COMP METH Sneha Nair fall 2024

Particle in Cell Method for a Plasma Simulator

Investigating 1D and 2D Effects

01

problem statement

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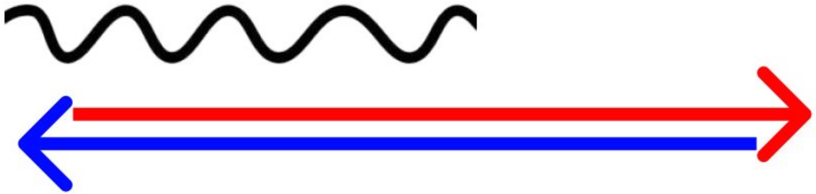
One Dimensional Analysis of Two Beam Instability

set up

In a one dimensional periodic box we split particles into two gaussian beams moving in opposite directions with a perturbation introduced to observe instability.

goal

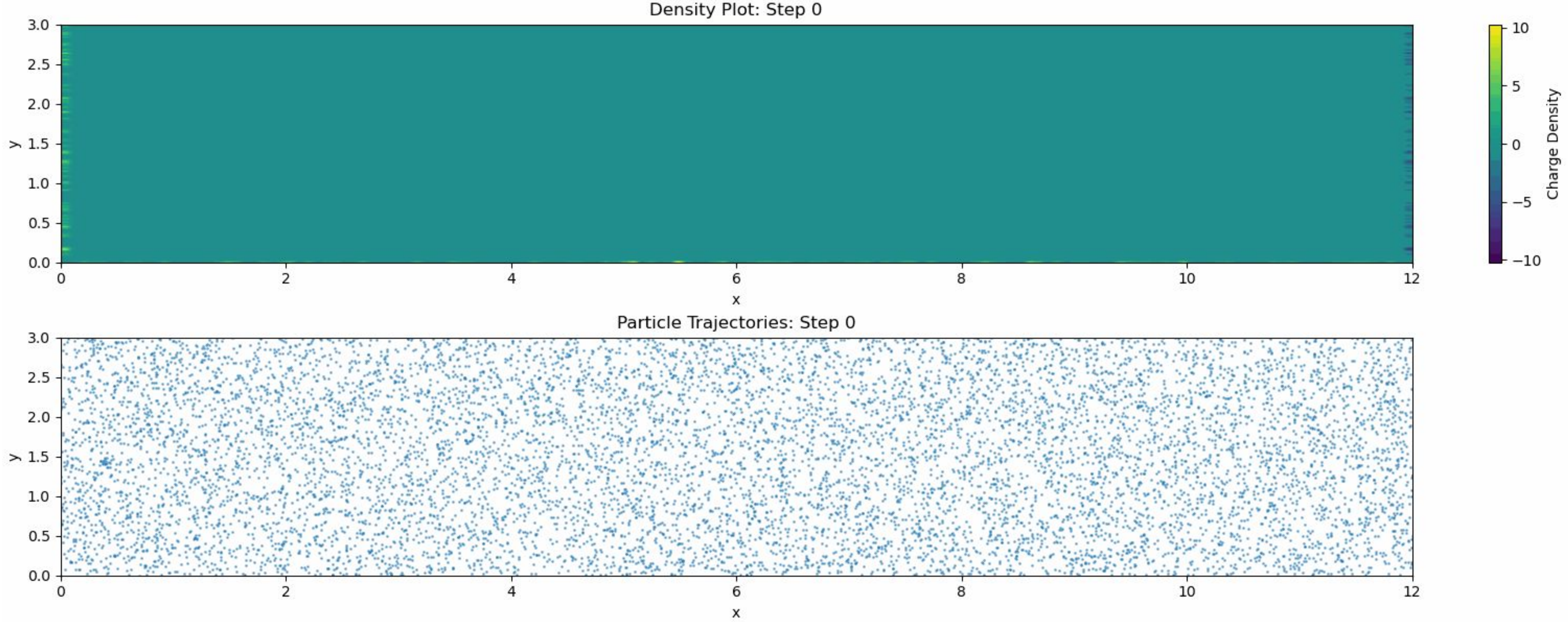


Map out the instability using the phase space primarily, but users can look at other crucial aspects of the code.

