Alternating Current

Introduction

It is the current which varies continuously in magnitude and periodically reverse in direction.

- When a coil is rotated in magnetic field, an alternating e.m.f. is generated in the coil. At any instant, the emf is given by
  $E = E_0 \sin \omega t$
  where E_0 = peak emf or maximum emf
  General equation of AC
  $V = V_0 \sin \omega t \quad \text{and} \quad I = I_0 \sin \omega t$
  $V = V_0 \sin 2\pi f t \quad \text{and} \quad I = I_0 \sin 2\pi f t$
- After starting from zero the time taken by current (or voltage) to reach maximum value is T/4.

Average value

Average value of AC for one complete cycle is zero.
Average value for half cycle:

$$I_{arg} = \frac{\int_0^{T/2} I \cdot dt}{\int_0^{T/2} dt}$$

For sinusoidal AC:

$$I_{av} = \frac{2I_0}{\pi} \text{ and } V_{av} = \frac{2V_0}{\pi}$$
$$= 0.637 I_0 = 63.7\% \text{ of } I_0$$

RMS value

$$I_{rms} = \frac{\int_0^T i^2 dt}{\int_0^T dt}$$

1. For sinusoidal AC:
$$I_{rms} = \frac{I_0}{\sqrt{2}} = 70.7\% \text{ of } I_0$$
$$V_{rms} = \frac{V_0}{\sqrt{2}} = 70.7\% \text{ of } V_0$$

2. If $I = I_1 \sin \omega t + \frac{I_2 \cos \omega t}{\sqrt{2}}$, then
$$\mathrm{I_{ms}} = \frac{\sqrt{\mathrm{I}_1^2 + \left(\frac{\mathrm{I}_2}{\sqrt{2}}\right)^2}}{\sqrt{2}}$$
$$\mathrm{I_{max}} = \sqrt{\mathrm{I}_1^2 + \left(\frac{\mathrm{I}_2}{\sqrt{2}}\right)^2}$$
$$\mathrm{I_{av}} = \frac{2}{\pi}\sqrt{\mathrm{I}_1^2 + \left(\frac{\mathrm{I}_2}{\sqrt{2}}\right)^2}$$

3. If $\mathrm{I} = \mathrm{I}_1$ sinωt + I_2 cosωt
$$I_{\max} = \sqrt{I_1^2 + I_2^2}$$
$$I_{\mathrm{rms}} = \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}}$$
$$I_{av} = \frac{2}{\pi}\sqrt{I_1^2 + I_2^2}$$

- AC is measured by hot wire instrument.
- AC ammeter and voltmeter measures rms value.
- Hot wire meter reads both AC and DC.

- Moving coil ammeter is not used with AC circuit because average value is zero.
- In AC mains voltage 220 V represents rms value of supply.
  V_{max} = V_0 = $\sqrt{2}$ rms
- Peak to peak voltage = $2V_0$ = $2\sqrt{2}$ rms

Form factor

It is the ratio of rms value to average value.
For sine value:
Form factor = 1.11

Impedance (Z)

It is the total opposing resistance in an AC circuit.

$$Z = \frac{E_{rms}}{I_{rms}} = \frac{E_0}{I_0}$$

Inductive reactance

(X_L) = ωL = $2\pi fL$

Capacitive reactance $(X_C) = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$

Reactance (X): The hindrance offered by inductance and capacitance in an AC circuit.

Susceptance (S)

It is the reciprocal of reactance.

$$S = \frac{1}{X}; \quad S_L = \frac{1}{X_L}; \quad S_C = \frac{1}{X_C}$$

$$[S] = M^{-1}L^{-2}T^3A^2$$

Admittance (γ)

It is the reciprocal of impedance.

$$\gamma = \frac{1}{Z}; \quad [\gamma] = M^{-1}L^2T^3A^2$$

- Power in AC circuit:
  P = $V_{rms} \times I_{rms} \times$ cosθ; cosθ is power factor.
- Power factor is defined as the ratio of true power to apparent power.

