

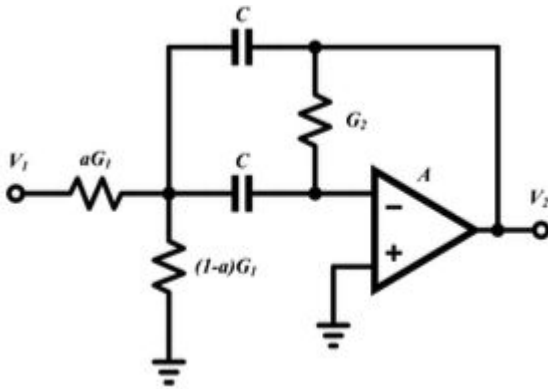
ECE 580 HW3

Yen-Chun Chen

934559635

1. Find the expressions in terms of the element values for the dc gain, the pole frequency (ω_0) and the pole Q (Q_p), for the Delyiannis and Rauch filters.

Delyiannis-Friend Filter:



Transfer Function with an ideal op-amp:

$$H(s) = -\frac{saCG_1}{s^2C^2 + 2sCG_2 + G_1G_2} = -\frac{\frac{saG_1}{C}}{s^2 + s\frac{2G_2}{C} + \frac{G_1G_2}{C^2}}$$

The DC Gain: $\lim_{s \rightarrow 0} H(s) = H(0) = 0$

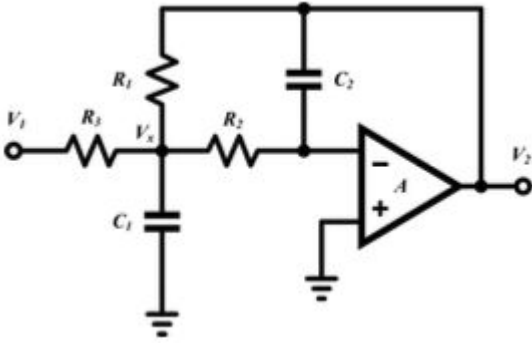
The transfer function for a general second order low-pass filter is given

$$H(s) = \frac{A_v \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}, \text{ we got that } \omega_0^2 = \frac{G_1G_2}{C^2} \text{ Thus, the pole frequency } \omega_0 = \frac{\sqrt{G_1G_2}}{C}$$

Pole Q is determined by the coefficient of s in the denominator

$$\text{So we got } \frac{\omega_0}{Q_p} = \frac{2G_2}{C} \rightarrow Q_p = \frac{\sqrt{G_1G_2}}{C} \frac{C}{2G_2} = \frac{1}{2} \sqrt{\frac{G_1}{G_2}}$$

Rauch Filter:



$$\text{Transfer Function: } H(s) = \frac{\frac{G_2 G_3}{C_1 C_2}}{s^2 + s \left(\frac{G_1 + G_2 + G_3}{C_1} \right) + \frac{G_1 G_2}{C_1 C_2} + \varepsilon}$$

$$\varepsilon = \frac{1}{A} \left[s^2 + s \left(\frac{G_1 + G_2 + G_3}{C_1} + \frac{G_2}{C_2} \right) + \frac{(G_1 + G_3)G_2}{C_1 C_2} \right]$$

For an ideal op-amp, $A \rightarrow \infty$ leads to $\varepsilon \rightarrow 0$. Thus, the DC Gain: $\lim_{s \rightarrow 0} H(s) = H(0) = \frac{\frac{G_2 G_3}{C_1 C_2}}{\frac{G_1 G_2}{C_1 C_2}} = \frac{G_3}{G_1}$

The transfer function for a general second order low-pass filter is given

$$H(s) = \frac{A_v \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}, \text{ we got that } \omega_0^2 = \frac{G_1 G_2}{C_1 C_2} \text{ Thus, the pole frequency } \omega_0 = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