Two Dimensional Analysis of Perturbations to Static Plasma

set up

1. Start with a plasma medium of unmagnetized slow moving/non moving ions.
2. Apply a disturbance event in the form of an injection of particles at a high speed or a stagnant block. (non-relativistic and no consideration of B-field)



goal

1. Observe how the system evolves towards shock conditions after the initial injection of particles via velocity, density, fields
2. Play around with the boundary conditions (sticky, reflective, periodic)

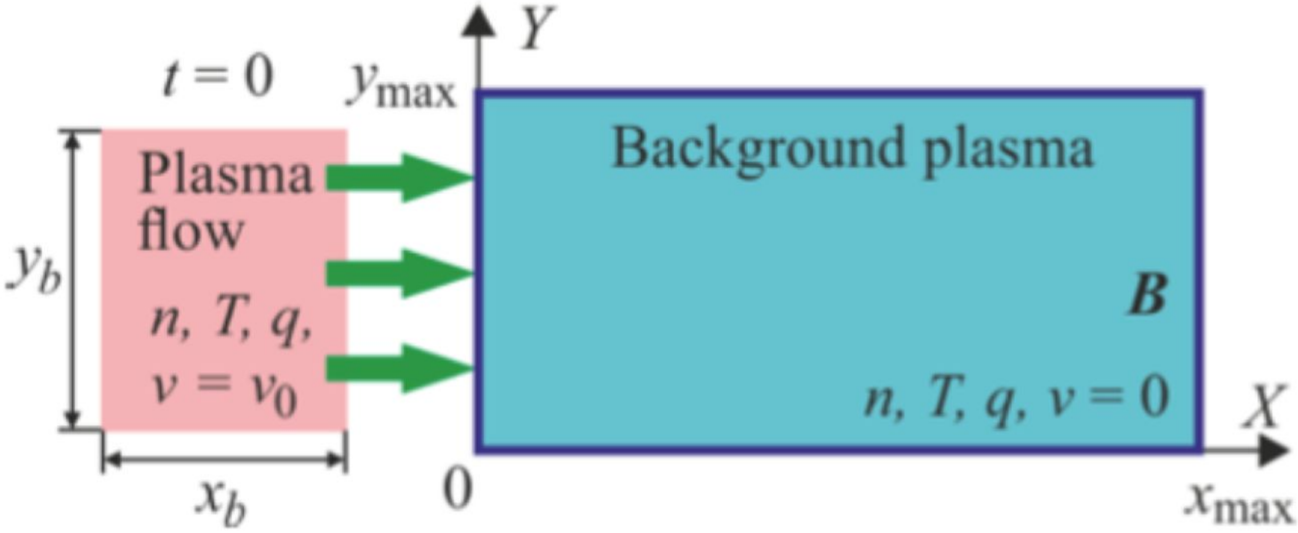


Figure 1

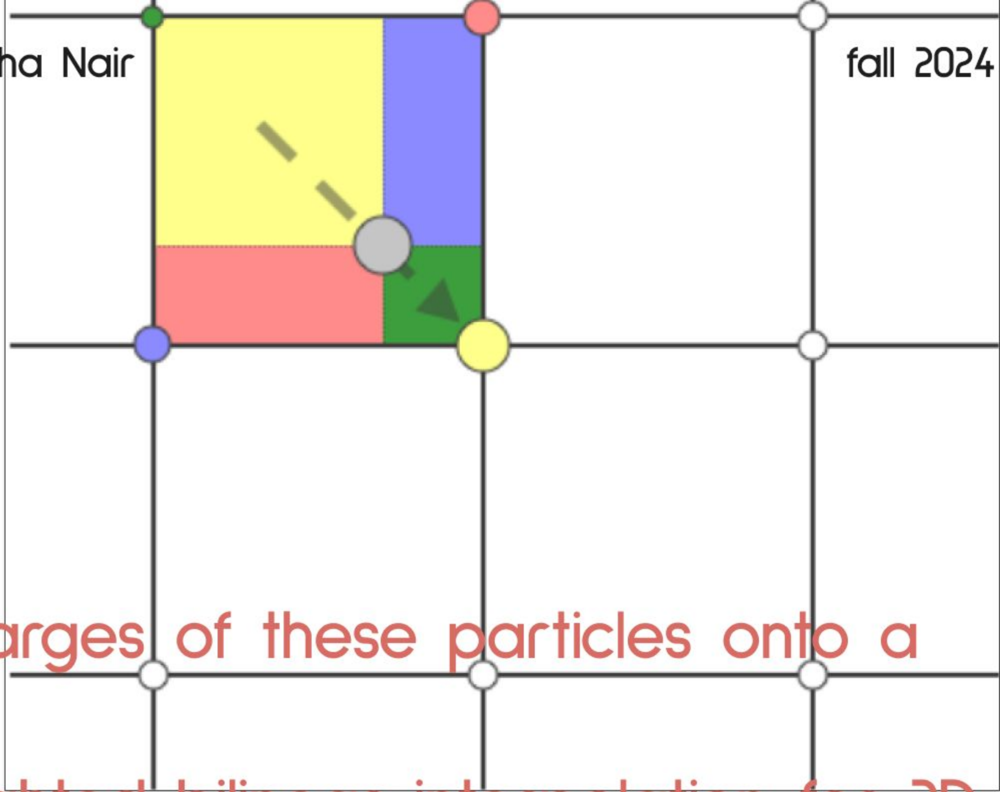
02

numerical methods used

step 1

Initialize the positions and velocities of heavy particles (ons) in a cell.
In this instance we set their initial velocities to zero.

step 2



Deposit the charges of these particles onto a grid.

Here I use weighted bilinear interpolation for 2D and Linear Interpolation for 1D

Particle In Cell Method

WHY Bi-Linear Interpolation

1. Built on the Linear Interpolation used in 1D.
2. Balanced computational time and accuracy.
