CS201

Mathematics for Computer Science I

Lecture 1

Instructor

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Contents

Week 1-2: Sets, Orders, and Proofs

- Sets, relations, functions, partial orders, equivalence classes, proof techniques

Week 3-5: Counting

- Permutations, combinations, binomial coefficients, partitions, generating functions, inclusion-exclusion, Ramsey theory

Week 6-7: Graph Theory

- Degree, paths, cycles, trees, planar graphs

Week 8-14: Algebra

- Groups, rings, fields, finite fields

Reference Books

- Discrete mathematics and its applications, by Kenneth Rosen.
- Discrete mathematics, by Norman Biggs.
- Introduction to combinatorial mathematics, by Chung Liu.
- Elementary number theory, by David Burton.

Grading

The course will have

- Midsem, weightage 25%
- Endsem, weightage 50%
- Assignments, weightage 25%
- 80+% marks $\Rightarrow$ A grade
- 20+% marks $\Rightarrow$ D or higher grade

Exams

- Exams will be take-home – no other good way to conduct them.
- Discussion is encouraged, but no copying.
- To ensure discussion, all students should form a group of size up to three. Each group needs to submit only one answer.

Copying Policy

- Any group caught copying in an assignment will get zero in that assignment.
- Any group caught copying in exams will get zero in that exam.

Schedule

- Discussion hours on Mon-Thu at 12.00 hrs, and Fri at 17.00 hrs. First one on Monday, September 7.
- Midsem during Oct 12-18, 2020
- Endsem during Dec 3-12, 2020

TAs

- Dhanish Kumar, dhanish@cse.iitk.ac.in
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Sets

- Sets are collections of objects.
- It can be any collection:
  - Collection of English alphabets
  - Collection of students in this class
  - Collection of countries in the world
  - Collection of molecules in the universe
  - Collection of objects satisfying any given property
- Objects in a set are called elements of the set.

Sets

- Sets are fundamental objects of mathematics.
  - Every mathematical statement can be expressed as properties of a set.
  - For example:
    Every integer can be uniquely expressed as product of prime numbers
    can be restated as:
    No element repeats in the set of products of prime numbers and the set equals the set of integers.

Sets

- Every mathematical object can be viewed as a set:
  - A geometric object, such as line, is viewed as the set of all points making up the object.
  - A mapping associating elements of one set to elements of another set can also be viewed as a set. If $f : A \mapsto B$ is one such mapping then the set representing f is:
  $$\{(a, f(a)) \mid a \in A\}.$$
  - Can numbers also be represented as a set?

Numbers as Sets

- Represent number 0 as null set $\{\}$, also written as $\emptyset$.
- Represent number 1 as set containing null set, that is, $\{\emptyset\}$.
- Represent number 2 as set containing null set and set representing 1, that is, $\{\emptyset, \{\emptyset\}\}$.
- And so on...
- Negative numbers can also be represented in similar way.
- Rational numbers can be represented as a set of two integers, corresponding to numerator and denominator.
- A real number can be represented as set of infinitely many rational numbers converging to the real number.

Why?

- Representing everything as sets allows for axiomatization of mathematics.
  - Axioms are a collection of basic assumptions from which all theorems can be derived.
  - The most popular axiomatization is called Zermalo-Fraenkel set theory, named after its inventors.
  - It has infinitely many axioms that can be grouped into nine groups.
  - One of the axioms is Axiom of Regularity: every non-empty set x contains an element y such that $x \cap y = \emptyset$.
  - Another is Axiom of Choice: given any set x whose every member is non-empty, there exists a mapping f such that $f(y) \in y$ for every $y \in x$.

Why?

- Such aximatization lead to the famous Godel's Incompleteness Theorem which states that for every axiomatic system, there exist statements which are true but not provable in the system.
- Godel's Second Incompleteness Theorem states that for any axiomatic system that is reasonably powerful, its consistency cannot be proven within the system unless it is inconsistent.

Why?

- In addition, it provides a uniform framework to study mathematical objects and their relationships.
- Note that there are multiple ways of representing an object as a set. We choose the one that is most convenient.
- Often, the set representation is implicitly assumed, e.g., for numbers.