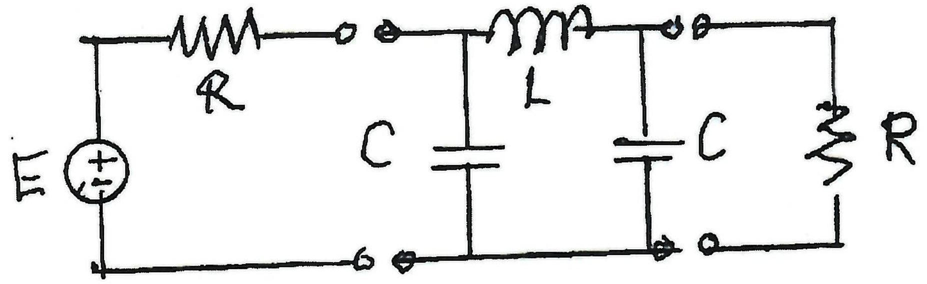
2023

1. (a) Find the chain matrix of the two-port shown below.

   (b) Find the voltage gain V_2/E if the two-port operates between two equal resistors R.



$V_1 = AV_2 - BI_2 \qquad I_1 = CV_2 - DI_2$

$A = (V_1/V_2)_{I_2=0} = \dfrac{sL + 1/sC}{1/sC} = s^2LC + 1$

$B = V_1/-I_2 \,|_{V_2=0} = sL$

$C = I_1/V_2 \,|_{I_2=0} = sC(s^2LC + 2)$

$D = I_1/-I_2 \,|_{V_2=0} = s^2LC + 1$

$AD - BC = (s^2LC + 1)^2 - s^2LC(s^2LC + 2) = 1$ ✓

Lecture notes p. 67:

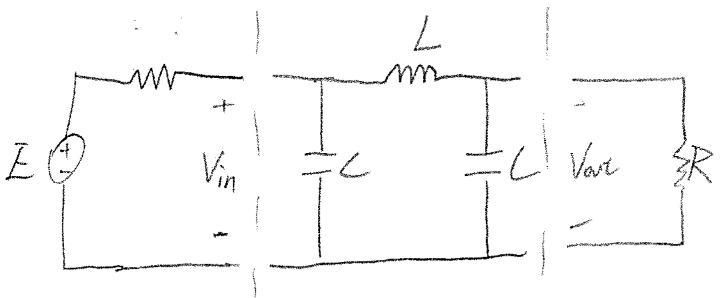
$\dfrac{V_2}{E} = \dfrac{1}{A + B/R + CR + D}$

$V_2/E = 1/[2(s^2LC+1) + sL/R + sRC(s^2LC + 2)$

$V_2/E = 1/[s^3RLC^2 + s^2 2LC + s(L/R + 2RC) + 2]$

2023 chain matrix A-B-C-D

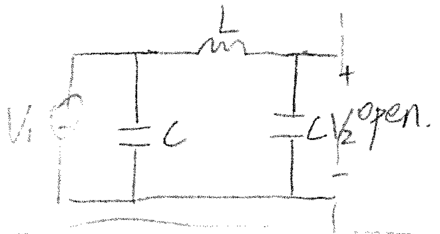
P37-38



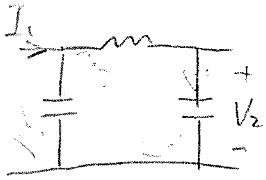
2-port network

$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} \Rightarrow \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} = \frac{sL + 1/sC}{1/sC} = s^2LC + 1$$



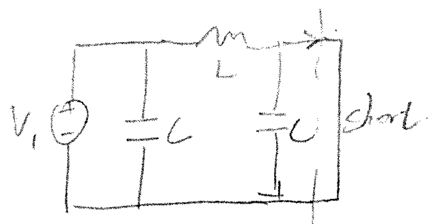
$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} = sC(s^2LC+2)$$



