Refraction at Spherical Surface & Prism

Refraction at single spherical surface

- **Sign convention**
  - All the distance are measured form the pole of the refracting surface.
  - Distance measured in the direction of incident light in taken positive and opposite to the direction of incident light is taken negative.
  - Height above the principle axis is positive and below principle axis is negative.

- **Object is in rarer medium**

  $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

  Where,
  μ_1 = R.I of rarer medium
  μ_2 = R.I of denser medium
  R = radius of curvature

  - **Transverse magnification**

    $m = I/O = v/u$

- **Object in denser medium**

  $$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Lenses

A lens is a portion of a transparent refracting medium bounded by two spherical surfaces or one spherical and other plane surface. Spherical lens is two types :

i. Convex or convergent lens
ii. Concave or divergent lens

Convex lens are three types

a. Biconvex
b. Planoconvex
c. Concavo-convex lens

Concave lens are three types

a. Biconcave
b. Planoconcave
c. Convexo concave

Lens formula

The relationship between objects distance, image distance & focal length of lens is lens formula.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Linear magnification

The ratio of height of image & height of object is called linear magnification.

$$m = I/O = v/u = \frac{v-f}{f} = \frac{f}{u-f}$$

Power of lens

The ability of lens to converge or diverge the parallel beam of light is called power of lens. It is equal to reciprocal of focal length in meter.

$$\therefore P=\frac{1}{f(\text{in m})}$$

It is measured in dioptre (D)

Newton's formula of lens

$$f=\sqrt{x_1 x_2}$$

x_1 & x_2 are the distance of object & real image from focus.

Combined focal length

When large number of lens of focal length $f_1, f_2, \ldots, f_n$ are placed in contact then the combined focal length f is:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\cdots+\frac{1}{f_n}$$

If two lens are placed at a distance d having focal length f_1 and f_2 then combinedf ocal length is:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

Focal length of convex lens by displacement method

Keeping the object & screen fixed the convex lens can form the real image of object at two position on screen. Then:

$$\mathrm{u}=\frac{D-x}{2}, v=\frac{D+x}{2}, f=\frac{D^2-x^2}{4D}$$
$$\mathrm{m}_1=\frac{I_1}{0}=\frac{v}{u}, m_2=\frac{I_2}{0}=\frac{u}{v}$$
$$\mathrm{m}_1 \times \mathrm{m}_2=1\&\mathrm{o}=\sqrt{I_1 \times I_2}$$
$$\therefore \mathrm{m}_1-\mathrm{m}_2=\mathrm{x}/\mathrm{f}$$

Silvering one surface of lens

If a lens is silvered on one of the surface of it then it will behave like a combination of two lens and a mirror. Therefore its focal length is:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_m}+\frac{1}{f_1}=\frac{2}{f_1}+\frac{1}{f_m}$$

- When one surface of **equiconvex** lens of radius of curvature R is silvered then:

$$\begin{aligned}\frac{1}{\mathrm{F}}&=\frac{2}{\mathrm{f}_1}+\frac{1}{\mathrm{f}_\mathrm{m}}=2(\mu-1)\times\frac{2}{\mathrm{R}}+\frac{2}{\mathrm{R}}\\&=(2\mu-1)\frac{2}{\mathrm{R}}\\ \therefore \mathrm{F}&=\frac{\mathrm{F}}{2(2\mu-1)}\text{ i.e. act as concave mirror}\end{aligned}$$

- When one surface of **equiconcave** lens of radius of curvature R is silvered then:

$$\frac{1}{\mathrm{F}}=\frac{2}{\mathrm{f}_{\mathrm{I}}}+\frac{1}{\mathrm{f}_{\mathrm{m}}}=2(\mu-1)\times\frac{2}{\mathrm{R}}+\frac{2}{\mathrm{R}}$$
$$=(2\mu-1)\frac{2}{\mathrm{R}}$$
$$\therefore \mathrm{F}=\frac{\mathrm{R}}{2(2\mu-1)} \text{ so act as convex mimor}$$

