Electric Field & Potential

Electric field intensity (E)

Electric field intensity at any point is defined as force experienced by a vanishingly small positive test charge placed at that point.

$$\vec{E} = \lim_{dq \to 0} \frac{\vec{F}}{dq}$$

If a charge q is palced in electric field $\vec{E}$, then force acting on it is:

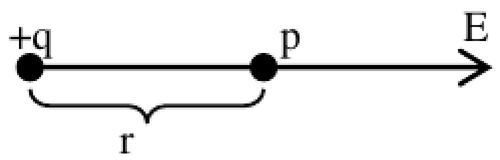
$$F = qE$$

Let q be a point producing electric field. The force on a positive test charge q_0 at a distance r from q is:

$$F = \frac{1}{4\pi\varepsilon_0} \frac{qq_0}{r^2}$$

$$\text{Field strength,} \quad E = \frac{F}{q_0} = \frac{qq_0}{4\pi\varepsilon_0 r^2 q_0}$$

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0 r^2}$$



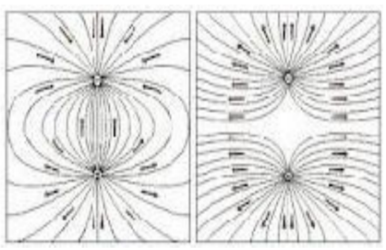
Faraday introduced the concept of electric lines of forces to understand electric field.

Electric lines of forces

A line of force is an imaginary curve the tangent to which at a point gives the direction of intensity at that point and number of lines of force per unit area normal to the surface surrounding that point gives the magnitude of intensity at that point.

Properties of electric lines of forces:

1. They are curved or straight.
2. Two electric lines do not cross to each other.
3. They start from +ve to terminate at −ve charge.
4. They contract longitudinally on account of attraction between unlike charges, and expand laterally.
5. A line of force is a path of +ve charge when it is initially at rest. If a positive charge has initial velocity in a direction different from line of force, it will not move along the line of force.



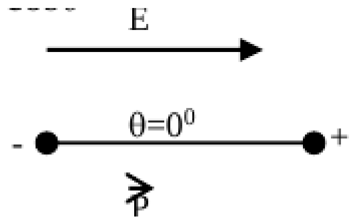
Electric Dipole

Two equal and opposite charges separated by a finite distance constitute an electric dipole. If *−q* and *q* are charges at distance *2l* apart, then dipole moment:
$\vec{p} = q2\vec{l}$

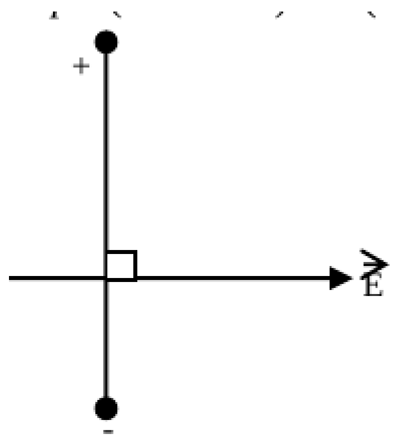
Its direction is directed from *−q* to *+q*. The torque on a dipole in uniform electric field
$\tau = pE\sin\theta$

In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$
The vector done in rotating the dipole from equilibrium through an angle *θ*
$W = pE(1 - \cos\theta)$

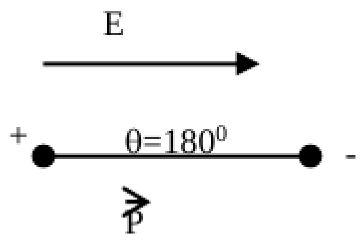
Potential energy of dipole
$U = -pE\cos\theta)$



$\tau = pE\sin 0^\circ = 0$ (min)
$W = pE(1 - \cos 0^\circ) = 0$ (min)



$\tau = pE\sin 90^\circ = pE$ (max)
$W = pE(1 - \cos 90^\circ) = pE$

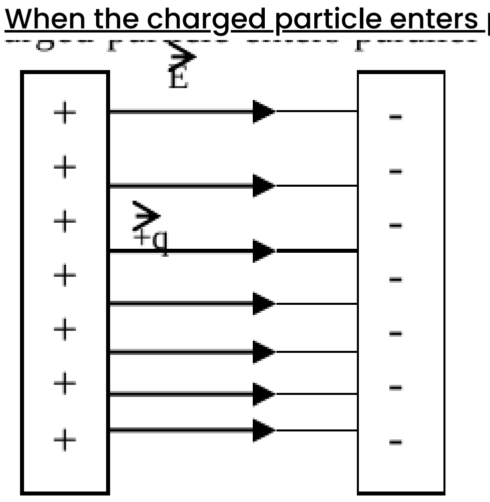


$\tau = pE\sin 180^\circ = 0$
$W = pE(1 - \cos 180^\circ) = 2pE$ (max)

From this, it is clear that work done is minimum ($= 0$) when dipole is parallel to field (*i.e.* $\theta = 0^o$) and maximum ($= 2pE$) when dipole is antiparallel to the field *i.e.* $\theta = 180^o$.

