Current Electricity

Electric Current

It is the rate of flow of charge, conventionally, the direction flow of positive charge is taken as the direction of electric current.

It is a scalar quantity and its unit Ampere (A).

If small amount of charge dq flows in time dt, then,

$$I = \frac{dq}{dt}$$

The current carriers in conductor are electron, ions in electrolytes and electron and holes in semiconductor.

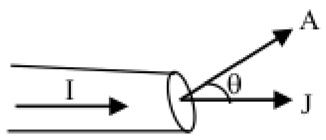
Current Density

Current density at a point inside the conductor is defined as the amount of current flowing per unit area around that point of the conductor, provided the area is held in a direction normal to that current.

i.e. current density $\vec{J} = \frac{I}{A}$

If area is not normal to current, but when the plane of small area A makes an angle θ with direction of current , then

$$J = \frac{I}{A\cos\theta}$$



$$\Rightarrow I = JA\cos\theta = \vec{J}\cdot\vec{A}$$

The unit of current density is A/m^2

Drift Velocity

When no potential is applied across the conductor, the electrons are in random motion. The average velocity of electrons is zero. The current in the conductor is zero.

When a potential difference is maintained across a conductor, the electrons gain some average velocity in the direction of +ve terminal. This average velocity is superimposed over the random velocity and is called as drift velocity.

$$v_d = \frac{I}{nAe}$$

where,

n = no. of electrons per unit volume

A = area of cross section of conductor

The order of drift velocity is 10^{-4} m/s.

Ohm's law & electric resistance

When a potential difference is applied across a conductor, a current I is set up in the conductor. According to Ohm's law:

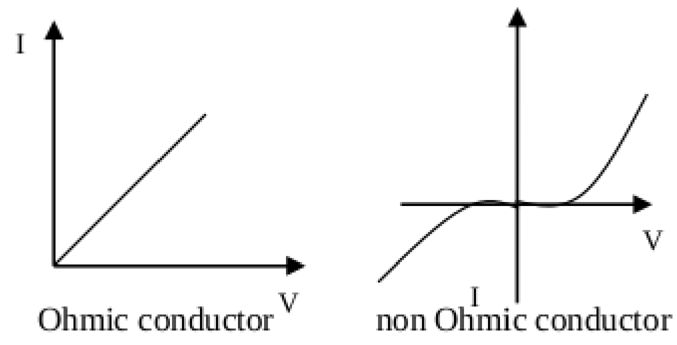
Under given physical conditions the current I produced in the conductor is proportional to the applied potential difference across the conductor.

i.e, $I \propto V$

Or, $I = KV$

where K is a constant of proportionality called conductance of given conductor.

If the substance follows the Ohm's law, then the substance is said to be Ohmic substance. Some substances does not follow Ohm's law, they are called non-Ohmic substance.



The resistance of a conductor depends upon the temperature, nature and dimensions of the material of conductor. The unit of resistance is volt/ampere or Ohm (Ω) and that of conductance is mho (Ω^{-1}) or siemon.

Resistivity and conductivity

For a given conductor of uniform cross-section A and of length l, the electric resistance R is directly proportional to length l and inversely proportional to cross-sectional A i.e.

$$R \propto \frac{I}{A}$$

$$\Rightarrow R = \rho \frac{I}{A}$$

where ρ is called specific resistance or resistivity of metal of the conductor at given temperature.

The specific resistance of a conductor depends upon the temperature and nature of the material of the conductor. It is independent of the dimensions of the conductor.

The reciprocal of resistivity (ρ) is called the conductivity (σ). The unit of conductivity is mho/meter.