- When plane surface of **plano convex** lens of radius of curvature R is silvered then:

$$\frac{1}{F}=\frac{2}{f_1}+\frac{1}{f_m}=2(\mu-1)\frac{1}{R}+\frac{1}{\infty}$$
$$\therefore F=\frac{R}{2(\mu-1)} \quad \text{i.e act as concave mirror}$$

- When convex surface of plano convex lens of radius R is silvered then:

$$\frac{1}{F}=\frac{2}{f_1}+\frac{1}{f_m}=2(\mu-1)\frac{1}{R}+\frac{2}{R}=\frac{2\mu}{R}$$
$$\therefore F=\frac{R}{2\mu} \quad \text{i.e act as convex mirror}$$

Lens maker's formula:

The formula relating the focal length of lens with R.I. of medium & radius of curvature of spherical surfaces by which lens is made.

$$\frac{1}{f}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

- When lens of focal lengthy f_a if air is immersed in liquid of refractive index μ_l then focal length in liquid is:

$$\mathrm{f}_{\mathrm{I}}=\frac{(\mu_{\mathrm{g}}-1)}{({}_l\mu_{\mathrm{g}}-1)}\mathrm{f}_{\mathrm{a}}$$

Defects of Images

The imperfection or defects of image are called aberration. Defect due to light is called chromatic aberration and due to spherical surface is called spherical aberration.

Chromatic aberration

The image of a white object formed by a lens is coloured and blurred. The defect is due to different focal length of a lens for different colours of light.
Violet light bend most and red bend least.

So, $f_r > f_v$

The difference in focal length of red and violet color is called longitudinal chromatic aberration.

$\therefore$ Chromatic abberation $= f_r - f_v = \omega \times f$

$\omega =$ dispersive power of material of lens

Achromatism

The combination of two lens which is free form chromatic aberration is called achromatism.

$$\therefore \frac{f_1}{f_2}=\frac{-\omega_1}{\omega_2}$$

Spherical aberration

The defect due to spherical shape of lens such that different part of lens are different. So focal length of central part of maximum & peripheral part is minimum. So parallel rays do not pass through a single focus and the image is blurred. The paraxial rays focus farthest and marginal rays are focused nearest.

$\therefore$ Longitudinal spherical aberration $= f_p - f_m$

Methods to reduce spherical aberration in lens:

i. reducing the parts of light entering
ii. using lens of large focal length
iii. using plano-convex lens

Scattering of Light

When light falls on the particle in the atmosphere whose dimension are of the order of wave length of light then it is splitted up into it's component. This phenomenon is called scattering of light. Intensity of scattered light with wave length λ is given by Rayleigh Law as:

$$I \propto \frac{1}{\lambda^4}$$

Prism

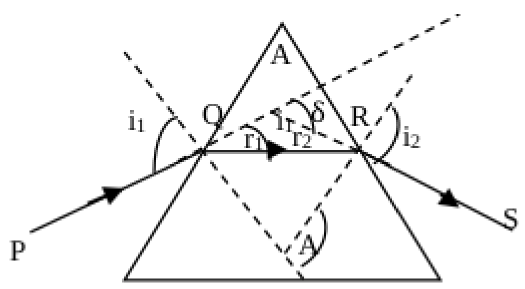
When light ray incident at an angle i_1 at one of the refracting face of prism then the angle of refraction at 1^{st} & 2^{nd} face arer_1 and r_2 emergence is i_2

$$A = r_1 + r_2$$

Deviation produced by prism

$$\begin{aligned}\delta &= (i_1 + i_2) - (r_1 + r_2)\\ &= (i_1 + i_2) - A\end{aligned}$$

