λ_{zero} after discarding the highest and lowest 10% of the measurements. The reported statistical error (1.4 pm) is twice the standard error of the mean of the trimmed data set. Figure 3 shows the 35 λ_{zero} measurements and the trimmed mean. Table I shows a summary of the error budget, and we discuss systematic errors associated with the laser system below.

We generated 2 W of laser light using a master oscillator power amplifier system [22,23]. We used a Littrow extended cavity diode laser with wavelength-dependent pointing compensation [24] to keep the seed light well coupled into a tapered amplifier over a 5 nm tuning range. A Bristol Instruments 621B wavelength meter calibrated against a saturated absorption signal in a vapor cell measured the vacuum wavelength of the seed laser with an uncertainty of 0.3 pm.

After spatial filtering with a single mode fiber, 1% of the power was in a broadband spectral component from spontaneous emission in the tapered amplifier [25]. To quantify the uncertainty in λ_{zero} caused by this broadband component, we characterized the laser spectrum with a grating spectrometer, and we accounted for the laser spectrum by modifying Eq. (3) with an additional integral over the frequency-dependent laser intensity. We calculated that the broadband light introduces an uncertainty of 0.5 pm to our measurement of λ_{zero}.

We also measured the crossing angle between the laser and atom beams and applied a 0.56(5) pm correction to λ_{zero} due to the Doppler shift. We note that our measurement was performed on an atom beam with a natural abundance of potassium isotopes. If we assume that R is the same for ^{39}K and ^{41}K, then the measured λ_{zero} is predicted to be 0.03 pm less than the ^{39}K λ_{zero}. Finally, we calculated that, at the intensity we are using, the hyperpolarizability of the ground state causes a shift for λ_{zero} on the order of 0.001 pm. This is negligible in our current experiment but suggests an interesting opportunity for future measurements of intensity-dependent shifts in λ_{zero} due to higher order effects.

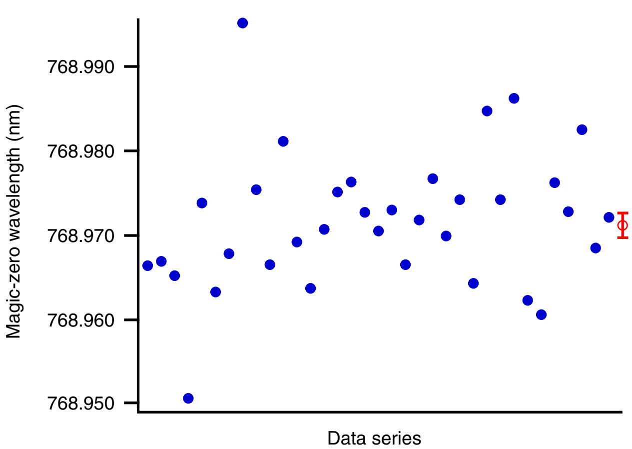

FIG. 3 (color online). The 35 separate λ_{zero} measurements (solid blue) and the trimmed mean (open red). We assumed that the statistical errors of all measurements were the same, and we report twice the standard error of the trimmed mean as the final statistical error.

Contrast loss due to several factors analogous to inhomogeneous broadening limits the precision with which λ_{zero} can be measured. Averaging over the width of the atom beam and accounting for $+1$ and -1 diffraction orders from the first nanograting explains most of the observed contrast loss in Fig. 1(b) [19]. The velocity spread of the atom beam ($\sigma_v \approx v_0/15$) slightly reduced the observable contrast as well. The small contrast loss due to light at λ_{zero} can be explained by unintended elliptical polarization of the laser beam. Circular polarization causes different Zeeman substates (m_F) to acquire different phase shifts even at λ_{zero}. Averaging over all eight $|F, m_F\rangle$ states in our experiment reduces the contrast but introduces little error to λ_{zero} thanks to the equal (thermally distributed) populations of all m_F in our atom beam. We allow for a conservative 0.1 pm uncertainty in λ_{zero} due to unaccounted-for effects such as quadratic Zeeman shifts or optical pumping compounded with the light polarization.

Because of the contrast loss from all these mechanisms, if we could optimize our experiment just by increasing the laser power without bound, we would choose only 10 times more power. Furthermore, this would result in only 5 times better sensitivity, approaching 50 pm$/\sqrt{\text{Hz}}$. If we had power to spare, one way to maintain higher contrast would be to use a triangular mask for a large area light beam. This would cause the differential phase shift to be independent of position in the atom beam.

Next, we explore how photon scattering, analogous to homogeneous broadening, imposes a fundamental limit on the precision with which any magic-zero wavelength can be measured, even in different types of experiments. Atom interferometers are, in principle, ideal tools for studying the small energy shifts that result from light near λ_{zero}. However, magic-zero wavelengths may also be measured with other methods. For example, atom loss rates in an optical dipole trap would increase near λ_{zero}. A Bose-Einstein condensate imprinted by a light beam redder

TABLE I. Magic-zero wavelength error budget.

Source of error	λ_{zero} error (pm)
Laser wavelength	0.3
Broadband light	0.5
Polarization	0.1
Doppler shift	0.05
Total systematic error	0.6
Total statistical error	1.4
Total error	1.5