Steady-State Productivity Equations for a Multiple-Wells System in Sector Fault Reservoirs

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Summary

Currently in the oil industry, pseudosteady-state productivity equations for a multiple-wells system are used in all reservoir systems, regardless of the outer boundary conditions. However, if the reservoir is under edgewaterdrive, pseudosteady state is no longer applicable. When the producing time is sufficiently long, productivity equations based on the steady state are required.

This paper presents steady-state productivity equations for a multiple-wells system in homogeneous, anisotropic sector fault reservoirs. Taking fully penetrating vertical wells as uniform line sinks, and solving a square matrix of dimension *n,* where *n* is the number of wells, simple, reasonably accurate multiple-wells-system productivity equations are obtained. The proposed equations relate the production-rate vector to the pressure-drawdown vector and are applicable to a multiple-wells system, which is located arbitrarily in a sector fault reservoir. The analytical solutions are verified with reservoir numerical simulation in several examples. This paper also gives an equation for calculating skin factors of each well in steady state.

The analytical solutions perform well and match well with numerical solutions, and the benefit of the analytical model can be emphasized when the reservoir data deviate from their idealistic representation. It is concluded that the proposed equations provide a fast analytical tool to evaluate the performance of a multiple-wells system in a sector fault reservoir.

Introduction

Well productivity is one of primary concerns in field development and provides the basis for field-development strategy. To determine the economic feasibility of drilling a well, petroleum engineers need reliable methods to estimate its expected productivity. We often relate the productivity evaluation to the long-time performance behavior of a well—that is, the behavior during pseudosteady-state or steady-state flow.

Substituting Darcy's equation into the equation of continuity, the productivity equation of a fully penetrating vertical well in a homogeneous, isotropic-permeability circular reservoir is

$$Q_w = F_D \frac{2\pi KH(P_e - P_w)/(\mu B)}{\ln(R_e/R_w)}, \qquad (1)$$

where P_e is the outer-boundary pressure, P_w is the flowing bottom-hole pressure, and F_D is the unit conversion factor (Butler 1994). In oilfield units, F_D=0.001127; in practical SI units, F_D=86.4. The introduction of field and practical SI units is listed at the end of the paper in a table denoted Appendix A; the units conversion factors can found be in a section called the SI Metric Conversion Factors.

Ge (1982) introduced Eq. 2, which accounts for asymmetrical positioning of a well within its circular drainage area, to calculate the productivity for an off-center well:

$$Q_w = F_D \frac{2\pi KH\left(P_e - P_w\right)/\left(\mu B\right)}{\ln\left[\left(R_e^2 - R_0^2\right)/\left(R_e R_w\right)\right]}, \qquad (2)$$

where R_0 is the off-center distance from the circular-drainage-area center.

Lu and Tiab (2008) extended Eq. 2 to a partially penetrating off-center vertical well in an anisotropic circular-cylinder-drainage reservoir.

Ge (1982) also introduced a productivity equation for a two-well system in a uniform-thickness isotropic circular cylinder reservoir, as shown in **Fig. 1.** The two wells are located at equal distance from the center of the circular system, and the wellbore radii are the same. The outer-boundary pressure (P_e) is assumed to be constant during production, such that the flow rates are equivalent. The steady-state productivity for each well is given by

$$Q_w = F_D \frac{2\pi KH(P_e - P_w)/(\mu B)}{\ln\left[\left(R_e^4 - R_0^4\right)/\left(2R_w R_0 R_e^2\right)\right]}. \qquad (3)$$

The performance of a multiple-wells system has received attention in the last decade. Camacho-V et al. (1999) gave a buildup solution in a system with multiple wells producing at constant wellbore pressures. Umnuayponwiwat and Ozkan (2000) presented equations of pressure-transient behavior and inflow performance of multiple vertical and horizontal wells in closed systems. Valko et al. (2000) presented pseudosteady-state productivity index for multiple wells producing from a closed rectangular reservoir. Marhaendrajana and Blasingame (2001) presented a solution and associated analysis method to evaluate single-well performance behavior in a multiple-wells reservoir system. Fokker et al. (2005) presented a semianalytical model for productivity testing of multiple wells; the interference effects and the presence of natural or induced fractures were accounted for. Busswell et al. (2006) presented a generalized analytical solution for reservoir problems with multiple wells and boundary conditions, assuming a cuboid-shaped reservoir using a method of integral transforms.

