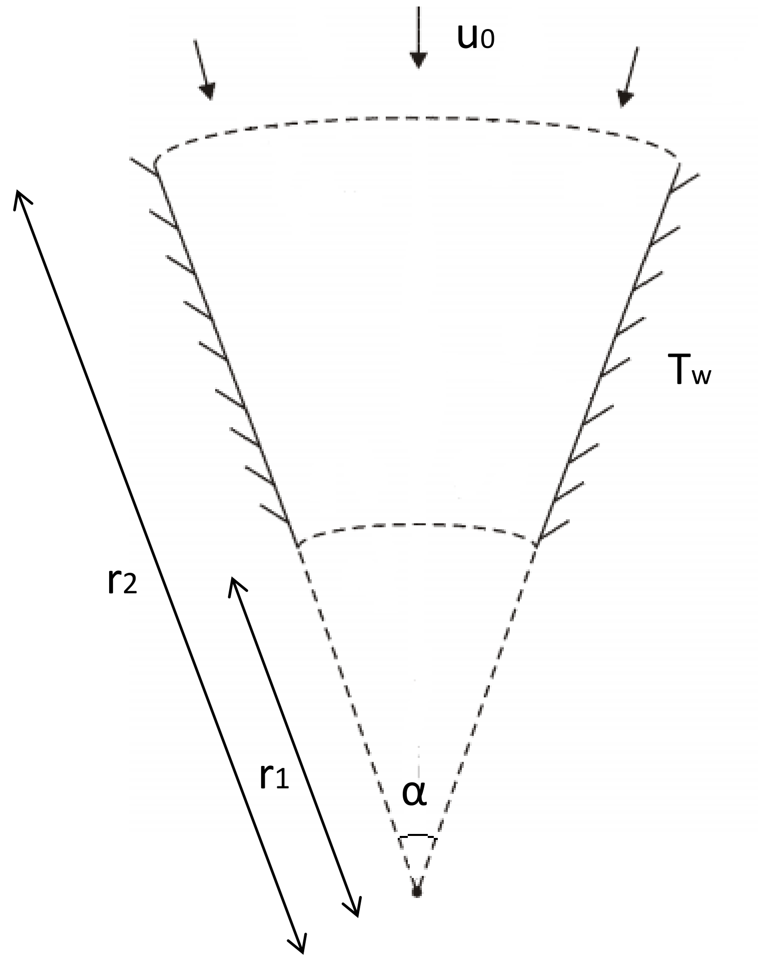
Physical background



Mathematical background
Main equations in polar coordinate system

$$\frac{\partial u_\varphi}{\partial \tau} + u_r \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_\varphi}{\partial \varphi} - \frac{u_\varphi u_r}{r} = -\frac{1}{r}\frac{\partial p}{\partial \varphi} + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u_\varphi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u_\varphi}{\partial \varphi^2} + \frac{1}{r}\frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \varphi}\right]$$

$$\frac{\partial u_r}{\partial \tau} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_r}{\partial \varphi} - \frac{u_\varphi^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \varphi^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\varphi}{\partial \varphi}\right] + \frac{Gr}{\mathrm{Re}^2}\theta$$

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} = 0$$

$$\frac{\partial \theta}{\partial \tau} + u_r \frac{\partial \theta}{\partial r} + \frac{u_\varphi}{r}\frac{\partial \theta}{\partial \varphi} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \theta}{\partial \varphi^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right)$$

Vorticity – Stream function formulation

$$u_r = -\frac{1}{r}\frac{\partial \psi}{\partial \varphi} \qquad u_\varphi = \frac{\partial \psi}{\partial r} \qquad \Omega = \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r} - \frac{1}{r}\frac{\partial u_r}{\partial \varphi}$$

$$\frac{\partial \psi}{\partial \tau} - \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \varphi^2} = -\Omega + \frac{1}{r}\frac{\partial \psi}{\partial r}$$

$$\frac{\partial \Omega}{\partial \tau} + u_r \frac{\partial \Omega}{\partial r} + \frac{u_\varphi}{r}\frac{\partial \Omega}{\partial \varphi} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \Omega}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \Omega}{\partial \varphi^2} + \frac{1}{r}\frac{\partial \Omega}{\partial r}\right) - \frac{Gr}{\mathrm{Re}^2}\frac{\partial \theta}{\partial \varphi}$$

$$\frac{\partial \theta}{\partial \tau} + u_r \frac{\partial \theta}{\partial r} + \frac{u_\varphi}{r}\frac{\partial \theta}{\partial \varphi} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \theta}{\partial \varphi^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right)$$

