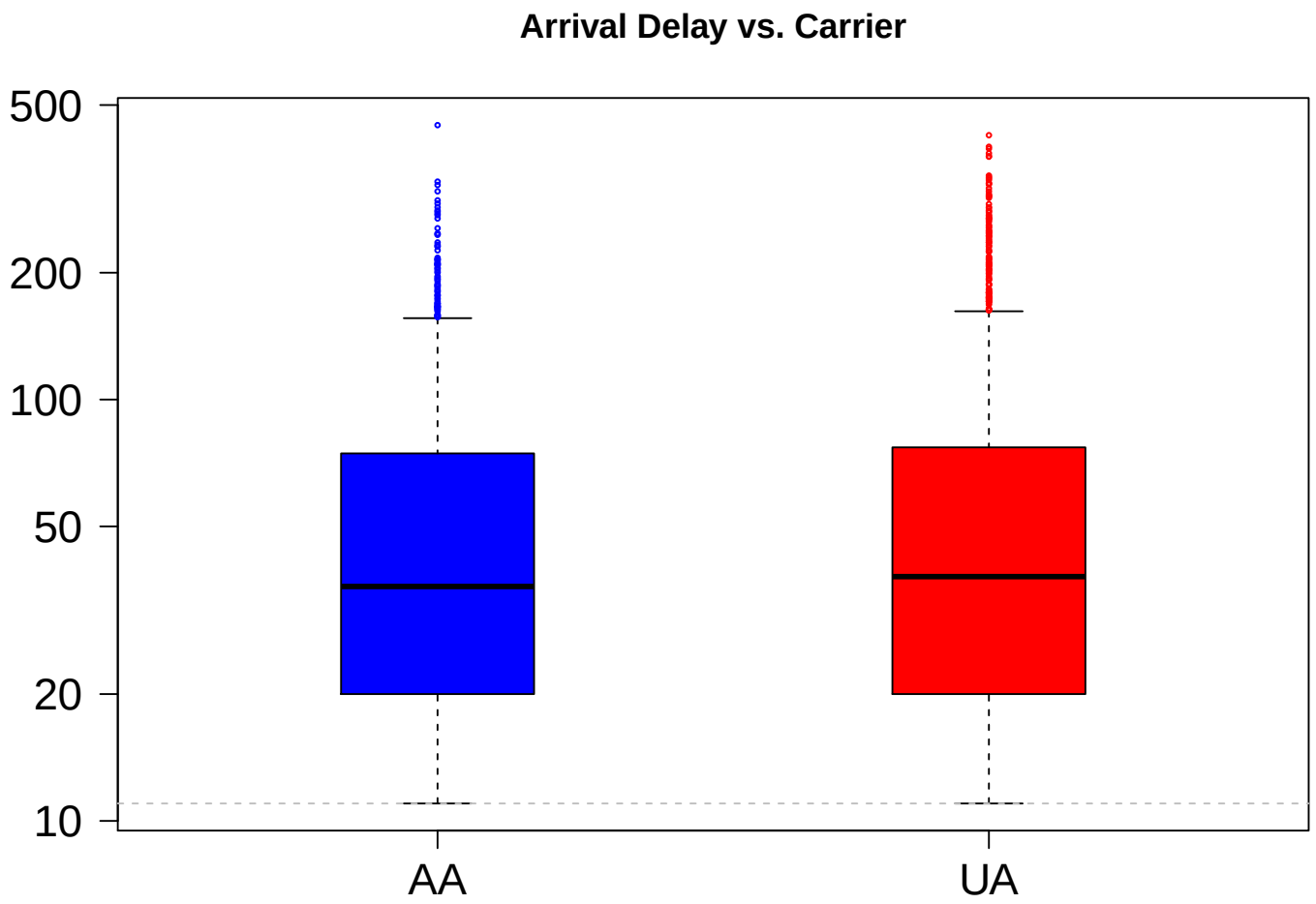
ORD Fligts Data Cont'd

carrier	origin	arr_delay
UA	EWR	12
AA	LGA	8
AA	LGA	14
AA	LGA	4
UA	LGA	20
UA	EWR	21

In this example, we have two categorical variables, `carrier`, `origin` and a numerical variable `arr_delay`, respectively. How to visualize, for example, `arr_delay` vs. `carrier`?

