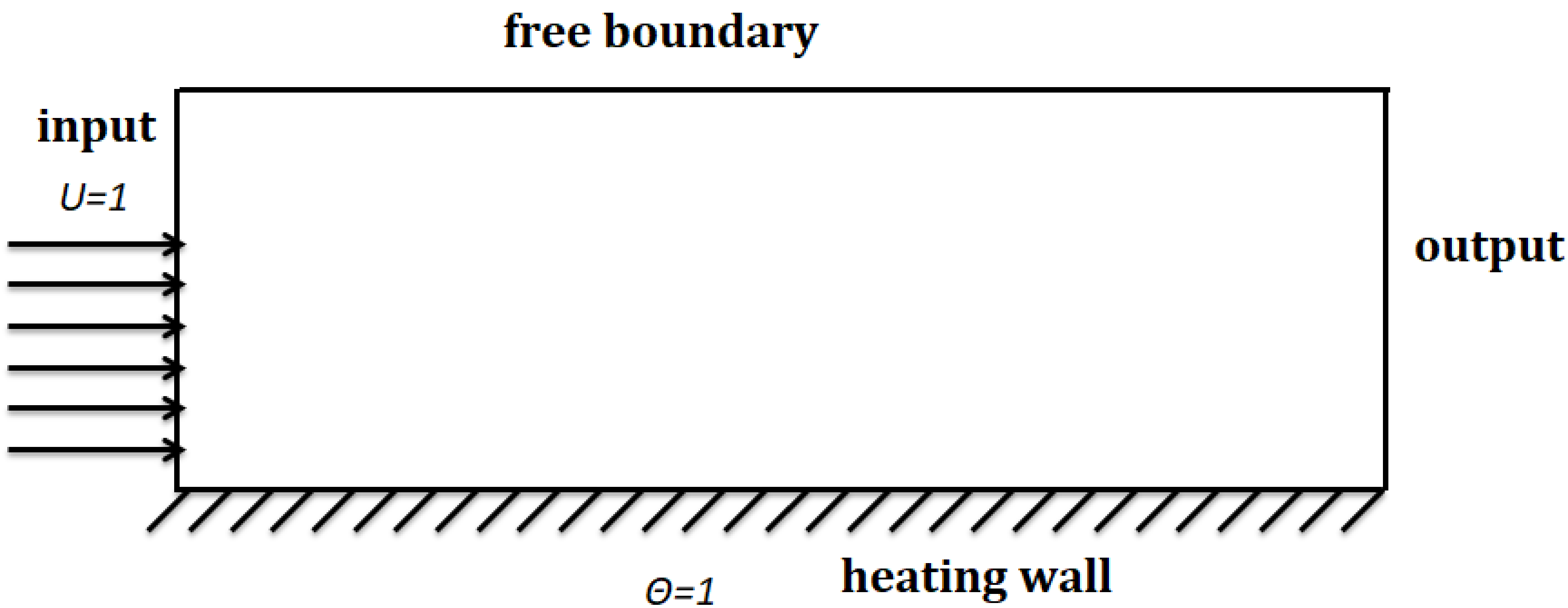
Физическая постановка



Mathematical background

$$\begin{cases} \dfrac{\partial U_x}{\partial \tau} + U_x \dfrac{\partial U_x}{\partial x} + U_y \dfrac{\partial U_x}{\partial y} = -\dfrac{\partial P}{\partial x} + \dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2 U_x}{\partial x^2} + \dfrac{\partial^2 U_x}{\partial y^2} \right); \\ \dfrac{\partial U_y}{\partial \tau} + U_x \dfrac{\partial U_y}{\partial x} + U_y \dfrac{\partial U_y}{\partial y} = -\dfrac{\partial P}{\partial y} + \dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2 U_y}{\partial x^2} + \dfrac{\partial^2 U_y}{\partial y^2} \right) + \dfrac{Gr}{\mathrm{Re}^2}\Theta; \\ \dfrac{\partial U_x}{\partial x} + \dfrac{\partial U_y}{\partial y} = 0. \end{cases}$$

