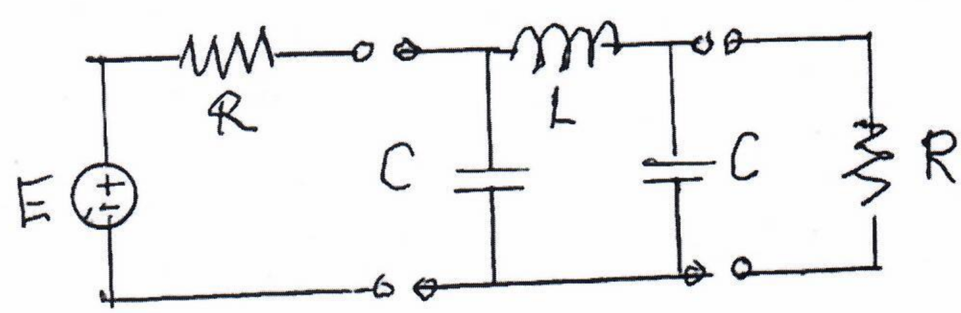
MIDTERM EXAMINATION

October 25, 2023 **Open Book**

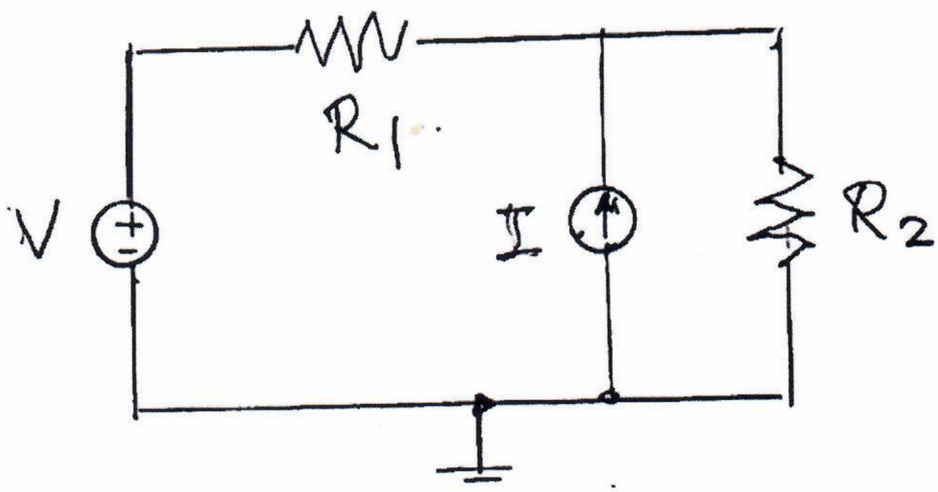
1. Find the chain matrix of the two-port shown below.

Extra Credit: Find the voltage gain V_2/E if the two-port operates between two equal resistors R. P. 67

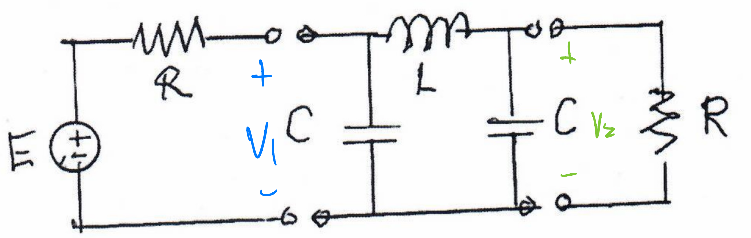


2. Construct the MNA matrix equations for the circuit shown below.

Extra Credit: Find all voltages and currents in the circuit. The element values are $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $V = 3$ V and $I = 1$ mA.



