

# STAT 8010–001 Statistical Methods I

## Homework 2

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**Due Date:** February 27, 09:30am

### Problem 1

The transportation department of a city remodeled one of its parking garages and increased the hourly parking rates. From the city's records, the average parking time over the past 5 years was 180 minutes. The department wants to know whether the remodeling and rate increases have changed the mean parking time. Over a 3-month period after the changes were made, a random sample of 100 cars had an average parking time of 168 minutes with a standard deviation of 45 minutes.

- (a) State the null and alternative (research) hypotheses for the study?

Let  $\mu$  be the true mean parking time in minutes.

$H_0 : \mu = 180$  vs.  $H_a : \mu \neq 180$

- (b) Construct a 95% confidence interval (using  $t_{0.025,99} = 1.984$ ) for the average parking time after the changes were made to the garage.

95% CI for  $\mu$  is  $\bar{X}_n \pm t_{0.025,df=99} \times \frac{s}{\sqrt{n}} = 168 \pm 1.984 \times \frac{45}{\sqrt{100}} = (159.072, 176.928)$

- (c) Do the data support the research hypothesis if  $\alpha = 0.05$ ?

Yes, because the 95% CI DOES NOT contain 180 ( $\mu_0$ ), therefore we reject  $H_0$  and we conclude that we have enough statistical evidence to support the research hypothesis that  $\mu \neq 180$  at 0.05 level.

- (d) What is the p-value of the test?

P-value =  $2 \times \mathbb{P}(t^* > |t_{obs}|)$  where  $t_{obs} = \frac{168-180}{\frac{45}{\sqrt{100}}} = -2.67$  and  $t^* \sim t_{df=99}$   
 $\Rightarrow$  P-value =  $2 \times \mathbb{P}(t^* > 2.67) = 0.0089$

## Problem 2

Answer “true” or “false” for each question.

(a) Given one particular random sample, if we form the 95% confidence interval for the sample mean, there is a 95% chance that the population mean lies in this confidence interval.

False

(b) If a larger number of random samples are selected and we form the 95% confidence interval for each sample mean, the population mean will lie in about 95% of these confidence intervals.

True

(c) The 95% confidence interval around a given sample mean is wider than the 90% confidence interval around that mean.

True

(d) If we reject the null hypothesis at the  $\alpha = 0.05$  level, then we should also reject it at the  $\alpha = 0.01$  level.

False

### Problem 3

Answer “true” or “false” for each question. If your answer is “false,” change the statement to make it true. Change only the underlined words.

- (a) A Type I error is committed when we fail to reject the null hypothesis  $H_0$  when  $H_0$  is actually false.

False, Type II error

- (b) If we make a Type II error, we have missed detecting an event or effect when there actually was one.

True

- (c) The probability of making a Type I error is equal to  $\beta$

False, Type II error

- (d) If we increase the probability of making a Type II error, we increase the probability of making a Type I error.

False, Decrease