Atom beam velocity measurements using phase choppers

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Abstract. We describe a new method to measure atom beam velocity in an atom interferometer using phase choppers. Phase choppers are analogous to mechanical chopping discs, but rather than being transmitted or blocked by mechanical choppers, an atom receives different differential phase shifts (e.g. zero or π radians) from phase choppers. Phase choppers yield 0.1% uncertainty measurements of beam velocity in our interferometer with 20 min of data and enable new measurements of polarizability with unprecedented precision.

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1. Introduction

More accurate methods for measuring atomic velocity are needed to support high-precision atom interferometry experiments. For example, atom beam velocity is the leading source of uncertainty in several measurements of atomic and molecular polarizabilities [1–4]. This is

because phase shifts to atomic de Broglie waves depend on the time of flight for atoms propagating through an interaction region. Additionally, improved measurements of beam velocity can increase the accuracy of space-domain matter-wave inertial sensors [5, 6]. In this paper, we describe a novel and highly accurate method to measure the flow velocity and velocity distribution of atoms in an atom beam interferometer.

Many techniques exist to measure atom beam velocities, but few provide the 0.1% accuracy we demonstrate here using a new method. For comparison and background, we review a handful of velocity measurement techniques. First, two spinning mechanical choppers (slotted disks) separated by a distance L and blocking the beam at frequency f can transmit atoms with velocity v = nLf, where n is an integer. Molecular beam velocity has been measured using this technique with 0.2% uncertainty [7, 8], but this requires moving parts inside a vacuum system and can cause unacceptable vibrations. Another approach uses the small gravitational free fall of an atom beam through separated apertures at different heights to define the velocity of transmitted atoms with 1% uncertainty [9]. Doppler shifts of an atomic transition observed with a resonant laser enable measurements of velocity with 0.8% uncertainty [3]. Similar uncertainty (0.8%) was obtained with Bragg diffraction from standing waves of light by analyzing rocking curves [3]. Atom diffraction using a nanograting has been used by our group to measure beam velocity with 0.3% uncertainty [1]. Finally, pulsed beams and time-resolved detection were recently used to achieve 0.03% uncertainty velocity measurements of pulsed metastable helium beams [10], but this technique is less applicable to continuous beams of ground state atoms.

Our new velocity measurement technique uses *phase choppers* to measure the velocity of atoms in an interferometer. Phase choppers do not block any atoms, do not require resolved diffraction, have no moving parts, work for continuous or pulsed beams, and work well for many types of atoms and molecules. Phase choppers are similar to the phase shifters described in [11] and their utility for measuring beam velocity was first proposed in [12]. This paper develops a significantly more thorough analysis of atom beam velocity measurements using phase choppers and we demonstrate velocity measurements with 0.1% uncertainty. We tested phase choppers with supersonic beams of Li, Na, K and Cs, and we use the velocity measurements as inputs to atomic polarizability measurements. To demonstrate the utility of phase choppers for precision measurements, we present consistent measurements of Cs polarizability with 0.1% precision using beams that had different velocities (spanning 925–1680 m s⁻¹).

2. Phase choppers theory

The principle behind phase choppers is similar to that behind mechanical choppers. An atom with velocity v will travel a distance L from the first chopper to the second chopper in a time $\tau = L/v$, corresponding to a fundamental chopping frequency $f_0 = v/L$. Mechanical choppers simply block or transmit atoms, leading to a maximum in the transmitted flux when the chopping frequency is any integer multiple of f_0 . In the method we present in this paper, phase choppers are switched on and off by a function generator to periodically apply phase shifts to atomic de Broglie waves in an interferometer. We will explain how this leads to a maximum in the interferometer contrast, instead of the flux, when the chopping frequency satisfies $f = nf_0$. Additionally, the ability to control wavefunction phase, rather than amplitude, allows atoms to contribute to the interference fringes in unique ways and provides new measurement possibilities, as we describe next.

