DGD 3

Q1. Which of the following are subspaces of $M_{2,2}(\mathbb{R})$?

$$A = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2,2}(\mathbb{R}) : a + d = 0 \right\}$$

$$B = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2,2}(\mathbb{R}) : ad = 1 \right\}$$

$$C = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2,2}(\mathbb{R}) : a, b, c, d \text{ are integers} \right\}$$

$$D = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2,2}(\mathbb{R}) : ad - bc = 0 \right\}$$

Q2. Which of the following are subspaces of $\mathscr{F} = \{f | f : \mathbb{R} \to \mathbb{R}\}$ (space of real-valued functions with domain $\mathbb{R}$) ?

$U = \{f \in \mathscr{F} : f(0)f(1) = 0\}$

$V = \{f \in \mathscr{F} : f(0) + f(1) = 0\}$

$W = \{f \in \mathscr{F} : f(x) = -2f(x) \text{ for all } x \in \mathbb{R}\}$

$X = \{f \in \mathscr{F} : f(1) \leq 0\}$

Q3. Which of the following are subspaces of $\mathbb{R}^3$? For the subspaces, find a spanning set.

$S = \{(x, x + y, x + 2y) : x, y \in \mathbb{R}\}$

$T = \{(x, y, z) \in \mathbb{R}^3 : x - 2 = y - 3 = z\}$

$Y = \{(x, y, z) \in \mathbb{R}^3 : xyz = 0\}$

$M = \{(x, y, z) \in \mathbb{R}^3 : x - y - z = 0\}$

Q4. Is the polynomial $1 + x^2$ a linear combination of $1 + x - x^2$ and x ?

Q5. Is the polynomial $1 - 5x$ a linear combination of $1 + x$ and $1 - x$?

Q6. Is the function $\cos(x + \pi/2)$ a linear combination of $\sin(x)$ and $\cos(x)$?

Q7. Does the function x^2 belong to $\operatorname{span}\{\sin(x), \cos(x)\}$?

Q8. Are the subsets $\{(1, 2)\}$ and $\operatorname{span}\{(1, 2)\}$ equal?

Q9. Give two distinct finite spanning sets for each of the following subspaces:

$U = \{(2x, x) : x \in \mathbb{R}\}$

$V = \{(x, y, z) \in \mathbb{R}^3 : x + y - 2z = 0\}$