Hydrostatics

Density (ρ)

The ratio of mass of body and its volume is called density

$\therefore$ Density $(\rho)=\dfrac{\text{mass}}{\text{volume}}$

Relative density or specific gravity

The ratio of wt of certain volume of body and wt of same volume of water at 4^0c is called specific gravity

$\therefore$ Specific gravity $=\dfrac{wt\text{ of body}}{\text{wt. of same volume of water at }4^{\circ}\text{c}}$

$=\dfrac{\text{density of body}}{\text{density of water at }4^{\circ}\text{c}}$

Here density of water at 4^0c = 1g/cc so

Specific gravity = Density of body

It is just ratio of two similar quantities so it has no unit

For mixture

i. When two liquids of equal mass are mixed whose density are ρ_1 and ρ_2 then

Density of mixture,

$$\rho=\frac{m+m}{\dfrac{m}{\rho_1}+\dfrac{m}{\rho_2}}=\frac{2\rho_1\rho_2}{\rho_1+\rho_2}$$

$\therefore$ Harmonic mean is density of mixture

ii. When two liquids of equal volume are mixed whose densities are ρ_1 and ρ_2 then

Density of mixture $(\rho)=\dfrac{v\rho_1+v\rho_2}{v+v}$

$=\dfrac{\rho_1+\rho_2}{2}$

$\therefore$ Arithmetic mean is density of mixture.

Archimede's principle

When a body is completely or partially immersed in liquid then, loss in wt of body is equal to wt of liquid displaced by body.

$\therefore$ Loss in wt = wt of liquid displaced

$\therefore$ Apparent wt of body = wt -upthrust

$\rightarrow$ If body is suspended by string and immersed in liquid then Tension (T)

= wt -upthrust

= v $(\rho$ -$\sigma)$ g.

Liquid pressure

Pressure at bottom of liquid column of height h and density ρ is

P = ρgh

Pascal's Law

Liquid transmit pressure equally in all direction placed in a closed vessel

$$\therefore \quad \frac{F_1}{A_1}=\frac{F_2}{A_2}$$

$\rightarrow$ It is used to multiply force

Principle of floatation

When body of density ρ is placed in liquid of density σ then

i. If wt $>$ upthrust ie. $\rho>\sigma$ then body sink in liquid

ii. If wt = upthrust ie $\rho = \sigma$ then body just sink in liquid or float in liquid

iii. If wt $<$ upthrust is $\rho<\sigma$ then body sink partially in liquid so that wt. of body become equal to the wt of liquid displaced by immersed portion

$$\therefore \mathrm{wt} = \text{upthrust}$$
$$\mathrm{v}\rho\mathrm{g} = \mathrm{v}'\sigma\mathrm{g}$$
$$\frac{\mathrm{v}'}{\mathrm{v}} = \frac{\rho}{\sigma} = \text{fraction immersed}$$

$$\therefore \text{fraction outside} = 1 - \frac{\rho}{\sigma}$$

When a body of density ρ_o is taken to a depth 'h' inside liquid of density 'ρ' then density of body become

$$\rho_h = \rho_o + \Delta\rho$$

$$= \rho_0\left(1 + \frac{\Delta\rho}{\rho_0}\right)$$
$$= \rho_0\left(1 + \frac{\rho g h}{B}\right) \quad \left[\because \frac{\Delta V}{V} = \frac{\Delta\rho}{\rho}\right]$$

A metal ball of weight w has density ρ is immersed in liquid of density σ has weight w/ then the volume of cavity in ball is

$$\mathrm{V_c} = \mathrm{V} - \mathrm{V_m} = \left(\frac{\mathrm{w} - \mathrm{w}'}{\sigma} - \frac{\mathrm{w}}{\rho}\right)$$

A body of density 'ρ' is released in liquid of density 'σ' where $\rho>\sigma$ then body accelerate downward.

a. The acceleration of body in liquid is
$$\mathrm{a} = \frac{\mathrm{V}\rho\mathrm{g} - \mathrm{V}\sigma\mathrm{g}}{\mathrm{V}\rho} = \left(1 - \frac{\sigma}{\rho}\right)\mathrm{g}$$

   If $\sigma > \rho$ then body accelerate upward with same acceleration i.e.
$$\mathrm{a} = \left(\frac{\sigma}{\rho} - 1\right)\mathrm{g}$$

b. The velocity of body in liquid after travelling a distance 'h'is

$$v^2 = 0^2 + 2ah$$

   or, $v = \sqrt{2ah} = \sqrt{2\left(1 - \frac{\sigma}{\rho}\right)gh}$

c. The time taken by body to travel a distance 'h' in liquid is

$$\mathrm{h} = 0 + \frac{1}{2}\mathrm{at}^2$$
   or, $\mathrm{t} = \sqrt{\frac{2\mathrm{h}}{\mathrm{a}}} = \sqrt{\frac{2\mathrm{h}}{\left(1 - \frac{\sigma}{\rho}\right)\mathrm{g}}}$

Efflux velocity

The velocity of liquid escaping through a hole from a vessel at a depth 'h' is

v = $\sqrt{2gh}$ is called efflux velocity.

i) If H be the height of liquid in vessel then time taken by liquid to reach the ground is

$$\mathrm{t} = \sqrt{\frac{2(\mathrm{H} - \mathrm{h})}{\mathrm{g}}}$$

ii) The distance at which liquid fall on ground from base of vessel is range so

$$
\begin{aligned}
R = vt &= \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} \\
&= \sqrt{4(H-h)h}
\end{aligned}
$$