To explain how phase choppers enable velocity measurements, we will describe how the atom interference pattern changes when the phase choppers are switched on and off at several

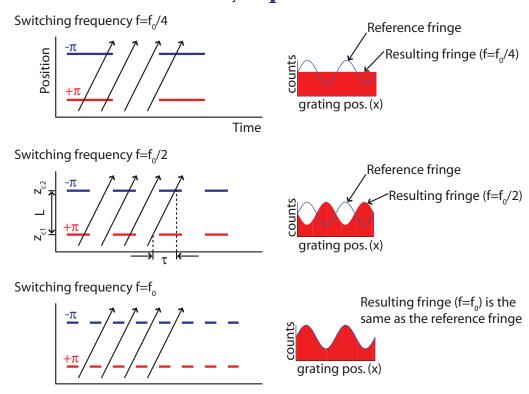


Figure 1. Atoms with different starting times will acquire 0, $+\pi$ (red) or $-\pi$ (blue) phase shifts as they pass through choppers 1 and 2, depending on their velocity v and the chopping frequency f. The time it takes an atom with velocity v to travel the distance L between the choppers is τ and $f_0 = 1/\tau$. We measure the average of the sinusoidal probability distributions formed by each atom interfering with itself, shown at the right. The reference interference pattern with the choppers off is shown in blue and the resulting interference patterns with the choppers on are shown in red.

particular frequencies. Before turning on the phase choppers, the atom interference pattern is given by $\langle N \rangle (1 + C_0 \sin(kx + \phi_0))$, where $\langle N \rangle$ is the average flux, C_0 is the reference contrast and ϕ_0 is the reference phase. This interference pattern represents a sum of the sinusoidal probability distributions of each detected atom (see [13, 14] for additional information). Depending on its start time and velocity and the chopping frequency, an atom will pass through the choppers in one of four possible pairs of conditions (off–off, on–off, off–on or on–on) and produce a probability distribution phase-shifted by an amount equal to the sum of the differential phase shifts applied by each phase chopper. For clarity, we specify the differential phase shift from chopper 1 to be π , the differential phase shift from chopper 2 to be $-\pi$, and the duty cycle to be 50%; however, we stress that phase choppers still enable velocity measurements when using other phase shifts and duty cycles. We now describe what happens to the interference pattern created by atoms with a single velocity v when the choppers are switched at four particular frequencies (see figure 1):

• $f \ll f_0$. Atoms experience the off-off (0 net differential phase shift) or on-on (0 net differential phase shift) pairs of conditions with equal likelihood, and all atoms emerge with 0 net phase shift. The contrast and phase of the detected ensemble remain unchanged.

- $f = f_0/4$. Atoms experience each of the four possible pairs of conditions, off-off (0), on-off (π) , off-on $(-\pi)$ and on-on (0), with equal likelihood. Therefore, half of the ensemble will acquire a 0 net differential phase shift, and half will acquire a π net differential phase shift. The ensemble contrast is 0 and phase is indeterminate. Contrast minima repeat at frequencies $f = (2n+1) f_0/4$, where n is an integer.
- $f = f_0/2$. Atoms experience on-off (π) and off-on $(-\pi)$ pairs of conditions with equal likelihood. The ensemble contrast remains unchanged, but the phase shifts by π (modulo 2π). Contrast revivals with π phase shifts repeat at frequencies $f = (2n+1)f_0/2$.
- $f = f_0$. Once again, all atoms experience the off-off (0) or on-on (0) states. The ensemble contrast and phase remain unchanged. Contrast revivals with no phase shift repeat at frequencies $f = nf_0$.

These simple cases show how by finding the value of f_0 one can find the velocity of an atom beam through the relation $v=Lf_0$. The contrast revivals and minima that occur at large n provide a way of leveraging small changes in velocity into large changes in revival/minima frequency. In practice, we find the velocity of our atom beam by measuring the contrast at many frequencies and fitting the contrast data to a model discussed below. Figure 3 shows fitted data from a typical chopper frequency scan using a more rigorous model that we develop next. The major corrections to the simple model include methods to account for velocity distribution, velocity-dependent phase shifts from the choppers, application of non- π average phase shifts, and velocity-dependent phase shifts due to the Sagnac effect.

