Lecture 8

Introduction to Design of Experiments

Reading: Oehlert Chapters 1, 2; Dean-Voss-Draguljić Chapters 1, 2

DSA 8020 Statistical Methods II

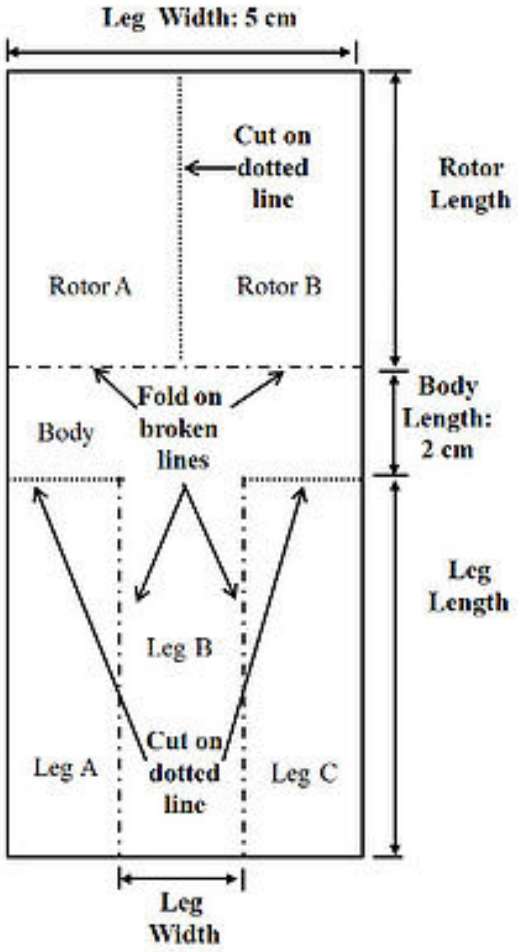
Whitney Huang
Clemson University

Agenda

1. Definitions and Preliminaries
2. History of Experimental Design
3. Fundamental Principles of Experimental Design

Paper Helicopter Experiment

Suppose we would like to investigate how long a paper helicopter can fly in the air.



Some of potential determining factors:

- Paper type (light, medium, and heavy)
- Rotor length (7.5cm or 8.5cm)
- Leg length (7.5cm or 12 cm)
- Leg width (3.2cm or 5cm)

Source:
https://blog.minitab.com/en/learning-design-of-experiments-with-paper-helicopters-and-minitab

Steps for Planning, Conducting and Analyzing an Experiment

- State the problem of interest
- Select the response variable and determine the factor(s)
- Choose the design and conduct the experiment
- Perform statistical analysis
- Draw conclusions

Battery Experiment [Dean-Voss-Draguljić p. 24]

- **Specific question**: How do battery types vary with respect to life-per-unit cost?
- **Response**: Time (per unit cost) to exhaust battery under standard load
- **Comparative**: Difference between 4 battery types
- **Controlled**: All compared using the same device
- **Replication**: Four batteries of each type tested

Some Definitions

- **Factor:** variable whose influence upon the response variable. Settings of factor are called **levels**
- **Treatments:** the procedures (set of values for all factors) to compare
- **Experimental units:** objects on which treatments are applied
- **Measurement units:** objects on which the response is measured. These may differ from the experimental units
- **Randomization:** using a known probabilistic mechanism to assign treatments to experimental units
- **Experimental error:** variation in response outcomes (modeled as random)

Main Elements of An Experiment

An experiment applies treatments to experimental units and measures responses.

- Want to learn about **treatments** (e.g., dose of drug; nano-tech coating for a fabric)
- **Responses** tell us how the treatment worked (patient get better; stain resistance)
- Experimenter assigns treatments to **experimental units** (e.g., a patient; a bolt of fabric)

Experimental Unit

- Perhaps the most important concept in statistical design
- The experimental unit is the unit (subject, plant, pot, animal) which is randomly assigned to a treatment
- The experimental unit *defines the unit to be replicated to increase degrees of freedom*

Experimental Units vs Measurement Units

If a group of “units” must have the same treatment, they are likely measurement units (MUs) rather than experimental units (EUs)

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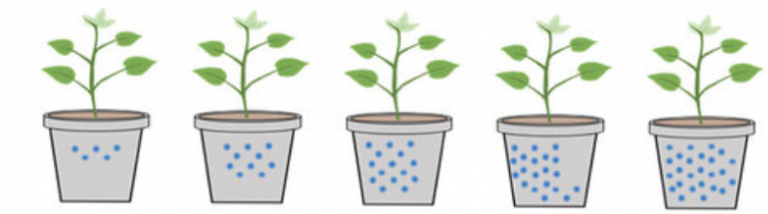

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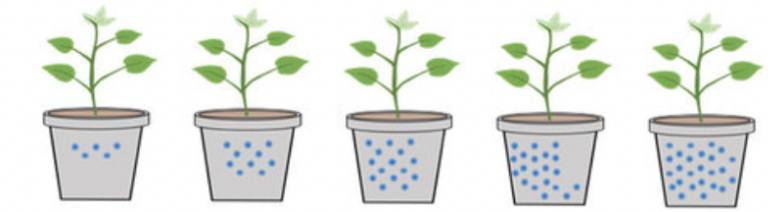


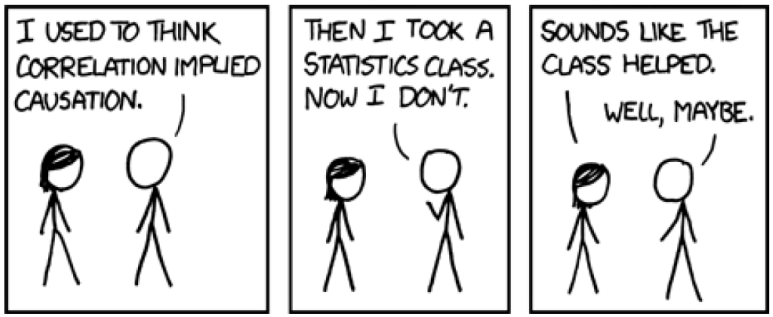
- Different food placed in tanks containing the fish. Fish are not the EUs

