ELEC 344 Applied Electronics And Electromechanics

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Magnetic Circuit

Amperes' Law

When a conductor carries current, a magnetic field is produced around it. The line integral of the magnetic field intensity H around a closed path is equal to the total current linked by the contour.

$$\oint Hdl = \sum i = i_1 + i_2 ...$$

where H is the magnetic field intensity at a point on the path. Note that H and dl are parallel in this case. Otherwise, $\oint H \cdot dl \cdot cos\theta = \sum i$.

Magnetic Flux Density B

The magnetic intensity H produces a magnetic flux density B everywhere it exists.

$$B = \mu H [Weber/m^2] or [Tesla]$$
$$\mu = \mu_o \mu_r$$

where

- μ is the permeability of the medium
- μ_o is the permeability of the free space $\sim 4\pi 10^{-7}$ [henry/meter]
- μ_r is the relative permeability of the medium

Example

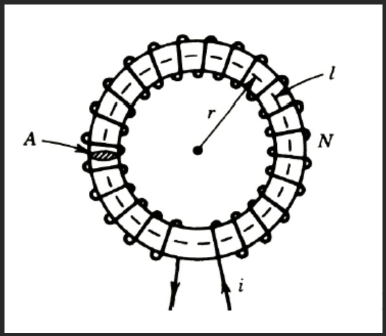


Figure 1: Toroid

Consider a toroid (a ring shaped magnetic core), with wires wrapped around the entire circumference as shown in Fig 1. When Current flow through the coil, the flux is mostly confined in the core material. The flux outside is called leakage flux and it can usually be neglected. Consider a path at a radius r. The magnetic intensity on this path is H.

$$\begin{aligned} \oint H \cdot dl &= Ni \\ Hl &= Ni \\ H \cdot 2\pi r &= Ni \end{aligned}$$

where $N \cdot i$ is the magnetomotive force mmf.

$$\begin{aligned} Hl &= Ni \\ H &= \frac{N}{l} \cdot i \\ B = \frac{\mu Ni}{l} &[Tesla] \end{aligned}$$

Assume that all the fluxes are confined in the toroid (no magnetic leakage). The magnetic flux across in the cross section of the toroid is,

$$\begin{aligned} \Phi &= \int BdA \\ \Phi &= BA[Wb] \end{aligned}$$

where B is the flux density in the core and A is the cross section area of the toroid. Let H be the magnetic intensity for this path,

$$\begin{aligned} \Phi = \frac{\mu Ni}{l} A &= \frac{Ni}{l/(\mu A)} \\ &= \frac{Ni}{R} \end{aligned}$$

where R is the reluctance of the magnetic path, and $\frac{1}{R}$ is called the permeance of the magnetic path.

Magnetizing Curve

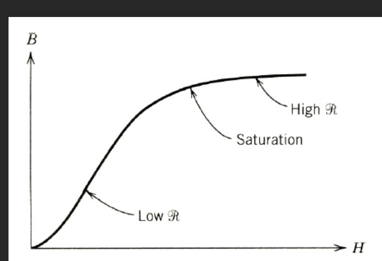


Figure 2: Magnetizing Curve

If the magnetic intensity increased by increasing the current, the flux density in the core changes as shown in Figure 2. Before saturation, B increases at a rate of $\mu = \mu_r \mu_o$. After saturation, B increases at a rate of μ_o.

Example

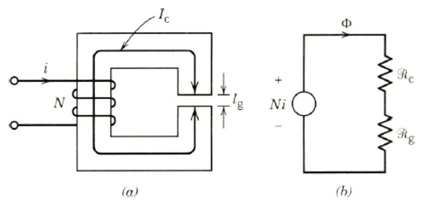


FIGURE 1.9 Composite structure. (*a*) Magnetic core with air gap. (*b*) Magnetic equivalent circuit.

$$\Phi = \frac{Ni}{\mathcal{R}_c + \mathcal{R}_g} \tag{1.17}$$

$$Ni = H_c l_c + H_g l_g \tag{1.18}$$

where l_c is the mean length of the core
l_g is the length of the air gap

Faraday's Law

A change in flux induces a voltage in the coil and the current flowing through the wire induces a flux that opposes the original flux.

