of multiple velocities traversing multiple interferometers. The resulting fringe pattern is described by

$$C_{\mathrm{sum}}(x)e^{i\phi_{\mathrm{sum}}(x)} = C_0 e^{i\phi_0} \sum_{j=-1,1} P_j \int_0^\infty P(v) e^{i\phi_{\alpha,j}(x,v)} dv, \tag{8}$$

where C_{sum} is the real-valued contrast of the fringe pattern, ϕ_{sum} is the phase of the fringe pattern, C_0 and ϕ_0 are the initial contrast and phase of the interferometer, j denotes the interferometer number (upper or lower diamond in Fig. 1), $P_j = 0.5$ is the probability of an atom being found in interferometer j, and $P(v)$ is the velocity distribution of the beam. In our experiment, the phase shift ϕ_{sum} is reduced by as much as 4% by performing the sum described in Eq. (8) compared to a simple weighted average of phases, and the contrast is reduced by more than 50%.

The Sagnac phase must also be accounted for in our experiment and modifies Eq. (8) [19,20]. Because the Sagnac phase is dispersive, ignoring it would lead to an error in polarizability of up to 1%. The Sagnac phase in our interferometer is given by

$$\phi_{\mathrm{Sag}}(v) = \frac{4\pi {L_g}^2 \Omega}{d_{\mathrm{g}} v}, \tag{9}$$

where L_{g} is the distance between adjacent nanogratings and Ω is the rotation rate of the Earth projected into the plane of the interferometer. At our latitude, the Sagnac phase is as much as 4.8 rad for our rubidium beam. The reference phase ϕ_{ref} and contrast C_{ref} of the interferometer are determined by the Sagnac phase in the absence of an electric field:

$$C_{\mathrm{ref}} e^{i\phi_{\mathrm{ref}}} = C_0 e^{i\phi_0} \sum_{j=-1,1} P_j \int_0^\infty P(v) e^{i\phi_{\mathrm{Sag}}(v)} dv. \tag{10}$$

We find the total phase and contrast of the interferometer in the presence of an electric field by adding the Sagnac phase to the polarizability phase shift before conducting the incoherent sum shown in Eq. (8). This procedure yields

$$C_{\mathrm{total}}(x)e^{i\phi_{\mathrm{total}}(x)} = C_0 e^{i\phi_0} \sum_{j=-1,1} P_j \int_0^\infty P(v) e^{i[\phi_{\alpha,j}(x,v)+\phi_{\mathrm{Sag}}(v)]} dv. \tag{11}$$

Finally, the measured phase shift and relative contrast are

$$\phi_{\mathrm{measured}}(x) = \phi_{\mathrm{total}}(x) - \phi_{\mathrm{ref}}, \tag{12}$$

$$C_{\mathrm{measured}}(x) = C_{\mathrm{total}}(x)/C_{\mathrm{ref}}. \tag{13}$$

As an alternative point of view, we may describe the measured phase shift in terms of a classical electrostatic force on the individual atomic dipoles instead of the quantum-mechanical phases acquired by an atom in the electric field. In the classical-mechanics picture, a neutral atom in an electric field experiences a force $\boldsymbol{F} = -\nabla U_{\mathrm{Stark}} = \alpha E \nabla E$. The deflection of the interferometer paths will cause the same displacement of the observed fringes as the phase-shift analysis discussed above.

III. VELOCITY MEASUREMENT

The velocity determines both the amount of time an atom interacts with the electric field and the spatial separation s of the paths inside the electric-field gradient. Therefore, an accurate determination of the beam velocity and the velocity distribution is essential for a precise polarizability measurement.

We determine the velocity of the atom beam by analyzing the far-field diffraction pattern from the first grating. The velocity distribution of the beam is modeled by

$$P(v)dv = Av^3 \exp\left[-(v-v_0)^2/\left(2\sigma_v^2\right)\right] dv, \tag{14}$$

where v is the velocity, v_0 is the flow velocity, σ_v describes the velocity distribution, and A is a normalization factor [21]. In the limit of a supersonic beam, $v_0/\sigma_v \gg 1$, the normalization factor can be written as $A = [\sqrt{2\pi} v_0 \sigma_v ({v_0}^2 + 3{\sigma_v}^2)]^{-1}$. The location of the nth diffraction order at the detector plane is given by

$$x_n = \frac{\lambda_{\mathrm{dB}}}{d_{\mathrm{g}}} n z_{\mathrm{det}} = \frac{hn}{mvd_{\mathrm{g}}} z_{\mathrm{det}}, \tag{15}$$

where the propagation distance z is equal to the distance from the first grating to the detector, z_{det}. We use $m = m_{\mathrm{avg}}$, which is the average mass of the atomic species, rather than calculating and adding the diffraction patterns for each isotope. A reanalysis of a subset of our data shows that this approximation yields a small difference in velocity (<0.02%) and polarizability (<0.05%) when isotopes are taken into account. Next, we rearrange Eq. (15) to find

$$v(x_n) = \frac{z_{\mathrm{det}} hn}{md_{\mathrm{g}} x_n} \tag{16}$$

and use this to transform $P(v)dv$ to $P(x)dx$. Finally, we sum over all diffraction orders, each weighted by c_n, and add the zeroth-order peak to obtain the diffraction pattern for an infinitesimally thin beam and detector:

$$P(x)dx = \left\{ c_0 \delta(x-0) + \sum_{n\neq 0} c_n A \left(\frac{z_{\mathrm{det}} hn}{md_{\mathrm{g}}}\right)^4 x^5 \times \exp\left[-\left(\frac{z_{\mathrm{det}} hn}{md_{\mathrm{g}} x} - v_0\right)^2 \Big/ \left(2\sigma_v^2\right)\right] \right\} dx. \tag{17}$$

The observed diffraction pattern (see Fig. 3) is a convolution of the spatial probability distribution given by Eq. (17) with the collimated beam and detector shapes. Two narrow collimating slits of width 20 and 10 μm separated by 890 mm determine the beam shape. We model the detector wire as a square aperture with width 70 μm. We fit the observed diffraction pattern to the convolution described above to find the flow velocity v_0. With four diffraction scans, we can determine v_0 with a statistical precision of 0.1%.

The diffraction orders are sufficiently close together, the beam is sufficiently broad, and the detector is sufficiently thick that we cannot use diffraction data alone to determine the velocity distribution σ_v with enough precision for the polarizability measurements. Instead, as discussed later, we find the velocity distribution parameter σ_v from