The alternative form of Ohm's law is

$$J = \sigma E$$

where,

J = current density

& E = electric field strength

Variation of resistance with temperature

The resistance of a conductor varies with temperature. If R_t be resistance of a conductor at t○C and R_0 its resistance at 0°C, then

R_t = R_0 (1 + αt)

where α is called the temperature coefficient of resistance.

$$\alpha = \frac{R_t - R_0}{R_0 t}$$

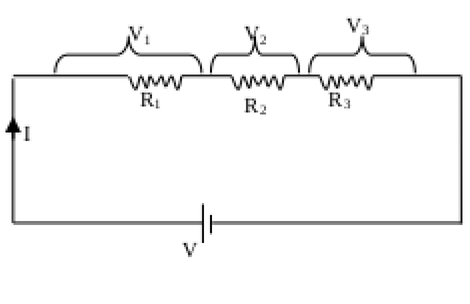
- unit of α is ^{0}c^{-1}
- α is positive for conductors
- α is negative for semiconductors, electrolytes, carbon and mica.
- α is zero for superconductors.

Combination of resistors

There are two arrangements for connecting a number of resistances.

- **Series combination:**

  In this arrangement the resistances are connected end to end in succession. In this combination:



  - The current in each resistor is same.
  - Total p.d. V across the combination is equal to the sum of the p.d. across individual resistances i.e.

    $$V = V_1 + V_2 + V_3$$

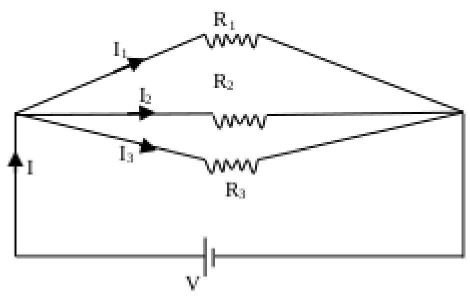
such that, $V_1 : V_2 : V_3 = R_1 : R_2 : R_3$

- The equivalent or effective resistance (R) of the combination is equal to the sum of individual resistances i.e.

  $R = R_1 + R_2 + R_3$

- **Parallel combination:**

  In this case, one end of each resistor is connected at one point and the other end of each to the other point.



- P.d. across each resistor is same.

- The current is different in different resistors.

  $I = I_1 + I_2 + I_3$

  $\& \; I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$

- The equivalent or effective resistance (R) of the combination is given by:

  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

  and effective conductance K is the sum of conductances of individual resistors i.e.

  $K = K_1 + K_2 + K_3$

Electromotive force (emf) and potential difference (V)

The emf of a source or a cell is the potential difference between the terminals of source when no current is drawn from it i.e. when the source is open or infinite resistance circuit. Alternatively the emf of a cell is defined as the work done in moving test charge in the entire closed circuit including the solution of the cell i.e.

$$E = \frac{W}{q_0}$$

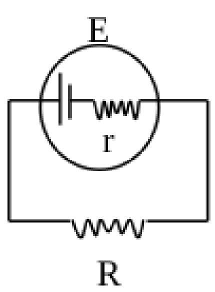
When the terminals of a cell are connected to an external resistance, the cell is said to be closed circuit. The potential difference across terminals of a cell in closed circuit is called the potential difference across external resistance. Alternatively, the p.d. across the external circuit is the work done in carrying per unit positive test charge from one terminal to another in external circuit i.e

$$V = \frac{W_{ext}}{q_0}$$

In general, $\mathrm{E} > \mathrm{V}$ but E may be less than V, if an opposite current flows in the cell e.g. when cell is being charged.

Internal resistance of cell (r)

It is the resistance offered by the solution of cell between its electrodes. It is denoted by r.



$$r = \frac{E - V}{I}$$

where,
$E =$ emf of cell
$V =$ terminal voltage of cell
$I =$ current

Terminal voltage:
$V = E - Ir$

Electrical power consumed due to heating effect:
$$P = \frac{E^2 R}{(R+r)^2}$$
P is maximum if $R = r$
and,
$$P_{max} = \frac{E^2}{4R} = \frac{E^2}{4r}$$

Superconductors

Those materials which offer least resistance to the flow of current through them are called superconductors. E.g. mercury at temperature 4.2 K, lead at 7.25 K.

