TEE 5122 Assignment 1

1(a) $$S_{TN(f)} = \frac{2h|f|}{\exp\left(\frac{h|f|}{KT}\right) - 1} \quad (1)$$

The power spectral density for low frequency is defined by $f \ll \frac{KT}{h}$

hence we may use the approximation $\exp\left(\frac{h|f|}{KT}\right) = 1 + \frac{h|f|}{KT}$ (2)

substitute eqn (2) in (1)

$$S_{TN(f)} = \frac{2h|f|}{1 + \frac{h|f|}{KT} - 1} = \frac{2h|f|}{\frac{h|f|}{KT}} = \frac{2h|f|KT}{h|f|}$$

$$\therefore S_{TN(f)} = 2KT \quad (3)$$

The mean square value of the thermal noise voltage measured across the terminals of the resistor

$$V_{TN}^2 = 2RB_N S_{TN}(f) \quad (4)$$

substituting eqn (3) in (4)

$$V_{TN}^2 = 2RB_N(2KT)$$

$$\therefore V_{TN}^2 = 4KTB_NR \text{ volts}^2$$

(b) $$S_{no} = |H(\omega)|^2 KT \quad (1)$$

from the circuit, transfere function ~~is~~

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1/sC}{R + 1/sC} = \frac{1/sC}{\frac{RsC+1}{sC}} = \frac{1}{1+RsC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}} \quad (2) \quad \therefore \cancel{|H(\omega)|^2 =}$$

$$|H(\omega)|^2 = \frac{1}{1+(\omega RC)^2} \quad (3)$$

The output spectral density, S_{no} decreases as the frequency increases, the total noise power is obtained by intergrating S_{no} over the frequency rang from 0 to ∞

$$P_{no} = \int_0^\infty S_{no}\, df \quad (4)$$

substituting for (3) in eqn (4)

$$P_{no} = \int_0^\infty \frac{1}{1+(wCR)^2} \cdot KT \cdot df$$

but $w = 2\pi f$

$\therefore$ let $2\pi f CR = x$

$$\frac{df}{dx} 2\pi f CR = 1$$

$$df = \frac{dx}{2\pi RC}$$

$$\therefore P_{no} = \int_0^\infty \frac{KT}{1+x^2} \cdot \frac{dx}{2\pi RC}$$

factoring out constants

$$P_{no} = \frac{KT}{1+x^2} \int_0^\infty \frac{1}{1+x^2}\, dx$$

$$P_{no} = \frac{KT}{2\pi RC} \left[\tan^{-1} x\right]_0^\infty$$

$$P_{no} = \frac{KT}{2\pi RC} \left[\frac{\pi}{2} - 0\right]$$

$$P_n = \frac{KT}{2RC} \cdot \frac{1}{2}$$

$\therefore$ The total noise power at the output is (5)

$$P_{no} = \frac{KT}{4RC}$$

deriving from (5) it can be seen that

$$B_N = \frac{1}{4RC}$$

where B_N is the effective noise bandwidth

2 (b) The auto corelation function is defined by $R(\tau)$

taking fourier transform of $R(\tau)$

$$FT[R(\tau)] \rightarrow S(f)$$

or Inverse fourier transform $IFT[S(f)] = R(\tau)$ (1)

~~hence~~

$$S(f) = \frac{N_0}{2}$$

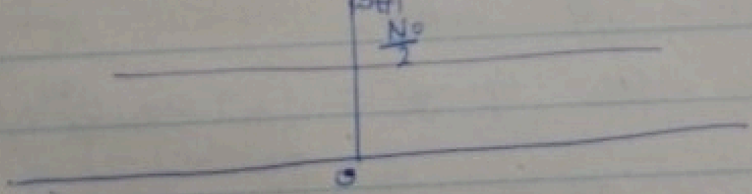
substituting $S(f)$ in eqn (1)

$$R(\tau) = IFT\left[\frac{N_0}{2}\right]$$

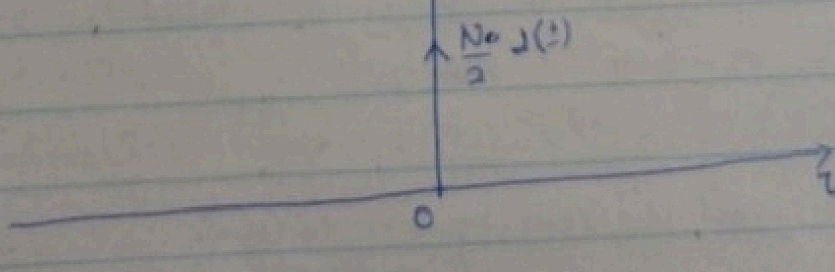
$$R(\tau) = \frac{N_0}{2}\delta(t)$$

white noise has power spectral density which is u

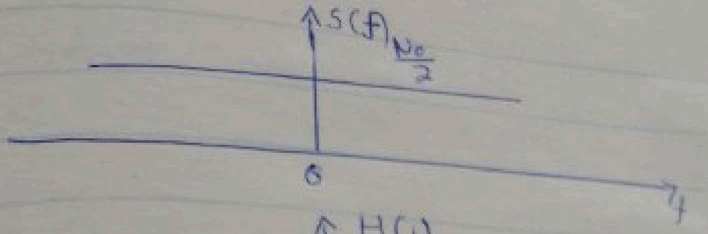
over the entire frequency range a shown by fig

