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Mathematical Model of Foraging in a Group of Robots: Effect of Interference

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Abstract. In multi-robot applications, such as foraging or collection tasks, interference, which results from competition for space between spatially extended robots, can significantly affect the performance of the group. We present a mathematical model of foraging in a homogeneous multi-robot system, with the goal of understanding quantitatively the effects of interference. We examine two foraging scenarios: a simplified collection task where the robots only collect objects, and a foraging task, where they find objects and deliver them to some pre-specified "home" location. In the first case we find that the overall group performance improves as the system size grows; however, interference causes this improvement to be sublinear, and as a result, each robot's individual performance decreases as the group size increases. We also examine the full foraging task where robots collect objects and deliver them home. We find an optimal group size that maximizes group performance. For larger group sizes, the group performance declines. However, again due to the effects of interference, the individual robot's performance is a monotonically decreasing function of the group size. We validate both models by comparing their predictions to results of sensor-based simulations in a multi-robot system and find good agreement between theory and simulations data.

Keywords: robotics, foraging, mathematical modeling

1. Introduction

Robot collection and foraging are two of the oldest and most studied problems in robotics. In these tasks a single robot or a group of robots (Goldberg and Matarić, 2000; Parker, 1994) has to collect objects scattered around the arena and to assemble them either in some random location (collection task (Beckers et al., 1994; Martinoli et al., 1999)) or a pre-specified "home" location (foraging task (Matarić, 1992; Goldberg and Matarić, 2000; Nitz et al., 1993)). These tasks have been studied under a wide variety of conditions and architectures, both experimentally and in simulation: in homogeneous (Goldberg and Matarić, 2000) and heterogeneous (Goldberg and Matarić, 2000; Parker, 1994) systems, using behavior-based (Matarić, 1992; Goldberg and Matarić, 2000) and hybrid control (Nitz et al., 1993), no communication (Goldberg and Matarić, 2000), direct communication (Nitz et al., 1993; Sugawara and Sano, 1997), as well as indirect communication through the environment (Holland and Melhuish, 2000; Vaughan et al., 2000b). The broad appeal of this problem is explained both by ubiquity of collection in general and foraging in particular in nature—as seen in the food gathering behavior of many insects—as well as its relevance to many military and industrial applications, such as de-mining, mapping and toxic waste clean-up.

There are several reasons to study foraging in a group of robots. Besides providing a test-bed for the design of physical robots and their controllers, foraging serves as a useful framework for exploring many issues in the design and implementation of multi-robot teams. Additionally, deploying a team of robots to perform a collection or a foraging task is often of practical importance: it introduces robustness and parallelism. Many robots working in parallel may complete the task faster. Additionally, the performance of the group may not be affected by individual robot failure. However, having a task done by a team of robots working together introduces problems not present in single robot systems. One of the most critical issues is the effect of interference, as manifested by collision avoidance, on the performance of the group. Interference has long been recognized as an important issue in multi-robot design (Fontan and Matarić, 1996; Sugawara and Sano, 1997). While most of the research has concentrated on minimizing interference (through communication, collaboration), few attempts (Nitz et al., 1993) have been made to characterize it and study it quantitatively.

Biological metaphor has been successfully applied to the design of controllers for collection and foraging tasks in reactive (Holland and Melhuish, 2000; Martinoli et al., 1999) and behavior-based (Matarić, 1992; Goldberg and Matarić, 2000) multi-robot systems. In addition to providing a distributed control mechanism, biological metaphor offers several other advantages for collective robotics systems over alternative designs: (i) scalability: each robot has the same controller whether the group is composed of ten or 10,000 robots; (ii) robustness: group performance is robust to individual agent failure; (iii) flexibility: robots can be dynamically added or removed without significantly affecting the performance of the system; (iv) local sensing: in many cases the desired collective behavior can be achieved via local interactions only; (v) adaptability: allows for simple learning that enables robots to operate in uncertain hostile environments.

A distributed control mechanism based on simple local interactions offers still another advantage over alternative designs: it can be mathematically modeled and analyzed. Mathematical analysis is an alternative to experiment and simulation, two principal tools used by the robotics community. While experiments allow researchers to study the systems's behavior in real environments, they are very costly and time-consuming, both to set-up and execute. Sensor-based simulations, such as Webots (Michel, 1998) and Player/Stage (Gerkey et al., 2001), recreate the experiments under realistic conditions, but they usually don't scale well as the size of the system grows, and are, therefore, impractical tools for a detailed investigation of the properties of some multi-robot systems. As a result, many of the questions remain unanswered: e.g., for a given task size, is there an optimal number of robots that will complete the task in the shortest period of time? Using mathematical analysis, on the other hand, we can efficiently study dynamics of even very large robot groups, predict their long term behavior, gain insight into system design: e.g., what parameters determine group behavior, optimize performance, prevent instabilities, etc. However, with the exception of the research by Sugawara and coworkers (Sugawara and Sano, 1997; Sugawara et al., 1997), multi-robot systems in general, and foraging in particular, have not been analyzed mathematically.

In this paper, we present a mathematical model of foraging in a homogeneous group of robots. We analyze the behavior of the system, focusing on quantitative characterization of the effects of interference on the performance of the group. In Section 3.1 we introduce and analyze a model of the simplified collection scenario in which the robots find and collect pucks but don't bring them home. In Section 3.2 we examine the full foraging scenario that includes delivering the pucks home. We validate both models by comparing their predictions to results of sensor-based simulations of foraging in a multi-robot system.

2. Foraging in a Group of Robots

We present a mathematical model of foraging in a homogeneous group of robots using behavior-based control, the type of system studied by Mataric and collaborators (Matarić, 1992; Goldberg and Matarić, 2000). Figure 1 is a snapshot of a typical experiment with four robots. The robots' task is to collect small pucks

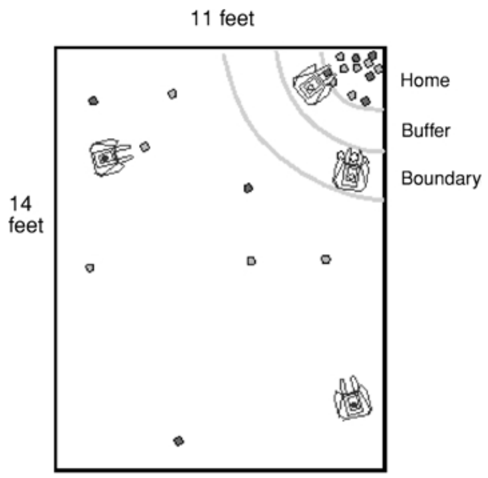


Figure 1. Diagram of the foraging arena (courtesy of D. Goldberg).

scattered randomly around the arena. The arena itself is divided into a search region and a small "home", or goal, region where the collected pucks are deposited. The "boundary" and "buffer" regions are part of the home region and are made necessary by limitations in the robots' sensing capabilities, as described below. Each robot has an identical set of behaviors governed by the same controller. The behaviors that arise in the collection task are (Goldberg and Matarić, 2000):

Avoiding obstacles, including other robots and boundaries. This behavior is critical to the safety of the robot.
Wandering or searching for pucks: robot moves forward and at random intervals turns left or right through a random arc. If the robot enters the Boundary region, it returns to the search region. This prevents the robot from collecting pucks that have already been delivered.
Detecting: a puck.
Grabbing: a puck.
Homing: if carrying a puck, move towards the home location.
Creeping: activated by entering Buffer region. The robot will start using the close-range detectors at this point to avoid the boundaries.
Home: robot drops the puck. This activates the exiting behavior.
Exiting: robot exits the home region and resumes search.

