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Nuclear Physics **A286** (1977) 53–66; © *North-Holland Publishing Co., Amsterdam*

SYSTEMATIC INVESTIGATION OF SCATTERING PROBLEMS WITH THE RESONATING-GROUP METHOD †

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Received 17 November 1976

Abstract: The resonating-group method is used to make a systematic study of the $\alpha+\alpha$, $N+\alpha$, $N+{}^{16}O$ and $N+{}^{40}Ca$ scattering problems. The nucleon-nucleon potential employed contains a weakly repulsive core and yields a satisfactory description of not only the two-nucleon low-energy scattering data but also the essential properties of the deuteron, triton and α-particle. The result shows that the calculated values do generally agree quite well with experimental data. In particular, it is found that, because of the use of totally antisymmetric wave functions, the scattering behavior at large angles is adequately described.

1. Introduction

The resonating-group method [1,2]), which employs totally antisymmetric wave functions, has been frequently utilized to describe the behavior of nuclear systems from a microscopic viewpoint. Especially in the analysis of scattering problems, this method is useful because rather accurate results can already be obtained with the adoption of a single-channel approximation, if proper precautions are taken. In this study, we wish to demonstrate this assertion by a systematic investigation of the $\alpha+\alpha$, $N+\alpha$, $N+{}^{16}O$ and $N+{}^{40}Ca$ systems †††.

In the $\alpha+\alpha$ case, our purpose is to show that a single-channel resonating-group calculation can yield a good agreement with empirical phase-shift values. Here the necessary precaution is to carefully take the specific distortion effect [7,8]) into account ‡. For this one needs to employ in the calculation a nucleon-nucleon potential which can give a satisfactory description of not only the two-nucleon low-energy scattering data but also the essential properties of light nuclei, in particular, those of the α-particle. In addition, it is of course imperative that the adoption of this potential does not cause resonating-group calculations to become

† Work supported in part by the US Energy Research and Development Administration.

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††† All these systems have been previously investigated [3–6]). In this study, we make the improvement of adopting a more realistic nucleon-nucleon potential, using a better α-cluster wave function, and taking in the $\alpha+\alpha$ case the specific distortion effect explicitly into account.

‡ A preliminary investigation [9]) of the specific distortion effect in $\alpha+\alpha$ scattering has been made. There it was found that, especially in $l = 0$ and 2 states, this effect does have a moderate, but significant, influence.

computationally unfeasible. Fortunately, such a nucleon-nucleon potential can indeed be found, and a description of which will be given in sect. 2 where a brief formulation of the $\alpha+\alpha$ problem will also be presented.

Next, we consider the $N+\alpha$ scattering problem. In this case, it is necessary to introduce a spin-orbit component into the nucleon-nucleon potential in order to split the calculated phase shifts. As will be seen in sect. 3, the result obtained is, as in the $\alpha+\alpha$ case, very satisfactory.

In sect. 4, the problems of $N+{}^{16}O$ and $N+{}^{40}Ca$ scattering are studied. Here the important points to observe are that the cluster internal functions must be chosen to yield correctly the rms radii of the corresponding clusters and reaction effects must be taken approximately into consideration.

Results are presented in individual sections and concluding remarks are given in sect. 5.

2. The $\alpha+\alpha$ scattering

2.1. FORMULATION

The formulation of the $\alpha+\alpha$ problem is very similar to that given in ref. [9]). The specific distortion effect is properly taken into account by the addition of square-integrable distortion functions into the no-distortion approximation. That is, we write the trial wave function as

$$\Psi_N = \Psi_0 + \sum_{i=1}^{N} \mathscr{A}[\tilde{\phi}_{1i}\tilde{\phi}_{2i}G_i(\boldsymbol{R}_1-\boldsymbol{R}_2)\xi(s,t)\chi(\boldsymbol{R}_{c.m.})], \tag{1}$$

where $\mathscr{A}$ is an antisymmetrization operator, N is the number of distortion functions, $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ denote the c.m. coordinates of the two α-clusters, $\xi(s, t)$ is an appropriate spin-isospin function, and $\chi(\boldsymbol{R}_{c.m.})$ is a normalizable function describing the c.m. motion of the entire system. The function Ψ_0 represents the usual no-distortion wave function; it is given by