ORD Example: Arrival Delay vs. Air Carrier



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Example: Max Heart Rate and Age

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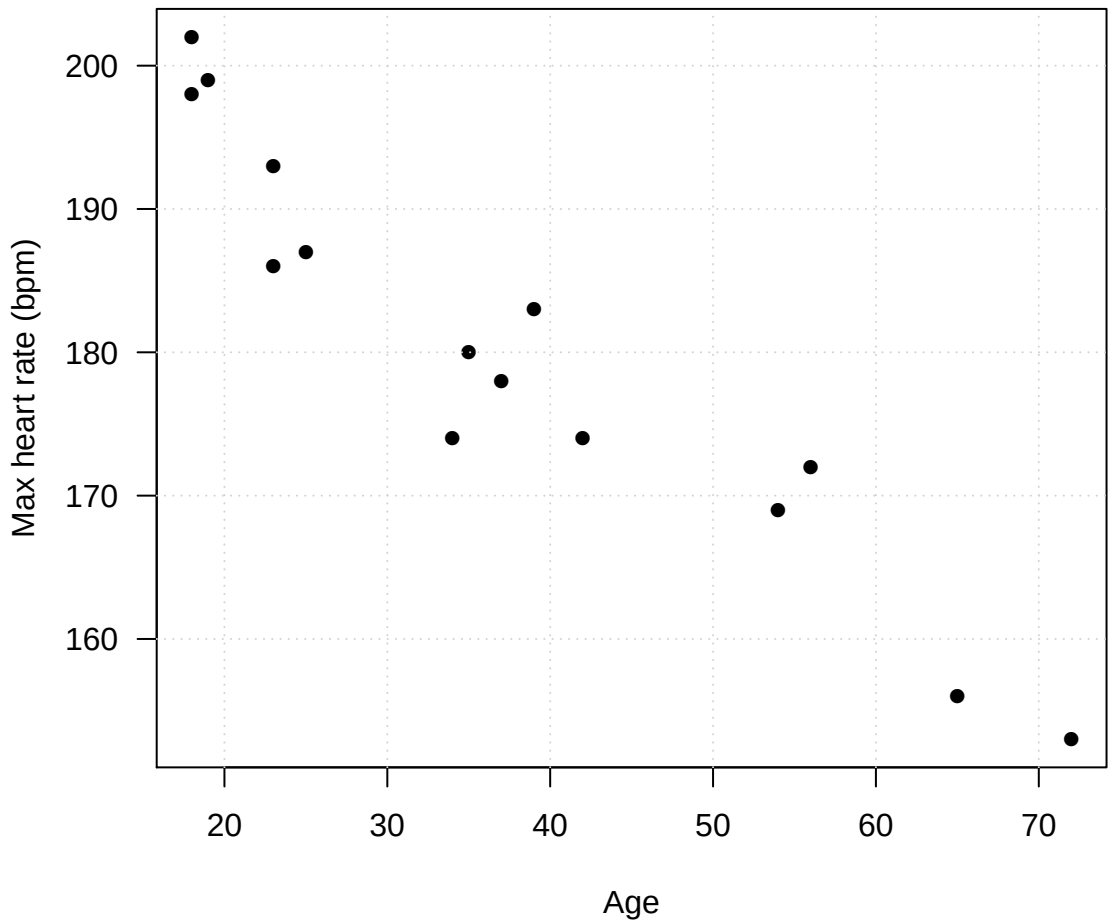
Suppose we have 15 people of varying ages are tested for their maximum heart rate (MHR)

Age	18	23	25	35	65	54	34	56	72	19	23	42	18	39	37
MHR	202	186	187	180	156	169	174	172	153	199	193	174	198	183	178

- How many variables do we have in this data set? What are the variable types?
- How to summarize these variables?