3. Higher order polynomial fits for a larger number of particles means higher error. (Runge Phenomenon)

step 3

$$E(x) = -\frac{d\phi}{dx}$$

$$\nabla^2 \phi = \frac{\rho - \rho_0}{\epsilon_0}$$

$$\frac{d^2\phi(x)}{dx^2} = n - n_0$$

$$\mathbf{E} = -\nabla\phi$$

Solve Poisson's equation for the electric potential using a finite difference (Central Difference) and Maxwell's Equations

WHY

1. Approximating gradients (first derivatives).
2. Discretizing the Laplacian operator (second derivatives) in Poisson's equation.

Central Differencing?

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\Delta x}$$

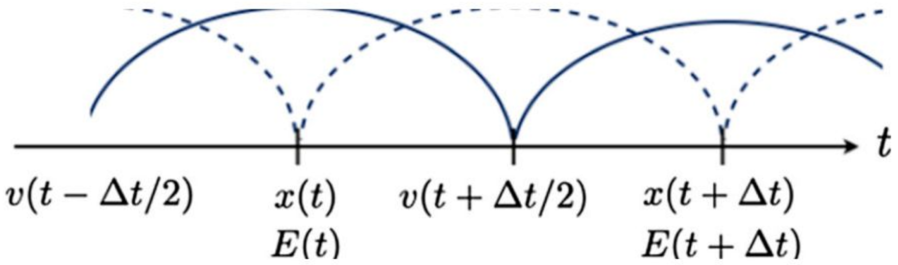
$$\frac{\partial \phi}{\partial y} \approx \frac{\phi_{i,j+1} - \phi_{i,j-1}}{2\Delta y}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \approx \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta x^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{\Delta y^2}$$

step 4

Map field quantities from to particle position to calculate the forces acting on particles. Again using Bi-linear interpolation for consistency. Linear for 1D