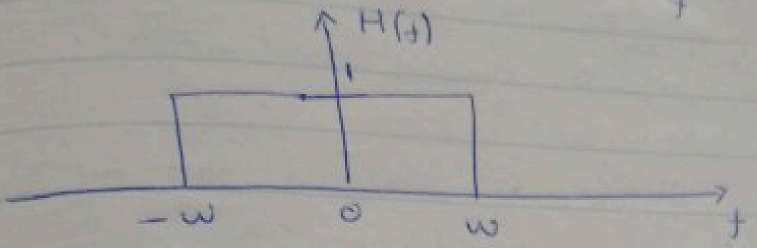


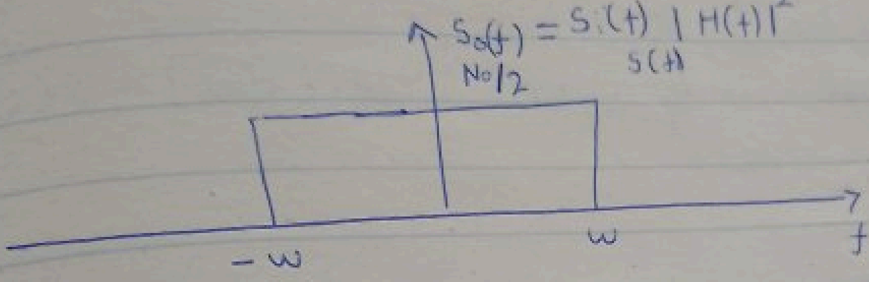
Autocorrelation function of white noise



2(c)







Output noise power N_o = Area of $S_o(f)$

$$= 2w \cdot \frac{N_o}{2}$$

$$= wN_o$$

The output of power spectral density

3(a) $S_y(f) = |H(f)|^2 \, S_x(f)$

given that $S_x(f) = 10^{-10}$ WHz

$|H(f)|^2$ = area under response $= \frac{1}{2} \times 20 \times 10^3 \times 1$

$$= 10^4$$

$$\therefore S_y(f) = 10^4 \times 10^{-10}$$

$$= 10^{-6} \text{ W}$$

Output power spectral density $= 10^{-6}$

3(b) Thee transfer fuction of RC low pass filter with a 3 dB Bandwidth of 8 kHz is given by

$$H(\omega) = \frac{1}{1 + j\frac{w}{w_o}} \quad \text{where } w_o = 2\pi(8\,000)$$

Given $\frac{N_o}{2} = 10^{-9}$ W/Hz

output noise power is

$$S_o(f) = S_i(f)\,|H(f)|^2$$

$$N_o = E[n_o^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\eta}{2}\,|H(\omega)|^2\,d\omega$$

$$= \frac{\eta}{2}\,\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+\left(\frac{w}{w_o}\right)^2}\,dw$$

$$= \frac{1}{4}\,\eta\, w_o \;\cancel{\mathcal{F}2\,zb}$$

$$= \frac{1.2(10^{-9})(2\pi)(8)(10^3)}{4}\ \text{W}$$

$$= 25{,}2\ \text{MW}$$

4 (a)

$$P_{s_1} = 150\ \text{MW}$$

$$P_{n_1} = 1{,}5\ \text{MW}$$

$$P_{s_o} = 1{,}5\ \text{W}$$

$$P_{n_o} = 45\ \text{mW}$$

$$\text{Noise Factor} = \frac{P_{s_1}}{P_{n_1}} \times \frac{P_{no}}{P_{so}}$$

$$= \frac{150\times10^{-6}}{1{,}5\times10^{-6}} \times \frac{45\times10^{-3}}{1{,}5}$$

$$= 3$$

$$\text{Noise Factor in dB} = 10\log_{10}(3)$$

$$= 4{,}77\ \text{dB}$$

4(b) $R_3 = 400k\Omega$

$R_2 = \frac{20 \cdot 80}{20+80} + 10 = 26k\Omega$

$R_1 = 500 + 2000 = 2500\Omega$

$g_1 = 10 \quad g_2 = 20$

Net noise resistanc can be found R_T

$$R_T = R_1 + \frac{R_2}{g_1^2} + \frac{R_3}{g_1^2 g_2^2}$$

$$= 2500 + \frac{26000}{10^2} + \frac{400\ 000}{(10 \times 20)^2}$$

$$= 2770\Omega$$

$$R_{eq} = R_T - R_N$$
$$= 2770 - 500$$
$$= 2270\Omega$$

$$F = 1 + \frac{2770(40+500)}{40 \times 500}$$

$$= 75,79$$

At room temperature $25^\circ C$ in K $273+25 = 298K$

2(a) $\therefore k = 1,38 \times 10^{-23}$

$kT = 1,38 \times 10^{-23} \times 298$

$kT = 4,1 \times 10^{-21}$

$I_{na} = \sqrt{1,6 \times 10^{-17}}$

$= 4 \times 10^{-9}$ A

Since source has resistance 150Ω noise voltage

$I_{na}R_s = 4 \times 10^{-9} \times 150 = 6 \times 10^{-7}$ V shot noise current does not develop a voltage across R_n

$\therefore$ the noise voltage generated by R_n

$$V_{nn} = \sqrt{4kTB_nR}$$
$$= \sqrt{4 \times 4,1 \times 10^{-21} \times 10 \times 10^{6} \times 300}$$
$$= 7 \times 10^{-6} \text{ V}$$

thermal noise voltage from source

$$V_{ns} = \sqrt{4 \times 4,1 \times 10^{-21} \times 10 \times 10^{6} \times 150}$$
$$= 5 \times 10^{-6} \text{ V}$$

The total noise voltage to amplifier becomes

$$V_n = \sqrt{7^2 + 5^2 + 0,6^2}$$
$$= 8,6 \times 10^{-6} \text{ V}$$
$$= 8,6 \ \mu\text{V}$$

$\therefore$ signal to noise ratio $S/N = 20 \log \frac{V_s}{V_n}$

$$= 1,31 \text{ dB}$$