Pole Q is determined by the coefficient of s in the denominator

$$\text{So we got } \frac{\omega_0}{Q_p} = \frac{G_1 + G_2 + G_3}{C_1} \rightarrow Q_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}} \left(\frac{C_1}{G_1 + G_2 + G_3} \right)$$

2. Design a Rauch filter for the following specifications:

$$R_1 = 1 \text{ k}\Omega, R_2 = 0.5 \text{ k}\Omega, DC \text{ gain} = 1, Q_p = 2^{-\frac{1}{2}}, \omega_0 = 40\pi \text{ krad/s}$$

Please output the performance characteristics of the filter (gain/frequency response/zero pole, etc.)

```
clear
close all
clc

R1 = 1e3;           % 1k ohms
R2 = 500;           % 500 ohms
DC_gain = 1;        % DC Gain
Q_p = 2^(-0.5);     % Quality factor
w_0 = 2*pi*20e3;    % Pole Frequency (rad/s)

G1 = 1/R1;
G2 = 1/R2;
G3 = G1*DC_gain; % H(0) = G3/G1

% Calculate C1 and C2
C1C2 = G1 * G2 / w_0^2;
C1 = Q_p * (G1 + G2 + G3) / w_0;
C2 = C1C2 / C1;
fprintf('G1 = %.d mho\nG2 = %.d mho\nG3 = %.d mho\nC1 = %.6e F\nC2 = %.6e F\nDC Gain = %.2f\nQ_p = %.4f\nw_0 = %.2f rad/s', G1, G2, G3, C1, C2, DC_gain, Q_p, w_0);
```

```
G1 = 1e-03 mho