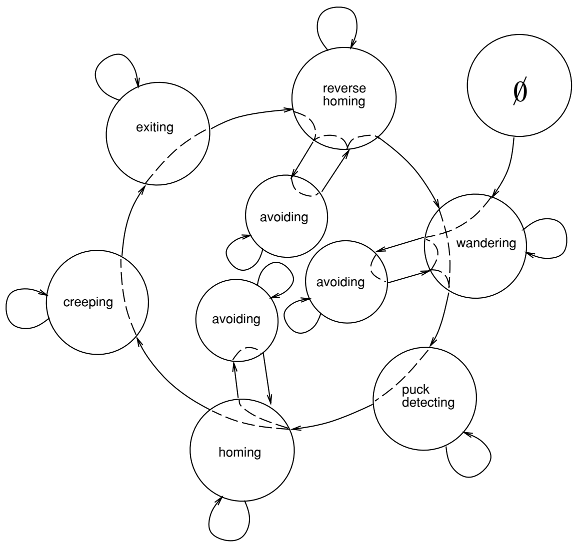


Figure 2. State diagram of the foraging robot behaviors from Goldberg et al. (Goldberg and Matarić, 1999). The diagram was constructed automatically by analyzing behavior data from the foraging experiments.

Figure 2 shows the sequence of behaviors that the robot engages in during the foraging task. This graph was constructed automatically by analyzing behavior data from foraging experiments (Goldberg and Matarić, 1999). The presence of three separate avoiding states is necessary to prevent a wandering (searching) robot from making a transition to the homing state through a common avoiding state.

2.1. *Interference*

In the foraging scenario outlined above, robots act completely independently, without communicating directly or through the environment. Interference is the only interaction between the robots, and it is caused by competition for space between spatially extended robots. When two robots find themselves within sensing distance of one another, they will execute obstacle avoiding maneuvers in order to reduce the risk of a potentially damaging collision. The robot stops, turns in place by some angle and moves forward. This behavior takes time to execute; therefore, avoidance increases the time it takes the robot to find pucks and deliver them home. Clearly, a single robot working alone will not experience interference from other robots. However, if a single robot fails, as is likely in a dynamic, hostile environment, the collection task will not be completed. A group of robots, on the other hand, is robust to an individual's failure. Indeed, many robots may fail but the performance of the group may be only moderately affected. Many robots working in parallel may also speed up the collection task. Of course, the larger the group, the greater the degree of interference—in the extreme case of a crowded arena, robots will spend all their time avoiding other robots and will not bring any pucks home.

Interference has long been recognized as a critical issue in multi-robot systems (Fontan and Matarić, 1996; Sugawara and Sano, 1997). Several approaches to minimize interference have been explored, including communication (Parker, 1998) and cooperative strategies such as trail formation (Vaughan et al., 2000a) and bucket brigade (Fontan and Matarić, 1996; Østergaard et al., 2001). In some cases, the effectiveness of the strategy to minimize interference will also depend on the group size (Østergaard et al., 2001). Therefore, it is important to quantitatively understand interference between robots and how it relates to the group and task sizes before choosing alternatives to the default strategy. For some tasks and a given controller, there may exist an optimal group size that maximizes the performance of the system (Nitz et al., 1993; Fontan and Matarić, 1996; Østergaard et al., 2001). Beyond this size the adverse effects of interference become more important than the benefits of increased robustness and parallelism, and it may become beneficial to choose an alternate foraging strategy. We will study interference mathematically and attempt to answer these questions.

2.2. *Player/Stage Multi-Robot Simulator*

We validate the mathematical model by comparing its predictions to the results of foraging simulations. We used Player/Stage to simulate the foraging task with groups of robots. Player/Stage is a client/server-based scalable multi-robot simulator developed at the USC Robotics Lab (Gerkey et al., 2001). Player is a network-based interface to the onboard sensors and actuators that constitute a robot, while Stage supports virtual Player robots, sensing and moving in a two-dimensional bitmapped world, that interact with simulated devices. Available sensor models include sonar, laser rangefinder, pan-tilt-zoom camera with color "blob" detection and odometry.

The Stage world consists of a circular arena, with robots and pucks initially randomly distributed around the arena. Each robot comes equipped with a ring of 16 sonars, evenly distributed around its perimeter, for the purpose of obstacle avoidance, a color camera and a vision system to locate "colored" pucks, a gripper for picking up the puck, and an odometry system to help robot find "home" and move towards it (used in Sec. 3.2 only). We simulated foraging task in groups of one to ten robots, each given a task to collect (or collect and deliver home) 20 pucks. For each group of robots, we averaged results of several, usually ten, simulations. Simulation parameters are listed in Table 1.

Table 1. Simulation parameters.

Parameter	Value	Parameter	Value
No. of robots	1–10	Avoid time	3 s
No. of pucks	20	Avoid dist	250 mm
Robot radius	0.2 m	Robot speed	300 mm/s
Puck radius	0.05 m	Min detect area	200 pixels
Arena radius	3 m	Rev. homing time	10 s
Home radius	0.75 m		

2.2.1. Behavior Structure. The robots' behavior structure closely replicates that of the robots studied in experiments (Goldberg and Matarić, 2000). Behavior-based control governs the actions of the simulated robots. The following behaviors were used:

0 *Search for pucks*: robot executes a random walk around the arena until a puck is found with a camera. The puck is "painted" some bright color, so that it can be seen with a color camera. The size of the puck in the robot's visual field must exceed some minimum detection area (in pixels), before the robot recognizes it as a puck.
1 *Collect pucks*: under this behavior the robot will visually servo towards the puck and collect it with a gripper. The gripper may fail to pick up a puck with some small probability, consistent with failure under experimental conditions due to unreliability of real grippers and sensor update rates.
2 *Go home*: after the puck has been collected, the robot will odometrically servo towards the home location and deposit the puck there. Home is a semicircular region centered on a point at the edge of the arena.
3 *Reverse homing*: the robot moves away from home a specified distance in a random direction.
4 *Avoid collisions*: If a close obstacle (another robot or arena wall) is sensed at any time, the robot will turn away from the obstacle in a random direction at 40 deg/s for a time specified by the avoid time parameter.

For purposes of analysis only, we split behavior **4** into two distinct behaviors: **4**—avoiding collisions while behaviors **0**, **1** and **3** are active, and **5**—avoiding collisions while homing, i.e., when behavior **2** is active.