The primary goal of this study is to present productivity equations for multiple fully penetrating vertical wells in steady state in homogeneous, anisotropic sector fault reservoirs.

Well and Reservoir Models

Fig. 2 is a schematic of a multiple-wells system which drains a sector fault reservoir with height *H* and radius R_e, and the *j*th well is R_j away from the vertex of the angle of the sector. In a polar-coordinates system, the *j*th well is located at (R_j, θ_j).

The following assumptions are made:

1. At time t=0, pressure is distributed uniformly in the reservoir, equal to the initial pressure P_{ini}.
2. All wells are parallel to the *z*-direction, with a producing length equal to the pay-zone thickness *H,* and the top and bottom reservoir boundaries are impermeable.
3. A single-phase fluid, of small and constant compressibility C_f, constant viscosity μ, and formation volume factor *B,* flows from the reservoir to the wells. Fluid properties are independent of pressure. Gravity forces are neglected.

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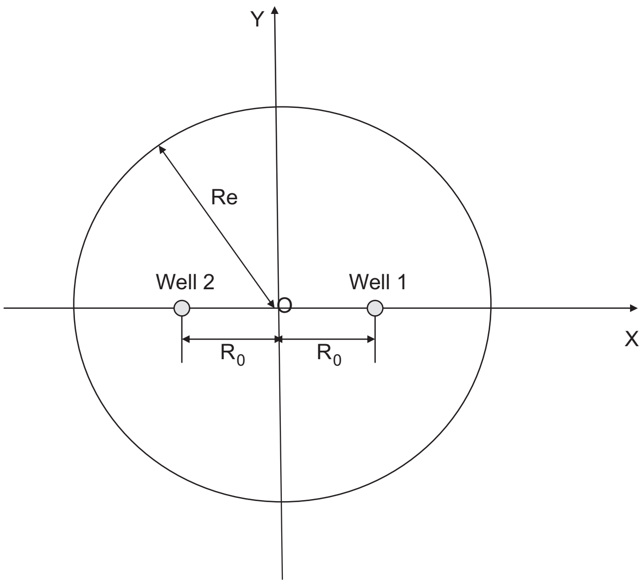

Fig. 1—Symmetric two-wells system.

4. A multiple-wells system is in a sector fault reservoir with constant horizontal permeability K_h, vertical permeability K_z, and thickness H. Horizontal permeability K_h is equal to radial permeability K_r. The outer boundary of the sector is always at constant pressure, P_e=P_{ini}, and the two sides of the angle of the sector are impermeable.

5. In any given time interval, the number of wells, their locations, the wellbore radii, and the mechanical skin factors are considered constant.

Diffusivity Equations and Boundary Conditions

In steady state, there holds

$$K_x \frac{\partial^2 P}{\partial x^2} + K_y \frac{\partial^2 P}{\partial y^2} + K_z \frac{\partial^2 P}{\partial z^2} = 0. \quad (4)$$

And for a sector fault reservoir, we assume

$$K_x = K_y = K_h = K_r. \quad (5)$$

The top and bottom reservoir boundaries are impermeable,

$$\frac{\partial P}{\partial z}\Big|_{z=0} = 0, \frac{\partial P}{\partial z}\Big|_{z=H} = 0. \quad (6)$$

As Fig. 2 shows, a system of fully penetrating multiple vertical wells is in a sector drainage domain

$$\Omega = \left[(R_e, \Phi) : \Phi = \pi / \tau\right], \quad (7)$$

where τ is an arbitrary real number that is greater than 0.5.

The two sides of the angle are impermeable,

$$\frac{\partial P}{\partial N}\Big|_{OA} = 0, \frac{\partial P}{\partial N}\Big|_{OB} = 0, \quad (8)$$

where $\frac{\partial P}{\partial N}\Big|_{OA,OB}$ are the exterior normal derivatives of pressure on the two sides of the angle of the sector area.

The outer boundary is with edgewater; during production, the pressure at the outer boundary is always equal to initial reservoir pressure P_{ini}.

$$P\big|_{r=R_e} = P_e = P_{\text{ini}}. \quad (9)$$

In order to solve the diffusivity equation in a dimensionless isotropic drainage domain, define the average permeability K_a as follows:

$$K_a = K_r^{2/3} K_z^{1/3}. \quad (10)$$

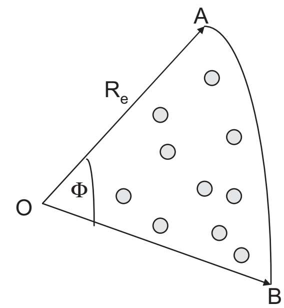

Fig. 2—Multiple-wells system in a sector-fault reservoir.

