Wave Optics

Nature of Light

Light is a form of energy which gives the sensation of sight. To explain the phenomena associated with light, various theories have been put forward.

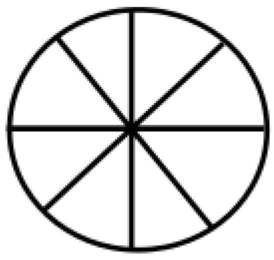
- ### Newton's Corpuscular theory:

  Light consist extermely small, very light material particles called corpuscle which travels with the velocity of light. This theory was able to explain reflection, refraction and rectilinear propagation of light. But the theory is unable to explain phenomena like interference, diffraction and polarization. According to the theory, velocity of light is more in denser medium rather than is rarer medium which is against the experimental observation of Focault.

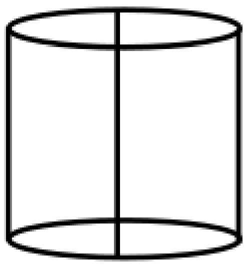
- ### Huygen's wave theory:

  Huygen proposed the wave theory of light. According to him,

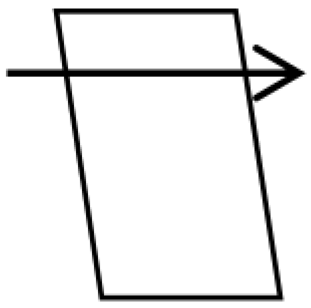
  - Each point source of light is a centre of disturbance from which waves spread in all directions. The locus of all particles vibrating in the same phase is called wave front. The shape of wave front depends on the source.



Spherical wavefront for point source, Intensity $\propto \frac{1}{r^2}$



Cylindrical wavefront source, linear source, Intensity $\propto \frac{1}{r}$



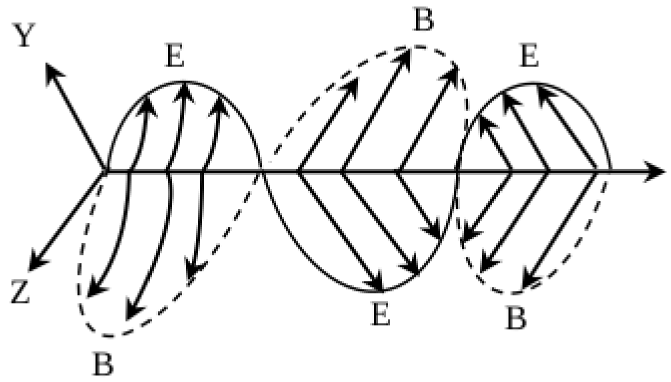
Plane wave front Source – Point or linear source at infinity.
Intensity does not depend on distance

  - Each point of a wavefront acts as a source of disturbance called secondarysource which give new wave front called wave-lets which travels with the speed of light.
  - The forword envelope of the secondary wave - lets at any instant gives the new wave- front.
  - In a homogenous medium, the wave front is always perpendicular to the direction of wave.

  To explain the propagation of wave, Huygen assumed light as a mechanical wave requires medium called ether. This theory explained to phenomenon like reflection, refraction, interference and diffraction but failed to explain Photoelectric effect, Compton effect and Raman effect.

- ### Maxwell's electro magnetic wave theory:

  According to Maxwell, the electro magnetic waves are those waves in which there are sinusoidal variation of electric and magnetic field vectors at right angle to each other as well as at right angles to the direction of wave propagation. Both these fields vary with time and space and have same frequency.



PROPERTIES OF E.M.W:

  - Produced by accelerated or oscillating charge.
  - Do not require any material medium for propagation.
  - Travel in free space with a speed $3 \times 10^8\ m/s$ given by the relation; $c = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}}$
  - The amplitude of the electric and magnetic fields in free space are related by: $c = \dfrac{E_0}{B_0}$
  - The direction of variation of electric and magnetic field vectors are perpendicular to each other as well is perpendicular to the direction of propagation of wave. Therefore e.m.w. are transverse in nature.
  - The energy in e.m.w. is divided equally between electric field and magnetic field vectors.
  - The e.m.w are not deflected by electric and magnetic fields.
    The speed of light was measured by optical method was found same as the speed of e.m.w. shown by Maxwell in the relation $c = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}}$ so light is also an e.m.w.
    This theory is able to explain most of the phenomenon related to light but fails to explain emission and absorption spectra.

