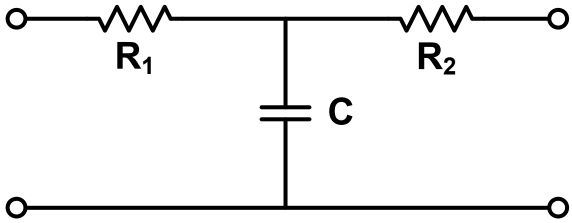
Yen - Chun Chen
934 559 635

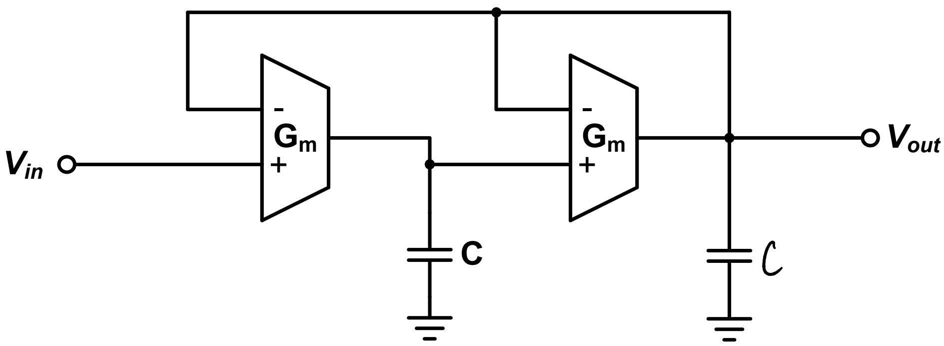
ECE580
Final Examination
Monday, December 9, 2024, 12-2pm

1. Find the two-port matrices of the circuit shown below. Assume R1 = R2 = 50 ohms, C = 10 pf and f=1000 MHz.
   (a) Find the Z, Y, T. matrices of the complete two-port.
   (b) Find the scattering matrix of the shunt C, with R1 and R2 regarded as terminations.



2. Find the gain $|H(j\omega)|$ of the Delyiannis-Friend filter at the pole frequency ω_o, The opamp is ideal.

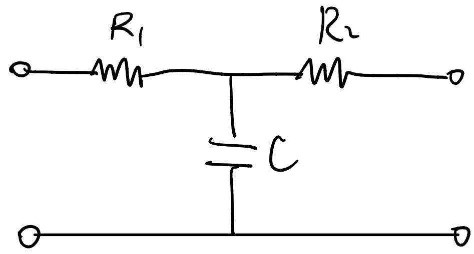
3. (a) Find the transfer function of the circuit shown for G_m = 1 mA/V and C = 1pF
   (b) Find the poles and zeros.
   (c) Find the pole frequency and pole Q.



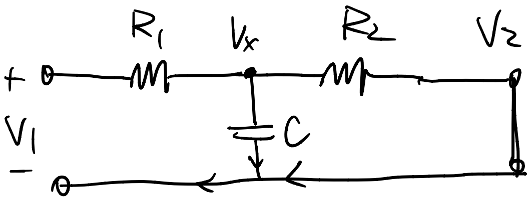
(Here the second capacitor is also C)

(1.a) For Y matrix ,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



$$\begin{cases} I_1 = Y_{11} V_1 + Y_{12} V_2 \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \end{cases}$$



$$\begin{cases} I_1 = (V_1 - V_X) Y_1 \\ Y_1 (V_1 - V_X) = Y_C V_X + Y_2 V_X \end{cases}$$

$$\rightarrow V_X (Y_1 + Y_2 + Y_C) = Y_1 V_1 + Y_2 V_2$$

$$\rightarrow I_1 = (Y_1 + Y_C) V_1 - Y_C V_2$$

$$\begin{cases} I_1 = (Y_1 + Y_C) V_1 - Y_C V_2 \\ I_2 = (Y_2 + Y_C) V_2 - Y_C V_1 \end{cases}$$

$$\rightarrow Y \text{ matrix} = \begin{bmatrix} Y_1 + Y_C & -Y_C \\ -Y_C & Y_2 + Y_C \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + SC & -SC \\ -SC & \frac{1}{R_2} + SC \end{bmatrix}$$

