Simple Harmonic Motion

Simple harmonic motion (SHM)

Period Motion

Motion which repeats after regualt interval of time is called periodic motion.

Simple Harmonic Motion

The back and forth motion of particle about a fixed position in a straight line in which acceleration is directly proportional to displacement form mean position and always directed towards mean position is called simple harmonic motion.

$\vec{a} \propto \vec{x}$

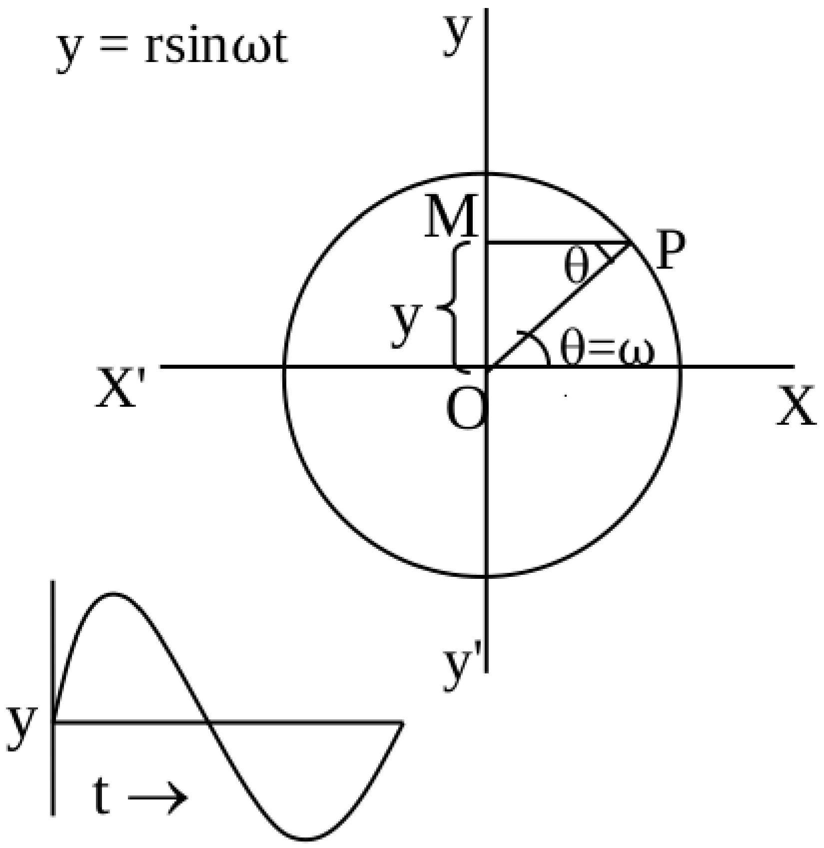
or, $\vec{a}$ = -k $\vec{x} \Rightarrow \vec{F}$ = -k $\vec{x}$

In case of angular motion of particle which oscillate about mean position in which restoring torque is proportional to the angular displacement.

ie. $\vec{\tau} \propto \vec{\theta} \Rightarrow \vec{\tau}$ = -k $\vec{\theta}$

Representation of SHM

A body is moving along the circumference of circular path of radius r with uniform angular velocity 'ω' then P be any position of body on the circumference of circle then perpendicular from P on yoy' is PM. The foot of perpendicular M moves forwards and backward about point O is called mean position. The displacement of M from O is



Characteristics of SHM

1. Velocity (v)
   Rate of change in displacement is called velocity of body in SHM.
   $\therefore$ velocity (v) = $\dfrac{dy}{dt}$ = rωcosωt
   = $\omega\sqrt{r^2 - y^2}$
   At mean position,
   y = 0 so v = rω i.e. Velocity of body in SHM is maximum at mean position.
   At extreme position,
   y = r so, v = 0 ie velocity of body in SHM is least at end of oscillation.
2. Acceleration (a)
   Rate of change in velocity of body is called acceleration.
   $\therefore$ Acceleration (a) = $\dfrac{dv}{dt}$ = -rω^2sinωt
   = -ω^2y
   $\therefore$ a = -ω^2y
   At mean position y = 0 so a = 0 ie acceleration of body in SHM is 0 at mean position
   At end of oscillation y = r so a = -ω^2r ie acceleration of body m SHM is maximum at end of oscillation.
3. Time period (T)
   Time taken to complete one oscillation

