Work, Energy and Power

Work

Work is said done on a body when an external force displaces the body in the direction of force.

When a force $\vec{F}$ produce a displacement $\vec{S}$ on a body then

Work done $(W) = \vec{F} \cdot \vec{S}$

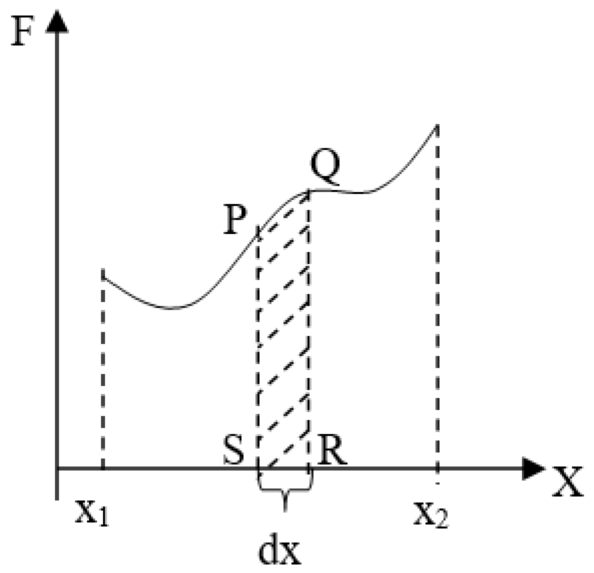
Work done $(W) = FS\cos\theta$

Let $\theta =$ angle between force and displacement.

- If θ = 0° then W = FS i.e. work done by force becomes maximum if force and displacement are in same direction.
- If θ = 90° then W = 0 i.e. no work is done by force if force and displacement are perpendicular to each other.
- If θ = 180° then W = -FS i.e. work done is negative so work is done against force.

Work done from force displacement graph:

When the body moves under the action of varying force then area of graph under force curve between displacement axis gives the work done.



$\therefore$ Small work done $(dW) = F \cdot dx$

Total work done $(W) = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} (\text{Area of strip PQRS })$

Power (P)

The work is done per unit time is called power.

$\therefore$ Power $(P) = \frac{W}{t} = \frac{F \cdot S}{t} = F \cdot v$

When a body moves against a force with uniform velocity then power is $P = \vec{F} \cdot \vec{v}$

The power at any instant of time is

$P = \vec{F} \cdot \vec{v}$ [Instantaneous power]

The practical unit of power is h.p. where 1 hp = 746 W.

Energy

The capacity of a body to do work is called energy. So work is done by the body on the expense of energy. There are various form of energy as mechanical energy, heat energy, light energy etc.

Mechanical energy is divided as

- **Potential energy** (PE):

  The energy possessed by a body by virtue of its position and configuration is called potential energy.

  $\therefore$ Potential energy (U) $= \int_{r_0}^{r} \overrightarrow{F_{ext}} \cdot \overrightarrow{dr}$

Potential energy (U) = $-\int_{r_0}^{r} \overrightarrow{F} \cdot \overrightarrow{dr}$

When $\overrightarrow{F_{ext}}$ = external force,

F = internal force

- **Gravitational potential energy**
  The gravitational potential energy of a body of mass `$\mathrm{m}$' at a height `h' relative to ground is

  Potential energy (PE) = mgh.

  The gravitational potential energy on the surface of earth is

  $$\mathrm{PE} = -\frac{GMm}{R}$$

- **Elastic potential energy:**
  The elastic potential energy stored on the spring of spring constant K when the elongation x is produced is

  $\therefore$ Elastic potential energy (PE) = Kx^2

  A spring is stretched by x_1 and it is further stretched by x_2, then amount of work done to stretch the spring further is

  Work done (W) = $\mathrm{K}\ (\mathrm{x}_1 + \mathrm{x}_2)^2$ $-\mathrm{Kx}_1{}^2$

- **Kinetic energy:**
  Energy possessed by body by virtue of its motion is called kinetic energy.