2.2.2. A Note on Calculating Parameters. In the mathematical models presented below, we will use a set of parameters to connect the model to experiments and simulations. The main parameters we will use are α_p, α_r, the rate at which a robot encounters a puck and another robot respectively. In principle, these parameters can be computed ab initio by taking into account the details of the robots dimensions and sensing capabilities in the following way: as a robot travels through the arena, it sweeps out some area during time interval dt and will detect objects that fall in that area. This detection area is $vw_i dt$, where v is robot's speed, and w_i is robot's detection width for object of type i. This number is the sum of the sizes of the robot and the object it is trying to detect, and the detection distance associated with the sensing hardware it is using to detect that object (e.g. sonar, camera resolution, etc.). If the arena radius is R with N_i objects of type i distributed uniformly around it, a robot will detect these objects at a rate $\alpha_i = v w_i N_i / \pi R^2$. This idealization is useful for roughly estimating model parameters, but because it omits all the details of the experiment (such as sensor errors and failures), it does not get them right. A better way is to estimate them by fitting the model to experimental data, or by calibrating the model by measuring these parameters experimentally or in simulation for a single robot and using this value in the calculations. In order to estimate α_r by calibration, for instance, we have to run the experiment or simulation for two robots in an empty arena, keeping track of the number of times each robot attempts collision avoidance maneuvers. Likewise, to estimate α_p, we have to run the experiment or simulation for a single robot and some pucks scattered around the arena, keeping track of the rate at which the robot picks them up. Although we did not perform these calibrations explicitly, we can estimate the parameters from the data in the first two rows of Table 2: $\alpha_r = n\ collisions/T_{total} = 0.06$ (note that this number includes wall collisions), and $\alpha_p = T_1/(20 \cdot T_{total}) = 0.02$. These numbers are very close to the values we used, which we determined (by eye) to give the best agreement between theory and simulations. Note that this calibration can be done in simulation for a single robot for an environment of arbitrary complexity, and the parameters can be used to study the performance of teams of robots quantitatively in the same complex environment.

Table 2. Average time (in seconds) each robot spends in the active behaviors during the simplified foraging task (**0**: search, **1**: collect, **4**: avoid) during the time it took the group of robots to collect all pucks.

Robots	**0**	**1**	**4**	Collisions
1	75.22	71.03	24.15	7.9
2	57.04	37.35	22.19	7.2
3	33.90	24.88	13.61	4.7
4	24.15	18.41	12.75	4.3
5	23.18	14.89	13.50	4.3
6	21.74	13.53	14.93	4.8
7	16.72	11.34	11.24	3.9
8	14.09	9.87	11.06	3.7
9	16.67	9.14	13.31	4.4
10	16.75	7.54	13.96	4.6

The last column gives the average number of obstacle avoidance maneuvers per robot during the same time period.

3. Mathematical Analysis of Foraging

As mentioned above, interference is the result of competition between two or more robots for the same resource, be it physical space, a puck both are trying to pick up, energy, communications channel, etc. In the collection and foraging tasks, competition for physical space, and the resulting avoidance of collisions with other robots, is the most common source of interference. In order to understand interference quantitatively, we will first examine the simplified foraging task that includes searching and avoiding only. This task can be implemented with a subset of robot behaviors listed in Section 1, namely searching, avoiding, detecting a puck and grabbing it. This scenario may be realized experimentally by allowing robots to pick up a puck and store it in a carrying pouch, for instance. Then we will examine the full foraging scenario, where robots are required to deliver collected pucks to a home location.

In Lerman and Galstyan (2001) we presented a methodology for constructing mathematical models that describe the dynamics of collective behavior of multi-agent systems. The methodology applies to Markov systems, in which each agent's state at a future time depends only on its present state and none of its past states. While this may seem as a restrictive criterion, it is satisfied by many behavior-based and reactive robot systems. In the context of robotics, *state* labels a set of related robot behaviors required to accomplish a task. Thus, the search state may consist of the *wandering* and *puck detecting* behaviors, or we may simply take each behavior to be a separate state. The mathematical model consists of a series of coupled differential equations, one for each state, each of which describes how the average number of agents in that state changes in time. The equations may be solved analytically or numerically, allowing us to quantitatively study the behavior of the multi-agent system. Below we construct and solve a mathematical model of two foraging scenarios, with an emphasis on analyzing the effects of interference.

3.1. Searching and Avoiding

In order to construct a mathematical model of the simplified foraging scenario, the "searching and avoiding"-only case, it is helpful to write down the macroscopic state diagram of the system. During some short time interval, every robot is either in the searching state or the avoiding state, as shown in Fig. 3. We incorporate in the

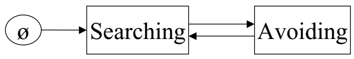


Figure 3. State diagram of the simplified foraging scenario in which robots search and collect pucks, but don't deliver them "home".

search state actions and behaviors such as searching for and collecting a puck (detecting a puck and grabbing it in experiments).

All the robots are initially in the searching state. The searching robots wander around the arena, looking for pucks. If a searching robot detects an obstacle, such as another robot or a wall, it executes avoiding behavior for a time period τ, after which it resumes the search. If, after avoiding for this time period, the robot still finds its path blocked by the obstacle, it will repeat the avoidance maneuver. This is a good approximation of the experimental realization of the avoiding behavior in which robot avoids by turning in place until it senses free space. If a robot encounters a puck, it picks it up and continues searching. This action does not change the robot's state. Let $N_s(t)$ be the number of robots in the search state at time t, and $N_a(t)$ the number of robots in the avoiding state at time t, with $N_s(t) + N_a(t) = N_0$, the total number of robots, a constant. We model the environment by letting $M(t)$ be the number of uncollected pucks at time t. Also, let α_r be the rate of detecting another robot and α_p the rate of detecting a puck. These parameters connect the model to the experiment, and they are related to the size of the robot and the puck, robot's detection distance and speed, as described in Sec. 2.2.2.

Initially, at $t = 0$, there are N_0 searching robots and M_0 pucks scattered around the arena. The following equations specify the dynamics of the system:

$$\frac{dN_s(t)}{dt} = -\alpha_r N_s(t)[N_s(t) + N_0] + \alpha_r N_s(t-\tau)[N_s(t-\tau) + N_0], \quad (1)$$

$$\frac{dM(t)}{dt} = -\alpha_p N_s(t) M(t). \quad (2)$$

The first equation describes how interference affects searching robots. The meaning of the equation is as follows: the number of searching robots decreases when two searching robots detect each other and commence avoiding maneuvers or when a searching robot detects another robot in the avoiding state; it increases when robots that started avoiding behavior at time $t - \tau$ exit the avoiding state and resume searching. We don't need an equation describing the dynamics of the avoiding robots, because we can compute this quantity using the conservation of the total number of robots N_0. Equation (2) says that the number of uncollected pucks decreases in time because searching robots encounter pucks and pick them up. Note, that the rate at which the pucks are collected is proportional to the number of robots in searching mode, $N_s(t)$.