$$e = -N \frac{d\Phi_{original}}{dt}$$

Self Inductance

$$L = \frac{N\Phi}{i}$$

$$Li = N\Phi \rightarrow \text{ flux linkage}$$

$$e = \frac{d(N\Phi)}{dt} = \frac{dLi}{dt}$$

$$= \frac{Ldi}{dt} + \frac{idL}{dt} = \frac{Ldi}{dt}$$

$$L = \frac{N\Phi}{i} = \frac{N^2}{Reluctance}$$

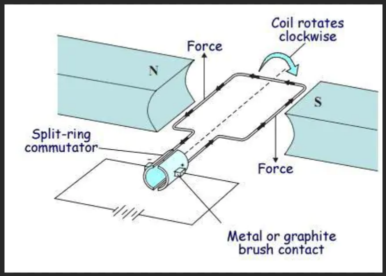
Lorentz Force

Moving a conductor in a magnetic field induces a current in that conductor. If the conductor is perpendicular to the magnetic field,

$$F = i \cdot l \cdot B$$

DC Machine

Brushed Motor



For dc motor, the flux Φ_f is established by the stator, this can be done with either a permanent magnet or a field winding. In the field winding's case, the flux is proportional to the current in the coils, where.

$$\Phi_f = k_f I_f$$

The Electromagnetic torque of the motor is given by,

$$T_{em} = F \cdot \frac{d}{2} \cdot 2$$
$$T_{em} = i \cdot l \cdot B \cdot d \cdot cos\theta$$
$$T_{em} = i \cdot A \cdot B \cdot cod\theta = i\Phi cos\theta$$
$$\boxed{T_{em} = k_t \Phi i_a} \text{ where } k_t \text{ is the motor torque constant}$$

Back EMF

Since the armature(a conductor) is moving in the magnetic field, a voltage is induced across the ends of the armature. At a speed of ω_m, the induced $emf(electromotiveforce)\, e_a$ is,

$$\boxed{e_a = lvB = AB\omega_m = k_e \Phi_f \omega_m}$$

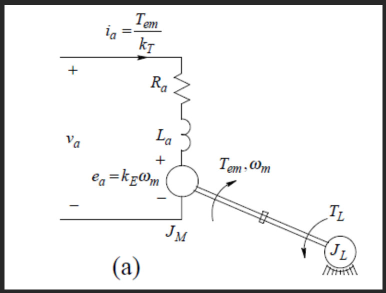
where k_e is the voltage constant of the motor.

Power

Ideally, the power from the electrical domain is $\%$ transferred to the mechanical domain where,

$$P_e = P_m$$
$$P_e = e_a i_a = k_e \Phi_f \omega_m i_a$$
$$P_m = \omega_m T_{em} = k_t \Phi_f \omega_m i_a$$

Electro-Mechanical Model



In practice, a controllable voltage v_t is applied to established i_a.

$$V_t = e_a + i_a R_a + L_a \frac{di_a}{dt}$$

where e_a is back emf, R_a is armature resistance, and L_a is armature inductance.

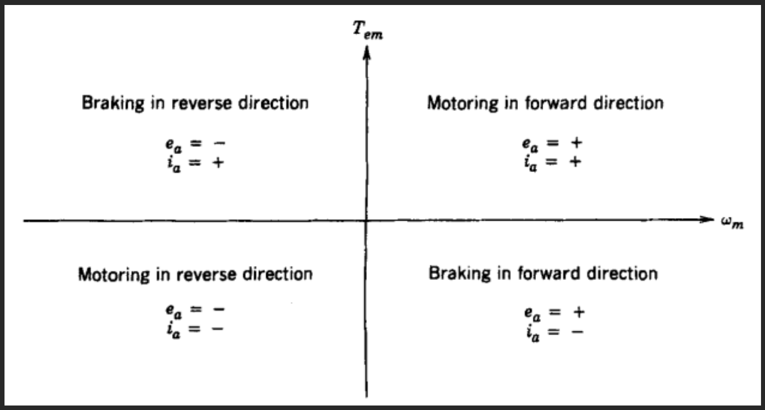
Torque is given by,

$$T_{em} = J_L \frac{d\omega_m}{dt} + B_L \omega_m + T_L(t)$$

where J_L and B_L are the total equivalent of inertia and damping, and T_L is the load.

