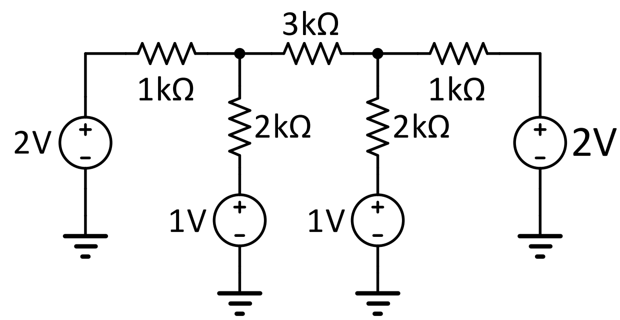
FINAL EXAMINATION

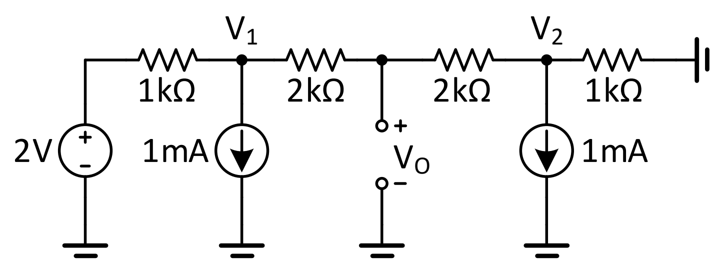
ECE 580

December 14, 2023

1. Analyze the circuit shown. ***Warning***: look for a shortcut!!



2. Analyze the circuit shown using inter-reciprocity.

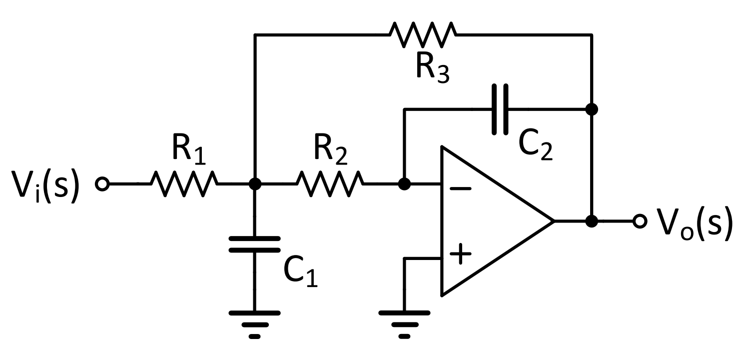


For extra credit, find V_1 and V_2.

3. The Rauch filter shown has a transfer function (assume ideal opamp)

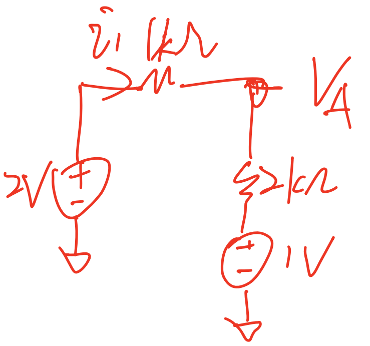
$$H(s) = \frac{1}{s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2 + s \cdot C_2 \cdot \left(R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}\right) + \frac{R_1}{R_3}}.$$

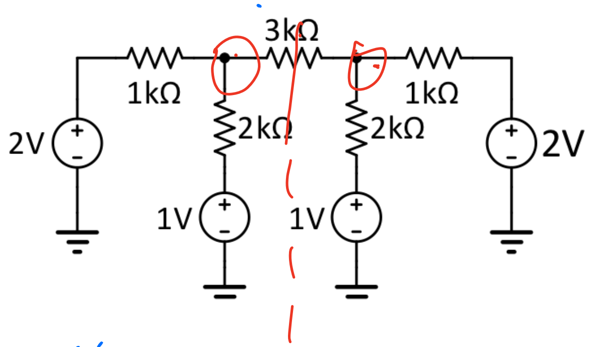
If $R_1 = 2R_2 = 2R_3$ and pole Q is $Q = 1/\sqrt{3}$, what should be the ratio C_1/C_2?



For extra credit, find the requirements for the DC gain A_0 and the pole Q so that the minimum capacitance spread (C_1/C_2 or C_2/C_1) can be 1. **Hint:** consider the requirements for A_0 and Q so that there is real solution for R_2/R_3 when $C_1 = C_2$.