  The kinetic energy of body of mass `$\mathrm{m}$' moving with velocity `v' is

  KE = $\frac{1}{2}\mathrm{mv}^2 = \frac{p^2}{2m}$

  where, p = momentum

Work energy theorem:

The work done by net force on body is equal to the change in kinetic energy of body is called work energy theorem.

$\therefore$ Work done = Change in KE

Conservation of energy:

Under the action of conservative forces the total mechanical energy of system remains conserved.

$\therefore$ PE + KE = constant

Conservative and non conservative forces

- Conservative force: The force is said conservative if the work done by force is independent to the actual path followed by body, depends on initial and final position of body and work done in round trip is zero. Examples of conservative forces are gravitational force, electrostatic force, elastic force i.e. central force are conservative in nature.

- Non conservative force: The work done by force depends on actual path followed by body, independent to initial and final position of body and work done by force in closed path is not equal to zero then the force is said non conservative force. Examples of non conservative forces are frictional force & viscous force.

Collision

The interaction of a body on another body due to which its momentum or kinetic energy or both changes is called collision. It is divided as

- **Elastic collision:**
  The collision in which total linear momentum of system of colliding bodies as well as the kinetic energy of system is conserved is called elastic collision. The examples of elastic collision are atomic collision, molecular collision etc.
  A body of mass m_1 moving with velocity u_1 collides elastically with another body of mass m_2 moving with velocity u_2 then their velocities changes to v_1 and v_2 respectively, then

  For conservation of linear momentum,

  $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

  For conservation of kinetic energy,

  $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

  Now, $u_1 - u_2 = v_2 - v_1$

  For one dimensional elastic collision the relative velocity of approach is equal to the relative velocity of separation.

Here, $e=-\left(\frac{v_1-v_2}{u_1-u_2}\right)$

Where the ratio of relative velocity after collision and relative velocity before collision is a fixed quantity is called coefficient of restitution. The value of e lies in between 0 to 1 in which value of e is 0 for perfectly inelastic collision and 1 for perfectly elastic collision. The velocities after collision of one dimensional elastic collision are given by

$$v_1=\left(\frac{m_1-m_2}{m_1+m_2}\right)u_1+\left(\frac{2m_2}{m_1+m_2}\right)u_2$$

$$v_2=\left(\frac{2m_1}{m_1+m_2}\right)u_1+\left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$$

- If $m_1=m_2$, then $v_1=u_2$ & $v_2=u_1$ i.e. body exchange their velocities.
- If $u_2=0$, then
  $v_1=\left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$
  $v_2=\left(\frac{2m_1}{m_1+m_2}\right)u_1$
- If $m_2>>m_1$, i.e. a light particle collides with heavy particle at rest then $v_1=-u_1$ & $v_2=0$ i.e. velocity of light particle is reversed keeping the heavy particle at rest.
- If $m_2>>m_1$, then $v_1=u_1$ & $v_2=2u_1$ i.e. a heavy particle collides with light particle at rest then velocity of heavy particle remains same but the velocity of light particle becomes double of velocity of heavy particle.

- **Inelastic collision:**
  The collision during which total linear momentum of system of colliding bodies remains conserved but the kinetic energy of system is not conserved. Here the total kinetic energy may be greater or less than initial kinetic energy.

When a ball is dropped from certain height

A ball is dropped from a height 'h' having coefficient of restitution 'e' then it rebound from ground then

Velocity of ball while hitting ground is

$$\mathrm{U}=\sqrt{2\mathrm{gh}}$$

- Velocity after nth bounce is
  $v_{\mathrm{h}}=\mathrm{e}^n\mathrm{u}$
- Height raised after nth bounce is
  $\mathrm{h_n}=\mathrm{e^{2nh}}$
- Fractional change in momentum
  $\left(\frac{\Delta P}{P}\right)=1-e^n$
- Fractional change in KE
  $\left(\frac{\Delta \mathrm{KE}}{\mathrm{KE}}\right)=1-\mathrm{e^{2n}}$