

DSA 8020 R Session 9: Completely Randomized Designs

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Completely Randomized Design (CRD)

Create the data set

```
r1 <- c(9.8, 8.8, 8.4, 9.5, 9.2)
r2 <- c(8.2, 6.9, 7.5, 7.1, 6.5)
r3 <- c(6.8, 6.6, 5.9, 7.3, 7.2)
r4 <- c(4.8, 5.2, 5.4, 5.9, 4.6)
times <- c(r1, r2, r3, r4)
trt <- rep(1:4, each = 5)
dat <- data.frame(y = times, trt = as.factor(trt))
```

Summary statistics by treatments

```
(means <- tapply(dat$y, dat$trt, mean))
```

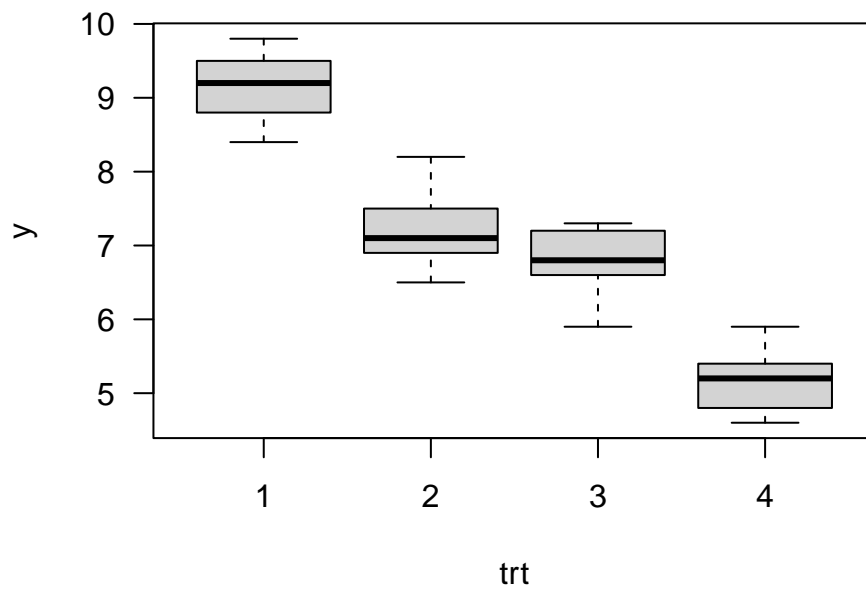
```
##      1      2      3      4  
## 9.14 7.24 6.76 5.18
```

```
(vars <- tapply(dat$y, dat$trt, var))
```

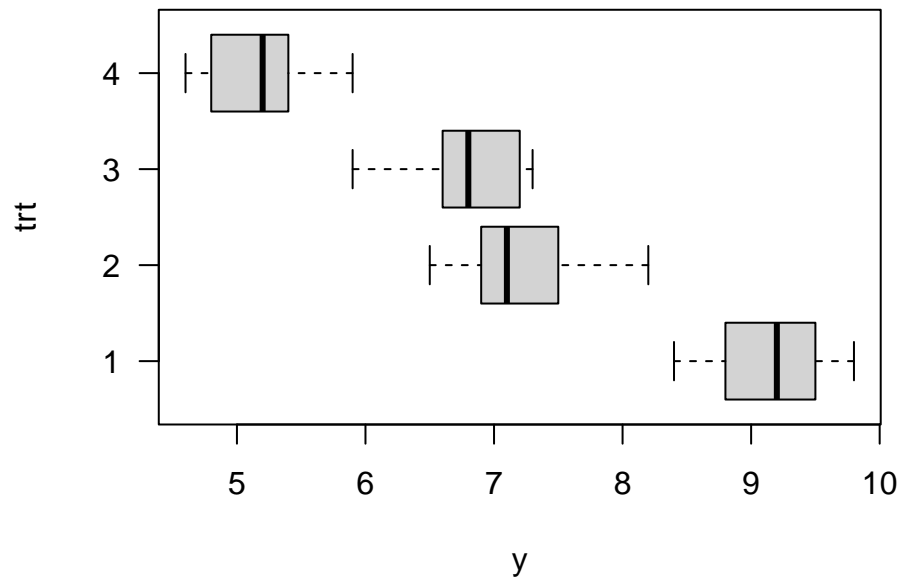
```
##      1      2      3      4  
## 0.308 0.418 0.313 0.262
```