AC through pure resistor (R)

$$I = I_0 \sin \omega t; I = \frac{V_0}{R}$$

$$P_{av} = V_{rms} \times I_{rms} = I_{rms}^2 R = \frac{V_0 I_0}{2} = \frac{V_0{}^2}{2R}$$

- There is no lending and no lagging between current and voltage in pure resistance.
  $\theta = 0$
  Power factor, cosθ = 1

AC through pure inductor (L)

Inductive reactance; X_L = ωL = $2\pi fL$

$$\text{If } V = V_0 \sin \omega t, \text{ then } I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\theta = -\frac{\pi}{2}$$

$$I_0 = \frac{V_0}{X_L} = \frac{V_0}{2\pi fL}$$

- There is a phase difference of 90○ produced between current and voltage.
- Current lags behind the voltage by $\pi/2$.
- For pure inductor, $\theta = \dfrac{\pi}{2}; \cos\theta = 0$
- Average power dissipation in pure inductor is zero.

AC through pure capacitance (C)

Capacitive reactance; $X_C = \omega C = \dfrac{1}{2\pi fC}$

$$\text{If } V = V_0 \sin \omega t, \text{ then, } I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_0 = \frac{V_0}{X_L} \quad \theta = +\frac{\pi}{2}; \cos\theta = 0$$

- Average power dissipation in pure capacitor is zero.
- Current leads the voltage by $\pi/2$.

AC with RC series circuit

Impedance:

$$Z = \sqrt{X_L^2 + R^2}; V = IZ$$

$$\cos\theta = \frac{R}{Z}$$

$$V = \sqrt{V_L^2 + V_R^2}$$

$$0 < \theta < \frac{\pi}{2}$$

AC with LR series circuit

$$X_L = \omega L = 2\pi fL$$

$$Z = \sqrt{R^2 + X_L^2}; \quad V = IZ$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$\tan\theta = \frac{\omega L}{R}$$

AC with LCR series circuit

Reactance: $X = X_L - X_C = \omega L - \dfrac{1}{\omega L}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$\tan\theta = \left(\frac{X_L - X_C}{R}\right) \text{ and } \cos\theta = \left(\frac{R}{Z}\right)$$

1. If $X_L > X_C$; then current lags behind the voltage.
2. If $X_L < X_C$; then current leads the voltage.
3. If $X_L = X_C$; then current and voltage are in same phase. This is called the case of resonance and resonant frequency is given by the condition
   $X_L = X_C$

- Resonant frequency $f = \dfrac{1}{2\pi\sqrt{LC}}$
- In series resonance circuit,
  a. Impedance is minimum; $Z = R$
  b. Power factor; $\cos\theta = 1$
  c. Current flowing is maximum; $I_{max} = \dfrac{V}{R}$
  d. Circuit is resistive only.
- Series resonant circuit is called **acceptor** circuit.
- In parallel resonance circuit, at resonance current is minimum and impedance is maximum. Parallel resonance circuit is called **rejector** circuit.
- $I = I_0 \cos\theta$ is called wattful component of current.
- $I = I_0 \sin\theta$ is called wattless component of current because it does not contribute any power.

Choke coil

Choke coil is simply a coil having large inductance but a small resistance.

- It is used to control current in AC circuit.
- It is used in fluorescent tubes.
- Power factor of the coil is nearly zero i.e. power loss in circuit is minimum.

- It is a simple LR circuit with impedance,
  $Z=\sqrt{R^2+\omega^2 L^2}$
- Quality factor of a coil;
  $\mathrm{Q.F.}=\dfrac{\omega L}{R}=\dfrac{2\pi f L}{R}=\dfrac{X_L}{R}$

Transformer

It a device for converting high voltage into low voltage and vice-versa.

- It is based on the principle of mutual induction.
- It always works on AC.
  It is of two types:
  a. **Step up transformer**: It converts low voltage into high voltage. In this transformer number of turn in secondary is more than the no. of turn in primary.
  b. **Step down transformer**: It converts high voltage into low voltage. In this transformer no. of turns in secondary is less than primary.
- The ratio of number of turns in secondary and primary is called turn ratio.

  Turn ratio: $\eta=\dfrac{n_s}{n_p}$
- If E_p and E_s are alternating voltages; i_p and i_s the alternating currents across primary and secondary terminals respectively then:
  $\dfrac{E_s}{E_p}=\dfrac{i_p}{i_s}=\dfrac{n_s}{n_p}=n$

Efficiency of transformer

$$\eta=\frac{\text{Output power}}{\text{Input power}}$$

$$\therefore \quad \frac{P_{\text{out}}}{P_{\text{in}}}=\frac{E_s \cdot i_s}{E_p \cdot i_p}$$

- Efficiencies of the order of 99% can be easily achieved.
- In transformer frequency is not change.
- Transmission of power at high voltage is economical.