Mechanical Skin Factor

It is not unusual during drilling, completion, or workover operations for materials such as mud filtrate, cement slurry, or clay particles to enter the formation and reduce the permeability around the wellbore. This effect is commonly referred to as "wellbore damage," and the region of altered permeability is called the "skin zone." This zone can extend from a few inches to several feet from the wellbore. Many other wells are stimulated by acidizing or fracturing, which, in effect, increases the permeability near the wellbore.

The effect of the skin zone is to alter the pressure distribution around the wellbore. In case of wellbore damage, the skin zone causes an additional pressure loss in the formation. In case of wellbore improvement, a result opposite to that of wellbore damage occurs.

If we refer to the pressure drop in the skin zone as ΔP_s, for an isotropic reservoir, there holds

$$\Delta P_s = \left(\frac{1}{F_D}\right)\left(\frac{\mu B Q}{2\pi K H}\right) S, \quad (11)$$

where S is the mechanical skin factor caused by formation damage or stimulation.

In an anisotropic sector fault reservoir, the additional pressure drop caused by the mechanical skin factor can be calculated by

$$\Delta P_s = \left(\frac{1}{F_D}\right)\left(\frac{\mu B Q}{2\pi K_r H}\right) S. \quad (12)$$

Multiple-Wells Productivity Equations for a Sector Fault Reservoir

The two basic vector quantities that we would like to relate are the pressure drawdown vector,

$$\vec{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} P_{\text{ini}} - P_{wf,1} \\ P_{\text{ini}} - P_{wf,2} \\ \vdots \\ P_{\text{ini}} - P_{wf,n} \end{bmatrix}, \quad (13)$$

and the surface-production-rate vector,

$$\vec{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix}. \quad (14)$$

The surface-production-rate vector is calculated by

$$\vec{Q} = \frac{F_D\left(2\pi K_r H\right)}{\mu B}\left([A] + [D_s]\right)^{-1} \vec{d}, \quad (15)$$

where the matrix $[A]$ is the influence matrix,

$$[A]=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad (16)$$

with elements a_{ij} representing the influence of Well j on the pressure at the circumference of Well i.

For a sector fault reservoir, if $i \neq j$,

$$a_{ij}=\left(\frac{1}{2}\right)\ln(\lambda_1\lambda_2), \quad (17)$$

where (Lu and Tiab 2009)

$$\lambda_1=\frac{R_{Di}^{2\tau}+R_{Dj}^{2\tau}-2(R_{Di}R_{Dj})^{\tau}\cos[\tau(\theta_i-\theta_j)]}{1-2(R_{Di}R_{Dj})^{\tau}\cos[\tau(\theta_i-\theta_j)]+(R_{Di}R_{Dj})^{2\tau}} \quad (18)$$

$$\lambda_2=\frac{R_{Di}^{2\tau}+R_{Dj}^{2\tau}-2(R_{Di}R_{Dj})^{\tau}\cos[\tau(\theta_i+\theta_j)]}{1-2(R_{Di}R_{Dj})^{\tau}\cos[\tau(\theta_i+\theta_j)]+(R_{Di}R_{Dj})^{2\tau}}. \quad (19)$$

For a sector fault reservoir, if $i=j$,

$$a_{ii}=\ln\left(\frac{\lambda_3}{\lambda_4}\right), \quad (20)$$

where (Lu and Tiab 2009)

$$\lambda_3=2\tau R_{wDi}R_{Di}^{2\tau-1}\sin(\tau\theta_i), \quad (21)$$

$$\lambda_4=(1-R_{Di}^{2\tau})\left[1-2R_{Di}^{2\tau}\cos(2\tau\theta_i)+R_{Di}^{4\tau}\right]^{1/2}. \quad (22)$$

All the definitions of dimensionless variables in the above equations can be found in Appendix B.

