Capacitor

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Every conductor has capacity to store charge which is numerically equal to ratio of charge given to it to the potential raised.

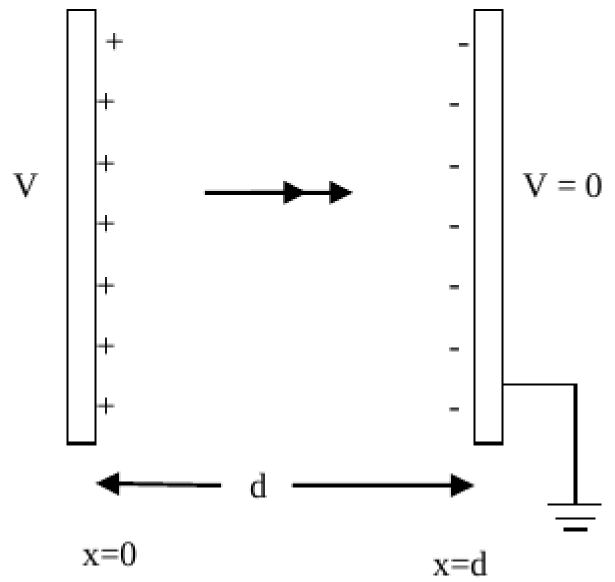
If the conductors (called plates) carry equal and opposite charges, the system is called a capacitor or condenser. The capacitance of a capacitor is defined as:

$$C = \frac{\text{Magnitude of charge on a plate}}{\text{PD between the plates}}$$

Parallel Plate Capacitor

Electric field between parallel plate capacitor is:

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$



$$\text{or,} \quad E = \frac{\sigma}{\varepsilon_0}$$

$$\text{or,} \quad \frac{-dV}{dx} = \frac{\sigma}{\varepsilon_0}$$

$$\text{or,} \quad -\int_V^0 dV = \int_0^d \frac{\sigma}{\varepsilon_0} dx$$

$$\text{or,} \quad V = \frac{\sigma}{\varepsilon_0} d$$

$$\text{So,} \quad C = \frac{q}{V} = \frac{q}{\frac{\sigma d}{\varepsilon_0}}$$

$$\text{or, } C = \frac{q}{\left(\frac{\sigma d}{\varepsilon_0}\right)} = \frac{q}{\frac{q}{A} \times \frac{d}{\varepsilon_0}} = \frac{\varepsilon_0 A}{d}$$

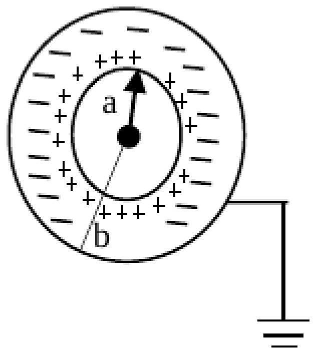
$$\therefore \quad C = \frac{\varepsilon_0 A}{d}$$

When dielectric slab of thickness t and dielectric constant K is introduced between the parallel plate capacitor, then

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

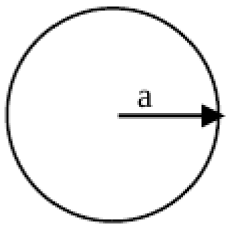
Spherical Capacitor (Concentric spheres)

$$C = 4\pi\varepsilon_0 K \frac{ab}{b-a}$$



where *a* is radius of inner sphere and *b* is radius of outer sphere.

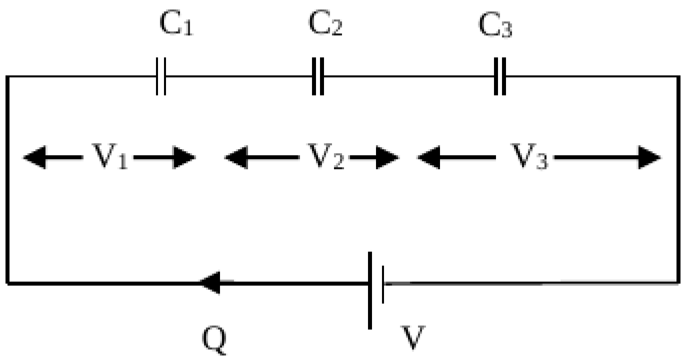
For single sphere, the capacitance can be written as:
$C = 4\pi\varepsilon_0 a$



where *a* is radius of sphere.