- ### Einstein's quantum theory:

  This theory was developed by Einstein using plank's quantum hypothesis of black body radiation. According to this theory light in emitted by the source is the form of energy bundles called photons. The energy associates with a photon is given by :
  $E = h\nu = \dfrac{hc}{\lambda}$
  On the basis of this theory photo electric effect, Compton effect & Raman effect was explained but failed to explain interference, diffraction and polarization.

- ### Dual nature theory:

  According to this theory light has dual nature. One wave nature and second corpuscles nature. This theory in able to explain all the phenomenon of light as.

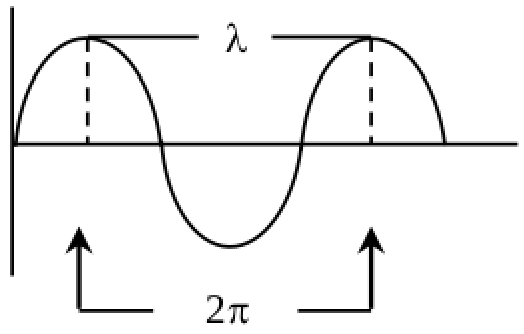
- ### PRINCIPLE OF SUPERPOSTION :

  According to this principle when two or more than two waves travelling in a medium. Superimpose, then the displacement of the resultant wave at any instant is equal to the vector sum of the displacement due to individual waves at that instant.
  i.e, $\vec{y} = \overrightarrow{y_1} + \overrightarrow{y_2} + \cdots + \overrightarrow{y_n}$

Interference

- ### Coherent sources of light:

  The sources of light which emit continuous light waves of the same wavelength, same amplitude and in same phase or having constant phase difference are called coherent sources.



<u>For interference, coherence of waves is a must.</u> Two independent sources of light cannot be coherent. So two coherent light sources can be obtained from a single source of light by reflection refraction etc. In young's double slit experiment, coherence is obtained by division of wave front and in thin film by division amplitude.

Relation between path difference and phase difference:

Path difference is the difference between the distance travelled by light to met at a point. For λ path difference the phase difference is 2π. For path difference of 'x' phase difference is:

$\phi = \frac{2\pi}{\lambda} \times x$

Resultant amplitude and intensity due to superposition of waves:

When two waves of intensities I_1 and I_2 and amplitude a_1 & a_2 superpose then the amplitude 'A' and intensity 'I' of resultant wave is given as:

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$
$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$
where ϕ is the phase difference between the two waves.

- ### Condition for maximum intensity:
  - <u>In terms of phase difference (ϕ) :</u>
    $\phi = 2\pi n$ where $n = 0, 1, 2, 3, \ldots$
  - <u>In terms of path difference (x) :</u>
    $x = n\lambda$
- ### Condition for minimum intensity:
  - <u>In terms of phase difference (ϕ) :</u>
    $\phi = (2n-1)\pi$ where $n = 1, 2, 3, \ldots$
  - <u>In terms of path difference (x) :</u>
    $x = (2n-1)\frac{\lambda}{2}$

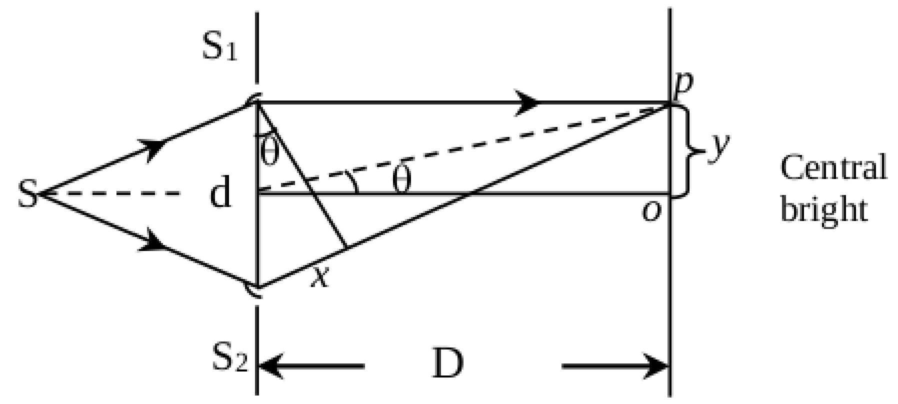
Here,
$\text{Intensity}(I) \propto (\text{Amplitude})^2$
So, $I_1 \propto a_1^2$
$I_2 \propto a_2^2$
$\therefore \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$
Also, $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}$

