ECE 580 HW3

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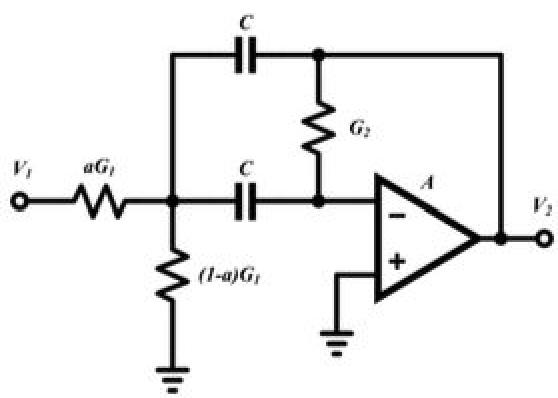
1. Find the expressions in terms of the element values for the dc gain, the pole frequency (ωo) and the pole Q (Qp), for the Delyiannis and Rauch filters.

Delyiannis-Friend Filter:



Transfer Function with an ideal op-amp:

$$H(s) = -\frac{\mathrm{saCG_1}}{s^2C^2 + 2\mathrm{sCG_2} + G_1G_2} = -\frac{\frac{\mathrm{saG_1}}{C}}{s^2 + s\frac{2G_2}{C} + \frac{G_1G_2}{C^2}}$$

The DC Gain: $\lim_{s\to 0} H(s) = H(0) = 0$

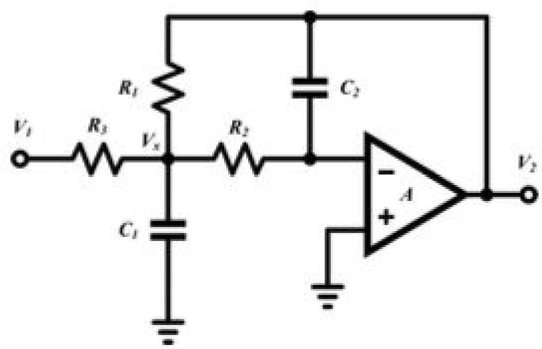
The transfer function for a general second order low-pass filter is given

$H(s) = \dfrac{A_v\,\omega_0{}^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0{}^2}$, we got that $\omega_0^2 = \dfrac{G_1G_2}{C^2}$ Thus, the pole frequency $\omega_0 = \dfrac{\sqrt{G_1G_2}}{C}$

Pole Q is determined by the coefficient of *s* in the denominator

So we got $\dfrac{\omega_0}{Q_p} = \dfrac{2G_2}{C} \rightarrow Q_p = \dfrac{\sqrt{G_1G_2}}{C}\dfrac{C}{2G_2} = \dfrac{1}{2}\sqrt{\dfrac{G_1}{G_2}}$

Rauch Filter:



Transfer Function: $H(s) = \dfrac{\dfrac{G_2G_3}{C_1C_2}}{s^2 + s\left(\dfrac{G_1+G_2+G_3}{C_1}\right) + \dfrac{G_1G_2}{C_1C_2} + \varepsilon}$

$$\varepsilon = \frac{1}{A}\left[s^2 + s\left(\frac{G_1+G_2+G_3}{C_1} + \frac{G_2}{C_2}\right) + \frac{(G_1+G_3)G_2}{C_1C_2}\right]$$

For an ideal op-amp, $A \to \infty$ leads to $\varepsilon \to 0$. Thus, the DC Gain: $\lim_{s\to 0} H(s) = H(0) = \dfrac{\dfrac{G_2G_3}{C_1C_2}}{\dfrac{G_1G_2}{C_1C_2}} = \dfrac{G_3}{G_1}$

The transfer function for a general second order low-pass filter is given

$H(s) = \dfrac{A_v\,\omega_0{}^2}{s^2 + \dfrac{\omega_0}{Q}s + \omega_0{}^2}$, we got that $\omega_0^2 = \dfrac{G_1G_2}{C_1C_2}$ Thus, the pole frequency $\omega_0 = \sqrt{\dfrac{G_1G_2}{C_1C_2}}$

