Trigonometric Equations and General Values

Trigonometric Equation:

An equation involving one or more trigonometric functions of a variable is known as trigonometric equation. The equation may be true for one or more values, but not for every value of a variable.

General Solution

The set of all possible solution of trigonometric equation is called general solution of the equation. The solution generalized by means of periodicity is known as general values.

Principal value:

The value of the trigonometric function between 0 to 2π is known as principal value. The general solution of some trigonometric equation are:

		Trigonometric equation	General solution
a.	(i)	$\sin\theta = 0$	$\theta = \mathrm{n}\pi$
	(ii)	$\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2}$
	(iii)	$\tan\theta = 0$	$\theta = \mathrm{n}\pi$
b.	(i)	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n\alpha$
		$\csc\theta = \csc\alpha$	$\theta = n\pi + (-1)^n\alpha$
	(ii)	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha$
		$\sec\theta = \sec\alpha$	$\theta = 2n\pi \pm \alpha$
	(iii)	$\tan\theta = \tan\alpha$	$\theta = \mathrm{n}\pi + \alpha$
		$\cot\theta = \cot\alpha$	$\theta = \mathrm{n}\pi + \alpha$
c.	(i)	$\sin\theta = -\sin\alpha$	$\theta = n\pi + (-1)^n(-\alpha)$
	(ii)	$\cos\theta = -\cos\alpha$	$\theta = 2n\pi + (\pi - \alpha)$
	(iii)	$\tan\theta = -\tan\alpha$	$\theta = n\pi + (-\alpha)$
d.	(i)	$\sin^2\theta = 1$	$\theta = n\pi + \frac{\pi}{2}$
	(ii)	$\cos^2\theta = 1$	$\theta = n\pi$
e.	(i)	$\sin^2\theta = \sin^2\alpha$	$\theta = \mathrm{n}\pi \pm \alpha$
	(ii)	$\cos^2\theta = \cos^2\alpha$	$\theta = \mathrm{n}\pi \pm \alpha$
	(iii)	$\tan^2\theta = \tan^2\alpha$	$\theta = \mathrm{n}\pi \pm \alpha$
f.	(i)	$\sin\theta = 1$	$\theta = (4n+1)\frac{\pi}{2}$
	(ii)	$\sin\theta = -1$	$\theta = (4n-1)\frac{\pi}{2}$
	(iii)	$\cos\theta = 1$	$\theta = 2\mathrm{n}\pi$
	(iv)	$\cos\theta = -1$	$\theta = (2n+1)\pi$

Note:

- Any value of x which satisfies the given condition is a root of the equation. But the value of x which satisfies the given condition for which $\infty=\infty$ is not the solution as it is indeterminate form.

- When we square a given equation for finding solution, then after finding roots, we have to check which roots satisfy the original equation.
- General values gives infinite solution.

Method to solve the Equation of the form $a \cos \theta+b \sin \theta=c$

i. Divide the equation by
$\sqrt{(\text { coeff of } \sin \theta)^{2}+(\text { coeff of } \cos \theta)^{2}}$
i.e. $\sqrt{a^{2}+b^{2}}$
i.e. $\frac{a}{\sqrt{a^{2}+b^{2}}} \cos \theta+\frac{b}{\sqrt{a^{2}+b^{2}}} \sin \theta$
$=\frac{c}{\sqrt{a^{2}+b^{2}}}$
$\sin (A+\theta)=\frac{c}{\sqrt{a^{2}+b^{2}}}=\sin \alpha(\text { suppose })$

ii. Use the formula for $\sin \theta=\sin \alpha$

iii. If $|c|>\sqrt{a^{2}+b^{2}}$, then the value of sine of an angle is greater than one, so it has no solution.

iv. If $|c| \leq \sqrt{a^{2}+b^{2}}$ then the value of sine of an angle is less than one and it has solution.

Note:

We have:

$$\left.\begin{array}{l}\sin ^{2} \theta=\sin ^{2} \alpha \\ \cos ^{2} \theta=\cos ^{2} \alpha \\ \tan ^{2} \theta=\tan ^{2} \alpha\end{array}\right] \Rightarrow \theta=n \pi \pm \alpha, n \in I$$

Proof of these results are:

i. $\sin ^{2} \theta=\sin ^{2} \alpha$
$\frac{1-\cos 2 \theta}{2}=\frac{1-\cos 2 \alpha}{2}$
$\cos 2 \theta=\cos 2 \alpha$
$2 \theta=2 n \pi \pm 2 \alpha$
$\theta=n \pi \pm \alpha, \mathrm{n} \in \mathrm{I}$

ii. $\cos ^{2} \theta=\cos ^{2} \alpha$
$\frac{1+\cos 2 \theta}{2}=\frac{1+\cos 2 \alpha}{2}$
$\cos 2 \theta=\cos 2 \alpha$
$2 \theta=2 \mathrm{n} \pi \pm 2 \alpha$
$\theta=n \pi \pm \alpha, n \in I$

iii. $\tan ^{2} \theta=\tan ^{2} \alpha$
$1+\tan ^{2} \theta=1+\tan ^{2} \alpha$
$\sec ^{2} \theta=\sec ^{2} \alpha$
$\cos ^{2} \theta=\cos ^{2} \alpha$
like in (ii), we get
$\theta=n \pi \pm \alpha$