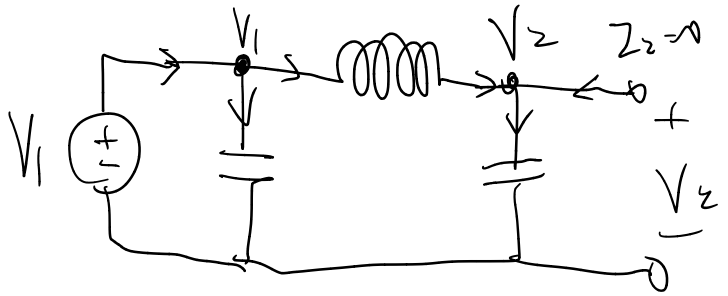
Chain Matrix



$AB - CD$

$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} \Rightarrow \begin{matrix} V_1 = AV_2 - BI_2 \\ V_2 = CV_2 - DI_2 \end{matrix}$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}^{\text{open}} = \frac{sL + \frac{1}{sC}}{1/sC} = s^2LC + 1$$

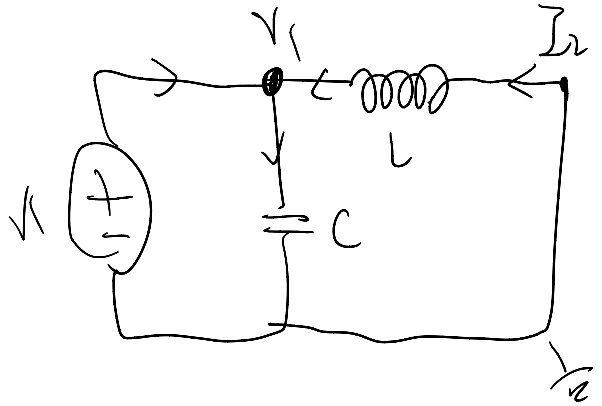


$$\frac{V_1 - V_2}{sL} = sCV_2$$

$$V_1 - V_2 = s^2CL \cdot V_2$$

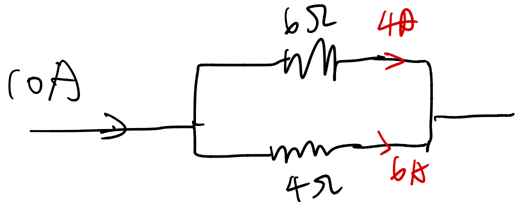
$$V_1 = V_2(s^2CL + 1)$$

$$B = \frac{V_1}{-I_2}\Big|_{V_2=0}^{\text{short}} = sL$$

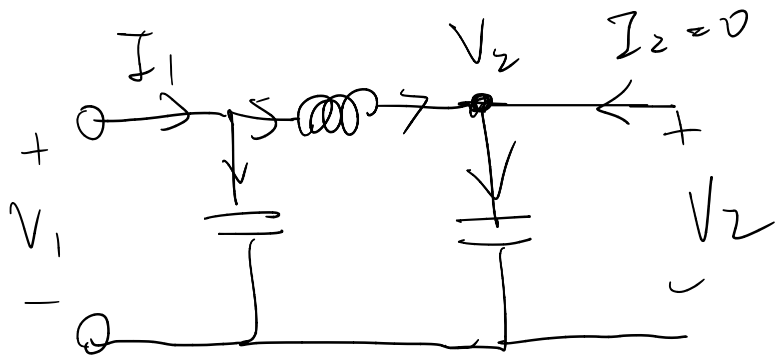


$$I_2 = \frac{0 - V_1}{sL}$$

$$\rightarrow sL = \frac{V_1}{-I_2}$$



$$C = \frac{I_1}{V_2}\Big|_{I_2=0}^{\text{open}}$$



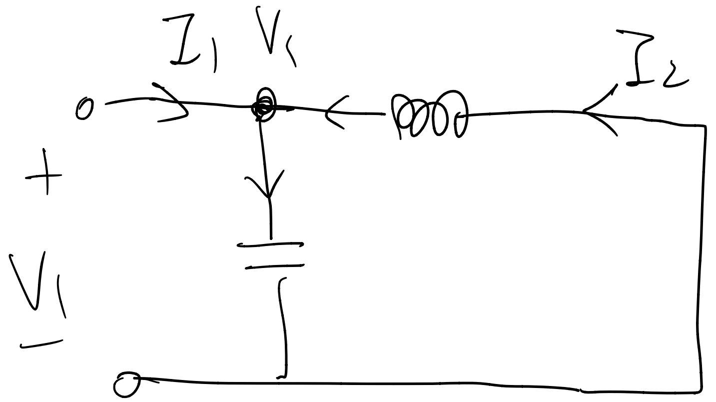
$$sC(s^2LC + 2)$$

$$I_1 \cdot \frac{\frac{1}{sC}}{sL + \frac{1}{sC} + \frac{1}{sC}} \times \frac{1}{sC} = V_2$$