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So we got $\dfrac{\omega_0}{Q_p} = \dfrac{G_1+G_2+G_3}{C_1} \to Q_p = \sqrt{\dfrac{G_1G_2}{C_1C_2}}\left(\dfrac{C_1}{G_1+G_2+G_3}\right)$

2. Design a Rauch filter for the following specifications:

$$R_1 = 1\ k\Omega,\ R_2 = 0.5\ k\Omega,\ DC\ gain = 1,\ Q_p = 2^{-\frac{1}{2}},\ \omega_0 = 40\pi\ krad/s$$

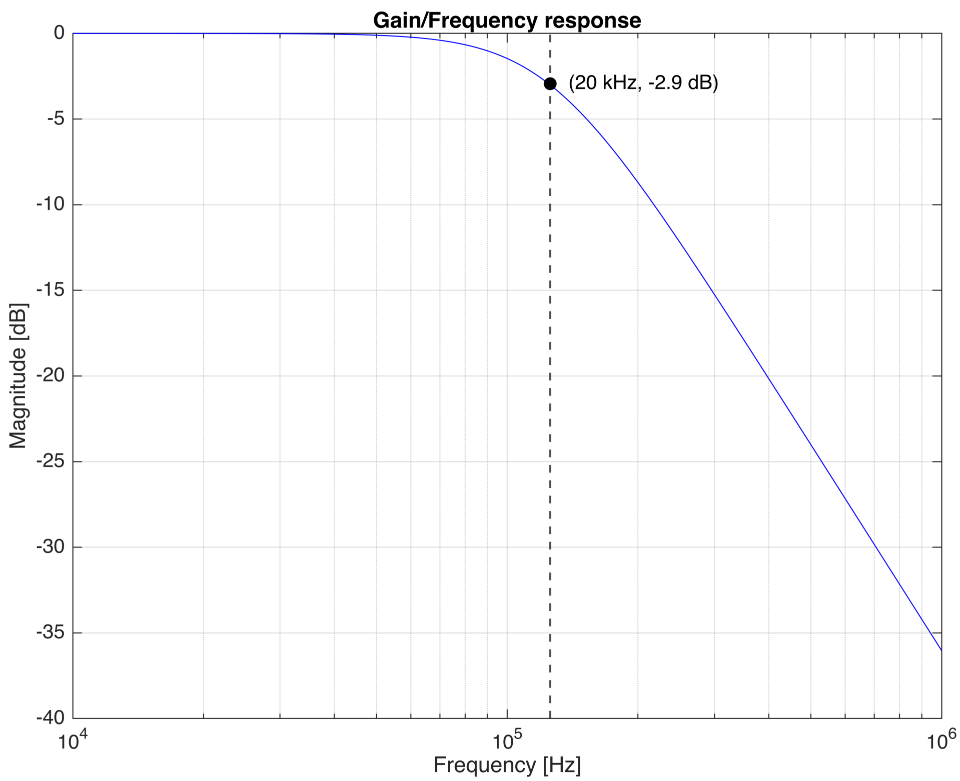
Please output the performance characteristics of the filter (gain/frequency response/zero pole, etc.)

```
clear
close all
clc

R1 = 1e3;         % 1k ohms
R2 = 500;         % 500 ohms
DC_gain = 1;      % DC Gain
Q_p = 2^(-0.5);   % Quality factor
w_0 = 2*pi*20e3;  % Pole Frequency (rad/s)

G1 = 1/R1;
G2 = 1/R2;
G3 = G1*DC_gain; % H(0) = G3/G1

% Calculate C1 and C2
C1C2 = G1 * G2 / w_0^2;
C1 = Q_p * (G1 + G2 + G3) / w_0;
C2 = C1C2 / C1;
fprintf('G1 = %.d mho\nG2 = %.d mho\nG3 = %.d mho\nC1 = %.6e F\nC2 = %.6e
F\nDC Gain = %.2f\nQ_p = %.4f\nw_0 = %.2f rad/s', G1, G2, G3, C1, C2,
DC_gain, Q_p, w_0);
```