Vorticity – stream function formulation

$$U_x = \frac{\partial \Psi}{\partial y}; \; U_y = -\frac{\partial \Psi}{\partial x};$$

$$\Omega = \frac{\partial U_x}{\partial y} - \frac{\partial U_y}{\partial x}$$

$$\frac{\partial \Omega}{\partial \tau} + U_x \frac{\partial \Omega}{\partial x} + U_y \frac{\partial \Omega}{\partial y} = \frac{1}{\mathrm{Re}} \left(\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right) - \frac{Gr}{\mathrm{Re}^2} \frac{\partial \Theta}{\partial x}$$

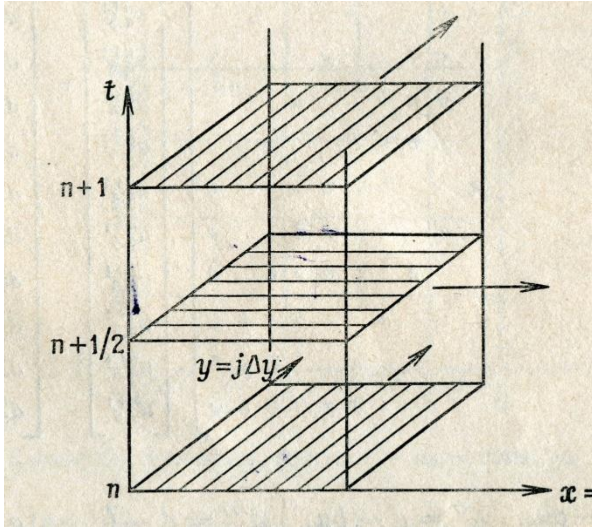
$$\frac{\partial \Theta}{\partial \tau} + U_x \frac{\partial \Theta}{\partial x} + U_y \frac{\partial \Theta}{\partial y} = \frac{1}{\mathrm{Re} \cdot \mathrm{Pr}} \left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \right)$$

Local one-dimensional scheme

$$\frac{\partial \vartheta}{\partial \tau} + u_x \frac{\partial \vartheta}{\partial x} + u_y \frac{\partial \vartheta}{\partial y} = \frac{1}{\Pr \mathrm{Re}} \left(\frac{\partial^2 \vartheta}{\partial x^2} + \frac{\partial^2 \vartheta}{\partial y^2} \right) + F$$

$$\frac{\vartheta^*_{i,j} - \vartheta^n_{i,j}}{\Delta \tau} + \left(u_x \frac{\partial \vartheta}{\partial x} \right)^*_{i,j} = \frac{1}{\Pr \mathrm{Re}} \left(\frac{\partial^2 \vartheta}{\partial x^2} \right)^*_{i,j} + F$$

$$\frac{\vartheta^{n+1}_{i,j} - \vartheta^*_{i,j}}{\Delta \tau} + \left(u_y \frac{\partial \vartheta}{\partial y} \right)^{n+1}_{i,j} = \frac{1}{\Pr \mathrm{Re}} \left(\frac{\partial^2 \vartheta}{\partial y^2} \right)^{n+1}_{i,j}$$