We rewrite the system of Eqs. (1) and (2) in dimensionless form using the following variable transformations: $n_s(t) = N_s(t)/N_0$ (fraction of searching robots), $m(t) = M(t)/M_0$ (fraction of uncollected pucks), $\alpha = \alpha_p/\alpha_r$, $t \to \alpha_r N_0 t$, $\tau \to \alpha_r N_0 \tau$(dimensionless time)

$$\frac{dn_s}{dt} = -n_s(t)[n_s(t) + 1] + n_s(t-\tau)[n_s(t-\tau) + 1], \quad (3)$$

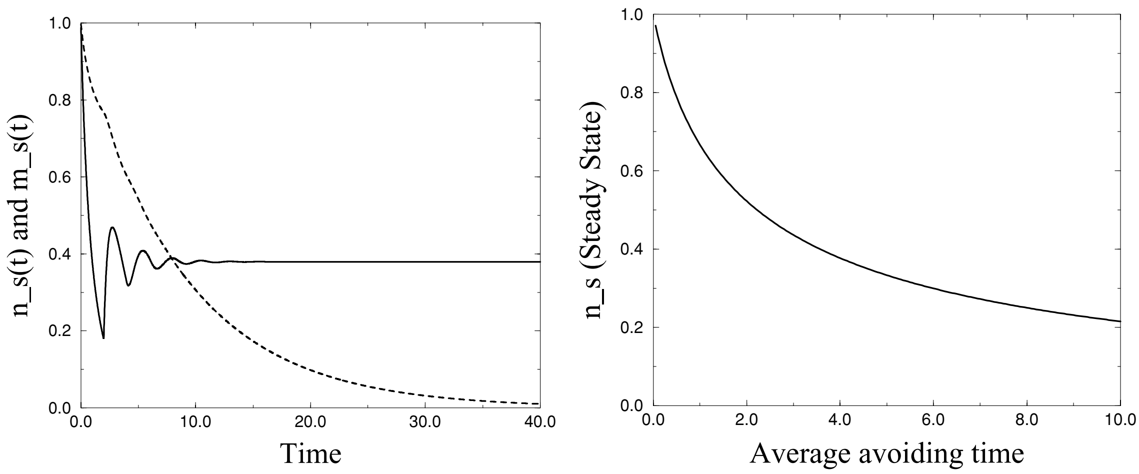
$$\frac{dm}{dt} = -\alpha n_s(t) m(t), \quad (4)$$

subject to initial conditions $n_s(0) = 1, m(0) = 1$. Note that of the experimental parameters α_r, α_p and τ, the first equation depends only on τ; therefore, its solutions will also depend only on τ.

Figure 4(a) shows typical solutions of Eqs. (3) and (4). The fraction of searching robots has a steady state value, which is reached after a period of transient oscillations characteristic of time-delay differential equations. Note, that the steady state value $n_s \equiv n_s(t \to \infty)$ is simply the fraction of time an individual robot spends in the searching mode. Because the avoiding time τ is the only parameter that appears in Eq. (3), the steady state solution is fully determined by τ. Figure 4(b) shows the dependence of the steady state solutions n_s on the avoiding time parameter. Naturally, as the avoiding time increases, a robot spends more time in the avoiding mode, hence decreasing n_s. The dependence of the steady state solution on τ can be obtained using a simple criterion for dynamical equilibrium. Namely, we note that in the steady state the number of robots returning to the searching mode per unit time can be estimated as $n_{av}/\tau \equiv (1 - n_s)/\tau$. Then, balancing the transition rates between two states (searching and avoiding) yields a quadratic equation for n_s with solution

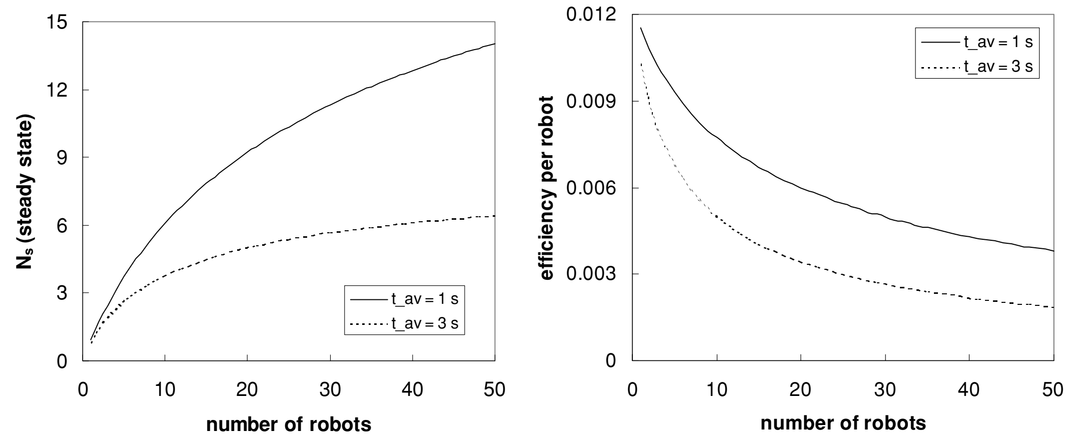
$$n_s = \frac{1}{2}(\sqrt{(1 + 1/\tau)^2 + 4/\tau} - 1 - 1/\tau) \quad (5)$$

Interference depends both on the total number of robots in the system and on the time it takes robots to execute obstacle avoidance behavior. Therefore, we examine next the dependence of the system's



(a) (b)

Figure 4. (a) Fraction of searching robots (solid line) and the fraction of uncollected pucks (dashed line) vs. time. (b) Fraction of searching robots in the steady state vs. avoiding time parameter τ (in dimensionless units).



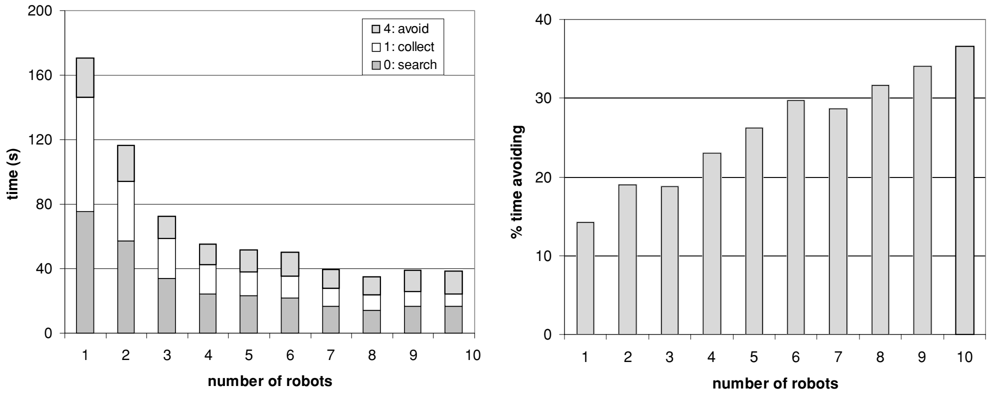
(a) (b)

Figure 5. (a) Number of searching robots vs. the total number of robots for two different values of the avoiding time parameter: $\tau = 1$ s (solid line) and $\tau = 3$ s (dashed line), for $\alpha_p = 0.02$ and $\alpha_r = 0.04$. (b) Efficiency per robot vs. system size for the same parameter values.

performance on the total number of robots. In order to improve performance, is it always beneficial to add robots to the system, or will the negative effects of interference outweigh the added benefits of more workers at some point?