Plot the data

```
boxplot(y ~ trt, data = dat, las = 1)
```



```
boxplot(y ~ trt, data = dat, las = 1, horizontal = T)
```



ANOVA

```
AOV <- aov(y ~ trt, data = dat)
summary(AOV)
```

```
##           Df Sum Sq Mean Sq F value    Pr(>F)
## trt         3  39.91  13.303    40.9 9.92e-08 ***
## Residuals   16   5.20   0.325
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Multiple Comparisons

```
# LSD
library(agricolae)
LSD_bon <- LSD.test(AOV, "trt", p.adj = "bonferroni")
LSD_bon$groups
```

```
##      y groups
## 1 9.14      a
## 2 7.24      b
## 3 6.76      b
## 4 5.18      c
```

```
# HSD
HSD <- TukeyHSD(AOV, conf.level = 0.95)
HSD$trt
```

```
##      diff      lwr      upr      p adj
## 2-1 -1.90 -2.931952 -0.868048 4.024593e-04
```

```
## 3-1 -2.38 -3.411952 -1.348048 3.310735e-05
## 4-1 -3.96 -4.991952 -2.928048 4.112087e-08
## 3-2 -0.48 -1.511952 0.551952 5.577630e-01
## 4-2 -2.06 -3.091952 -1.028048 1.708962e-04
## 4-3 -1.58 -2.611952 -0.548048 2.363679e-03
```

Model Assumptions

Example: Balloon Experiment (taken from Dean and Voss Exercise 3.12)

The experimenter (Meily Lin) had observed that some colors of birthday balloons seem to be harder to inflate than others. She ran this experiment to determine whether balloons of different colors are similar in terms of the time taken for inflation to a diameter of 7 inches. Four colors were selected from a single manufacturer. An assistant blew up the balloons and the experimenter recorded the times with a stop watch. The data, in the order collected, are given in Table 3.13, where the codes 1, 2, 3, 4 denote the colors pink, yellow, orange, blue, respectively.

Table 3.13 Times (in seconds) for the balloon experiment

Time order	1	2	3	4	5	6	7	8
Coded color	1	3	1	4	3	2	2	2
Inflation time	22.0	24.6	20.3	19.8	24.3	22.2	28.5	25.7
Time order	9	10	11	12	13	14	15	16
Coded color	3	1	2	4	4	4	3	1
Inflation time	20.2	19.6	28.8	24.0	17.1	19.3	24.2	15.8
Time order	17	18	19	20	21	22	23	24
Coded color	2	1	4	3	1	4	4	2
Inflation time	18.3	17.5	18.7	22.9	16.3	14.0	16.6	18.1
Time order	25	26	27	28	29	30	31	32
Coded color	2	4	2	3	3	1	1	3
Inflation time	18.9	16.0	20.1	22.5	16.0	19.3	15.9	20.3

Figure 1: Source: Table 3.13 of Dean and Voss Exercise 3.12

Read the data into R

```
balloon <- read.csv("cr_assumptions.csv", header = T)
head(balloon)
```

```
##   ORDER COLOR TIME
## 1     1     1 22.0
## 2     2     3 24.6
## 3     3     1 20.3
## 4     4     4 19.8
## 5     5     3 24.3
## 6     6     2 22.2
```

```
summary(balloon)
```

```
##      ORDER      COLOR      TIME
## Min.   : 1.00   Min.   :1.00   Min.   :14.00
## 1st Qu.: 8.75   1st Qu.:1.75   1st Qu.:17.40
## Median :16.50   Median :2.50   Median :19.70
## Mean   :16.50   Mean   :2.50   Mean   :20.24
## 3rd Qu.:24.25   3rd Qu.:3.25   3rd Qu.:22.60
## Max.   :32.00   Max.   :4.00   Max.   :28.80
```

```
head(balloon, 10)
```

```
##      ORDER COLOR TIME
## 1         1     1 22.0
## 2         2     3 24.6
## 3         3     1 20.3
## 4         4     4 19.8
## 5         5     3 24.3
## 6         6     2 22.2
## 7         7     2 28.5
## 8         8     2 25.7
## 9         9     3 20.2
## 10        10     1 19.6
```

Convert variable *COLOR* to a factor

```
attach(balloon)
colorf <- as.factor(COLOR)
colorf
```

```
## [1] 1 3 1 4 3 2 2 2 3 1 2 4 4 4 3 1 2 1 4 3 1 4 4 2 2 4 2 3 3 1 1 3
## Levels: 1 2 3 4
```

Model Fitting and Residuals

