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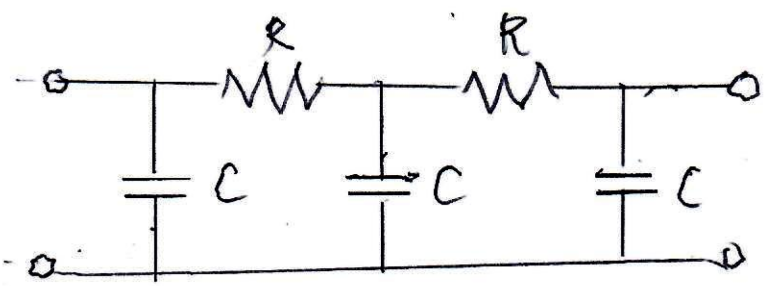
HOMEWORK 1

ECE 580

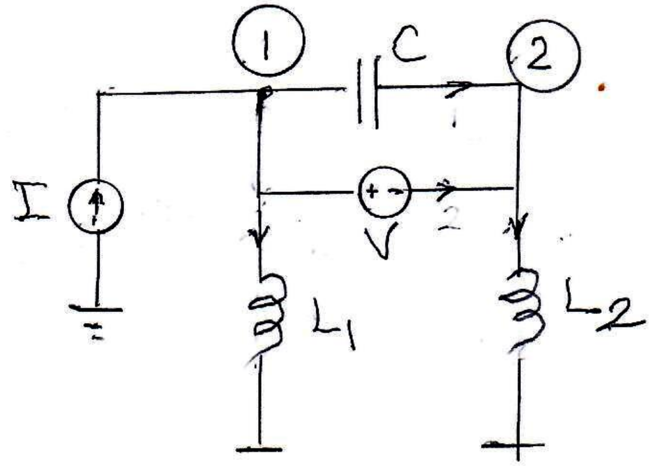
Due October 14, 2024

1. In the circuit shown, R = 1 kΩ , C = 2 pF and the frequency is 10 Mhz.

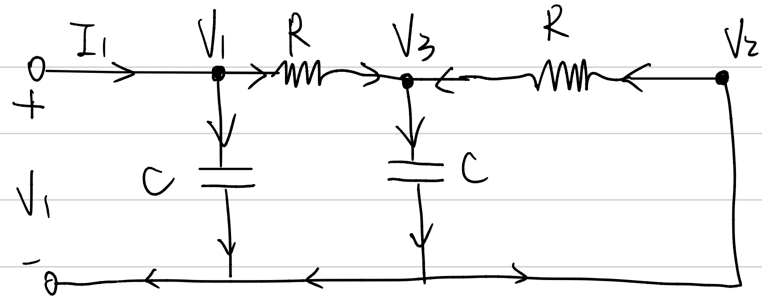
Find the short-circuit admittances.



2. Write the MNA equations in the Laplace domain for the circuit shown.



1.



$Z_C = \frac{1}{sC}$, $Y_C = sC$

$Y_R = \frac{1}{R}$

Apply KCL :

$$\begin{cases} I_1 = (V_1 - V_3) Y_R + V_1 Y_C \\ (V_1 - V_3) Y_R + (\overset{0}{\cancel{V_2}} - V_3) Y_R = V_3 \cdot Y_C \\ I_2 = (\underset{0}{\cancel{V_2}} - V_3) \cdot Y_R \rightarrow V_3 = \frac{-I_2}{Y_R} \end{cases} \rightarrow \begin{cases} I_1 = V_1 Y_R + I_2 + V_1 Y_C \\ V_1 Y_R + I_2 + I_2 = -I_2 \cdot \frac{Y_C}{Y_R} \\ V_1 Y_R = -\left(2 + \frac{Y_C}{Y_R}\right) \cdot I_2 \end{cases}$$

$$\rightarrow \begin{cases} I_2 = \frac{-Y_R^2}{2Y_R + Y_C} \cdot V_1 \\ I_1 = V_1 (Y_R + Y_C) + \frac{-Y_R^2}{2Y_R + Y_C} V_1 = V_1 \left(\frac{Y_R^2 + 3 Y_R Y_C + Y_C^2}{2 Y_R + Y_C} \right) \end{cases}$$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2 = 0} = \frac{Y_R^2 + 3 \cdot Y_R Y_C + Y_C^2}{2 Y_R + Y_C}$$