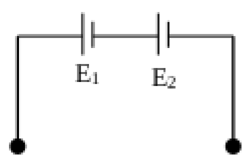
A metal conductor behaves as a superconductor at a temperature called critical temperature, which is different for different materials.

Superconductors are used

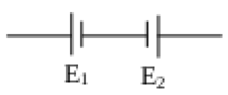
- in making very strong electromagnets.
- to produce very high speed computers.
- in transmission of electric power.
- in the study of high energy particle physics.

Grouping of cells

- **Series grouping:**
  - If +ve terminal of 1^{st} cell is connected with −ve terminal of 2^{nd} cell, then total emf:
    $E_t = E_1 + E_2$

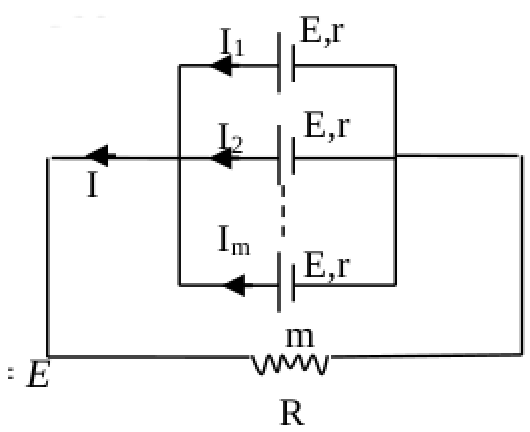


  - If +ve terminal of 1^{st} cell is connected with +ve terminal of 2^{nd} cell and −ve terminal of 1^{st} cell is connected to −ve terminal of 2^{nd} cell then:



    $E_t = E_1 - E_2$ if $E_1 > E_2$
    $E_t = E_2 - E_1$ if $E_2 > E_1$
    $E_t = 0$ if $E_1 = E_2$
  - If n identical cells are connected in series total emf:
    $E_t = nE$

- **Parallel grouping:**



  - Total emf = E
  - Net resistance $= R + \dfrac{r}{m}$

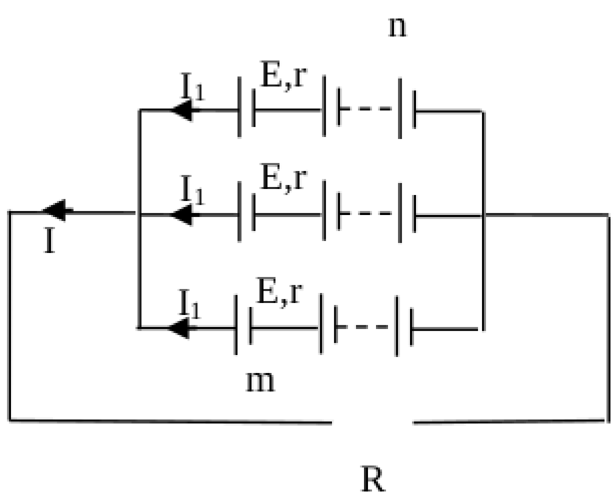
- Current in resistor R:

$$I = \frac{\text{total emf}}{\text{total resistance}}$$

$$I = \frac{E}{R + \frac{r}{m}}$$

  - If $R \ll \frac{r}{m}$ then $i = \frac{mE}{r}$
  $i \approx m \times \text{current due to one cell}$
  - If $R \gg \frac{r}{m}$ then $i = \frac{E}{R}$
  $i \approx \text{current due to one cell}$
  - i.e, for maximum current, the cells should be connected in parallel when net internal resistance >> net external resistance.

- Mixed grouping:



Let there be n cells in series in one row and m such rows be in parallel then:

- Total no. of cells $= n \times m = nm$
- Total emf $= nE$
- Total internal resistance:

$$r = \frac{nE}{R + \frac{nr}{m}}$$

- In mixed grouping, current will be maximum when total external resistance $(R) =$ total internal resistance $\left(\frac{nr}{m}\right)$.

$$\therefore \text{maximum current}(I_{max}) = \frac{nE}{2R} = \frac{mE}{2r}$$