Scatterplot

A scatterplot is a useful tool to graphically display the relationship between two numerical variables. Each dot on the scatterplot represents one observation from the data



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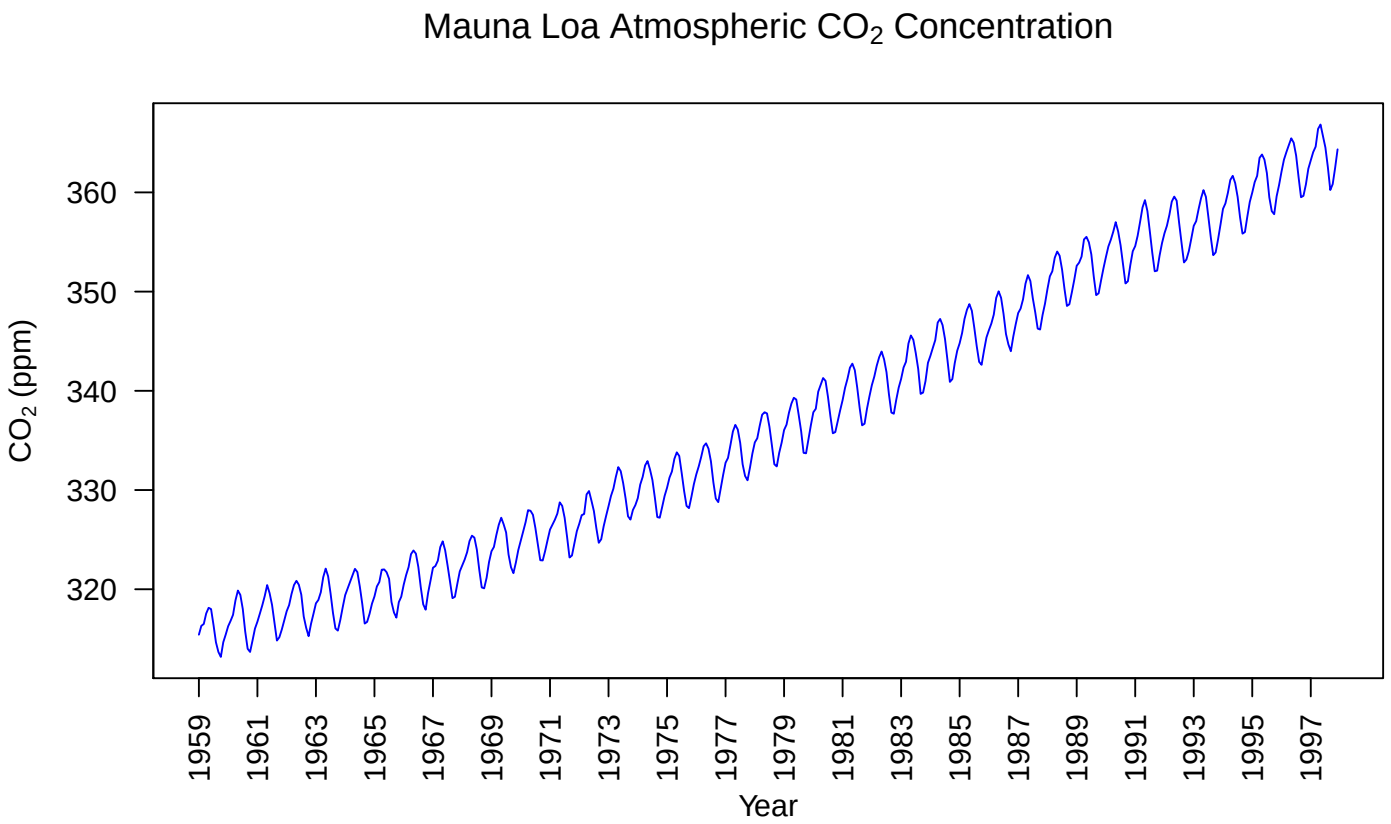
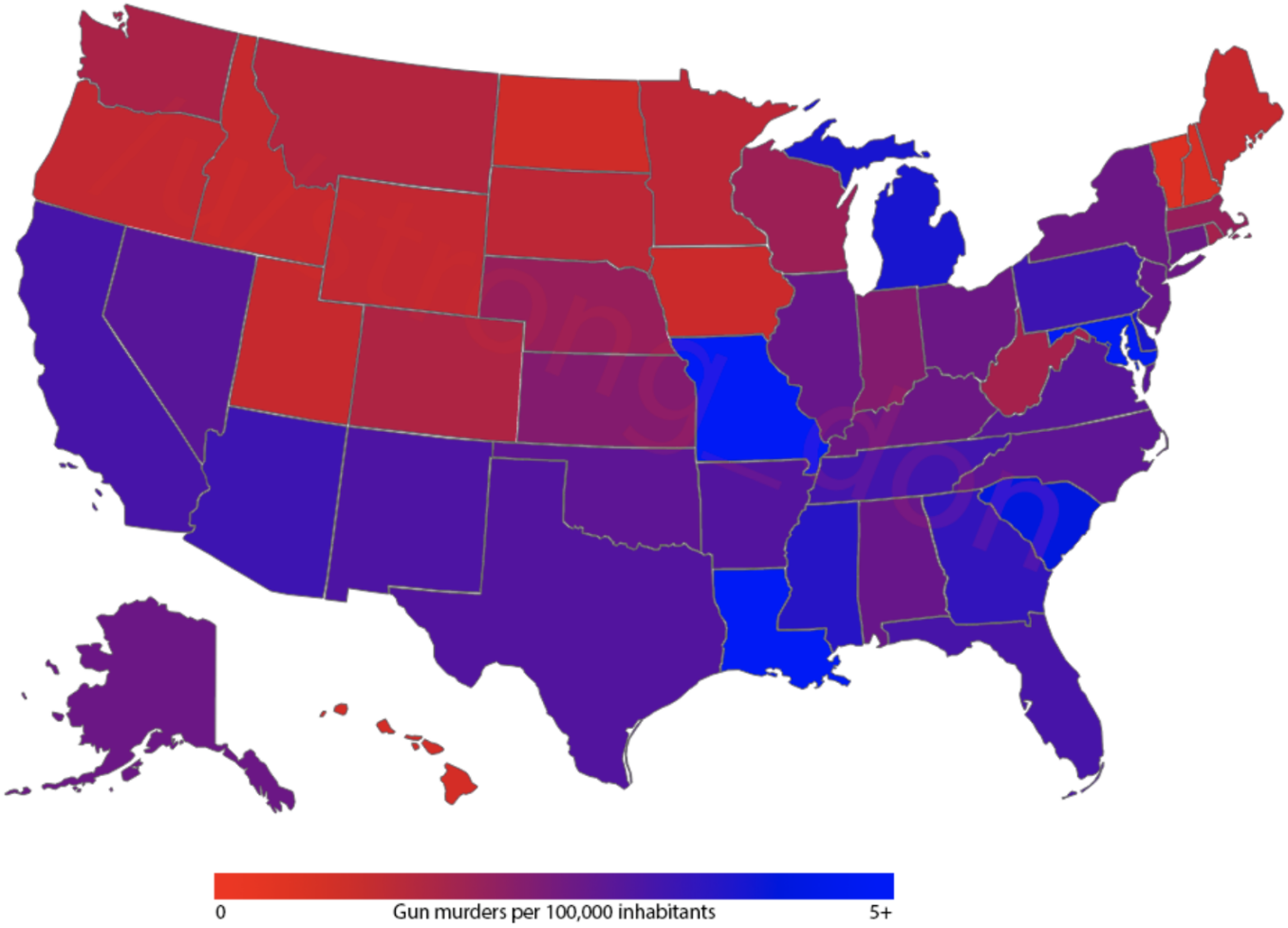
Visualizing Time Series Data