Boundary conditions

- input: $\Omega = 0;\ \Theta = 0;\ U_x = 1;\ \Psi = y\Big|_0^{y_{\max}}$

- On heating wall: $\Psi = 0;\ \Theta = 1;$

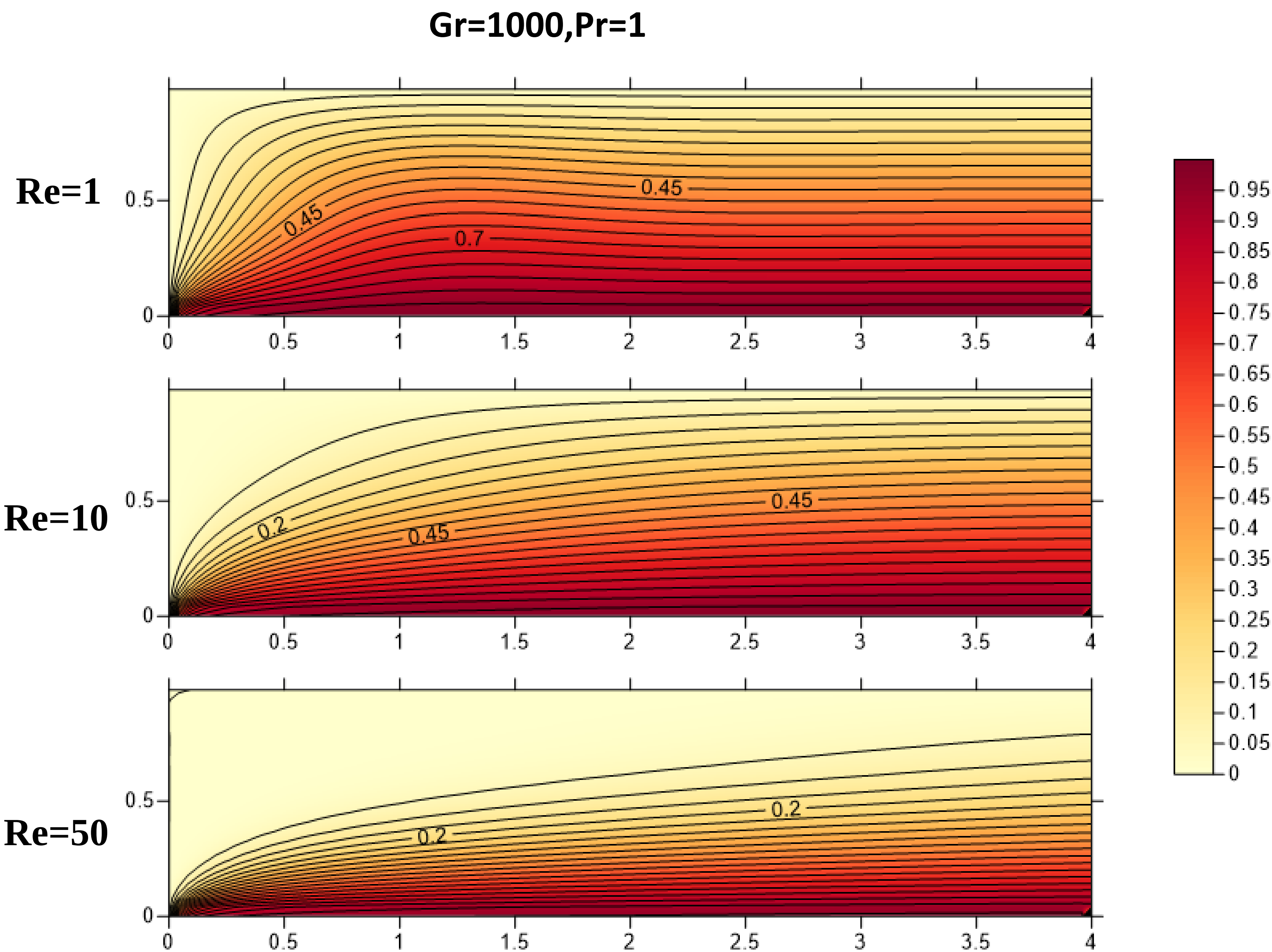
$$\Omega = \frac{2(\Psi_2 - \Psi_1)}{\Delta y^2}.$$