Figure 5(a) shows the number of searching robots in the steady state for two values of the avoiding time parameter as the system size (the total number of robots) grows. The number of searching robots is a monotonically increasing function of the size of the system—no optimal size effect is evident. Moreover, the longer avoiding takes, the fewer searching robots there are at some fixed system size. Figure 5(b) shows the relative efficiency, or the time it takes to collect 80% of pucks per robot, under the same conditions as in (a). We can clearly see that interference adversely affects each robot's *individual* efficiency. The greater the interference effect, either because of the larger group size or the bigger avoiding time parameter, the worse the per robot performance is in the foraging task. Thus adding one robot to the system may increase the overall performance of the group, but it will decrease each robot's individual performance.

3.1.1. Comparison with Simulations. We ran simulations of the simplified scenario in groups of one to



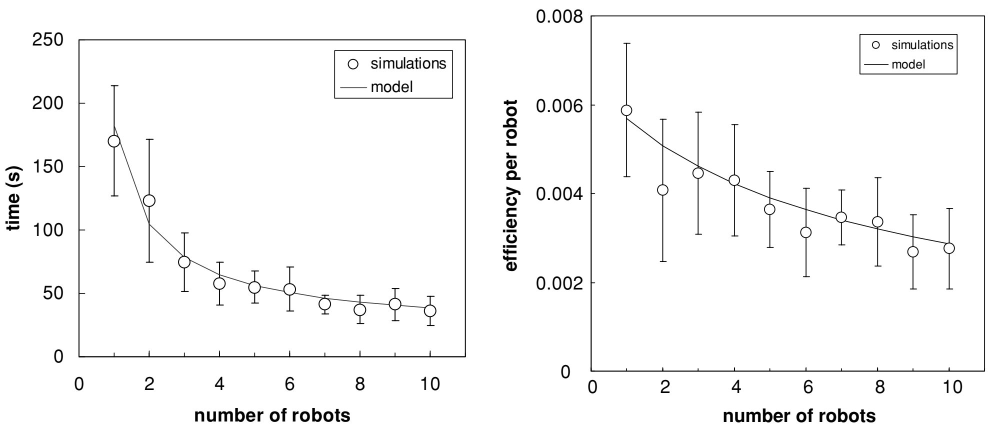
(a) (b)

Figure 6. (a) Average time each robot spent in the active behaviors during the time it took the group to collect all pucks vs robot group size. (b) Percentage of time each robot spent in the avoiding behavior as a function of group size.

ten robots, where the robots had to find and collect twenty pucks. During these simulations only behaviors **0** (search), **1** (collect), and **4** (avoid) were active (see Section 2.2). Note that the searching state of the mathematical model corresponds to behaviors **0** and **1**, while the avoiding state maps directly to the behavior **4**. We ran the simulation ten times for each group of robots and averaged the results. Each simulation ran until the last of the twenty pucks was collected.

Table 2 lists the average amount of time (in seconds) each robot spent in the active behaviors during the time it took the group to collect all twenty pucks. These values are shown graphically in Fig. 6(a). Although it appears at first that robots spend less time avoiding other robots as the size of the group grows, because the total time to collect the pucks also decreases, the relative amount of time each robot spends in the avoiding behavior goes up as a function of the group size, as expected (Fig. 6(b)). The last column gives the average number of times a robot attempted to avoid collisions during the same time period. Note the finite number of collisions for a single robot, which is the result of the robot attempting to avoid collisions with walls. Though this number for a single robot is the largest in the column, the rate, or the number per unit time, of obstacle avoidance maneuvers quickly increases as the size of the group grows.

Figure 7(a) shows the time taken by each group of robots to collect all twenty pucks. The solid line gives the prediction of the mathematical model for $\tau = 3$ s, $\alpha_p = 0.02$ and $\alpha_r = 0.04$. The model agrees with the simulations well within experimental error.



(a) (b)

Figure 7. (a) Time taken by each group of robots to collect all pucks. The solid line is the prediction of the model with $\tau = 3$ s, $\alpha_p = 0.02$ and $\alpha_r = 0.04$. (b) Efficiency per robot.

There is no significant difference in the quality of the fit when wall avoidance by robots is taken into account. Because the time it takes a group of robots to complete the task decreases, the group's efficiency, defined as the inverse of the time it takes the robots to complete the task, increases. However, as pointed out earlier, *efficiency per robot* decreases as the group size grows (cf. Fig. 5(b)). The per-robot efficiency, along with the model's prediction, is plotted in Fig. 7(b).

3.2. Searching, Homing and Avoiding

In the previous section we considered a very simple model of the foraging task consisting of two elementary behaviors, searching for pucks and avoiding obstacles. Our main conclusion was that interference effectively decreases the number of searching robots. We also showed that adding robots always increases overall performance of the system but leads to a deterioration of the individual robot performance. In this section we examine the scenario where robots are required to collect pucks and bring them to a specified "home" location.

Figure 8 shows the state diagram for foraging with homing. Initially the robots are in the search state. When a searching robot encounters a puck, it picks it up and moves toward the "home" region. Execution of the homing behavior requires a period of time τ_h. At the end of this period, the robot deposits the puck at home and resumes the search for more pucks. While a robot is either searching or homing, it will encounter and try to avoid obstacles for a time period τ after which it returns to its previous state. There are two separate avoiding states to preclude robots from moving from the searching to the homing state, or *vice versa*, through the common avoiding state.

Each state in the diagram corresponds to a dynamic variable. Let $N_s(t)$, $N_h(t)$, $N_s^{av}(t)$, $N_h^{av}(t)$ be the number of searching, homing, avoiding while searching and avoiding while homing robots at time t, with the total number of robots, $N_0 = N_s(t) + N_h(t) + N_s^{av}(t) + N_h^{av}(t)$, a constant. We model the environment by letting $M(t)$ be the number of undelivered pucks at time t.

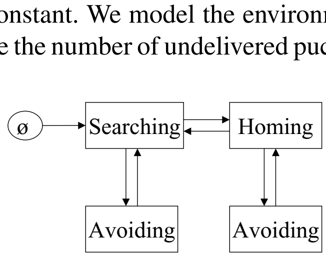


Figure 8. State diagram of a multi-robot foraging system with homing.

