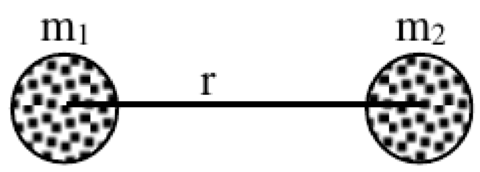
Gravity and Gravitation

Newton's law of Gravitation

Every body in the universe attract other bodies towards it's centre. The force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of distance between their centers



F $\propto \frac{m_1 m_2}{r^2}$

F = $G\frac{m_1 m_2}{r^2}$

Where G = 6.67 $\times$ 10^{-11} Nm^2/kg^2 is called universal gravitational constant whose value remain same at all the places and for all bodies.

Gravity

The force by which any body is attracted by earth towards its centre is called gravity of earth.

$\text{Acceleration due to gravity (g)}$

The acceleration produced on any body due to gravity of earth is called acceleration due to gravity.

When a body is placed on the surface of earth of mass M and radius R then

F = $\frac{GMm}{R^2}$ = mg

$\therefore$ g = $\frac{GM}{R^2}$ is the expression

Factors affecting the value of g

(a) Shape of earth (R)

Earth is not perfectly spherical in which polar region is flat and equatorial region is bulge so R$_p <$ R$_e$ ie value of g is maximum at pole and least at equator on the surface of earth.

(b) Altitude or height (h)

The acceleration due to gravity on surface of earth is

g = $\frac{GM}{R^2}$ ---(i)

At any point at a height 'h' from surface of earth

g' = $\frac{GM}{(R+h)^2}$ ---(ii)

Dividing

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2$$

For any point close to surface of earth

g' = g $\left(1 - \frac{2h}{R}\right)$

(c) Depth (x)

The acceleration due to gravity at a point a depth x from surface of earth is

g' = g $\left(1 - \frac{x}{R}\right)$

At centre of earth x = R then g' = 0 ie. acceleration due to gravity is zero at centre of earth

(d) Rotation of earth

When earth rotate about its axis with constant angular velocity ω then the value of g at a latitude θ is

g' = g$\left(1-\frac{R\omega^2}{R}\cos^2\theta\right)$

At equator, θ = 0^0 so g_e = g -Rω^2 ie variation of g is maximum at equator due to rotation of earth.

At pole θ = 90^0 so g_p = g ie there is no variation in g due to rotation of earth

Gravitational field, potential and potential energy

i. Gravitational field

The space around earth up to which the influence of attraction of can be felt by any other bodies is called gravitational field of earth

The gravitational field strength at any point is defined and force experienced by a body of mass 1kg placed at the point

$\therefore$ Field strength (E) = $\frac{F}{m}$= $\frac{GMm}{R^2m}$= g

Hence the gravitational field strength is numerically equal to acceleration due to gravity,

ii. Gravitational potential (V)

The gravitational potential at any point in gravitational field is defined as the work done to bring a body of mass 1kg from infinity to that point

$\therefore$ Gravitational potential (V) = -$\frac{GM}{r}$

-ve sign indicates the gravitational field is attractive in nature

$\therefore$ E = -$\frac{dV}{dr}$

iii. Gravitational potential energy (PE)

The gravitational potential energy of any body in gravitational field is defined as the total amount of work done to bring that body from infinity to that point

$\therefore$ potential energy (PE) = -$\frac{GMm}{r}$

For system of n masses

Potential energy (PE) = $\sum_{i=1}^{n}\sum_{j=1}^{n}$=$\frac{Gm_im_j}{r_{ij}}$

Satellite

The heavenly body that revolve around planet in its own orbit is called satellite

i. Orbital velocity (V_o)

The velocity by which a satellite revolve around planet in it's own orbit is called orbital velocity. For a satellite of mass m revolving around earth of mass M & radius R in an orbit of radius 'r' at a height 'h' then

$$\frac{GMm}{r}=\frac{mv_o^2}{r}$$

V_o =$\sqrt{\frac{GM}{r}}$

For an orbit close to surface of earth r = R so V_o = $\sqrt{\frac{GM}{R}}$=$\sqrt{gR}$

For an orbit of height 'h' from surface of earth

R = R + h so

V_o = $\sqrt{\frac{GM}{R+h}}$

Time period (T) =$\frac{2\pi r}{v_o}$

$$= \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

$\therefore T^2 \propto r^3$

ii. Escape velocity (v_e)

The minimum velocity of a satellite by which it is projected from surface of earth and will be able to escape from gravitational field of earth

$\therefore v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

iii. Total energy of satellite

The sum of PE and KE of a satellite is equal to the total energy of satellite

$\therefore$ Total energy (E_T) = PE + KE

$= -\frac{GMm}{r} + \frac{1}{2} m{v_o}^2$

$$= -\frac{GMm}{r} + \frac{GMm}{2r}$$

$$= -\frac{GMm}{2r}$$

$\therefore E_T = -\frac{GMm}{2r}$

-ve sign indicates the satellite remain in gravitational field of earth (planet)

Geostationary satellite and parking orbit

When the time period of rotation of earth about its axis become equal to time period of revolution of satellite around earth, kept above equatorial plane and the direction of rotation of earth become same as direction of revolution of satellite then a satellite seems to be stationary is called geo stationary satellite and orbit of such satellite is called parking orbit.

$\rightarrow$ Geostationary satellites are used for transmission, communication broad casting etc.

$\rightarrow$ Time period for geostationary satellite is 24hrs

$\rightarrow$ Height of geostationary satellite is about 36000km from surface of earth.