$$I_1 \frac{\left(\frac{1}{sC}\right)^2}{\frac{s^2CL+2}{sC}} = V_2 \rightarrow \frac{I_1}{V_2} = sC(s^2CL + 2)$$

$$1 + s^2LC$$

$$D = \frac{I_1}{-I_2}\Big|_{V_2=0}^{\text{short}}$$



$$\begin{cases} I_2 = (0 - V_1) \cdot \frac{1}{sL} \rightarrow V_1 = -sL \cdot I_2 \\ I_1 + I_2 = sCV_1 = -s^2CL \cdot I_2 \end{cases}$$

$$\rightarrow I_1 = (-1 - s^2CL)I_2$$

$$\rightarrow \frac{I_1}{-I_2} = 1 + s^2CL$$

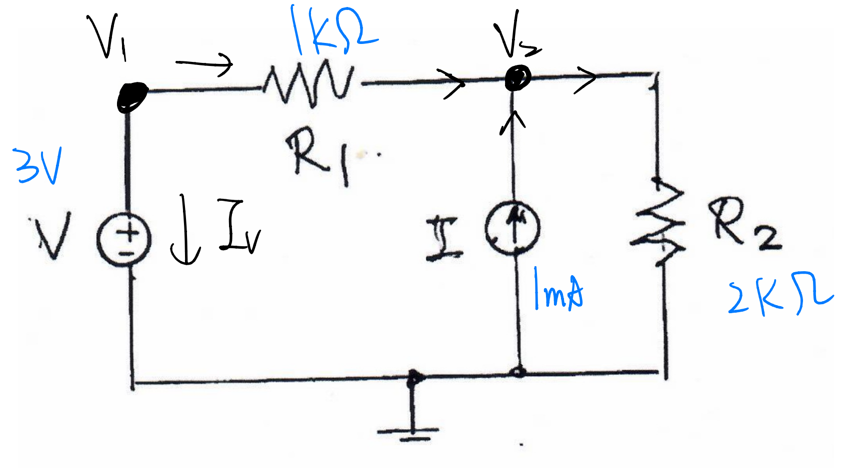
(B)

$$A_V = \frac{V_2}{E} = \frac{R_L}{AR_L + B + CR_GR_L + DR_G} = \frac{1}{A + \frac{B}{R} + CR + D}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} s^2LC + 1 & sL \\ sC(s^2CL + 2) & s^2CL + 1 \end{bmatrix}$$

$$= \frac{R}{(s^2LC+1)R + sL + sC(s^2CL+2)\cdot R^2 + s^2CLR + R}$$

$$= \frac{R}{s^2LCR + R + sL + s^3C^2LR^2 + 2sCR^2 + s^2CLR + R}$$



$$\begin{bmatrix} V_1 \\ V_2 \\ I_V \end{bmatrix}$$

For node 1: $\frac{V_1}{R_1} - \frac{V_2}{R_1} + I_V = 0$

For node 2: $\frac{V_1}{R_1} - \frac{V_2}{R_1} + I = \frac{V_2}{R_2} \rightarrow \frac{-V_1}{R_1} + \frac{V_2}{R_1} + \frac{V_2}{R_2} = I$

For Voltage source: $V_1 = V$

$$\begin{pmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ I_V \end{pmatrix} = \begin{pmatrix} 0 \\ I \\ V \end{pmatrix}$$

$V_1 = 3V$

$3 \times 10^{-3} - 5 \times 10^{-4} V_2 + I_V = 0$

$(10^{-3} + 5 \times 10^{-4}) V_2 = 10^{-3} + 3 \times 10^{-3}$

10×10^{-4}

$V_2 = \frac{4 \times 10^{-3}}{15 \times 10^{-4}} = \frac{40}{15} = \frac{8}{3} V$

$\sim 2.67V$

$\frac{40}{3}$

$30 \times 10^{-4} - \frac{40}{3} \times 10^{-4} + I_V = 0$

$I_V = \frac{-5}{3} \times 10^{-3}$

$= -1.67mA$