$\therefore$ a = ω^2y

or, $\omega = \sqrt{\frac{a}{y}}$

or, T = $2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{Displacement}{Acceleration}}$

4. Frequency (f)
   No. of complete oscillations made by body in one second is called frequency
   $\therefore$ a = ω^2y
   or, $\omega = \sqrt{\frac{a}{y}}$
   or, f = $\frac{1}{2\pi}\sqrt{\frac{a}{y}} = \frac{1}{2\pi}\sqrt{\frac{Acceleration}{Displacement}}$

Total energy of SHM

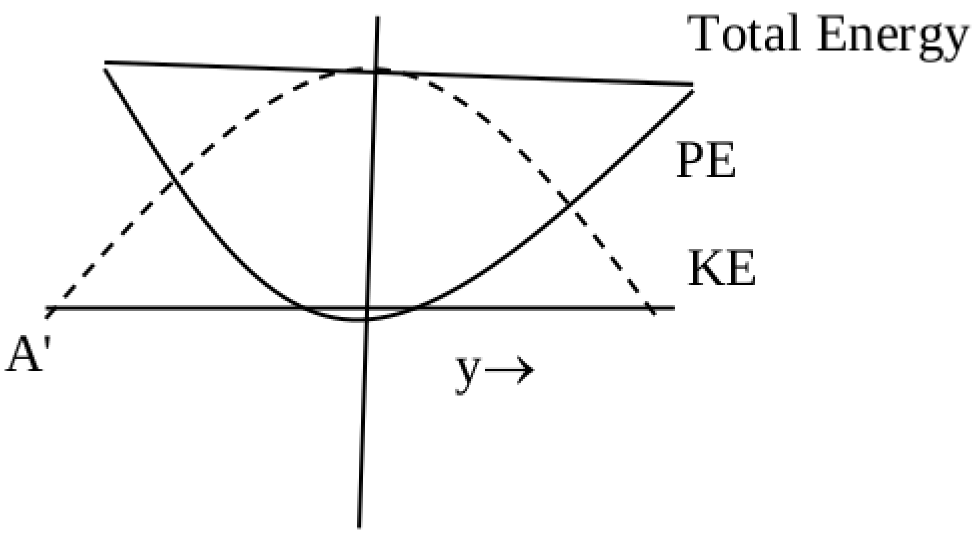
When a body is in SHM then the total energy of body remain constant at any position but the KE changes in PE while moving towards extreme position from mean position and vice versa. The friction and air resistance is neglected in ideal case.

At any position

KE = $\frac{1}{2}mr^2\omega^2\cos^2\omega t = \frac{1}{2}m\omega^2(r^2 - y^2)$

PE = $\frac{1}{2}ky^2 = \frac{1}{2}m\omega^2y^2$

Total energy (E_T) = $\frac{1}{2}m\omega^2r^2$



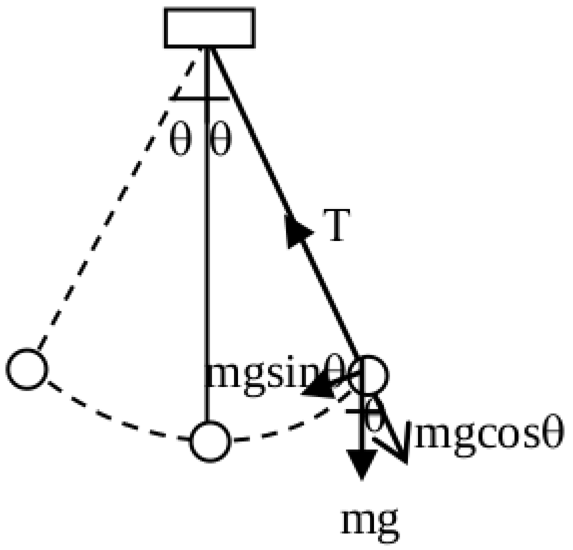
Harmonic Oscillators

(a) Simple pendulum

Heavy point mass is suspended by light, inextensible, perfectly flexible string from rigid support then it is called simple pendulum

Time period (T) = $2\pi\sqrt{\frac{l}{g}}$

Hence time period is in dependent to mass, shape, material of bob and amplitude of motion.