$$\therefore \delta + A = i_1 + i_2$$



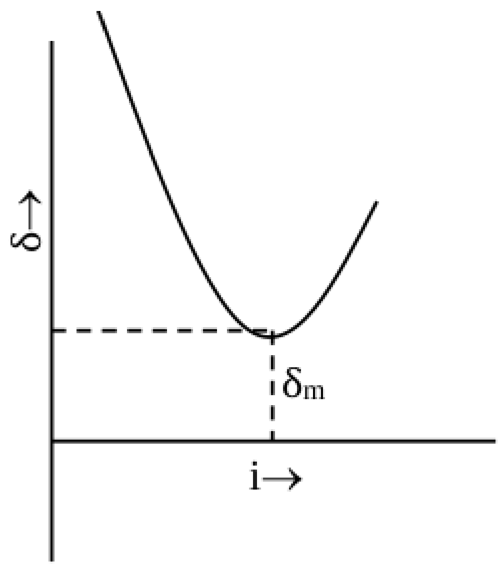
Minimum deviation

On increasing the angle of incidence, at 1st face of prism the angle of emergence decreases & for certain value of angle of incidence the deviation of light through a prism become least is called minimum deviation at which light rays become symmetrical about two refracting face.

At minimum deviation

$$\begin{aligned}&\mathbf{i}_1 = \mathrm{i}_2 = \mathrm{i} \ \& \ \mathrm{r}_1 = \mathrm{r}_2 = \mathrm{r}\\ &\therefore \delta_\mathrm{m} = 2\mathrm{i} - \mathrm{A}\\ &\mathrm{Or, i} = (\mathrm{A} + \delta_\mathrm{m})/2\\ &\& \ \mathrm{r} = \mathrm{A}/2\end{aligned}$$

$\therefore$ refractive index $(\mu) = \dfrac{\sin i}{\sin r} = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$



Deviation due to small angled prism

The prism having refracting angle less than 10 o (A≤10) is called small angled prism.

The deviation due to small angled prism is:

$\delta=(\mu-1)A$

Hence, deviation is independent to angle of incident.

Limiting angle of prism

The greatest refracting angle for a prism is $A=2C$ is called limiting angle of prism.

Where, $C=\text{critical angle}$

If $A>2C$ then light can't emerge from second refracting face.

Dispersion of light

When light is passed through a prism then it split into its' constituent colour is called dispersion. The display of different colour of light is called spectrum.

- **Angular dispersion (θ):**

  The angle between violet and red colour of light in spectrum is called angular dispersion.
  $$\begin{aligned}\text{Angular dispersion}(\theta) &= \delta_v-\delta_r \\ &= (\mu_v-\mu_r)A\end{aligned}$$

- **Dispersive power (ω):**

  Ratio of angular dispersion and deviation for mean colour of light is called dispersive power.
  $$\therefore \text{Dispersive Power}(\omega)=\frac{\theta}{\delta}=\frac{(\mu_v-\mu_r)}{\mu-1}$$

Deviation without dispersion

A single prism dispersion as well as deviate the light. The combination of two prism such that combination is free from dispersion is called dispersion without deviation.

Condition for no dispersion is:

$$\frac{\mathrm{A}}{\mathrm{A}'}=-\left(\frac{\mu_{\mathrm{v}}-\mu_{\mathrm{r}}'}{\mu_{\mathrm{v}}-\mu_{\mathrm{r}}}\right)$$

or, $\omega\delta+\omega'\delta=0$

Net deviation $(\delta')=\delta\left(1-\frac{\omega}{\omega'}\right)$

Dispersion without deviation

When white light passes through a single prism then dispersion as well as deviation takes place. The combination of two prisms which is free from deviation but there is some dispersion is called dispersion without deviation.
Condition for no deviation

$$\frac{\mathrm{A}}{\mathrm{A}'}=-\frac{(\mu'-1)}{\mu-1}$$
Net dispersion $=\delta\left(\omega-\omega'\right)$