Vector

Scalar

A physical quantity having magnitude but no direction is called a scalar. Distance, mass, time, temperature etc. are the examples of scalars.

Vector

Vectors are the physical quantities having both magnitude and direction. Velocity, acceleration, force, weight etc are the examples of vectors.

Modulus of a vector

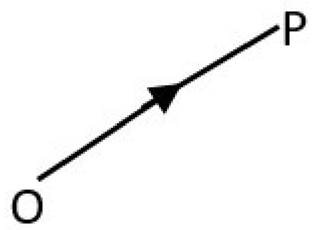
The modulus of a vector is a non-negative number which is the measure of the line segment representing the vector.

If $\overrightarrow{\mathrm{AB}} = x\overrightarrow{\mathrm{i}} + y\overrightarrow{\mathrm{j}}$ then $|\overrightarrow{\mathrm{AB}}| = \sqrt{x^2+y^2}$

If $\overrightarrow{\mathrm{AB}} = (x, y, z)$ be any vector in space then, $|\overrightarrow{\mathrm{AB}}| = \sqrt{x^2+y^2+z^2}$

Position vector

Let $0(0,0,0)$ be origin and $\mathrm{P}(x, y, z)$ be any point. Then vector $\overrightarrow{\mathrm{OP}} = (x, y, z)$ is called position vector of point P.



Let O be the origin:- The position vector of the point P is the vector $\overrightarrow{OP}$. If $\vec{a}$ and $\vec{b}$ are the position vectors of the points A and B respectively, then the vector $\overrightarrow{AB}$ is given by $\vec{b}-\vec{a}$. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = b - a$

Section formula:-

The position vector of a point which divides internally the line segment joining two points A, B with position vectors $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ in the ratio $m : n$ is $\dfrac{m\vec{b}+n\vec{a}}{m+n}$ The section formula for external division is $\dfrac{m\vec{b}-n\vec{a}}{m-n}$

Mid point formula:-

The position vector of the midpoint of $\overrightarrow{\mathrm{AB}}$ is $\frac{1}{2}(\overrightarrow{\mathrm{a}} + \overrightarrow{\mathrm{b}})$, where $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ are position vectors of A and B respectively.

Centroid formula:-

If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of a triangle, then the position vector of the centroid if the triangle is

$$\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

Types of vectors:-

- <u>Null vector (zero vector)</u>
  A vector having magnitude zero is called zero vector. It is denoted by O.
  In other words, it is a vector having same initial and terminal point.
  We have, |O| = 0
  The direction of a null vector is indeterminate and is geometrically represented by a point.

- <u>Unit vector</u>

  A vector whose magnitude is unity is called unit vector. The unit vector in the direction of a non zero vector $\overrightarrow{\mathrm{a}}$ denoted by $\hat{a}$ and is given by:

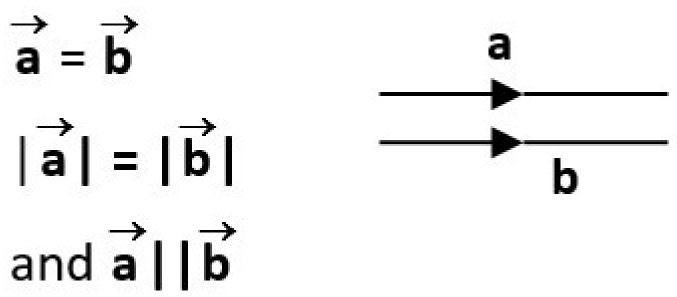
$\hat{a} = \dfrac{\vec{a}}{|a|}$ where $(|\vec{a}| \neq 0)$

- <u>Negative of a vector</u>

  A vector having the same magnitude of a given vector $\overrightarrow{\mathrm{OA}}$ but the opposite direction is denoted by
  $-\overrightarrow{\mathrm{OA}}$ or $\overrightarrow{\mathrm{AO}}$ and is called the negative of the vector $\overrightarrow{\mathrm{OA}}$.
  $\overrightarrow{\mathrm{AO}} = -\overrightarrow{\mathrm{AO}}$
  $\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AO}} = 0$