Where $Y_R = \frac{1}{R} = 10^{-3}$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2 = 0} = \frac{-Y_R^2}{2 Y_R + Y_C}$$

$$Y_C = sC = 2 \times 10^{-12} s$$
$$= 2 \times 10^{-12} j\omega$$
$$= 2 \times 10^{-12} \times j \times 2\pi \cdot 10^{7}$$
$$= 4\pi \times 10^{-5} j$$

$$\begin{bmatrix} Y_{11} \\ Y_{12} \end{bmatrix} = \begin{bmatrix} \dfrac{10^{-6} + 12\pi \times 10^{-8} j + 16\pi^2 \cdot 10^{-10} \cdot (-1)}{2 \times 10^{-3} + 4\pi \times 10^{-5} j} \\ \\ \dfrac{-10^{-6}}{2 \times 10^{-3} + 4\pi \times 10^{-5} j} \end{bmatrix}$$

Since it's a reciprocal and symmetrical network,

therefore $Y_{12} = Y_{21}$, $Y_{11} = Y_{22}$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{10^{-6} + 12\pi\times10^{-8}j + 16\pi^2\cdot10^{-10}\cdot(-1)}{2\times10^{-3} + 4\pi\times10^{-5}j} & \dfrac{-10^{-6}}{2\times10^{-3} + 4\pi\times10^{-5}j} \\ \dfrac{-10^{-6}}{2\times10^{-3} + 4\pi\times10^{-5}j} & \dfrac{10^{-6} + 12\pi\times10^{-8}j + 16\pi^2\cdot10^{-10}\cdot(-1)}{2\times10^{-3} + 4\pi\times10^{-5}j} \end{bmatrix}$$

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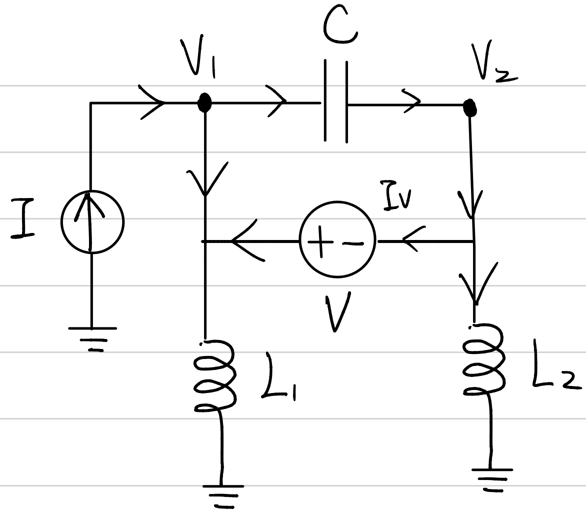
2.



$$Z_C = \frac{1}{sC}$$

$$Z_L = sL$$

For node 1: $\frac{V_1}{sL_1} + sC \cdot V_1 - sC \cdot V_2 = I$

For node 2: $sC \cdot V_1 - sC \cdot V_2 = \frac{V_2}{sL_2} \rightarrow sC \cdot V_1 - \left(sC + \frac{1}{sL_2}\right)V_2 = 0$

For voltage source: $V_1 - V_2 = V$

Apply MNA equations with variables matrix $\begin{bmatrix} V_1 \\ V_2 \\ I_V \end{bmatrix}$

$$\rightarrow \begin{bmatrix} \frac{1}{sL} + sC & -sC & 0 \\ sC & -sC - \frac{1}{sL_2} & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_V \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ V \end{bmatrix}$$

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