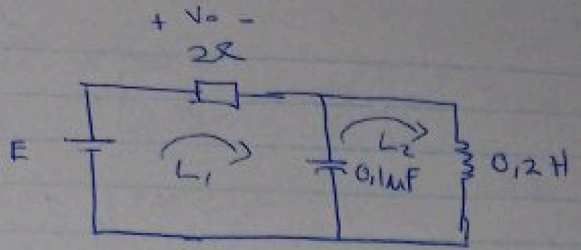
1



for L_1

$E - V_0 - V_c = 0$

$U - V_0 - V_c = 0$

$U - IR - X_2 = 0$

but $I = I_L + I_C$

$I_L = \cancel{L\frac{di}{dt}}\, X_1$ and $I_C = C\frac{dV_c}{dt}$

$U - \left(L\frac{di}{dt} + C\frac{dV_c}{dt}\right)R - X_2 = 0$

$\frac{di}{dt} = \dot{X}_1 \qquad \frac{dV_c}{dt} = \dot{X}_2$

$U - L\dot{X}_1 R - C\dot{X}_2 R - X_2 = 0$

but $\dot{X}_1 = X_2$

$U - L X_2 R - C\dot{X}_2 R - X_2 = 0$

$C\dot{X}_2 R = U - L X_2 R - X_2$

$C = 0,1\ \mu F$

$R - 2\ \Omega$

$L = 0,2\ H$

$\dot{X}_2 = \frac{U}{CR} - \frac{LR X_2}{CR} - \frac{X_2}{CR}$

$= \frac{U}{CR} - \frac{L}{C} X_2 - \frac{X_2}{CR}$

$\dot{X}_2 = \frac{U}{(0,1\times10^{-6})2} - \frac{0,2}{(0,1\times10^{-6})} X_2 - \frac{X_2}{(0,1\times10^{-6})(2)}$

$$\dot{x}_2 = \frac{u}{CL} - x_2\left[\frac{L}{C} + \frac{1}{CR}\right]$$

$$\dot{x}_2 = \frac{u}{2\times10^7} - x_2(7\times10^6)$$

from L_2

$$V_c - V_L = 0$$

$$X_2$$

$$u - \left(X_1 + C\frac{dv_c}{dt}\right)R - X_2 = 0$$

$$u - (X_1 + C\dot{X}_2)R - X_2 = 0$$

$$u - X_1R + RC\dot{X}_2 - X_2 = 0$$

$$RC\dot{X}_2 = -X_2 + u + X_1R$$

$$\dot{X}_2 = -\frac{X_2}{RC} + \frac{u}{RC} + \frac{X_1R}{RC}$$

$$\dot{X}_2 = \frac{X_2}{RC} - \frac{u}{RC} + \frac{X_1}{C}$$

from LR

$$V_c - V_L = 0$$

$$X_2 - L\frac{di}{dt} = 0$$

$$X_2 - L\dot{X}_1 = 0$$

$$L\dot{X}_1 = X_2$$

$$\dot{x}_1 = \frac{x_2}{L}$$

$\therefore$ state matrix $\bar{A}$

$$\bar{A} = \begin{bmatrix} 0 & 1/L \\ -1/C & -1/RC \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 0 \\ +1/RC \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$$\bar{D} = [0]$$

$$V_o = U - x_2$$

2. Find Transfer function

$$\bar{A} = \begin{bmatrix} -3 & 1 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \quad \bar{C} = [1\ 0\ 0]$$

$$\bar{D} = [0]$$

$$SI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} s+3 & -1 & 0 \\ 2 & s & 0 \\ -1 & 0 & s-1 \end{bmatrix}$$

$$|SI - A| = (s+3)\left[s(s-1) - 0\right] - (-1)\left[2(s-1) - 0\right]$$

$$= (s+3)(s^2 - s) + 1\left[2s - 2\right]$$

$$= s^3 - s^2 + 3s^2 - 3s + 2s - 2$$

$$= s^3 + 2s^2 - s - 2$$

adj $SI - A$ $\quad c_{11} = \begin{vmatrix} s & 0 \\ 0 & s-1 \end{vmatrix} \Rightarrow s(s-1) - 0 = s^2 - s$

$$c_{12} = -\begin{vmatrix} 2 & 0 \\ -1 & s-1 \end{vmatrix} \Rightarrow 2(s-1) - 0 = 2s - 2$$

$$c_{13} = \begin{vmatrix} 2 & s \\ -1 & 0 \end{vmatrix} \Rightarrow 0 - s(-1) = s$$

$$c_{21} = -\begin{vmatrix} -1 & 0 \\ 0 & s-1 \end{vmatrix} \Rightarrow -\left[-s+1-0\right] = s - 1$$

$$c_{22} = \begin{vmatrix} s+3 & 0 \\ -1 & s-1 \end{vmatrix} \Rightarrow (s+3)(s-1) =$$

$$c_{22} = -\begin{vmatrix} s+3 & -1 \\ -1 & 0 \end{vmatrix} \Rightarrow -\left[0-(-1)(-1)\right] = 1$$

$$c_{31} = \begin{vmatrix} -1 & 0 \\ s & 0 \end{vmatrix} \Rightarrow 0 \quad = 0$$

$$c_{32} = -\begin{vmatrix} s+3 & 0 \\ 2 & 0 \end{vmatrix} \Rightarrow 0$$

$$c_{33} = \begin{vmatrix} s+3 & -1 \\ 2 & s \end{vmatrix} \rightarrow s(s+3)-2(-1) \Rightarrow s^2+3s+2$$

$$\text{adj } sI-A = \begin{bmatrix} s^2-s & s-1 & 0 \\ 2s-2 & (s+3)(s-1) & 0 \\ s & 1 & s^2+3s+2 \end{bmatrix}$$