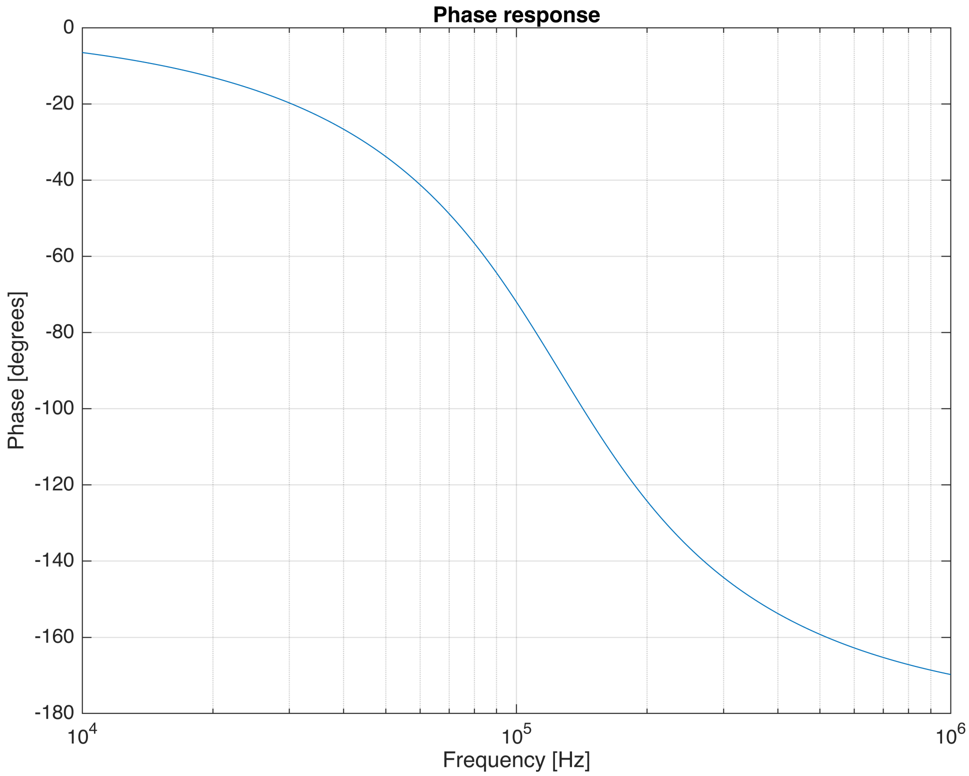
```
G1 = 1e-03 mho
G2 = 2e-03 mho
G3 = 1e-03 mho
C1 = 2.250791e-08 F
C2 = 5.626977e-09 F
DC Gain = 1.00
Q_p = 0.7071
w_0 = 125663.71 rad/s
```

```
num = [0 , 0 , (G2 * G3)/(C1 * C2)];          % Numerator
den = [1 , (G1+G2+G3)/C1 , (G1*G2)/(C1*C2)]; % Denominator
tf_sys = tf(num, den);
[H, w] = freqs(num, den);

% Magnitude response
mag_dB = 20*log10(abs(H));
[~, idx] = min(abs(w - w_0)); % Find the magnitude at w_0
mag_at_w0 = mag_dB(idx);

figure(1);
semilogx(w, mag_dB, 'b');
% Plot the intersection point
hold on;
```

```
xline(w_0, 'k--', 'LineWidth', 1);
plot(w_0, mag_at_w0, 'ko', 'MarkerSize', 6, 'MarkerFaceColor', 'k');
text(w_0*1.1, mag_at_w0, sprintf('(%0.d kHz, %0.1f dB)', w_0/(2*pi)/1000,
mag_at_w0));
hold off;
title("Gain/Frequency response");
ylabel("Magnitude [dB]");
xlabel("Frequency [Hz]");
grid on;
```