```
mod1 <- lm(TIME ~ colorf)
summary(mod1)
```

```
##
## Call:
## lm(formula = TIME ~ colorf)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -5.8750 -2.2500  0.0687  2.0531  6.2250
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   18.337      1.162   15.778 1.83e-15 ***
## colorf2         4.237      1.644    2.578  0.0155 *
## colorf3         3.538      1.644    2.152  0.0401 *
```

```
## colorf4      -0.150      1.644  -0.091   0.9279
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.287 on 28 degrees of freedom
## Multiple R-squared:  0.2967, Adjusted R-squared:  0.2214
## F-statistic: 3.938 on 3 and 28 DF,  p-value: 0.01836
```

```
anova(mod1)
```

```
## Analysis of Variance Table
##
## Response: TIME
##           Df Sum Sq Mean Sq F value    Pr(>F)
## colorf      3 127.66  42.554   3.9379 0.01836 *
## Residuals  28 302.58  10.806
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Residuals
r <- residuals(mod1)
s <- rstandard(mod1)
var(s)
```

```
## [1] 1.032258
```

```
t <- rstudent(mod1)
```

Assess Equal Variance

```
# Levene's test for equal variance
library(lawstat)
levene.test(TIME, colorf, location = "mean")
```

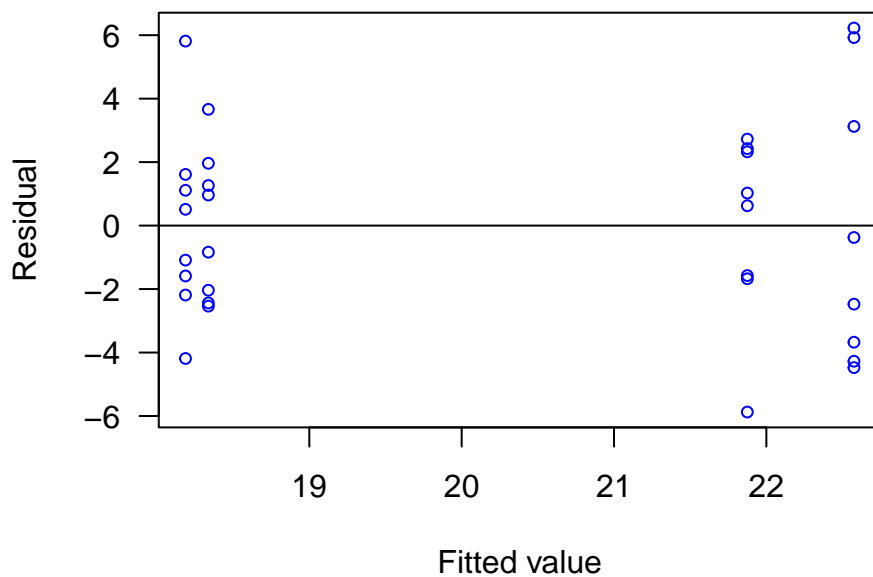
```
##
## Classical Levene's test based on the absolute deviations from the mean
## ( none not applied because the location is not set to median )
##
## data:  TIME
## Test Statistic = 2.1682, p-value = 0.1141
```

```
# Brown-Forsythe test
levene.test(TIME, colorf, location = "median")
```

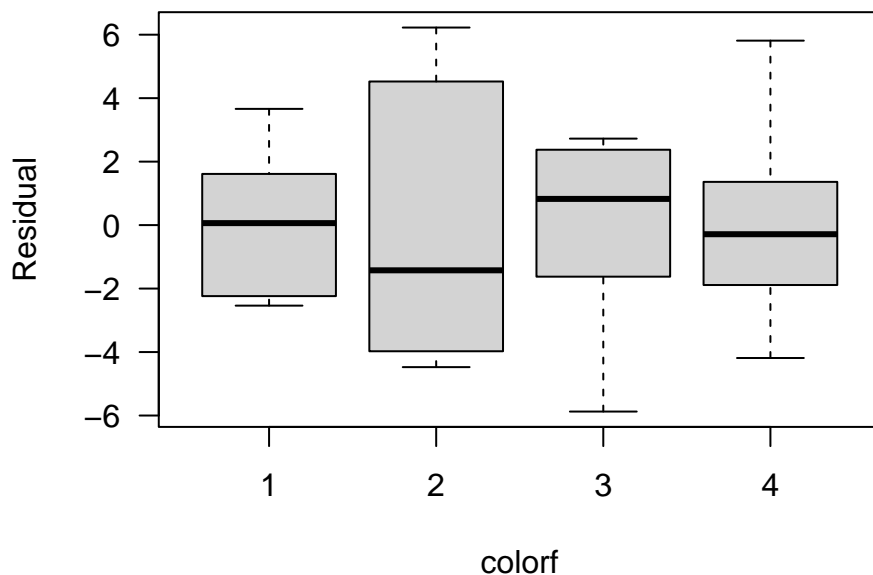
```
##
## Modified robust Brown-Forsythe Levene-type test based on the absolute
## deviations from the median
##
## data:  TIME
## Test Statistic = 1.3975, p-value = 0.2642
```

Plot r_{ij} versus \hat{y}_i and treatments