Regenerative Braking

While the motor is in steady state, $V_t > E_a$ and i_a is positive. If V_t is decreased while the motor is still moving forward, when $V_t < E_a$, i_a is negative, creating a torque that opposes the forward spinning, which decreases the motor speed. Since the speed of the motor is decreasing, e_a also decreases until the motor is completely stopped. When i_a is negative, the motor is a generator because the inertial energy of the motor is converted to current.



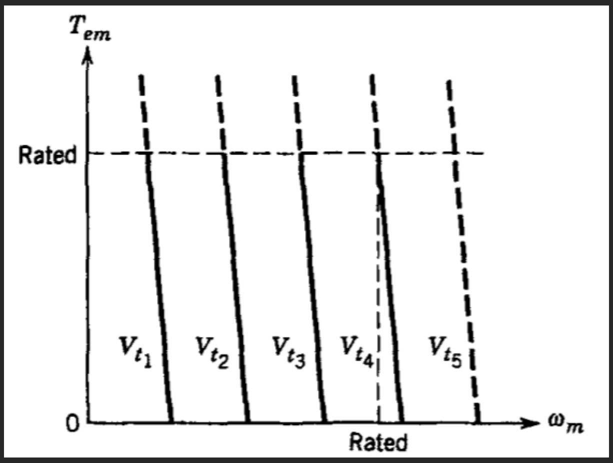
Generating Magnetic Field

Permanent Magnet

$$\Phi = \text{constant}$$
$$e_a = k_e \Phi_f \omega_m = k_E \omega_m$$
$$T_{em} = k_t \Phi_f i_a = k_T i_a$$

In steady state, $\frac{di_a}{t} = 0$,

$$V_t = e_a + i_a R_a = k_E \omega_m + \frac{T_{em}}{k_T} R_a$$
$$\omega_m = \frac{1}{k_E}(V_t - \frac{R_a}{k_T} T_{em})$$



The armature current cannot exceeds the rated current, and therefore torque cannot exceeds the rated value. On the other hand, speed cannot exceeds the rated value because it requires an input voltage that exceeds the rated value. The rated values can be exceeded on a short-term bases but not long term.

Field Winding

$$\Phi_f = k_f i_f$$
$$e_a = k_e k_f i_f \omega_m$$
$$T_{em} = k_t k_f i_f i_a$$

$$V_t = e_a + i_a R_a = k_E \omega_m + \frac{T_{em}}{k_T} R_a$$
$$\omega_m = \frac{1}{\Phi_f k_e}(V_t - \frac{R_a}{\Phi_f k_t} T_{em})$$

Induction Motor

Induction motor is consisted of a stator and a rotor. The stator has three coils and when a three phase balanced power is supplied to the coils, a rotating magnetic field is induced. A rotor is a current carrying loop of conductor placed within the rotating magnetic field. The rotating magnetic field cuts through the rotor creating an emf on the loop causing the loop to rotate.

If the rotor rotates at the same speed as the magnetic field, the field no longer cuts through the conductors, and the rotor will decrease in speed until the magnetic field is cutting through the conductors again. The rotor will never be able to catch up to the rotating field.

Number of Poles

$$rpm = \frac{120 f_{in}}{p}$$

Synchronous Speed

The speed the magnetic field rotates at is called the synchronous speed.

$$\omega_s = \frac{2\pi/(p/2)}{1/f} = \frac{2}{p}\omega_{in}$$

where ω_s is the synchronous speed, p is the number of poles, and ω_{in} is the source frequency.

$$rpm = 60 * \frac{\omega_s}{2\pi} = \frac{120}{f_{in}}$$

Air Gap Voltage

The rotating magnetic field causes an emf on the stator windings, E_{ag}, an air gap voltage is developed in each of the windings. Assuming no leakage and no resistance in the stator windings.

$$e_{ag} = N_s \frac{d\Phi_{ag}(t)}{dt} = N_s \omega \hat{\Phi}_{ag} cos(\omega t)$$

where N_s is the number of turns per phase of the stator winding and $\Phi_{ag}(t)$ is the rotating flux and $\hat{\Phi}_{ag}$ is the peak flux.

$$\Phi_{ag}(t) = \hat{\Phi}_{ag} sin(\omega t)$$
$$\frac{d\Phi_{ag}}{dt} = \hat{\Phi}_{ag}(t) cos(\omega t)$$

The rms air gap voltage is then,

$$E_{ag} = kf\Phi_{ag}$$

where k is a constant.

