STAT 8010–003 Statistical Methods I
Homework 3

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Due Date: October 16, 1:25pm

Problem 1

The transportation department of a city remodeled one of its parking garages and increased the hourly parking rates. From the citys records, the average parking time over the past 5 years was 180 minutes. The department wants to know whether the remodeling and rate increases have changed the mean parking time. Over a 3-month period after the changes were made, a random sample of 100 cars had an average parking time of 168 minutes with a standard deviation of 45 minutes.

(a) State the null and alternative (research) hypotheses for the study?

(b) Construct a 95% confidence interval (using $t(0.025, 99) = 1.984$) for the average parking time after the changes were made to the garage.

(c) Do the data support the research hypothesis if $\alpha = 0.05$?

(d) What is the p-value of the test?

Problem 2

Answer "true" or "false" for each question.

(a) Given any particular random sample, if we form the 95% confidence interval for the sample mean, there is a 95% chance that the population mean lies in this confidence interval.

(b) If a larger number of random samples are selected and we form the 95% confidence interval for each sample mean, the population mean will lie in about 95% of these confidence intervals.

(c) The 95% confidence interval around a given sample mean is wider than the 90% confidence interval around that mean.

(d) If we reject the null hypothesis at the $\alpha = 0.05$ level, then we should also reject it at the $\alpha = 0.01$ level.

Problem 3

Answer "true" or "false" for each question. If your answer is "false," change the statement to make it true. Change only the underlined words.

(a) A Type I error is committed when we fail to reject the null hypothesis H_0 when H_0 is actually false.

(b) If we make a Type II error, we have missed detecting an event or effect when there actually was one.

(c) The probability of making a Type I error is equal to β

(d) If we increase the probability of making a Type II error, we increase the probability of making a Type I error.

Problem 4

A researcher wanted to test the hypotheses $H_0 : \mu \leq 38$ against $H_a : \mu \geq 38$ with $\alpha = 0.05$. The researcher knows in advance that $\sigma = 6$. A random sample of 50 measurements from a population yielded $\bar{X} = 40.01$.

(a) What conclusions can you make about the hypotheses?

(b) Could you have made a Type II error in this situation? Explain.

(c) Calculate the probability of a Type II error if the actual value of μ is 39.

Problem 5

A study was conducted of 90 adult male patients following a new treatment for congestive heart failure. One of the variables measured on the patients was the increase in exercise capacity (in minutes) over a 4-week treatment period. The previous treatment regime had produced an average increase of $\mu = 2$ minutes. The researchers wanted to evaluate whether the new treatment had increased the value of μ in comparison to the previous treatment. The data yielded $\bar{x} = 2.25$. Suppose we do know the population variance $\sigma^2 = 1$.

(a) Using $\alpha = 0.05$, what conclusions can you draw about the research hypothesis?

(b) What is the probability of making a Type II error if the actual value of μ is 2.1?