- output: $\frac{\partial}{\partial x} = 0.$

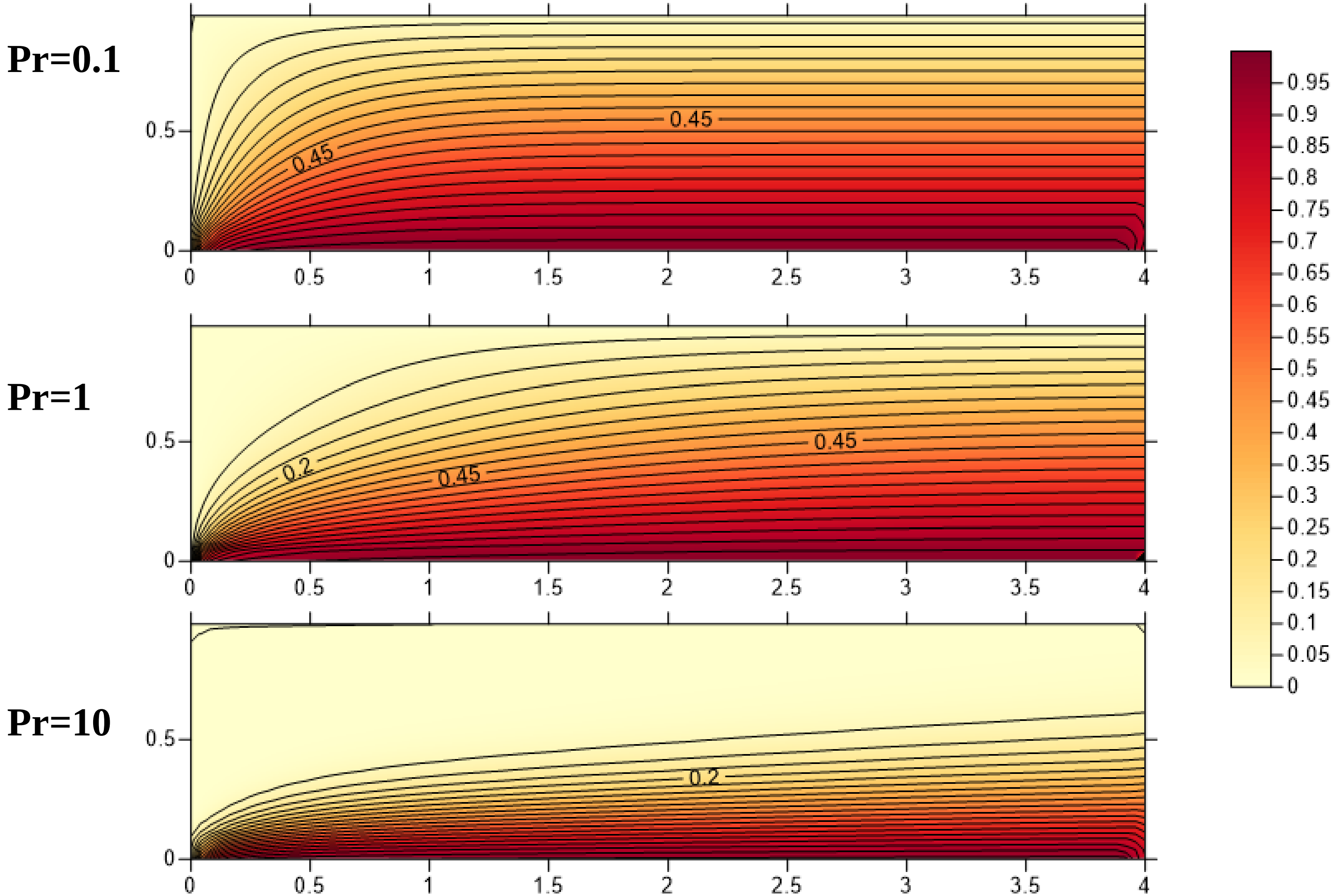
- on upper wall:

$$\Psi = y_{\max};\quad \Omega = \frac{2(\Psi_{w-1} - \Psi_w + U\Delta y)}{\Delta y^2};\quad \frac{\partial \Theta}{\partial y} = 0.$$