The contrast and phase of the measured interference pattern are given by an average of the fringe patterns formed by atoms with different start times, t, and velocities, v, weighted by the velocity probability distribution P(v):

$$C(f)e^{i\phi(f)} = C_0e^{i\phi_0}f \int_{t=0}^{1/f} \int_{v=0}^{\infty} P(v)e^{i(\phi_1(v,t)+\phi_2(v,t+L/v))} dt dv,$$
(1)

where C(f) and $\phi(f)$ are the contrast and phase of the measured fringe pattern, and $\phi_1(v, t)$ and $\phi_2(v, t)$ are the differential phase shifts applied by choppers 1 and 2.

The nonzero width of the velocity distribution of the atom beam modifies the chopper revivals in two ways. Firstly, different velocity classes correspond to different fundamental frequencies f_0 and this causes a decay of the contrast revival envelope. Secondly, the differential phase shift acquired by an atom passing through a chopper is velocity dependent, and therefore it is impossible to apply the same differential phase shift to all atoms. This phase dispersion decreases the contrast at $f = (2n+1)f_0/2$ revivals and increases the contrast at $f = nf_0$ revivals.

We model the velocity distribution of the supersonic atom beam used in our interferometer by

$$P(v, v_0, r) dv = Av^3 e^{-\frac{r^2}{2}(v/v_0 - 1)^2} dv,$$
(2)

where v is the velocity, v_0 is the flow velocity, r describes the sharpness of the velocity distribution and A is a normalization factor [7]. For sharp velocity distributions, $r \gg 1$, the normalization factor can be written as $A = (\sqrt{2\pi} v_0^4 (r^{-1} + 3r^{-3}))^{-1}$. In the next section, we describe how we build and operate the phase choppers and fit the measured contrast versus chopping frequency to find v_0 and r (see figure 3).

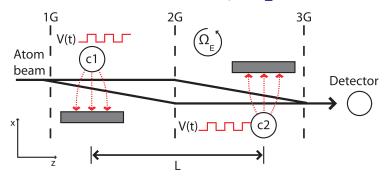


Figure 2. Nanogratings 1G, 2G and 3G form a Mach–Zehnder interferometer. Two phase choppers (c1 and c2) are placed a distance L=1270.68(25) mm apart. A voltage V(t) applied across the choppers creates an electric field (dashed lines). Atoms with velocity v passing through choppers 1 and 2 acquire net differential phase shifts $\phi_1(t) + \phi_2(t + L/v)$. The Earth rotation rate Ω_E modifies the measured contrast and phase via the Sagnac effect, especially for slow beams. A hot-wire detector counts the atoms. Diagram not to scale.

3. Experimental design

Before describing in detail how the phase choppers enable velocity measurements, we briefly review our atom interferometer and the construction of the phase choppers. We use three 100 nm period nanogratings to diffract a supersonic beam of atoms and create a Mach–Zehnder interferometer (see figure 2). An atom diffracted by the first and second gratings will be found with a sinusoidal probability distribution at the plane of the third grating. The third grating acts as a mask of this interference pattern. We measure the flux as a function of grating position to determine the phase and contrast of the fringe pattern. The gratings are each separated by 940 mm. We detect $\langle N \rangle \approx 10^5$ atoms s⁻¹ with a typical contrast of $C_0 \approx 25\%$ using a hot-wire detector 0.5 m beyond the third grating. See [1, 13, 14] for additional information.

We implemented phase choppers with electric field gradients switched on and off at a frequency f ranging from 0 to 30 kHz. We create the electric field gradient by periodically applying a voltage V(t) of 1–5 kV to a D=1.57 mm diameter copper wire at a distance a=1 mm from a grounded aluminum strip. See [11], figure 3 for a schematic of the phase chopper. The high voltage is switched on and off in less than 200 ns with a DEI PVX-4130 pulse generator controlled by an SRS DS345 function generator. We place chopper 1 at a distance of approximately 300 mm after the first grating (and chopper 2 at a similar distance before the third grating). We measured the chopper 1 to chopper 2 distance as L=1270.68(25) mm. Two translation stages allow us to move the choppers perpendicular to the beam.

