Unit, Dimension and Error Analysis

Measurement

Physical quantities

Different quantities needed to describe the physical phenomenon or object are called physical quantities Examples: speed, length, mass, density, etc.

The physical quantities are divided in two groups.

1. **Fundamental quantities:** Those physical quantities which are independent to any other physical quantities. Eg. Mass, length, time etc.
2. **Derived quantities:** Those physical quantities which depend on other physical quantities and obtained by multiplying and dividing the fundamental quantities are called derived quantities. Eg. Density, velocity, acceleration, force etc.

Units

The physical quantities are measured by comparing with some standard measurement of same kinds are called units. The units are divided in two groups.

1. Fundamental units: Units of fundamental quantities are called fundamental unit. These are independent to any other units.
2. Derived unit: Units of derived quantities are called derived unit. These units are obtained by multiplying and dividing the fundamental units.

System of measurement:

1. **CGS system:** The system of measurement in which 3 fundamental quantities mass, length and time are measured in g, cm and s respectively. All other derived quantities are also measured in terms of these units of measurement.
2. **MKS system:** The system of measurement in which 3 fundamental quantities mass, length and time are measured in kg, m, s respectively. All other derived quantities are measured in terms of these units of measurement is called MKS system.
3. **FPS system:** the system of measurement in which 3 fundamental quantities mass, length and time are measured in pound (*lb*), foot (ft) ad second (s) respectively. All other derived quantities are also measured in terms of these units of measurement.
4. **International system of units (SI):** SI is an abbreviation "Le systeme International d' unites' which is French and equivalent of international system of units. It is used widely throughout the world in which seven different quantities are introduced as fundamental quantities and their units as fundamental units.

	Quantity	Unit	Symbol
1.	Mass	Kilogram	kg
2.	Length	Meter	m
3.	Time	Second	S
4.	Temperature	Kelvin	K
5.	Electric Current	Ampere	A
6.	Luminous intensity	Candela	cd
7.	Amount of substance	Mole	mol.

Two more quantities are introduced as supplementary quantities and their units as supplementary unit.

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2.	Solid angle	Steradian	Sr

Prefixes of power ten:

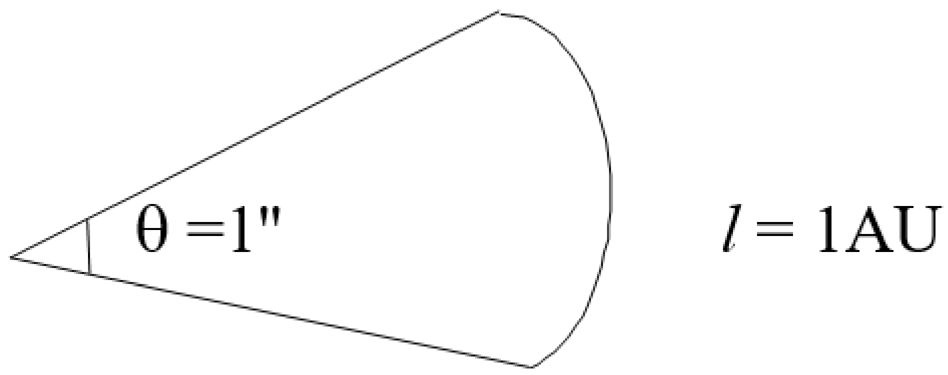
Prefix	Abbreviation	Power of ten
Atto	a	10^{-18}
Femto	f	10^{-15}
Pico	p	10^{-12}
Nano	n	10^{-9}
Micro	m	10^{-6}
Milli	m	10^{-3}
Centi	c	10^{-2}
Deci	d	10^{-1}
Kilo	k	10^{3}
Mega	M	10^{6}
Giga	G	10^{9}
Tera	T	10^{12}
Peta	P	10^{15}
Exa	E	10^{18}

Some useful practical units:

1. Astronomical unit (AU): Average distance between centre of earth and centre of sun.

$\therefore 1\text{AU} = 1.496 \times 10^{11} = 1.5 \times 10^{11}\text{m}$

2. Par Sec used to measure long distance and represents a parallactic second. One par sec is the distance at which an arc of 1 AU long subtends an angle of 1".



$\therefore$ 1 Par sec $= 3.1 \times 10^{16}$ m

3. Light year (ly) –
Distance traveled by light is vacuum in one year.
$\therefore$ 1 ly $= 9.46 \times 10^{15}$ m

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1 yard = 91.44 cm
1 mile = 1.609×10^3 m
1 nautical mile = 1.852 ´ 10^3 m
1 angstrom (1Å) = 10^{-10} m
1 Fermi = 1 femtometre = 10^{-15} m

For area:

1 barn = 10^{-28} m^2

1 acre = 4047 m^2

1 hectare = 10^4 m^2

For mass:

1 tonne or metric ton = 1000 kg

1 quintal = 100 kg

1 slug = 14.57 kg

1 *lb* = 0.4536 kg

1 chandra shekhar limit (CSL) = 1.4 times mass of sun

1 amu (atomic mass Unit)=1.67´10^{-27} kg.

