Expansion of Solid and Liquid

Expansion of Solid

On heating solid, its dimension increases is called expansion

- **One dimensional expansion of solid is called linear expansion**
  coefficient of linear expansion or linear expansivity (α). The change in length per unit original length per degree change in temperature is constant for a matter. Its unit is $/^oC \text{ or } K^{-1}$.
  $\therefore \alpha = \dfrac{\Delta l}{l\Delta\theta} \text{ or } l_\theta = l_o(1+\alpha\Delta\theta)$
- **Two dimensional expansion is called superficial expansion**
  coefficient of superficial expansion or superficial expansivity (β).The change in area per unit original area per degree change in temperature is constant for a matter. Its unit is $/^oC \text{ or } K^{-1}$.
  $\therefore \beta = \dfrac{\Delta A}{A\Delta\theta} \text{ or } \mathrm{A}_\theta = \mathrm{A}_0(1+\beta\Delta\theta)$
- **Three dimensional expansion is called cubical expansion**
  coefficient of cubical expansion or cubical expansivity (γ). The change in volume per unit original volume per degree change in temperature is constant for a matter. Its unit is $/^oC \text{ or } K^{-1}$.
  $\therefore \gamma = \dfrac{\Delta V}{V\Delta\theta} \text{ or } \mathrm{V}_\theta = \mathrm{V}_\mathrm{o}(1+\gamma\Delta\theta)$

Relationship between $\alpha, \beta \text{ and } \gamma$

For a body (isotropic body)

$\alpha = \beta/2 = \gamma/3$

$\therefore \alpha : \beta : \gamma = 1 : 2 : 3$

For anisotropic body

$\beta = \alpha_1 + \alpha_2 \text{ and } \gamma = \alpha_1 + \alpha_2 + \alpha_3$

Pendulum Clock

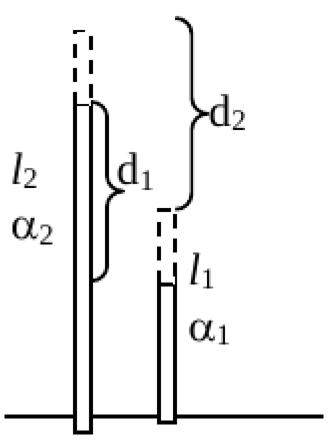
The time period of a pendulum clock is given by

$$\mathrm{T} = 2\pi\sqrt{\frac{l}{g}}$$

- When a pendulum clock keep correct time then time period of it must be 2 sec
- During summer season, temperature increases so the length of pendulum clock also increases and it lose time or become late
- During winter length of pendulum clock decreases so it gain time or become fast
- To keep correct time at all temperature pendulum clock is made by invar' since value of linear expansivity for it is least
- Change in time per sec $= \dfrac{1}{2}\alpha\,\Delta\,\theta$

Differential expansion

Two rods of lengths l_2 and l_1 of linear expansivity α_2 and α_1 have difference d_1 at $\theta_1^o C$ and d_2 at $\theta_2^o C$ then,



At $\theta_1^o C$

$$d_1 = l_2 - l_1$$

At $\theta_2^o C$

$$d_2 = l_2' - l_1'$$

$$= l_2 \{1 + \alpha_2 (\theta_2 - \theta_1)\} - l_1 \{1 + \alpha_1 (\theta_2 - \theta_1)\}$$
$$= l_2 + l_2\alpha_2 (\theta_2 - \theta_1) - l_1 - l_1\alpha_1 (\theta_2 - \theta_1)$$
$$= \mathrm{d}_1 + (l_2\alpha_2 - l_1\alpha_1)(\theta_2 - \theta_1)$$
$$\therefore \mathrm{d}_2 = \mathrm{d}_1 + (l_2\alpha_2 - l_1\alpha_1)(\theta_2 - \theta_1)$$

Force due to expansion or contraction

When a rod placed between two rigid supports is not free to expand on heating and not free to contract on cooling then the force or tension is developed along it.

Force (F) = $YA\alpha \, \Delta \, \theta$

Thermal stress $\left(\frac{F}{A}\right) = Y\alpha \, \Delta \, \theta$

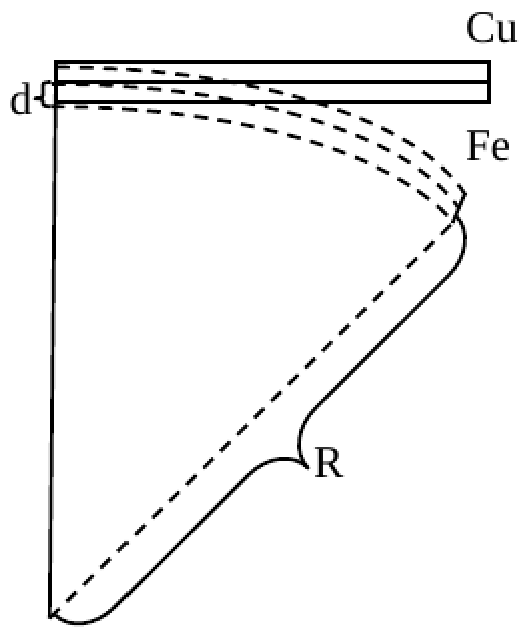
Thermal strain $\left(\frac{\Delta l}{l}\right) = \alpha \, \Delta \, \theta$