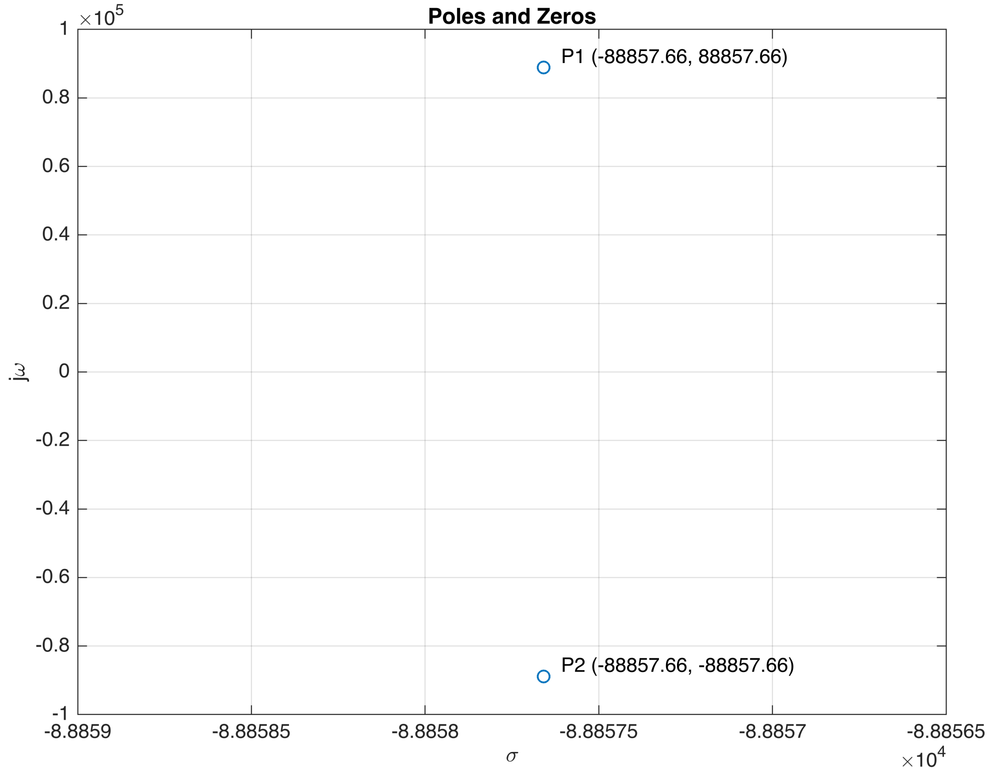

```
%Phase Response
figure(2);
semilogx(w, 180/pi*unwrap(angle(H))); % Plot phase in degrees
title("Phase response");
ylabel("Phase [degrees]");
xlabel("Frequency [Hz]");
grid on
```



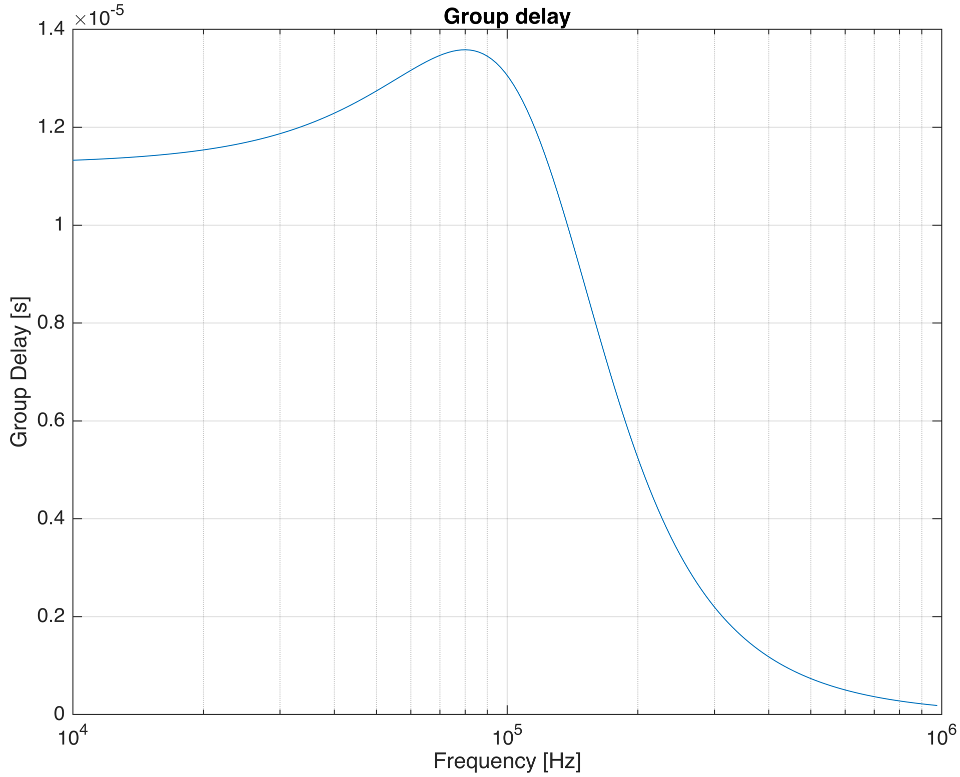

```
% Poles and Zeros
[z, p, k] = tf2zp(num, den)
```

```
z =
  0×1 empty double column vector
p = 2×1 complex
10^4 ×
  -8.8858 + 8.8858i
  -8.8858 - 8.8858i
k =
1.5791e+10
```