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi}=0 \qquad u_r=-\frac{1}{r}\frac{\partial \psi}{\partial \varphi} \qquad u_\varphi=\frac{\partial \psi}{\partial r}$$

$$\frac{\partial}{\partial r}\left(-\frac{1}{r}\frac{\partial \psi}{\partial \varphi}\right)+\frac{1}{r}\left(-\frac{1}{r}\frac{\partial \psi}{\partial \varphi}\right)+\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{\partial \psi}{\partial r}\right)=0$$

$$\frac{1}{r^2}\frac{\partial \psi}{\partial \varphi}-\frac{1}{r}\frac{\partial^2 \psi}{\partial \varphi \partial r}-\frac{1}{r^2}\frac{\partial \psi}{\partial \varphi}+\frac{1}{r}\frac{\partial^2 \psi}{\partial \varphi \partial r}\equiv 0$$

Numerical method

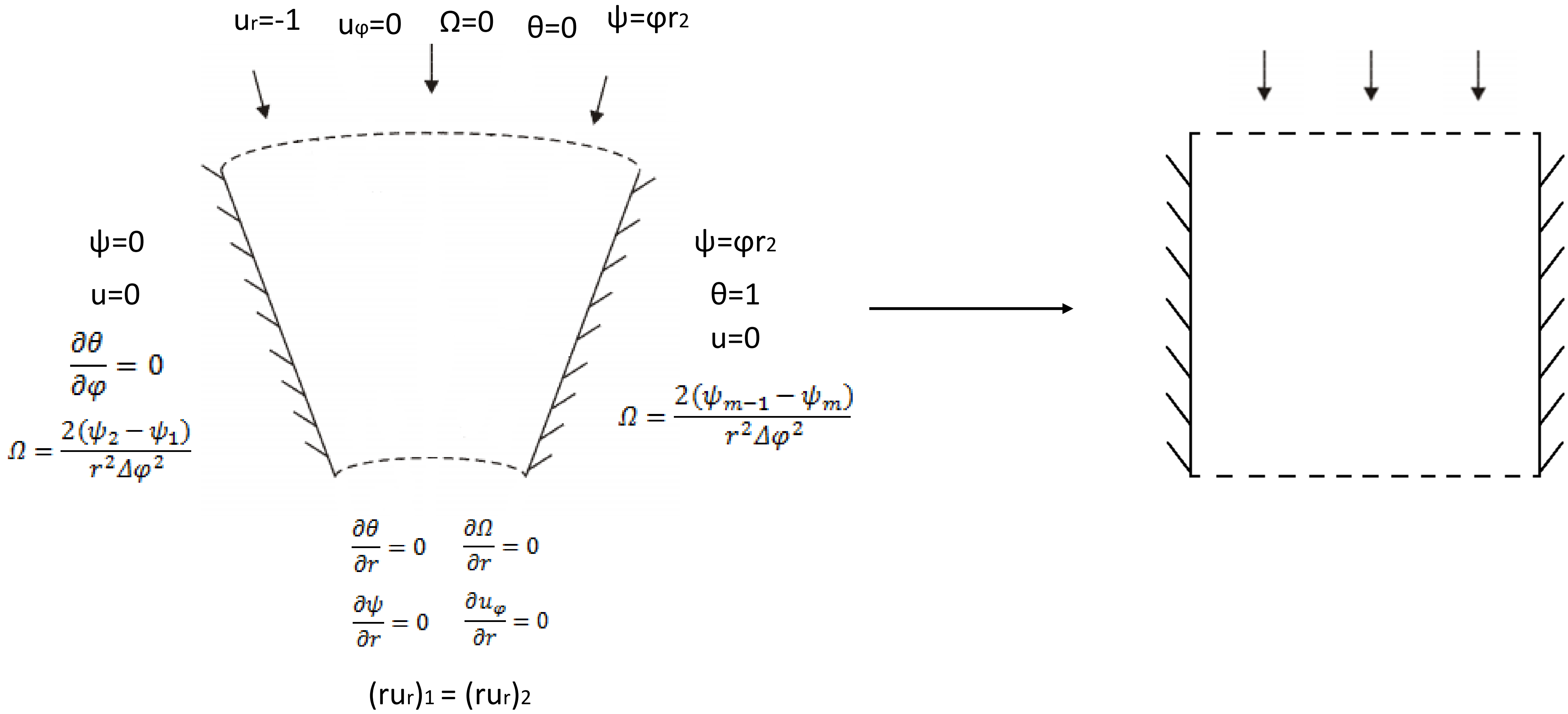
$$\frac{\partial\theta}{\partial\tau}+u_r\frac{\partial\theta}{\partial r}+\frac{u_\varphi}{r}\frac{\partial\theta}{\partial\varphi}=\frac{1}{\Pr\mathrm{Re}}\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\theta}{\partial\varphi^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right)$$

local one-dimensional scheme

$$\frac{\theta^*_{i,j}-\theta^n_{i,j}}{\Delta\tau}+\left(u_r\frac{\partial\theta}{\partial r}\right)^*_{i,j}=\frac{1}{\Pr\mathrm{Re}}\left(\left(\frac{\partial^2\theta}{\partial r^2}\right)^*_{i,j}+\left(\frac{1}{r}\frac{\partial\theta}{\partial r}\right)^n_{i,j}\right)$$

