The line strengths S_1 and S_2, and thus R, can also be determined from state lifetime measurements. To our knowledge, the most precise *independent* measurements of the $4p_{1/2}$ and $4p_{3/2}$ state lifetimes were performed by Volz and Schmoranzer using beam-gas-laser spectroscopy [16]. They reported lifetime uncertainties of 0.25% and a similar uncertainty for R (which leads to a 2 pm uncertainty in $\lambda_{\rm zero}$). In comparison, our measurement of R has an uncertainty of 0.20%. State lifetimes can also be derived from molecular or photoassociation spectroscopy [17,18]. However, these spectroscopy experiments [17,18] do not distinguish between the $4p_{1/2}$ and $4p_{3/2}$ state lifetimes (they depend on an average), so they cannot be used to determine R or $\lambda_{\rm zero}$.

To measure the magic-zero wavelength, we focused 500 mW of laser light asymmetrically on the paths of our three grating Mach-Zehnder atom interferometer [19–21]. Atom waves propagating along each interferometer path acquired a phase shift $\phi(\omega)$ proportional to the dynamic polarizability $\alpha(\omega)$ at the laser frequency ω. We found the laser frequency $\omega_{\rm zero} = 2\pi c/\lambda_{\rm zero}$ at which the dynamic polarizability vanishes by measuring the phase shift as a function of laser wavelength.

The phase shift $\phi_0(\omega)$ along one interferometer path is given by

$$\phi_0(\omega) = \frac{\alpha(\omega)}{2\epsilon_0 c\hbar v}\int_{-\infty}^{\infty} I(x,z)dz, \qquad (3)$$

where $v \approx 1600$ m/s is the atom velocity, $I(x, z)$ is the laser beam intensity (assumed to be monochromatic for now), x is the transverse coordinate in the plane of the interferometer, and z is the longitudinal coordinate. The laser beam intensity was 400 W/cm^2 (500 mW focused to a beam waist of ≈ 200 μm). We measure the differential phase shift $\phi(\omega)$ for two components of the atomic wave functions that are separated by 60 μm in our atom interferometer. Figure 1 shows the differential phase shift and contrast of the interferometer as the laser wavelength is scanned 5 nm across the $D1$ and $D2$ lines.

Equation (3) is useful for understanding the origin of the phase shift, similar to $\phi(\omega)$ shown in Refs. [11,13]. But our measurements of $\lambda_{\rm zero}$ do not depend on precise knowledge of the atom beam velocity or the focused laser beam irradiance. Changes in these parameters would affect only the magnitude of the phase shift, not the zero crossing. Therefore, we reduce Eq. (3) to simply

$$\phi(\omega) = b\alpha(\omega), \qquad (4)$$

where b is a parameter proportional to the laser beam intensity and the interaction time. To precisely measure $\lambda_{\rm zero}$, we studied phase shifts within 100 pm of $\lambda_{\rm zero}$, as shown in Fig. 2. The laser power changed with wavelength and drifted over time, so we monitored the power incident on the atom beam and normalized the measured phase shifts. We reproduced this 1 h experiment 35 times over a period of 5 d. We fit these data to Eqs. (2) and (4), with R and b as the only free parameters. The precision with which we can determine $\lambda_{\rm zero}$ is determined by the slope $d\phi/d\lambda$. This slope is typically 1 mrad/pm, and our phase uncertainty from shot noise is $\delta\phi \approx 1$ mrad with 5 min of data.

Our reported measurement of the magic-zero wavelength is the average of 35 individual measurements of

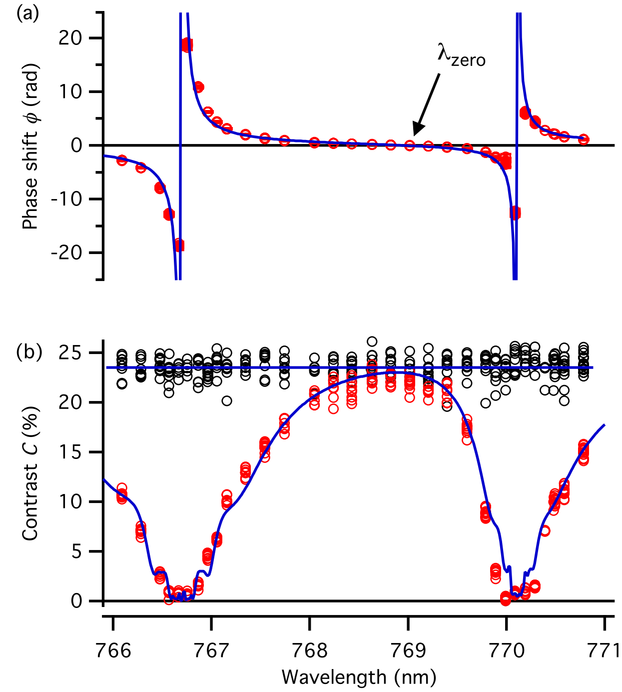

FIG. 1 (color online). Measurements of the interferometer (a) phase shift ϕ and (b) contrast C as a function of laser wavelength. The measured phase shifts are normalized by the laser power at that wavelength. The reference contrast C_0 is shown as black circles.

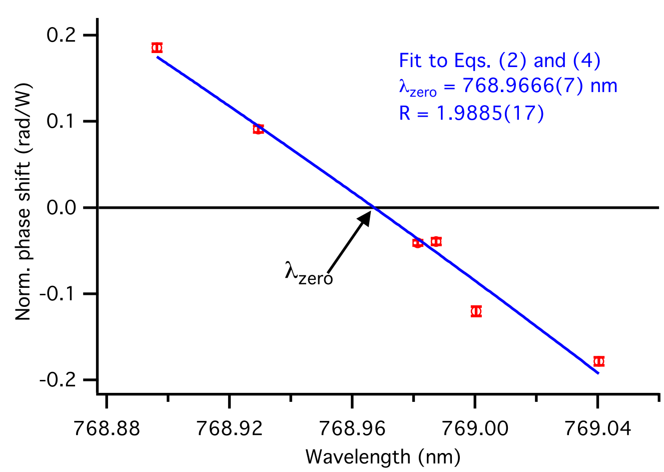

FIG. 2 (color online). Measurements of phase shift and laser wavelength. Each point represents 5 min of data. The fit uses Eqs. (2) and (4) described in the text, with free parameters R and b. R determines $\lambda_{\rm zero}$.