The diagonal matrix $[D_s]$ is constructed from the vector of skin factors as

$$[D_S]=\begin{bmatrix} S_1 & 0 & \cdots & 0 \\ 0 & S_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_n \end{bmatrix}. \quad (23)$$

By rearranging Eq. 15, we can express the skin-factor vector as

$$\vec{S}=[D_q]^{-1}\left\{\frac{F_D(2\pi K_r H)}{\mu B}\vec{d}-[A]\vec{Q}\right\}, \quad (24)$$

where

$$[D_q]^{-1}=\begin{bmatrix} Q_1^{-1} & 0 & \cdots & 0 \\ 0 & Q_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_n^{-1} \end{bmatrix}. \quad (25)$$

Step-by-Step Procedure

Given the locations of each well in a sector fault reservoir, and that the outer-boundary pressure, flowing bottomhole pressure, reservoir- and fluid-properties data are also known, we may calculate the production rate or mechanical skin factor of each well.

Case One: Skin factors of each well are known; calculate production-rate vector.

Step 1. Calculate pressure-drawdown vector $\vec{d}$ in Eq. 13 and the number $F_D(2\pi K_r H)/(\mu B)$ in Eq. 15.

Step 2. Using Eqs. 17 and 20, calculate the influence matrix [A]; then use Eq. 23 to obtain the matrix $([A]+[D_s])$.

Step 3. Calculate the matrix $([A]+[D_s])^{-1}$.

Step 4. Using Eq. 15, calculate the production-rate vector $\vec{Q}$.

Case Two: Production rates of each well are known, calculate the skin-factor vector.

Skin factor is usually calculated from pressure-drawdown or -buildup analysis, but in steady state, pressure-transient data are not available; the following procedure provides another approach to obtain skin factor.

Step 1. Calculate pressure-drawdown vector $\vec{d}$ in Eq. 13 and the number $F_D(2\pi K_r H)/(\mu B)$, then obtain $[F_D(2\pi K_r H)/(\mu B)]\vec{d}$ in Eq. 24.

Step 2. Using Eqs. 17 and 20, calculate the influence matrix [A]; then calculate the vector $[A]\vec{Q}$.

Step 3. Using Eq. 25, calculate the matrix $[D_q]^{-1}$.

Step 4. Using Eq. 24, calculate the skin-factor vector.

Example One: Consider an anisotropic sector fault reservoir containing eight wells. The polar coordinates of these wells (R_j, θ_j) are presented in **Table 1.** The outer-boundary pressure, flowing bottomhole pressures, wellbore radii, and reservoir- and fluid-properties data in practical SI units are also shown in Table 1. Assuming the wellbore radii are identical and skin factors of each well are also identical in the following cases, calculate surface production rates for the eight-wells system and compare the analytical solutions with the numerical solutions obtained by the ECLIPSE 100™ black-oil simulator.

Case 1: $\Phi = 50°$, $S = 0, 5, 10$.

Case 2: $S = 0$, $\Phi = 60, 90, 120°$.

Case 3: $S = 5$, $\Phi = 50°$, $H = 18, 20, 22$m.

Solution, Case 1: When $S = 5$, the step-by-step procedure is as shown below:

Step 1: First, the value of τ is obtained: $\tau = 180°/\Phi = 180/50° = 3.6$.

The pressure-drawdown vector $\vec{d}$ is

$$\vec{d}=[12.9 \quad 12.8 \quad 11.6 \quad 12.5 \quad 13.2 \quad 9.3 \quad 12.2 \quad 8.7]^T$$

The unit for each element in $\vec{d}$ is MPa, and the superscript T stands for matrix transpose.

TABLE 1—WELL LOCATIONS AND RESERVOIR AND FLUID PROPERTIES FOR EXAMPLE 1

Outer boundary pressure, Pe	18.0 MPa
Well 1, $P_{wf,1}$	5.1 MPa
Well 2, $P_{wf,2}$	5.2 MPa
Well 3, $P_{wf,3}$	6.4 MPa
Well 4, $P_{wf,4}$	5.5 MPa
Well 5, $P_{wf,5}$	4.8 MPa
Well 6, $P_{wf,6}$	8.7 MPa
Well 7, $P_{wf,7}$	5.8 MPa
Well 8, $P_{wf,8}$	9.3 MPa
Well 1, (R_1, θ_1)	(300 m, 5°)
Well 2, (R_2, θ_2)	(400 m, 10°)
Well 3, (R_3, θ_3)	(800 m, 15°)
Well 4, (R_4, θ_4)	(1000 m, 20°)
Well 5, (R_5, θ_5)	(200 m, 25°)
Well 6, (R_6, θ_6)	(1500 m, 30°)
Well 7, (R_7, θ_7)	(600 m, 35°)
Well 8, (R_8, θ_8)	(2000 m, 40°)
Wellbore radius, R_w	0.1 m
Drainage radius, R_e	2500 m
Sector angle, Φ	50°
Payzone thickness, H	20 m
Radial permeability, K_r	0.1 μm^2
Vertical permeability, K_z	0.025 μm^2
Oil viscosity, μ	5.0 mPa.s
Formation volume factor, B	1.25 Rm^3/Sm^3

