CS201: Midsem Examination

September 21, 2023

Duration: Two Hours **Maximum Marks**: 50

Question 1. (10 marks) Consider $\mathbb{R}^2$, the usual two-dimensional space. A *triangle* $T \subset \mathbb{R}^2$ is the set of all points inside the boundary defined by lines joining three given points. In other words,

$$T = \{(x, y) \in \mathbb{R}^2 \mid (x, y) = \alpha(x_1, y_1) + \beta(x_2, y_2) + \gamma(x_3, y_3), 0 \leq \alpha, \beta, \gamma \leq 1, \alpha + \beta + \gamma = 1\}.$$

Let $\mathbb{T}$ be the set of all triangles in $\mathbb{R}^2$. Prove that $|\mathbb{T}| = |\mathbb{R}|$.

Question 2. (10 marks) Recall the definition of set of numbers with addition and multiplication $(N, +, *)$ given in the assignment. Consider such a set of numbers N that is finite. Let m be multiplicity of $(N, +, *)$. Prove that m divides $|N|$.

Question 3. (10+10 marks) Define polynomial $Q_k(y) = 0^k + 1^k y + 2^k y^2 + 3^k y^3 + \cdots + n^k y^n = \sum_{i=0}^{n} i^k y^i$, for $k \geq 0$. Generating function for polynomials $Q_k(y)$ is $G(x) = \sum_{k \geq 0} Q_k(y) x^k$. Derive a formula for $G(x)$.
Let $Q(y) = Q_\infty(y) = \sum_{i \geq 0} i^k y^i$. Derive a formula for generating function $Q(y)$.

Question 4. (10 marks) Consider the following C function:

```
int f(int *A, int n)  {
    int i;
    int value;

    for (i = 1, value = A[0]; i < n; i++) {
        value += f(A+i, n-i);
    }
    return value;
}
```

Derive a formula for time complexity of function `f`.