$$\Psi_0 = \mathscr{A}[\phi_{\alpha 1}\phi_{\alpha 2}F(\boldsymbol{R}_1-\boldsymbol{R}_2)\xi(s,t)\chi(\boldsymbol{R}_{c.m.})], \tag{2}$$

with $F(\boldsymbol{R}_1-\boldsymbol{R}_2)$ being a relative-motion function and $\phi_{\alpha k}$ ($k = 1$ or 2) describing the spatial structure of a free α-particle. The function $\phi_{\alpha 1}$ is chosen as

$$\phi_{\alpha 1} = \sum_{i=1}^{N_0} A_i \exp\left[-\tfrac{1}{2}\alpha_i \sum_{j=1}^{4} (\boldsymbol{r}_j-\boldsymbol{R}_1)^2\right], \tag{3}$$

and similarly for $\phi_{\alpha 2}$. The parameters α_i and A_i ($i = 1$ to N_0) are determined by minimizing the expectation value of the α-particle Hamiltonian; their values will be given below in subsect. 2.2 where our choice of a nucleon-nucleon potential is also discussed.

The functions $\tilde{\phi}_{1i}$ and $\tilde{\phi}_{2i}$ in eq. (1) are chosen as

$$\tilde{\phi}_{1i} = \exp\left[-\tfrac{1}{2}\tilde{\alpha}_i \sum_{j=1}^{4} (\boldsymbol{r}_j - \boldsymbol{R}_1)^2\right],$$

$$\tilde{\phi}_{2i} = \exp\left[-\tfrac{1}{2}\tilde{\alpha}_i \sum_{j=5}^{8} (\boldsymbol{r}_j - \boldsymbol{R}_2)^2\right], \tag{4}$$

with the choice of $\tilde{\alpha}_i$ to be discussed below. As for the function $G_l(\boldsymbol{R}_1 - \boldsymbol{R}_2)$ in the distortion-function term of Ψ_N, we make the following expansion:

$$G_l(\boldsymbol{R}_1 - \boldsymbol{R}_2) = \sum_{l=0}^{\infty} \tilde{A}_{li} \frac{1}{R} g_{li}(R) P_l(\cos\theta), \tag{5}$$

where

$$g_{li}(R) = R^{n+1} \exp(-\tilde{\beta}_i R^2), \tag{6}$$

with

$$n = \begin{cases} 4 & \text{for } l = 0, 2 \\ l & \text{for } l \geqq 4. \end{cases}$$

The choice of $\tilde{\beta}_i$ will also be discussed below.

The variational amplitudes $\tilde{A}_{li}$ as well as the variational function $F(\boldsymbol{R}_1 - \boldsymbol{R}_2)$ are determined by solving the projection equation [7])

$$\langle \delta\Psi_N | H - E_T | \Psi_N \rangle = 0, \tag{7}$$

where E_T is the total energy of the system composed of the internal energies of the two α-particles and their relative energy E in the c.m. frame. The translationally invariant Hamiltonian H in eq. (7) has the form

$$H = -\frac{\hbar^2}{2M} \sum_{i=1}^{8} \nabla_i^2 + \sum_{i<j=1}^{8} V_{ij} - T_{\text{c.m.}}, \tag{8}$$

where $T_{\text{c.m.}}$ is the c.m. kinetic-energy operator and V_{ij} is a nucleon-nucleon potential to be described in the next subsection.

2.2. NUCLEON-NUCLEON POTENTIAL

The nucleon-nucleon potential is assumed as †

$$V_{ij} = (V_R + \tfrac{1}{2}(1 + P_{ij}^{\sigma})V_t + \tfrac{1}{2}(1 - P_{ij}^{\sigma})V_s)(\tfrac{1}{2}u + \tfrac{1}{2}(2-u)P_{ij}^{r}) + \tfrac{1}{2}(1 + \tau_{iz})\tfrac{1}{2}(1 + \tau_{jz})e^2/r_{ij}, \tag{9}$$

where u (≈ 1) is an exchange-mixture parameter, chosen in a way to be discussed

† In contrast to our previous $\alpha + \alpha$ calculation [4]), the exchange-Coulomb interaction is fully taken into consideration.