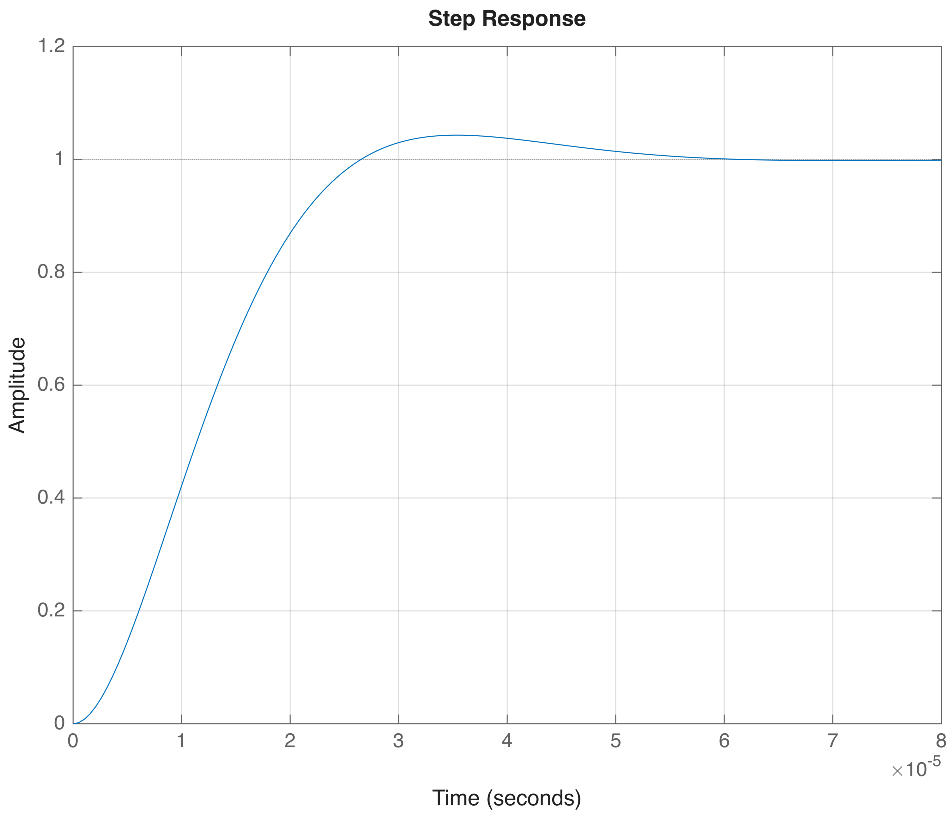
```
figure(3);
plot(z,'o');
plot(p,'o', 'LineWidth',2);
title("Poles and Zeros");
ylabel("j\omega");
xlabel("\sigma");
for i = 1:length(p)
    text(real(p(i)) + 0.05, imag(p(i)) + 0.05, ...
        sprintf('P%d (%.2f, %.2f)', i, real(p(i)), imag(p(i))), ...
        'HorizontalAlignment', 'left', 'VerticalAlignment', 'bottom');
end
grid on
```




```
% Group delay
gd = -diff(unwrap(angle(H)))./diff(w);
figure(4);
semilogx(w(1:length(w)-1), gd);
ylabel("Group Delay [s]");
xlabel("Frequency [Hz]");
title("Group delay");
grid on
```




```
% Step Response
figure(5);
step(tf_sys);
title("Step Response");
grid on
```


Step Response
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8
×10^-5