Observational vs. Experimental Studies

- An observational study has the same triple of treatment, unit, and response, but one **observes** the assignment of treatments to units (e.g., human health studies on cigarette smoke and adverse health effects)

- What makes an experimental study special is **control**. The experimenter gets to control the assignment of treatments to the experimental units

- Experiments can make causal inference while observational studies find association



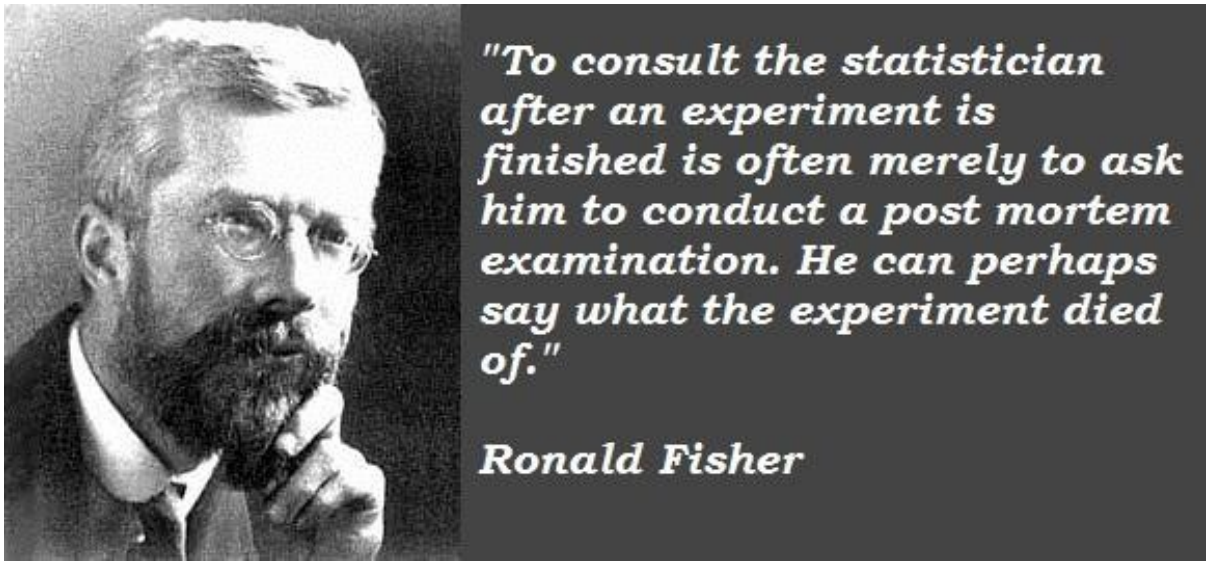
Source: Slide 5 at `http://users.stat.umn.edu/~gary/classes/5303/lectures/Introduction.pdf`

Why Designed Experiments?

- Design for direct comparison of treatments
- Design to reduce bias in comparisons (avoid systematic errors)
- Design to reduce variability (be precise)
- Experiments support causual inference

A Brief History of Experimental Design: Agricultural Era

- R. A. Fisher and his co-workers, Rothamsted Agricultural Experimental Station (1930, England)
- Introduced statistical experimental design and data analysis. Summarized the fundamental principles: **replication**, **randomization**, and **blocking**
- Factorial designs, ANOVA



A Brief History of Experimental Design: Industrial Eras

- The first industrial era, 1951 - late 1970s
  - Process modeling and optimization
  - G. E. P. Box & K. B. Wilson and coworkers in chemical industries and other processing industries
  - Empirical modeling, response surface methodologies, central composite design
- The second industrial era, late 1970s - 1990
  - Quality improvement and variation reduction
  - G. Taguchi and robust parameter design

A Brief History of Experimental Design: Modern Era

- Popular outside statistics, and an indispensable tool in many scientific/engineering endeavors
- New challenges:
  - Large and complex experiments, e.g., screening design in pharmaceutical industry, experimental design in biotechnology
  - **Computer experiments:** efficient ways to model complex systems based on computer simulation
  - ...

Fundamental Principles: Replication, Randomization, and Blocking

Replication

- Each treatment is applied to (experimental) units that are representative of the population
- Enable the estimation/quantification of **experimental error** using standard deviation
- Decrease variance of estimates and increase the power to detect significant differences: for independent $y_i's$,

$$\mathrm{Var}(\frac{1}{n}\sum_{i=1}^{n} y_i) = \frac{1}{n}\mathrm{Var}(y_1)$$

Randomization

Use of a chance mechanism such as random number generators to assign treatments to (experimental) units. It has the following advantages:

- Protect against latent variables or “lurking” variables
- Reduce influence of subjective bias in treatment assignments (e.g., clinical trials)
- Ensure validity of statistical inference

Blocking

A **block** refers to a collection of homogeneous units. Effective blocking : larger between-block variations than within-block variations.

Examples: hours, batches, lots.

- Run and compare treatments within the same blocks to eliminate block-block variation and reduce variability of treatment effects estimates
- Block what you can and randomize what you cannot

Summary

This slides cover:

- Basic concepts of design of experiments (DOE):
- A brief history of DOE
- Fundamental principles: randomization, blocking, replication

Plans for the Next Three Weeks

- Completely Randomized Designs
- Randomized Complete Block Designs, Factorial Designs, and Split-Plot Designs
- Random and Mixed Effects Models
- Computer Experiments