Wave motion and Velocity of Sound Wave

Mechanical wave

The waves which require material medium for their propagation are known as mechanical waves.

e.g. sound wave, waves in string etc.

Electromagnetic wave

The wave which do not require material medium for their propagation is known as electromagnetic wave.

e.g. light wave, x-rays, γ-rays etc.

Transverse wave

Particles of the medium vibrate perpendicular to the direction of propagation of wave.

e.g. light wave, heat wave, all electromagnetic wave, waves in string.

- For transverse wave medium requires shear modulus, hence medium must be rigid.
- Shear modulus of liquid and gas are zero.
- Only energy is transferred.
- It can be polarized.

Longitudinal wave

Particle of medium vibrates in the direction of propagation of wave.

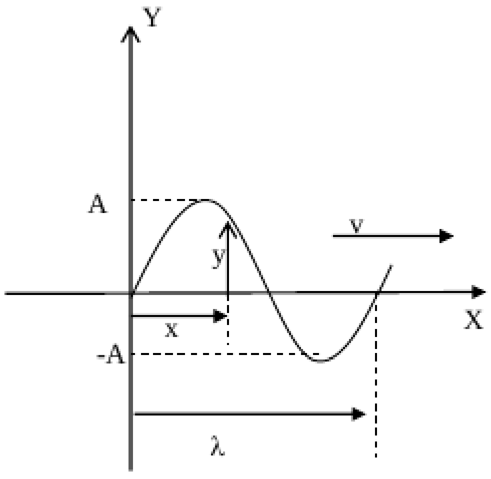
- It cannot be polarized.
- Mechanical wave transfer energy and momentum.
- In longitudinal wave density of medium changes.

Sound wave

i. Material medium is required for their propagation.
ii. Sound waves are longitudinal in nature.
iii. Sound travels in rocks in the form of longitudinal and transverse wave both.
iv. Speed of sound in air at NTP is 332 m/sec.
v. Sound wave cannot be polarized.
vi. Sound wave is audible between 20 Hz to 20 KHz.

Equation of progressive wave

Equation of plane progressive wave:



The equation in the form of

$y\ (x,t)$ = frepresents progressive wave.

where $y\ (x,t)$ = displacement of particle at position x at time `t'

v = wave velocity

i. $y\ (x,t) = f$; along $(+)$ve direction of x-axis.

ii. y (x,t) = f; along (-)ve direction of x-axis.

Equation of different forms of plane harmonic progressive wave

a. y = A sin $(\omega t \pm Kx)$

b. y = A sin $\frac{2\pi}{\lambda}$ $(vt \pm x)$

c. y = A sin $2\pi\left(\frac{t}{\lambda} \pm \frac{x}{\lambda}\right)$

where,
A = Amplitude
ω = $2\pi f$ = $\frac{2\pi}{T}$ = Angular frequency
K = $\frac{2\pi}{\lambda}$ = Wave number

Wave velocity:

$$V = \frac{\omega}{K}$$

Unit: m/sec or cm/sec

Particle velocity:

V_P = $\frac{dy}{dt}$

Maximum particle velocity: $(V_P)_{max}$ = $A\omega$

Wavelength:

It is the distance between particles oscillating in same phase.

$$\lambda = \frac{2\pi}{K}$$

Phase and phase difference:

Phase (ϕ) = ωt -Kx

i. *Phase difference with time:*
At constant position: x = constant
$\Delta\phi$= $\omega.\Delta t$ = $2\pi f.\Delta t$ = $\frac{2\pi}{T}$ Δt

ii. *Phase difference with position:*
At constant time: t = constant
$\Delta\phi$ = -$K.\Delta x$
where (-)ve sign indicates second particle is lagging behind the first.
$\therefore \Delta$ ϕ= phase difference = δ (say)
Δx = path difference
$\delta = \frac{2\pi}{\lambda} = \frac{2\pi f}{V}. \Delta x$

