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Labeling Proof Rules with Construction

$c : \forall x\ \phi \qquad c \in \prod_{x \in A} \phi(x) \qquad$ gener^n of fn^n

$c : \exists x\ \phi \qquad c \in \sum_{x \in A} \phi(x) \qquad$ gen^n of product

$$\exists\mathbf{i}\ \frac{\Gamma \vdash a : \phi[t/x]}{\Gamma \vdash \langle t, a \rangle : \exists x.\phi}$$

$$\exists\mathbf{e}\ \frac{\Gamma \vdash a : \exists x.\phi \qquad \Gamma', v : \phi[y/x] \vdash c : \psi}{\Gamma, \Gamma' \vdash \text{let } a = \langle d, e \rangle \text{ in } c[d/y, e/v] : \psi} \qquad y \notin FV(\Gamma', \psi, \exists x.\phi)$$

$$\forall\mathbf{i}\ \frac{\Gamma \vdash a : \phi[y/x]}{\Gamma \vdash \lambda y.a : \forall x.\phi} \qquad y \notin FV(\Gamma, \forall x.\phi)$$

$$\forall\mathbf{e}\ \frac{\Gamma \vdash a : \forall x.\phi}{\Gamma \vdash at : \phi[t/x]}$$

Q. $\exists x\, \phi \longleftrightarrow \neg A x \neg \phi$

Axiom

$$\dfrac{b : \forall x \neg P \vdash b : \forall x . \neg P}{b : \forall x \neg P \vdash b\,t : \neg P[t/x]}\ \forall_e$$

Axiom

$$c : P[t/x] \vdash c : P[t/x]$$

$$\dfrac{b : \forall x \neg P \vdash b\,t : \neg P[t/x] \qquad c : P[t/x] \vdash c : P[t/x]}{b : \forall x \neg P,\ c : P[t/x] \vdash (b\,t)\,c : \bot}\ (\rightarrow e)$$

Axioms

$$a : \exists x\, P(x) \vdash a : \exists x\, P(x)$$

$$\dfrac{a : \exists x\, P(x) \vdash a : \exists x\, P(x) \qquad b : \forall x \neg P,\ c : P[t/x] \vdash (b\,t)\,c : \bot}{a : \exists x\, P(x),\ b : \forall x \neg P \vdash \text{let } a = \langle d, e \rangle \text{ in } (bt)c\,[d/t,\ e/c] : \bot}\ \exists_e$$

$$a : \exists x\, P(x) \vdash \lambda b : \forall x \neg P,\ \text{let } a = \langle d, e \rangle \text{ in } (bt)c\,[d/t,\ e/c] : \neg \forall x \neg P(x)$$

Non-Classical Logic

Minimal, Intuitionistic Logic are examples of Non-Classical Logics.

We wish to capture mathematical reasoning.

material implication, $A \to B \equiv \neg A \vee B$

$$\neg A \models A \to B$$
$$B \models A \to B$$

this does not capture, conditional in english language

$A \wedge B \to C \models (A \to C) \vee (B \to C)$, holds for material implication.

b: $\forall x \neg P_i$ c: $\beta[t/x]$

If switch A is on, and switch B is on, then this implies when A is on, or when B is on, the lights are on.

Paraconsistency

The idea of paraconsistency is that coherence is possible even without consistency. Put another way, a paraconsistent logician can say that a theory is inconsistent without meaning that the theory is incoherent, or absurd.