below. The potentials V_R, V_t and V_s are taken to have the following Gaussian forms:

$$V_R = V_{0R}\exp(-\kappa_R r_{ij}^2), \qquad V_t = -V_{0t}\exp(-\kappa_t r_{ij}^2), \qquad V_s = -V_{0s}\exp(-\kappa_s r_{ij}^2), \quad (10)$$

with

$$\begin{aligned} V_{0R} &= 200.0 \text{ MeV}, & \kappa_R &= 1.487 \text{ fm}^{-2}, \\ V_{0t} &= 178.0 \text{ MeV}, & \kappa_t &= 0.639 \text{ fm}^{-2}, \\ V_{0s} &= 91.85 \text{ MeV}, & \kappa_s &= 0.465 \text{ fm}^{-2}. \end{aligned} \quad (11)$$

With these values, the n-p triplet and p-p singlet s-wave effective-range parameters are

$$\begin{aligned} a_t &= 5.426 \text{ fm}, & r_{0t} &= 1.764 \text{ fm}, \\ a_s &= -7.819 \text{ fm}, & r_{0s} &= 2.745 \text{ fm}, \end{aligned} \quad (12)$$

which agree well with the values determined experimentally [10,11]). The α-particle wave function is obtained by optimizing the expectation value E_α of the α-particle Hamiltonian. With N_0 in eq. (3) equal to 1, 2, 3 and 10, the results obtained for E_α and the α-particle rms radius R_α are given in table 1 †. From this table it is noted that, already for $N_0 = 2$, the values for E_α and R_α are quite close to those obtained with $N_0 = 10$. Therefore, for computational simplicity, we shall always adopt $N_0 = 2$ in the following resonating-group calculations.

TABLE 1

Results for E_x and R_x

N_0	A_1	A_2	A_3	α_1 (fm^{-2})	α_2 (fm^{-2})	α_3 (fm^{-2})	E_x (MeV)	R_x (fm)
1	1.0			0.606			−24.69	1.362
2	1.0	16.057		0.305	0.708		−25.39	1.415
3	1.0	9.478	−8.992	0.355	0.795	2.66	−25.60	1.407
10 *)							−25.61	1.409

*) The values of α_i used in the $N_0 = 10$ case range from 0.1 to 5.16 fm^{-2}.

The nucleon-nucleon potential of eqs. (9)–(11) is an appropriate choice, because the values of E_α and R_α are rather close to the corresponding experimental values of −28.3 MeV and 1.48 ± 0.06 fm [refs. [12,13])]. In addition, it should be emphasized that the repulsive component in the nuclear part of V_{ij}, represented by the term V_R, is rather weak. This is important, since otherwise one will have to introduce an

† With a space-symmetric function given by eq. (3), the value of E_x is independent of u. Also, it should be mentioned that the nucleon-nucleon potential of eqs. (9)–(11) yields quite satisfactory results for the binding energies and rms radii of the deuteron and the triton.

anti-correlation Jastrow factor [1]) into the trial function of eq. (1), which will make resonating-group calculations highly complicated from a computational viewpoint †.