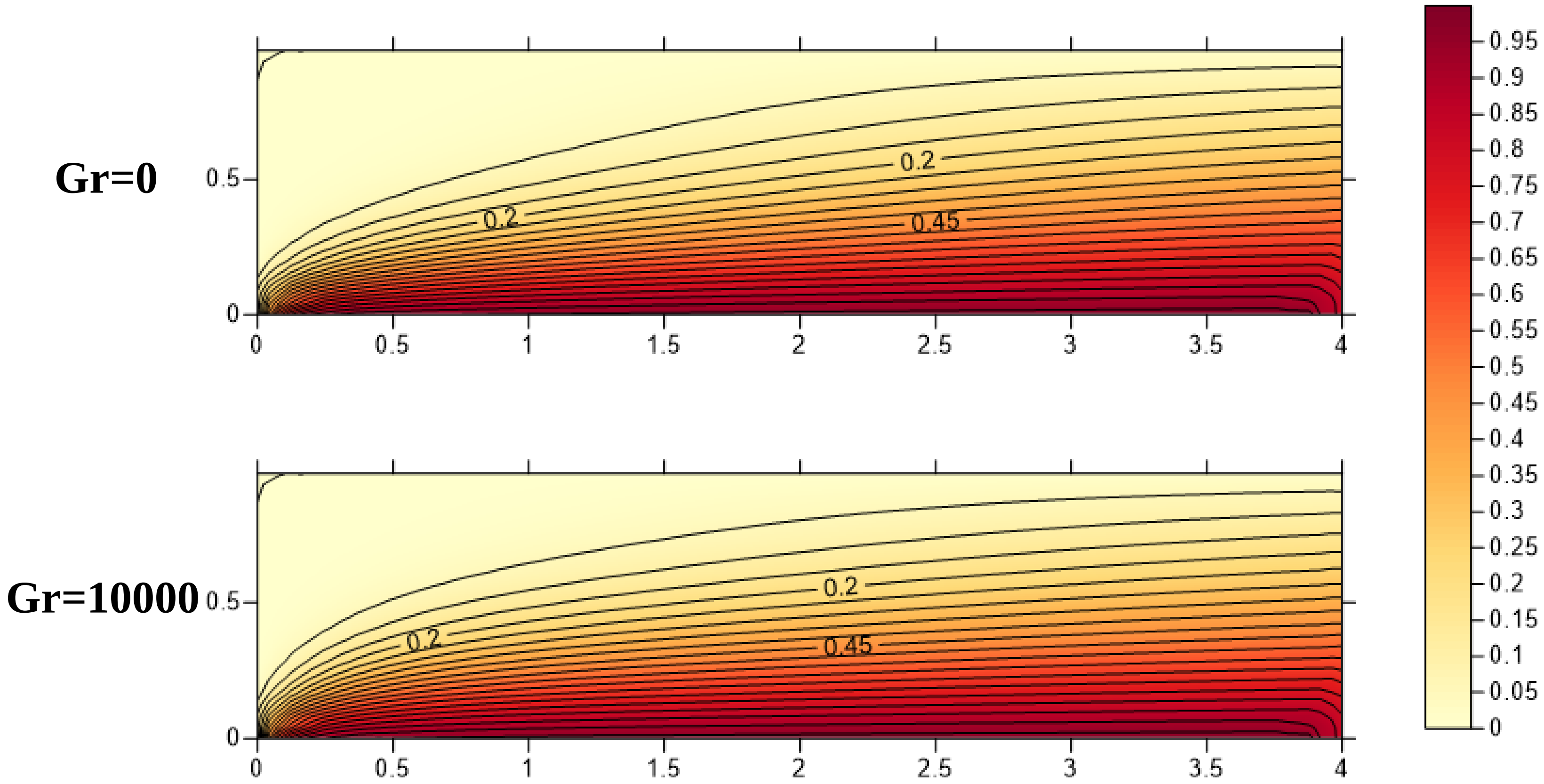
Results


Gr=1000,Pr=1
Re=1
Re=10
Re=50
0.45
0.7
0.2
0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

Gr=1000,Re=10:

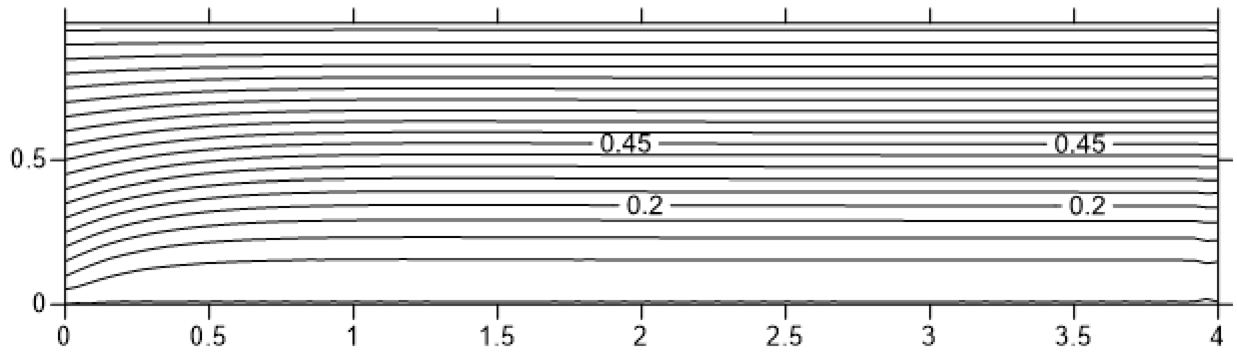


Re=25,Pr=1:

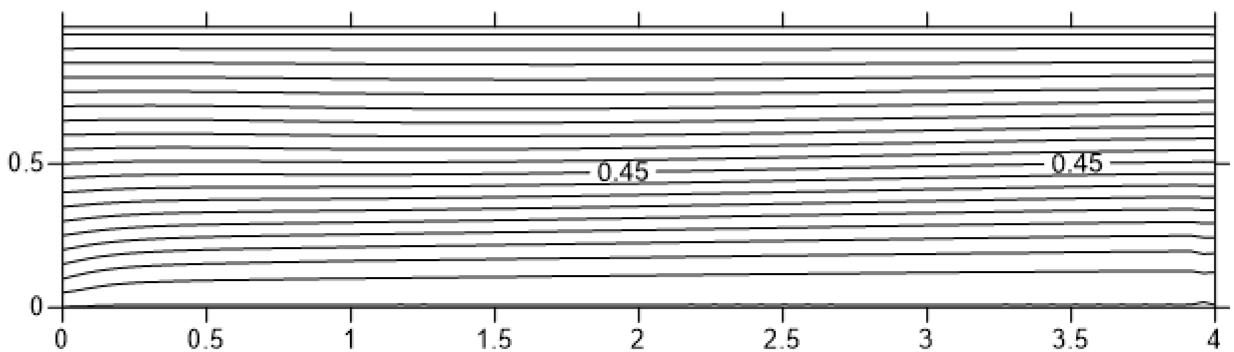

Gr=0
Gr=10000
0.2
0.2
0.45
0
0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

Stream function for Re=25;Pr=1;