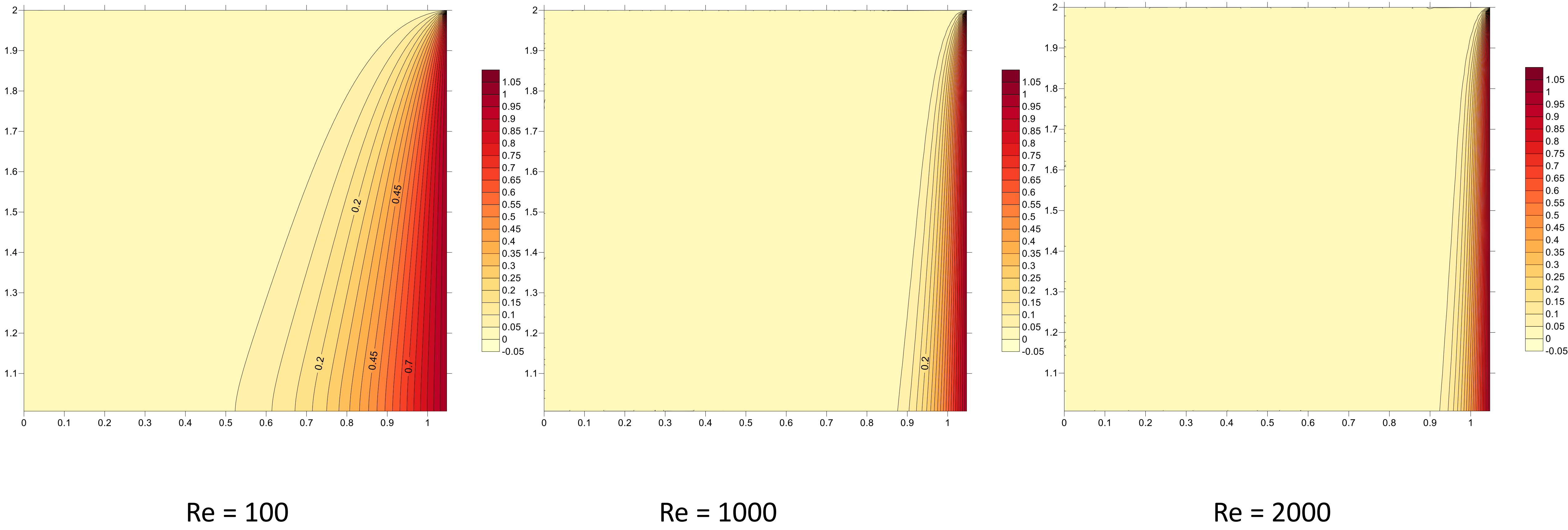
$$\frac{\theta^{n+1}_{i,j}-\theta^*_{i,j}}{\Delta\tau}+\left(\frac{u_\varphi}{r}\frac{\partial\theta}{\partial\varphi}\right)^{n+1}_{i,j}=\frac{1}{\Pr\mathrm{Re}}\left(\frac{1}{r^2}\frac{\partial^2\theta}{\partial\varphi^2}\right)^{n+1}_{i,j}$$