Slip Speed

$$\omega_{slip} = \omega_{synchronous} - \omega_{rotor}$$

$$\text{Slip} = \text{s} = \frac{\omega_{slip}}{\omega_{synchronous}}$$

Rotor Voltage

The induced voltage on the rotor is proportional to the slip speed.

$$E_r = kf_{slip}\Phi_{ag}$$

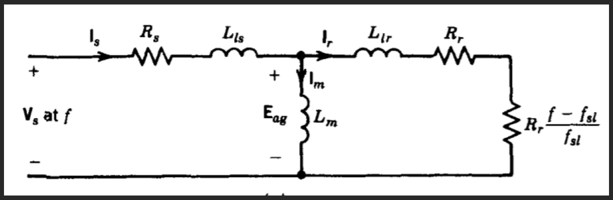
which is similar to E_{ag} but the speed is the slip speed which is the relative speed between the rotor and the rotating flux. k is the same constant as the k that appeared in E_{ag}.

$$E_{ag} = sE_r$$

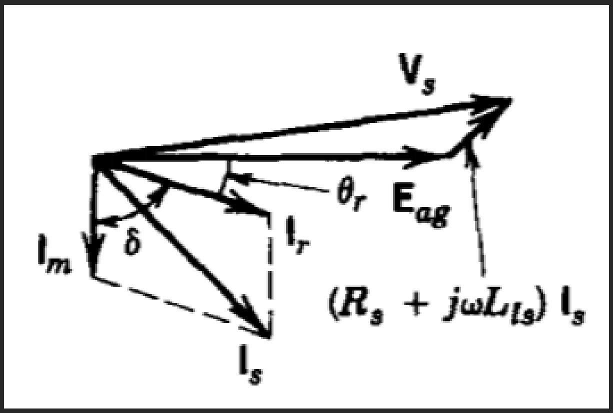
The rotor voltage induces a current at the slip frequency in the the rotor, and therefore

$$E_r = R_{rotor} i_{rotor} + j2\pi f_{slip} L_{leakage_rotor} i_{rotor}$$

Per Phase Rotor-Stator Circuit



The total current drawn by the stator is the sum of the magnetizing current and the rotor current. The magnetizing current produces the rotating flux and the rotor current produces electromagnetic torque.



$$\begin{aligned} \text{power factor } &= \theta_{rotor} = tan^{-1}(\frac{2\pi f_{slip} L_{leakage_rotor}}{R_{rotor}}) \\ \text{torque angle } &= \delta = 90^\circ + \theta_{rotor} \end{aligned}$$

$$V_{source} = E_{ag} + (R_s + j2\pi f L_{leakage_stator}) I_{stator}$$

$$T_{em} = k_4 \Phi_{ag} I_{rotor} sin(\delta)$$

Usually, $2\pi f_{slip} L_{leakage_rotor} \ll R_{rotor}$ and therefore, $\delta \sim 90^\circ$

$$\begin{aligned} T_{em} &= k_4 \Phi_{ag} I_{rotor} \\ I_{rotor} &\sim k_5 \Phi_{ag} f_{slip} \\ T_{em} &= k_6 \Phi_{ag}^2 f_{slip} \end{aligned}$$

Power

$$P_{rotor} = 3R_{rotor}I_{rotor}^2$$
$$P_{ag} = 3\frac{f_{input}}{f_{slip}}R_{rotor}I_{rotor}^2$$
$$P_{em} = P_{ag} - P_{rotor}$$

$$T_{em} = \frac{P_{em}}{\omega_{rotor}} = \frac{P_{ag}}{\omega_{synchronous}}$$