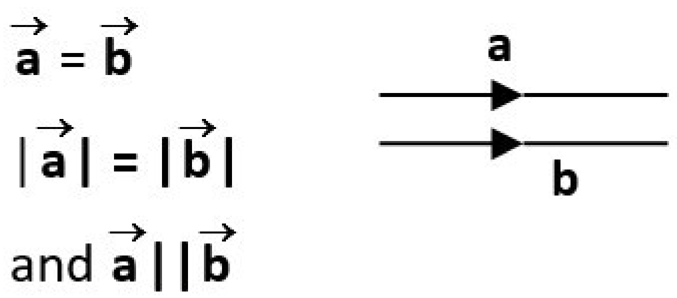
- <u>Equal vectors</u>

  Two vectors $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ are said to be equal if they have equal magnitude and same sense of direction.



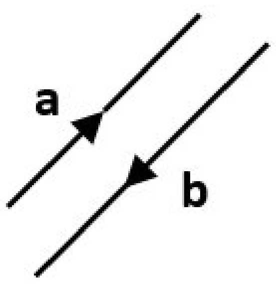
- <u>Like vectors</u>

  Two vectors are said to be like vectors if they have same direction.



- <u>Unlike vectors</u>

  Two vectors are said to be unlike vectors if they have opposite direction.



- <u>Localised vector</u>
  A vector which passes through a given point and which is parallel to the given vector is said to be a localized vector.

- <u>Collinear vectors</u>
  Two or more vectors which lie along the same line or which are parallel are called collinear vectors.
  Any vector $\overrightarrow{\mathrm{r}}$ collinear with $\overrightarrow{\mathrm{a}}$ can be written as $\overrightarrow{\mathrm{r}} = \lambda \overrightarrow{\mathrm{a}}$ where λ is any scalar.

Note:

i. For the collinear vectors $(\mathrm{x}_1, \mathrm{y}_1)$, $(\mathrm{x}_2, \mathrm{y}_2)$ and $(\mathrm{x}_3, \mathrm{y}_3)$ in a plane

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

- <u>Coplanar vectors:</u>
  Two or more than two vectors which lie in a same plane are called coplanar vectors otherwise they are non-coplanar vectors.

  Any vector $\overrightarrow{\mathrm{r}}$ in the space can uniquely be expressed as a linear combination of three non- coplanar vectors $\overrightarrow{\mathrm{a}}$, $\overrightarrow{\mathrm{b}}$ and $\overrightarrow{\mathrm{c}}$ as

