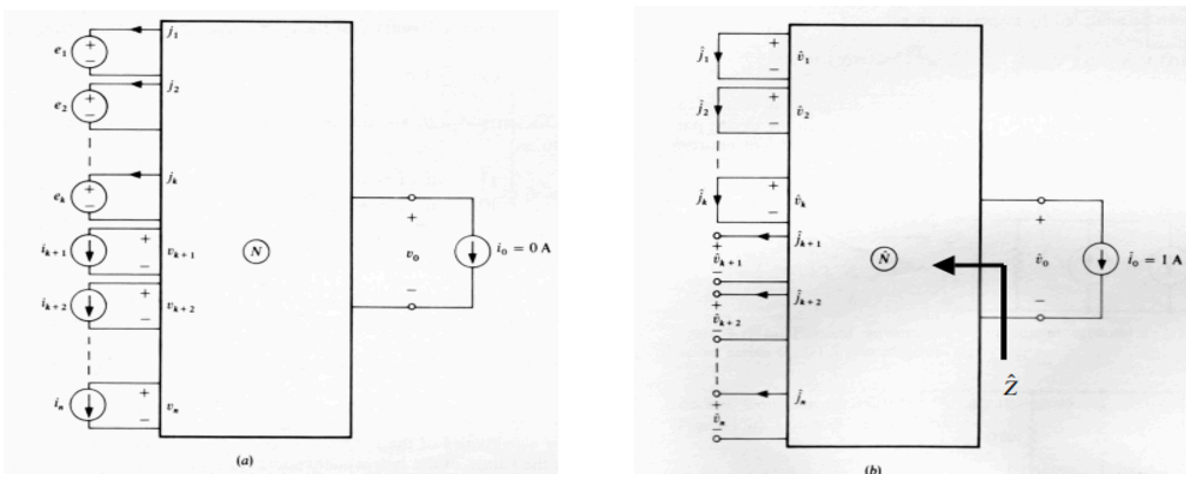
ANALYSIS OF MULTI-SOURCE CIRCUITS

***Problem*: analysis of multi-source circuit. Applications: tolerance analysis, noise analysis.**



Solution using Tellegen's Theorem: consider a conductance multi-port with one output v_o . It may contain resistors and voltage-controlled voltage sources, such as CMOS transistors for incremental signals. In the figure, N is the physical network, N' the *inter-reciprocal (adjoint*) one. Then *j = G.v,* and *j' = G'.v'.* N' has the same configuration, but different elements than N.

By Tellegen's theorem, for all branches

$\boldsymbol{v}^T . \boldsymbol{j}' = \boldsymbol{v}'^T . \boldsymbol{j} = 0.$

Splitting the set of branches into *port branches* and *internal branches*, and assuming only resistors and transconductances in the multi-port circuit **N**, Tellegen's theorem gives

$$[\boldsymbol{v}^T.\ \boldsymbol{j'}]_{\text{port}} + [\boldsymbol{v}^T.\ \boldsymbol{G'}.\boldsymbol{v'}]_{\text{int}} = [\boldsymbol{v'}^T.\ \boldsymbol{j}]_{\text{port}} + [\boldsymbol{v'}^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}}$$

Since $[\boldsymbol{v'}^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}}$ is a scalar, it is equal to its transpose, and hence

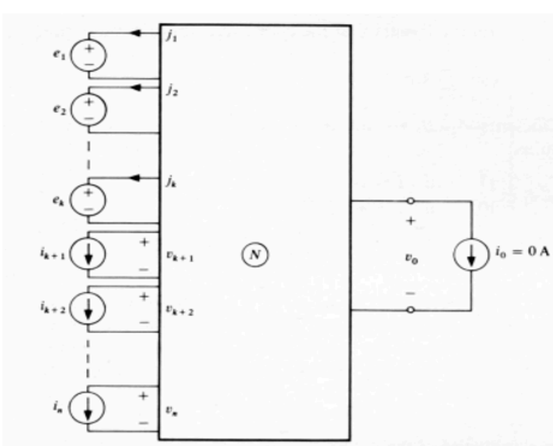
$$[\boldsymbol{v'}^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}} = [\boldsymbol{v'}^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}}^T = [\boldsymbol{v}^T.\ \boldsymbol{G}^T.\boldsymbol{v'}]_{\text{int}} = \boldsymbol{v}^T.\ \boldsymbol{G'}.\boldsymbol{v'}]_{\text{int}}$$

Thus, if we want the internal $\boldsymbol{v}^T.\boldsymbol{j}$ products to be the same for N and N' so that **N'** is the adjoint of **N,** then we have to choose **G' = G**^{T}. Then also