Also, let α_r be the rate of detecting another robot and α_p the rate of detecting a puck. These parameters connect the model to the experiment, and they are related to the size of the robot and the puck, robot's detection radius and the speed of the robot. It was shown experimentally (Goldberg and Matarić, 2000) that interference is most pronounced near the home region, because the density of robots is, on average, greater there. Therefore, we expect the rate of encountering other robots to be greater near the home region and introduce α_r', the rate of detecting another robot while homing. The following equations describe the time evolution of the dynamic variables[1]:

$$\begin{aligned}\frac{dN_s(t)}{dt} &= -\alpha_p N_s(t)\big[M(t) - N_h(t) - N_h^{av}(t)\big] \\ &\quad - \alpha_r N_s(t)[N_s(t) + N_0] \\ &\quad + \frac{1}{\tau_h} N_h(t) + \frac{1}{\tau} N_s^{av}(t), \end{aligned} \tag{6}$$

$$\begin{aligned}\frac{dN_h(t)}{dt} &= \alpha_p N_s(t)\big[M(t) - N_h(t) - N_h^{av}(t)\big] \\ &\quad - \alpha_r' N_h(t)[N_h(t) + N_0] \\ &\quad + \frac{1}{\tau} N_h^{av}(t) - \frac{1}{\tau_h} N_h(t), \end{aligned} \tag{7}$$

$$\frac{dN_h^{av}(t)}{dt} = \alpha_r' N_h(t)[N_h(t) + N_0] - \frac{1}{\tau} N_h^{av}(t), \tag{8}$$

$$\frac{dM(t)}{dt} = -\frac{1}{\tau_h} N_h(t). \tag{9}$$

The first two terms in Eq. (6) account for a decrease in the number of searching robots when robots find pucks and start homing, or when searching robots encounter and attempt to avoid other robots. The number of available pucks is just the number of pucks in the arena less the pucks held by homing robots. When a searching robot encounters another searching robot, both start executing avoidance maneuvers, decreasing the number of searching robots by two; while when a searching robot encounters a homing or either of the avoiding robots, the number of searching robots decreases by one. The total decrease is, therefore, proportional to $2N_s + N_h + N_s^{av} + N_h^{av} = N_s + N_0$. The last two terms in the equation require more explanation. We assume that it takes on average τ_h time for a robot to reach home after grabbing a puck. Then the average number of robots that deliver pucks during a short time interval dt and return to the searching state can be approximated as $dt N_h/\tau_h$. Likewise, in a period of time dt, $dt N_s^{av}/\tau$ robots leave the avoiding state and resume searching. Interference will increase the homing time for each robot; therefore, in general, homing time will be a function of N_0, τ and τ_h^0, the average homing time in the absence of collisions with other robots. For low to moderate robot densities, it is reasonable to assume the increase will be linear in the interference strength. The effective homing time can, therefore, be modeled as

$$\tau_h = \tau_h^0[1 + \alpha_r' \tau N_0]. \qquad (10)$$

The remaining equations have similar interpretations. We can take advantage of the conservation of the total number of robots to compute $N_h^{av}(t)$. Equations (6)–(9) are solved numerically under the conditions that initially, at $t = 0$, there are M_0 pucks and N_0 searching robots. Note that these equations are approximations of the time delay differential equations presented earlier (cf. Eqs. (1) and (2)). This approximation is valid when looking for long time solutions to the equations, which is exactly the regime we are exploring.

Figure 9 shows the time evolution of the fraction of searching robots and pucks for $M_0 = 20$, $N_0 = 5$, $\tau = 3$ s, $\tau_h^0 = 16$ s. The number of searching robots (solid line) first quickly decreases as robots find pucks and carry them home, but then it increases and saturates at some steady state value as the number of undelivered pucks approaches zero (dashed line). The fraction of searching robots in the steady state is inversely proportional to the avoiding time parameter.

In order to compare the performance of different size groups, we define the efficiency of the system as the inverse time required for the group to collect 80% of the pucks ($M(T_{80\%})/M_0 = 0.2$ in Fig. 9(a)). Figure 10(a) shows efficiency of the group vs. group size for two different interference strengths, as measured by τ. For both cases the efficiency of the group peaks for some group size, indicating an optimal group size for the task. The efficiency is less for the group with a higher interference strength, or larger avoiding time parameter (solid line). Moreover, the greater the effect of interference, the smaller the optimal group size. However, unlike the searching-and-avoiding task, in this case efficiency has a maximum, indicating an optimal group size for the task. Moreover, the greater the effect

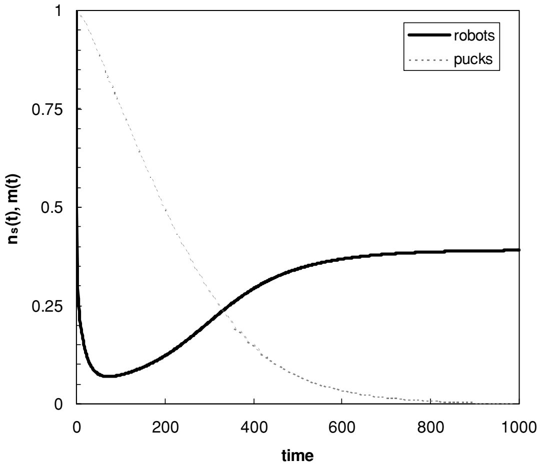


Figure 9. Time evolution of the fraction of searching robots (solid line) and undelivered pucks (dashed line) for $\tau = 3$ s, $\alpha_p = 0.02$, $\alpha_r = 0.04$, and $\alpha_r' = 0.08$.

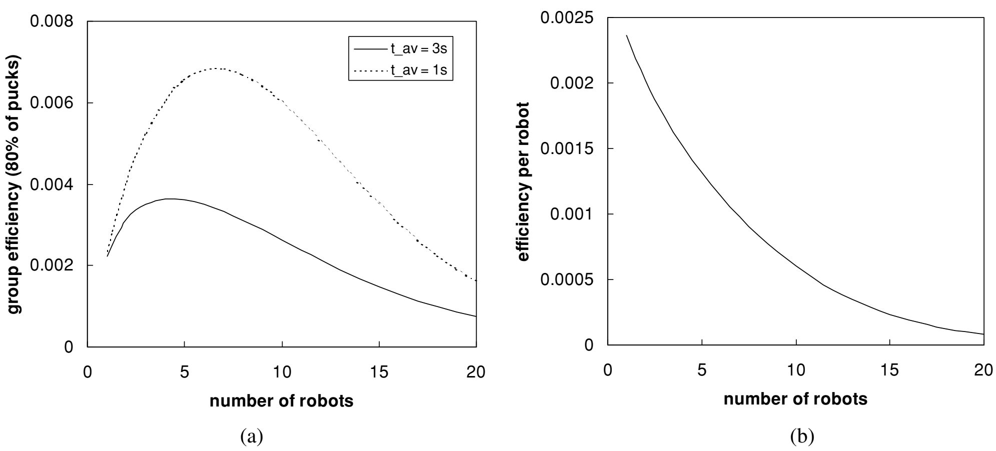


Figure 10. (a) Efficiency of different size robot groups defined as the inverse of the time it takes the group to collect 80% of the pucks in the arena for $\tau = 3$ s (solid line) and $\tau = 1$ s (dashed line) and $\tau_h^0 = 16$ s, $\alpha_p = 0.02$, $\alpha_r = 0.04$, $\alpha_r' = 0.08$. (b) Efficiency per robot for different group sizes.

of interference (larger τ), the smaller the optimal group size.