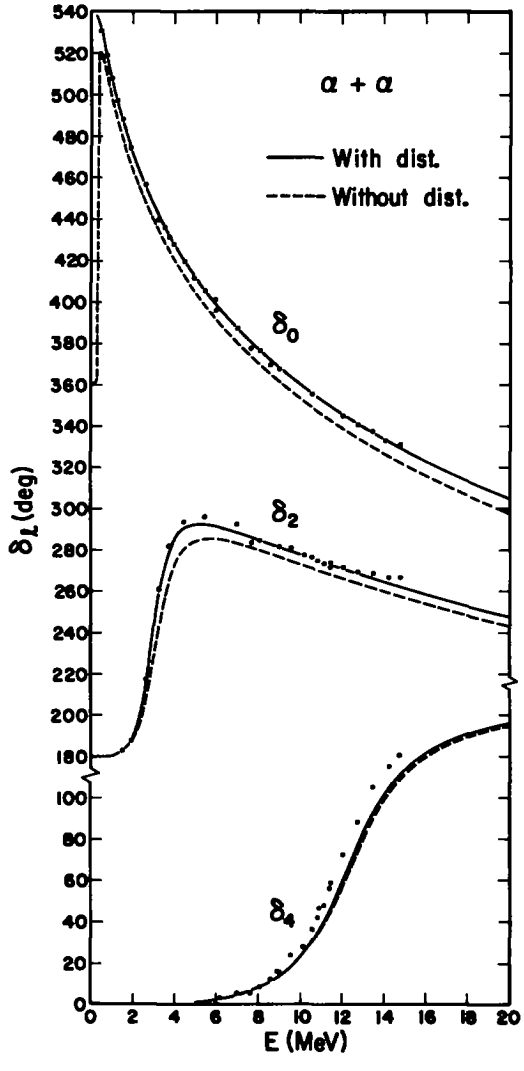

Fig. 1. The $\alpha+\alpha$ scattering with and without specific distortion effect. Empirical data shown are those of ref. [14]).

† In table 1 it is noted that, even with $N_0 = 1$, the value of E_x is still fairly close to the $N_0 = 10$ result. This is an indication that, with our choice of a weakly repulsive core in the nucleon-nucleon potential, the omission of the anti-correlation factor is not likely to introduce serious errors into the resonating-group calculation.

2.3. RESULTS AND DISCUSSION

The procedure for choosing the number N of distortion functions and the nonlinear parameters $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ ($i = 1$ to N) is discussed in detail in refs. [7,9]). In the present case of $\alpha+\alpha$ scattering, it is found that, for a value of u close to but less than 1, a choice of $N = 3$ is sufficient and the corresponding $(\tilde{\alpha}_i, \tilde{\beta}_i)$ sets can be taken as (0.7 fm^{-2}, 0.32 fm^{-2}), (0.7 fm^{-2}, 0.48 fm^{-2}) and (0.7 fm^{-2}, 0.64 fm^{-2}). The value of u is then chosen such that the resonance energy of the $l = 0$ state is correctly obtained. The result is $u = 0.950$.

In fig. 1 we show $l = 0$, 2 and 4 $\alpha+\alpha$ phase shifts, calculated with $u = 0.950$, at c.m. energies from 0 to 20 MeV. The solid curves show the result obtained with the three distortion functions mentioned above, while the dashed curves show the no-distortion ($N = 0$) result. The empirical data points are those of ref. [14]). As is seen, there is a good over-all agreement between the calculated result with $N = 3$ and the empirical result. In the $l = 4$ states, the calculated phase shifts are generally somewhat too small. This can probably be attributed to the fact that, in our calculation, a rather simple nucleon-nucleon potential containing a weakly repulsive core has been adopted.

From fig. 1 one further notes that, even though the specific distortion effect has only a minor influence on the phase shift in the $l = 4$ state, it does have a significant influence on both $l = 0$ and $l = 2$ phase shifts. Therefore, for a proper description of the behavior of the $\alpha+\alpha$ system in the strong-interaction region, it is important that this effect should be carefully taken into consideration.

3. The N+α scattering

3.1. FORMULATION

The N+α scattering problem is somewhat simpler to solve than the $\alpha+\alpha$ scattering problem. Because the α-cluster feels the polarizing influence of only a single nucleon, the specific distortion effect is expected to be rather unimportant †. Therefore, in our present calculation, we shall simply consider this problem in the no-distortion approximation, using the α-cluster wave function given by eq. (3) with $N_0 = 2$.

To split the calculated phase shifts in various orbital-angular-momentum states, we add a spin-orbit potential into the nucleon-nucleon potential of eq. (9). This potential has the form

$$u_{ij} = -\frac{1}{2\hbar} V_\lambda \exp(-\lambda r_{ij}^2)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot(\boldsymbol{r}_i-\boldsymbol{r}_j)\times(\boldsymbol{p}_i-\boldsymbol{p}_j). \tag{13}$$

For simplicity in calculation, we shall consider only the limit case where the range

† In the n+t scattering problem [15]) where the specific distortion effect has been explicitly considered, it was found that, even though the triton cluster is more compressible than the α-cluster, this effect has a negligible influence.

parameter λ approaches infinity. This is a reasonable assumption, because it was found that in light systems [3,16]) the scattering results are rather insensitive to the value of λ, as long as it is not chosen to have a very small value and the depth parameter V_λ is correspondingly adjusted.