Where $R_1 = R_2 = 50\Omega$, $C = 10pF$, $f = 10^9 Hz$

$$\rightarrow W = 2\pi f = 2\pi \times 10^9 \text{ rad/s}$$

$$Y_1 = \frac{1}{50} = 0.02 \text{ S} = Y_2$$

$$Y_C = jWC = j \cdot 2\pi \times 10^9 \cdot 10^{-11} = j \cdot (0.0628) \text{ S}$$

$$\rightarrow Y = \begin{bmatrix} 0.02 + 0.0628j & -0.0628j \\ -0.0628j & 0.02 + 0.0628j \end{bmatrix} \#$$

(1. a) Z matrix . According to textbook P. 42

$$Z = \frac{1}{|y|}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$= \frac{1}{y_{11} \cdot y_{22} - y_{12}y_{21}}\begin{bmatrix} 0.02+0.0628j & 0.0628j \\ 0.0628j & 0.02+0.0628j \end{bmatrix}$$

$$= \frac{1}{0.0004 + 0.002512j}\begin{bmatrix} 0.02+0.0628j & 0.0628j \\ 0.0628j & 0.02+0.0628j \end{bmatrix}$$

#

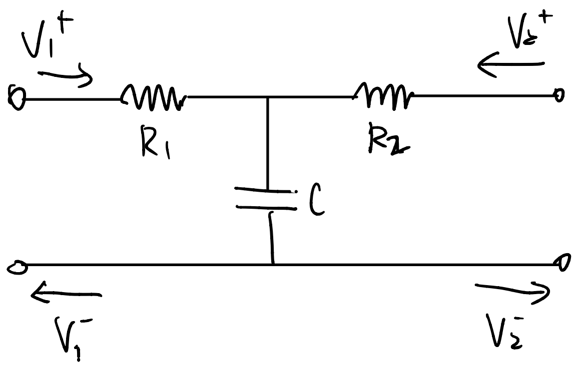
For T matrix,

$$T = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{|Z|}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix} = \begin{bmatrix} \frac{0.02+0.0628j}{0.0628j} & \frac{|Z|}{0.0628j} \\ \frac{1}{0.0628j} & \frac{0.02+0.0628j}{0.0628j} \end{bmatrix}$$

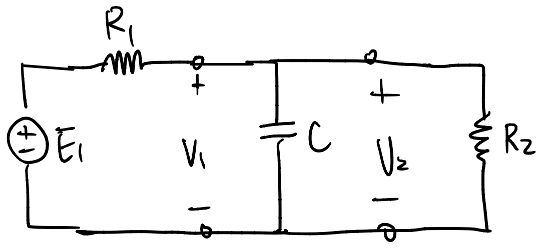
#

(1.b) Scattering Matrix

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$



$$\rightarrow \begin{cases} V_1^- = S_{11} V_1^+ + S_{12} V_2^+ \\ V_2^- = S_{21} V_1^+ + S_{22} V_2^+ \end{cases}$$

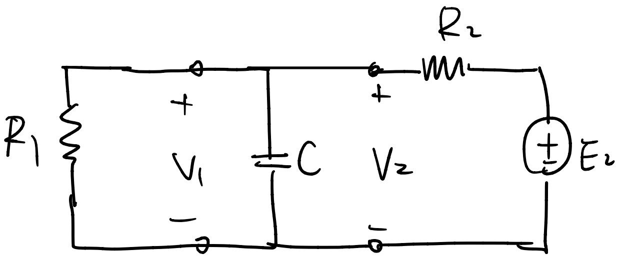


$$S_{11} = \frac{V_1^-}{V_1^+} = \frac{\left(\frac{1}{SC} /\!/ R_2\right) - R_1}{\left(\frac{1}{SC} /\!/ R_2\right) + R_1} = \frac{\frac{R_2}{1+SR_2C} - R_1}{\frac{R_2}{1+SR_2C} + R_1}$$