The final plot (Fig. 10(b)) shows that for this variant of the foraging task interference causes the per-robot efficiency to monotonically decrease with group size—adding a robot to the group decreases the performance of all robots, though if the initial group size was less than the optimal size, adding a robot will increase the overall efficiency of the group.

3.2.1. Comparison with Simulations. We ran foraging simulations for groups of one to ten robots and twenty pucks randomly scattered around the arena. In the results presented below, we split the avoiding behavior into two behaviors: **4**—avoiding while searching, collecting pucks and reverse homing, and **5**—avoiding while the homing behavior is active.

Table 3. Average time (in seconds) each robot spends in the active behaviors during the foraging task (**0**: search, **1**: collect, **2**: home, **3**: reverse home, **4**: avoid, **5**: avoid while homing) as a function of robot group size.

Rbts	**0**	**1**	**2**	**3**	**4**	**5**	Colls	Hcolls
1	307.64	156.90	265.11	225.84	73.69	21.99	23.7	7.1
2	118.68	81.07	170.02	101.89	46.70	45.08	15.1	13.5
3	94.80	61.48	143.22	61.54	57.65	71.78	17.8	22.1
4	50.98	39.71	131.51	34.06	55.84	99.85	15.9	29.5
5	53.14	29.59	126.84	24.27	69.52	150.66	18.9	41.3
6	67.05	28.89	139.68	20.32	94.26	224.40	22.0	53.3
7	137.90	58.11	111.32	23.69	130.21	184.20	37.0	43.1
8	80.94	32.94	133.35	17.06	123.05	265.56	30.1	62.3
9	74.62	31.36	153.58	15.96	130.10	299.18	33.7	77.7

The last two columns give, respectively, the average number of avoidance maneuvers per robot while searching/collecting/reverse homing and while homing.

Table 3 lists the average amount of time (in seconds) each robot spent in the active behaviors during the time it took the group collect the pucks and deliver them home. The last two columns list the average number of times a robot attempted to avoid collisions, both while engaging in the non-homing behaviors and while homing, during the time it took the group to complete the task. Although in all cases all twenty pucks were collected, robots were only able to deliver on average 19.14 ± 0.53 of them. This was caused by excessive crowding near the home location. In the current implementation of the simulator, robots see the already delivered pucks, and if there are no other pucks left in the arena, the robots will all go home. Although reverse homing acts to disperse robots, and eventually all puck should be delivered, we did not run the simulations long enough for this to happen. The total time in the results presented below is, therefore, the time the last of the pucks was delivered.

Figure 11(a) graphically displays the average amount of time each robot spent in the active behaviors while foraging. Figure 11(b) shows the fraction of the total task time the robot was homing (behaviors **2** and **5** active). Note that the rate of increase in the homing time per robot as a function of group size appears to justify our assumption, Eq. (10), that the homing time increases with the size of the group.

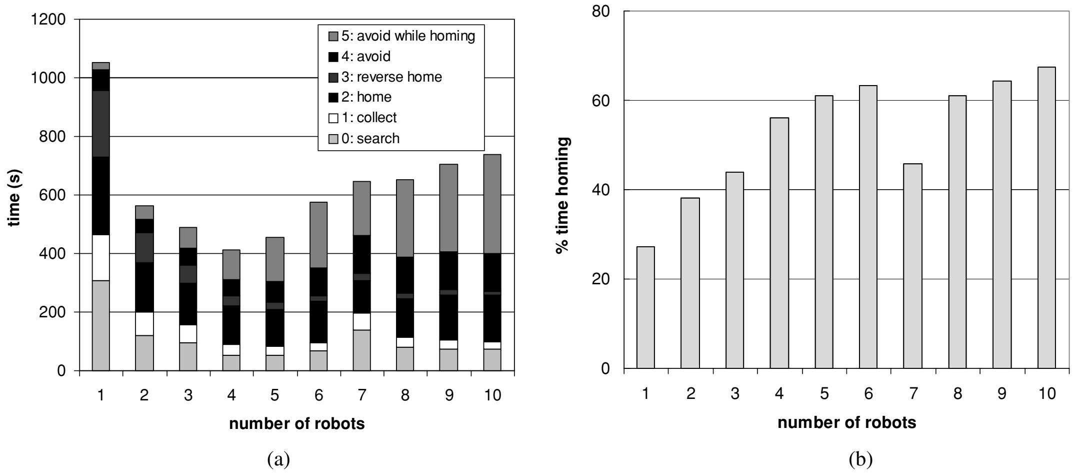
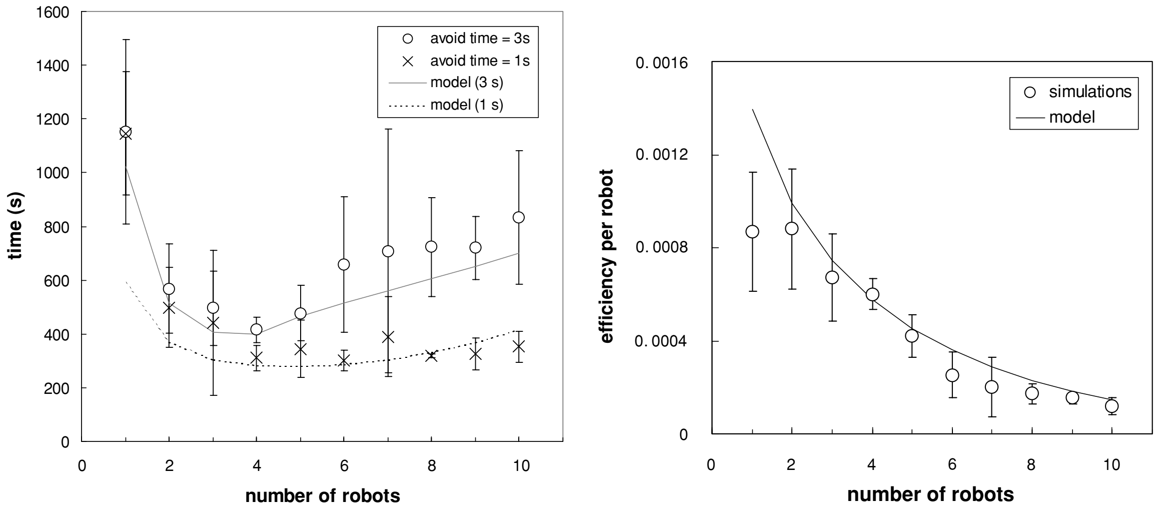


Figure 11. (a) Average time each robot spent in the active behaviors during the time it took the group to deliver all pucks vs robot group size. (b) Percentage of time each robot was homing as a function of group size.