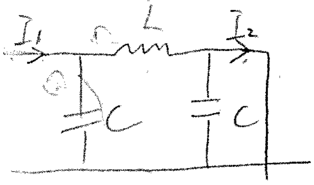
$$I_1 = \frac{\frac{1}{sC}+sL}{1/sC} V_2 \cdot sC + V_2 \cdot sC$$

$$(2+s^2LC)sC$$

$$B = \frac{V_1}{-I_2}\bigg|_{V_2=0} = sL$$



$$D = \frac{I_1}{-I_2}\bigg|_{V_2=0} = 1 + s^2LC$$



$$I_1 = -I_2 + -I_2 \cdot sL \cdot sC$$

$$\Delta \begin{bmatrix} A & B \\ C & D \end{bmatrix} = AD - BC = (s^2LC+1)^2 - sL \cdot sC(s^2LC+2) = 1 \Rightarrow \text{reciprocal two-ports}$$

$$s^4L^2C^2 + 2s^2LC + 1 - s^4L^2C^2 - 2s^2LC$$

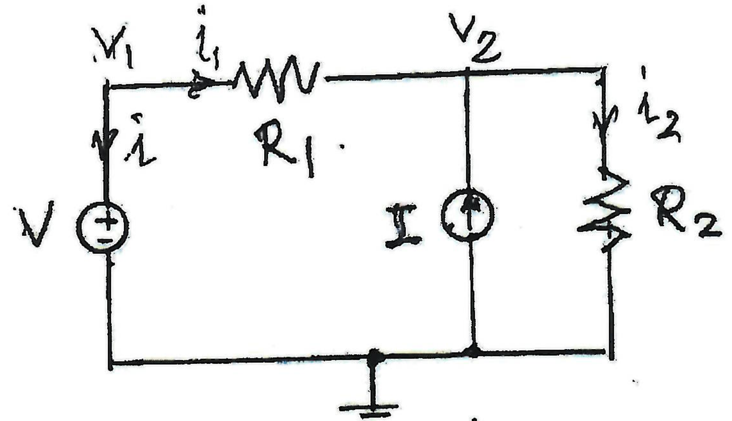
(b) P_{62} / P_{67}

$$A_v = \frac{V_2}{E} = \frac{R_L}{AR_L + B + CR_G R_L + DR_G}$$

$$= \frac{R}{s^2RLC + R + sL + s^3R^2LC^2 + s2R^2C + s^2RLC + R}$$

$$= \frac{1}{s^3RLC^2 + s^2 2LC + s(L/R + 2RC) + 2}$$

2. Find all voltages and currents in the circuit below. The element values are $R_1 = 1\ k\Omega$, $R_2 = 2\ k\Omega$, $V = 3\ V$ and $I = 1\ mA$.



Note : Use MNA !

KCL1 : $(V_1 - V_2)/R_1 + i = 0$

KCL2 : $(V_2 - V_1)/R_1 + V_2/R_2 = I$.

KVL : $V_1 = V$

$$\begin{bmatrix} G_1 & -G_1 & 1 \\ -G_1 & G_1+G_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ I \\ V \end{bmatrix}$$

Solution:

$-G_1 V_2 + i = -G_1 V$

$-G_1 V + (G_1 + G_2) V_2 = I$

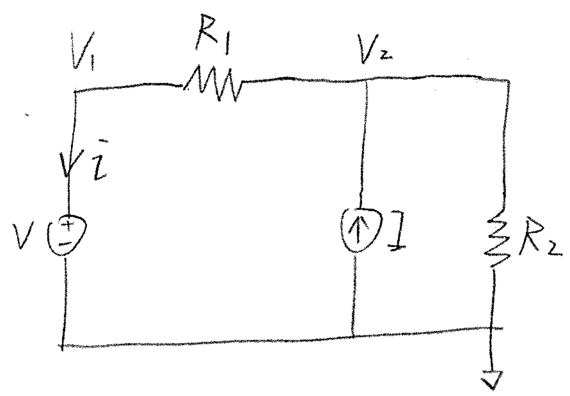
$V_2 = (I + G_1 V)/(G_1 + G_2) = \dfrac{10^{-3} + 3 \times 10^{-3}}{1.5 \times 10^{-3}}$

$V_2 = 8/3\ V$, $i = +10^{-3}(8/3 - 3) = -\frac{1}{3}$ mA

$i_2 = I - i = 4/3$ mA $\quad V_2 = 8/3\ V$

2:

$\boxed{\text{MNA}}$



$$KCL1: \frac{(V_1 - V_2)}{R_1} + i = 0$$

$$KCL2: \frac{(V_2 - V_1)}{R_1} + \frac{V_2}{R_2} = I$$

$$KVL: \quad V_1 = V$$

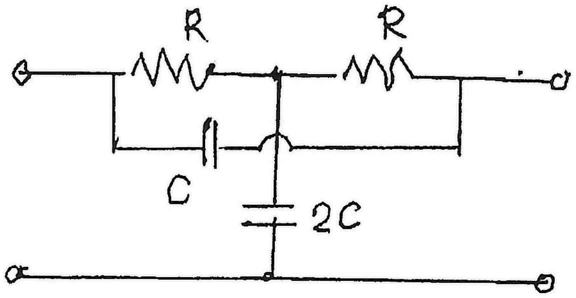
$$\begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 0 \\ -\frac{1}{R} & \frac{1}{R_1}+\frac{1}{R_2} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ I \\ V \end{bmatrix}$$

