One Dimension Kinematics

Introduction

Motion and rest

Motion: A body changes its position with time with respect to objects of its surrounding is said to be in motion.

Rest: A body does not change its position with time with respect to objects of its surrounding is said to be in rest.

Motion along a straight line (one dimension)

Distance: The actual path followed by body is called distance. It is a scalar quantity.

Displacement: The shortest distance between initial and final position of body is called displacement. It is a vector quantity.

Hence, distance ≥ displacement.

Speed and Velocity

Speed: Rate of change in distance is called speed.

The instantaneous speed $(v) = \frac{\lim}{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

Average speed $= \frac{\text{Total distace travelled}}{\text{total time}} = \frac{\text{s}}{\text{t}}$

Velocity: The rate of change in displacement of a body is called velocity. It is a vector quantity of unit m/s.

Instantaneous velocity $(\vec{v}) = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta \text{s}}}{\Delta t} = \frac{\overrightarrow{ds}}{dt}$

Average velocity $\overrightarrow{(\vec{v})} = \frac{\text{Total displacement}}{\text{total time}} = \frac{\vec{S}}{t}$

Acceleration

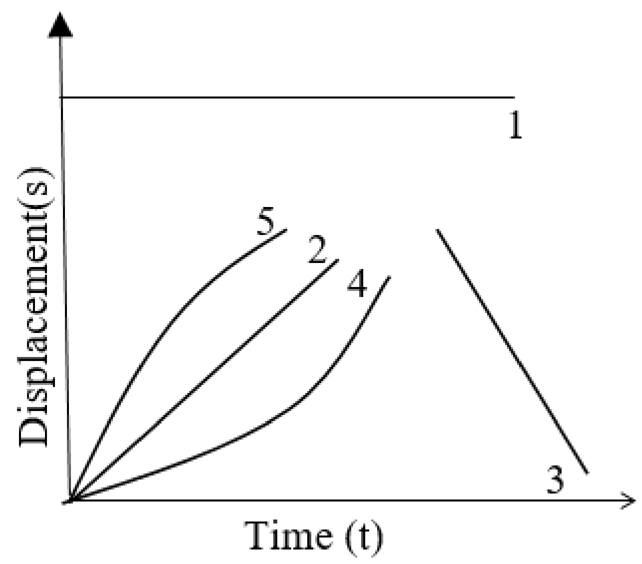
The rate of change in velocity of a body is called acceleration. If is a vector quantity of unit m/s^2.

If $\overrightarrow{\Delta v}$ be the total change in velocity in time Δt then,

Average acceleration $(\vec{a}) = \frac{\overrightarrow{\Delta v}}{\Delta t}$

Motion Time Graph

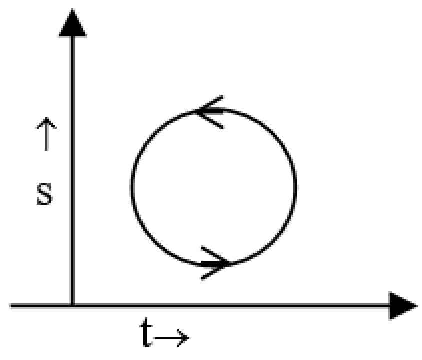
Displacement time graph



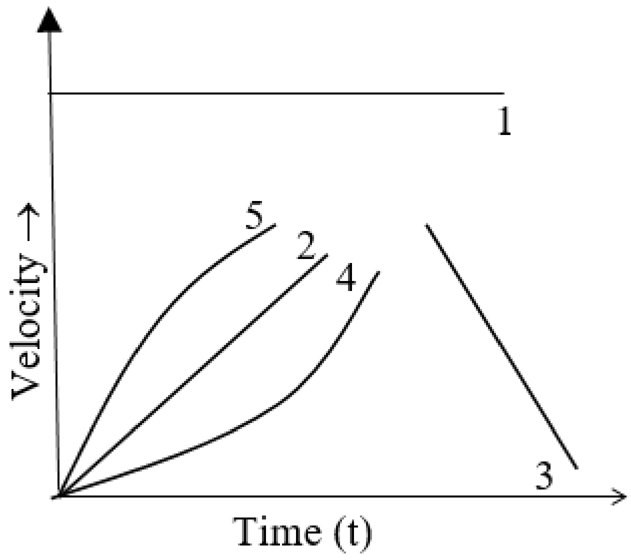
When displacement of body is ploted along *y* axis and time along *x* axis then graph is displacement time graph. It has following characters:

- The diplacement time graph is parallel to time axis then velocity of body is zero as in (1).
- The displacement time graph is a straight line as shown in fig (2) and (3) then the velocity of body is constant.
- When displacement time graph is a curve in which slope increases then velocity of body is increasing is shown in (4) and the distance time graph is curve in which slope decreases then velocity of body is decreasing as in (5).

- Two displacement at a time is impossible.



Velocity time graph



When velocity of body is plotted along *y* axis and time along *x* axis then graph is called velocity time graph. The velocity time graph has following characters.