Boundary Conditions

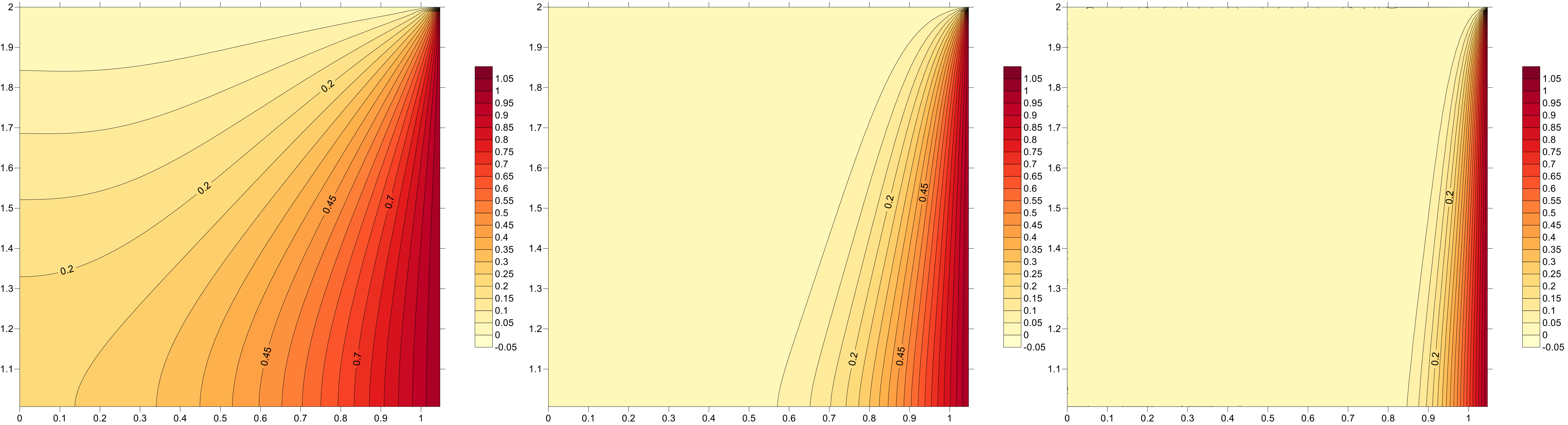


Results

Pr = 0.1; Gr = 1000; α = 30

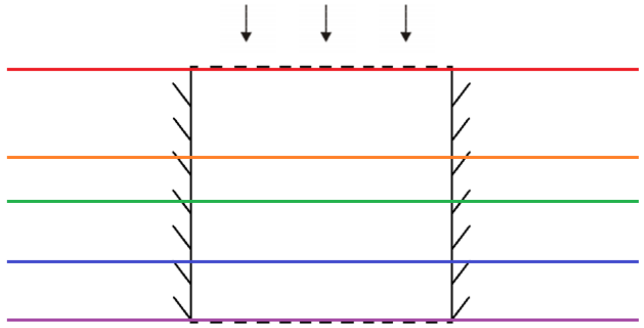


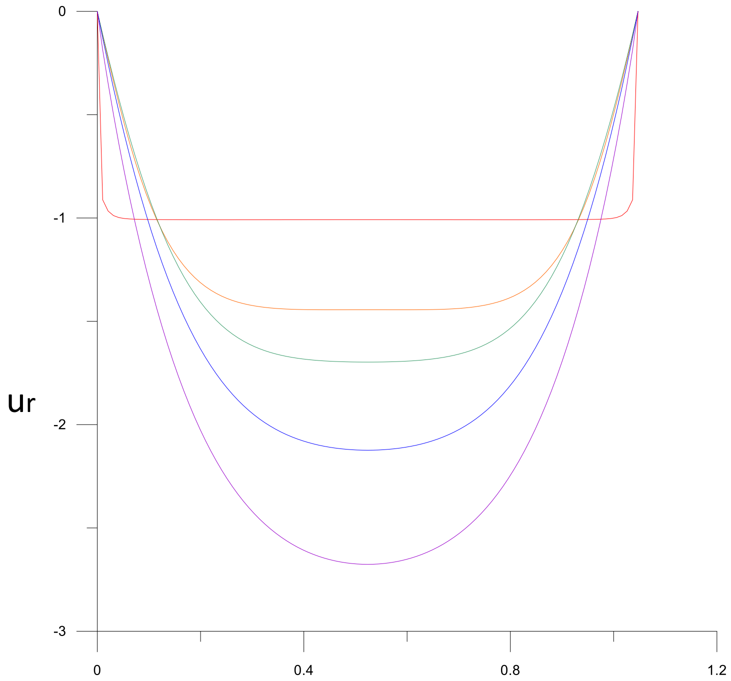