2022

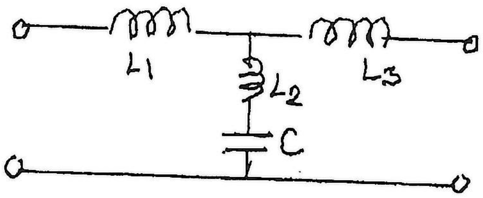
MIDTERM EXAMINATION

October 19,2022 Open Book

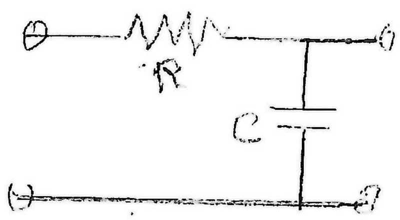
1. Find the short-circuit parameters of the circuit shown below. Assume R = 5 kΩ, C = 10 pF and f = 3 MHz.

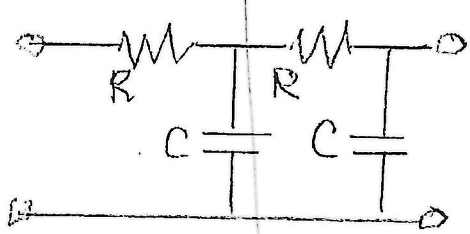


2. Two of the short-circuit admittances of the circuit shown are $y_{11} = y_{22} = (7s^2 + 1)/(16s^3 + 4s)$. Find all element values.



3. **Find the chain matrices of the two-ports shown below.**

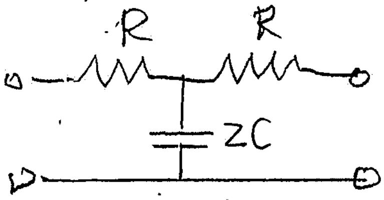




PTO

Solutions.

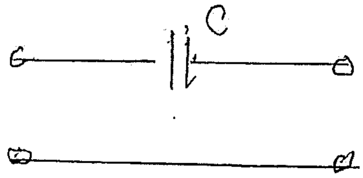
1. Splitting into parallel two-ports.



$$y_{11a} = y_{22a} = \frac{G}{2}\,\frac{2sC + \frac{G}{2}}{sC + \frac{G}{2}}$$

$$y_{12a} = y_{21a} = -\frac{G}{2}\,\frac{G}{sC + G}$$

$$\frac{1}{2sR^2C + 2R}$$



$$y_{11b} = y_{22b} = -y_{12b} = -y_{21b} = j\omega C$$

$$\frac{G}{2}$$

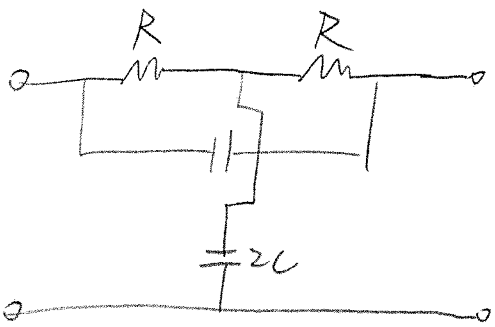
$$\underset{\sim}{Y} = \begin{bmatrix} 1.47 + j2.38 & -0.53 - j1.38 \\ & \end{bmatrix} \times 0.1\,\mathrm{mS}$$

10^{-4}

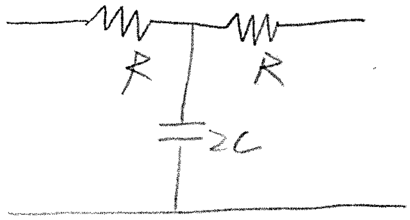
2. Next sheet
3. Next sheet

2022.

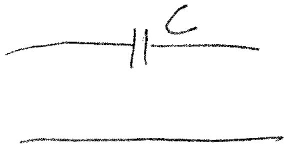
1.



