System Modeling

Finite State Machine

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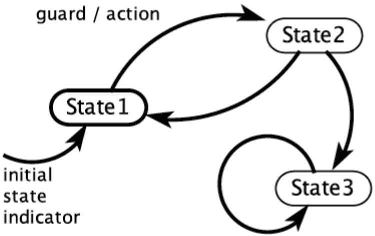
State Machine Models

- Summarize the behavior of an object or a subsystem in response to messages and events
- Shows how the object instance changes state depending on the messages that it receives
- Usually need a state diagram for the complex objects in the system

Finite State Machine

State Machine: Model of a system with discrete dynamics

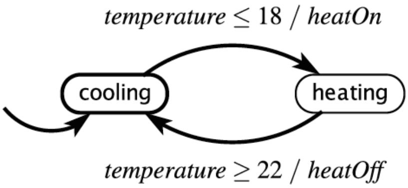
Finite-State Machine (FSM): A state machine where the set *States* of possible states is finite



- State
- Initial State
- Transition
  - Self transition
  - Guard and Action

Example: FSM for Thermostat

input: *temperature* : $\mathbb{R}$
outputs: *heatOn*, *heatOff* : pure



Chattering: the heater would turn on and off rapidly when the temperature is close to the set-point temperature

Solution: hysteresis strategy, dwell time

Event-triggered vs. time-triggered transitions

Mathematical Notation of a State Machine

A finite state machine is represented as a five-tuple

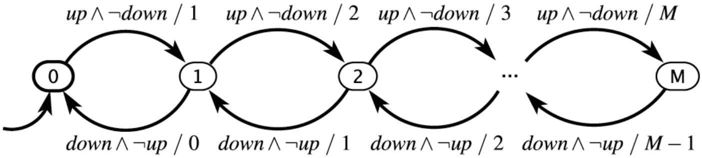
$$(States, Inputs, Outputs, update, initialState)$$

where,

- *States* is a finite set of states
- *Inputs* is a set of input valuations
- *Outputs* is a set of output valuations
- $update : States \times Inputs \rightarrow States \times Outputs$ is an update function, mapping a state and an input valuation to a next state and an output valuation
- *initialState* is the initial state

Example: Garage Counter

inputs: *up*, *down* : pure
output: *count* : $\{0, \cdots, M\}$

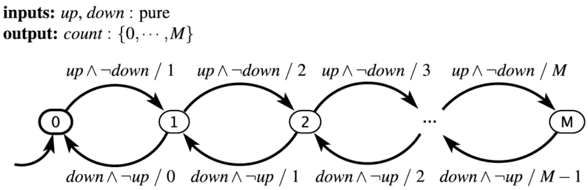


$$\begin{aligned} States &= \{0, 1, \ldots, M\} \\ Inputs &= (\{up, down\} \to \{present, absent\}) \\ Outputs &= (\{count\} \to \{0, 1, \ldots, M, absent\}) \\ initialState &= 0 \end{aligned}$$

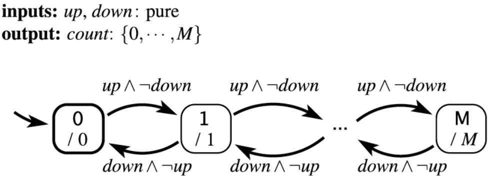
Update function:

$$update(s, i) = \begin{cases} (s+1, s+1) & \text{if } s < M \wedge up \wedge \neg down \\ (s-1, s-1) & \text{if } s > 0 \wedge \neg up \wedge down \\ (s, absent) & \text{otherwise} \end{cases}$$

Mealy Machines and Moore Machines



Mealy Machine: Produces outputs when a transition is taken

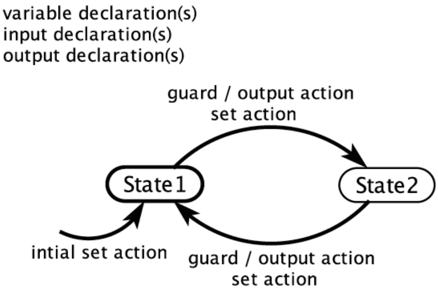


Moore Machine: Produces outputs when the machine is in a state

Extended State Machine

Augments the FSM Model with variables

Variables may be read and written while taking a transition



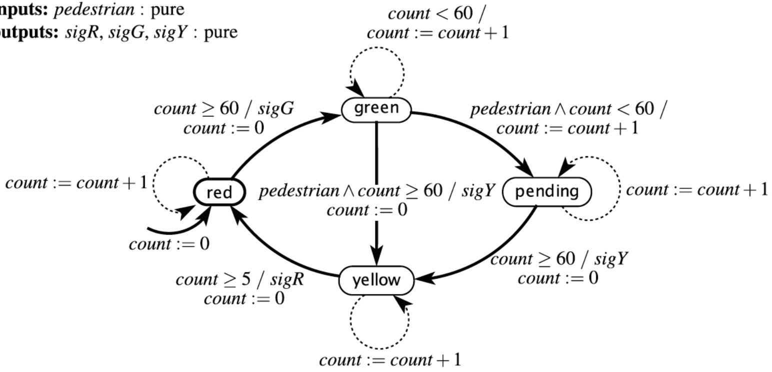
The number of states. can be quite large, or even infinite

n discrete states, *m* variables each of which can have one of *p* possible values

$$|\ States\ | = np^m$$

Example: ESM for Traffic Light

variable: *count*: $\{0, \cdots, 60\}$
inputs: *pedestrian* : pure
outputs: *sigR*, *sigG*, *sigY* : pure



Reachable States

All states that can be reached from the initial state on some input sequence

May be smaller than the set of states

Traffic Light:

Total number of states = 244

Total number of reachable states = 189

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