- If phase difference between two particles is positive, then 2nd particle is leading ahead the first.
- If phase difference between two particles is, $\delta = 2n\pi$; n = 0, $\pm$ 1, $\pm$ 2, $\pm$ ....
  then two particles are said to be **inphase** i.e **same phase**.
  In this case path difference, $\Delta x = n\lambda$
- If the phase difference between two particles is,
  $\delta = (2n \pm 1)\pi$; n = 0, $\pm$ 1, $\pm$ 2, $\pm$ ......
  then two particles are said to be **out of phase**.
  In this case path difference,
  $\Delta x = (2n \pm 1)\frac{\lambda}{2}$

For wave pulse,

$$y(x,t) = f\left(t \pm \frac{x}{v}\right)$$

At t = 0; y = $g(x)$ gives the equation of wave pulse.

e.g. $y = \dfrac{a}{b + (x \pm vt)^2}$

Differential equation of wave motion:

$\frac{dy}{dt}$ = -v. $\frac{dy}{dx}$

$\frac{dy}{dt}$ = particle velocity

$\frac{dy}{dt}$ = slope of tangent or relative deformation

Velocity of oscillating particle = $-\mathrm{v}\times$ slope of tangent

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

Equation of progressive wave or travelling wave in pressure variation form:

When sound wave propagates in medium then some regions gets compressed and rarefied that is there is change in pressure. The equation is,

$$P = P_0 \cos(\omega t - Kx)$$

$$P = P_0 \sin\left(\omega t - Kx + \frac{\pi}{2}\right)$$

where;

P_0 = pressure amplitude (max^m pressure)

P_0 = BKA = $\rho\mathrm{v}^2\mathrm{KA}$; $\left(v = \sqrt{\frac{B}{\rho}}\right)$

- The phase difference between displacement amplitude and pressure amplitude is $\frac{\pi}{2}$ i.e. where displacement is maximum pressure is minimum and vice-versa.

Velocity of wave on string:

$\mathrm{V} = \sqrt{\frac{T}{\mu}}$;

where, T = tension in string or wire

where, μ = mass per unit length = $\frac{m}{l}$

where, μ = linear mass density

If `ρ' is density of material of wire then,

μ = $\frac{m}{l}$ = $\frac{\rho \times A \times l}{l}$ = $\rho\mathrm{A}$ = $\rho\times\pi\mathrm{r}^2$

where, r = radius of cylindrical wire,

$$V = \sqrt{\frac{T}{\rho A}} = \frac{1}{r}\sqrt{\frac{T}{\rho\pi}}$$

- It does not depend on shape and size of wave pulse.
- It depends on elasticity and inertia.
  i. $\mathrm{v}\propto \sqrt{T}$
  ii. $\mathrm{v}\propto \frac{1}{\sqrt{\mu}}$
  iii. $\mathrm{v}\propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{r} \propto \frac{1}{\sqrt{A}}$
- Velocity of transverse wave in wire hung from rigid support at a distance `x' from lower end.

  V = $\sqrt{\frac{\frac{M}{L}xg}{\frac{M}{L}}}$ = $\sqrt{xg}$

Velocity of sound wave:

V = $\sqrt{\frac{Elasticity}{Density}} = \sqrt{\frac{E}{\rho}}$

i. **In case of solid:** E = γ = Young's modulus of elasticity
V = $\sqrt{\frac{\gamma}{\rho}}$

ii. **In case of liquid:** E = B = Bulk modulus of elasticity
V = $\sqrt{\frac{B}{\rho}}$

iii. **In case of gaseous medium:**

a. **Newton's formula:** Newton assumed that the propagation of sound wave is an ***isothermal*** process. i.e. temperature is constant. The modulus of elasticity is equal to the pressure of the gas.
E = P
$\therefore$ V = $\sqrt{\frac{P}{\rho}}$
In case of air at N.T.P.:
P = 1.013 $\times$ 10^5 N/m^2
ρ = 1.293 kg/m^3
Velocity of sound, V = $\sqrt{\frac{1.013}{1.293}}$ $\times$ 10^5
Velocity of sound, V = 280 m/sec
Experimentally; V = 332 m/sec
So it is not correct.

