- $\boldsymbol{f = f_0/4}$. Atoms experience each of the four possible pairs of conditions, off–off (0), on–off (π), off–on ($-\pi$) and on–on (0), with equal likelihood. Therefore, half of the ensemble will acquire a 0 net differential phase shift, and half will acquire a π net differential phase shift. The ensemble contrast is 0 and phase is indeterminate. Contrast minima repeat at frequencies $f = (2n+1) f_0/4$, where n is an integer.
- $\boldsymbol{f = f_0/2}$. Atoms experience on–off (π) and off–on ($-\pi$) pairs of conditions with equal likelihood. The ensemble contrast remains unchanged, but the phase shifts by π (modulo 2π). Contrast revivals with π phase shifts repeat at frequencies $f = (2n+1) f_0/2$.
- $\boldsymbol{f = f_0}$. Once again, all atoms experience the off–off (0) or on–on (0) states. The ensemble contrast and phase remain unchanged. Contrast revivals with no phase shift repeat at frequencies $f = nf_0$.

These simple cases show how by finding the value of f_0 one can find the velocity of an atom beam through the relation $v = Lf_0$. The contrast revivals and minima that occur at large n provide a way of leveraging small changes in velocity into large changes in revival/minima frequency. In practice, we find the velocity of our atom beam by measuring the contrast at many frequencies and fitting the contrast data to a model discussed below. Figure 3 shows fitted data from a typical chopper frequency scan using a more rigorous model that we develop next. The major corrections to the simple model include methods to account for velocity distribution, velocity-dependent phase shifts from the choppers, application of non-π average phase shifts, and velocity-dependent phase shifts due to the Sagnac effect.

The contrast and phase of the measured interference pattern are given by an average of the fringe patterns formed by atoms with different start times, t, and velocities, v, weighted by the velocity probability distribution $P(v)$:

$$C(f)e^{i\phi(f)} = C_0 e^{i\phi_0} f \int_{t=0}^{1/f} \int_{v=0}^{\infty} P(v) e^{i(\phi_1(v,t)+\phi_2(v,t+L/v))}\, dt\, dv, \tag{1}$$

where $C(f)$ and $\phi(f)$ are the contrast and phase of the measured fringe pattern, and $\phi_1(v,t)$ and $\phi_2(v,t)$ are the differential phase shifts applied by choppers 1 and 2.

The nonzero width of the velocity distribution of the atom beam modifies the chopper revivals in two ways. Firstly, different velocity classes correspond to different fundamental frequencies f_0 and this causes a decay of the contrast revival envelope. Secondly, the differential phase shift acquired by an atom passing through a chopper is velocity dependent, and therefore it is impossible to apply the same differential phase shift to all atoms. This phase dispersion decreases the contrast at $f = (2n+1) f_0/2$ revivals and increases the contrast at $f = nf_0$ revivals.

We model the velocity distribution of the supersonic atom beam used in our interferometer by

$$P(v, v_0, r)dv = Av^3 e^{-\frac{r^2}{2}(v/v_0-1)^2}\, dv, \tag{2}$$

where v is the velocity, v_0 is the flow velocity, r describes the sharpness of the velocity distribution and A is a normalization factor [7]. For sharp velocity distributions, $r \gg 1$, the normalization factor can be written as $A = (\sqrt{2\pi} v_0^4 (r^{-1} + 3r^{-3}))^{-1}$. In the next section, we describe how we build and operate the phase choppers and fit the measured contrast versus chopping frequency to find v_0 and r (see figure 3).