The formulation of the N+α problem is similar to that given in ref. [3]); hence, only a very brief description will be given here. In the limit where λ approaches infinity, the intercluster spin-orbit interaction is characterized by a single parameter $J_\lambda = V_\lambda \lambda^{-\frac{1}{2}}$, and the following relation holds:

$$\int_0^\infty k_l^{\text{s.o.}}(R, R') f_{Jl}(R') \mathrm{d}R' = C V_{\text{s.o.}}(R) f_{Jl}(R), \tag{14}$$

where $V_{\text{s.o.}}(R)$ and $k_l^{\text{s.o.}}(R, R')$ represent, respectively, the direct and exchange parts of the intercluster spin-orbit potential, $f_{Jl}(R)$ is the radial part of the relative-motion wave function in a particular (J, l) state, and C is equal to 0.5 in the N+α case. Thus, in each (J, l) state, we obtain the function $f_{Jl}(R)$ by solving the integrodifferential equation †

$$\left\{\frac{\hbar^2}{2\mu}\left[\frac{\mathrm{d}^2}{\mathrm{d}R^2} - \frac{l(l+1)}{R^2}\right] + E - V_{\text{N}}(R) - V_{\text{C}}(R) - \eta_{Jl} V_{\text{s.o.}}(R)\right\} f_{Jl}(R)$$

$$= \int_0^\infty [k_l^{\text{N}}(R, R') + k_l^{\text{C}}(R, R')] f_{Jl}(R') \mathrm{d}R', \tag{15}$$

where V_{N} and V_{C} are the direct nuclear central and direct Coulomb potentials, respectively, and k_l^{N} and k_l^{C} represent the kernel functions arising from the exchange character in the nucleon-nucleon potential and the antisymmetrization procedure. The quantity η_{Jl} is given by

$$\eta_{l+\frac{1}{2},l} = l, \qquad \eta_{l-\frac{1}{2},l} = -(l+1), \qquad \eta_{\frac{1}{2},0} = 0. \tag{16}$$

The parameter J_λ in $V_{\text{s.o.}}(R)$ is then phenomenologically adjusted in a way as discussed below.

It should be noted that, because of the relation expressed by eq. (14) the integrodifferential equation given by eq. (15) is entirely equivalent to that given by eq. (8) in ref. [3]). The resultant, phenomenologically determined value of J_λ will contain, of course, the effect of the exchange part of the intercluster spin-orbit potential. We choose to write the integrodifferential equation in the form of eq. (15), rather than in the form of eq. (8) in ref. [3]), because in the more complicated problems of N+^{16}O and N+^{40}Ca scattering ††, we shall adopt the simplifying assumption of not explicitly

† In the derivation of this equation, we have used the expectation value of the cluster Hamiltonian for the internal energy of the cluster.

†† We should mention that, in the N+^{16}O and N+^{40}Ca scattering problems, the relation expressed by eq. (14) does not strictly hold even in the limit case where λ approaches infinity.

deriving the exchange part of the intercluster spin-orbit interaction, but simply taking its effect into consideration by phenomenologically adjusting the parameter J_λ.

3.2. RESULTS AND DISCUSSION

The exchange-mixture parameter u and the spin-orbit strength parameter J_λ are adjusted to yield a best fit with the empirical p+α phase-shift data of Arndt, Roper and Shotwell [17]). The values so obtained are $u = 0.970$ and $J_\lambda = 50$ MeV · fm^5. In fig. 2, the solid curves show the calculated results for δ_0, δ_1^+, and δ_1^-, while the solid dots represent the empirical values for these phase shifts. As is seen, the agreement between the calculated and empirical values is indeed quite good. This is somewhat to be expected, because in the energy region below the reaction threshold, the p+α system should be a rather ideal case for a single-channel resonating-group calculation.

In part (a) of figs. 3 and 4, we show the calculated n + α differential cross section and polarization at a c.m. energy of 13.12 MeV where partial waves up to $l = 3$ make significant contributions. The cross-section data shown are those of Shamu and Jenkin [18]), while the polarization data shown are those of May, Walter and Barschall [19]). From these figures one sees that there is a good agreement between calculation and experiment, thus further verifying the usefulness of a resonating-group calculation in a light system.