- ### Interference

  Re-distribution of energy due to the superposition of waves is called interference.
  If the phase difference between two waves reaching at the point is, $\phi = 2n$ where $n = 0, 1, 2, 3, \ldots$, then we get constructive interference and if the phase difference is $\phi = (2n-1)\pi$ then we get destructive interference.

  <u>Condition for sustained interference :</u>
  - Two sources must be coherent.
  - Two sources must emit continuous waves of the same wavelength and frequency.

- ### Young's double slit experiment



Narrow slit S 1 & S 2 equi distance from mono chromatic source 'S' acts as two coherent sources which are separated by a distance 'd' and the screen is at a distance of D from the slit.

O is the point of central maxima because the path difference for point O is zero. For any point P at a distance of 'y' from O, the path difference is 'x'

From figure:

$$\sin\theta = \frac{x}{d}$$
$$\text{Or, } x = d\sin\theta$$
And, $\tan\theta = \dfrac{y}{D}$

For small angle $\theta, \sin\theta \approx \tan\theta \approx \theta$

$$\therefore \frac{x}{d} = \frac{y}{D}$$
$$\text{Or, } y = \frac{xD}{d}$$

If p is maxima then $x = n\lambda$

$$\therefore y_n = \frac{n\lambda D}{d}$$

If p is minimum then $x = (2n-1)\dfrac{\lambda}{2}$

$$\therefore y_n = \frac{(2n-1)\lambda D}{2d}$$

Fringe width (β):

The distance between any two consecutive maxima or minima is called fringe width.

$$\beta = y_n - y_{n-1}$$
$$\Rightarrow \beta = \frac{\lambda D}{d}$$

Fringe breadth is independent of the order of fringe and is same for bright and dark fringe.
If the surrounding medium has R.I. μ then:

$$\beta_m = \lambda_m D/d$$

For air,

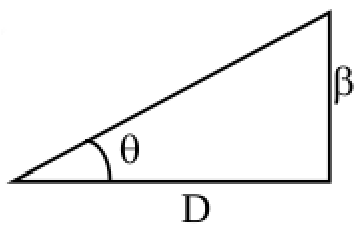
$$\beta_a = \lambda_a D/d$$

Here,
$\mu = \lambda_a/\lambda_m$
$\therefore \beta_m = \beta_a/\mu$

Angular fringe width:

Angular fringe width is:

$$\tan\theta \approx \theta = \beta/D = \frac{\lambda}{d}$$



- If n_1 fringes are visible in a field of view with light of wave length λ_1 and n_2 fringes are visible with light of wave length λ_2 in the same field then :
  $n_1\lambda_1 = n_2\lambda_2$
- When transparent sheet of thickness t having refractive index μ is introduced on path of one of the interfering wave then path difference is introduced by $t(\mu-1)$ due to which n th fringe shift by:
  $t(\mu-1) = n\lambda$
- When w_1 and w_2 be the width of slits then the intensities and amplitude are related with width as:
  $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{w_1}{w_2}\right)$

- **Points to Remember:**
  - All dark and bright fringe has same breadth.
  - Brightness of all bright fringe is same if white bright is used in place of monochromatic light then central fringe is white with colored fringe on the side of central fringe with red at the outer edge and violet at the inner edge.
  - $\beta_{red} > \beta_{orange} > \cdots > \beta_{violet}$
  - In the air, the fringe width is maximum and decreases as the R.I. of medium increases.
  - If one slit is closed then for slit of the order of λ, we get diffraction pattern.
  - If the slit width are unequal, the minima will not be completely dark.
  - If the distance between the slits is less then the wavelength of light then no interference pattern will be observed.
  - If a transparent sheet of R.I. μ is kept in front of one of the slit then the fringe width remain the same but whole fringe will shift towards the side in which the sheet is kept.