Visualizing Cross-Sectional Data



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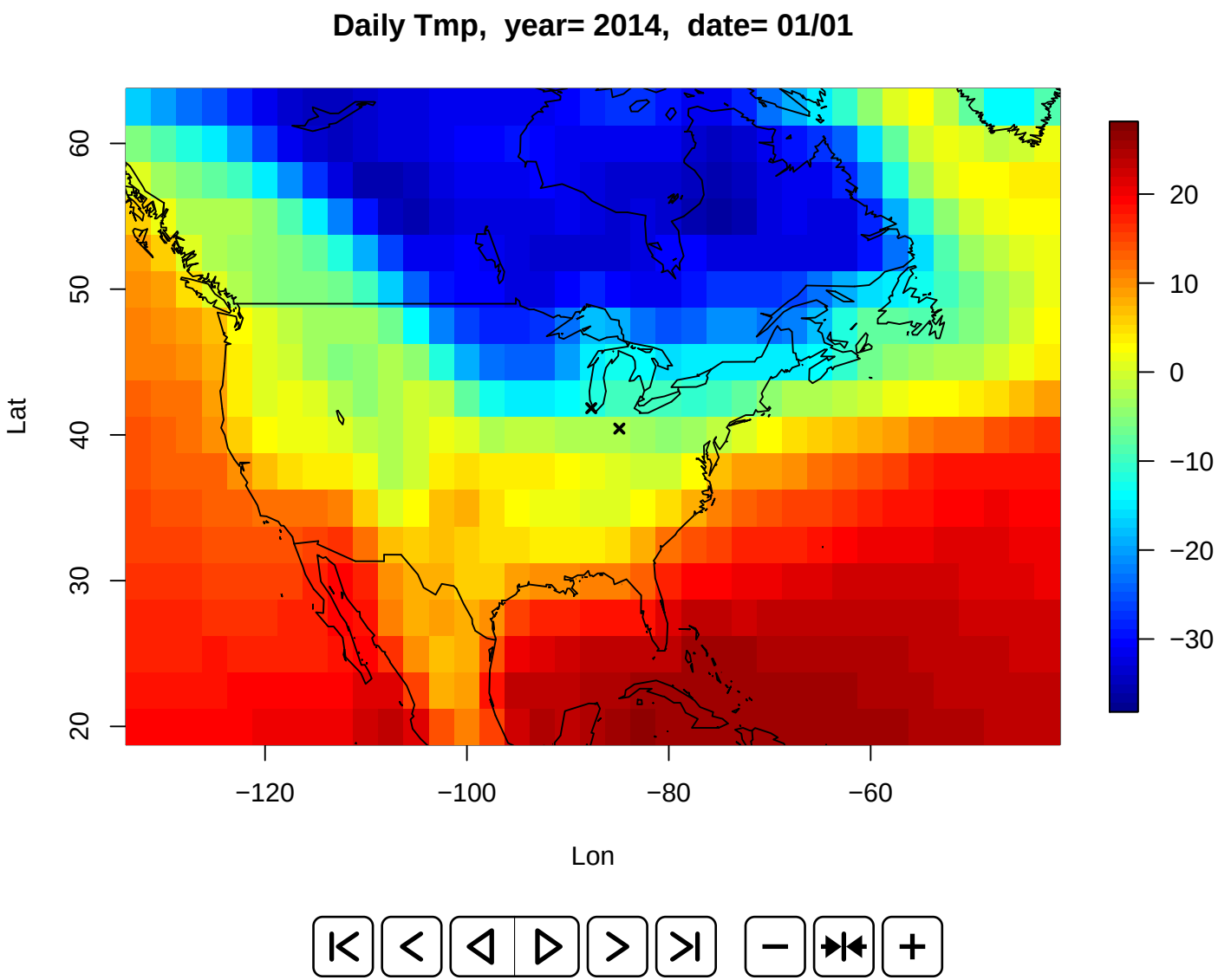
Visualizing Spatio-Temporal Data



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Summary

In this lecture, we learned

- How to summarize numerical variable
- How to visualize two variables simultaneously
- How to visualize time series, cross-sectional, spatio-temporal data sets

We will talk about Probability in the next few weeks