Friction

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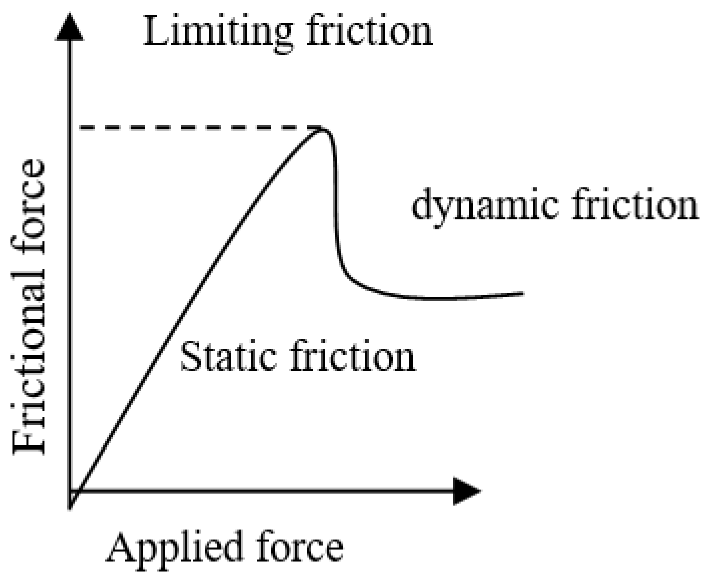
When two surfaces in contact come in relative motion then a force is developed between two surfaces which try to oppose the relative motion between two surfaces and always act in opposite direction of applied force. This force is called frictional force. The frictional force oppose the applied force and exist even the two surfaces tends to move. The frictional force is due to molecular interaction between two surfaces.

Static and kinetic friction

The frictional force between two surfaces before the relative motion starts is called static friction. The frictional force between two surfaces which are in relative motion is called kinetic friction.

Static frictional force is self adjusting force since the magnitude as well as direction of static friction adjust inself with magnitude and direction of applied force.



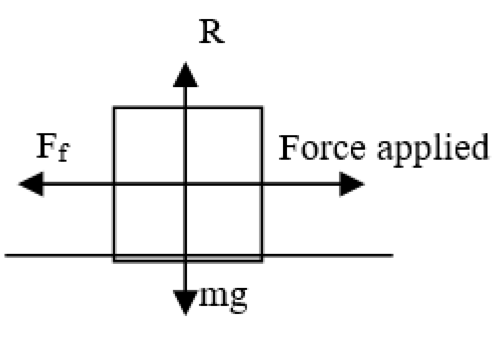
The frictional force increases as the applied force increases and become maximum at which body just start to move is called limiting force of friction. The body now comes in motion and frictional force decreases is called kinetic force of friction.

Angle of friction, coefficient of friction and angle of repose

Coefficient of friction (μ):

The limiting frictional force between two surfaces in contact is directly proportional to the normal reaction.

$F_f \propto R$



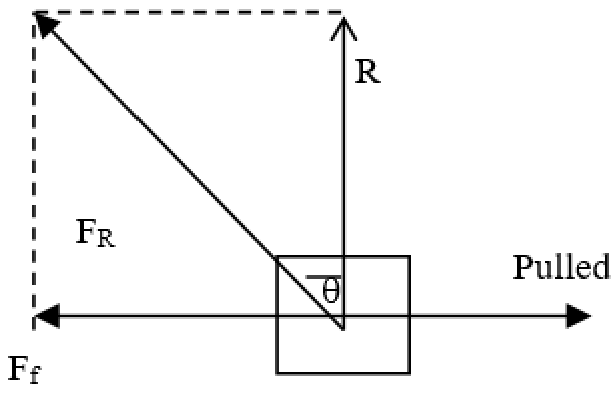
$\therefore F_f = \mu R$

where μ is constant is called coefficient of friction which depends on nature of two surfaces in contact.

The coefficient of friction when a body start to slide on a surface is called static friction, coefficient of friction when body is moving on a surface is called kinetic friction and coefficient of friction when body is rolling on a surface is called rolling friction where $\mu_s > \mu_k > \mu_r$.

Angle of friction (θ):

The angle made by resultant of normal reaction and limiting force of friction with normal reaction is called angle of friction.