Diffraction

- **Diffraction**

  The phenomena of bending of light around the corner and penetration into the geometrical shadow is called diffraction. Diffraction is shown by wave nature of light. It is shown both by transverse and longitudinal wave. Diffraction is possible only when the size of object is comparable to the wave length.

  According to Fresnel, diffraction occurs due to the mutual interference of secondary wave lets starting from portion of the wave front which are not blocked by the obstacle or from portions of the wave front which are allowed to pass through the aperture.

  ### Types of Diffraction:

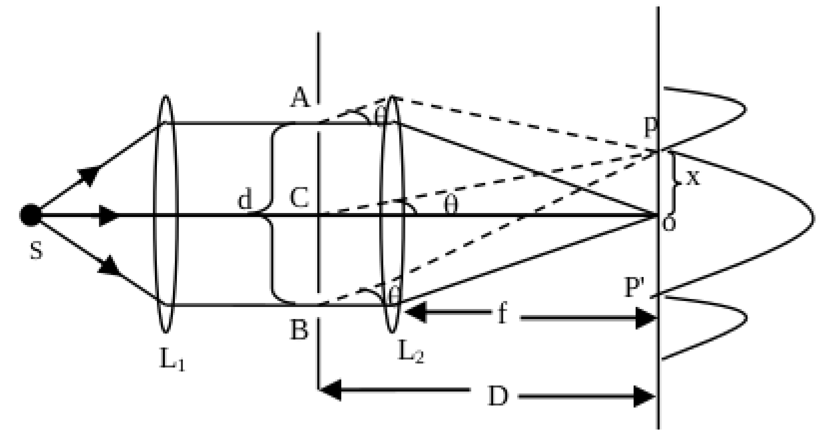
  The diffraction phenomena is divided into two types.

  - **Fresnel diffraction:**

    In this type of diffraction source and screen are at finite distance from the slit. The incident wave front are either spherical or cylindrical.

  - **Fraunhofer diffraction:**

    In the diffraction, the source and the screen are at infinite distance from the slit and the incident wave front is plane wave front. Diffraction of light at a single slit.



S is a monochromatic light source at the focus of lens L 1 such that plane wave front is produced which falls on the slit AB. The lens L 2 focuses the diffraction pattern on the screen.

The diffraction pattern obtained on the screen has a central bright band and alternate dark and bright bands. The bright band p to p' is called central maxima. The intensity of other maxima on both sides decreases from central maxima.

For any point 'P' the path difference is:
$x = d \sin\theta$

If P is minima then:
$d \sin\theta = n\lambda \ [n = 0, 1, 2, 3, \ldots.]$

If P is maxima then:
$d \sin\theta = (2n+1)\frac{\lambda}{2} \ [n = 1, 2, 3, \ldots..]$

Width of central maxima:

The width of central maximum is the distance between first secondary minimum on either side of O.
For small θ,

$$\sin\theta \approx \theta = \frac{\lambda}{d}$$

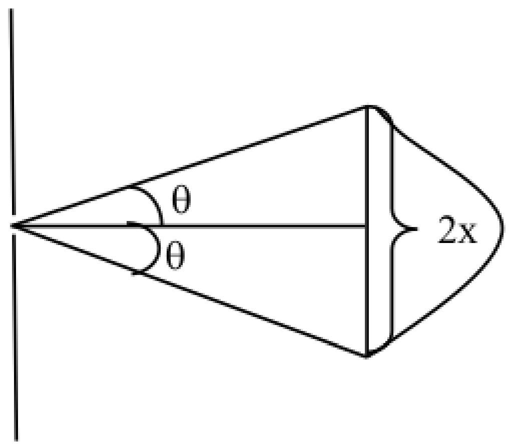
Angular width of central maxima is :

$$2\theta = \frac{2\lambda}{d}$$

$$\frac{x}{D(f)} = \frac{\lambda}{d}$$

$$\Rightarrow x = \frac{D\lambda}{d}$$

$$\therefore 2x = \frac{2D\lambda}{d}$$



Width of a secondary maxima of minima is half of the central maxima.