For time:

1 shake = 10^{-8} sec

1 solar year = 365.25 days

For pressure:

1 bar = 1 atmospheric pressure = 10^5 N/m^2

1 torr = 1mm of Hg pressure

1 atmospheric pressure = 760 mm of Hg

1 bar = 760 torr

Dimension

The power of fundamental quantity involved in any physical quantity is called dimension of that physical quantity. The representation of physical quantity in terms of power of fundamental quantities involved in it is called dimensional formula of that physical quantity. Three fundamental quantities mass, length, and time are represented by [M], [L] and [T] respectively. All other derived quantities are also expressed in terms of these representations.

E.g. Force = ma = kgm/s^2 = [MLT^{-2}]

The dimension of mass is 1, length is 1 and time is −2 in force and the representation [MLT^{-2}] is called dimensional formula of force.

Dimensional formula of some quantities:

S.N.	Physical quantity	Formula	Dimen-sional formula	Unit
1.	Density	$\frac{\text{mass}}{\text{volume}}$	[$ML^{-3}T^{\circ}$]	Kgm^{-3}
2.	Specific gravity	$\frac{\text{Density of body}}{\text{Density of water at } 4^{\circ}\text{ C}}$	[$M^{\circ}L^{\circ}T^{\circ}$]	-
3.	Linear momentum	mv	[MLT^{-1}]	Kgm s^{-1}
4.	Impulse	F × t	[MLT^{-1}]	Ns
5.	Pressure	F/A	[$ML^{-1}T^{-2}$]	Nm^{-2}

7.	Work	F × d	$[ML^2T^{-2}]$	Kgm^2/s^2
8.	Moment of force	F × r	$[ML^2T^{-2}]$	Nm
9.	Power	$\frac{W}{t}$	$[ML^2T^{-3}]$	w
10.	Surface tension	$\frac{F}{l}$	$[ML^oT^{-2}]$	Nm^{-1}
11.	Surface energy	Energy	$[ML^2T^{-2}]$	J
12.	Force constant	$K = \frac{F}{x}$	$[MT^{-2}]$	Nm^{-1}
13.	Thrust	Force	$[MLT^{-2}]$	N
14.	Stress	$\frac{F}{A}$	$[ML^{-1}T^{-2}]$	N/m^2
15.	Strain	$\frac{e}{L}$	$[M^oL^oT^o]$	-
16.	Modulus of elasticity	$\frac{\text{stress}}{\text{strain}}$	$[ML^{-1}T^{-2}]$	N/m^2
17.	Radius of gyration	Length	$[M^oL^1T^o]$	m
18.	Moment of inertia	Mr^2	$[ML^2T^o]$	Kgm^2
19.	Angle	$\theta = \frac{l}{r}$	$[M^oL^oT^o]$	radian
20.	Angular velocity	$\omega = \frac{\omega}{t}$	$[M^oL^oT^{-1}]$	rad/s
21.	Angular acceleration	$\alpha = \frac{\omega}{t}$	$[M^oL^oT^{-2}]$	rad/s^2
22.	Angular momentum	$L = I\omega$	$[ML^2T^{-1}]$	Kgm^2/s
23.	Torque	$\tau = I\alpha$	$[ML^2T^{-2}]$	Nm
24.	Frequency	$f = \frac{1}{T}$	$[M^oL^oT^{-1}]$	s^{-1} or Hertz (Hz)
25.	Velocity gradient	$\frac{dv}{dx}$	$[M^oL^oT^{-1}]$	s^{-1}
26.	Rate of flow	$\frac{v}{t}$	$[M^oL^3T^{-1}]$	m^3/s
27.	Planck's constant	$h = \frac{E}{f}$	$[ML^2T^{-1}]$	Js

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29.	Specific heat capacity	$S=\frac{Q}{m\Delta\theta}$	$[M^0L^2T^{-2}K^{-1}]$	J/kg-k
30.	Specific latent heat	$L=\frac{Q}{m}$	$[M^0L^2T^{-2}]$	J/kg
31.	Thermal conductivity	$\mathrm{K}=\frac{\mathrm{Q}}{\mathrm{t}\cdot\mathrm{A}\left(\frac{\mathrm{d}\theta}{\mathrm{dt}}\right)}$	$[MLT^{-3}K^{-1}]$	$wm^{-1}K^{-1}$
32.	Universal gas constant	$R=\frac{PV}{nT}$	$[ML^2T^{-2}K^{-1}mol^{-1}]$	$Jmol^{-1}K^{-1}$
33.	Boltzman constant	$k=\frac{R}{N_A}$	$[ML^2T^{-2}K^{-1}]$	JK^{-1}
34.	Entropy	$S=\frac{dQ}{T}$	$[ML^2T^{-2}K^{-1}]$	JK^{-1}
35.	Charge	$Q=It$	[AT]	C
36.	Electric dipole moment	$q\times 2l$	[ALT]	Cm
37.	Current density	$J=I/A$	$[AL^{-2}]$	Am^{-2}
38.	Permittivity	$\left(\varepsilon_0=\frac{\mathrm{Cd}}{\mathrm{A}}\right)$	$[M^{-1}L^{-3}T^4A^2]$	Fm^{-1}
39.	Capacitance	$C=4\pi\varepsilon_0 r$	$[M^{-1}L^{-2}T^4A^2]$	F
40.	Resistance	$R=\frac{V}{I}$	$[ML^2T^{-3}A^{-2}]$	W
41.	Magnetic field induction	$B=\frac{F}{Il}$	$[MT^{-2}A^{-1}]$	T
42.	Magnetic flux	$\phi=BA$	$[ML^2T^{-2}A^{-1}]$	Wb
43.	Permeability	$\mu=\frac{2BR}{I}$	$[MLT^{-2}A^{-2}]$	Hm^{-1}
44.	Inductance	$L=\frac{E}{\frac{dI}{dt}}$	$[ML^2T^{-2}A^{-2}]$	H
45.	Resistance×Capacitance	RC	[T]	S
46.	$\frac{\text{Inductance}}{\text{Resistance}}$	$\frac{L}{R}$	[T]	S
47.	$\sqrt{\text{Inductance}\times\text{Capacitance}}$	$\sqrt{LC}$	[T]	S

