Elasticity

Elasticity

It is the property of matter to regain its original configuration on removing the deforming force applied on it called elasticity

- When a body does not gain original configuration on removing the deforming force applied, what ever small it may be is called plastic body.
- Quartz fibre is perfectly elastic and mud is plastic

Restoring force:

- When body is deformed then a force is developed inside body which try to bring the body in original configuration is called restoring force

Deforming force:

- The external force applied on body which try to change the configuration body is called deforming force
  Within elastic limit
  Deforming force = Restoring force

Stress

The restoring force developed on unit area of deformed body is called stress.

$\therefore$ Stress = $\frac{F}{A}$

F = force applied

Strain

The change in dimension of body per unit original dimension is called strain. It is divided as

1. Longitudinal strain :
   Change in length per unit original length
2. Volumetric strain :
   Change in volume per unit original volume.
3. Shear strain :
   Change in position per unit original position

Hooke's Law

With in elastic limit stress is directly proportional to strain

$\therefore$ stress $\propto$ strain

or, $\frac{Stress}{Strain}$ = K

Where K is constant is called modulus of elasticity

It is divided as :

i. Young's modulus of elasticity (Y)

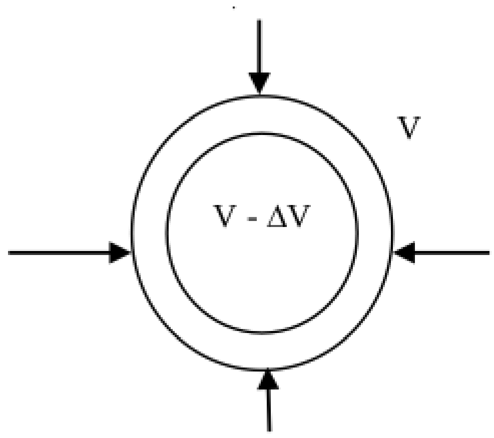
The ratio of normal stress and longitudinal strain is called Young's modulus of elasticity.

$\therefore$ Young's modulus (Y) = (F/A)/(e/L) = FL/Ae

$$\therefore F = \frac{YAe}{L}$$

ii. Bulk modulus (B)

The ratio of normal stress and volumetric strain is called Bulk modulus when force applied on surface area A normally of volume V which decreases by ΔV then



Bulk modulus (B) = $\frac{F/A}{-\frac{\Delta V}{V}} = \frac{PV}{-\Delta V}$

-ve sign indicates volume decreases on increasing pressure

Compressibility (C)

The reciprocal of Bulk modulus is called compressibility

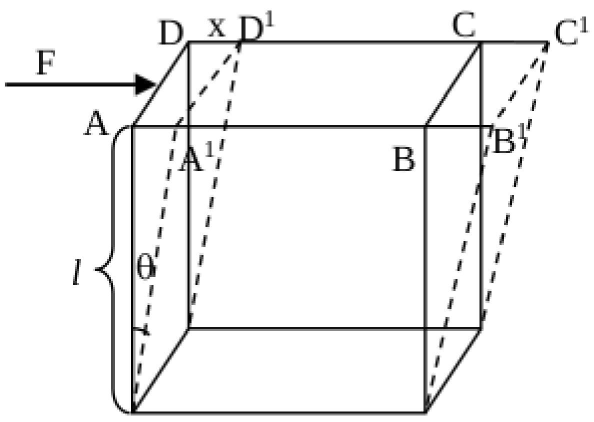
$$C = \frac{1}{B}$$

iii. Modulus of rigidity (η)

The ratio of tangential stress to shear strain is constant is called modulus of rigidity.

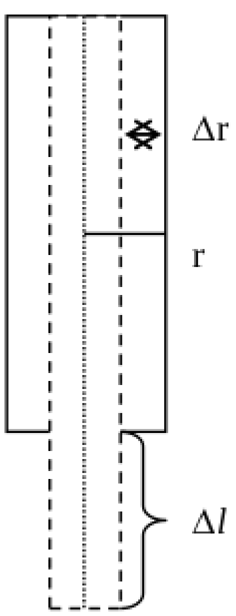
Due to tangential force F applied on force ABCD of area A, which is displaced by x with respect to lower face at a distance l then shear strain (θ) = $\frac{x}{l}$

$\therefore$ Modulus of rigidity (η) = $\frac{F}{A\theta}$



Poission's ratio (σ)

When force is applied along the length of wire then its length increases and diameter decreases.



Lateral strain (β)

The ratio of change in diameter per unit original diameter

Lateral strain $= -\frac{\Delta D}{D} = -\frac{\Delta r}{r}$

-ve sign indicates the diameter decrease.

Longitudinal strain (α)
The ratio of change in length per unit original length.

$\therefore$ longitudinal strain = $\frac{\Delta l}{l}$

The ratio of lateral strain and longitudinal strain is constant is called poission's ratio.

$\therefore \sigma$ = $\frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\frac{-\Delta r}{r}}{\frac{\Delta l}{l}}$

$\rightarrow$ The theoretical value of between -1 and 0.5 while practical value is 0 to 0.5

$\rightarrow$ If value of σ is 0.5 then fractional change in volume is

$\therefore$ Fractional change in volume $\frac{\Delta V}{V}$

= (1 -2σ).$\frac{\Delta l}{l}$

Relationship between elastic constant

$\mathrm{Y} = 3\mathrm{B}(1 - 2\sigma)$

$\mathrm{Y} = 2\mathrm{n}(1 + \sigma)$

$\frac{9}{\mathrm{Y}} = \frac{1}{\mathrm{B}} + \frac{3}{\eta}$ or, $\mathrm{Y} = \frac{9\mathrm{B}\eta}{3\mathrm{B} + \eta}$

$\sigma = \frac{3\mathrm{B} - 2\eta}{6\mathrm{B} + 2\eta}$

Energy stored in a stretched wire

The work done against restoring force is stored as elastic potential energy on a stretched wire

$\therefore$ Energy stored (E) = 1/2 Fe = $\frac{YAe^2}{2L}$

= 1/2 stress $\times$ strain $\times$ volume

$\therefore$ Energy density $\left(\frac{E}{V}\right)$ = 1/2 stress $\times$ strain

The elasticity of matter decreases on increasing temperature but elasticity of invar remain almost constant.

On adding more elastic impurity elasticity increases and on adding less elastic impurity elasticity decreases