Gr=0

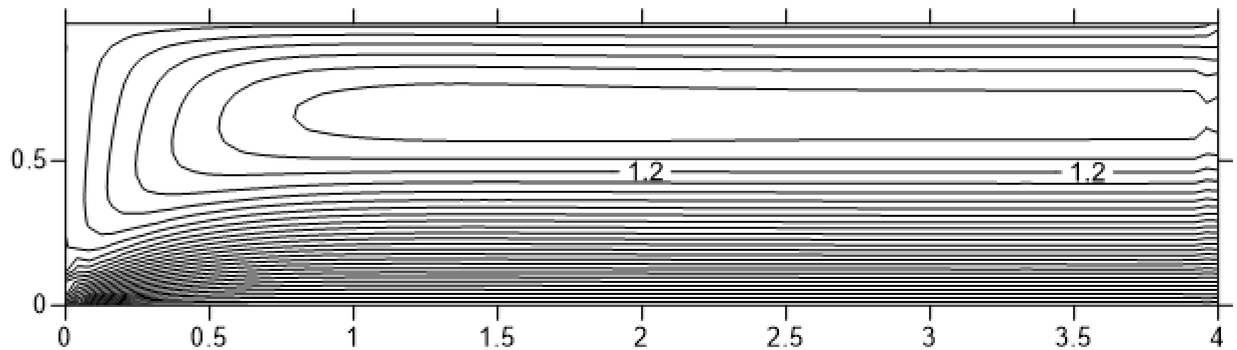


Gr=10000

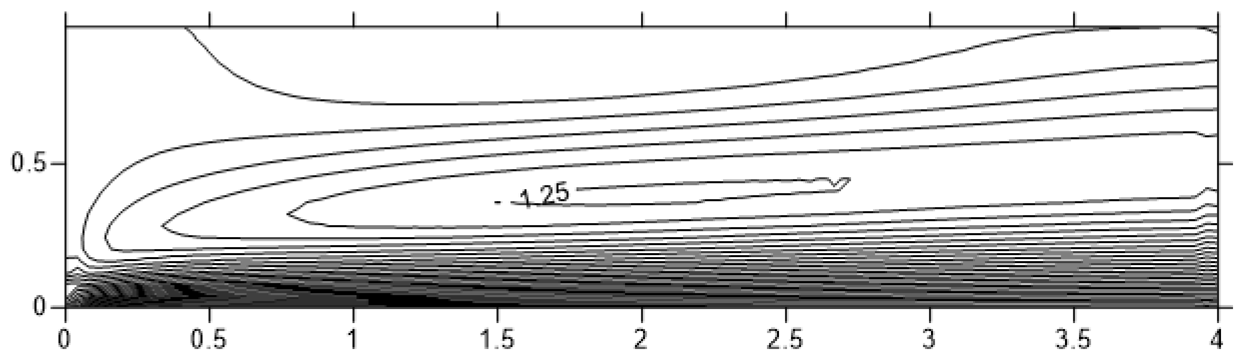


Velocities distribution for Re=25;Pr=1;

