DGD 1

Q1. Let $\mathbf{u} \in \mathbb{R}^n$ and let $c, d \in \mathbb{R}$. Prove that $(c+d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$.

Q2. Show that every vector in $\mathbb{R}^3$ can be expressed as a linear combination of the vectors $\hat{\mathbf{i}} = (1,0,0)$, $\hat{\mathbf{j}} = (0,1,0)$, $\hat{\mathbf{k}} = (0,0,1)$.

Q3. Let $\mathbf{u} = (1,-1,0)$ and $\mathbf{v} = (2,-1,-2)$ be two vectors in $\mathbb{R}^3$.

a) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

b) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

c) Find a vector in $\mathbb{R}^3$ that is orthogonal to $\mathbf{v}$.

Q4. Let $\mathbf{u}, \mathbf{v}$ be two orthogonal vectors in $\mathbb{R}^n$. Show that $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$ (i.e. show that the Theorem of Pythagoras holds in $\mathbb{R}^n$).

Q5. Find the intersection of the lines $L_1 = \{(4,13,-7) + t(-1,-6,4) : t \in \mathbb{R}\}$ and $L_2 = \{(2,0,2) + t(0,1,-1) : t \in \mathbb{R}\}$.

Q6. Find a vector parametric equation for the line $2x + 3y - 13 = 0$ of $\mathbb{R}^2$.

Q7. Find a vector parametric equation and an Cartesian equation for the line passing through $(1,3)$ and $(2,7)$.

Q8. Find all values of k for which $(k,k,1)$ and $(k,-2,-3)$ are orthogonal.

Q9. Find the point of intersection of the plane $P = \{(x,y,z) : 2x + 2y - z = 5\}$ and the line with parametric equations $x = 4 - t$, $y = 13 - 6t$, $z = -7 + 4t$.

Q10. Find an equation for the plane that passes through the point $(1,1,1)$ and which is perpendicular to the line with parametric equations

$$x = -6 + 2t, y = 1 - 4t, z = -3 + 3t \quad t \in \mathbb{R}$$

Q11. Find an equation for the line containing $(-5,0,1)$ and which is parallel to the two planes with Cartesian equations $2x - 4y + z = 0$ and $x - 3y - 2z = 1$.

Q12. Find a Cartesian equation for the plane $W = \{(0,2,-2) + s(1,-1,2) + t(2,-4,-1) : s,t \in \mathbb{R}\}$

Q13. Find the area of the triangle whose vertices are the points $P(3,0,-2)$, $Q(5,2,-1)$ and $R(5,9,0)$.

Q14. Find the volume of the parallelepiped determined by $\mathbf{u} = (1,-2,3)$, $\mathbf{v} = (1,3,1)$ and $\mathbf{w} = (2,1,2)$.