CS202M Final Exam

Feb 21, 2024
Duration: 2 hrs
Max Marks: 65

Closed Book and Open Lecture Notes and Open Printouts of Course Website Material

Instructions

1. The exam is closed book and open lecture-notes and open printouts of course website material.
2. Your answers should be as complete as in closed notes exam.
3. You are not allowed to have electronic devices such as mobile phones, laptops or tablet PCs with you during the exam.
4. Answers should be clear, and to the point. Also write your answers neatly and strike out any rough work.
5. In your derivations, for each inference rule applied please write its name alongside.

Q1(marks-4+8)

(i) In a lattice $\mathbf{L} = (L, \leq, \wedge, \vee, 0, 1)$ which of the two identities always holds? Prove it.

$(a \vee b) \wedge (a \vee c) \leq a \vee (b \wedge c)$

$a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

(ii) Show that if $\mathbf{L}$ above is finite and distributive then one can define '$\rightarrow$' in it to make it a Heyting algebra. Prove the correctness of your construction.

Q2(marks-3+4)

(i) Give a four element sub-algebra of Boolean algebra $(P(\{1,2,3\}), \subseteq, \cap, \cup, \neg, \emptyset, \{1,2,3\})$.

(ii) Which of the following hold in four valued para-consistent logic of Belnap? Prove your answers. [operator '$\to$' is material implication and designated values are b and 1]

(a) $\models p \to p$

(b) $\neg(p \wedge q) \models \neg p \vee \neg q$

Q3(marks-3+8+5)

(i) Let $\mathbf{B}_1 = (B_1, \leq_1, \wedge_1, \vee_1, \neg_1, 0_1, 1_1)$ and $\mathbf{B}_2 = (B_2, \leq_2, \wedge_2, \vee_2, \neg_2, 0_2, 1_2)$ be Boolean algebras. Structure $\mathbf{B}_1 \times \mathbf{B}_2$ is a Boolean algebra with domain $B_1 \times B_2$ and component-wise definitions of operations. Show the operations and constants of $\mathbf{B}_1 \times \mathbf{B}_2$ in terms of operations and constants of $\mathbf{B}_1$ and $\mathbf{B}_2$. (You do not need to prove that $\mathbf{B}_1 \times \mathbf{B}_2$ is a Boolean algebra under these operations.)

(ii) Let $\mathbf{B} = (B, \leq, \wedge, \vee, \neg, 0, 1)$ be a Boolean algebra and let $a \in B$, be s.t. $0 < a < 1$. In a question of an earlier exam, we saw that $\mathbf{B} \upharpoonright a = (B \upharpoonright a, \leq, \wedge, \vee, \neg', 0, a)$, where $B \upharpoonright a = \{x \in B \mid x \leq a\}$, $\neg'(x) = \neg x \wedge a$ is a Boolean algebra. Consider Boolean algebra $(\mathbf{B} \upharpoonright a) \times (\mathbf{B} \upharpoonright \neg a)$.

Show that the function $f : B \to (B \upharpoonright a) \times (B \upharpoonright \neg a)$, defined as $f(x) = (x \wedge a, x \wedge \neg a)$ is 1-1 and onto.

(iii) The function f in **(ii)** is in fact an isomorphism of Boolean algebras. For the purpose of this part you may simply assume it without proving it. That is, you may assume that Boolean algebras $\mathbf{B}$ and $(\mathbf{B} \upharpoonright a) \times (\mathbf{B} \upharpoonright \neg a)$ are isomorphic. Using this assumption argue that every finite Boolean algebra can be expressed as a (finite) product of Boolean algebras $\mathbf{2}$.

Q4(marks-5)

Show that CNF satisfiability problem, in which each clause has at most one negative literal can be solved in polynomial time.

Q5(marks-20) For each of the following judgment either derive it in the weakest system (minimal, intuitionistic or classical) needed to derive it or give a falsifying interpretation to show that it is not true in classical logic. Your derivations should be complete in themselves, without making reference to class-notes, but details of purely propositional reasoning may be suppressed. Quantifier steps should be shown clearly.

(i) $\exists x \forall y. A(x, y) \vdash \forall y \exists x. A(x, y)$

(ii) $\exists x.(C(x) \rightarrow D(x)) \vdash \forall x. C(x) \rightarrow D(x)$

(iii) $\forall y \exists x. A(x, y) \vdash \exists x \forall y. A(x, y)$

(iv) $\phi \rightarrow \exists x. \psi(x) \vdash \exists x.(\phi \rightarrow \psi(x))$, $x \notin FV(\phi)$

(v) $\forall x A(x) \vdash \exists x A(x)$

[**Note:** Not all parts of this question carry equal marks. Actual break-up of marks is not shown to avoid hinting at the relative difficulty.]

Q6(marks-5) Which of the following hold in Fuzzy logic (with threshold 1). Prove your answers.

(i) $\models_1 (p \rightarrow q) \rightarrow \neg p \vee q$

(ii) $\models_1 \neg p \vee q \rightarrow (p \rightarrow q)$

(iii) $p \rightarrow q \models_1 (p \wedge r \rightarrow q)$

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