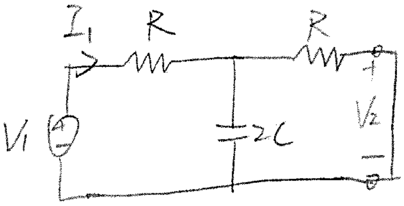
$\Rightarrow$



T-type.



$$y_{11a} = y_{22a} = \left.\frac{I_1(s)}{V_1(s)}\right|_{V_2=0} = \frac{2sRC+1}{2sR^2C+2R}$$

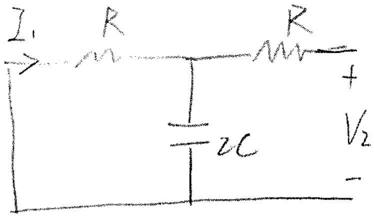


$$R \parallel \frac{1}{2sC} + R = \frac{R \cdot \frac{1}{2sC}}{R + \frac{1}{2sC}} + R$$

$$= \frac{2sR^2C+2R}{2sRC+1} \xrightarrow{(\ )^{-1}}$$

$$y_{12a} = y_{21a} = \left.\frac{I_1(s)}{V_2(s)}\right|_{V_1=0} = -\frac{1}{2sR^2C+2R}$$

$$\frac{R}{2sRC+1}$$



$$V_2 = -I_1 R + \frac{-I_1 R}{\left(R \parallel \frac{1}{2sC}\right)} R$$

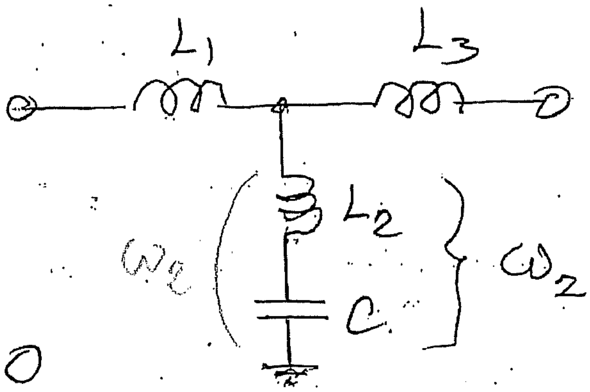
$$= -I_1 R(1 + 2sRC+1)$$

$$\underline{y_{11b} = y_{22b} = -y_{12b} = -y_{21b}} = sC$$

$R = 5\times10^3\,\Omega$, $C = 10^{-11}F$, $f = 3\times10^6\,Hz$. $s = j\omega = j\cdot 2\pi f \doteq 1.88\times10^7 j$

$$\begin{bmatrix} y_{11a}+y_{11b} & y_{12a}+y_{12b} \\ y_{21a}+y_{21b} & y_{22a}+y_{22b} \end{bmatrix} = \begin{bmatrix} 1.47+2.38j & -0.53-1.38j \\ & \end{bmatrix} \times 10^{-4}$$

3. Find the element values of the two-port shown from $y_{11} = y_{22} = (7s^2+1)/(16s^3+4s)$.



For $s = 0$

$$y_{11}^{-1} = y_{22}^{-1} = s(L_1 + L_3) \Rightarrow L_1 + L_3 = 4H$$

For $s = j\omega_2$

$$y_{11}^{-1} = y_{22}^{-1} = j\omega_2 L_1 + j\omega_2 L_3 \Rightarrow L_1 = L_3 = 2H$$

For $s \to \infty$

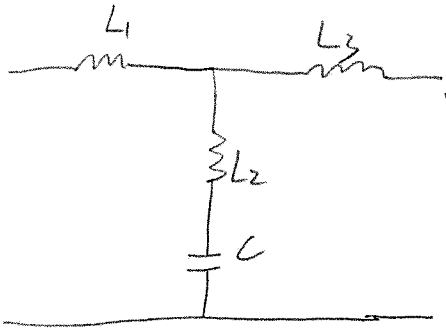
$$y_{11}^{-1} \to sL_1 + s\frac{L_1 L_2}{L_1 + L_2} = sL_1\left(1 + \frac{L_2}{L_1 + L_2}\right) = s\frac{16}{7}$$

$L_1/L_2 = 6$, $L_2 = 1/3\,H$

$$y_{11}^{-1} - sL_1 = \frac{16s^3 + 4s}{7s^2 + 1} - s2 = \frac{2s^3 + 2s}{7s^2 + 1}$$

Hence, $\omega_2 = \pm 1$, $C_2 = 1/L_2 = 3F$

2.