1. Analyze the circuit shown. ***Warning***: look for a shortcut!!





$$\frac{2-1}{3} = \frac{1}{3}\, mA$$

$$V_A = 1 + \frac{1}{3}\times 2 = \frac{5}{3}\, V$$

$$\frac{2-V_A}{1} = i_1$$

$$\frac{1-V_A}{2} = i_2$$

$$\frac{V_A-V_B}{3} = i_3$$

$$i_1 + i_2 = i_3$$

$$\frac{2-V_B}{1} = i_6$$

$$\frac{1-V_B}{2} = i_5$$

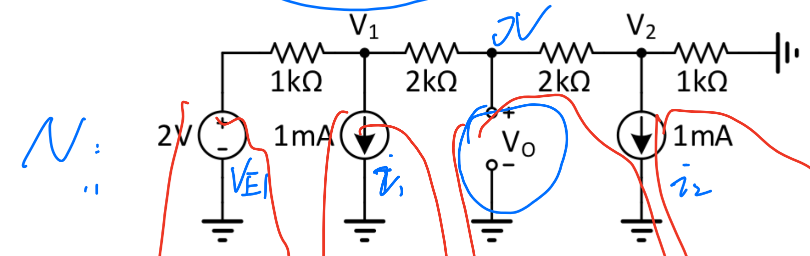
$$\frac{V_B-V_A}{3} = -i_4$$

$$i_4 + i_5 + i_6 = 0$$

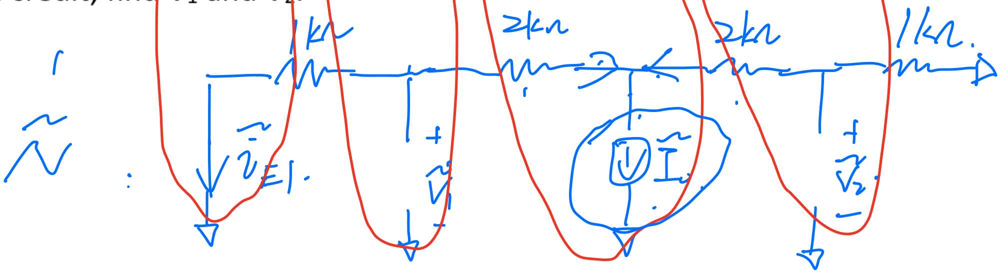
$$V_A = V_B$$

$$V_A = \frac{5}{3}\, V$$

2. Analyze the circuit shown using inter-reciprocity.



For extra credit, find V_1 and V_2.



$$\Longrightarrow V_o \cdot \tilde{I}_o + V_{E1} \cdot \tilde{i}_{E1} = i_1 \cdot \tilde{V}_1 + i_2 \tilde{V}_2$$

$$\begin{cases} \tilde{i}_{E1} = -\frac{1}{2}\tilde{I}_0 \\ \tilde{V}_1 = -\frac{1}{2}\tilde{I}_0 R_1 \\ \tilde{V}_2 = -\frac{1}{2}\tilde{I}_0 R_1 \end{cases}$$

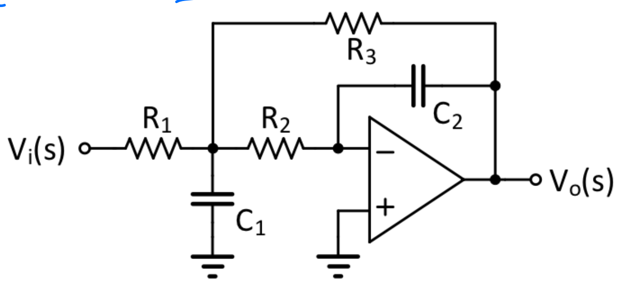
$$V_o = -V_{E1}\frac{\tilde{i}_{E1}}{\tilde{i}_o} + i_1 \frac{\tilde{V}_1}{\tilde{I}_o} + i_2 \frac{\tilde{V}_2}{\tilde{i}_o}$$

$$= 0V$$

3. The Rauch filter shown has a transfer function (assume ideal opamp)

$$H(s) = \frac{1}{s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2 + s \cdot C_2 \cdot \left(R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}\right) + \frac{R_1}{R_3}}.$$

If $R_1 = 2R_2 = 2R_3$ and pole Q is $Q = 1/\sqrt{3}$, what should be the ratio C_1/C_2?