G2 = 2e-03 mho
G3 = 1e-03 mho
C1 = 2.250791e-08 F
C2 = 5.626977e-09 F
DC Gain = 1.00
Q_p = 0.7071
w_0 = 125663.71 rad/s
```

```
num = [0 , 0 , (G2 * G3)/(C1 * C2)]; % Numerator
den = [1 , (G1+G2+G3)/C1 , (G1*G2)/(C1*C2)]; % Denominator
tf_sys = tf(num, den);
[H, w] = freqs(num, den);
% Magnitude response
```

```
[H, w] = freqs(num, den);
mag_dB = 20*log10(abs(H));

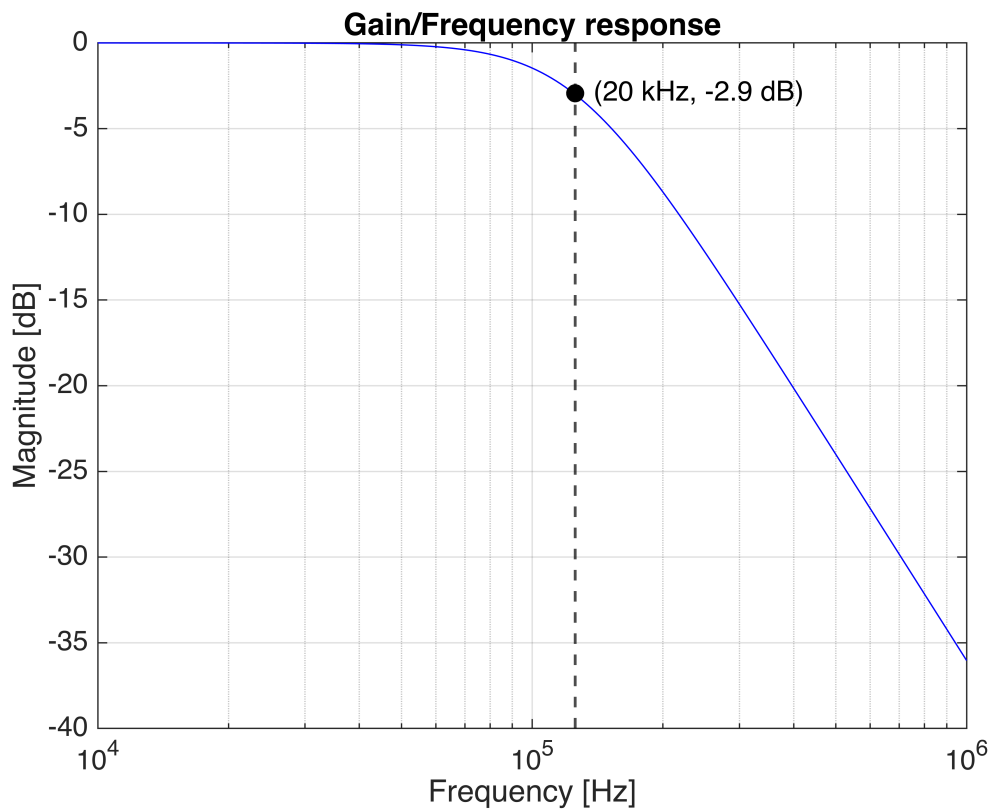
% Find the magnitude at w_0
[~, idx] = min(abs(w - w_0));
mag_at_w0 = mag_dB(idx);
```

```

figure;
semilogx(w, mag_dB, 'b');
grid on;
title("Gain/Frequency response");
ylabel("Magnitude [dB]");
xlabel("Frequency [Hz]");

% Plot the intersection point
hold on;
xline(w_0, 'k--', 'LineWidth', 1);
plot(w_0, mag_at_w0, 'ko', 'MarkerSize', 6, 'MarkerFaceColor', 'k');
text(w_0*1.1, mag_at_w0, sprintf('(%0.1f kHz, %0.1f dB)', w_0/(2*pi)/1000,
mag_at_w0));

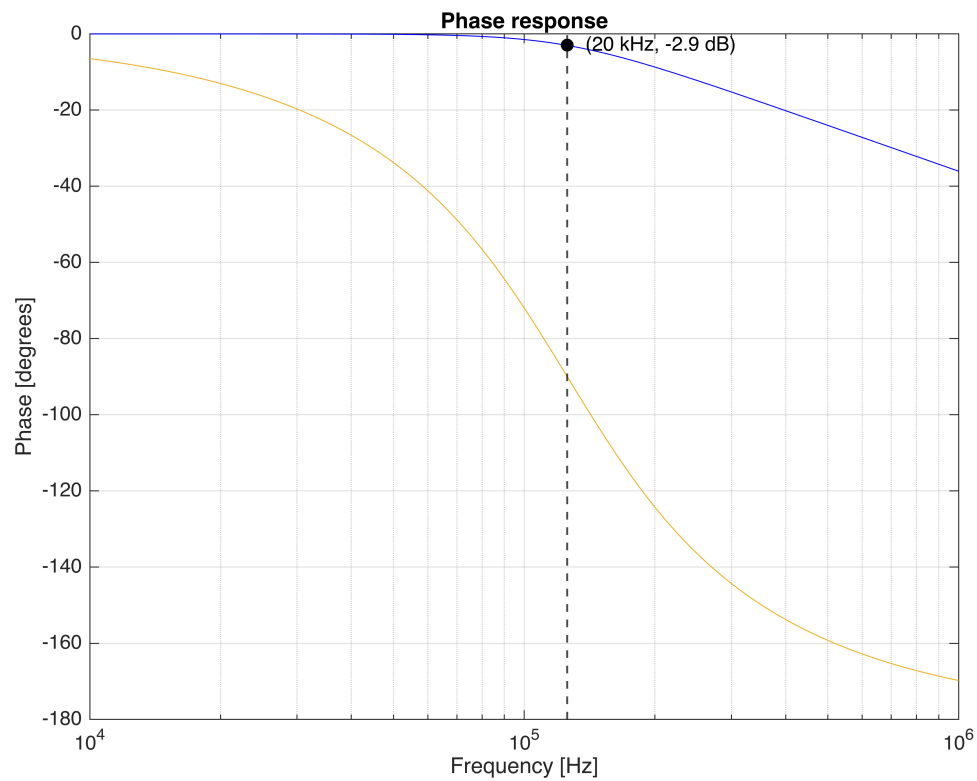
```



```

%Phase Response
semilogx(w, 180/pi*unwrap(angle(H))); % Plot phase in degrees
title("Phase response");
ylabel("Phase [degrees]");
xlabel("Frequency [Hz]");
grid on