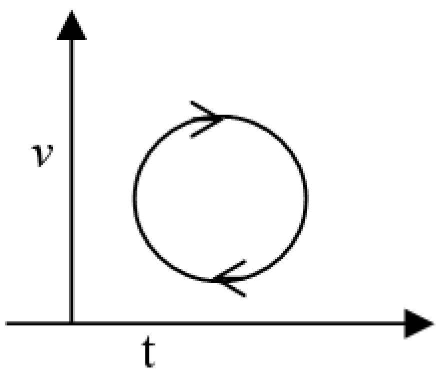
- If velocity time graph is a straight line parallel to time axis then velocity is constant and acceleration is zero as in (1).
- If velocity time graph is a oblique straight line with positive slope then body is uniformly accelerating as shown in (2) and velocity time graph is a oblique straight line with negative slope then body is uniformly decelerating as shown in (3).
- If velocity lime graph is a curve with increasing slope then body is accelerating as shown in curve (4) and velocity time graph is a curve with decreasing slope then body is decelerating as shown in curve (5).
- The area under curve in velocity time graph give the displacement of body.

$$\therefore \vec{v} = \frac{\overrightarrow{\mathrm{ds}}}{\mathrm{dt}}$$

$$\Rightarrow \overrightarrow{\mathrm{ds}} = \vec{v}\mathrm{dt}$$

$\therefore$ Displacement $(\mathrm{s}) = \int_{t_1}^{t_2} v dt$

- Velocity time graph is a curve as in fig. is not possible which give two velocity at a time.



Equation of 1D motion

A body moving with initial velocity 'u' accelerate uniformly at 'a' then final velocity become '*V*, the equations of motion are

- $v = \mathrm{u} + \mathrm{at}$

- $\mathrm{s}=\mathrm{ut}+\frac{1}{2}\mathrm{at}^2$
- $v^2=\mathrm{u}^2+2\mathrm{as}$
- $\mathrm{s}_{\mathrm{nth}}=\mathrm{u}+(2\mathrm{n}-1)\mathrm{a}/2$

Motion under gravity:

- $v=\mathrm{u}+\mathrm{gt}$
- $\mathrm{h}=\mathrm{ut}+\frac{1}{2}\mathrm{gt}^2$
- $\mathrm{v}^2=\mathrm{u}^2+2\mathrm{gh}$
- $\mathrm{h}_{\mathrm{nth}}=\mathrm{u}+\frac{\mathrm{g}}{2}(2\mathrm{n}-1)$

When body is projected against gravity then equations of motion changes as

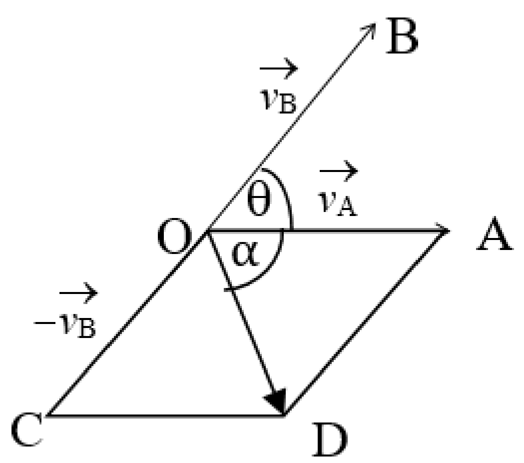
- $v=\mathrm{u}-\mathrm{gt}$
- $\mathrm{h}=\mathrm{ut}-\frac{1}{2}\mathrm{gt}^2$
- $\mathrm{v}^2=\mathrm{u}^2-2\mathrm{gh}$
- $\mathrm{h}_{\mathrm{nth}}=\mathrm{u}-\frac{\mathrm{g}}{2}(2\mathrm{n}-1)$

Relative Velocity

When two bodies A and B are moving with velocity $\vec{v}_A$ and $\vec{v}_B$ at an angle θ then velocity of A relative to B is writen as

$\overrightarrow{v_{\mathrm{AB}}}=\overrightarrow{v_{\mathrm{A}}}-\overrightarrow{v_{\mathrm{B}}}=\overrightarrow{v_A}+(-\overrightarrow{v_B})$

Magnitude of $\vec{v}_{AB}$ is



$v_{\mathrm{AB}}=\sqrt{v^2\mathrm{A}-2v_{\mathrm{A}}v_{\mathrm{B}}\cos\theta+v_{\mathrm{B}}^2}$

Direction of $\vec{v}_{AB}$ is

Let α be angle of relative velocity with $\vec{v}_A$, then

$\tan\alpha=\frac{v_{\mathrm{B}}\sin\theta}{v_{\mathrm{A}}-v_{\mathrm{B}}\cos\theta}$

Some tips

- Displacement of a body can be positive, zero and negative.
- Displacement of a body between two position has unique value.
- Displacement of body can never be distance travelled by body.
- For body moving with uniform speed, the distance travelled at the end of 1st, 2nd, 3rd, --------- second will be in the ratio of 1:2:3.