Energy denstity $\left(\frac{E}{V}\right) = \frac{1}{2} Stress \times Strain$

$$= \frac{1}{2} Y (\alpha \, \Delta \, \theta)^2$$

Thermostat or bimetallic strip

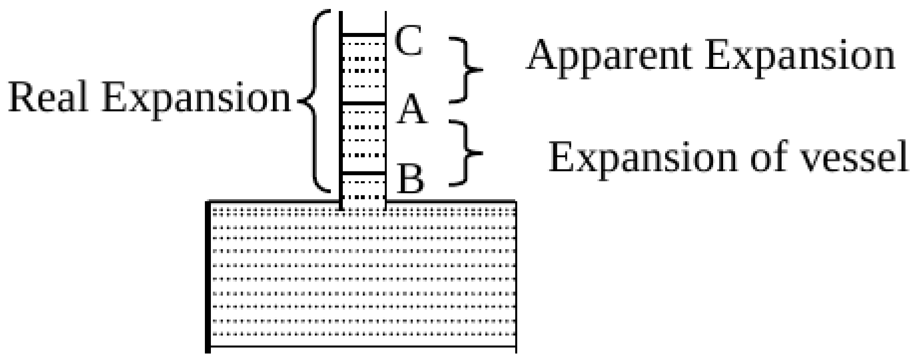
When two different metal strips are welded together then on heating, it bends towards one of lower linear expansivity



- It is used in automatic electric switch to control temperature
- Metal of higher linear expansivity lies on convex side and lower lines on concave side
- Radius of curvature $(R) = \frac{d}{(\alpha_1 - \alpha_2) \, \Delta \, \theta}$

Expansion of liquid

In case of liquid linear and superficial expansivities are absent. Liquid is heated in a vessel which expand itself so the expansion that we observe is called apparent expansion is always less than real or absolute expansion due to expansion of vessel.



- **Coefficient of apparent expansion or apparent expansivity (γ_a):**
  The apparent change in volume per unit original volume per degree change in temperature is called apparent expansivity.
  $\gamma_\mathrm{a} = \frac{(\Delta V)_a}{V \Delta \theta}$ or, $(\Delta \mathrm{V})_\mathrm{a} = \mathrm{V}\gamma_\mathrm{a} \Delta \theta$

- **Coefficient of real expansion or real expansivity (γ_r):**
  Real Change in volume per unit original volume per degree change in temperature is constant for a liquid
  $$\therefore \gamma_r = \frac{(\Delta V)_r}{V\Delta\theta}$$
  $$\text{or, } (\Delta V)_r = V\gamma_r\Delta\theta$$
  Now,
  Real expansion = Apparent expansion + Expansion of vessel
  or, $\gamma_r = \gamma_a + \gamma_g = \gamma_a + 3\alpha_g$
  γ_g = Cubical expansivity of vessel
  - A vessel completely filled by liquid is heated then volume of liquid overflow or on cooling volume of empty space above liquid is $(\Delta V)_a = V_o\gamma_a\Delta\theta$
  - A liquid is placed in a vessel upto certain mark so that level of liquid remain at same mark at all temperature then $(\Delta V)_1 = (\Delta V)_g \rightarrow \gamma_r = \gamma_g = 3\alpha_g$
  - Liquid is placed in a vessel so that volume of empty space above liquid remain same at all temperature then $(\Delta V)_l = (\Delta V)_g \rightarrow V_l\gamma_r = V_g\gamma_g$

Change in density

On heating matter, volume increases at constant mass so the density of matter decreases on increasing temperature.

$$M = V_o\rho_o = V_\theta\rho_\theta$$
$$\text{Or, } V_o\rho_o = V_o(1+\gamma\Delta\theta)\rho_\theta$$
$$\text{Or, } \rho_\theta = \frac{\rho_o}{1+\gamma\Delta\theta}$$
$$\text{Or, } \rho_\theta = \rho_o(1-\gamma\Delta\theta)$$

- If a body just rink in liquid or float in liquid then $wt = \text{ upthrust } \implies \rho_b^\theta = \rho_l^\theta$

Correction of Hg barometer

When the height of barometer is observed *h_{scale}* at $\theta^o C$ then true height is iven by: $h_{true} = h_{scale}(1+\alpha\theta)$, $\alpha =$ linear expansivity of scale.

The correct height is measured at 0^oC then reading at θ^oC is observed h_{scale} then $h_{cor} = h_{scale}1-(\gamma-\alpha)\theta$

Dulong and Petits' experiment

When h_θ and h_0 be the heights of liquid column in two sides of U shaped tube at θ^oc and 0^oc then

Absolute expansivity $(\gamma) = \dfrac{h_\theta - h_o}{h_o\theta}$

Anomalous behavior of water

Every matter expands on heating but water contract on heating from *0^oC* to *4^oC* so

- Water has greatest density and least volume at *4^oC*
- Water freezes from top of pond
- Water at *4^oC* is present at bottom of pond whose surface freezes
- Expansivity of water is *−ve* from 0^oC to *4^oC*
- Mean or average expansivity of water is *0* at *4^oC*
- For water