$$= \frac{R_2 - R_1 - SR_2R_1C}{R_2 + R_1 + SR_1R_2C}$$

$$\begin{cases} V_1 = V_1^+ + V_1^- = V_1^+ (1+S_{11}) \\ V_2 = V_2^- = V_1 \end{cases}$$

$$S_{21} = \frac{V_2^-}{V_1^+} = (1+S_{11}) = \frac{2R_2}{R_2 + R_1 + SR_1R_2C}$$



$$S_{22} = \frac{V_2^-}{V_2^+} = \frac{\left(\frac{1}{SC} /\!/ R_1\right) - R_2}{\left(\frac{1}{SC} /\!/ R_1\right) + R_2}$$

$$= \frac{R_1 - R_2 - SCR_1R_2}{R_1 + R_2 + SCR_1R_2}$$

$$\begin{cases} V_2 = V_2^+ + V_2^- = V_2^+ (1+S_{22}) \\ V_1 = V_1^- = V_2 \end{cases}$$

$$S_{12} = \frac{V_1^-}{V_2^+} = \frac{2R_1}{R_1 + R_2 + SR_1R_2C}$$

$$\text{Scattering Matrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

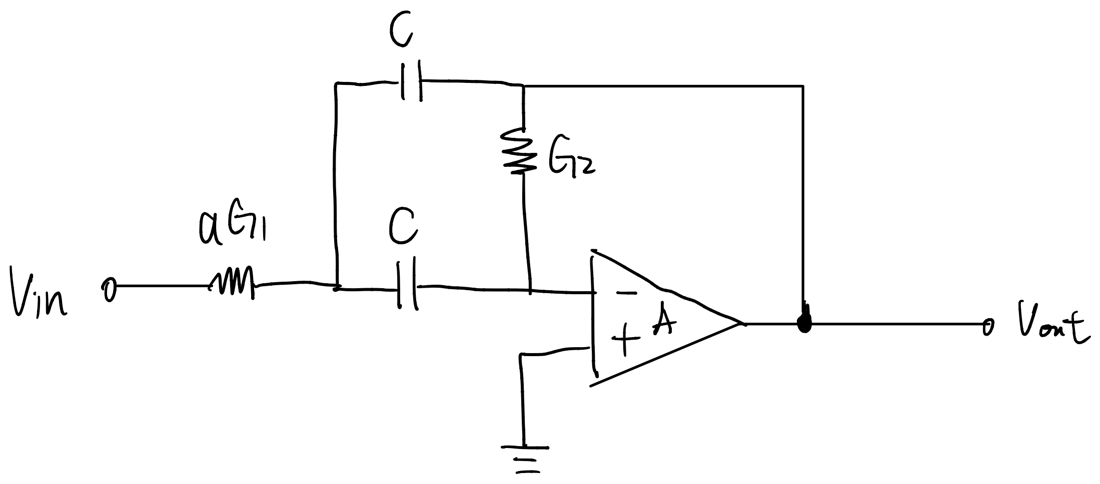
$$= \frac{1}{R_1 + R_2 + SR_1R_2C} \begin{bmatrix} R_2 - R_1 - SR_1R_2C & 2R_1 \\ 2R_2 & R_1 - R_2 - SR_1R_2C \end{bmatrix}$$

Where $R_1 = R_2 = 50\Omega$, $C = 10pF$, $f = 10^9 Hz$

$\rightarrow w = 2\pi f = 2\pi \times 10^9$ rad/s

sub into S matrix then get the final answer #

2. Deliyannis - Friend filter



For ideal opamp, transfer function

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{-sa\,{G_1}/{C}}{s^2 + s\frac{2G_2}{C} + \frac{G_1 G_2}{C^2}}$$

