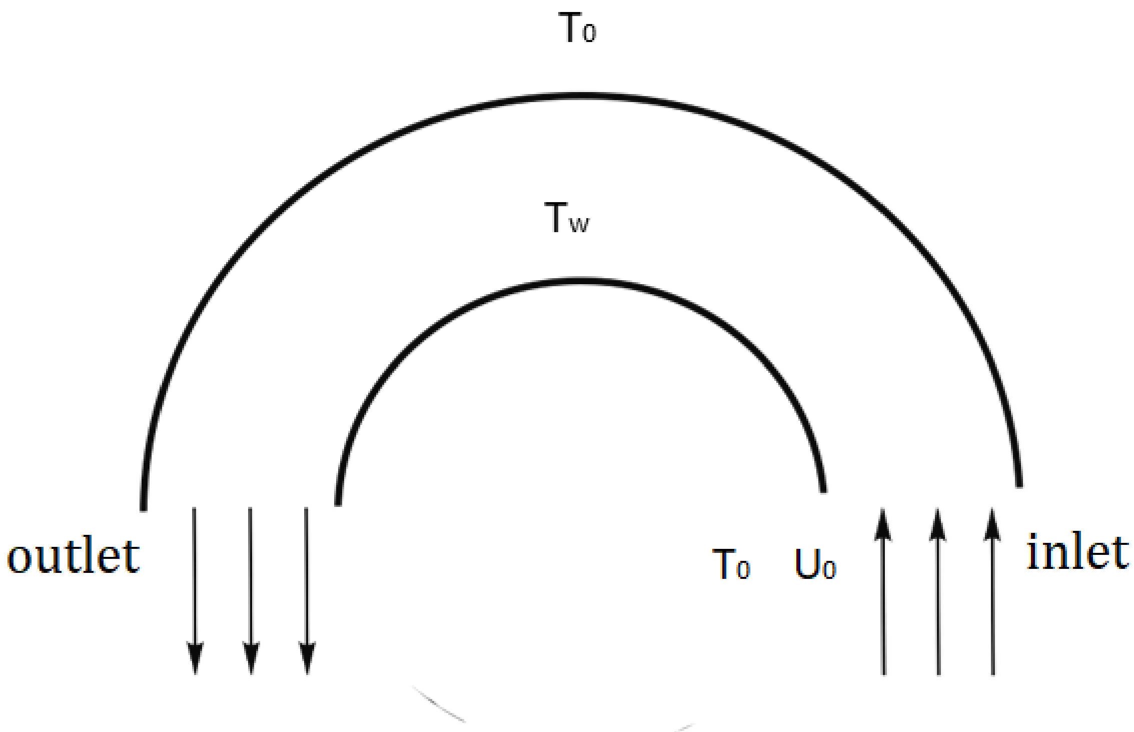
Physical Background



Mathematical background

Main equations in polar coordinate system

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\varphi}{r}\frac{\partial u_r}{\partial \varphi}-\frac{u_\varphi^2}{r}=-\frac{\partial p}{\partial r}+\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \varphi^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\varphi}{\partial \varphi}\right]$$

$$\frac{\partial u_\varphi}{\partial t}+u_r\frac{\partial u_\varphi}{\partial r}+\frac{u_\varphi}{r}\frac{\partial u_\varphi}{\partial \varphi}-\frac{u_\varphi u_r}{r}=-\frac{1}{r}\frac{\partial p}{\partial \varphi}+\left[\frac{\partial^2 u_\phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 u_\varphi}{\partial \varphi^2}+\frac{1}{r}\frac{\partial u_\varphi}{\partial r}-\frac{u_\varphi}{r^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \varphi}\right]$$

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi}=0$$

$$\frac{\partial \theta}{\partial \tau}+u_r\frac{\partial \theta}{\partial r}-\frac{u_\varphi}{r}\frac{\partial \theta}{\partial \varphi}=\frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 \theta}{\partial \varphi^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}\right)$$

Vorticity – Stream function formulation

$$u_r = -\frac{1}{r}\frac{\partial \psi}{\partial \varphi} \qquad u_\varphi = \frac{\partial \psi}{\partial r} \qquad \Omega = \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r} - \frac{1}{r}\frac{\partial u_r}{\partial \varphi}$$

$$\frac{\partial \psi}{\partial t} - \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \varphi^2} = -\Omega + \frac{1}{r}\frac{\partial \psi}{\partial r}$$

$$\frac{\partial \Omega}{\partial t} + u_r\frac{\partial \Omega}{\partial r} + \frac{u_\varphi}{r}\frac{\partial \Omega}{\partial \varphi} = +\frac{1}{Re}\left[\frac{\partial^2 \Omega}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \Omega}{\partial \varphi^2} + \frac{1}{r}\frac{\partial \Omega}{\partial r}\right] - \frac{Gr}{Re^2}\frac{\partial \theta}{\partial \varphi}$$