In comparing with our previous N+α calculation [3]) where a nucleon-nucleon potential of simpler form was used, we note that the quality of fit obtained here is

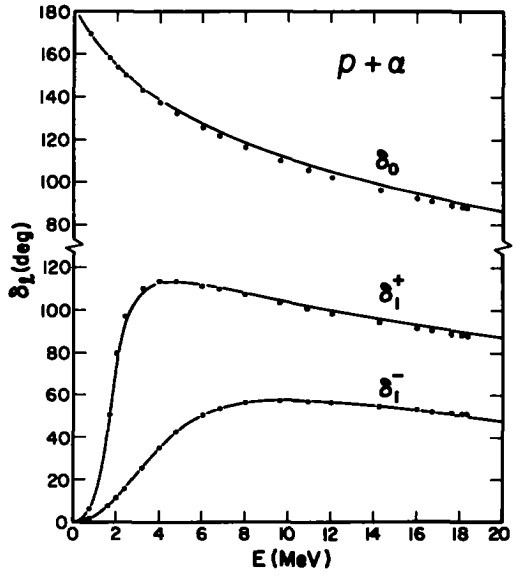

Fig. 2. Phase shifts for p+α scattering in $l = 0$ and 1 states. Empirical data shown are those of ref. [17]).

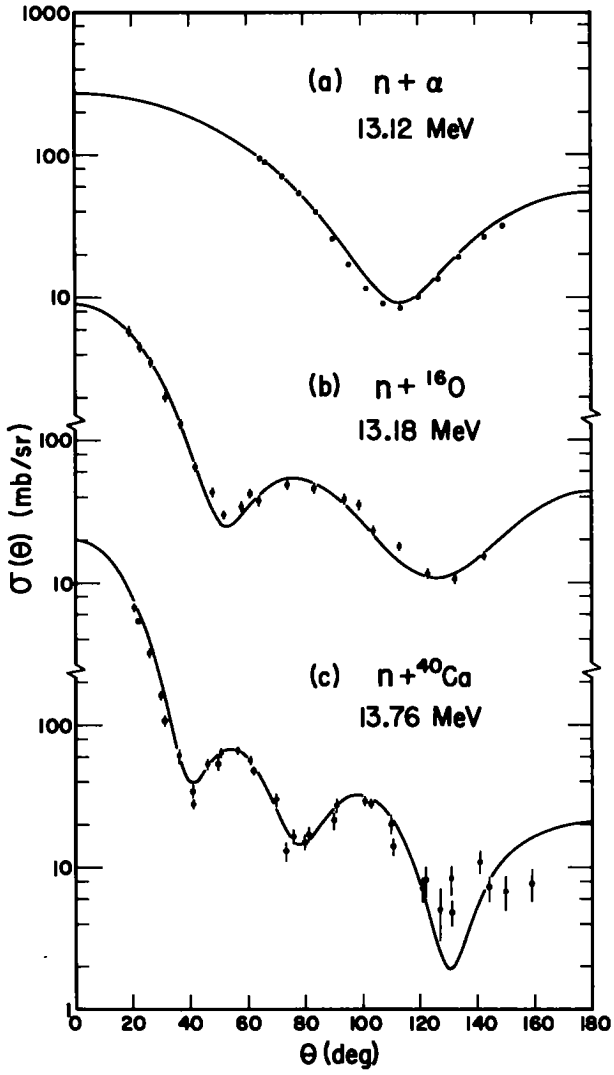

Fig. 3. Comparison of calculated and experimental differential cross sections for n+α, n+^{16}O and n+^{40}Ca scattering at indicated energies. Experimental data shown are those of refs. [18,23-25]).

only slightly better. This indicates that, at least for resonating-group calculations at low and medium energies, the most important requirement on a nucleon-nucleon potential is that it should fit satisfactorily the low-energy two-nucleon scattering data. However, it should be mentioned that the present nucleon-nucleon potential does have the advantage that it yields quite correctly the binding energies and rms radii of the deuteron, triton and α-particle; therefore, it can be employed in calculations where specific distortions of these clusters play an important role.