When $s = jw_0$,

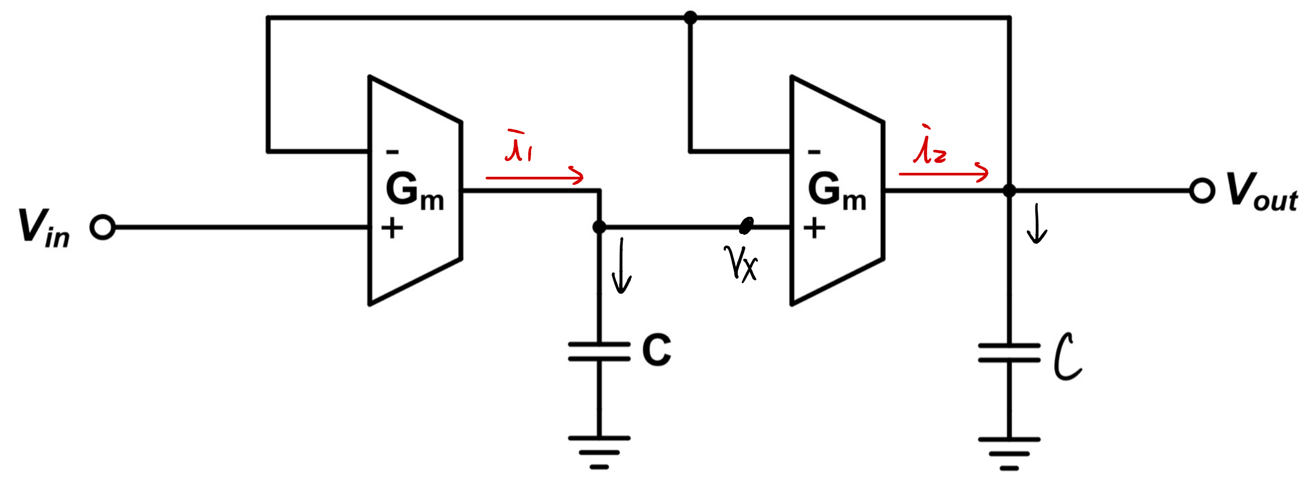
$$H(jw_0) = \frac{-jw_0 \cdot a \cdot \frac{G_1}{C}}{(jw_0)^2 + jw_0 \cdot \frac{2G_2}{C} + \frac{G_1 G_2}{C^2}}$$

$$= -\frac{jw_0 \cdot a\frac{G_1}{C}}{jw \cdot \frac{2G_2}{C}}$$

$$= -\frac{aG_1}{2G_2}$$

$$|H(jw_0)| = \left|-\frac{aG_1}{2G_2}\right| = \frac{aR_2}{2R_1}$$ #

3.



$$\begin{cases} i_1 = V_{in} \cdot G_m \\ V_x = \frac{i_1}{sC} = \frac{V_{in} \cdot G_m}{sC} \\ i_2 = V_x \cdot G_m = \frac{V_{in} \cdot G_m \cdot G_m}{sC} \\ V_{out} = \frac{i_2}{sC} = \frac{V_{in} \cdot G_m^2}{s^2 C^2} \end{cases}$$

(a) $V_{out} = V_{in} \cdot \frac{G_m^2}{s^2 \cdot C^2}$

$$\rightarrow H(s) = \frac{V_{out}}{V_{in}} = \frac{G_m^2}{s^2 \cdot C^2} = \frac{1}{s^2} \frac{10^{-3} \cdot 10^{-3}}{10^{-24}} = \frac{1e18}{s^2} \quad \#$$

(b)

Since $H(s) = \frac{1e18}{s^2}$

There is no zeros. Double poles at $s = 0$ #

(C) General transfer function for second-order filter is

$$H(s) = \frac{A_v \cdot W_0^2}{s^2 + \frac{W_0}{Q} s + W_0^2}$$

There is no coefficient of s and constant term.

$\rightarrow$ pole frequency $= 0$

pole $Q = 0$ #