Motion of charged particle in electric field

i. **When the charged particle enters parallel to electric field:**



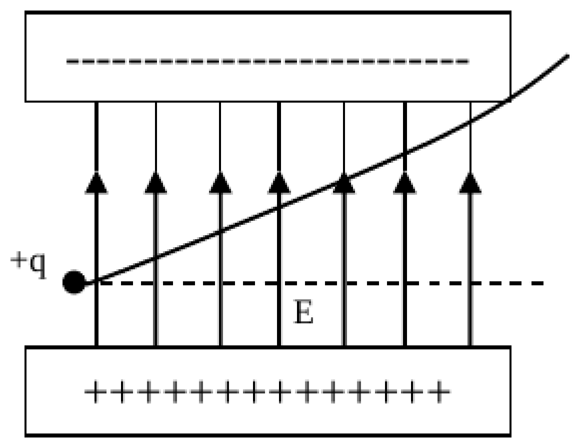
Let a particle of mass *m* and charge *q* is initially at rest in uniform electric field *E*. The force acted on charge *q* is:
$F = qE$
This force accelerates charge *q* by acceleration *a*
$a = \frac{F}{m} = \frac{qE}{m}$
If the particle is at rest at *t = 0*, then velocity of charged particle after time *t* is:
$v = u + at$
$v = 0 + \frac{qE}{m}t$
$\therefore v = \frac{qE}{m}t$

Distance travelled by the particle is:
$s = ut + \frac{1}{2}at^2$
$s = 0 + \frac{1}{2}\frac{qE}{m}t^2$
$\therefore s = \frac{1}{2}\frac{qE}{m}t^2$

KE gained by the particle:

$$K = \frac{1}{2}mv^2$$
$$K = \frac{1}{2}m\left(\frac{qE}{m}t\right)^2$$
$$\therefore K = \frac{1}{2}\frac{q^2E^2}{m}t^2$$

ii. <u>If the particle is projected perpendicular to the field with an initial velocity V_0</u>



Motion of charge particle q along x-axis:
$v_x = v_0 = constant$
$x = v_0t$ ....(1)
(No acceleration along x-axis)

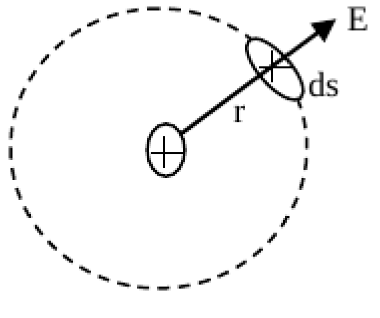
Motion of particle q along y-aixs:
$$v_y = u_0 = at$$
$$v_y = 0 + \frac{qE}{m}t$$
$$\therefore v_y = \frac{qE}{m}t$$

Distance travelled along y-axis:
$$y = u_yt + \frac{1}{2}at^2$$
$$= \frac{1}{2}\frac{qE}{m}t^2$$
$$= \frac{1}{2}\frac{qE}{m}\left(\frac{x}{V_0}\right)^2$$
$$\therefore y = \frac{qE}{2mV_0^2}x^2 \qquad [from (1) t = \frac{x}{V_0}]$$
i.e. the path is a parabola.

Gauss's Law

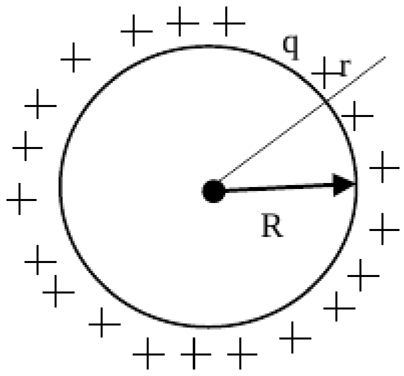
The total flux linked with a closed surface is $\frac{1}{\epsilon_0}$ times the charge enclosed by closed surface i.e.

$$\phi = \int_s \overrightarrow{E} \cdot \overrightarrow{ds}$$



Electric field intensities for some special cases:

i. <u>For hollow or solid conducting sphere:</u>
a. At the point outside the sphere (r>R)
$$E = \frac{q}{4\pi\epsilon_0 r^2}$$



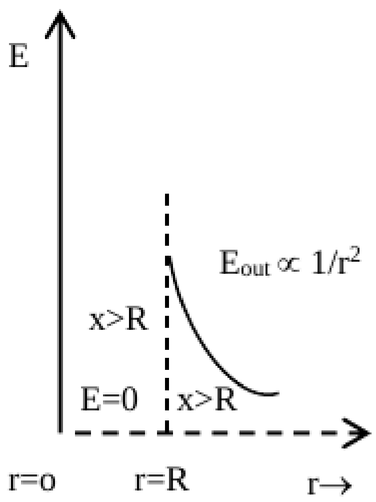
b. On the surface of sphere (r = R)

