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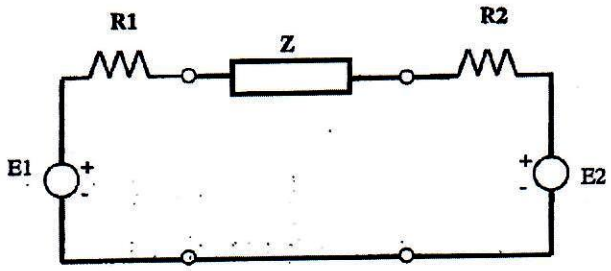
Homework 2

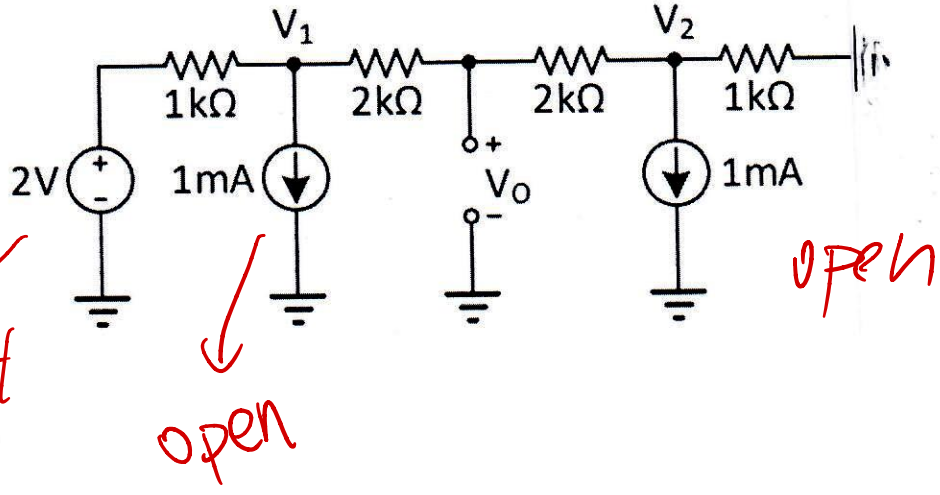
ECE 580

Due November 6, 2024

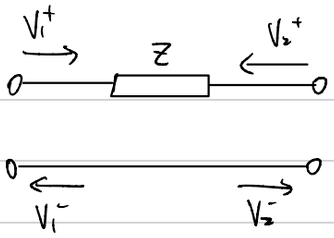
1. Find the scattering matrix of a single-element two-port with terminations R1 and R2.



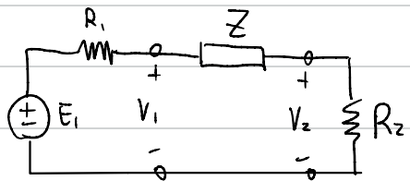
2. Find all voltages in the circuit shown using inter-reciprocity.



1.



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \rightarrow \begin{aligned} V_1^- &= S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ \\ V_2^- &= S_{21} V_1^+ + S_{22} V_2^+ \end{aligned}$$

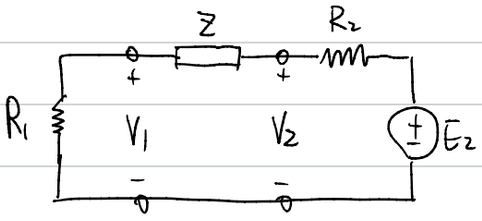


$$S_{11} = \frac{V_1^-}{V_1^+} = \frac{Z + R_2 - R_1}{Z + R_2 + R_1}$$

$$S_{21} = \frac{V_2^-}{V_1^+} = (1 + S_{11}) \cdot \frac{R_2}{Z + R_2} = \frac{R_2}{Z + R_2} \cdot \frac{Z + R_2 - R_1 + Z + R_2 + R_1}{Z + R_2 + R_1}$$

$$= \frac{2R_2 (Z + R_2)}{(Z + R_2)(Z + R_2 + R_1)}$$

$$\begin{cases} V_1 = V_1^+ + V_1^- = V_1^+ (1 + S_{11}) \\ V_2 = V_2^- = V_1 \cdot \frac{R_2}{Z + R_2} \end{cases} \Rightarrow V_2^- = V_1^+ (1 + S_{11}) \cdot \frac{R_2}{Z + R_2} = \frac{2R_2}{Z + R_2 + R_1}$$