```
plot(mod1$fitted, mod1$resid, las = 1, xlab = "Fitted value", ylab = "Residual",
     cex = 0.75, col = "blue")
abline(h = 0)
```

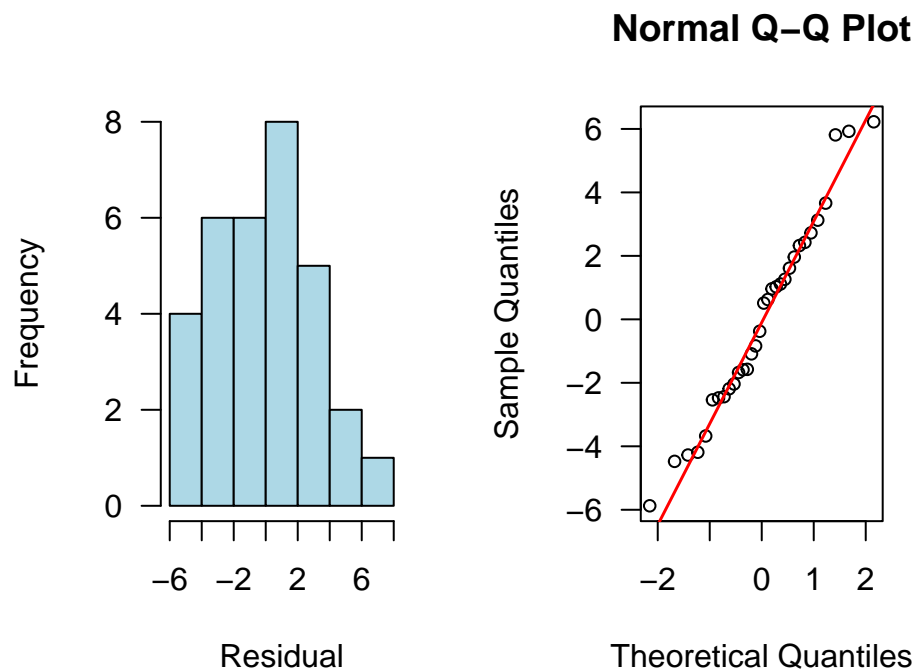


```
plot(mod1$resid ~ colorf, ylab = "Residual", las = 1)
```



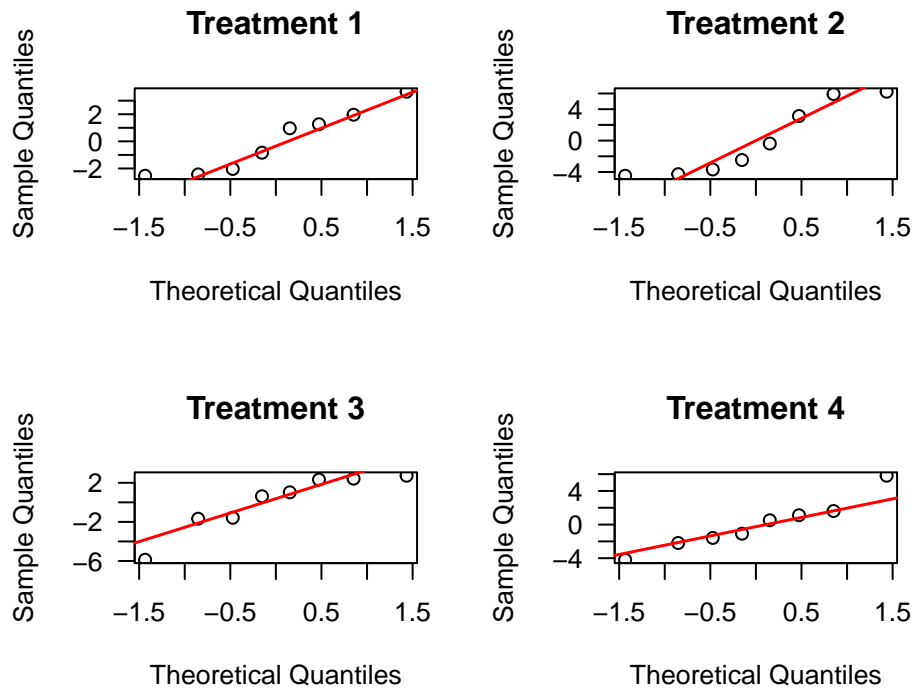
Assess Normality