- A particle travel a distance S_1, S_2, S_3 etc with speed v_1, v_2, v_3 respectively then
  Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}} = \dfrac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3} = \dfrac{s_1 + s_2 + s_3}{\dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \dfrac{s_3}{v_3}}$

  For only two distance,

  - If $s_1 = s_2 = s$, then average speed $= \dfrac{s + s}{\dfrac{s}{v_1} + \dfrac{s}{v_2}} = \dfrac{2v_1v_2}{v_1 + v_2}$

    $\therefore$ Average speed is equal to harmonic mean speed

  - If $t_1 = t_2 = t$, then average speed $= \dfrac{v_1t + v_2t}{t + t} = \dfrac{v_1 + v_2}{2}$
    $\therefore$ Average speed is equal to arithmetic mean speed.

- Velocity of body can be zero, negative and positive.

- Velocity of body can never be greater than speed.
- Acceleration of a body can be zero, negative and positive.

- Acceleration of a body is positive means velocity is increasing and acceleration is negative mean velocity is decreasing.
- When a body start from rest uniformly accelerate then velocity after successive seconds will be 1:2:3:4... and distance travelled will be 1:3:5:7:.........

- Total distance traveled after first, 2^{nd}, 3^{rd} ...... second by uniformly accelerating body will be 1:4:9......

- When a body is moving with variable acceleration then the average acceleration is the ratio of total change in velocity and total time taken.

  If a be the acceleration for time t_1 and a_2 be the acceleration for time t_2 then

  Average acceleration: $(a_{av}) = \dfrac{a_1t_1 + a_2t_2}{t_1 + t_2}$
- The time taken to reach successive points at equal distance will be $1 : \sqrt{2} : \sqrt{3}$

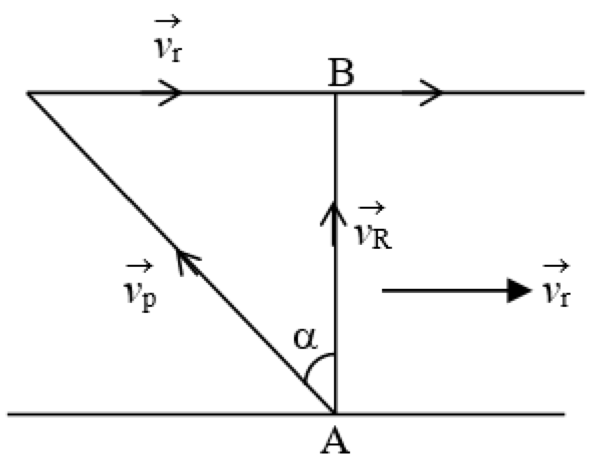
- The time taken to cover successive equal distance from starting point will be in the ratio of $1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

  When a body is released along inclined plane of inclination q of length '/and height 'h' then,

  - Velocity at bottom of plane is
    $v = \sqrt{2gh} = \sqrt{2gl\sin\theta}$
  - Time to reach at bottom of plane is
    $t = \sqrt{\dfrac{2l}{g\sin\theta}} = \sqrt{\dfrac{2h}{g}} \times \dfrac{1}{\sin\theta}$

  - For plane of same height and difference inclination time is in the ratio of $\text{cosec}\theta_1 : \text{cosec}\theta_2$.
  - For plane of same length and different inclination time is in the ratio of
    $\sqrt{\text{cosec}\theta_1} : \sqrt{\text{cosec}\theta_2} : \sqrt{\text{cosec}\theta_3}$

Crossing the river



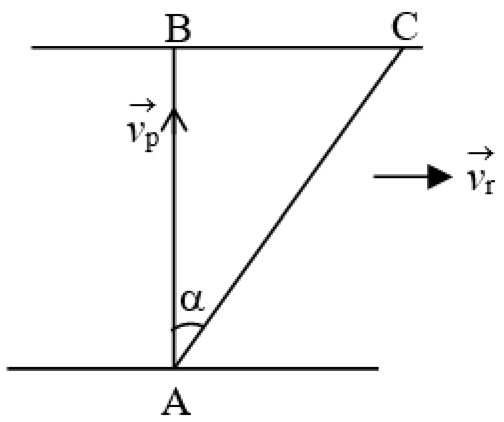
A river of width AB = d flowing with $\vec{v}_r$ in which a person wish to cross the river through shortest path AB then person must swim at an angle a with AB so that resultant of $\vec{v}_p$ and $\vec{v}_r$ lies along AB.

- For direction with AB,

  $\sin\alpha = \dfrac{v_r}{v_p}$

- For time,
$$t = \frac{AB}{v_R} = \frac{d}{\sqrt{v_p^2 - v_r^2}}$$

Person wish to cross the river in minimum time then time will be least if component of $\vec{p}$ along AB is maximum i.e. swimming of along AB.



- $\text{Time}(t) = \dfrac{AB}{v_p \cos\theta}$

  For minimum time, $\cos\theta = 1$

  $\therefore$ minimum time $(t) = \dfrac{AB}{v_p}$
- While swimming along AB, reaches at C in opposite bank so,

$$\tan\alpha = \frac{v_r}{v_p} = \frac{BC}{AB}$$

$$BC = \left(\frac{v_r}{v_p}\right) AB$$