→ If the angular displacement is not small then sin? = $\theta - \frac{\theta^3}{3!}$

so

T = $\dfrac{2\pi\sqrt{\dfrac{l}{g}}}{1-\dfrac{\theta^2}{16}}$ = $\dfrac{T_o}{1-\dfrac{\theta^2}{16}}$

Where

θ = maximum angular displacement in radian.

$\rightarrow$ For pendulum of sufficient length where l is comparable to radius of earth

$$\mathrm{T} = 2\pi\sqrt{\frac{R}{g(1+R/I)}}$$

i. If l = ∞ then T = $2\pi\sqrt{\dfrac{R}{g}}$ = 84.6 min

ii. If l = R then T = $2\pi\sqrt{\dfrac{R}{2g}}$ = 59.8 min

$\rightarrow$ When pendulum is hanging in lift which is accelerating then

a) If lift move upward with acceleration 'a' then

T = $2\pi\sqrt{\dfrac{l}{g+a}}$

b) If lift move downward with acceleration 'a' then

T = $2\pi\sqrt{\dfrac{l}{g-a}}$

$\rightarrow$ When pendulum is oscillating in a non viscous liquid of density σ where $\sigma < \rho$ then

T = $2\pi\sqrt{\dfrac{l}{1-\dfrac{\sigma}{\rho}g}}$

$\rightarrow$ When pendulum is in a car accelerating with 'a' then

T = $2\pi\sqrt{\dfrac{l}{(g^2+a^2)^{1/2}}}$

$\rightarrow$ Second pendulum

Pendulum of time period 2 sec is called second pendulum

(b) Spring Mass

i. Horizontal system

A small mass is attached to the end of spring of spring constant K then

Restoring force (F) = -kx

Time period (T) = $2\pi\sqrt{\dfrac{m}{k}}$

ii. Vertical system

A small mass is attached to the end of spring which can oscillate vertically then

Time period (T)

$\rightarrow$ If mass of spring is also calculated then

T = $2\pi\sqrt{\dfrac{m+\dfrac{m_s}{3}}{k}}$

$\rightarrow$ If two mass m_1 and m_2 are connected at either end of spring then.

Time period (T) = $2\pi\sqrt{\frac{\mu}{k}}$

Where

$\mu = \frac{m_1 m_2}{m_1 + m_2}$ is called reduced mass

$\rightarrow$ If a spring of spring constant K is divided in 'n' parts and each is connected with mass 'm' then

Time period (T) = $2\pi\sqrt{\frac{m}{nk}}$

$\rightarrow$ if a mass m is connected with two springs placed in parallel of spring constant k_1 and k_2 then

Time period (T) = $2\pi\sqrt{\frac{m}{k_1 + k_2}}$

$\rightarrow$ If a mass 'm' is connected by two springs of spring constant k_1 and k_2 placed in series then

Time period (T) = $2\pi\sqrt{\frac{m}{keq}}$

Where, $K_{eq} = \frac{k_1 k_2}{k_1 + k_2}$

(c) Compound pendulum

Any rigid body if suspended then it is capable to swing in a vertical plane about axis passing through it then it is called compound pendulum.

Time period (T) = $2\pi\sqrt{\frac{I}{mgd}}$

I = moment of inertia

d = distance of CG from point of suspension

(d) Torsional pendulum

Rigid body is capable to show angular simple harmonic motion about axis passing through body then it is called tensional pendulum.

Time period (T) = $2\pi\sqrt{\frac{I}{C}}$

C = Torsional rigidity

Damped harmonic oscillator

The harmonic oscillator in which energy is used to do the work against friction air resistance so that energy decreases is called damped harmonic oscillator.

Damping force is directly proportional to velocity so F = -kx -bv

Angular velocity (ω) = $\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$