MIDTERM EXAMINATION

October 19,2022 Open Book

1. Find the short-circuit parameters of the circuit shown below. Assume R = 5 kΩ, C = 10 pF and f = 3 MHz.

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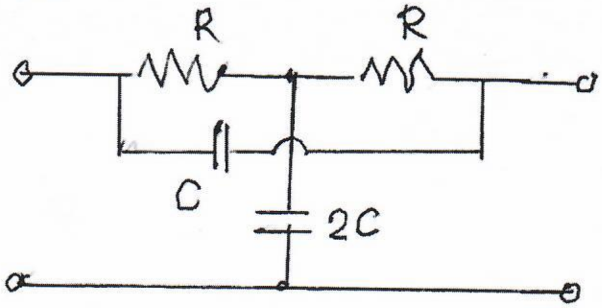
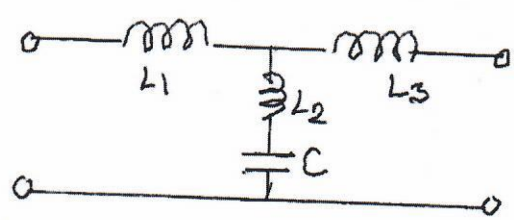
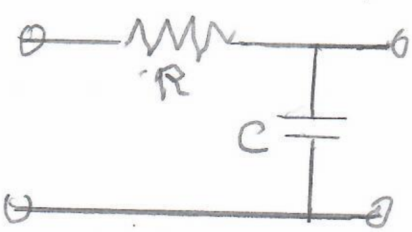
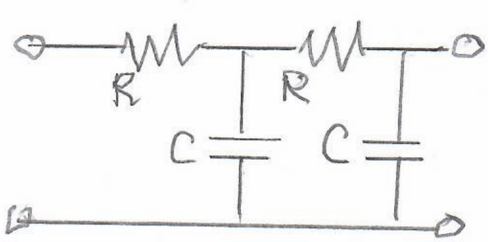


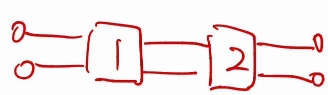
2. Two of the short-circuit admittances of the circuit shown are $y_{11} = y_{22} = (7s^2 + 1)/(16s^3 + 4s)$. Find all element values.



3. Find the chain matrices of the two-ports shown below.

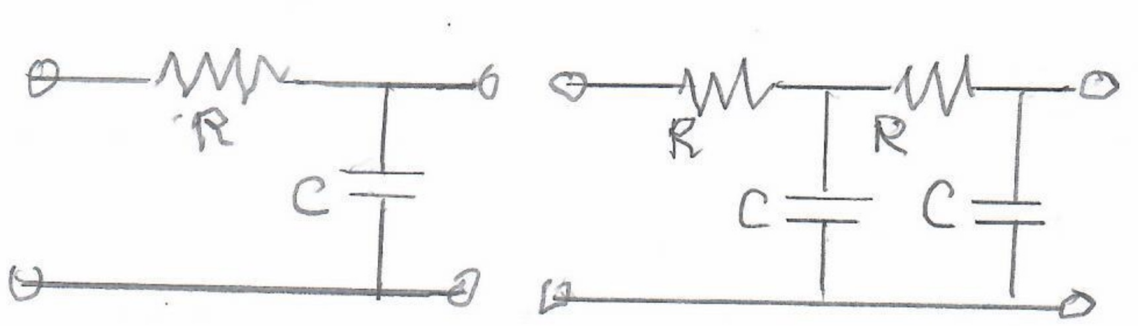






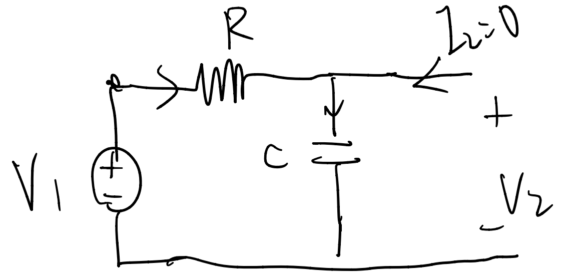
3.