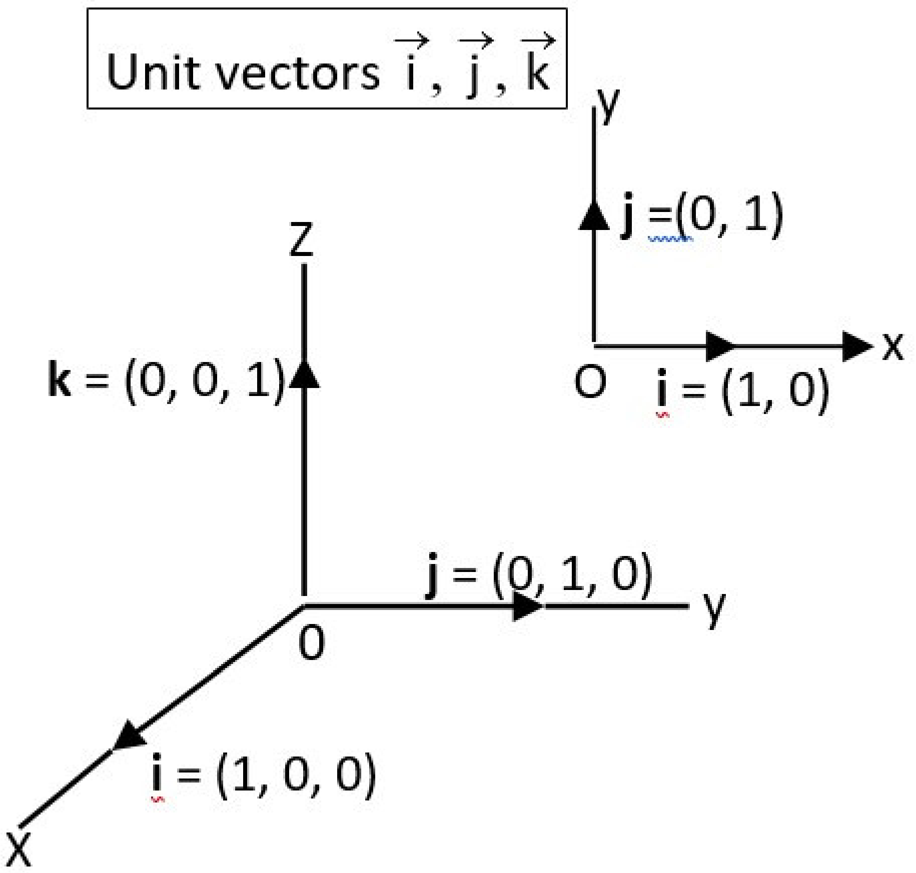
  i.e. $\overrightarrow{\mathrm{r}} = x\overrightarrow{\mathrm{a}} + y\overrightarrow{\mathrm{b}} + z\overrightarrow{\mathrm{c}}$, where x, y, z are scalars.

Note:

i. Set of vectors $\overrightarrow{r}_1, \overrightarrow{r}_2, \overrightarrow{r_3}$ are said to be coplanar if $\overrightarrow{x\overrightarrow{r_1}} + \overrightarrow{yr_2} + \overrightarrow{zr_3} = \overrightarrow{0}$ for at least one (x or y or z) is not equal to zero. Or one of the vectors can be represented as the linear combination of other two.
i.e., $\overrightarrow{r_1} = x\overrightarrow{r_2} + \overrightarrow{yr_3}$

ii. If the vectors represented by the points (a_1, a_2, a_3), (b_1, b_2, b_3) and (c_1, c_2, c_3) are coplanar, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$



Direction cosines of a vector:-

Let $\vec{a} = \left(x_1\vec{i} + x_2\vec{j} + x_3\vec{k}\right)$ be any vector in space.

$|\vec{a}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$

unit vector of $\vec{a}$ is $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$= \frac{x_1\overrightarrow{i} + x_2\overrightarrow{j} + x_3\overrightarrow{k}}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$$

$$= \frac{x_1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}\vec{i} + \frac{x_2}{\sqrt{x_1^2 + x_2^2 + x_3^2}}\vec{j} + \frac{x_3}{\sqrt{x_1^2 + x_2^2 + x_3^2}}\vec{k} = \overrightarrow{l_1} + m\vec{j} + n\vec{k}$$

where (l, m, n) are the direction cosines of the vector $\vec{a}$

Linear combination of vectors

A vector $\overrightarrow{r}$ is said to be in a linear combination of vectors $\overrightarrow{r_1}, \overrightarrow{r}_2, \overrightarrow{r_3}, \ldots \overrightarrow{r_n}$ if there exist scalars $x_1\ x_2, x_3, \ldots\ldots x_n$ such that $\overrightarrow{r} = x_1\overrightarrow{r_1} + x_2 r_2 + x_3\overrightarrow{r_3} + \ldots + x_n\overrightarrow{r_n}$