Re = 10; Gr = -1000; α = 30



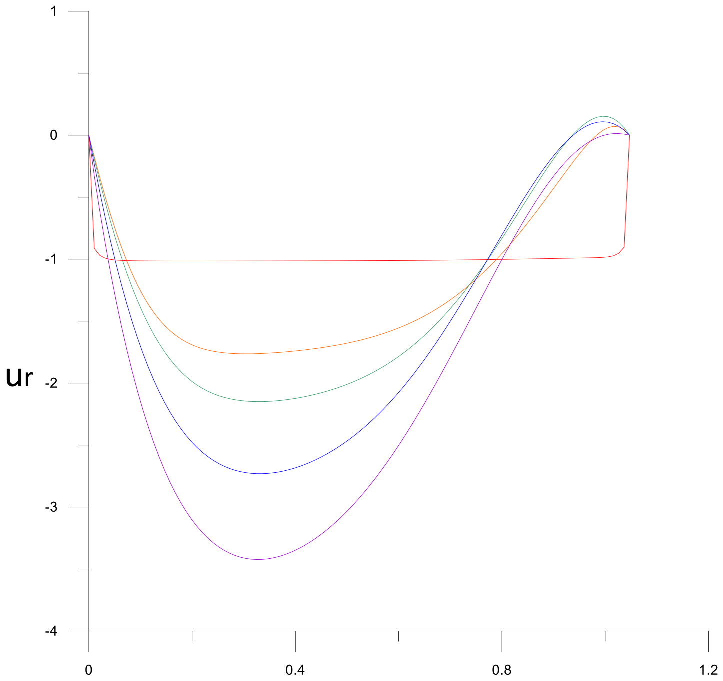
Pr = 0.1 | Pr = 1 | Pr = 10

Re = 10; Pr = 0.1; α =30

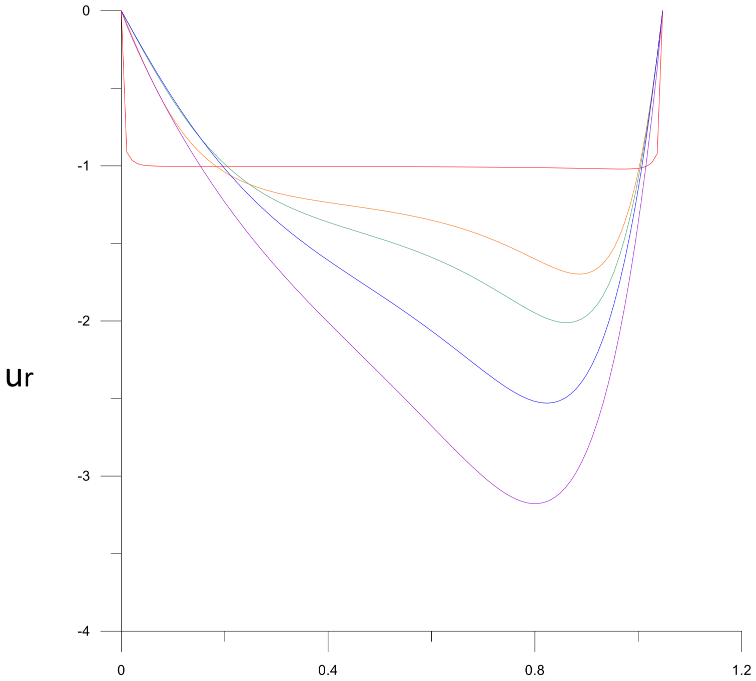