3. Find the chain matrices of the two-ports shown below.



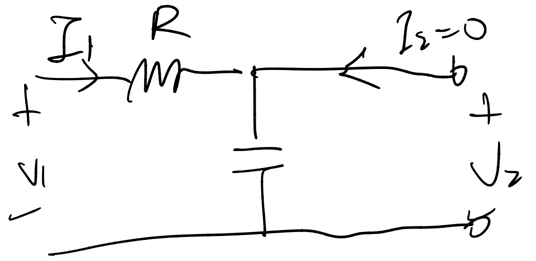
$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} sCR+1 & R \\ sC & 1 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = sCR+1$$



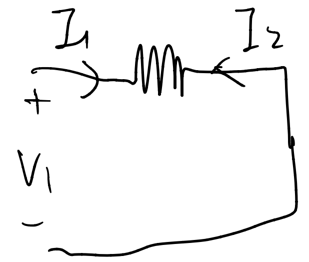
$$\frac{V_1 - V_2}{\cancel{R}} = sCV_2R$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = sC$$



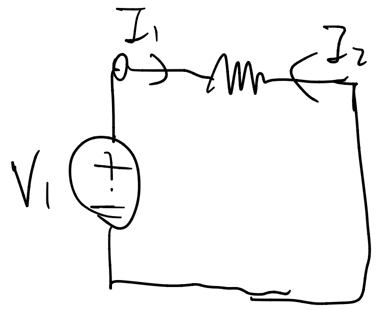
$$I_1 = V_2 \cdot sC$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = R$$



$$\frac{V_1}{R} = -I_2$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$



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Since it's cascade connection and two R has same value, two C same value chain matrices will be T^2

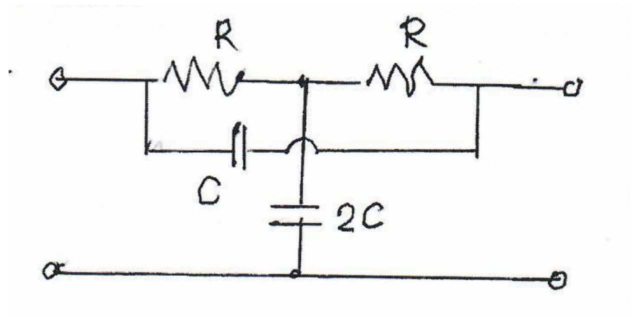
$$T^2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A^2+BC & AB+BD \\ AC+DC & BC+D^2 \end{bmatrix}$$

$$= \begin{bmatrix} (sCR+1)^2 + RsC & (sCR+1)R + R \\ (sCR+1)sC + sC & sCR+1 \end{bmatrix}$$

$$s^2C^2R + 2sC$$

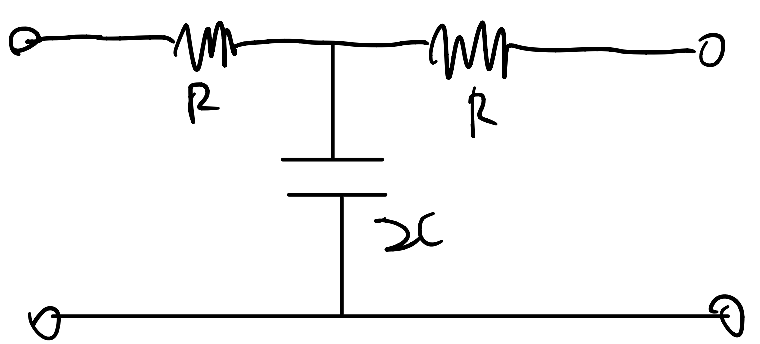
$$sCR^2 + 2R$$

1

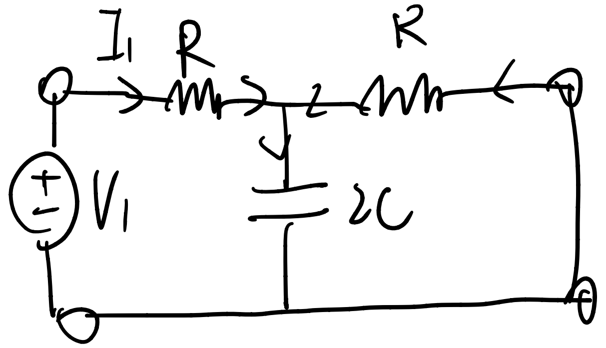


$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow \begin{array}{l} I_1 = y_{11} \cdot V_1 + y_{12} V_2 \\ I_2 = y_{21} \cdot V_1 + y_{22} V_2 \end{array}$$