An atom with velocity v passing through the 1 mm gap in phase chopper i will acquire a differential phase shift

$$\phi_i(v,t) = c \frac{\alpha V(t)^2}{v} \frac{s(v)(2x_{0i} + s(v))}{(b^2 - x_{0i}^2)(b^2 - (x_{0i} + s(v))^2)},$$
(3)

where $c = 8\pi^2 b/h \ln^{-2}((a+D/2+b)/(a+D/2-b))$, α is the atomic polarizability, v is the velocity of the atom, x_0 is the beam position relative to the ground plane, m is the mass of the atom, $b = a\sqrt{1+D/a}$, $s(v) = hL_{\rm gc}/mvd_{\rm g}$ is the path separation at the chopper, $d_{\rm g}$ is the

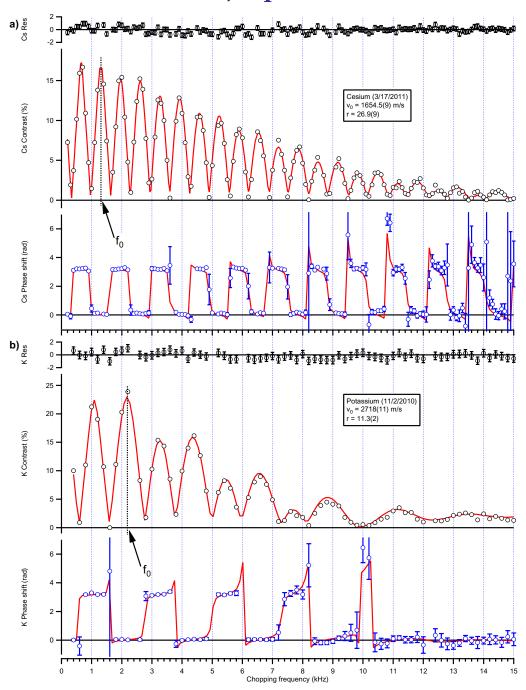


Figure 3. Phase chopper data (circles) and corresponding best-fit functions (red curves) for (a) a cesium atom beam with a 70% helium, 30% argon carrier gas and (b) a potassium atom beam with a 100% helium carrier gas and using ϕ_1 and ϕ_2 as free parameters in the least-squares fit rather than fixed at the measured values ϕ_{iDC} . f_0 for each data set is labeled. We fit the contrast (black) to find the flow velocity v_0 and velocity ratio r. The statistical error for each contrast measurement is shown in fit residuals. The measured phase (blue) is also shown, but is not fit. Each point is derived from 5 s of data.

grating period, $L_{\rm gc}$ is the distance from the first grating to chopper 1 or from the third grating to chopper 2, and h is Planck's constant. Classically, one may describe the differential phase shift by a transverse deflection in an electric field where the force is given by $F = \alpha E \nabla E$. See [1] for a full derivation of the acquired phase with a similar electrode geometry.

While equation (3) is useful for designing the chopper geometry for an expected atom beam velocity, we cannot use it directly to measure beam velocity. Doing so would require not only more accurate knowledge of the chopper geometry, but also knowledge of polarizability—the very quantity that we would like to eventually measure. Instead, we empirically tune the choppers to induce π and $-\pi$ differential phase shifts by adjusting the voltage V applied to both choppers and the position x_0 of each chopper individually. We refer to the actual induced differential phase shift as ϕ_{iDC} , where i is the chopper number. For 0.1% uncertainty velocity measurements, we can tolerate phase differences $(\phi_{iDC} - \pi)/\pi$ as large as 5%, provided that the uncertainty in the measured phase shift is less than 0.5%.

