Electromagnetic Induction

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The phenomenon of production of e.m.f. or current with the help of magnetic field is known as electromagnetic induction.

Michel Faraday's Laws of ElectromagneticInduction

i. When a time varying (changing) magnetic flux is linked with a conductor or coil, an e.m.f. induces it
ii. The magnitude of induced emf is directly proportional to the rate of change of magnetic flux.

$$\mathrm{E}=-\mathrm{N}\frac{d\phi}{2t}$$

$$\mathrm{E}=-\mathrm{N}\frac{(\phi_2-\phi_1)}{t}=-\mathrm{NA}\frac{(B_2-B_1)}{t}$$

Where, N = no. of turn in the coil

$\rightarrow$ Faraday's law is the law of conservation of energy.

$$\mathrm{i}=\frac{E}{R+r}$$

$\rightarrow$ If a resistance of the coil 'r' then current flows in the circuit
i = E/r

Methods of producing Induced e.m.f

i. Changing the magnetic field B
ii. Changing the area 'A' of the loop
iii. Changing the relative orientation of the coil with respect to the magnetic field.

Lenz's Law

The direction of induced (and also e.m.f) is such that is always opposes the cause (change) by which it is produced.

Lenz's law is based upon the law of conservation of energy. It gives the direction of induced current or induced e.m.f.

Production of induced e.m.f. in a coil in magnetic field

i. If a coil is rotated through an angle θ from its initial position in time 't' then induced e.m.f

$$\frac{-NBA(1-\cos\theta)}{t}$$

ii. If a coil is rotated through an angle 180^0 from its initial position then
$E=2NAB/t$
iii. If a coil is rotated through an angle 90^0 from its initial position then.
$E=-NAB/t$

iv. If a circular coil is put in a magnetic flux density then,

$$\mathrm{E}=-\mathrm{NA}\frac{(B_2-B_1)}{t}$$

v. If the magnetic flux is constant but area of the coil is changing then,

$$E=\frac{-NB\left(A_2-A_1\right)}{t}$$

vi. If both magnetic flux and area are changing then,

$$E=\frac{-N\left(B_2A_2-B_1A_1\right)}{l}$$

Fleming's Right hand rule

Stretch the thumb and the first two fingers of your right hand in mutually perpendicular directions. If the first finger points in the direction of the magnetic field, the thumb in the direction of motion of the conductor, then middle finger points in the direction of the induced current or emf in the conductor.

$\rightarrow$ Induced emf in a wire of length 'l' moving with velocity 'v' in magnetic flux density 'B' at an angle 'θ from the magnetic field. $e=B/v\sin\theta$

i. The emf produced in a wire by its motion across a magnetic field does not upon the composition and the diameter of the wire.

ii. The induced emf in a circuit may increase or decrease the magnetic flux through the circuit.

$\Rightarrow$ If an aeroplane moving with velocity 'v' at a place where earth's magnetic field 'B' and dip angle is θ then the emf develops across the two points on its wings will be
$\mathrm{E} = (\mathrm{B}\sin\theta)l\mathrm{v} = \mathrm{B_v}/\mathrm{v} \quad \mathrm{volt}$
Where, B_v is the vertical component of earth's magnetic field.

$\Rightarrow$ Emf developed in a rod, rotating about its one end perpendicular to the magnetic field
$\mathrm{E} = 1/2\mathrm{Bwl}^2$

$\Rightarrow$ Emf developed in a wheel of radius 'R' and N spokes, rotating about its center, perpendicular to the magnetic field.
$\mathrm{E} = 1/2\mathrm{BWR}^2$
Note:-emf does not depend upon the number of spokes 'N'.

$\Rightarrow$ Emf developed in a conducting disc of radius 'R' is rotating perpendicular to the magnetic field.
$\mathrm{E} = 1/2\mathrm{BWR}^2$

Instantaneous Induced emf in a rotating coil

$$\mathrm{E} = \mathrm{E_o}\ \mathrm{sinwt}\ = \mathrm{NABw\ sinwt\ volts}$$
$$\mathrm{E_{max}} = \mathrm{E_o} = \mathrm{NABW}$$
$$\mathrm{N} = \ \text{No. of turns in coil}$$
$$\mathrm{w} = \ \text{Angular frequency; } \mathrm{w} = 2\pi\mathrm{f} = 2\pi/\mathrm{T}$$

Eddy currents

Currents induced in a thick conductor when conductor is placed in a changing magnetic field. Due to eddy current excess heat is produced.

$\rightarrow$ To minimize eddy current the conductor (core of transformer) is laminated

Self-Induction (L)

The emf induced in a coil by changing magnetic flux produced by the coil itself is called self inductance. It opposes the change of electric current in the circuit.