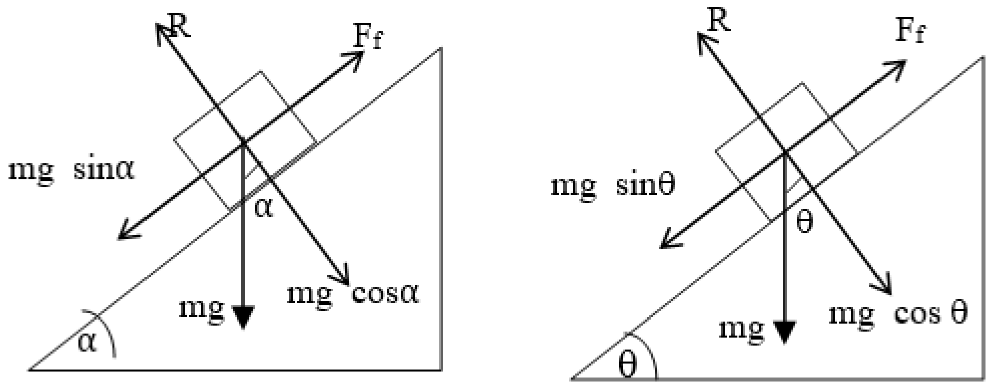


The angle made by F_R with R is θ so

$$\tan\theta = \frac{F_f}{R} = \frac{\mu R}{R} = \mu$$
$$\therefore \tan\theta = \mu$$

Angle of repose (α):

Angle made by inclined plane with horizontal at which an object placed on it just start to slide down the plane is called angle of repose.



$$F_f = \mu R$$
$$mg\sin\alpha = \mu mg\cos\alpha$$
$$\tan\alpha = \mu$$

Laws of limiting and kinetic force of friction

- The frictional force depends on the nature of two surfaces in contact. It increases on increasing roughness and decreases on decreasing roughness.

  The frictional force between two surfaces again increases if all the molecules on one surface comes in inter molecular range with all the molecules on another surface i.e. frictional force increases between very very smooth surfaces.

- Limiting force of friction is always greater than kinetic or dynamic force of friction.

- The limiting force of friction is independent to area of contact if nature and normal reaction of two surfaces remain same.

- The rolling friction is always less than the sliding friction.
- The limiting force of friction is directly proportional to normal reaction.

Acceleration of a body along inclined plane

When a block is placed on inclined plane of inclination θ then the block slide down the plane if the component mg sinθ is greater than kinetic force of friction so

Resultant force $(F_R) = mg\sin\theta - F_f$

$$= mg\sin\theta - \mu_k mg\cos\theta$$
$$= mg(\sin\theta - \mu_k\cos\theta)$$

where μ_k= coefficient of kinetic force of friction.

$\therefore$ Acceleration $(a) = \dfrac{F_R}{m}$

$$= g(\sin\theta - \mu_k\cos\theta)$$

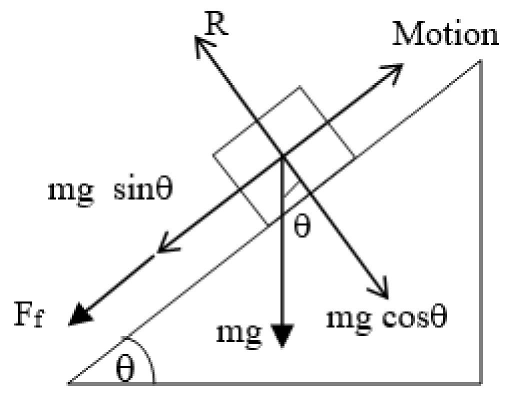
$$\therefore a = g(\sin\theta - \mu_k\cos\theta)$$

$\rightarrow$ When the block is moving up the inclined plane then acceleration of block is

$$a = g(\sin\theta + \mu_k\cos\theta)$$

When a block is pushed up along the inclined plane

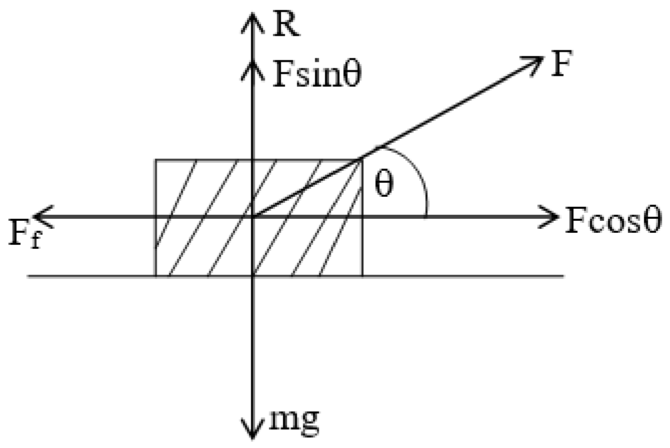
A block of mass m is pushed up along the inclined plane of inclination θ, then total force along the inclined plane is



$\therefore$ Force $(F_T) = mg\sin\theta + F_f$
$\Rightarrow F_T = mg\sin\theta + \mu_k mg\cos\theta$

Pulling & pushing a block on a surface

- **Pulling:** A block of mass 'm' is pulled by a force 'F' inclined at an angle θ with horizontal then block will move if



