Birla Institute of Technology & Science Pilani,
Second Semester 2024-25
MATH F113: Probability and Statistics
Tutorial Sheet 3

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1. A certain type of flashlight requires two type-D batteries, and the flashlight will work only if both its batteries have acceptable voltages. Suppose that 90% of all the batteries from a certain supplier have acceptable voltages. Among ten randomly chosen flashlights, what is the probability that at least nine will work? What assumptions did you make in the course of answering the question posed? **[Section 3.4, Ex 57]**

2. (a) Show that $b(x; n, 1-p) = b(n-x; n, p)$.
   (b) Show that $B(x; n; 1-p) = 1 - B(n-x-1; n, p)$. **[Section 3.4, Ex 63]**

3. Customers at a gas station pay with a credit card (A), debit card (B), or cash (C). Assume that successive customers make independent choices, with $P(A) = 0.5$, $P(B) = 0.2$ and $P(C) = 0.3$. **[Section 3.4, Ex 65]**
   (a) Among the next 100 customers, what are the mean and variance of the number who pay with a debit card? Explain your reasoning.
   (b) Answer part (a) for the number among the 100 who don't pay with cash.

4. A geologist has collected 10 specimens of basaltic rock and 10 specimens of granite. The geologist instructs a laboratory assistant to randomly select 15 of the specimens for analysis. **[Section 3.5, Ex 71]**
   (a) What is the pmf of the number of granite specimens selected for analysis?
   (b) What is the probability that all specimens of one of the two types of rocks are selected for analysis?
   (c) What is the probability that the number of granite specimens selected for analysis is within 1 standard deviation of its mean value?

5. Suppose small aircraft arrive at a certain airport according to a Poisson process with rate $\alpha = 8$ per hour, so that the number of arrivals during a time period of t hours is a Poisson rv with parameter $\mu = 8t$. **[Section 3.6, Ex 85]**
   (a) What is the probability that exactly 6 small aircraft arrive during a 1-hour period? At least 6? At least 10?
   (b) What are the expected value and standard deviation of the number of small aircraft that arrive during a 90-minute period?
   (c) What is the probability that at least 20 small aircraft arrive during a 2.5-hour period? That at most 10 arrive during this period?

6. Suppose trees are distributed in a forest according to a two-dimensional Poisson process with parameter α, the expected number of trees per hectare , equal to 80. **[Section 3.6, Ex 91]**
   (a) What is the probability that in a certain quarter-hectare plot, there will be at most 16 trees?
   (b) If the forest covers 85,000 hectares, what is the expected number of trees in the forest?
   (c) Suppose you select a point in the forest and construct a circle of radius 0.1km. Let Y = the number of trees within that circular region. What is the pmf of Y? [Hint: 1 sq km = 100 hectares.]

7. The Centers for Disease Control and Prevention reported in 2012 that 1 in 88 American children had been diagnosed with an autism spectrum disorder (ASD). **[Section 3.6, Ex 84]**
   (a) If a random sample of 200 American children is selected, what are the expected value and standard deviation of the number who have been diagnosed with ASD?
   (b) Referring back to (a), calculate the approximate probability that at least 2 children in the sample have been diaognosed with ASD?

8. The literacy rate for a nation measures the proportion of people age 15 and over who can read and write. The literacy rate for women in that nation is 12%. Let $X =$ the number of women you ask until one says that she is literate.

   (a) What is the probability distribution of X?

   (b) What is the probability that you ask five women before one says she is literate?

   (c) Find the (i) mean and (ii) standard deviation of X.

   (d) Find the moment generating function of X and use it to verify the results in part (c).