```
par(mfrow = c(1, 2), las = 1)
hist(mod1$resid, 8, main = "", xlab = "Residual", col = "lightblue")
qqnorm(mod1$resid, cex = 0.8)
qqline(mod1$resid, col = "red", lwd = 1.5)
```

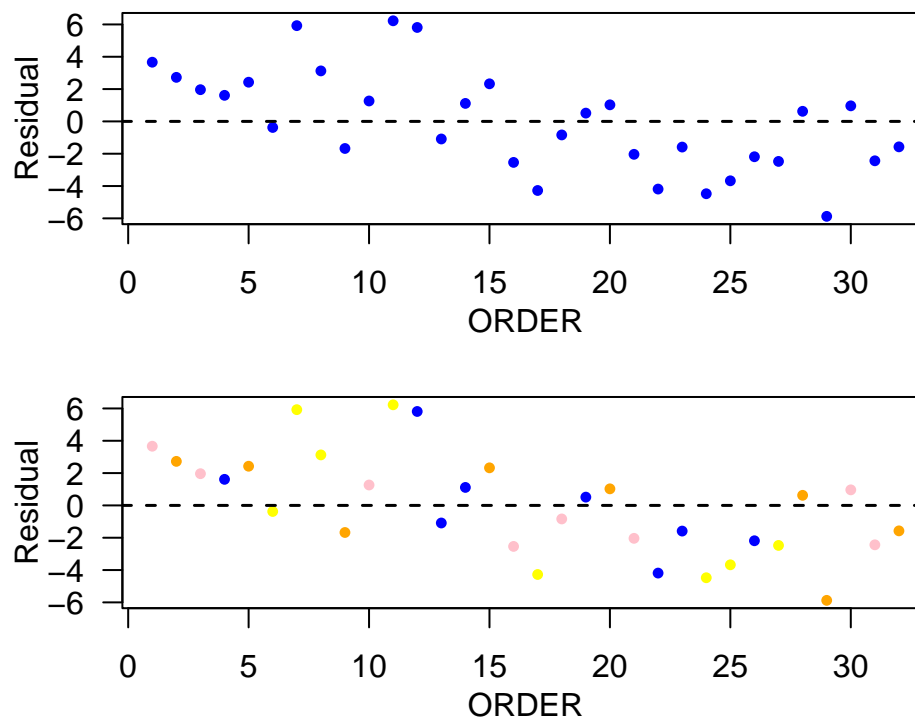


QQ-plot by treatment

```
par(mfrow = c(2, 2), las = 1)
new <- data.frame(colorf, mod1$resid)
trt <- paste("Treatment", 1:4)
for (i in 1:4){
  newc1 <- new[colorf == i,]
  qqnorm(newc1$mod1.resid, las = 1,
    main = trt[i])
  qqline(newc1$mod1.resid,
    col = "red", lwd = 1.5)
}
```

Assess Independence



Durbin-Watson test

```
library(lmtest)
dwtest(TIME ~ colorf, data = balloon)
```

```
##
## Durbin-Watson test
##
## data: TIME ~ colorf
## DW = 1.1617, p-value = 0.006005
## alternative hypothesis: true autocorrelation is greater than 0
```

Fit a model with correlated AR(1) error

```
library(nlme)
mod2 <- gls(TIME ~ colorf, correlation = corARMA(p = 1, q = 0))
mod2
```

```
## Generalized least squares fit by REML
## Model: TIME ~ colorf
## Data: NULL
## Log-restricted-likelihood: -74.42885
##
## Coefficients:
## (Intercept)      colorf2      colorf3      colorf4
## 18.5860865    3.7248742    3.4233901   -0.3578644
##
## Correlation Structure: AR(1)
## Formula: ~1
## Parameter estimate(s):
##      Phi
## 0.4285025
## Degrees of freedom: 32 total; 28 residual
## Residual standard error: 3.321057
```