TABLE 2—ANALYTICAL SOLUTIONS OF CASE 1

	Flow Rate (std m^3/d)								
Skin	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6	Well 7	Well 8	Total
0	9.92	16.58	21.60	64.38	15.62	43.59	21.48	76.96	270.13
5	13.41	18.01	23.94	49.76	16.67	36.95	22.51	55.89	237.12
10	15.00	18.48	23.51	41.81	17.26	31.95	22.14	44.54	214.68

In practical SI units, $F_D = 86.4$; then

$$F_D(2\pi K_r H)/(\mu B) = (2\pi)\times 86.4\times 0.1\times 20/(5\times 1.25)$$
$$= 173.718.$$

Step 2: Using Eqs. 17 and 20, when $S = 5$, the matrix $[A]+[D_s]$ is

$$[A]+[D_S] = \begin{bmatrix} 28.846 & 15.429 & 9.900 & 8.268 & 16.886 & 5.340 & 11.845 & 3.269 \\ 15.429 & 26.420 & 9.945 & 8.279 & 14.856 & 5.337 & 11.711 & 3.266 \\ 9.900 & 9.945 & 21.803 & 8.514 & 9.867 & 5.306 & 9.613 & 3.236 \\ 8.268 & 8.279 & 8.514 & 20.257 & 8.261 & 5.327 & 8.207 & 3.229 \\ 16.886 & 14.856 & 9.867 & 8.261 & 30.186 & 5.341 & 11.939 & 3.270 \\ 5.340 & 5.337 & 5.306 & 5.327 & 5.341 & 17.743 & 5.355 & 3.413 \\ 11.845 & 11.711 & 9.613 & 8.207 & 11.939 & 5.355 & 23.586 & 3.282 \\ 3.269 & 3.266 & 3.236 & 3.229 & 3.270 & 3.413 & 3.282 & 16.391 \end{bmatrix}$$

Step 3: Calculating the matrix $([A]+[D_s])^{-1}$, we obtain

$$\left([A]+\left[D_s\right]\right)^{-1} = \begin{bmatrix} 0.063 & -0.018 & -0.006 & -0.004 & -0.020 & -0.002 & -0.008 & -0.001 \\ -0.018 & 0.066 & -0.008 & -0.005 & -0.013 & -0.003 & -0.011 & -0.001 \\ -0.006 & -0.008 & 0.067 & -0.014 & -0.005 & -0.006 & -0.011 & -0.003 \\ -0.004 & -0.005 & -0.014 & 0.067 & -0.004 & -0.009 & -0.009 & -0.004 \\ -0.020 & -0.013 & -0.005 & -0.004 & 0.057 & -0.002 & -0.009 & -0.001 \\ -0.002 & -0.003 & -0.006 & -0.009 & -0.002 & 0.066 & -0.005 & -0.008 \\ -0.008 & -0.011 & -0.011 & -0.009 & -0.009 & -0.005 & 0.066 & -0.003 \\ -0.001 & -0.001 & -0.003 & -0.004 & -0.001 & -0.008 & -0.003 & 0.065 \end{bmatrix}$$

Step 4: Using Eq. 15, the surface-production-rate vector $\vec{Q}$ is obtained:

$$\vec{Q} = \left[Q_{w1} \quad Q_{w2} \quad Q_{w3} \quad Q_{w4} \quad Q_{w5} \quad Q_{w6} \quad Q_{w7} \quad Q_{w8}\right]^T$$
$$= [13.41 \quad 18.01 \quad 23.94 \quad 49.76 \quad 16.67 \quad 36.95 \quad 22.51 \quad 55.89]^T$$

The unit for each element in $\vec{Q}$ is std m^3/d.

When $S = 5$, the total flow rate of the multiple wells system is $Q_t = \sum_{j=1}^{8} Q_{w,j} = 237.12(\text{std } m^3/d)$.

