grating period, L_{gc} is the distance from the first grating to chopper 1 or from the third grating to chopper 2, and h is Planck's constant. Classically, one may describe the differential phase shift by a transverse deflection in an electric field where the force is given by $\boldsymbol{F} = \alpha \boldsymbol{E} \nabla \boldsymbol{E}$. See [1] for a full derivation of the acquired phase with a similar electrode geometry.

While equation (3) is useful for designing the chopper geometry for an expected atom beam velocity, we cannot use it directly to measure beam velocity. Doing so would require not only more accurate knowledge of the chopper geometry, but also knowledge of polarizability—the very quantity that we would like to eventually measure. Instead, we empirically tune the choppers to induce π and $-\pi$ differential phase shifts by adjusting the voltage V applied to both choppers and the position x_0 of each chopper individually. We refer to the actual induced differential phase shift as $\phi_{i\mathrm{DC}}$, where i is the chopper number. For 0.1% uncertainty velocity measurements, we can tolerate phase differences $(\phi_{i\mathrm{DC}} - \pi)/\pi$ as large as 5%, provided that the uncertainty in the measured phase shift is less than 0.5%.

After tuning and accurately measuring the chopper 1 and chopper 2 DC differential phase shifts $\phi_{i\mathrm{DC}}$, we are nearly ready to substitute them into equation (1) and measure contrast versus frequency to find velocity. First, however, we must make a correction to undo the effect of the velocity spread on the measurement of $\phi_{i\mathrm{DC}}$. The proper phase shift to input into equation (1) for velocity measurements is ϕ_{i0}, defined by

$$C_{i\mathrm{DC}} \mathrm{e}^{\mathrm{i}\phi_{i\mathrm{DC}}} = C_0 \int_{v=0}^{\infty} P(v, v_0, r) \mathrm{e}^{\mathrm{i}\phi_{i0}(v_0/v)^2} \, \mathrm{d}v. \tag{4}$$

Using the parameter ϕ_{i0} and the fact that $s(v) \ll x_0$, equation (3) can now be well approximated by

$$\phi_i(v, t) \approx \phi_{i0} \left(\frac{v_0}{v}\right)^2 \left(\frac{V(t)}{V_{\mathrm{DC}}}\right)^2. \tag{5}$$

Note that the $1/v^2$ dispersion comes from the fact that $s(v)$ in equation (3) is inversely proportional to v. In the limit of a very sharp velocity distribution, $r \to \infty$ and $\phi_{i0} \to \phi_{i\mathrm{DC}}$. Ignoring this correction and assuming $\phi_{i0} = \phi_{\mathrm{DC}}$ results in an error in v_0 of 0.1% for beams with $r = 10$, and 0.01% for beams with $r = 40$.

We proceed to measure the contrast and phase of the interference pattern at a series of chopping frequencies. We perform a least-squares fit of measured contrast using equation (1). Because ϕ_i depends on $P(v)$ through equations (4) and (5), we must numerically solve equation (4) for each iteration of the fit routine. The measured phase $\phi(f)$ provides a consistency check of the results of the contrast fit, but by itself is a less sensitive measure of velocity.

4. Results and errors

Figure 3(a) shows a chopper frequency scan from a beam with a slow v_0 and sharp r, and figure 3(b) shows a chopper frequency scan from a beam with a fast v_0 and broad r. We perform a least-squares fit of the contrast data to equation (1) to find v_0 and r. The error budget of the measurement is shown in table 1 and each parameter is discussed below. We estimate the uncertainty in v_0 and r due to each parameter by performing fits of the data at a parameter's central value and +/− its uncertainty. We also tested the stability of the least-squares fits with respect to uncertainty in each parameter by halving and doubling the uncertainties.