For extra credit, find the requirements for the DC gain A_0 and the pole Q so that the minimum capacitance spread (C_1/C_2 or C_2/C_1) can be 1. **Hint:** consider the requirements for A_0 and Q so that there is real solution for R_2/R_3 when $C_1 = C_2$.

$$H(s) = \frac{1}{s^2 + \frac{W_0}{Q} s + W_0^2} \qquad \div \frac{1}{R_1 R_2 C_1 C_2}$$

$$= \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \frac{R_1 + R_2 + \frac{R_1 R_2}{R_3}}{R_1 R_2 C_1} s + \frac{1}{R_2 R_3 C_1 C_2}}$$

$$\begin{cases} \frac{R_1 + R_2 + \frac{R_1 R_2}{R_3}}{R_1 R_2 C_1} = \frac{W_0}{Q} & (1) \\ \frac{1}{R_2 R_3 C_1 C_2} = W_0^2 & (2) \\ R_1 = 2R_2 = 2R_3, \quad Q = \frac{1}{\sqrt{3}} & (3) \end{cases}$$

$$\omega_0 = \frac{1}{\sqrt{R_3^{\,2} C_1 C_2}}$$

$$\frac{\omega_0}{1/\sqrt{3}} = \frac{\omega_0}{Q} = \frac{R_1 \cancel{R_3} + \cancel{R_2} R_3 + R_1 \cancel{R_2}}{R_1 \cancel{R_2} R_3 C_1}$$

$$= \frac{5}{2 \cancel{R_2} C_1} = \frac{\sqrt{3}}{\cancel{R_2} \sqrt{C_1 C_2}}$$

$$\frac{25}{4 C_1^{\cancel{2}}} = \frac{3}{\cancel{C_1} C_2}$$

$$\frac{C_1}{C_2} = \frac{25}{12}$$

$$H(s) = \frac{\dfrac{1}{R_1 R_2 C_1 C_2}}{s^2 + \dfrac{R_1 + R_2 + \dfrac{R_1 R_2}{R_3}}{R_1 R_2 C_1} s + \dfrac{1}{R_2 R_3 C_1 C_2}}$$

$$\text{DC gain} = \frac{\dfrac{1}{R_1 R_2 C_1 C_2}}{\dfrac{1}{R_2 R_3 C_1 C_2}}$$

$$= \frac{R_3}{R_1}$$

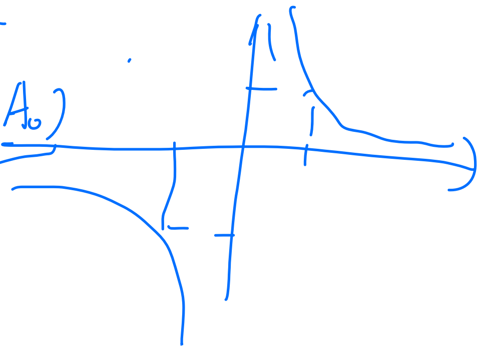
$$\begin{cases} \dfrac{R_1+R_2+\frac{R_1R_2}{R_3}}{R_1R_2C_1} = \dfrac{\omega_0}{Q} \\[2ex] \dfrac{1}{R_2R_3C_1C_2} = \omega_0^2 \\[2ex] A_0 = \dfrac{R_3}{R_1} \quad \leftarrow \end{cases}$$

$$(1+A_0)^2\frac{R_2}{R_3} + \frac{R_3}{R_2} + 2(1+A_0) = \frac{1}{Q^2}\frac{C_1}{C_2}. \quad \leftarrow$$

if $\frac{C_1}{C_2} = 1$ possible, $\frac{R_2}{R_3} = X$

$$f(x) = \underline{(1+A_0)^2 x + \frac{1}{X}} + 2(1+A_0) \quad x + \frac{1}{x}$$

$X = \frac{1}{1+A_0}$. $f(x) \geq 4(1+A_0)$



$$\frac{1}{Q^2} \geq (1+A_0)4$$

$$Q \leq \frac{1}{2\sqrt{1+A_0}}$$