Linearly independent

A set of non-zero vectors $\overrightarrow{a_1}, \overrightarrow{a_2}, \ldots \overrightarrow{a_n}$ is said to be **linearly independent** if $x_1\overrightarrow{a_1} + x_2\overrightarrow{a_2} + \ldots\ldots + x_n\overrightarrow{a_n} = 0$ holds iff

$x_1 = 0, x_2 = 0, \ldots\ldots x_n = 0$

Linearly dependent

A set of vectors $\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3}, \ldots \overrightarrow{a_n}$ is said to be linearly dependent if there exist scalars $x_1, x_2, \ldots x_n$ not all zero i.e. at least one of them is not zero, such that $x_1\vec{a}_1 + x_2\vec{a}_2 + \ldots + x_n\vec{a}_n = 0$. All coplanar vectors are **linearly dependent.**

Laws of vector addition:-

i. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$
$\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = 0 \quad (\text{ triangle law })$

$\text{So}, \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

ii. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative law)

iii. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})(\text{ associative law })$

iv. $m(n\overrightarrow{a}) = (mn)\overrightarrow{a}$ $(m, n$ being scalar)

v. $(m+n)\vec{a} = m\vec{a} + n\vec{a}$ $(\text{ distributive law })$

vi. $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$ (distributive law)

Scalar or dot product (inner product)

Let $\overrightarrow{a} = (a_1, a_2)$ and $\overrightarrow{b} = (b_1, b_2)$ be any two vectors and θ be the angle between them.

Then the dot product of $\overrightarrow{a}$ and $\overrightarrow{b}$ denoted by $\overrightarrow{a}.\overrightarrow{b}$ is given by

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2$

where $0 \leq \theta \leq \pi$

Scalar or dot product (inner product)

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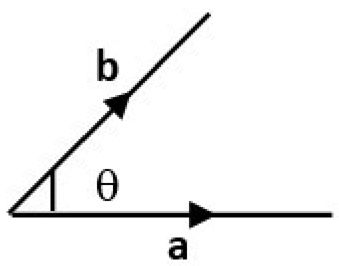
where $0 \leq \theta \leq \pi$

Angle between two vectors

If θ is the angle between two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, then $(0 \leq \theta \leq \pi)$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$



$\text{If } \theta = 90^\circ, \cos 90^\circ = 0$

$$0 = \frac{\mathbf{ab}}{|\mathbf{a}||\mathbf{b}|} \qquad \text{i.e. } \overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} = \mathbf{0}$$

Two vectors are orthogonal if $\overrightarrow{a}.\overrightarrow{b} = 0$

Properties of scalar product:-

i. Two vectors are perpendicular if $\overrightarrow{a}.\overrightarrow{b} = 0$

ii. $\vec{a} \cdot \vec{a} = \vec{a}^2 = |\vec{a}|^2 = a^2$

iii. $\overrightarrow{a} \cdot \overrightarrow{b} = 0$ either $\overrightarrow{a} = 0$ or $\overrightarrow{b} = 0$ or $\theta = 90^\circ$

iv. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}(\text{ commutative law })$

v. $\overrightarrow{i} \cdot \overrightarrow{i} = \overrightarrow{j} \cdot \overrightarrow{j} = \overrightarrow{k} \cdot \overrightarrow{k} = 1$
i.e. $\overrightarrow{i}^2 = \overrightarrow{j}^2 = \overrightarrow{k}^2 = 1$

vi. $\overrightarrow{i} \cdot \overrightarrow{j} = \overrightarrow{j} \cdot \overrightarrow{k} = \overrightarrow{k} \cdot \overrightarrow{i} = 0$, where $\overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k}$ are unit vectors in three mutually perpendicular directions.

vii. $m(\vec{a} \cdot \vec{b}) = (m\vec{a}) \cdot \vec{b} = a \cdot (m\vec{b})$

viii. $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}(\text{ distributive law })$

ix. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a}^2 - \vec{b}^2 = a^2 - b^2$

x. $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, if $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