The analytical solutions and the numerical solutions of Case 1 are shown in **Tables 2 and 3,** respectively.

Case 2: The results are shown in **Tables 4 and 5.**

Case 3: The results are shown in **Tables 6 and 7.**

From the preceding calculation results, it can be found that single-well productivity and total productivity of the multiple-wells system are increasing functions of the sector angle; there are no significant differences between the analytical results and its corresponding numerical result, which implies that the proposed analytical model is reliable.

Example Two: The following production-rate vector is given:

$$\vec{Q} = [Q_{w1} \quad Q_{w2} \quad Q_{w3} \quad Q_{w4} \quad Q_{w5} \quad Q_{w6} \quad Q_{w7} \quad Q_{w8}]^T$$
$$= [17.91 \quad 10.80 \quad 39.74 \quad 39.36 \quad 12.03 \quad 32.59 \quad 34.03 \quad 52.45]^T$$

TABLE 3—NUMERICAL SOLUTIONS OF CASE 1

	Flow Rate (std m^3/d)								
Skin	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6	Well 7	Well 8	Total
0	9.99	15.90	23.69	58.21	14.27	48.79	21.89	80.39	273.11
5	12.95	17.23	24.48	46.28	15.51	39.87	22.30	58.39	237.01
10	14.31	17.62	23.53	39.27	16.13	33.86	21.68	46.33	212.73

TABLE 4—ANALYTICAL SOLUTIONS OF CASE 2

	Flow Rate (std m^3/d)								
Sector Angle	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6	Well 7	Well 8	Total
$\Phi = 60°$	12.81	19.97	25.67	69.04	20.23	49.53	28.12	84.29	309.64
$\Phi = 90°$	21.90	29.47	34.59	78.38	36.46	59.31	44.29	93.44	397.83
$\Phi = 120°$	29.74	36.72	39.91	83.44	52.08	63.82	54.78	97.05	457.54

TABLE 5— NUMERICAL SOLUTIONS OF CASE 2

	Flow Rate (std m^3/d)								
Sector Angle	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6	Well 7	Well 8	Total
$\Phi = 60°$	12.73	19.01	26.97	61.65	18.49	52.37	27.57	83.86	302.66
$\Phi = 90°$	21.19	27.81	35.58	70.91	32.80	63.08	41.42	95.65	388.43
$\Phi = 120°$	26.39	32.54	39.59	75.31	42.84	69.11	47.42	104.20	437.50

TABLE 6—ANALYTICAL SOLUTIONS OF CASE 3

Thickness (m)	Flow Rate (std m^3/d)								
	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6	Well 7	Well 8	Total
18	12.06	16.21	21.54	44.79	15.00	33.25	20.26	50.30	213.41
20	13.41	18.01	23.94	49.76	16.67	36.95	22.51	55.89	237.12
22	14.75	19.81	26.33	54.74	18.33	40.64	24.76	61.48	260.83

TABLE 7—NUMERICAL SOLUTIONS OF CASE 3

Thickness (m)	Flow Rate (std m^3/d)								
	Well 1	Well 2	Well 3	Well 4	Well 5	Well 6	Well 7	Well 8	Total
18	11.73	15.62	22.26	41.97	14.03	36.40	20.25	53.30	215.55
20	12.95	17.23	24.48	46.28	15.51	39.87	22.30	58.39	237.01
22	14.15	18.83	26.65	50.53	16.97	43.24	24.32	63.34	258.02

The unit for each element in $\vec{Q}$ is std m^3/d.

The well locations, outer boundary pressure, flowing bottomhole pressures, wellbore radii, and reservoir- and fluid-properties data are the same as shown in Table 1. Calculate skin factors for the eight-wells system.

Solution Step 1: The pressure-drawdown vector $\vec{d}$ and the number $F_D(2\pi K_r H)/(\mu B)$ are already obtained in Example 1; then

$$[F_D(2\pi K_r H)/(\mu B)]\times\vec{d}$$
$$=\left[2240.96\ \ 2223.59\ \ 2015.13\ \ 2171.48\ \ 2293.08\ \ 1615.58\ \ 1945.64\ \ 1511.35\right]^T$$

Step 2: The influence matrix $[A]$ is already obtained in Example 1; then the vector $[A]\vec{Q}$ can be obtained as

$$[A]\vec{Q}$$
$$=[2264.04\ \ 2151.04\ \ 2075.91\ \ 1897.90\ \ 2234.83\ \ 1414.50\ \ 2166.31\ \ 1209.21]^T$$