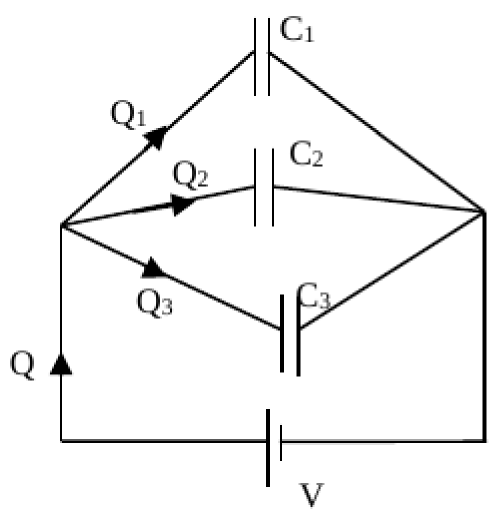
Combination of capacitors

1. Series Combination
   - Charge remains constant i.e. all the capacitors have same charge.
     i.e. $Q_1 = Q_2 = Q_3 = Q$



   - Potential across the capacitors is different. If *V1, V2, V3* are the potential differences across the capacitors, then applied potential difference
     $V = V_1 + V_2 + V_3$
   - If C_S is the effective capacitance, then
     $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots\ldots\ldots + \frac{1}{C_n}$$
   - The effective capacitance is less than the least of the individual capacitance.
   - If *n* identical capacitors each of capacitance *C* joined in series,
     $$C_s = \frac{C}{n}$$

2. Parallel Combination:
   - Potential difference across the capacitors is same.
     i.e. $V_1 = V_2 = V_3 = V$



   - Charged stored in capacitors is different. If Q_1, Q_2 & Q_3 are charges stored in capacitors and *Q* is the total charge, then
     $Q = Q_1 + Q_2 + Q_3$
   - *If* C_p *is the effective capacitance, then*
     $C = C_1 + C_2 + \ldots\ldots + C_n$
   - The effective capacitance is greater than the greatest of the individual capacitance.

- If *n* identical capacitors each of capacity *C* are joined in parallel,
  $C_p = nC$

Energy stored in capacitors

If *dq* is charge given to a capacitor at potential *V*

$$dW = dqV$$

$$\text{Or,} \quad W = \int_0^q V dq$$

$$\text{Or,} \quad W = \int_0^V \frac{q}{C} dq$$

$$W = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

This work is stored as electrical potential energy i.e. a capacitor stores electrical energy.

$$U = W = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}$$

Energy density of capacitor:

$$U_E = \frac{U}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\frac{\varepsilon_0 A}{dAd}V^2$$

$$\therefore \quad U_E = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

$$\because E = \frac{V}{d}$$

Charging and discharging

When a voltage is applied across the terminals of a capacitor, the potential cannot rise to its final value instantaneously but it rises exponentially as:

$$q = q_0\left(1 - e^{-t/CR}\right)$$
$$V = V_0\left(1 - e^{-t/RC}\right)$$
$$\& \quad I = I_0\left(1 - e^{-l/RC}\right)$$

Thus a capacitor is charged through a resistor. When a charged capacitor is allowed to discharged through resistance R, then the voltage across the capacitor and the current in the circuit falls exponentially as:

$$q = q_0 e^{-t/RC}$$
$$V = V_0 e^{-t/RC}$$
$$\& I = I_0 e^{-t/RC}$$

Cylindrical Capacitor

It consists of two-coaxial cylindrical conductors of radii *a* and *b*, the outer surface of outer conductor being earthed. The space between the two is filled with a dielectric of dielectric constant *K*.
The capacitance of cylindrical condenser of length *l*:

$$C = \frac{2\pi\varepsilon_0 Kl}{\log_e\left(\frac{\mathrm{b}}{a}\right)}$$