ELEC 344 Applied Electronics And Electromechanics

Kelvin Hsu

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Magnetic Circuit

Amperes' Law

When a conductor carries current, a magnetic field is produced around it. The line integral of the magnetic field intensity H around a closed path is equal to the total current linked by the contour.

$$\oint Hdl = \sum i = i_1 + i_2 ...$$

where H is the magnetic field intensity at a point on the path. Note that H and dl are parallel in this case. Otherwise, $\oint H \cdot dl \cdot cos\theta = \sum i$.

Magnetic Flux Density B

The magnetic intensity H produces a magnetic flux density B everywhere it exists.

$$B = \mu H [Weber/m^2] or [Tesla]$$
$$\mu = \mu_o \mu_r$$

where

- μ is the permeability of the medium
- μ_o is the permeability of the free space $\sim 4\pi 10^{-7}$ [henry/meter]
- μ_r is the relative permeability of the medium

Example

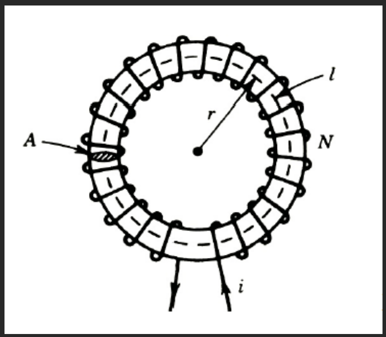


Figure 1: Toroid

Consider a toroid (a ring shaped magnetic core), with wires wrapped around the entire circumference as shown in Fig 1. When Current flow through the coil, the flux is mostly confined in the core material. The flux outside is called leakage flux and it can usually be neglected. Consider a path at a radius r. The magnetic intensity on this path is H.

$$\oint H \cdot dl = Ni$$
$$Hl = Ni$$
$$H \cdot 2\pi r = Ni$$

where $N \cdot i$ is the magnetomotive force mmf.

$$Hl = Ni$$
$$H = \frac{N}{l} \cdot i$$
$$B = \frac{\mu Ni}{l}[Tesla]$$

Assume that all the fluxes are confined in the toroid (no magnetic leakage). The magnetic flux across in the cross section of the toroid is,

$$\Phi = \int BdA$$
$$\Phi = BA[Wb]$$

where B is the flux density in the core and A is the cross section area of the toroid. Let H be the magnetic intensity for this path,

$$\Phi = \frac{\mu Ni}{l}A = \frac{Ni}{l/(\mu A)}$$
$$= \frac{Ni}{R}$$

where R is the reluctance of the magnetic path, and $\frac{1}{R}$ is called the permeance of the magnetic path.

Magnetizing Curve

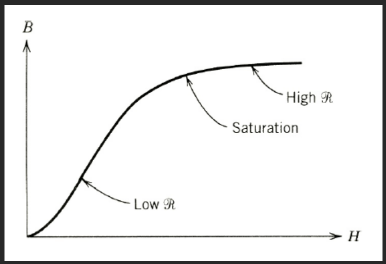


Figure 2: Magnetizing Curve

If the magnetic intensity increased by increasing the current, the flux density in the core changes as shown in Figure 2. Before saturation, B increases at a rate of $\mu = \mu_r \mu_o$. After saturation, B increases at a rate of μ_o.

Example

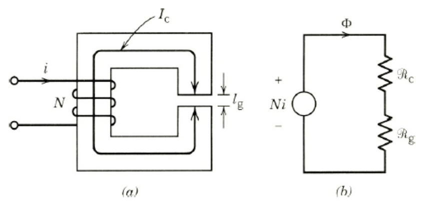


FIGURE 1.9 Composite structure. (*a*) Magnetic core with air gap. (*b*) Magnetic equivalent circuit.

$$\Phi = \frac{Ni}{\mathscr{R}_c + \mathscr{R}_g} \tag{1.17}$$

$$Ni = H_c l_c + H_g l_g \tag{1.18}$$

where l_c is the mean length of the core
l_g is the length of the air gap

Faraday's Law

A change in flux induces a voltage in the coil and the current flowing through the wire induces a flux that opposes the original flux.

$$e = -N\frac{d\Phi_{original}}{dt}$$

Self Inductance

$$L = \frac{N\Phi}{i}$$
$$Li = N\Phi \rightarrow \text{ flux linkage}$$

$$e = \frac{d(N\Phi)}{dt} = \frac{dLi}{dt}$$
$$= \frac{Ldi}{dt} + \frac{idL}{dt} = \frac{Ldi}{dt}$$

$$L = \frac{N\Phi}{i} = \frac{N^2}{Reluctance}$$

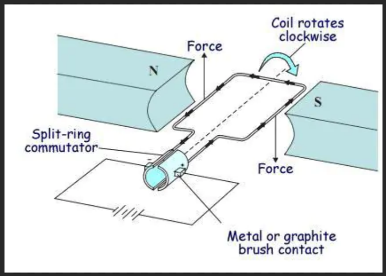
Lorentz Force

Moving a conductor in a magnetic field induces a current in that conductor. If the conductor is perpendicular to the magnetic field,

$$F = i \cdot l \cdot B$$

DC Machine

Brushed Motor



Torque of the motor is given by,

$$T = F \cdot \frac{d}{2} \cdot 2$$
$$T = i \cdot l \cdot B \cdot d \cdot cos\theta$$
$$T = i \cdot A \cdot B \cdot cod\theta = i\Phi cos\theta$$
$$T = k_T i_a \Phi \text{ where } k_T \text{ is the motor torque constant}$$

Back EMF

Since the conductor is moving in the magnetic field, a voltage is induced across the brushes. At a speed of ω_m, the induced $emf(electromotiveforce)$ e is,

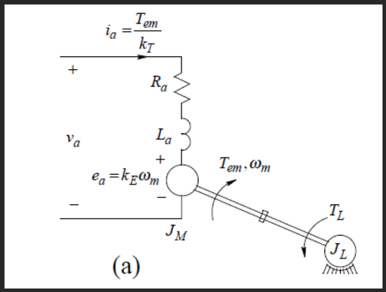
$$e_a = lvB = AB\omega_m = \Phi\omega_m$$
$$e_{avg} = k_E \omega_m \Phi$$

where n_a is the number of conductors inside the magnetic field.

Power

$$P_e = P_m \rightarrow \text{ideal}$$
$$P_e = e_a i_a = k_e \Phi \omega_m i_a$$
$$P_m = \omega_m T_e = \omega_m k_T \Phi i_a$$

Electro-Mechanical Model



$$V = e_a + i_a R_a + L_a \frac{di_a}{dt}$$

$$T_e = J_L \frac{d\omega_m}{dt} + B_L \omega_m T_L$$

Regenerative Braking

While the motor is in steady state, $V > E_a$ and i_a is positive. If V is decreased while the motor is still moving forward, when $V < E_a$, i_a is negative, creating a torque that opposes the forward spinning, which decreases the motor speed.

Generating Magnetic Field

Permanent Magnet

$$\Phi = \text{constant}$$

$$e_a = k_e \Phi \omega_m = k_E \omega_m$$

$$T_e = k_t \Phi \omega_m = k_T \omega_m$$

Field Winding

$$\Phi = k_f i_f$$

$$e_a = k_v k_f i_f \omega_m$$

$$T_e = k_t k_f i_f i_a$$