```

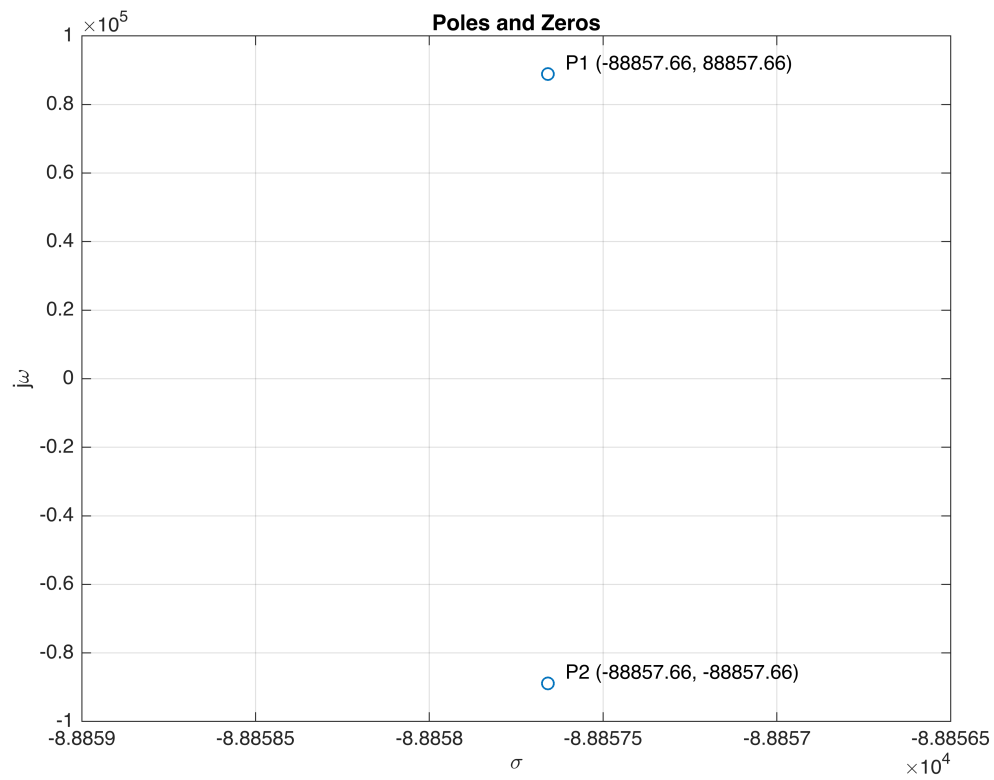


% Poles and Zeros

```
[z, p, k] = tf2zp(num, den)
```

```
z =  
    0×1 empty double column vector  
p = 2×1 complex  
104 ×  
    -8.8858 + 8.8858i  
    -8.8858 - 8.8858i  
k =  
1.5791e+10
```

```
figure;  
plot(z,'o');  
plot(p,'o', 'LineWidth',2);  
title("Poles and Zeros");  
ylabel("j\omega");  
xlabel("\sigma");  
for i = 1:length(p)  
    text(real(p(i)) + 0.05, imag(p(i)) + 0.05, ...  
        sprintf('P%d (%.2f, %.2f)', i, real(p(i)), imag(p(i))), ...  
        'HorizontalAlignment', 'left', 'VerticalAlignment', 'bottom');  
end  
grid on
```



```
% Step Response  
step(tf_sys);  
title("Step Response");  
grid on
```

