Yen-Chun Chen
934559635

MIDTERM EXAMINATION

ECE 580

October 16, 2024

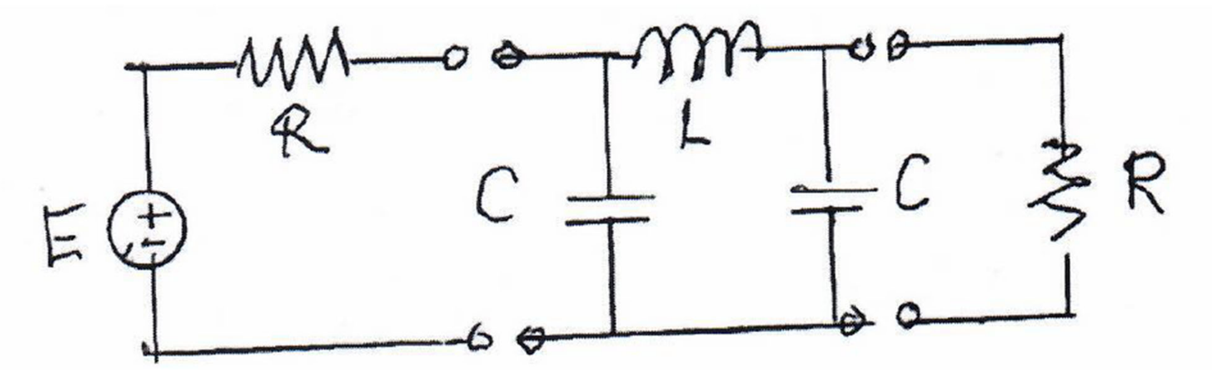
Open Book

1. a. Find the impedance parameters of the LC two-port shown.

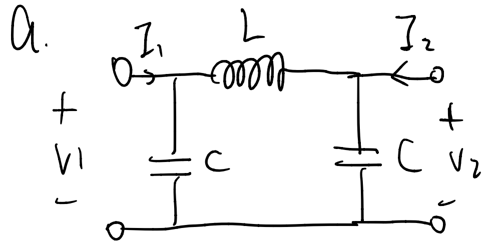
   b. Find the voltage gain $A_v(s)$ and the transducer factor $H(s)$ for the doubly terminated two-port.

   c , Find the reflection coefficient $\rho_1(s)$ and $\rho_2(s)$.

For extra credit: find the values of $H(j\omega)$ and $\rho_1(j\omega)$ for $R = 50\ \Omega$, $\omega = 100$ Mrad/s, $L = 10$ nH and $C = 1$ pF.



a.



From midterm 2023, we got the chain matrix is

$$T=\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{bmatrix} s^2LC+1 & sL \\ sC(s^2LC+2) & s^2LC+1 \end{bmatrix}$$

we can convert T to Z

$$\rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{AD-BC}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix} = \begin{bmatrix} \frac{s^2LC+1}{sC(s^2LC+2)} & \frac{(s^2LC+1)^2 - s^2CL(s^2LC+2)}{sC(s^2LC+2)} \\ \frac{1}{sC(s^2LC+2)} & \frac{s^2LC+1}{sC(s^2LC+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s^2LC+1}{sC(s^2LC+2)} & \frac{1}{sC(s^2LC+2)} \\ \frac{1}{sC(s^2LC+2)} & \frac{s^2LC+1}{sC(s^2LC+2)} \end{bmatrix} \#$$

b.

$$A_V(s) = \frac{V_2(s)}{E(s)} = \frac{R_L}{AR_L + B + CR_GR_L + DR_G}$$

$$= \frac{R}{(s^2LC+1)R + sL + sC(s^2CL+2)R^2 + s^2CLR + R}$$

$$= \frac{1}{s^3RLC^2 + 2s^2LC + s(L/R + 2RC) + 2} \#$$

$$H(s) = \frac{\sqrt{R_L/R_G}}{2A_v} = \frac{\sqrt{R/R}}{2A_v}$$

$$= s^3RLC^2 + 2s^2LC + s(L/R + 2RC) + 2 \quad \#$$

Extra Credits: $R = 50\Omega$, $W = 10^8$ rad/s, $L = 10^{-8}H$, $C = 10^{-12}F$

$$H(s) = (10^8 j)^3 \cdot 50 \times 10^{-8} \times 10^{-12} + 2 \cdot (10^8 j)^2 \cdot 10^{-8} \times 10^{-12}$$
$$+ (j \times 10^8)\left(10^{-8} \cdot \frac{1}{50} + 2 \cdot 50 \times 10^{-12}\right) + 2$$

$$\approx 2 + 0.03j \quad \#$$

c. $$\rho(s) = \frac{Z_{in}(s) - Z_0}{Z_{in}(s) + Z_0}, \quad Z = \begin{bmatrix} \frac{s^2LC+1}{sC(s^2LC+2)} & \frac{1}{sC(s^2LC+2)} \\ \frac{1}{sC(s^2LC+2)} & \frac{s^2LC+1}{sC(s^2LC+2)} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\rho_1(s) = \frac{Z_{in} - Z_{source}}{Z_{in} + Z_{source}} = \frac{Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}}{Z_{11} + \frac{Z_{12}Z_{21}}{Z_{22}}} \quad \#$$

$$Z_{in}(s) = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_L}$$

$$\rho_2(s) = \frac{Z_{out}(s) - Z_L}{Z_{out}(s) + Z_L} = \frac{Z_{22} - \dfrac{Z_{12} Z_{21}}{Z_{11}}}{Z_{22} + \dfrac{Z_{12} Z_{21}}{Z_{11}}}$$

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$$Z_{out}(s) = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_s}$$