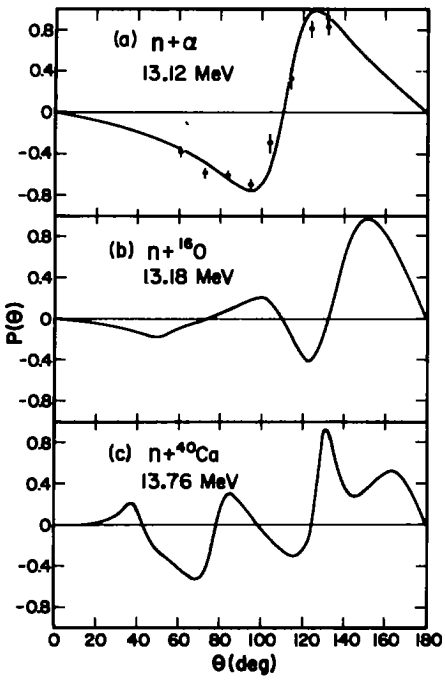

Fig. 4. Calculated polarizations for n+α, n+^{16}O and n+^{40}Ca scattering at indicated energies. Experimental data shown are those of ref. [19]).

4. The N+^{16}O and N+^{40}Ca scattering

4.1. FORMULATION

The formulations of the N+^{16}O and N+^{40}Ca problems are essentially the same as those given in refs. [5,6]); hence, they will not be further described here. The only notable difference is that in our present calculation we employ the nucleon-nucleon potential of eqs. (9) and (13), rather than the nucleon-nucleon potential given by eq. (5) of ref. [3]).

For calculations at energies where reaction channels are open, we add, as in previous resonating-group calculations [13]), a phenomenological imaginary potential $iW(R)$ into the formulation. In other words, we replace $V_N(R)$ in eq. (15) by $V_N(R)+iW(R)$. For $W(R)$ we use a surface-derivative form, i.e.,

$$W(R) = -4W_s \exp[(R-R_i)/a_i]\{1+\exp[(R-R_i)/a_i]\}^{-2}. \tag{17}$$

The geometry parameters R_i and a_i are chosen as

$$R_i = 3.2 \text{ fm}, \qquad a_i = 0.6 \text{ fm}, \tag{18}$$

in $N+^{16}O$ scattering and

$$R_i = 4.3 \text{ fm}, \qquad a_i = 0.6 \text{ fm}, \tag{19}$$

in $N+^{40}Ca$ scattering. The parameter W_s is then adjusted at each energy to obtain a best fit with experiment.

4.2. RESULTS AND DISCUSSION

The values of u and J_λ are obtained in the same way as described in refs. [5,6]). In the $n+^{16}O$ case, we determine the u-value by fitting the neutron separation energy of 3.26 MeV in the $l = 0$, $\frac{1}{2}^+$ first excited state of ^{17}O [ref. [20])]. This yields a value for u equal to 0.924. In the $n+^{40}Ca$ case, we solve eq. (15) in the $l = 1$ state with bound-state boundary condition and $V_{s.o.}(R)$ set equal to zero, and adjust u until the calculation yields a neutron separation energy of 5.75 MeV. This value corresponds to the $\boldsymbol{l}\cdot\boldsymbol{s}$ weighted average of the neutron separation energies in the 1.94 MeV, $\frac{3}{2}^-$ state and the 3.94 MeV, $\frac{1}{2}^-$ state of ^{41}Ca [refs. [21,22])]. The value of u so obtained is 0.856. As for the J_λ value, we find that, in both $n+^{16}O$ and $n+^{40}Ca$ cases, a choice of $J_\lambda = 50$ MeV · fm^5, the same value as that used in the $N+\alpha$ case, yields adequate results for the level splittings.