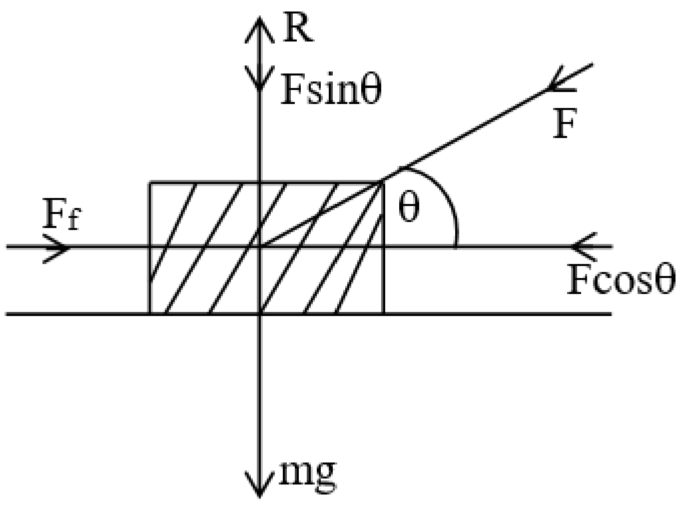
$$\mathrm{F}\cos\theta \geq \mu\mathrm{R}$$
$$\mathrm{F} \geq \frac{\mu\mathrm{mg}}{\cos\theta + \mu\sin\theta}$$

For minimum force, $\mathrm{F} = \dfrac{\mu\mathrm{mg}}{\cos\theta + \mu\sin\theta}$
F will be least if $\cos\theta + \mu\sin\theta$ is maximum so

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta + \mu\sin\theta) = 0$$
$$\Rightarrow \sin\theta + \mu\cos\theta = 0$$
$$\Rightarrow \tan\theta = \mu$$

$$\therefore \sin\theta = \frac{\mu}{\sqrt{\mu^2+1}}, \quad \cos\theta = \frac{1}{\sqrt{\mu^2+1}}$$
$$\therefore \quad F_{\min} = \frac{\mu\mathrm{mg}}{\sqrt{\mu^2+1}}$$

- **Pushing:** A block of mass 'm' is pushed on a horizontal surface by a force 'F' inclined at an angle θ with horizontal then



$$\mathrm{F}\cos\theta \geq \mu\mathrm{R}$$
$$\Rightarrow \mathrm{F}\cos\theta \geq \mu(\mathrm{mg} + \mathrm{F}\sin\theta)$$

$$\Rightarrow \mathrm{F} \geq \frac{\mu \mathrm{mg}}{\cos\theta - \mu\sin\theta}$$

For, minimum force, F_{min} = $\frac{\mu \mathrm{mg}}{\cos\theta - \mu\sin\theta}$

- Block does not move whatever great force may if angle with horizontal is $\geq$ angle of friction.

Stopping distance

When a body moving with velocity 'v', comes to rest after travelling a distance 's' on removing the driving force then

- $\frac{1}{2}\mathrm{m}v^2 = \mathrm{F_f S}$

  $\Rightarrow \frac{1}{2}\mathrm{m}v^2 = \mu \mathrm{mgs}$

  $\Rightarrow \mathrm{s} = \frac{v^2}{2\mu \mathrm{g}}$ i.e. $\mathrm{s} \propto v^2$
- Body comes to rest after time 't' then

  $0 = v - \mu \mathrm{gt}$

  $\Rightarrow t = \frac{v}{\mu g}$ i.e. $\mathrm{t} \propto v$

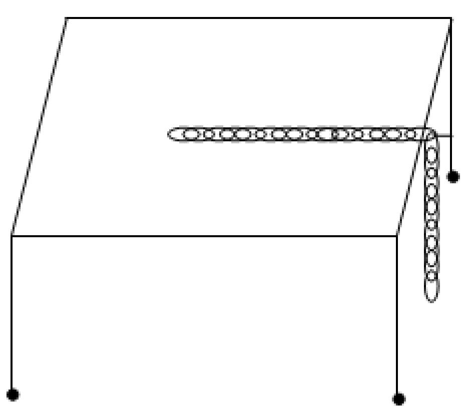
  If coefficient of friction changes then

  $t \propto \frac{1}{\mu}$

A uniform chain is hanging over the table

A uniform chain of mass M & length 'L' is hanging over the edge of table with 'x' length then greatest part hanging over the edge if

Wt of part hanging = frictional force



$$\frac{\mathrm{M}}{\mathrm{L}}\mathrm{xg} = \mu\left(\frac{\mathrm{M}}{\mathrm{L}}\right)(\mathrm{L}-\mathrm{x})\mathrm{g}$$

$$\Rightarrow \mu = \frac{\mathrm{x}}{\mathrm{L}-\mathrm{x}}$$

And greatest fraction hanging is

$$\Rightarrow \frac{\mathrm{L}-\mathrm{x}}{\mathrm{x}} = \frac{1}{\mu}$$

$$\Rightarrow \frac{\mathrm{L}}{\mathrm{x}} = \frac{1}{\mu} + 1$$

$$\Rightarrow \frac{\mathrm{x}}{\mathrm{L}} = \frac{\mu}{\mu+1}$$