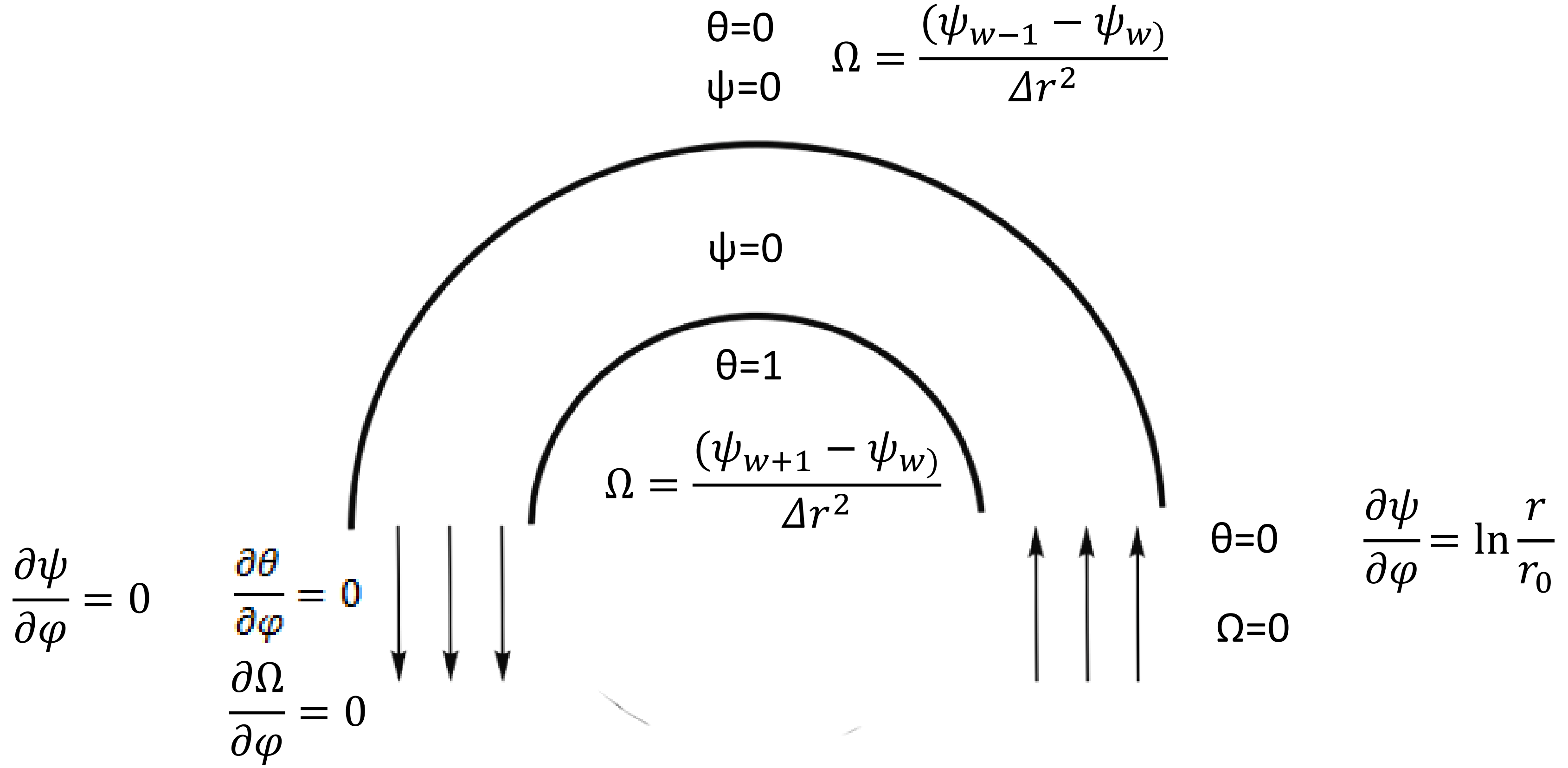
$$\frac{\partial \theta}{\partial \tau} + u_r\frac{\partial \theta}{\partial r} + \frac{u_\varphi}{r}\frac{\partial \theta}{\partial \varphi} = \frac{1}{\Pr \mathrm{Re}}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \theta}{\partial \varphi^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right)$$

