Superposition and waves in string and air columns

Introduction

- When two waves of same frequency travelling with same velocity in the same direction this gives rise to the phenomenon of interference of waves.
- Two waves of identical frequency and amplitude travelling along same path with same velocity in opposite directions. This gives rise to the phenomen of stationary waves.
- Two waves of slightly difference frequency moving with same velocity in same direction gives rise to the phenomenon of beats.

Principle of superposition

y = $y_1 + y_2 + y_3 + \dots\dots\dots\dots + y_n$

let, $y_1 = a_1 \sin(\omega t - kx)$

and $y_2 = a_2 \sin(\omega t - kx + \delta)$ are superposed then the, resulting wave is given by

$y = y_1 + y_2$

$y = A \sin(\omega t - kx + \beta)$

where, A = amplitude of resulting wave

$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta}$

and $\tan\beta = \dfrac{a_2 \sin\delta}{a_1 + a_2 \cos\delta}$

δ = phase difference of two waves

$\therefore$ intensity (I) $\propto a^2$

Resulting intensity is given b

$$\mathrm{I_R} = \mathrm{I_1} + \mathrm{I_2} + 2\sqrt{I_1 I_2}\cos\delta$$

For constructive interference

Cosδ = 1

$\delta = 2n\pi$

Path diff; $\Delta x = n\lambda$; x = 0, ± 1, ± 2.........

$\rightarrow$ Resultant intensity is maximum

$$\mathrm{I_{max}} = (\sqrt{I_1} + \sqrt{I_2})^2$$

Resultant amplitude is maximum

$A_{max} = (a_1 + a_2)$

For destructive interference

Cosδ = -1

$\delta = (2n-1)\pi$ or $(2n+1)\pi$

Path difference $\Delta x = (2n-1)\lambda/2$ or $(2n+1)\lambda/2$

$\rightarrow$ Resulting intensity is minimum

$$\mathrm{I_{min}} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$\rightarrow$ Resulting amplitude is minimum

$A_{min} = |a_1 - a_2|$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

Stationary wave:

y_1 = a sin(ωt -kx)

y_2 = a sin(ωt + kx)

y = y_1 +y_2

y = 2acoskx.sinwt $\Rightarrow$ represents eq^n. of stationary wave

y = A sinωt

where, A = 2acoskx = Amplitude of stationary waves

$\rightarrow$ The points where amplitude of stationary wave is minimum is called NODE

$\rightarrow$ The points where amplitude of stationary wave is maximum is called *ANTINODE*

NODE

The position of nodes

x = (2n -1) $\lambda/4$ = odd multiple of $\lambda/4$

x = $\lambda/4$, 3$\lambda/4$, 5$\lambda/4$, ----

i. Distance between two consecutive node is $\lambda/2$.

ii. Displacement and velocity are minimum.

iii. Strain is maximum

iv. pressure is maximum.

v. Density is maximum

Antinodes

The positions of antiondes

x = n$\lambda/2$

x =0, $\lambda/2$, λ, 3$\lambda/2$, -----

i. Distances between two consecutive antinode is $\lambda/2$

ii. Displacement and velocity are maximum

iii. Strain is minimum

iv. Pressure is minimum

v. Density is minimum

$\rightarrow$ The distance between node and adjacent antinode is $\lambda/4$

$\rightarrow$ Phase difference between two particles of a medium lying just opposite side of node is π.

$\rightarrow$ In stationary wave/standing wave all particles an the string may cross their mean position simultaneously

$\rightarrow$ All the particles oscillating between two consecutive node oscillate in same phase.

$\rightarrow$ In interference pattern conservation of energy is not violated. Energy is redistributed according to phase difference. -

BEATS

No. of beats/sec = $|f_1 - f_2|$

Beats period $(\mathrm{T}) = \dfrac{1}{|f_1 - f_2|}$

$\rightarrow$ Time interval between two consecutive maximum and minimum is $\dfrac{1}{|f_1 - f_2|}$

$\rightarrow$ Waxing means frequency decreases ($\downarrow$)

$\rightarrow$ Winning means frequency increases ($\uparrow$)

$\rightarrow$ When tuning fork is loaded, then f $\downarrow$ sec

$\rightarrow$ When tunning fork is filed, then f $\uparrow$ sec

$\rightarrow$ When prongs cuts then f $\uparrow$ sec

$\rightarrow$ When f -m, f, f + m, are sounded together then no. of beats = m

$\rightarrow$ When N tuning forks are arranged in increasing order of frequency such that any two consecutive produce 'm' beats, the frequency of last fork is 'x' times the first then,

$$\mathrm{f}_1=\frac{(N-1)m}{(x-1)}$$

and f_n = xf_1

Vibration of string fixed at both ends

Equation of stationary wave string fixed at both ends is given by

$$y=2\,\mathrm{a}\sin kx.\cos\omega t$$

$\rightarrow$ At x = 0, and x = L node is formed

No. of loops (n) = 2L $/\lambda$

Frequency of vibration is given by

$$\mathrm{f}=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

where,

T = tension in the string

μ = mass per unit length or linear mass density

$$\mu=\mathrm{m/L}=\frac{\rho\times\pi r^2 L}{L}=\rho\pi r^2$$

$$\mathrm{f}\propto\sqrt{T}$$
$$\mathrm{f}\propto 1/\mathrm{r}$$
$$\mathrm{f}\propto 1/l$$