Any physical equation will be correct only when the dimensions of all terms on left side become equal to the dimensions of all terms on right side. The physical equation which is dimensionally correct may be physically incorrect but the equation which is dimensionally in correct is always incorrect.

- *To convert the physical quantity from one system to another system:*
  Dimensional method is used to convert physical quantity from one system to another system if the unit of measurement in those systems are known. It is based on the value of physical quantity remain same whatever be the system of measurement. i.e.

  $n_1 u_1 = n_2 u_2$

  Where n_1 and n_2 are numerical values in two system and u_1 and u_2 are the units in two system

  It can be written as

  $$n_1 \left[M_1^a L_1^b T_1^c\right] = n_2 \left[M_2^a L_2^b T_2^c\right]$$
  $$n_1 = n_2 \left[\frac{M_2}{m_1}\right]^a \left[\frac{L_2}{L_1}\right]^b \left[\frac{T_2}{T_1}\right]^c$$

- *To derive the physical equation:*

  Dimensional method is used to derive the physical equation if the dependence of a physical quantity on other quantities is known.

  Any quantity 'X' depends on other quantities P, Q and R then

  $X \propto P^a Q^b R^c$

  Or, $X = k P^a Q^b R^c$, where k is constant without dimension

Limitations of dimensional analysis

- It does not give any information about numerical constants used in physical equation.
- Dimensional method can't be used to derive the physical equation which depends on more than three quantities.
- Dimensional method can't be used to derive the physical equation having trigonometrical function, logarithmic function, exponential function etc.
- Dimensional method can't be used to derive the physical equation containing more than two terms in one side of equation
- Dimensional method does not give any information whether the quantity is vector or scalar.

Principle of homogeneity of dimension:

The dimensional equation of each term in a physical equation must be equal if the physical equation is correct. In a physical equation A = B + C, the dimensional equation of A = Dimensional equation of B = Dimensional equation of C.

Error Analysis

Least count (LC):

The smallest measurement that can be taken by an instrument is called least count.

Eg. least count of scale graduates in mm is 1 mm.

Absolute error:

The different between true value and measured value is called absolute error:

- **Error in sum of quantities:**
  Let x = a + b be a relation in which Δa be absolute error in a and Δb be the absolute error in b then maximum error in x is

  $$\Delta x = \pm(\Delta a + \Delta b)$$

- **Error in difference of quantities:**
  When x = a - b be a relation in which Δa & Δb be the absolute error while measuring a and b then the maximum error in x is

  $$\Delta x = \pm(\Delta a + \Delta b)$$

- **Error in product of quantities:**

  When x = a × b be a relation in which Δa and Δb be absolute error while measuring a and b then maximum fractional error in measuring x is

  $$\frac{\Delta \mathbf{x}}{\mathbf{x}} = \pm\left(\frac{\Delta \mathbf{a}}{\mathbf{a}} + \frac{\Delta \mathbf{b}}{\mathbf{b}}\right)$$

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Let x = $\frac{a}{b}$ be a physical relationship in which Δa & Δb be the error in measuring a and b & Δx be the error which measuring x then.

Maximum fractional error is

$$\frac{\Delta \mathbf{x}}{\mathbf{x}} = \pm\left(\frac{\Delta \mathbf{a}}{\mathbf{a}} + \frac{\Delta \mathbf{b}}{\mathbf{b}}\right)$$

- **Error in quantity raised to some power :**

Let x = $\frac{a^n}{b^m}$ be a relationship in which Da & Db be the error in measuring a and b & Dx be the error in measuring x then

Maximum fractional error is

$$\frac{\Delta \mathbf{x}}{\mathbf{x}} = \pm\left(n\frac{\Delta \mathbf{a}}{\mathbf{a}} + m\frac{\Delta \mathbf{b}}{\mathbf{b}}\right)$$