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

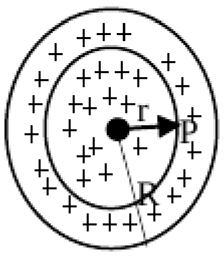
c. At the point inside the sphere (r<R)

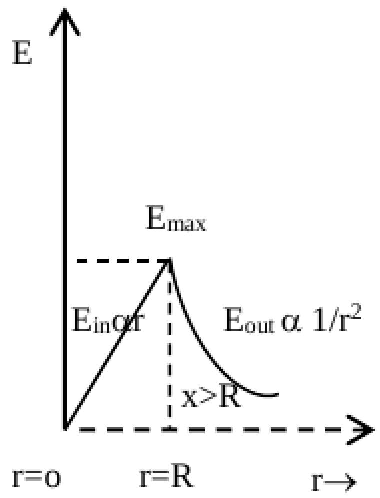
$E = 0$

There is no charge inside the sphere so electric field is zero everywhere.



ii. **For uniformly charged non-conducting sphere or spherical volume distribution of charge:**





Intensity at internal point P (r < R) will be due to the charge contained in sphere of radius r. Let this charge be Q'. And if Q be the total charge distributed in sphere

$$Q = \frac{4}{3}\pi R^3 \rho \quad \text{so, } Q' = \frac{4}{3}\pi r^3 \rho$$

$$\text{i.e. } Q' = \frac{Q}{R^3} r^3$$

$$\text{So, } E_{\text{in}} = \frac{Q'}{4\pi\varepsilon_0 r^2} = \frac{Q}{R^3}\frac{r^3}{4\pi\varepsilon_0 r^2}$$

$$\therefore \quad E_{\text{in}} = \frac{Q}{4\pi\varepsilon_0 R^3} r$$

$$\text{Or, } E_{\text{in}} \propto r$$

iii. **For infinite line of charge, infinite plane of charge and charged conductor**

a. Infinite line charge

$$F = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}$$

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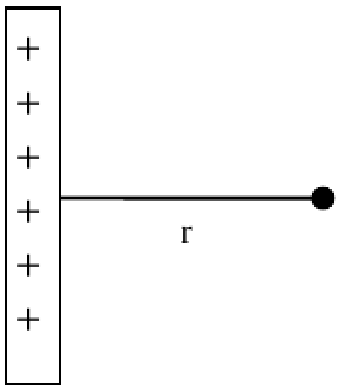

where λ is linear charge density i.e.

$$\lambda = \frac{q}{l}$$

b. Infinite plane charge

$$E = \frac{\sigma}{2\epsilon_0}$$

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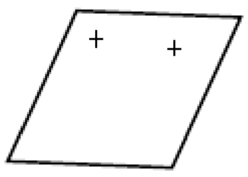


c. Charged conductor

$$E = \frac{\sigma}{epsilon_0}$$

where $\sigma = \frac{q}{A}$ is surface charge density.

Electric Potential

The potential at any point is defined as amount of work done in taking a unit positive charge from infinity to that point. This work done in electric field remains as potential energy. Thus when a charge *qc* is placed in electric field at a point, where potential is *V* then *PE* is *qV* Joule.

The potential energy may be positive r negative depending on whether the work done against electric force or by the electric force during transport respectively.

If *q* is a point charge, then at a distance r from the charge potential can be given as:

$$V = \frac{q}{4\pi\epsilon_0 r}$$

If q_1, q_2, q_3 ....*qn* point charges distributed in air and *P* is the point at distances $r_1, r_2, r_3 ... r_n$ from point charges respectively, then resultant potential at P is:

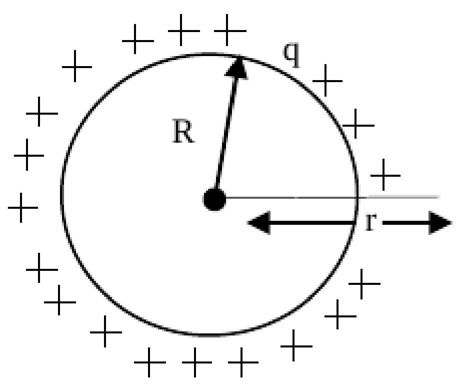
$$V_p = V_1 + V_2 + V_3 + \ldots\ldots\ldots V_n$$

$$V_p = \frac{1}{4\pi\epsilon_0}(q_1 + q_2 + q_3 + \ldots\ldots\ldots q_n)$$

1. **Equipotential surface**
   Equipotential surface is that surface where the potential at any point of the surface has the same value. The electric lines of forces and the equipotential surfaces are mutually perpendicular. No work is done in moving a charge from one point to another.

2. **Electric potential on a spherical charge shell**
   i. **Outside the shell (r>R)**

$$V = \frac{q}{4\pi\epsilon r}$$



ii. **On the surface of shell (r = R)**

$$V = \frac{q}{4\pi\epsilon R}$$

iii. **Inside shell (r<R)**

$$V = \frac{q}{4\pi\epsilon R}$$

Inside the charge shell, electric potential is same as potential on the surface.