$$\therefore (sI-A)^{-1} = \frac{1}{s^3+2s^2-s-2} \text{ adj } sI-A$$

$$= \begin{bmatrix} \frac{s^2-s}{s^3+2s^2-s-2} & \frac{s-1}{s^3+2s^2-s-2} & 0 \\ \frac{2s-2}{s^3+2s^2-s-2} & \frac{(s+3)(s-1)}{s^3+2s^2-s-2} & 0 \\ \frac{s}{s^3+2s^2-s-2} & \frac{1}{s^3+2s^2-s-2} & \frac{s^2+3s+2}{s^3+2s^2-s-2} \end{bmatrix}$$

$$TF = \bar{C}(sI-A)^{-1}\bar{B}$$

$$\bar{c}\,(sI-A)^{-1} = \left[\frac{s^2-s}{s^3+2s^2-s-2} \quad \frac{s-1}{s^3+2s^2-s-2} \right.$$

$$\bar{c}\,(sI-A)^{-1}\bar{B} = \frac{2\,(s^2-s)}{(s+1)\,(s^2+s-2)} + \frac{4\,(s-1)}{(s+1)\,(s^2+s-2}$$

$$\frac{2s^2-2s+4s-4}{(s+1)\,(s^2+s-2)}$$

$$= \frac{2s^2+2s-4}{(s+1)\,(s^2+s-2)}$$

$$= \frac{2\,(s^2+s-2)}{(s+1)\,(s^2+s-2)}$$

$$\bar{c}\,(sI-A)^{-1}\bar{B} = \frac{2}{s+1}$$

3. $$g(s) = \frac{2s+10}{s^2+3s+2}$$

factorsing

$$s^2+3s+2 \Rightarrow (s+1)(s+2)$$

partial fractions

$$\frac{2s+10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$2s+10 = A(s+2) + B(s+1)$$

if $s=-2$

$$2(-2)+10 = 0 + B(-2+1)$$
$$-4+10 = -B$$
$$6 = -B$$
$$B = -6$$

if $s=-1$

$$2(-1)+10 = A(-1+2)+0$$
$$-2+10 = A$$
$$8 = A$$

$$\frac{2s+10}{(s+1)(s+2)} = \frac{8}{s+1} - \frac{6}{s+2}$$

$$X_1(s) = \frac{1}{s+1}U(s) \qquad (1)$$

$$X_2(s) = \frac{1}{s+2}U(s) \qquad (2)$$

from (1)

$$X_1(s)(s+1) = U(s)$$
$$sX_1(s) + X_1(s) = U(s)$$
$$\dot{X}_1 + X_1 = U$$

in time domain

$$\dot{X}_1 = U - X_1$$

from ②

$$X_2(s)\,(s+2) = U(s)$$

$$sX_2(s) + 2X_2(s) = U(s)$$

in time domain

$$\dot{X}_2 + 2X_2 = U$$

$$\dot{X}_2 = U - 2X_2$$

∴ from

$$\dot{X}_1 = U - X_1$$

$$\dot{X}_2 = U - 2X_2$$

$$\bar{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

4. $\bar{A} = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}$ $\bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\bar{C} = [1 \ 0]$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s & 0 \\ 0 & s+2 \end{bmatrix}$$

$$|sI - A| = s(s+2) + 0$$
$$= s^2 + 2s$$

$$\text{adj } sI - A = \begin{bmatrix} s+2 & 0 \\ 0 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s(s+2)} \begin{bmatrix} s+2 & 0 \\ 0 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+2}{s(s+1)} & 0 \\ 0 & \frac{s}{s(s+2)} \end{bmatrix}$$

$$\bar{\Phi}(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\therefore \bar{\Phi}(t) = L^{-1} \bar{\Phi}(s) = L^{-1} \left\{ \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \right\}$$

$$\bar{\Phi}(t) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-2t} \end{bmatrix}$$

b) states $\bar{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$\bar{x}(t) = \bar{\Phi}(t)\,\bar{x}(0)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\bar{x}(t) = \begin{bmatrix} 2 \\ e^{-2t} \end{bmatrix}$$

5. $\bar{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\bar{C} = [1 \ \ 0]$

controllability matrix Q

$Q = [\bar{B} \ \ \bar{A}\bar{B}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$Q^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

closed loop poles at $s = -4, -6$

$(s+4)(s+6) = s^2 + 10s + 24$

using Ackermann's formula $\bar{k} = [0 \ \ 1] \, Q^{-1} \Delta_d(\bar{A})$

$\bar{k} = [0 \ \ 1] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^2 + 10 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + 24 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right.$

$= [1 \ \ 0] \left\{ \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix} + \begin{bmatrix} 24 & 0 \\ 0 & 24 \end{bmatrix} \right.$

$= [1 \ \ 0] \begin{bmatrix} 23 & 10 \\ -10 & 23 \end{bmatrix}$

$\bar{k} = [23 \ \ 10]$

k gain $g(s) = 0$ $\frac{1}{s^2+1} = \frac{1}{0^2+1} = 1$

$k = 1$

$\therefore$ $\bar{k} = [23 \ \ 10]$ $K = 1$

6 1. Design a suitable observer to estimate $\bar{x}$. The observer should be sufficiently fast so as to give it a transient perfomance much faster than ~~tat~~ that of the plant

2. feedback elements, $\bar{k}$ to estimated states so that the closed loop system has the required dynamic performance

b) Guidlines when selecting poles to a system in feedback design

1. If a pole or a pair of poles is in the left hand plane gives an acceptable response then leave them

2. Only move unstable poles or poles very near the imaginary axis

3. Unstable poles should be reflected in the imaginary axis