Gr = 0

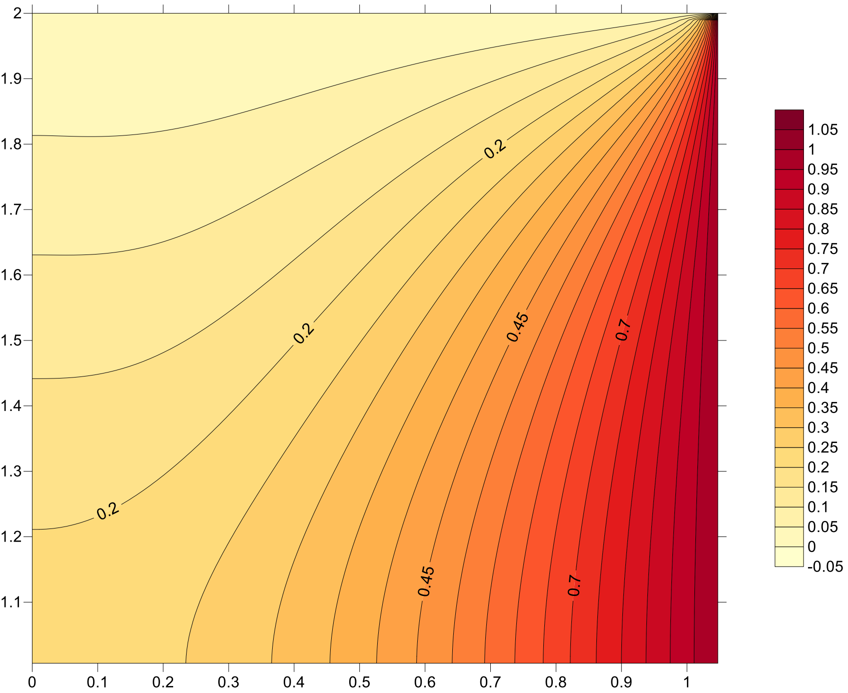


Gr = 1000

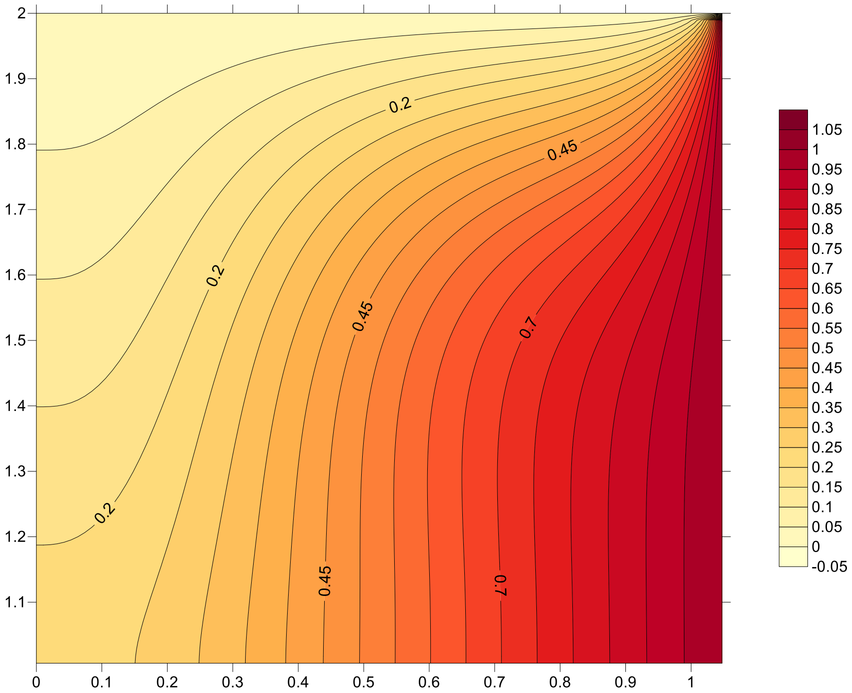


Gr = -1000

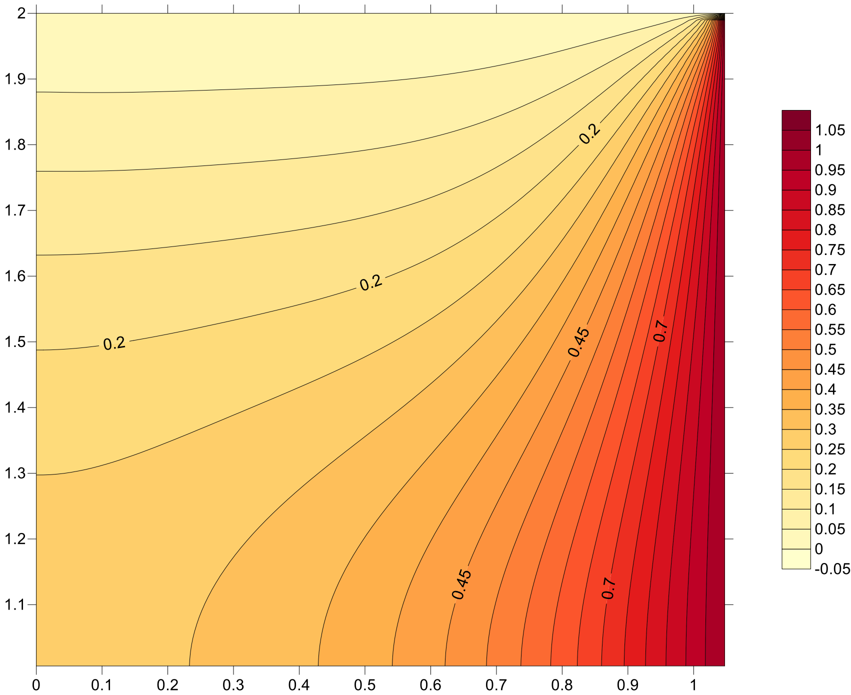
Re = 10; Pr = 0.1; α =30



