Elec341 System And Control

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The Stability of Linear Feedback System

Types of Stability

- Absolute Stability: A closed loop feedback system that is either stable or not stable.
- Relative Stability: Given a closed loop system is stable, we can further specify the degree of stability.

Absolute Stability

Absolute Stability of a system can be found by determining that all the poles for the transfer function lie in the left-half s-plane, or equivalently, that all the eigenvalues of the system matrix A lie in the left-half s-plane.

Relative Stability

Relative Stability can be determine by examining the relative locations of all the poles or eigenvalues.

Stable System

A stable system is a dynamic system with a bounded response to a bounded input. A linear system is stable if and only if the absolute value of its impulse response integrated over an infinite range, is infinite.

Let g(t) be the impulse response of the system.

$$\int_0^{\inf} |g(t)|dt = \inf$$

Locations of the poles determine whether the system would have an increasing, neutral, or decreasing response for a disturbance input.

- Poles in left-hand portion of s-plane $\rightarrow$ decreasing response
- Poles on $j\omega$ axis $\rightarrow$ neutral response
- Poles in right-hand portion of s-plane $\rightarrow$ increasing response

For a feedback system to be stable, all the poles need to have negative real parts.

For a feedback system with poles on the $j\omega$ axis and left-hand s-plane, the response of the system would be oscillations for a bounded input, unless the input is a sinusoid whose frequency is equal to the magnitude of the $j\omega$ roots. Such a system is called marginally stable since only certain bounded input would cause the system to be unstable.

For a feedback system with any poles in the right-hand portion, the system would have unbounded output response.

Methods to Determine Stability of a System

- The s-plane approach
- The frequency plane approach
- The time domain approach

Routh-Hurwitz Criterion

A criterion to determine the stability of linear systems.
The characteristic equation can be written in the form,

$$a_n s^n + a_{n-1} s^{n-1} .... + a_0 = 0$$

The coefficients of the characteristic equations are put into an array of the form,

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} \cdots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} \cdots \\ s^{n-2} & b_{n-1} & b_{n-3} & b_{n-5} \cdots \\ s^{n-3} & c_{n-1} & a_{n-3} & a_{n-5} \cdots \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ s^0 & h_{n-1} & & \end{array}$$

where,

$$b_{n-1} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}$$

$$b_{n-3} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}$$

$$c_{n-1} = \frac{-1}{b_{n-1}} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix}$$

The Routh-Hurwitz criterion states that the number of roots of the characteristic equation q(s) with positive real parts is equal to the number of changes in sign of the first column of the Routh array.

There are 4 other conditions that need to be considered.

1. No element in the first column is zero
2. There is a zero in the first column, but some other elements of the row containing the zero in the first column are nonzero.
3. There is a zero in the first column, and the other elements of the row containing the zero are also zero
4. Repeated roots of the characteristic equation on the $j\omega$ axis

TODO!!!

PID Control

$$\int_{\text{Beginning of semester}}^{\text{End of semester}} \text{Nathan's Effort} + \text{Dario's Effort}\ dt = 0$$