$\rightarrow$ It is also called Electrical Inertial

$$\mathrm{E} = \frac{-Ldi}{dt} = \frac{-L\Delta i}{\Delta t}$$

L = Self inductance or coefficient of self inductance.

Units of self inductance

i. In C.G.S system; -ab-henry
ii. In SI system; -Henry (H)
1 henry = 10^9 ab-henry
1H = 1 wb A^{-1}

$\rightarrow$ Magnetic flux linked with a coil; ϕ_t = Nϕ

$$\therefore L = \frac{\phi N}{I}$$

Energy stored in the magnetic field associated with the coil
$U = 1/2LI^2$

1. **Self inductance of a circular coil**

$$\mathrm{L} = \frac{\mu_o\mu_r\pi N^2R}{2} = \frac{\mu_o\mu_r N^2A}{2R}$$
$$\text{Where R} = \text{radius coil}$$
$$\text{i.e. } \mathrm{L} \propto \mathrm{N}^2$$
$$\mathrm{L} \propto \mathrm{R}$$

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2. **Self inductance of solenoid**

$$\mathrm{L} = \frac{\mu_o\mu_r N^2A}{l}$$

Where,
l = length of solenoid (m)
N = Total number of turns in solenoid
A = Area of cross section of solenoid

3. **Self-inductance of Toroidal coil**
$$L=\frac{\mu_o\mu_r N^2 t}{2\pi}\log_e(b/a)$$
Where b= outer radius, a = inner radius of toroid

Mutual Inductance

It is the phenomenon of production of induced e.m.f. in one coil due to varying current in the neighbouring coil.

$$\mathrm{E}_2=-M\frac{di_1}{dt}=-\mathrm{M}\frac{\Delta i_1}{\Delta t}$$
M = coefficient of mutual inductance
$\rightarrow$ Its SI unit is Henry (H)

Magnetic flux linked with second coil
$$\phi_2=\frac{MI_1}{N_2}\Rightarrow \mathrm{M}=\frac{N_2\phi_1}{I_1}$$
$\Rightarrow$ Mutual inductance of two circular coils
$$\mathrm{M}=\frac{\mu_o\mu_r\pi N_1N_2A_2}{2r_1}=\frac{\mu_o\mu_r\pi N_1N_2\pi r_2^2}{2r_1}$$
$$\mathrm{M}\propto\frac{r_2^2}{r_1}$$

$\Rightarrow$ Mutual inductance of two solenoid
$$\mathbf{M}=\frac{\mu_o\mu_r N_1N_2A}{l}$$
Where l = common length of solenoids

Combination of Inductance

i. Series combination of two coils (current in two coils in same direction)
$L = L_1 + L_2 + 2M$

ii. Series combination of two coils (current in the two coils in opposite direction)
$L = L_1 + L_2 - 2M$

iii. Parallel combination (For same direction of current)
$$\mathrm{L}=\frac{L_1L_2-M^2}{L_1+L_2+2M}$$

iv. Parallel combination (for opposite direction of current)
$$\mathrm{L}=\frac{L_1L_2-M^2}{L_1+L_2-2M}$$

Coefficient of coupling
$\mathrm{K}=\mathrm{M}/\sqrt{L_1L_2}$

i. For uncoupled coils; k = 0, M = 0
ii. For tightly couple; k = 1, M = (Max^m)
iii. For loose coupling; K $<$ 1, and M decreases

Rise of current in an inductor

i. Growth of current in L-R circuit is given by
$$\mathrm{I}=\mathrm{I_o}\left(1-e^{-\frac{Rt}{L}}\right)=\mathrm{I_o}\left(1-\mathrm{e}^{-\mathrm{t}T}\right)$$

ii. Instantaneous Potential; V = V_o e$^{-t/T}$
where, T = L/R = Time constant

Decay of current
I = I_o (1 -e$^{-t/T}$)
$\rightarrow$ Electric Generator or Dynamo is a device which converts mechanical energy into electrical energy.

Electric Motor

D.C. Motor
A D.C. motor is the converse of a DC generator. It is a device in which electric energy is converted into mechanical energy.
Efficiency of motor; $\eta=\frac{\text{Output}}{\text{Input}}=\frac{E_b}{E}$
For max^m ~efficiency; E_b = E/2

Where
E_b = Back emf
E = Applied emf

Back emf

When coil of motor rotates in a magnetic field an emf is induced which opposes the supply current to flow into the coil of the rotor, known as back emf.

→ Armature current is given by

$$I_a = \frac{(E - E_b)}{R_a}$$

Where R_a = resistance of armature