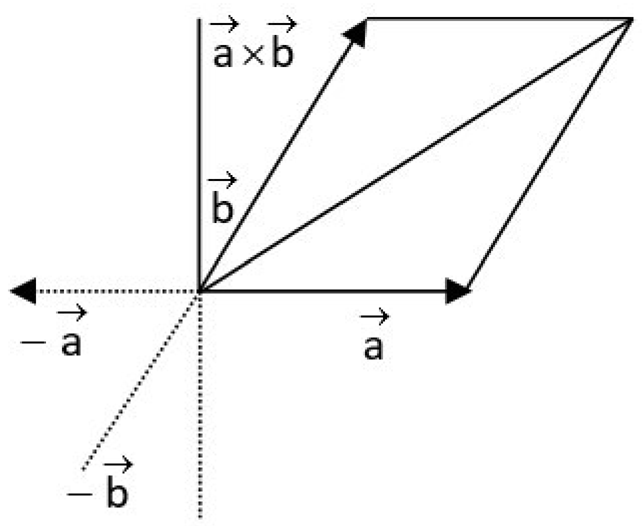
xi. $$\begin{aligned} \overrightarrow{\mathrm{a}} &= a_1 \overrightarrow{\mathrm{i}} + a_2 \overrightarrow{\mathrm{j}} + a_3 \overrightarrow{\mathrm{k}} \\ &= (\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{i}})\overrightarrow{\mathrm{i}} + (\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{j}})\overrightarrow{\mathrm{j}} + (\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{k}})\overrightarrow{\mathrm{k}} \end{aligned}$$

where $(\overrightarrow{\mathrm{a}} . \overrightarrow{\mathrm{i}}) = \mathrm{a}_1$
$\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{j}} = \mathrm{a}_2$
$\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{k}} = \mathrm{a}_3$

Vector product or cross product

The vector product of two vectors $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ denoted by $\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}$ is given by
$\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}} = \mathrm{ab}\sin\theta\hat{\eta}$

Where θ is the angle between $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ and $\hat{\eta}$ is the unit vector perpendicular to both $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ as given by the right handed screw rule from $\overrightarrow{\mathrm{a}}$ to $\overrightarrow{\mathrm{b}}$.



Let $\overrightarrow{\mathrm{a}} = (a_1, a_2)$ and $\overrightarrow{\mathrm{b}} = (b_1, b_2)$ be two vectors in the cartesian palne (i.e. XY plane) then the vector product of the two vectors $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ denoted by $\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}$, is defined by

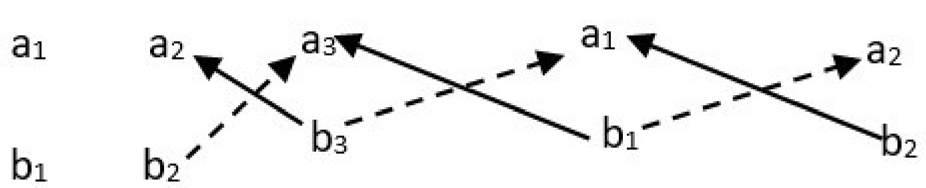
$\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}} = (a_1b_2 - a_2b_1)\overrightarrow{\mathrm{k}}$

where $\overrightarrow{\mathrm{k}}$ is the standard unit vector along positive z-axis.
The vector product of two space vectors

$\overrightarrow{\mathrm{a}} = (\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3)$ and $\overrightarrow{\mathrm{b}} = (\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3)$ is

$$\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}} = \begin{vmatrix} \overrightarrow{\mathrm{i}} & \overrightarrow{\mathrm{j}} & \overrightarrow{\mathrm{k}} \\ \mathrm{a}_1 & \mathrm{a}_2 & \mathrm{a}_3 \\ \mathrm{b}_1 & \mathrm{b}_2 & \mathrm{b}_3 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$