First Part



$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{2SRC+1}{2SR^2C+2R} = y_{22}$$



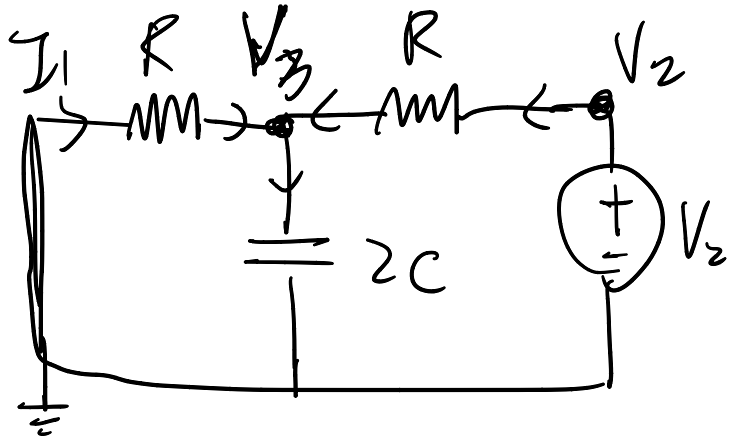
$$I_1 = \frac{V_1}{R + (R // \frac{1}{2SC})}$$

$$I_1 = \frac{V_1}{\frac{2SR^2C+2R}{2SRC+1}}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = \frac{1}{2SR^2C+2R}$$

$$\|$$

$$y_{21}$$



$$\begin{cases} V_3 = -I_1 R \\ I_1 + \frac{V_2 - V_3}{R} = V_3 SC2 \end{cases}$$

$$\left\{ V_2 = -I_1 R + \frac{-I_1 R}{(R // \frac{1}{SC})} \right.$$

$$Y_a = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2SRC+1}{2SR^2C+R} & \frac{1}{2SR^2C+2R} \\ \frac{1}{2SR^2C+2R} & \frac{2SRC+1}{2SR^2C+R} \end{bmatrix}$$

$$R+\left(R//\frac{1}{2SC}\right) = \frac{\frac{R}{2SC}}{R+\frac{1}{2SC}} = \frac{R}{\cancel{2SC}\cdot\left(\frac{2S\quad +1}{\cancel{SC}}\right)}$$

$$I_1 + \frac{V_2}{R} + \frac{I_1\cancel{R}}{\cancel{R}} = -I_1RSC\cdot 2$$

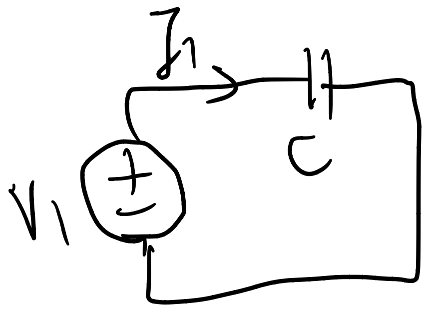
$$2I_1 + 2SRC\,I_1 + V_2/R = 0$$

$$2(SRC+1)R\,\frac{I_1}{V_2} + \cancel{V_2} = 0$$

$$= \frac{R}{2SRC+1} = \frac{2SR^2C+2R}{2SRC+1}$$

$$\frac{I_1}{V_2} = \frac{1}{2SR^2C+2R}$$

Second Part



$$y_{11b} = \frac{I_1}{V_1}\Bigg|_{V_2=0} = SC = y_{12b} = -y_{21b} = -y_{22b} \rightarrow y_b = \begin{bmatrix} SC & -SC \\ -SC & SC \end{bmatrix}$$