Fresnel distance:

The distance traversed by beam at which the spreading due to diffraction becomes greater than the size of 'd' of the aperture is called the Fresnel distance ['D_f'].

$$\frac{D_F\lambda}{d} = d \Rightarrow D_F = \frac{d^2}{\lambda}$$

- **Diffraction Grating:**

  It consists of large number of equally spaced parallel slits. When light incident normally on a transmission rating the direction of principal maxima is given by :

  $$(a+b)\sin\theta = n\lambda$$

  $(a+b)$ is called grating element and,

  $$\frac{1}{a+b} = N$$

  where, $N = \text{No. of lines per unit length}$

Resolving power of optical instrument

A large number of images are formed as a consequence of light diffraction from a source. If the source are separated such that the central maxima do not overlap, then the images can be distinguished and are said to be resolved. Resolving power of an optical instrument is the power or ability of the instrument to produce distinctly separate image of the close objects.

- **For Miscroscope:**

  Limit of resolution :

  $$d = \frac{\lambda}{2\mu\sin\theta}$$

  $$R.P. = \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$$

- **For telescope:**

  Limit of resolution :

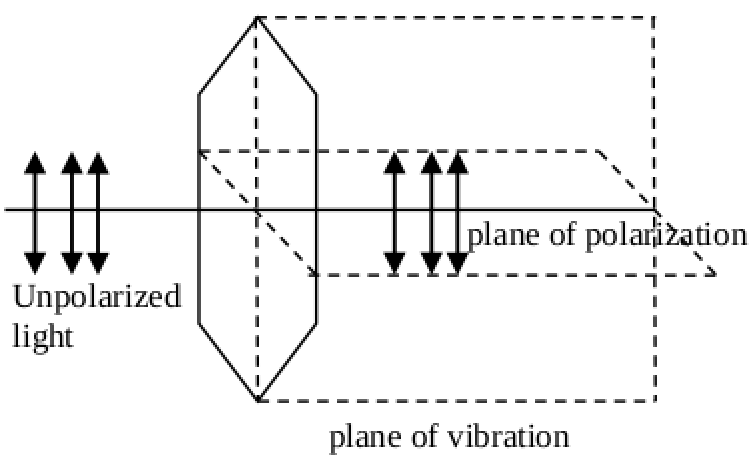
  $$d\theta = \frac{1.22\lambda}{D}$$

  $$R.P. = \frac{D}{1.22\lambda}$$

- **Examples of diffraction:**
  - Silver lining surrounding the profile of a mountain just before sunrise.
  - Spider web can give diffraction pattern.
  - Columned spectra that one sees while viewing a distance source of light through a fine cloth.
  - Sound produce in one room can be heard in the nearby room.

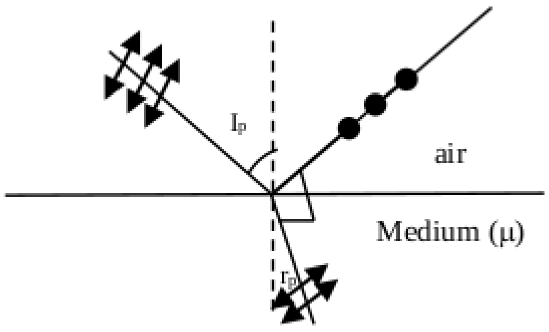
Polarization

The phenomenon of confining the vibration of a wave in a specific direction perpendicular to the direction of wave motion is called polarization. The plane containing the direction of vibration and wave motion is called plane of polarization.



- **Brewster's Law**

According to this law, when polarized light is incident at polarizing angle, i_p on an interface separating air from a medium of R.I. μ, then the reflected light is fully polarized provided:
$\mu = \tan i_p$



- ## Law of Malus

  According to this law, when a beam of completely polarized light is incident on an analyser, the resultant intensity of light (I) of transmitted light from the analyser varies directly as the square of the cosine of the angle (θ) between plane of transmission of analyze and ploarizer.
  i.e, $I \propto \cos^2 \theta$

- ## Polaroids:

  A Polaroid is a material which polarizes light. Tourmaline is a natural polarizing material Polaroids are now artificially made ex – quinine idosulphate.

- ## Optical rotates:

  It is the phenomenon of rotating the plane of polarization light about the direction of propagation of light when passed through solution contain crystal.

  The substances which rotate the plane of polarization are said to be optically active.
  Examples: quartz, sugar crystal, sodium chloride , turpentine oil etc.