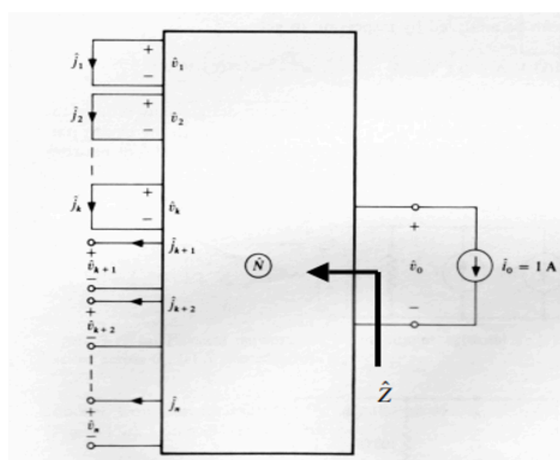
$$[\boldsymbol{v}^T.\ \boldsymbol{j'}]_{\text{port}} = [\boldsymbol{v'}^T.\boldsymbol{j}\ ]_{\text{port}}.$$

Since the resistors enter the branch admittance matrix *G* only in its diagonal, they will remain in place in N'. However, a VCCS described by the relation $j_k = g_{mk}.v_l$ will be turned around in N', to satisfy $j_l = g_{mk}.v_k$.

To analyze a multi-source circuit N with an open-circuit output voltage v_o, choose the source at the output port of N'as 1 A. If the output of N is a short circuit current j_o choose the source of N' as -1 V. All other sources should be set equal to zero in N'.



(a)



(b)

The resulting output is v_o = $\boldsymbol{V.I_v'} + \boldsymbol{J.V_j'}$, where $\boldsymbol{V}$ and $\boldsymbol{J}$ are the sources in **N,** and $\boldsymbol{I_v'}$ and $\boldsymbol{V_j'}$ are the signals in the same branches of **N'.** This also gives all internal gains from the sources to the output in N.

In addition, the output impedance of the physical network with all its sources set to zero can be found from Tellegen's Theorem. It is given by

$Z = -v_o/i_o = -v_o'/i_o' = v_o'$.

Thus, the complete Thevenin equivalent of the physical circuit **N** can be found from a single analysis of **N'**.

Application to Resistor Sensitivity Analysis: Consider an N-port containing resistors and voltage-controlled current sources, with a single output v_o. Let the resistor R_k carrying a current j_k have an error ΔR_k. The error voltage generated will be $j_k.\Delta R_k$ This can be regarded as an additional source of circuit N. Hence, its effect on v_o can be found in the following steps:

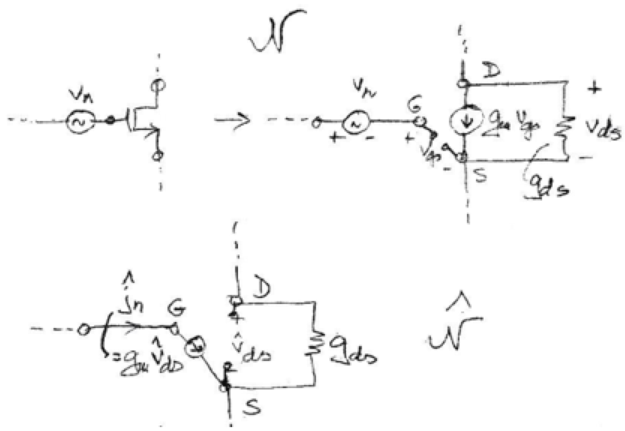
1. Generate N' from $G' = G^T$. This is done by keeping all resistors in place and turning all VCCS around. Set the values of all sources in N to zero in N'.
2. Find the current j_k' in branch k of N'.
3. The contribution of the resistor error on the output will be $j_k\, j_k' \Delta R_{k.}$

Application to Transconductor Sensitivity Analysis: Let N contain in branches k and l a VCCS with the relation $j_k = g_{mk}.v_l$. Assume that the transconductance has an error $\Delta g_{mk\,.}$ This will create an error current $\Delta g_{mk}.v_l$ in branch *l.* This can be regarded as a current source present in branch *l* of N, and hence its effect on the output can be found from the expression $v_l.\ \Delta g_{mk}.v_l'$.

Application to Noise Analysis: Both resistors and CMOS transistors are subject to thermal noise. It can usually be represented by a noise voltage source v_n connected in series with each resistor, and to the gate terminal of the transistor. Since the transistor may be modelled for small signals and low frequency as a voltage-controlled voltage source, inter-reciprocity can be used.

The process is illustrated below. Figure A shows the noisy transistor and its small-signal model; Figure B its image in N'. As discussed above, an error Δg_m introduces a current error in the drain current $\Delta g_m v_{gs}$, which contributes a term $v_{ds}.\Delta g_m.v_{lds}'$ to the output.

In a circuit containing random noise sources at the output it is the noise powers (the mean square values) of the individual noises that need to be added. So the term derived above needs to be squared in the output..



References

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