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi}=0 \qquad u_r=-\frac{1}{r}\frac{\partial \psi}{\partial \varphi} \qquad u_\varphi=\frac{\partial \psi}{\partial r}$$

$$\frac{\partial}{\partial r}\left(-\frac{1}{r}\frac{\partial \psi}{\partial \varphi}\right)+\frac{1}{r}\left(-\frac{1}{r}\frac{\partial \psi}{\partial \varphi}\right)+\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{\partial \psi}{\partial r}\right)=0$$

$$\frac{1}{r^2}\frac{\partial \psi}{\partial \varphi}-\frac{1}{r}\frac{\partial^2 \psi}{\partial \varphi \partial r}-\frac{1}{r^2}\frac{\partial \psi}{\partial \varphi}+\frac{1}{r}\frac{\partial^2 \psi}{\partial \varphi \partial r}\equiv 0$$

Boundary conditions

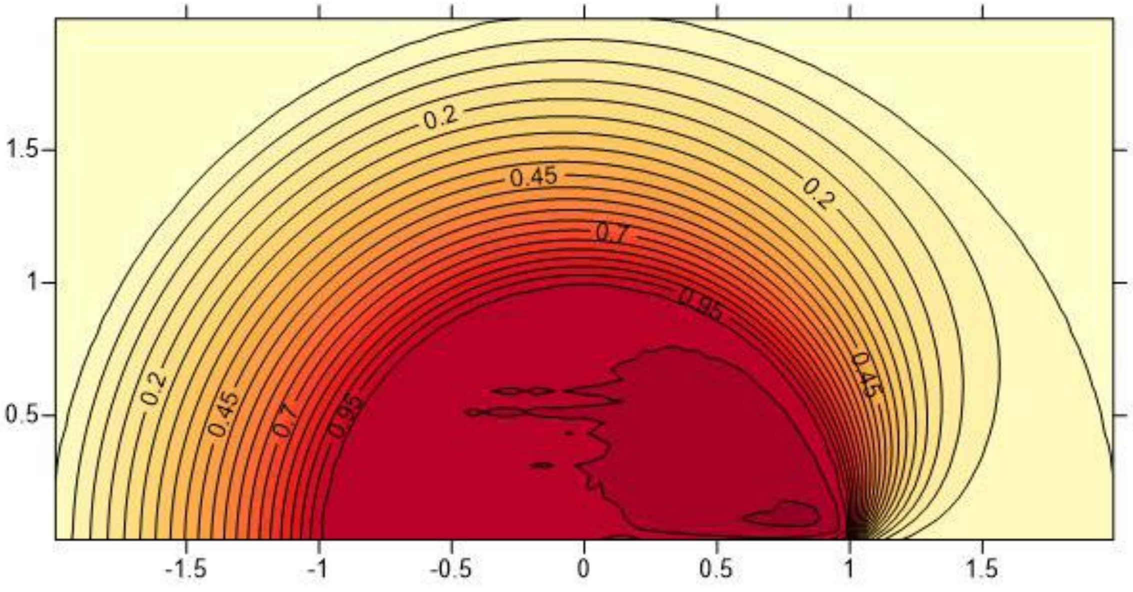


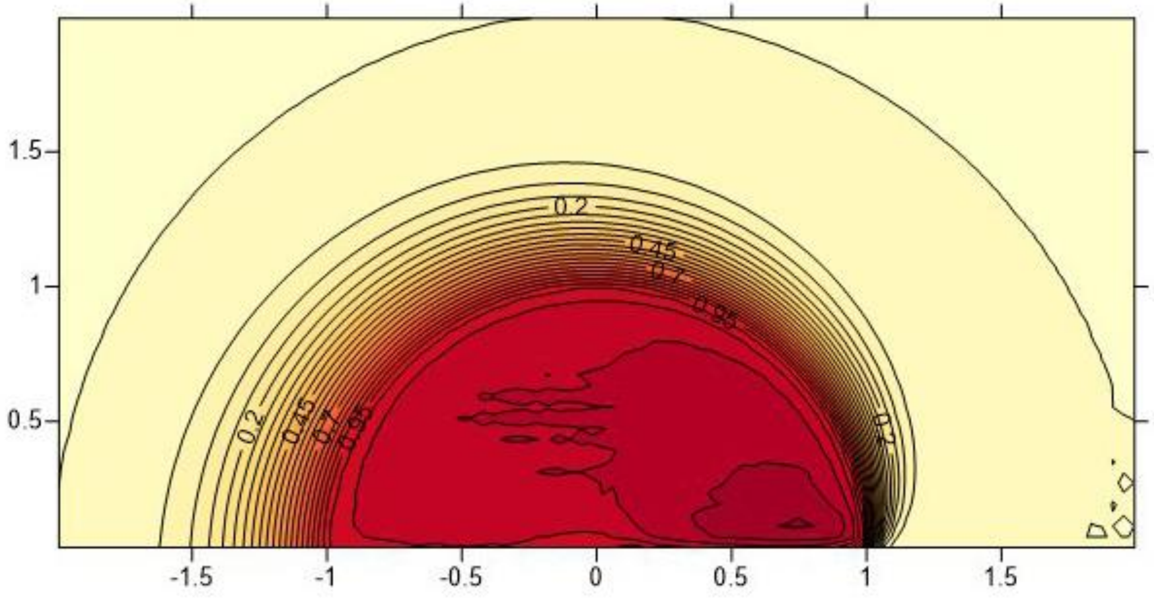
Result

Re = 10

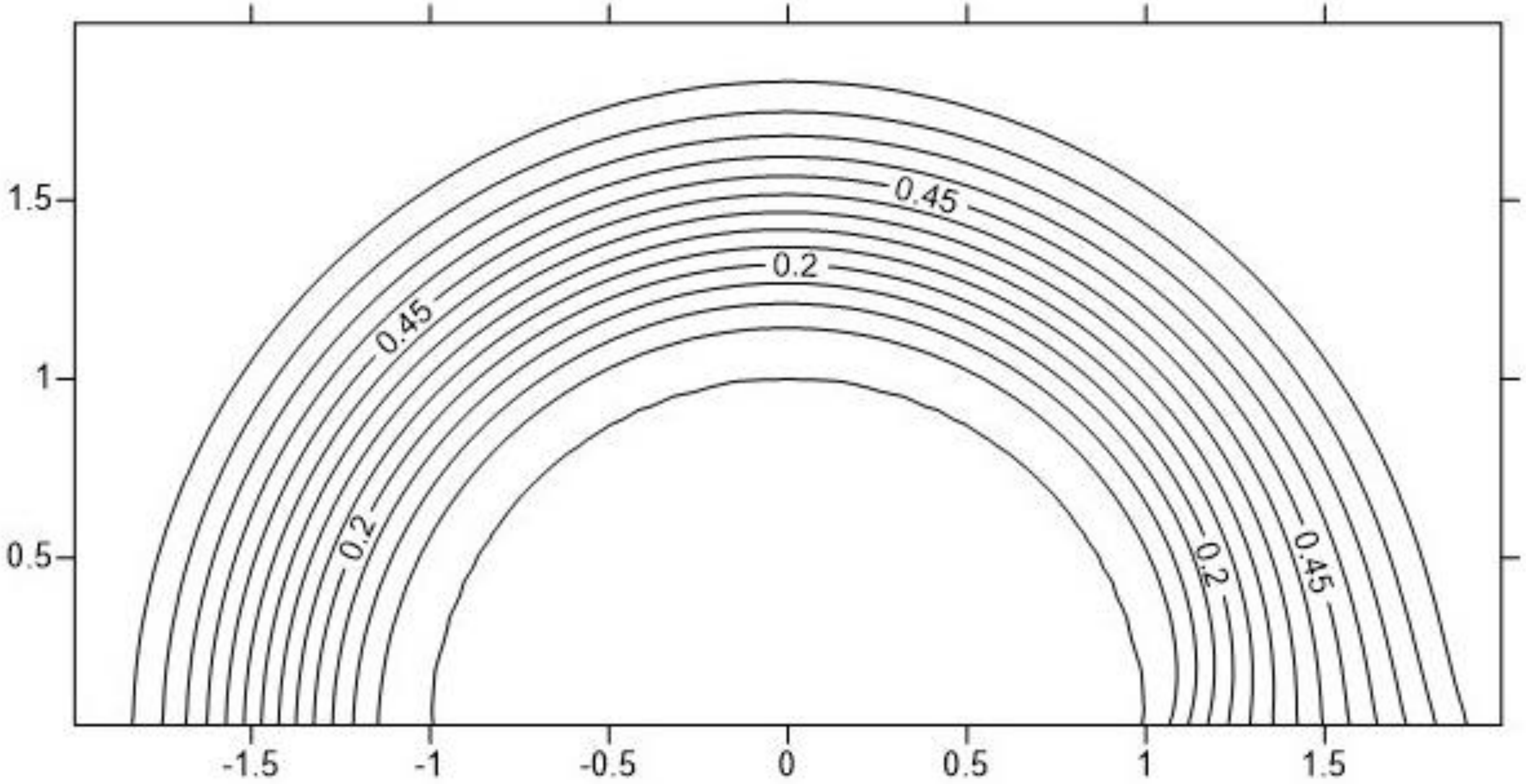
Pr = 1;

Re = 100





Stream function



Re = 10;

Pr = 0.1 Pr = 1 Pr = 10