After tuning and accurately measuring the chopper 1 and chopper 2 DC differential phase shifts ϕ_{iDC} , we are nearly ready to substitute them into equation (1) and measure contrast versus frequency to find velocity. First, however, we must make a correction to undo the effect of the velocity spread on the measurement of ϕ_{iDC} . The proper phase shift to input into equation (1) for velocity measurements is ϕ_{i0} , defined by

$$C_{i\text{DC}}e^{i\phi_{i\text{DC}}} = C_0 \int_{v=0}^{\infty} P(v, v_0, r)e^{i\phi_{i0}(v_0/v)^2} dv.$$
(4)

Using the parameter ϕ_{i0} and the fact that $s(v) \ll x_0$, equation (3) can now be well approximated by

$$\phi_i(v,t) \approx \phi_{i0} \left(\frac{v_0}{v}\right)^2 \left(\frac{V(t)}{V_{DC}}\right)^2.$$
 (5)

Note that the $1/v^2$ dispersion comes from the fact that s(v) in equation (3) is inversely proportional to v. In the limit of a very sharp velocity distribution, $r \to \infty$ and $\phi_{i0} \to \phi_{iDC}$. Ignoring this correction and assuming $\phi_{i0} = \phi_{DC}$ results in an error in v_0 of 0.1% for beams with r = 10, and 0.01% for beams with r = 40.

We proceed to measure the contrast and phase of the interference pattern at a series of chopping frequencies. We perform a least-squares fit of measured contrast using equation (1). Because ϕ_i depends on P(v) through equations (4) and (5), we must numerically solve equation (4) for each iteration of the fit routine. The measured phase $\phi(f)$ provides a consistency check of the results of the contrast fit, but by itself is a less sensitive measure of velocity.

4. Results and errors

Figure 3(a) shows a chopper frequency scan from a beam with a slow v_0 and sharp r, and figure 3(b) shows a chopper frequency scan from a beam with a fast v_0 and broad r. We perform a least-squares fit of the contrast data to equation (1) to find v_0 and r. The error budget of the measurement is shown in table 1 and each parameter is discussed below. We estimate the uncertainty in v_0 and r due to each parameter by performing fits of the data at a parameter's central value and +/- its uncertainty. We also tested the stability of the least-squares fits with respect to uncertainty in each parameter by halving and doubling the uncertainties.

Total

	71	
Parameter	Value (Unc.)	Uncertainty in v_0 (%)
Stat error in v		0.04
L	1270.68(25) mm	0.02
$\phi_{ m 1DC}/\pi$	1.000(5)	0.01
$\phi_{ m 2DC}/\pi$	1.000(5)	0.01
Beam width	$100(30) \mu \text{m}$	0.04
Initial contrast C_0	25(2)%	0.04
Transit time	$2 \mu \mathrm{s}$	0.005
Switching time	200 ns	0.001
Duty cycle	50.0(3)%	0.005
Sagnac phase shift	1.46(2) rad	0.005
Molecules	10(10)%	0.02

Table 1. Error budget for a typical velocity measurement with phase choppers.

0.08

We measured the distance between the two choppers, L, by inserting the ends of a calibrated tape measure between the wire and ground plane of each chopper. We subdivided the millimeter markings on the tape measure by analyzing high-resolution digital photographs of the tape measure inserted into each chopper. We took care to take the pictures with the chopper wire centered in the frame and from normal incidence to the ground plane. Thermal expansion may change the distance between choppers by no more than $150\,\mu\mathrm{m}$ (0.01%), significantly less than the measurement uncertainty in L.

Adding the Sagnac phase shift from the Earth's rotation into the analysis also yields small corrections to the best fit v_0 and r. The corrections to v_0 and r can be as large as 0.2 and 4%, respectively, for slower beams with smaller r. Holmgren $et\ al\ [1]$ explains how we incorporate the Sagnac phase shift in our interferometer model. The uncertainty in this correction is negligible.

We add a correction to the phase chopper model to account for the nonzero width of the atom beam and additional interferometers formed by other diffraction orders. We model this correction by making $\phi_i(v, t)$ (equation (3)) a function of x_0 as well and then introducing an additional average over beam width (x_0) in equation (1). For the widest beams, the correction to v_0 and r is less than 0.1%, and the uncertainty in v_0 due to the beam width is less than 0.04%.

Despite accounting for the averaging of fringe patterns from the velocity distribution and beam width, the measured reference contrast is systematically 1–2% larger than the best-fit contrast parameter C_0 . Fixing C_0 at its measured value in a least-squares fit of the chopper frequency scan typically results in an unrealistically small r, which then requires slightly slower v_0 to fit the data well. We estimate the uncertainty from this parameter by taking the full difference in fitted v_0 when using the two different initial contrasts. Future work is needed to discover the source of this uncertainty.

