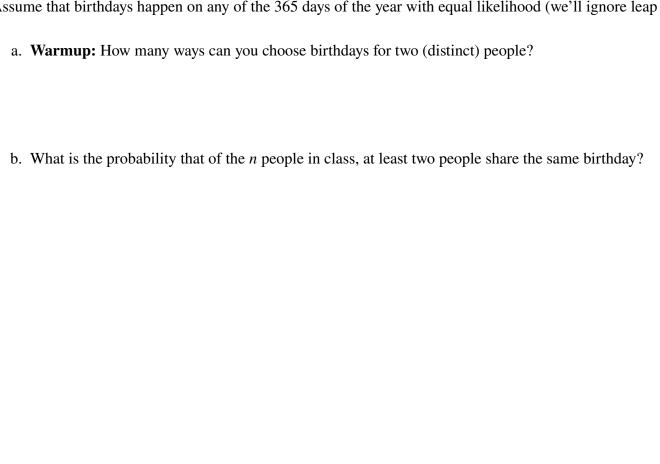
# Section 1: Analytic Probability

Chris Piech and the CS109 teaching teams

#### The Birthday Problem 1

When solving a counting problem, it can often be useful to come up with a generative process, a series of steps that "generates" examples. A correct generative process to count the elements of set A will (1) generate every element of A and (2) not generate any element of A more than once. If our process has the added property that (3) any given step always has the same number of possible outcomes, then we can use the product rule of counting.

Assume that birthdays happen on any of the 365 days of the year with equal likelihood (we'll ignore leap years).



c. What is the probability that this class contains exactly one pair of people who share a birthday?

### 2 Conditional Probability Warmup

What is the difference between these two terms P(B|A) and  $P(A \cap B)$ ? Imagine that B is the event that a student "correctly answer a multiple choice question" and A is the event that the same student "guesses randomly". Provide an explanation as well as a mathematical relationship between the two.

## 3 Self-Driving Car

A self-driving car has a 60% belief that there is a motorcycle to its left based on all the information it has received up until this point in time. Then, it receives a new, independent report from its left camera. The camera reports that there is **no** motorcycle. What is the updated belief that there is a motorcycle to the left of the car? The camera is an imperfect instrument. When there is truly no motorcycle, the camera will report "no motorcycle" 90% of the time. When there actually is a motorcycle, the camera will report "no motorcycle" 5% of the time.

## 4 Extra Practice: Axioms of Probability

Decide whether each of the three statements below is true or false:

a. 
$$P(A) + P(A^C) = 1$$
. Recall that  $A^C$  means  $A$  "complement" or "not"  $A$ 

b. 
$$P(A \cap B) + P(A \cap B^C) = 1$$
. Recall that  $\cap$  means "and"

c. If 
$$P(A) = 0.4$$
 and  $P(B) = 0.6$  then it must be the case that  $A = B^C$