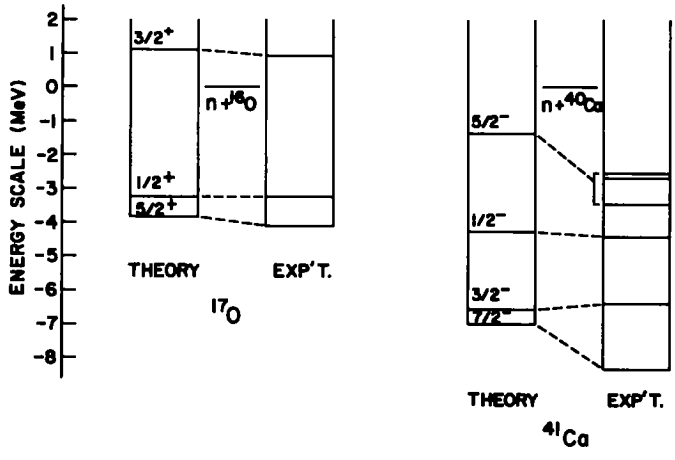

Fig. 5. Comparison of calculated and experimental energy spectra of ^{17}O and ^{41}Ca. The levels shown are those which have predominantly $n+^{16}O$ and $n+^{40}Ca$ cluster configurations.

Calculated levels in the compound nuclei ^{17}O and ^{41}Ca are shown in fig. 5, together with those experimental levels which have predominantly $n+^{16}O$ and $n+^{40}Ca$ cluster configurations. Here one sees that, even though the agreement between theory and experiment is generally quite reasonable, the calculated $l = 3$ levels in ^{41}Ca do lie somewhat higher than the corresponding experimental levels.

Very probably, this is caused by the fact that in our calculation a relatively simple ^{40}Ca wave function has been used, which does not describe the surface region of this nucleus quite properly.

A comparison between calculated and experimental[23-25]) differential cross sections for n + ^{16}O scattering at 13.18 MeV and n + ^{40}Ca scattering at 13.76 MeV is shown in parts (b) and (c) of fig. 3. To obtain the calculated curves, we have used $W_s = 4.7$ MeV in the n + ^{16}O case and $W_s = 9.0$ MeV in the n + ^{40}Ca case. As is seen from this figure, the agreement is quite satisfactory. The calculated n + ^{16}O reaction and integrated-elastic cross sections are equal to 639 and 883 mb, respectively, while the calculated n + ^{40}Ca reaction and integrated-elastic cross sections are equal to 1184 and 1020 mb, respectively. All these calculated values agree very well with corresponding measured values[23,24]).

Calculated polarizations for n + ^{16}O scattering at 13.18 MeV and n + ^{40}Ca scattering at 13.76 MeV are depicted in parts b and c of fig. 4. Experimental data are not yet available in these cases, so only calculated curves are shown for interest.

5. Conclusion

In this investigation, we have made a systematic study of the $\alpha+\alpha$, N + α, N + ^{16}O and N + ^{40}Ca systems using the resonating-group method in the single-channel approximation. The nucleon-nucleon potential employed contains a weakly repulsive core and gives a satisfactory description of not only the two-nucleon low-energy scattering data but also the essential properties of the deuteron, triton and α-particle. The result shows that, with proper precautions, the calculated values do generally agree quite well with experimental data. In particular, it is found that the fit to experiment in the backward angular region is satisfactory, which is of course a consequence of the fact that in resonating-group calculations totally antisymmetric wave functions are used.

There is, however, one undesirable feature which should be mentioned. From our calculations it is found that the required value of the exchange-mixture parameter u is somewhat different in different systems. This is shown in table 2, where one sees that the value of u decreases as the total number of nucleons in the compound system increases. This is a reflection of the fact that the nucleon-nucleon potential used here

TABLE 2

Variation of u with A, the total number of nucleons in the compound system

System	A	u
N + α	5	0.970
$\alpha+\alpha$	8	0.950
N + ^{16}O	17	0.924
N + ^{40}Ca	41	0.856

does not quite satisfy the requirement of saturation. To remedy this, one could of course simply adopt a nucleon-nucleon potential which contains a core of larger repulsive strength. But this would necessitate the introduction of a Jastrow anti-correlation factor into the resonating-group trial wave function and, consequently, would complicate the computational procedure [26,27]) to a very large extent †.

In conclusion, we feel that this investigation has convincingly demonstrated the usefulness of the resonating-group method as a microscopic method of treating nuclear systems. Therefore, it should be extensively used in the future to study those nuclear problems which have, until now, only been treated by macroscopic means.

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† Alternatively, one could use a nucleon-nucleon potential containing a strong tensor component. However, this would also cause great computational difficulties, if the tensor component is to be properly handled.

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