$$S_{22} = \frac{V_2^-}{V_2^+} = \frac{Z + R_1 - R_2}{Z + R_1 + R_2}$$

$$S_{12} = \frac{V_1^-}{V_2^+} = (1 + S_{22}) \frac{R_1}{Z + R_1} = \frac{R_1}{Z + R_1} \cdot \frac{Z + R_1 + R_2 + Z + R_1 - R_2}{Z + R_1 + R_2}$$

$$= \frac{2R_1 (Z + R_1)}{(Z + R_1 + R_2)(Z + R_1)}$$

$$\begin{cases} V_2 = V_2^+ + V_2^- = V_2^+ (1 + S_{22}) \\ V_1 = V_1^- = V_2 \cdot \frac{R_1}{Z + R_1} \end{cases} \Rightarrow V_1^- = V_2^+ (1 + S_{22}) \cdot \frac{R_1}{Z + R_1}$$

$$= \frac{2R_1}{Z + R_1 + R_2}$$

Scattering matrix $= \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

$$= \begin{bmatrix} \frac{Z + R_2 - R_1}{Z + R_2 + R_1} & \frac{2R_2}{Z + R_2 + R_1} \\ \frac{2R_1}{Z + R_1 + R_2} & \frac{Z + R_1 - R_2}{Z + R_1 + R_2} \end{bmatrix} = \frac{1}{Z + R_1 + R_2} \begin{bmatrix} Z + R_2 - R_1 & 2R_2 \\ 2R_1 & Z + R_1 - R_2 \end{bmatrix}$$

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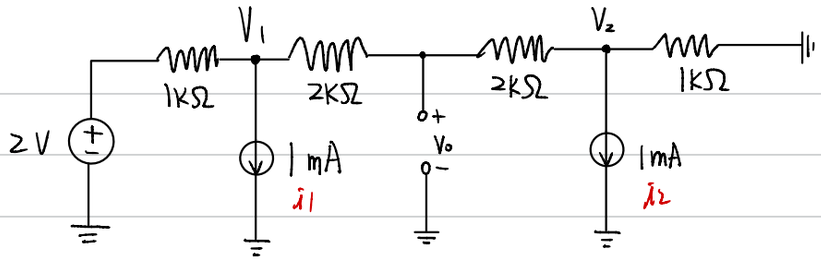
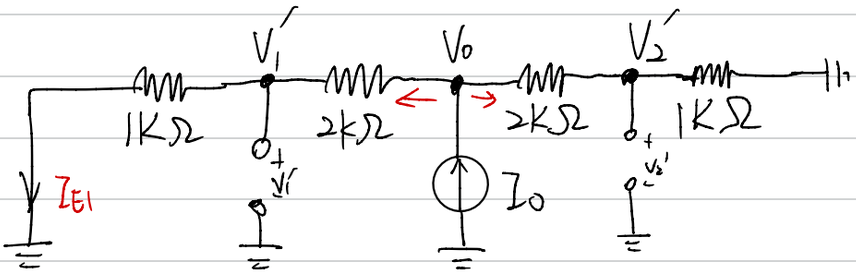
2.





$$\begin{cases} I_{E1} = \frac{1}{2} I_o \\ V_1' = \frac{1}{2} I_o \cdot 1K = 500 I_o \\ V_2' = \frac{1}{2} I_o \cdot 1K = 500 I_o \end{cases}$$

By Inter-Reciprocity: $V_o I_o + V_{E1} \cdot I_{E1} + V_1 I_1 + V_2 I_2 = 0$

$$\rightarrow V_o = -V_{E1} \cdot \frac{I_{E1}}{I_o} + I_1 \cdot \frac{V_1'}{I_o} + I_2 \frac{V_2'}{I_o}$$

$$= (-2) \cdot \frac{1}{2} + 1mA \cdot 500 + 1mA \cdot 500$$

$$= -1 + 0.5 + 0.5 = 0 \ V$$

KCL for V_1: $\frac{V_1 - 2}{1K} + \frac{V_1}{2K} + 1mA = 0$

$$\rightarrow 2V_1 - 4 + V_1 + 2 = 0 \rightarrow V_1 = \frac{2}{3} V$$

KCL for V_2: $\frac{V_2}{2K} + \frac{V_2}{1K} + 1mA = 0$

$$\rightarrow V_2 + 2V_2 + 2 = 0 \rightarrow V_2 = \frac{-2}{3} V$$

$$\begin{cases} V_o = 0V \\ V_1 = \frac{2}{3} V \\ V_2 = -\frac{2}{3} V \end{cases}$$

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