The voltage switching time and the transit time for atoms through the choppers introduce additional mechanisms by which the phase shifts may be different than desired. The switching time can be thought of as making the phase shifts depicted in figure 1 have fuzzy edges in time, while the finite extent of the electric field causes fuzzy edges in space. Both these corrections are negligible at our level of precision (see table 1).

Alkali dimer molecules slightly influence the chopper data as well. The rotationally averaged alkali dimer polarizabilities are slightly less than twice the atomic polarizabilities [15], but the de Broglie wavelength, and hence path separation s(v), for the dimers is half as large. Therefore, dimers acquire slightly smaller differential phase shifts than atoms. We estimate the number of alkali dimers in the atom beam in three ways. Firstly, we examine the far-field diffraction pattern from the first grating when the diffraction angle (determined by the atoms' mass and velocity) is sufficiently large to observe resolved diffraction orders. Molecules produce diffraction peaks at 1/2 the diffraction angle of atoms and we can determine the fraction of molecules in the beam by comparing diffraction order intensities between atoms and molecules. Secondly, although we cannot use a measurement of the DC phase shift, ϕ_{iDC} , to determine atomic polarizabilities, as discussed in section 3, we can use it to estimate the number of molecules in the beam. Molecule fractions greater than 10% would produce DC phase shifts inconsistent with what we expect from the phase choppers at a given position and voltage. Finally, the quality of the least-squares fit to the chopper data noticeably decreases when the molecule fraction rises above 10%.

We tested phase chopper velocity measurements against less precise velocity measurements using diffraction from a nanograting. We found no discrepancy between the two methods. However, as described in [1], we must account for a small difference between the velocity distribution of the entire beam (as measured by diffraction) and the velocity distribution of the atoms detected in the interferometer (as measured by phase choppers). The difference exists because the dispersive nature of the nanogratings, combined with detector size and beam width, leads to a velocity distribution in the atom interferometer that depends on the position of the detector. For precision measurements, it is preferable to directly measure the velocities of atoms that contribute to the detected interference pattern and thus avoid the correction (0.25% in $v_0)$ and its associated uncertainty (0.1%). In this respect, phase choppers are better than nanograting diffraction for measuring beam velocity in an atom interferometer.

We also tested the phase choppers by measuring the static ground state polarizability of cesium using beams with three very different flow velocities on three different days. Each day we alternated between measurements of beam velocity and polarizability every hour to account for small changes in velocity (<0.5%) over the course of a day due to instability in beam source temperature. The statistical error of each measurement of velocity was less than 0.1%. To measure polarizability, we used a third electric field gradient region that will be described in a future publication. We find the cesium polarizability (stat. unc.) to be 59.84(4), 59.71(7) and 59.85(8) ų at flow velocities of 925, 1345 and 1680 m s $^{-1}$. These polarizability measurements are subject to a systematic correction due to the third gradient region, but the consistency of the polarizability measurements provides strong evidence that our velocity measurements using phase choppers are reproducible at the 0.1% level.

Finally, we have also modeled and tested the phase choppers in the $\phi_{\rm 1DC}=+\pi$ and $\phi_{\rm 2DC}=+\pi$ mode. In the limit of an infinitely sharp velocity distribution $(r\to\infty)$, there is no difference between the $+\pi$, $+\pi$ configuration and the $+\pi$, $-\pi$ configuration. However, the finite velocity spread and the fact that $\phi_{\rm iDC}$ never exactly equals $\pm\pi$ results in the $+\pi$, $-\pi$ configuration yielding slightly more precise results than the $+\pi$, $+\pi$ configuration.

5. Conclusion

We used phase choppers to measure the velocity of atoms in an atom interferometer with 0.1% uncertainty. Phase choppers work for continuous or pulsed beams, do not require resolved diffraction, have no moving parts, and work well for many types of atoms and molecules. These velocity measurements enable high-precision absolute and ratio measurements of atomic polarizabilities. This work was supported by NSF Award no. 0969348.

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