Rotaional Dynamics

Moment of inertia (I)

The sum of product of mass and square of distance of all particles from axis of rotation is called moment of inertia which determine the inertia of rotating body. It's unit is kgm^2

∴ Moment of inertia (I) = Σmr^2

Radius of gyration (k)

The distance of a point from axis of rotation at which whole mass of system is assumed to be concentrated will produce same moment of inertia as distribution of mass produce about same axis

∴ Moment of inertia (I) = MK^2

KE of rotating body

The KE of a body of moment of inertia I rotating about any axis with angular velocity w is

KE_{rot} = $\frac{1}{2}$ Iω^2

Equations in rotational dynamics

i. $\omega = \omega_0 + \alpha t$
i. $\theta = \omega_0 t + 1/2\alpha t^2$
iii. $\omega = \omega_0^2 + 2\alpha\theta$
iv. $KE_{\mathrm{rot}} = 1/\omega^2$
v. Torque $(\tau) = I\alpha$
vi. Angular momentum (L) $= mvr = I\omega$
vii. Work $(W) = \tau.\theta$
viii. Power $(P) = \tau.\omega$

$\mathrm{Torque}\ (\tau)$

The moment of force acting on a body about point of turning is called torque

∴ Torque (τ) = $\overrightarrow{r} \times \overrightarrow{F}$ = rF sinθ

Angular momentum ($\overrightarrow{L}$)

Moment of momentum of body is called angular momentum.

∴ Angular momentum ($\overrightarrow{L}$) = $\overrightarrow{r} \times \overrightarrow{mv}$

= mvr = Iω

Now Rate of change in angular momentum is called torque

∴ Torque (τ) = $\frac{dL}{dt}$

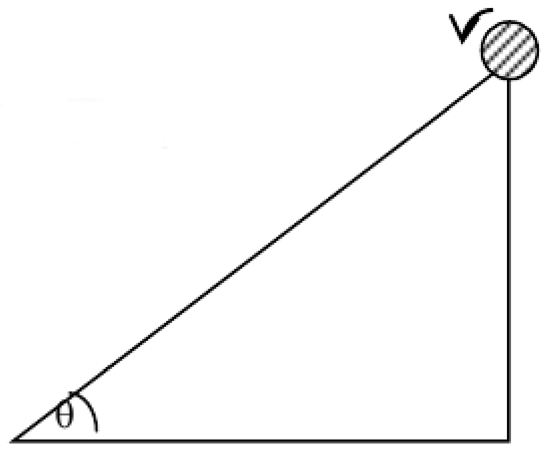
=$I\alpha$

Acceleration of rolling body along the inclined plane.

The acceleration of a rolling body along the inclined plane of inclination ? with horizontal is

$$\mathrm{a} = \frac{g\sin\theta}{1 + k^2/r^2}$$

where θ is angle of slope



Theorem's on moment of inertia

1. i. Parallel axis theorem

   The moment of inertia of a body about any axis parallel to axis through centre of mass is equal to the sum of moment of inertia about axis through centre of mass and product of mass and square of distance between two axis.
   ie. $I = I_{cm} + Mh^2$
   h = distance between two axis

2. ii. Perpendicular axis theorem

   Moment of inertia of lamina about axis perpendicular to lamina is equal to the sum of moment of inertia about two perpendicular axis on lamina from same point

   $\therefore I_z = I_x + I_y$

Principle of conservation of angular momentum.

If no external torque acts on the system of rotating body then total angular momentum remain conserved

i.e. $I_1\omega_2 = I_2\omega_2$

Moment of inertia of some bodies

S.N	Body	axis	Moment of Inertia
1.	Uniform bar	i. axis through centre & perpendicular ii. axis perpendicular to rod at end	$\frac{1}{12}ml^2$ $\frac{1}{3}ml^2$
2.	Rectangular lamina	Through CG & perpendicular	$\left(\frac{l^2+b^2}{12}\right)m$
3.	Ring	i. Through centre & perpendicular ii. About diameter	MR^2 $\frac{MR^2}{2}$
4.	Disc	i. Through centre & perpendicular to plane ii. About diameter	$\frac{MR^2}{2}$ $\frac{MR^2}{4}$
5.	Solid cylinder	About diameter	$\frac{MR^2}{2}$
6.	Sphere (solid)		$\frac{2}{5}MR^2$
7.	hollow sphere		$\frac{2}{3}MR^2$