Step 3: Calculating the matrix $[D_q]^{-1}$ in Eq. 25 and assuming that the column vector $[D_d]$ below consists of the diagonal elements of the matrix $[D_q]^{-1}$, we have

$$\left[D_d\right]$$
$$=\left[0.0559\ \ 0.0926\ \ 0.0252\ \ 0.0254\ \ 0.0831\ \ 0.0307\ \ 0.0294\ \ 0.0191\right]^T$$

The nondiagonal elements in $[D_q]^{-1}$ are zero.

Step 4: Using Eq. 24, the skin-factor vector is given as

$$\vec{S}=\left[S_1\ \ S_2\ \ S_3\ \ S_4\ \ S_5\ \ S_6\ \ S_7\ \ S_8\right]^T$$
$$=\left[-1.29\ \ 6.72\ \ -1.53\ \ 6.95\ \ 4.84\ \ 6.17\ \ -1.38\ \ 5.76\right]^T$$

Conclusions

Assuming no water encroachment and no multiphase-flow effects and taking edgewater as a constant-pressure boundary and assuming that top and bottom reservoir boundaries are impermeable, the following conclusions are reached:

1. The proposed multiple-wells productivity equation provides a simple, reasonably accurate and fast analytical tool to evaluate well performance in steady state.
2. For a given number of wells, well pattern, anisotropic permeabilities, skin factor, and pressure difference between reservoir outer boundary and flowing bottomhole pressure have significant effects on single-well productivity and total productivity of the multiple-wells system.
3. In a sector-fault reservoir, single-well productivity and total productivity of the multiple-wells system are increasing functions of the sector angle.

Nomenclature

B = formation volume factor, L^3/L^3
F_D = unit conversion factor, dimensionless
H = pay-zone thickness, L
K = effective permeability, L^2
n = number of wells
P_e = reservoir outer-boundary pressure, m/(Lt^2)
P_{ini} = initial reservoir pressure, m/(Lt^2)
P_w = flowing bottomhole pressure, m/(Lt^2)
PI = single-well productivity index, L^{4}t/m
PIT = total productivity index of the multiple-wells system, L^{4}t/m
Q_w = single-well production rate, L^3/t
Q_t = total production rate of the multiple-wells system, L^3/t
R_e = radius of the sector-fault reservoir, L
R_j = off-vertex distance of jth well in the multiple-wells system, L
R_w = wellbore radius, L
θ_j = wellbore location angle of jth well in the multiple wells system, radians
μ = fluid viscosity, m/(Lt)
λ_1 = a function defined by Eq. 18
λ_2 = a function defined by Eq. 19
λ_3 = a function defined by Eq. 21
λ_4 = a function defined by Eq. 22
Φ = angle of the sector reservoir, radians
Ω = drainage domain

Subscripts

D = dimensionless
e = external
h = horizontal
ini = initial
i, j = well index
r = radial
t = total
w = well
x, y, z = coordinate indicators

Superscripts

T = transposed

Vectors and Matrices

$\vec{d}$ = pressure-drawdown vector, m/(Lt^2)
$\vec{Q}$ = production-rate vector, L^3/t
$\vec{S}$ = skin-factor vector, dimensionless
$[A]$ = influence matrix, dimensionless
$[D_s]$ = diagonal matrix of skin factors, dimensionless
$[D_q]$ = diagonal matrix of surface production rates, L^3/t

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References

Busswell, G., Banerjee, R., Thambynayagam, R.K.M., and Spath, J. 2006. Generalized Analytical Solution for Reservoir Problems With Multiple Wells and Boundary Conditions. Paper SPE 99288 presented at the Intelligent Energy Conference and Exhibition, Amsterdam, 11–13 April. doi: 10.2118/99288-MS.

Butler, R.M. 1994. *Horizontal Wells for the Recovery of Oil, Gas and Bitumen*. Monograph Series, SPE, Richardson, Texas, USA.

Camacho-V., R., Galindo-N., A., and Prats, M. 1999. Buildup Solution in a System with Multiple Wells Producing at Constant Wellbore Pressures. Paper SPE 53936 presented at the Latin American and Caribbean Petroleum Engineering Conference, Caracas, 21–23 April. doi: 10.2118/53936-MS.