Gr = 1000

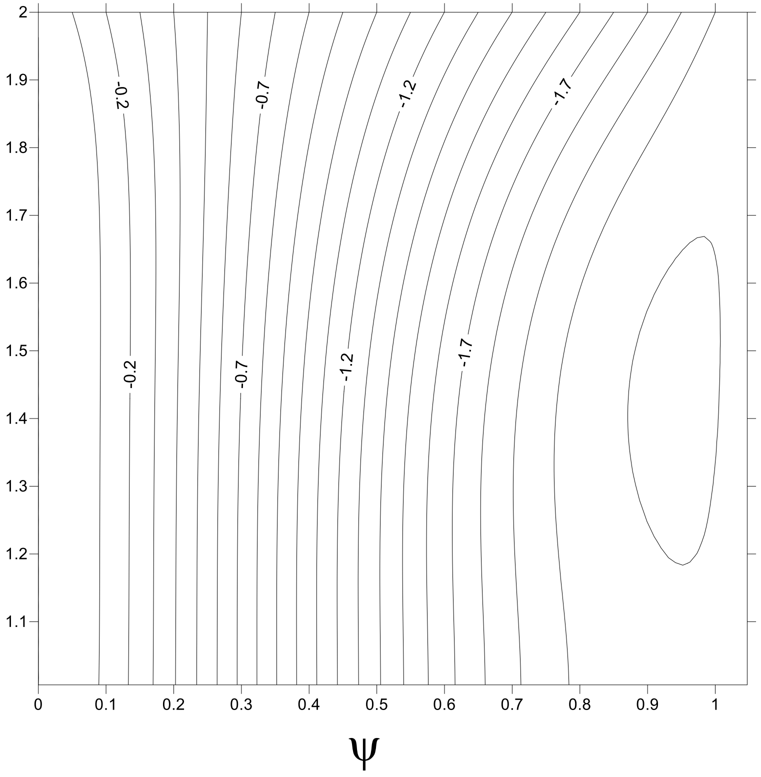


Gr = 10000

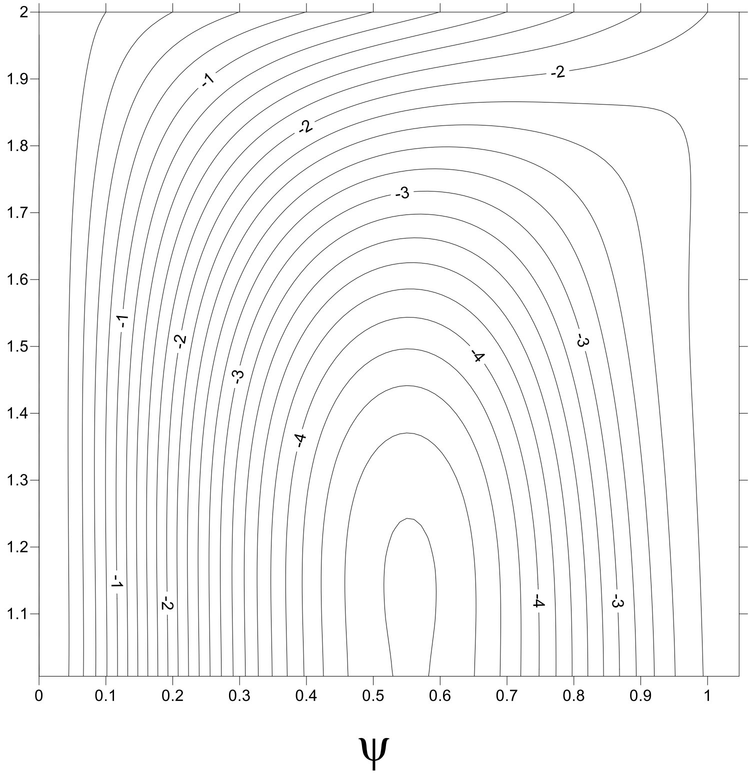


Gr = -10000

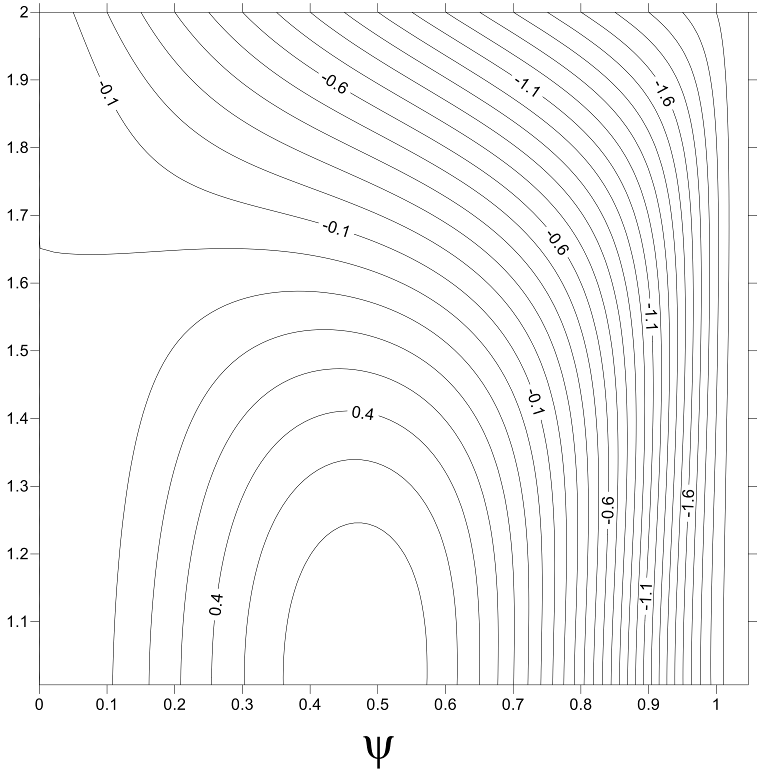
Re = 10; Pr = 0.1; α =30