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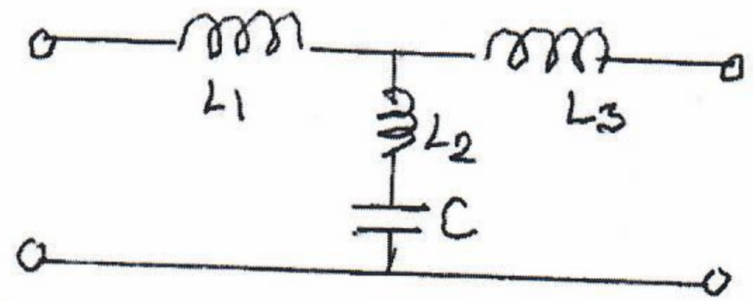
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} y_{11b} & y_{12b} \\ y_{21b} & y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} y_{11a}+y_{11b} & y_{12a}+y_{12b} \\ y_{21a}+y_{21b} & y_{22a}+y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$R = 5k\Omega$, $C = 10\,pF$, $f = 3MHz$, $SC = j\omega C = 2\pi f C = 2\pi \times 3 \times \cancel{10^6} \times \cancel{10} \times \cancel{10^{-12}} = 6\pi \times 10^{-5}$

$$\begin{bmatrix} \dfrac{2SRC+1}{2SR^2C+R} + SC & \dfrac{1}{2SR^2C+2R} - SC \\ \dfrac{1}{2SR^2C+2R} - SC & \dfrac{2SRC+1}{2SR^2C+R} + SC \end{bmatrix}$$

$$\frac{2 \times 6\pi \times 10^{-5} \times 5 \times 10^3 + 1}{2 \cdot 6\pi \times 10^{-5} \times 25 \times 10^6 + 5 \times 10^3} + 6\pi \times 10^{-5}$$

2. Two of the short-circuit admittances of the circuit shown are $y_{11} = y_{22} = (7s^2+1)/(16s^3+4s)$. Find all element values.



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

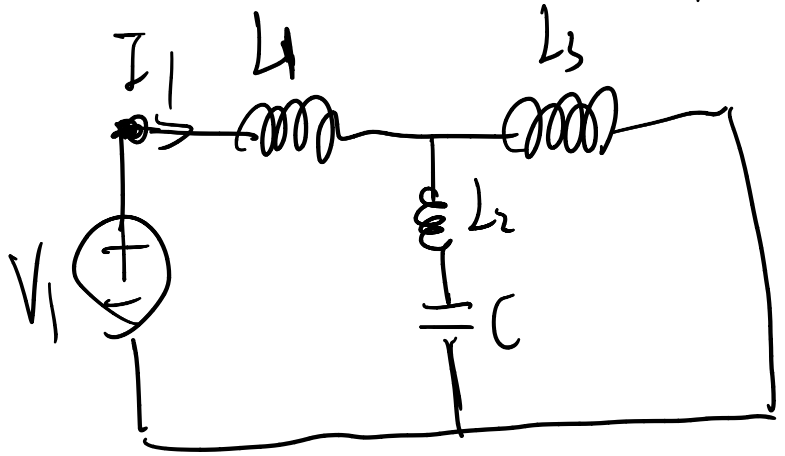
$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$y_{11} = y_{22} = \frac{7s^2+1}{s(16s^2+4)}$$

$$sL_2 + \frac{1}{sC} = \frac{s^2 C L_2 + 1}{sC}$$

$$y_{11} = y_{22} = \frac{I_1}{V_1}\Big|_{V_2=0}$$



$$I_1 = \frac{V_1}{sL_1 + \left[\left(sL_2 + \frac{1}{sC}\right) // sL_3\right]}$$

$$\left(sL_2 + \frac{1}{sC}\right) // sL_3 = \frac{s^3 C L_2 L_3 + sL_3}{s^2 C (L_2 + L_3) + 1}$$

$$\left[\left(sL_2 + \frac{1}{sC}\right) // sL_3\right] + sL_1 =$$

$$\frac{s^3 C L_2 L_3 + sL_3 + s^3 C (L_2+L_3) L_1 + sL_1}{s^2 C (L_2+L_3)+1} = \frac{s\left[s^2 C (L_1 L_2 + L_1 L_3 + L_2 L_3) + (L_1 + L_3)\right]}{s^2 C (L_2 + L_3) + 1}$$

$$= \frac{s(16s^2+4)}{7s^2+1}$$

$$
\begin{cases}
C(L_2+L_3) = 7 \\
L_1 + L_3 = 4 \\
L_1L_2 + L_1L_3 + L_2L_3 = 16 \\
L_1 = L_3
\end{cases}
\rightarrow
\begin{cases}
L_1 = L_3 = 2\,H \\
C(L_2+2) = 7 \rightarrow C = \dfrac{7}{5} = 1.4\,F \\
4L_2 = 12 \rightarrow L_2 = 3\,H
\end{cases}
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