Gr=0

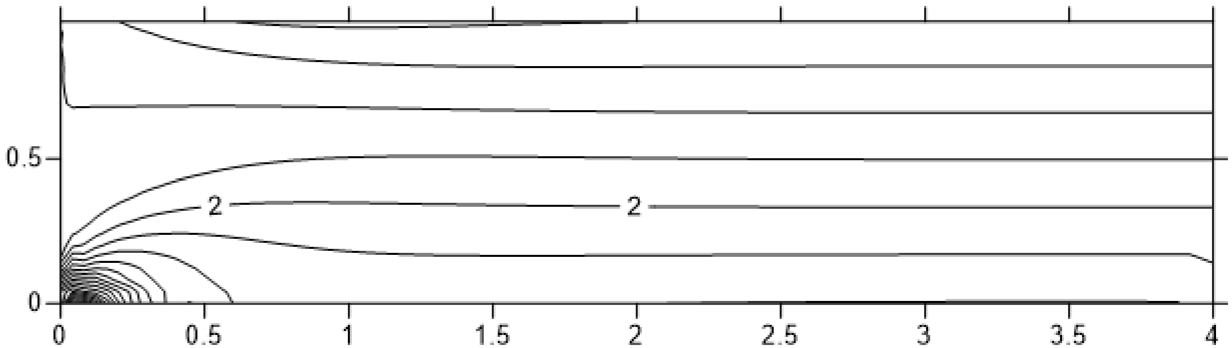


Gr=10000

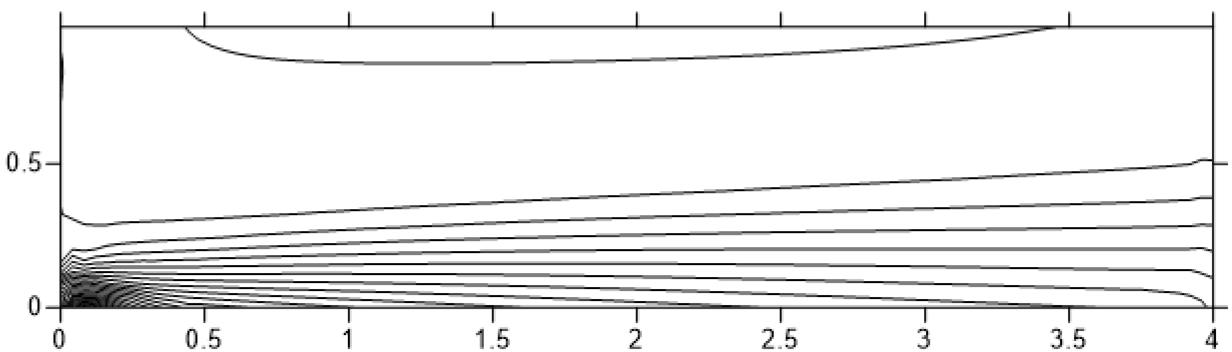


Vorticity distribution for Re=25;Pr=1;

Gr=0

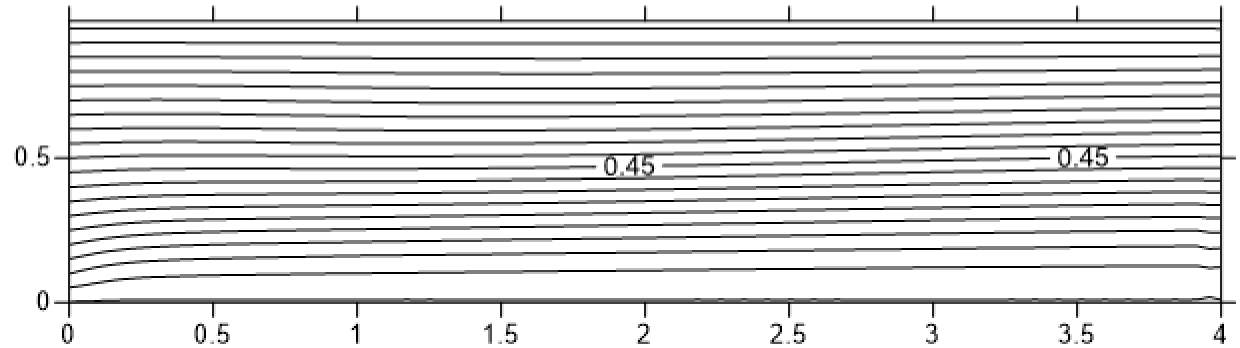


Gr=10000

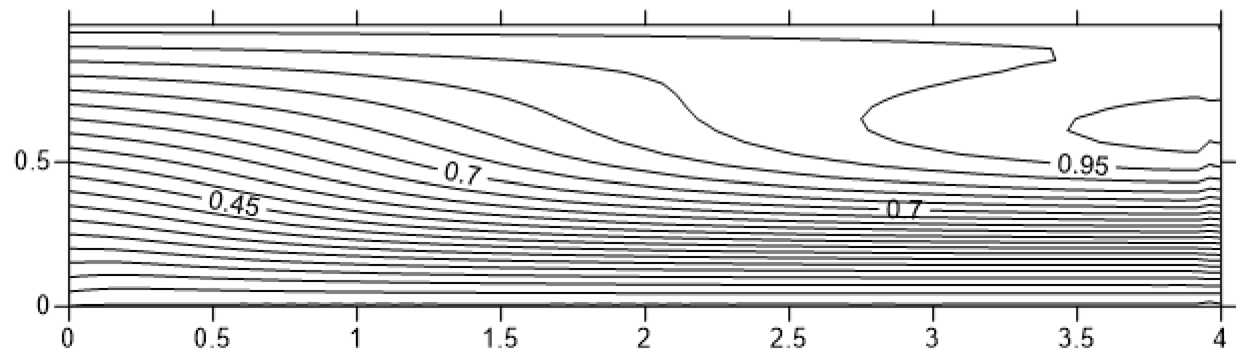


Influence of the angle of gravity
on the stream function distribution for Pr=1, Re=25, Gr=10000

vertical g



angled g



Influence of the angle of gravity
on the temperature distribution for Pr=1, Re=25, Gr=10000

vertical g

angled g