Note:

i. The area of a parallelogram whose adjacent sides are represented by $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ is $|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}|$

ii. The area of the triangle OAB in which $\overrightarrow{\mathrm{OA}} = \overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{OB}} = \overrightarrow{\mathrm{b}}$ is $\frac{1}{2}|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}|$

iii. If the diagonals of parallelogram are $\overrightarrow{\mathrm{c}}$ and $\overrightarrow{\mathrm{d}}$, then its sides $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ are $\vec{a} = \left(\frac{\vec{c} - \vec{d}}{2}\right)$ and $\vec{b} = \left(\frac{\vec{c} + \vec{d}}{2}\right)$

iv. If the position vectors of the vertices of a triangle are $\overrightarrow{\mathrm{a}}, \overrightarrow{\mathrm{b}}, \overrightarrow{\mathrm{c}}$ then $\Delta = \frac{1}{2}|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}} + \overrightarrow{\mathrm{b}} \times \overrightarrow{\mathrm{c}} + \overrightarrow{\mathrm{c}} \times \overrightarrow{\mathrm{a}}|$

v. The area of the parallelogram with diagonals **c** and **d** is $\frac{1}{2}\lceil \vec{c} \times \vec{d}|$

Properties of vector product:-

i. $(\vec{a} \times \vec{b}) = -(\vec{b} \times \vec{a})$

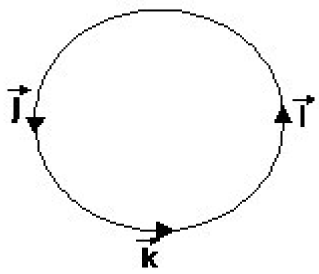
ii. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

iii. $\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}} = 0 \Rightarrow \overrightarrow{\mathrm{a}} = 0$ or $\overrightarrow{\mathrm{b}} = 0$ or $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ are parallel

iv. $(n\vec{a}) \times \vec{b} = n(\vec{a} \times \vec{b}) = \vec{a} \times n\vec{b}$

v.
$$\overrightarrow{\mathrm{i}} \times \overrightarrow{\mathrm{i}} = 0, \overrightarrow{\mathrm{j}} \times \overrightarrow{\mathrm{j}} = 0, \overrightarrow{\mathrm{k}} \times \overrightarrow{\mathrm{k}} = 0$$
$$\overrightarrow{\mathrm{i}} \times \overrightarrow{\mathrm{i}} = (1,0,0) \times (1,0,0)$$
$$= (0,0,0)$$
$$= 0$$

vi.
$$\overrightarrow{\mathrm{i}} \times \overrightarrow{\mathrm{j}} = \overrightarrow{\mathrm{k}}$$
$$\overrightarrow{\mathrm{j}} \times \overrightarrow{\mathrm{k}} = \overrightarrow{\mathrm{i}}$$
$$-\overrightarrow{\mathrm{k}} \times \overrightarrow{\mathrm{i}} = \overrightarrow{\mathrm{j}}$$
and,
$$\overrightarrow{\mathbf{j}} \times \overrightarrow{\mathbf{i}} = -\overrightarrow{\mathbf{k}}$$
$$\overrightarrow{\mathbf{k}} \times \overrightarrow{\mathbf{j}} = -\overrightarrow{\mathbf{i}}$$
$$\overrightarrow{\mathbf{i}} \times \overrightarrow{\mathbf{k}} = \overrightarrow{-j}$$



vii. A vector perpendicular to both $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ is $(\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}})$

viii. Unit vectors perpendicular to both $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ is

$$\pm\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

ix. $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Area of parallelogram

i. If the diagonals of parallelogram are $\overrightarrow{\mathrm{c}}$ and $\overrightarrow{\mathrm{d}}$, its sides $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ are $\vec{a} = \frac{\vec{c} - \vec{d}}{2} \& \vec{b} = \frac{\vec{c} + \vec{d}}{2}$

ii. The area of a parallelogram whose adjacent sides are represented by $\overrightarrow{a}$ and $\overrightarrow{b}$ is $|\overrightarrow{a} \times \overrightarrow{b}|$

iii. The area of a parallogram whose diagonals are $\overrightarrow{a} \& \overrightarrow{b}$ is $\frac{1}{2}|\overrightarrow{a} \times \overrightarrow{b}|$

