Lecture-17 Main Points

- Many Valued Logics
  - $(\mathcal{V}, f_{c_1}, \ldots, f_{c_n}, \mathcal{D})$
    * $\mathcal{V}$: Set of values
    * $f_{c_i} : \mathcal{V}^k \to \mathcal{V}$: if c_i is a $k-$ary connective.
    * $\mathcal{D} \subseteq \mathcal{V}$: set of designated values.
  - Generalizes classical logic (BA **2**)
    $(\{0,1\}, f_\wedge, f_\vee, f_\neg, \{1\})$
  - Consequence relation: $\models$
    * Binary relation. Notation $\Sigma \models A$,
      where Σ is a set of propositions and A is a proposition.
    * $\Sigma \models A$ holds iff for every valuation v,
      [(for every $B \in \Sigma$, $v(B) \in \mathcal{D}$) $\Rightarrow v(A) \in \mathcal{D}$]

- Kleene's three valued logic K_3
  - $(\{0, i, 1\}, f_\wedge, f_\vee, f_\neg f_\to, \{1\})$
  -

$\wedge$	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

$\vee$	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

x	$\neg x$
0	1
i	i
1	0

  - Implication ($\to$) is defined as per material definition
    $a \to b = \neg a \vee b$
  - Truth value i is thought of as *undecided*/*indeterminate*
    (It could be 0 or 1, we do not know which)

- No tautologies. (consider $v(A) = i$, for every atomic A)
- Some examples of consequence relation
  * $A, \neg A \models B$
  * $A, A \to B \models B$
  * $A \to B \models \neg B \to \neg A$
  * $A \not\models B \vee \neg B$
    Consider valuation v, $v(A) = 1$ and $v(B) = i$

- LK_3 (Logic of paradoxes)
  - Same as Kleene except that $\mathcal{D} = \{i, 1\}$.
  - Truth value i is now thought of as both true *and* false.
  - Connective definitions of K_3 are consistent with this reading of i also.
  - Truth value i can be assigned to paradoxical sentences.
  - Tautologies examples:
    $A \to A$, $(A \to B) \to \neg B \to \neg A$ etc.
  - Some examples of consequence relation
    * $A, \neg A \not\models B$
      consider valuation v, $v(A) = i$ and $v(B) = 0$
    * $A, A \to B \not\models B$
      consider valuation v, $v(A) = i$ and $v(B) = 0$
    * $A \to B \models \neg B \to \neg A$
    * $A \models B \vee \neg B$

- Belnap's four valued para-consistent logic
  - $(\{0, b, n, 1\}, f_\wedge, f_\vee, f_\neg f_\to, \{b.1\})$
  - The values are to be thought of as information about truth of a proposition p.
    * 0 [only information is that p is false.]
    * 1 [only information is that p is true.]
    * b [Contradictory information. p is true according to some sources and false according to some other sources.]
    * n [No information available about p.]

- Meaning of connectives $\wedge, \vee$ are given by considering a lattice structure on values in which $0, 1$ are the smallest and the largest elements repectively of the lattice and elements b, n are incomparable.
- $\neg b = b$ and $\neg n = n$.
- $\rightarrow$ is defined as a material implication.
- **Exercise:**

  Give some examples of tautologies and consequence relation in this logic.

- Fuzzy logics
  - Sorties paradox

    Problem of qualitative/vague predicates (examples: small, tall, young etc.)
  - Resolution

    Continuous truth values.
  - Logic F_ϵ

    $([0, 1], f_\wedge, f_\vee, f_\neg f_\rightarrow, \mathcal{D}_\epsilon = \{x \in [0, 1] | x \geq \epsilon\})$
  - $a \wedge b = min(a, b)$, $a \vee b = max(a, b)$, $\neg a = 1 - a$.
  - $a \rightarrow b = \begin{cases} 1 & \text{if } a \leq b \\ 1 - (a - b) & \text{if } a > b \end{cases}$
  - Associated consequence relation: $\models_\epsilon$
  - Consequence relation: $\Sigma \models A$

    $\Sigma \models A$ holds iff for every $\epsilon \in [0, 1]$, $\Sigma \models_\epsilon A$.
  - **Exercises:**
    1. Show that if $\Sigma = \{A_1, \ldots, A_m\}$ then $\Sigma \models A$ iff $\models_1 A_1 \wedge \ldots \wedge A_m \rightarrow A$
    2. Let A, B be atomic propositions. Give a valuation v and an ϵ s.t.
       $A, A \rightarrow B \not\models_\epsilon B$.
    3. Use Exercise-2 above to show that $\Sigma \models_1 A \not\Rightarrow \Sigma \models A$.

References

[1] G. Priest, *An Introduction to Non-Classical Logic* [parts of chapters 7,8 and 11].