$y_{22} = y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

$sL_2 + \frac{1}{sC_2} = \frac{s^2L_2C_2+1}{sC_2}$

$\left(\frac{s^2L_2C_2+1}{sC_2} \,||\, sL_3\right) + sL_1$

$\frac{\frac{s^2L_2C_2+1}{sC_2} sL_3}{\frac{s^2L_2C_2+1}{sC_2} + sL_3} = \frac{s^3L_2L_3C_2 + sL_3}{s^2(L_2C_2+L_3C_2)+1}$

$s=0$, C open

$y_{11}^{-1} = y_{22}^{-1} = s(L_1+L_3) = 4$

$+sL_1 \quad \stackrel{-1}{=} \quad \frac{s^2(L_2+L_3)C_2+1}{s^3L_2L_3C_2+sL_3+s^3L_1(L_2+L_3)C_2+sL_1}$

$s=j\omega_2$. $L_2 \cdot C_2$ short

$y_{11}^{-1} = y_{22}^{-1} = j\omega_2 L_1 = j\omega_2 L_3$

$L_1 = L_3 = 2$

$\begin{cases} (L_2+L_3)C_2 = 7 \\ L_2L_3C_2 + L_1(L_2+L_3)C_2 = 16 \\ L_1 + L_3 = 4 \\ L_1 = L_3 \end{cases}$

$L_1 = L_3 = 2$

$\begin{cases} (L_2+2)C_2 = 7 \\ L_2 = \frac{1}{3} \\ C_2 = \end{cases}$

$(L_2+2)C_2 = 7$

$(2L_2+2L_2+4)\frac{7}{L_2+2} = 16$

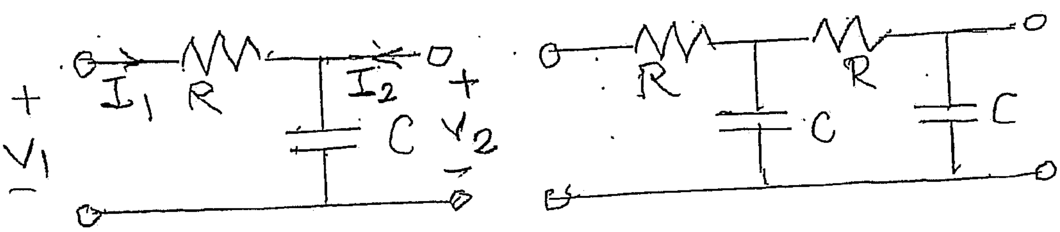
$28L_2 + 28 = 16L_2 + 32$

$L_2 = 4$

L_2

$C_2 =$

2.



Solutions

1a. $I_1 = sCV_2 - I_2$

$$V_1 = RI_1 + V_2 = R(sCV_2 - I_2) + V_2$$

$$\underset{\sim}{T} = \begin{bmatrix} sRC + 1 & +R \\ sC & +1 \end{bmatrix}$$

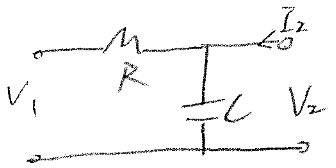
b. Find $\underset{\sim}{T}^2 = \begin{bmatrix} (sRC+1)^2 + sRC & R(sRC+1) + R \\ sC(sRC+1) + sC & sRC + 1 \end{bmatrix}$

3. A B C D

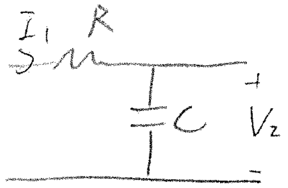
$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = sRC+1$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = sC$$

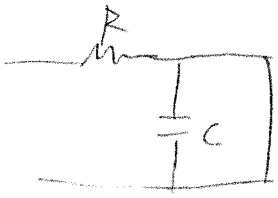


$$V_1 \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = V_2$$

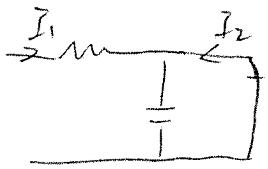


$$\frac{V_2}{\frac{1}{sC}} = I_1$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = R$$



$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$



$$T = \begin{bmatrix} sRC+1 & R \\ sC & 1 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} (sRC+1)^2 + sRC & R(sRC+1) + R \\ (sRC+1)sC + sC & sRC+1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} AA + BC & AB + BD \\ AC + CD & BC + D \cdot D \end{bmatrix}$$