Gr = 1000

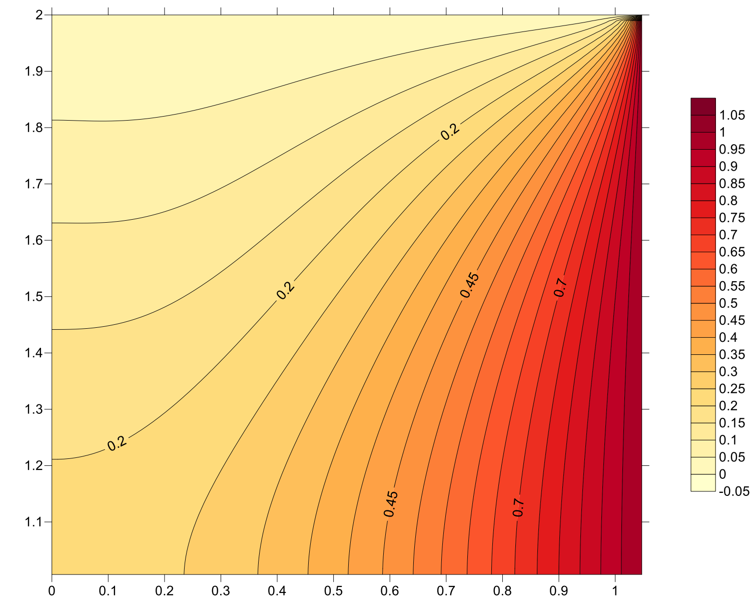


Gr = 10000

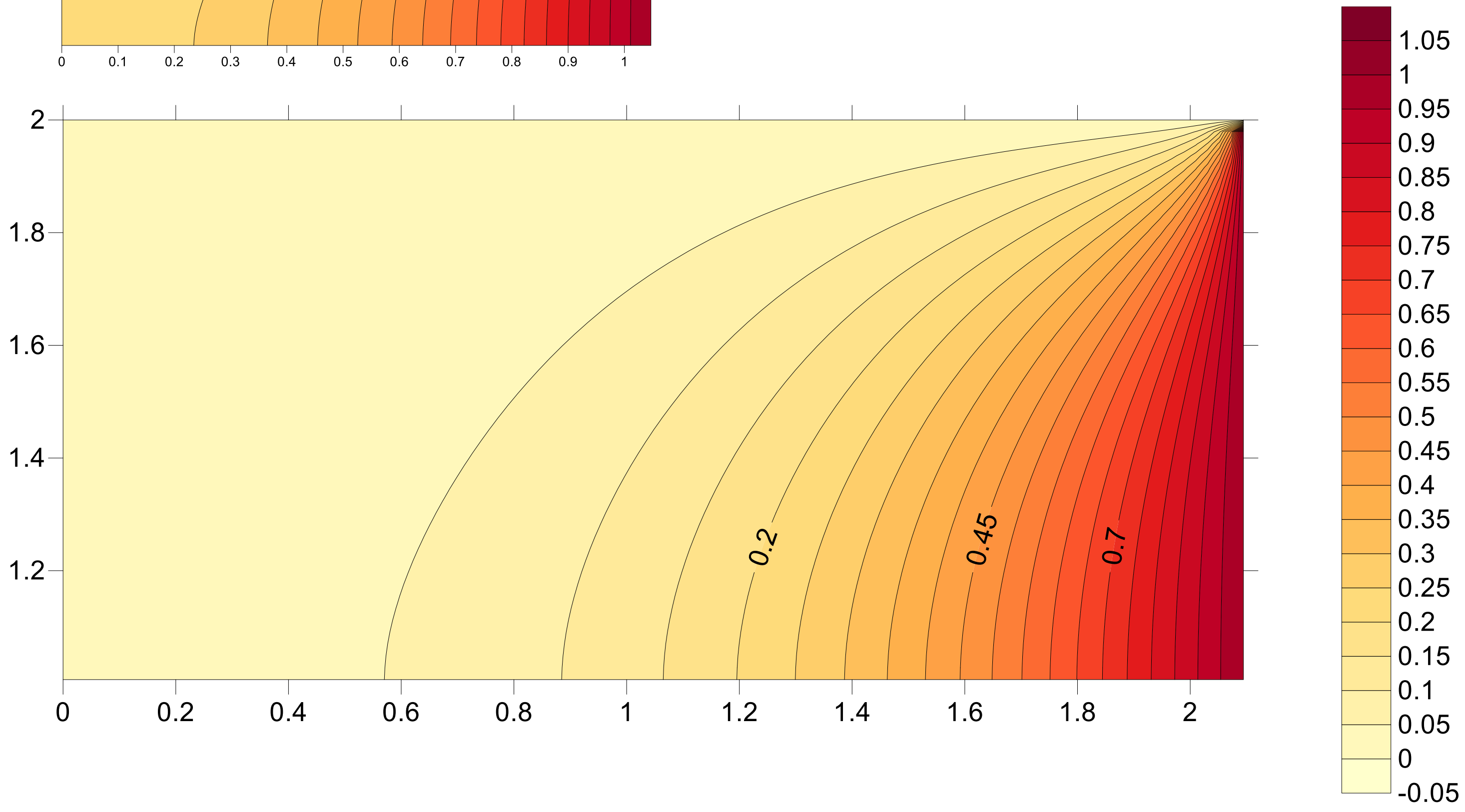


Gr = -10000

Re = 10; Pr = 0.1; Gr=1000



α = 30



α = 60