b. **Laplace's correction:** Laplace's assumed that propagation of sound wave in gaseous medium is an ***adiabatic process***. The modulus of elasticity of gas;
E = γ P
V = $\sqrt{\frac{\gamma RT}{M}}$
In case of air,
P = 1.013 $\times$ 10^5
ρ = 1.293 kg/m^2
γ= 1.41
V = 332 m/sec

Factors affecting velocity of sound:

a. With temperature:
V = $\sqrt{\frac{\gamma RT}{M}}$
$\mathrm{V} \propto \sqrt{T}$

- If V_0 and V_t are the velocity of sound in 0°C and t$^\circ$C then,

  $\frac{V_t}{V_0} = \sqrt{\frac{273+t}{273}}$
  For small temperature,
  $V_t = V_0 + 0.61t$
  Similarly,
  $\frac{V_{t_2}}{V_{t_1}} = \sqrt{\frac{273+t_2}{273+t_1}}$
- With increase in temperature, velocity of sound increases.
- If V_0 is the velocity of sound at 0°C and V_t is the velocity of sound at t$^\circ$C such that V_t = nV_0 then, temperature,

  $t = (n^2-1) \times 273^oC$
  For double the velocity of sound;
  n = 2, t = 819°C

b. With pressure:
If the temperature of the gas remains constant, the velocity of sound does not change with a change of pressure.
$\mathrm{P} \propto \rho$
$\frac{P}{\rho}$ = constant

With humidity:
Atmospheric air = dry air + water vapour
$\rho_{dry\ air} > \rho_{water\ vapour}$

- As humidity increases; $\rho_{atm.air}$ decreases; hence velocity of sound increases.
  Note: The density of hydrogen gas increases with increase in humidity, so velocity of sound in hydrogen decreases with increase in humidity.

- If two gases of same atomicity are mixed at same temperature then velocity of sound in mixture,

  $$\begin{array}{lcc} \text{Gas} & A & B \\ \text{Mol. wt.} & m_A & m_B \\ \text{Amount (ratio)} & x & y \end{array}$$

  $$\frac{V_A}{V_{mix}} = \sqrt{\frac{(xm_A + ym_B)}{(x+y)m_B}}$$

- Relation between RMS velocity of gas and velocity of sound

  $V_{RMS} = \sqrt{\frac{3RT}{M}}$ and

  $V_{sound} = \sqrt{\frac{\gamma RT}{M}}$

  $\therefore V_{sound} = \sqrt{\frac{\gamma}{3}} \times V_{RMS}$

- When a man on the ground finds that when he sees a jet plane just over his head the sound heard at angle `θ' with vertical then velocity of jet plane

  $V_{jet} = V_{sound} \times \sin\theta$

- When elevation of cloud is `$\theta$' above the horizon, a thunder is heard after time `t' observation of lighting. The speed of sound is `V'. The vertical height of cloud from ground,

  $h = Vt\sin\theta$

- If a stone is dropped into a well of height `h' above the water level, then splash in water is heard after time,

  $$t = \sqrt{\frac{2h}{g}} + \frac{h}{V}$$

Intensity (I):

$$I = \frac{Energy}{Area \times time}$$

Its unit is watt/m^2.

$I = 2\pi^2 f^2 A^2 \rho v$

$I \propto A^2 f^2$

Intensity in terms of pressure amplitude

$$I = \frac{P_0^2}{2\rho v}$$

$\therefore I \propto P_0{}^2$

i. For plane wave:

   $A \propto r^0$

ii. For spherical wave:

   $I \propto \frac{1}{r^2} \propto A^2$

   i.e. $A \propto \frac{1}{r}$

   Equation: $y = \frac{c}{r} \sin(\omega t - Kx)$

iii. For cylindrical wave,

   $I \propto \frac{1}{r} \propto A^2$

   i.e. $A \propto \frac{1}{\sqrt{r}}$

   Equation: $y = \frac{c}{\sqrt{r}} \sin(\omega t - Kx)$

- The speed of supersonic body is measured in **Mach number**. The Mach number is the ratio of speed of supersonic body to the speed of sound in air.

  $$\text{Mach number} = \frac{V_{body}}{V_{sound}}$$