ANALYSIS OF MULTI-SOURCE CIRCUITS

Tellegen's Theorem: conductance multi-port with one output.

For all branches

$\boldsymbol{v}^T.\ \boldsymbol{j}' = \boldsymbol{v}'^T.\ \boldsymbol{j} = 0.$

Splitting port branches and internal branches, and assuming only resistors and transconductances in the multi-port circuit **N**, Tellegen's theorem gives

$[\boldsymbol{v}^T.\ \boldsymbol{j}']_{\text{port}} + [\boldsymbol{v}^T.\ \boldsymbol{G}'.\boldsymbol{v}']_{\text{int}} = [\boldsymbol{v}'^T.\ \boldsymbol{j}]_{\text{port}} + [\boldsymbol{v}'^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}}$

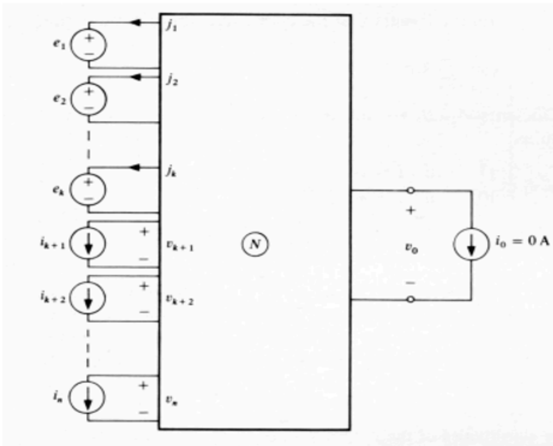
If **N'** is the adjoint of **N,** then $\mathbf{G}' = \mathbf{G}^T$. Since $[\boldsymbol{v}'^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}}$ is a scalar, it is equal to its transpose, and hence

$[\boldsymbol{v}'^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}} = [\boldsymbol{v}'^T.\ \boldsymbol{G}.\boldsymbol{v}]_{\text{int}}^T = [\boldsymbol{v}^T.\ \boldsymbol{G}'.\boldsymbol{v}']_{\text{int}}$.

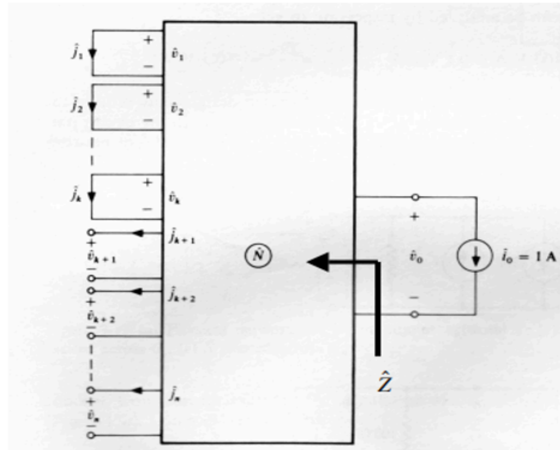
Therefore, also

$[\boldsymbol{v}^T.\ \boldsymbol{j}']_{\text{port}} = [\boldsymbol{v}'^T.\boldsymbol{j}\ ]_{\text{port}}$

To analyze the multi-source circuit **N**, in **N'** choose the source at the output port as 1 A (if we want an output voltage v_o) or -1 V (for i_o), and all sources equal to zero.



(a)



(b)

Then v_o = $\boldsymbol{V.I_v'} + \boldsymbol{J.V_j'}$, where $\boldsymbol{V}$ and $\boldsymbol{J}$ are the sources in **N,** and $\boldsymbol{I_v'}$ and $\boldsymbol{V_j'}$ are in the same branches of **N'.** This also gives all gains from the sources to the output.

In addition, the output impedance of the physical network for $\boldsymbol{V} = \mathbf{0}$ and $\boldsymbol{J} = \mathbf{0}$ is given by

$Z = -v_o/i_o = -v_o'/i_o' = v_o'$.

Thus, the complete Thevenin equivalent of the physical circuit **N** can be found from a single analysis of **N'**.