Area of Triangle:

i. Area of ΔOAB where $\overrightarrow{OA} = \overrightarrow{a} \& \overrightarrow{OB} = \overrightarrow{b}$ is $\frac{1}{2}|\overrightarrow{a} \times \overrightarrow{b}|$

ii. If position vectors of the vertices of a triangle are $\vec{a}, \vec{b}, \vec{c}$ then

$$\Delta = \frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

Scalar triple product (box product)

For any vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ where $\overrightarrow{a} = (a_1, a_2, a_3)$, $\overrightarrow{b} = (b_1, b_2, b_3)$ and $\overrightarrow{c} = (c_1, c_2, c_3)$
$\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c})$ is defined as scalar triple product and it is denoted by $[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{c}]$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Geometrically $[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{c}]$ means volume of parallelopiped with $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ as coterminus edges.

Note:-

i. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar then the volume of the parallelepiped formed by them is zero, i.e.

$$[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{c}] = \overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

ii. $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}($ dot and cross can be interchanged$)$

iii. $[\vec{a}\vec{b}\vec{c}] = \vec{b}\vec{c}\vec{a}] = [\vec{c}\vec{a}\vec{b}]$

iv. $[\vec{a}\vec{b}\vec{c}] = -[\vec{b}\vec{a}\vec{c}]\ [\vec{c}\vec{a}\vec{b}] = -[\vec{a}\vec{c}\vec{b}]$

v. $[\overrightarrow{i}\,\overrightarrow{j}\,\overrightarrow{k}] = [\overrightarrow{j}\,\overrightarrow{k}\,\overrightarrow{i}] = [\overrightarrow{k}\,\overrightarrow{i}\,\overrightarrow{j}] = 1$

Vector triple product:-

For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{a} \times (\vec{b} \times \vec{c})$ is defined as the vector triple product.
$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Note:

$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$

In general, vector triple product is not associative.

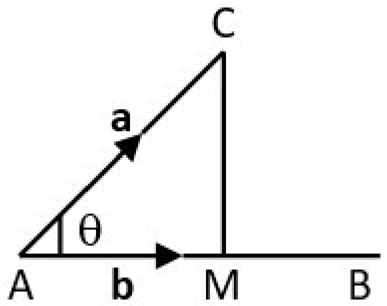
$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c} \Leftrightarrow \vec{a}$ and $\vec{c}$ are collinear.

Vector equation of a straight line:-

i. The vector equation of a line passing through a given point $\overrightarrow{a}$ and parallel to a given vector $\overrightarrow{b}$ is
$\vec{r} = \vec{a} + t\vec{b}$

ii. The vector equation of a line passing through the given points $\overrightarrow{a}$ and $\overrightarrow{b}$ is
$\overrightarrow{r} = (1 - \lambda)\overrightarrow{a} + \lambda\overrightarrow{b}$

Projection of a vector:-



$$\begin{aligned}\text{Projection of AC on AB} &= \text{AM}\\ &= \text{AC}\cos\theta\\ &= |\overrightarrow{\text{a}}|\frac{\overrightarrow{\text{a}}\cdot\overrightarrow{\text{b}}}{|\overrightarrow{\text{a}}||\overrightarrow{\text{b}}|}\\ &= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\end{aligned}$$

i. projection $\vec{a}$ upon $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

ii. projection of $\overrightarrow{\text{b}}$ upon $\overrightarrow{\text{a}} = \dfrac{\overrightarrow{\text{a}}\cdot\overrightarrow{\text{b}}}{|\overrightarrow{\text{a}}|}$