The newsvendor Problem

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The demand is stochastic. (Situation is under risk).

Selling season is very short.

Ordering well in advance of the season

There is only one ordering opportunity.

The excess quantity after the selling season is salvaged.

The newsvendor problem helps to model the inventory decisions for perisheable products.

Ordering too much : results in salvaging the unsold items
loss due to unsold items.

Ordering less : opportunity loss (of making profit).

Trade-off is between stocking more & stocking less.

Notations.

p = selling price / unit

c = purchase cost / unit

v = salvage value / unit

B = stockout cost / unit

x = Demand, pdf $f(x)$ and cdf $F(x)$. $x \in [a, b]$

π = profit function

Decision variable.

Q = Order quantity

Assumption.

$p > c > v \geq 0$; $B \geq 0$

Situation is under risk ; agents are considered to be risk neutral.

Profit function

$$\pi(x, Q) = \begin{cases} px - cQ + v(Q-x) \longleftarrow x \leq Q \\ pQ - cQ - B(x-Q) \longleftarrow x > Q \end{cases}$$

Expected profit

$$= E(\pi(x,Q)) = \int_a^Q (px - cQ + v(Q-x)) f(x)\,dx$$

$$+ \int_Q^b (pQ - cQ - B(x-Q)) f(x)\,dx \longrightarrow ①$$

$$\frac{d}{dQ} E(\pi(x,Q)) = 0$$

Leibniz integral rule.

$$\frac{d}{dy} \int_{a(y)}^{b(y)} f(x,y)\,dx = \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x,y)\,dx + f(b(y), y) \cdot \frac{db(y)}{dy} - f(a(y), y) \cdot \frac{da(y)}{dy}$$

$$① = (-c+v) \int_a^Q f(x)\,dx + (p-\cancel{c})Q f(Q) \cdot 1 - 0$$

$$+ (p-c+B) \int_Q^b f(x)\,dx + 0 - (p-\cancel{c})Q f(Q) \cdot 1$$

$$= (-c+v) \int_a^Q f(x)\,dx + (p-c+B) \int_Q^b f(x)\,dx$$

$$= (-c+v) F(Q) + (p-c+B)\left[1 - F(Q)\right] = 0$$

$$\Rightarrow F(Q) = \frac{p-c+B}{p-v+B} \longrightarrow ②$$

Second order condition

$$\frac{d^2 E(\pi(x,Q))}{d^2 Q} = (-c+v) f(Q) + (p-c+B)(-f(Q))$$

$$= -(p-v+B) f(Q) < 0$$

$\Rightarrow E(\pi(x,Q))$ is concave (strictly)

$\Rightarrow$ ② corresponds to unique optimal order quantity

$$\therefore ② \Rightarrow F(Q^*) = \left(\frac{p-c+B}{p-v+B}\right)$$

$$\text{Critical fractile} = F(Q^*) = \frac{C_u}{C_u + C_o}$$

C_o = overage cost = $c - v$

C_u = underage cost = $p - c + B$

Expected sales.

$$\int_a^Q x f(x)\,dx + \int_Q^b Q f(x)\,dx.$$

$$F(Q^*) = \frac{C_u}{C_u + C_o} = \frac{p-c+B}{p-v+B}. \qquad 0 < F(Q^*) < 1$$

Service level 1.

Probability that the firm ends the season having satisfied all the demand.
also known as in-stock probability $\rightarrow$ probability the firm has stock available for every customer.

$$\text{S.L.1.} = \frac{C_u}{C_u + C_o}.$$

Fill rate (Service level 2)

Percentage of demand that is satisfied.

$$\text{Fill rate} = \frac{\text{Expected sales}}{\text{Expected demand}}.$$