(a) (b)

Figure 12. (a) The time it takes the group to collect and deliver pucks home $\tau_h^0 = 16$ s, $\alpha_p = 0.02$, $\alpha_r = 0.04$, $\alpha_r' = 0.08$ and $\alpha_w = 0.04$, and the avoiding times $\tau = 3$ s, and $\tau = 1.5$ s. (b) Efficiency per robot for $\tau = 3$ s.

Figure 12 shows the total time required to complete the task for two different interference strengths. The solid line is the result of the model's prediction for $\tau = 3$ s, and the dotted line for $\tau = 1.5$ s,[2] and $\tau_h^0 = 16$ s, $\alpha_p = 0.02$, $\alpha_r = 0.04$, and $\alpha_r' = 0.08$. We have also taken into account the effect of wall avoidance, it's strength given by $\alpha_w = 0.04$. The simulations data (Fig. 13) shows that the average avoiding time per collision increases as the group size grows (although it remains essentially constant for the no-homing scenario). The increase is due to multiple avoidance moves per each collision avoidance attempt, caused by an increase in the local density of robots. Therefore, we model the avoiding time parameter τ as a linearly increasing function of N_0, with the initial value of $\tau^0 = 3$ s (or 1.5 s). Note that there are minor differences between the values of the parameters that best fit the data and the values we compute from the simulations: we took $\tau_h^0 = 16.00$ s (the data shows the average homing time for a single robot is 14.95 ± 0.98), and $\alpha_r = 0.06$ (although this value includes wall avoidance). Aside from these slight differences in the parameter values, the agreement between model and simulations is good. Note, that because efficiency is defined as the inverse time to collect pucks, minimum in Fig. 12 corresponds to a peak in efficiency (cf. Fig. 10). The per-robot efficiency is a decreasing function of the group size, as predicted by our model.

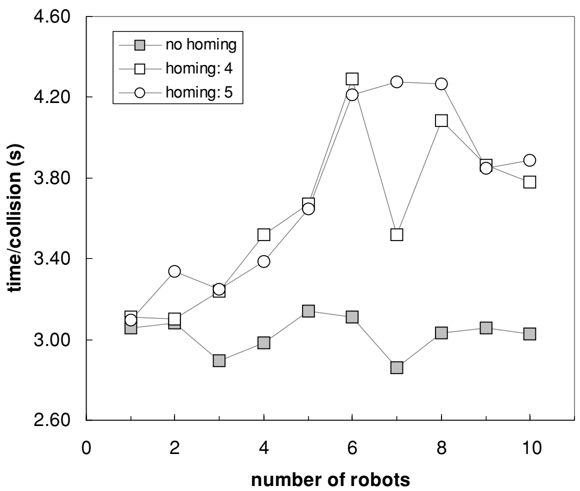


Figure 13. Average time per collision avoidance for the search-and-avoid task (grey squares) and the foraging task, both for the non-homing robots (white squares) and the homing robots (white circles).

4. Related Work

Nitz et al. (1993) briefly addressed the question of what is an appropriate number of robots for a foraging task in a given environment. By simulating foraging in groups of up to five communicating robots, they observed an increase in performance when adding one to three robots as compared to a single worker. However, the performance seemed to level off and even degrade with further additions. Performance of non-communicating robots seemed to improve as the group size grew, at least up to a group size of five. No simulations for larger group sizes were carried out. A similar trend is seen in the Beckers et al. (1994) experiments in which a group of non-communicating reactive robots collected a set of pucks distributed around the arena into a single cluster. In those experiments, the time required to gather all pucks into one cluster attained a minimum for three robots, and increased slowly thereafter up to a group size of five (ratio of the dimensions of the robot to arena were similar to the ones we used). Interestingly, this trend was reproduced in numeric and sensor-based simulations by Martinoli et al. (1999).

Sugawara and coworkers (Sugawara and Sano, 1997; Sugawara et al., 1998) carried out quantitative studies of foraging in groups of communicating and non-communicating robots in different environments. They have developed a simple mathematical model of foraging, similar to ours, and analyzed it under different conditions. In their system when a robot finds a puck, it may broadcast a signal to other robots to move towards it. For non-communicating robots they found that the inverse of the task completion time is linear in the number of robots. This result was independent of whether the puck distribution was homogeneous or localized. However, in the model describing time evolution of the system, avoiding terms appear only in the equations describing communication between robots. Thus, interference is not taken into account for non-communicating robots. This probably explains the difference between their conclusion and ours. We believe our paper presents a more complete and accurate model of foraging in a homogeneous, albeit non-communicating, multi-robot system.

5. Conclusion

Interference, which results from the inevitable competition for space between spatially extended robots, is an important issue in group robotics. Although we studied it only in the context of foraging by a group of robots, our analysis and, to some extent, conclusions, may apply to other multi-robot tasks. In order to quantitatively characterize the effects of interference on the group performance, we presented and analyzed a mathematical model of foraging in a group of robots, the type of system studied experimentally by Matarić and collaborators.

We analyzed two foraging scenarios in a homogeneous group of non-communicating robots: the simplified collection task where the robots search for and collect pucks only, and the full foraging task, where they find pucks and deliver them to a pre-specified home location. In the simplified collection task we found that increasing the robot group size reduces the total time required by the group to complete the task, thereby increasing the overall system performance. However, this improvement is sub-linear, and the relative, or the per-robot, foraging efficiency decreases as the size of the group grows. This decrease in the relative performance is due to the effects of interference.

Next, we looked at the system where robots collect pucks and deliver them home. We included the effects of interference both in the dynamic variables, and in the effect it has on parameters of the model, such as the homing time, or the average time it takes robots to reach the home location. This was motivated by the experimental observation that the density of robots is higher near home than elsewhere. We found that there is an optimal group size that maximizes the group's foraging efficiency. System performance decreases for groups larger than optimal size. The value of the optimal group size depends on the experimental parameters, and it is smaller the longer it takes the robots to execute obstacle avoidance maneuvers. Although the total group efficiency increases at first as the size of the group grows, the individual robot efficiency decreases monotonically, again due to the effects of interference.

We ran a number of simulations of both foraging scenarios using Player/Stage, a sensor-based multi-robot simulator and measured how long it took groups of different sizes to complete the foraging task. We found good agreement between the results of the model and simulations.

The model of foraging presented in this paper is an example of our approach to quantitative mathematical analysis of collective behavior in multi-agent systems. We have applied it to study a number of systems, including coalition formation in electronic marketplaces (Lerman and Shehory, 2000), platoon formation in traffic flow (Galstyan and Lerman, 2001), and collaboration in robots (Lerman et al., 2001). This class of models describe very simple multi-agent systems in which each agent's future state depends only on its present state and no past states. These models, therefore, cannot take into account memory, learning, and complex decision making abilities. We are extending the mathematical approach to include more complex agents.

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Notes

1. For simplicity, we do not include wall avoidance in the equations, but do take it into account when fitting model to the data.
2. Although in the simulations we specified the avoidance time to be 1 s, multiple collisions caused the average avoiding time per collision to be slightly higher.

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