Permutations and Combinations

Permutation:-

The arrangements of a number of objects taken some or all of them at a time are called permutations. The total number of permutations of n distinct things taking r ($1 \le r \le n$) at a time is denoted by ^nP_r or by P(n.r).

The number of permutations of n distinct objects taken r at a time is given by

$$^n\mathrm{P}_r \text{ or } \mathrm{P(n,r)} = \frac{n!}{(n-r)!} \text{ where } r \le n$$

Factorials

n! = n . (n -1). (n – 2) 3.2.1
5! = 1. 2. 3. 4. 5 = 120
6! = 1. 2. 3. 4. 5. 6 = 720
0! = 1
n! = n(n -1) !
10! = 10 . 9!
1 ! = 1

Note:-

i. $^n\mathrm{P}_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$
ii. $^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!\ [\because 0! = 1]$
iii. $^n\mathrm{P}_1 = \frac{n!}{(n-1)!} = \frac{n \cdot (n-1)!}{(n-1)!} = n[n! = n(n-1)!]$
iv. $^{n-1}P_r + r \cdot ^{n-1}P_{r-1} =^n P_r$

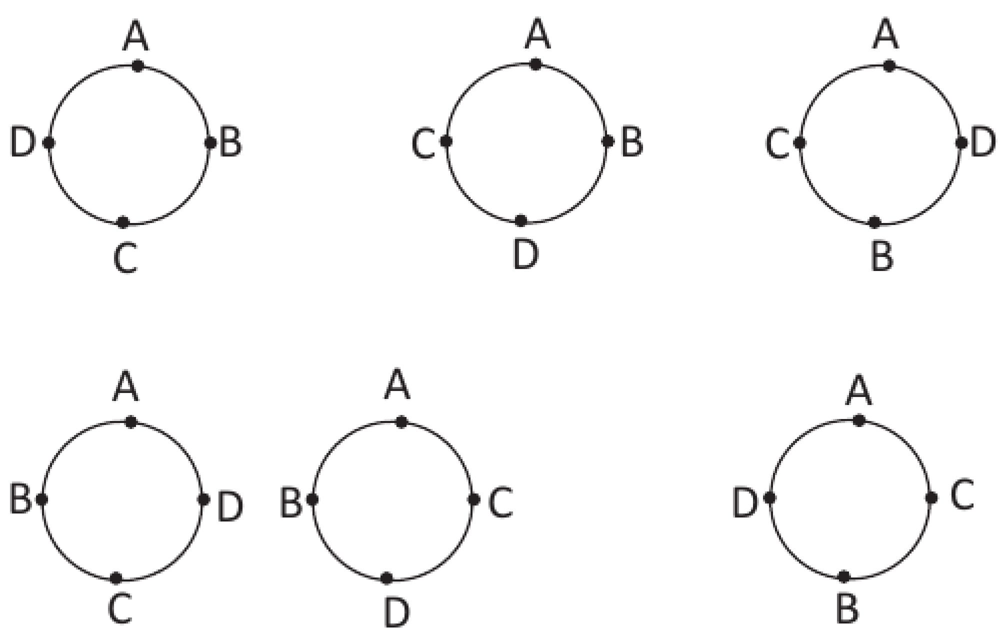
Note:

The number of permutations of n objects taken all at a time when p objects are of one kind, q objects are of second kind and r objects are of third kind is

$$N = \frac{n!}{p!q!r!}$$

Circular permutations

Circular permutation of four letters A, B, C, D are shown below:



4 objects can be arranged in (4-1)! = 3! Ways
As the cyclic orders ABCD, BCDA, CDAB, DABC are same

Note:

Number of permutations of n distinct objects taken r at a time when the repetition of objects is allowed is n^r.

Combinations

The selections (groups) of a number of things taken some or all of them at a time are called combinations. In combination, the order is not considered. The total number of combinations of n distinct things taking r ($1 \le r \le n$) at a time is denoted by ^nC_r or by C (n, r).

- The number of combination of n distinct objects taken r at a time is given by

  $$^nC_r = \frac{n!}{(n-r)!r!}$$

Properties:

- $^nC_0 = ^n C_n = 1$
- $^nC_1 = ^n C_{n-1} = n$
- $^nC_r = ^n C_s \Rightarrow r = s \text{ or } r + s = n$
- $^nC_r +^n C_{r-1} =^{n+1} C_r$
- $\dfrac{^nC_r}{^nC_{r-1}} = \dfrac{n-r+1}{r}$
- $^nC_0 +^n C_1 +^n C_2 + \cdots\cdots +^n C_n = 2^n$
  - Number of ways of selecting one or more objects out of n objects is $(2^n - 1)$.
  - When n is even:
    The greatest value of ^nC_r is $^nC_{n/2}$. [put r = n/2]
  - When n is odd:
    The greatest value of ^nC_r is at $r = \left(\dfrac{n-1}{2}\right)$ or $r = \left(\dfrac{n+1}{2}\right)$
  - The combination of n objects taking at a time if q(
  - Combination of n objects taking r at a time if q (< r) particular objects are always included is C(n-q, r - q).

Relationship between Permutation and Combination:-

Permutation is the arrangement of objects in order and combination is the selection of objects without order.

$$^{\mathrm{n}}\mathrm{P}_{\mathrm{r}} = \frac{n!}{n-r!}$$

$$^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = \frac{n!}{(n-r)!r!}$$

$$^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} =^{\mathrm{n}} \mathrm{P}_{\mathrm{r}} \cdot \frac{1}{r!}$$

$$^{n}\mathrm{P}_{\mathrm{r}} = \mathrm{r}! \cdot^{\mathrm{n}} \mathrm{C}_{\mathrm{r}}$$

i. The number of combinations of n distinct objects taken r at a time in which p particular things always occur is $^{n-p}C_{r-p}$.
ii. The number of combinations of n distinct objects taken r at a time in which p particular things never occur is $^{n-p}C_r$.

De-arrangements

If n things are arranged in a row, the number of ways in which they can be de-arranged so that none of them occupies its original place is

$$n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + - - - - - - + (-1)^n \frac{1}{n!}\right)$$