$\rightarrow$ When spring is plucked at a position then antinodes is formed at that position

$\rightarrow$ A string is fixed at one end and block of density ρ_b is hanging from string, when block is immersed in liquid of density ρ_l then ratio of frequency after and before.

$$\frac{f_2}{f_1}=\sqrt{\frac{\rho_b-\rho_l}{\rho_b}}\quad\therefore\mathrm{f}\propto\sqrt{T}$$

Where, ρ_l = density of liquid

$$\mathrm{f}=\frac{n}{2l}\sqrt{T/\mu}$$

If n = 1, f = f_o = fundamental frequency = $\frac{1}{2l}\sqrt{T/\mu}$

If n = 2 : f = $2\mathrm{f}_o$ = 2^{nd} Harmonics, 1^{st} overtone

If n = 3, f = $3\mathrm{f}_o$ = 3^{rd} Harmonics, 2^{nd} overtone

$\rightarrow$ No. of Harmonics = Integral multiple of fundamental frequency

$\rightarrow$ No. of overtones = No. of loops -1

$f\times$ l = constant

$\mathrm{f}_1\mathrm{l}_1$ = $\mathrm{f}_2\mathrm{l}_2$

$\rightarrow$ At fixed ends node is formed

No. of nodes = no. of loops + 1

No. of antinodes = no. of loops

Waves in pipe

Open organ pipe

$\rightarrow$ Open at both ends

No. of loops (n) = $2\mathrm{l}/\lambda$

Frequency ; $\mathrm{f} = \mathrm{n} \times \frac{v}{2l}$

Where, v = velocity of sound in air

l = length of open pipe

Fundamental frequency n = 1, $f_o = \frac{v}{2l}$

n = 2, f = $2f_o$; 2^{nd} Harmonic, 1^{st} overtone

n = 3, f = $3f_o$; 3^{rd} Harmonic, 2^{nd} overtone

$\rightarrow$ At open ends antinodes are formed

No. of antinodes = No. of loops + 1

No. of nodes = No. of loops

Closed organ pipe

$\rightarrow$ Closed at one end

No. of loops $(\mathrm{n}) = \frac{2l}{\lambda} + \frac{1}{2}$

$$\rightarrow \quad \text{frequency}\ , \mathrm{f} = (2\mathrm{n} - 1) \times \frac{v}{4l}$$

If n = 1; f = f_o = fundamental frequency

$\frac{v}{4l}$

If n = 2; f = $3f_o$ = 3^{rd} harmonics, 1^{st} overtone

If n = 3; f = $5f_o$ = 5^{th} harmonics, 2^{nd} overtone

$\rightarrow$ At closed end Node is formed

No. of nodes = No. of loops = No. of antinodes

Resonance tube and End Correction (e)

The length of n^{th} resonance is given by

$$l_\mathrm{n} + \mathrm{e} = (2\mathrm{n} - 1)\frac{\lambda}{4}$$

$\rightarrow$ If l_1 and l_2 are the lengths of first and second resonating lengths then, end correction is given by

$$\mathrm{e} = \left(\frac{l_2 - 3l_1}{2}\right)$$

$\rightarrow$ Wavelength of wave, λ = $2(\mathrm{l}_2 - \mathrm{l}_1)$

Velocity of sound: v = 2f $(\mathrm{l}_2 - \mathrm{l}_1)$

$\rightarrow$ End correction in terms of radius of organs pipe

e = 0.6 r = 0.3 D

Where r = radius of tube,

Effective length of open organ pipe;

l_{eff} = l_o +2e

Effective length of closed pipe l_{eff} = l_c + e

Melde's Law

$$n\sqrt{T} = constant$$

Where T = constant; n = no of loops

$\rightarrow$ When a person standing between two parallel hills, fires a gun and hears the first echo and second echo after t_1 and t_2, then the distance between the hills is

$$d = \frac{v}{2}(t_1 + t_2)$$

$\rightarrow$ When a vehicle approaching a cliff with 'u' and blow horn at a distance 'd' from cliff and its echo is heard after time 't' then

$$d = \left(\frac{u+v}{2}\right) t$$

The distance of the point form where echo is heard

$$x = \left(\frac{u-v}{2}\right) t$$

Where v = velocity of sound.