Dietz, D.N. 1965. Determination of Average Reservoir Pressure From Build-Up Surveys. *J Pet Technol* **17** (8): 955–959; *Trans.,* AIME, **234.** SPE-1156-PA. doi: 10.2118/1156-PA.

Fokker, P.A., Brouwer, G.K., Verga, F., and Ferrero, D. 2005. A Semi-Analytical Model for Productivity Testing of Multiple Wells. Paper SPE 94153 presented at the SPE Europec/EAGE Annual Conference, Madrid, Spain, 13–16 June. doi: 10.2118/94153-MS.

Ge, J.L. 1982. *Mechanics of Fluids Flow in Oil Reservoir*. Beijing: Petroleum Industry Publishing Company.

Lu, J. and Tiab, D. 2008. Productivity equations for an off-center partially penetrating vertical well in an anisotropic reservoir. *J. Pet. Sci. Eng.* **60** (1): 18–30. doi: 10.1016/j.petrol.2007.04.011.

Lu, J. and Tiab, D. 2009. *Productivity Equations for Oil Wells: New Solutions Based on Three Dimensional Models*. Saarbrücken, Germany: VDM Verlag Publishing Company.

Marhaendrajana, T. and Blasingame, T.A. 2001. Decline Curve Analysis Using Type Curves—Evaluation of Well Performance Behavior in a Multiwell Reservoir System. Paper SPE 71517 presented at the SPE Annual Technical Conference and Exhibition, New Orleans, 30 September–3 October. doi: 10.2118/71517-MS.

Umnuayponwiwat, S. and Ozkan, E. 2000. Evaluation of Inflow Performance of Multiple Horizontal Wells in Closed Systems. *J. Energy Resour. Technol.* **122** (1): 8–13. doi: 10.1115/1.483155.

Umnuayponwiwat, S., Ozkan, E., and Raghavan, R. 2000. Pressure Transient Behavior and Inflow Performance of Multiple Wells in Closed Systems. Paper SPE 62988 presented at the SPE Annual Technical Conference and Exhibition, Dallas, 1–4 October. doi: 10.2118/62988-MS.

Valko, P.P., Doublet, L.E., and Blasingame, T.A. 2000. Development and Application of the Multiwell Productivity Index (MPI). *SPE J.* **5** (1): 21–31. SPE-51793-PA. doi: 10.2118/51793-PA.

APPENDIX A—DIMENSIONAL FACTORS TABLE

	Dimension	SI	Field	Practical SI
Distance	L	m	ft	m
Area	L^2	m^2	ft^2	m^2
Pressure	mL^{-1}t^{-2}	Pa	psi	MPa
Permeability	L^2	m^2	md	μm^2
Oil viscosity	mL^{-1}t^{-1}	Pa.s	cp	mPa.s
Flow rate	L^{3}t^{-1}	m^{3}s^{-1}	bbl/D	m^3/D

Appendix B—Definition of Dimensionless Variables

For sector fault drainage region,

$$x_D = \left(\frac{x}{R_e}\right)\left(\frac{K_a}{K_r}\right)^{1/2}$$

$$y_D = \left(\frac{y}{R_e}\right)\left(\frac{K_a}{K_r}\right)^{1/2}$$

$$z_D = \left(\frac{z}{R_e}\right)\left(\frac{K_a}{K_z}\right)^{1/2}, \quad \text{(B-1)}$$

$$H_D = \left(\frac{H}{R_e}\right)\left(\frac{K_a}{K_z}\right)^{1/2}, \quad \text{(B-2)}$$

$$R_{eD} = (K_a / K_r)^{1/2}$$

$$R_{jD} = (R_j / R_e)(K_a / K_r)^{1/2}, \quad \text{(B-3)}$$

and

$$R_{wD} = (R_w / R_e)[(K_r / K_z)^{1/4} + (K_z / K_r)^{1/4}] / 2. \quad \text{(B-4)}$$

SI Metric Conversion Factors

bar	× 1.0*	E+05	= Pa
bbl	× 1.589 873	E–01	= m^3
cp	× 1.0*	E–03	= Pa·s
ft	× 3.048*	E–01	= m
ft^3	× 2.831 685	E–02	= m^3
°F	(°F – 32)/1.8		= °C
md	× 9.86923×10^{-16}		= m^2
psi	× 6.894 757	E+00	= kPa

*Conversion factor is exact.

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