step 5



$$m\frac{d\mathbf{v}}{dt} = \mathbf{F}, \quad \mathbf{F} = q\mathbf{E}$$

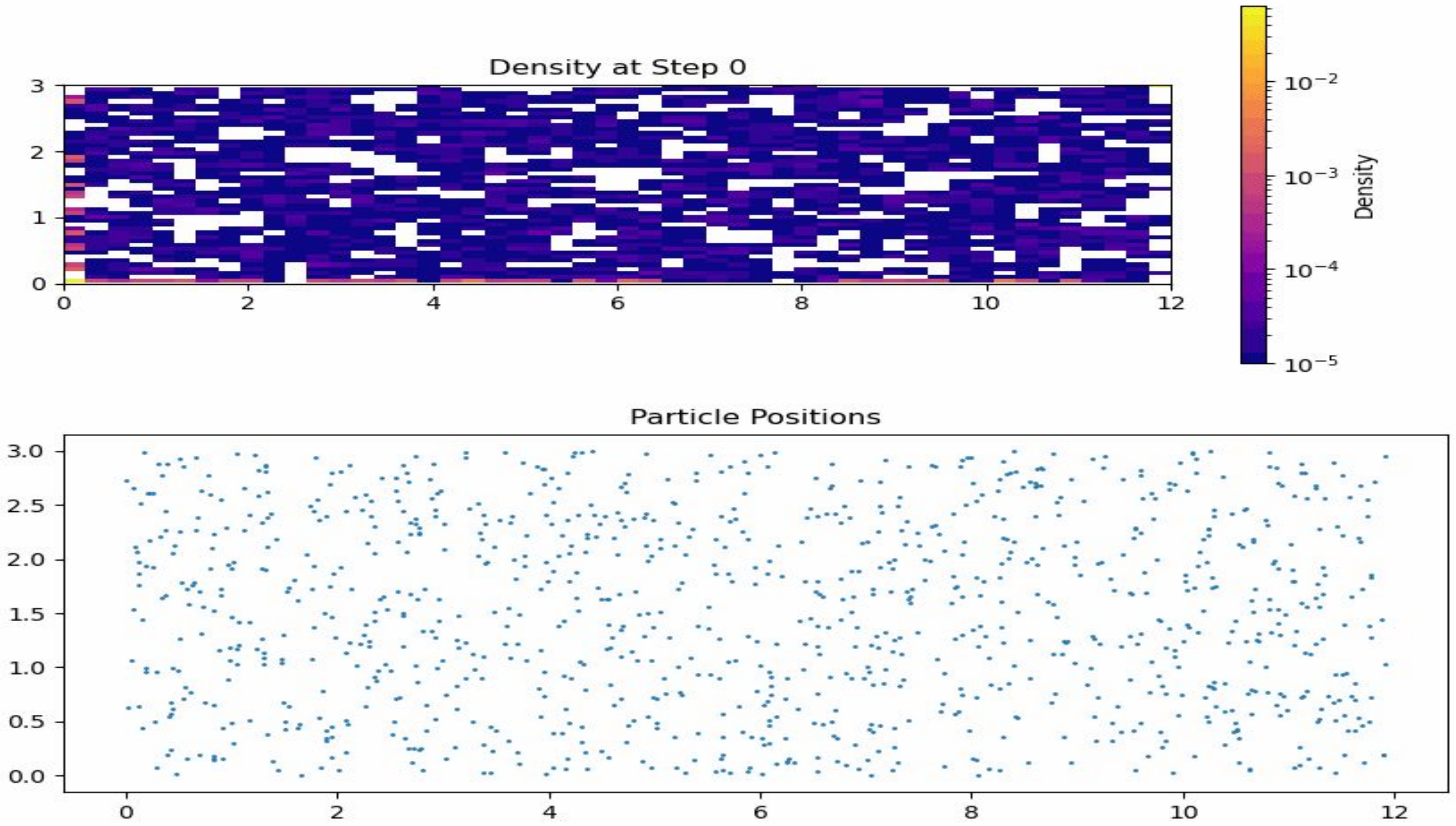
$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{E}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}$$

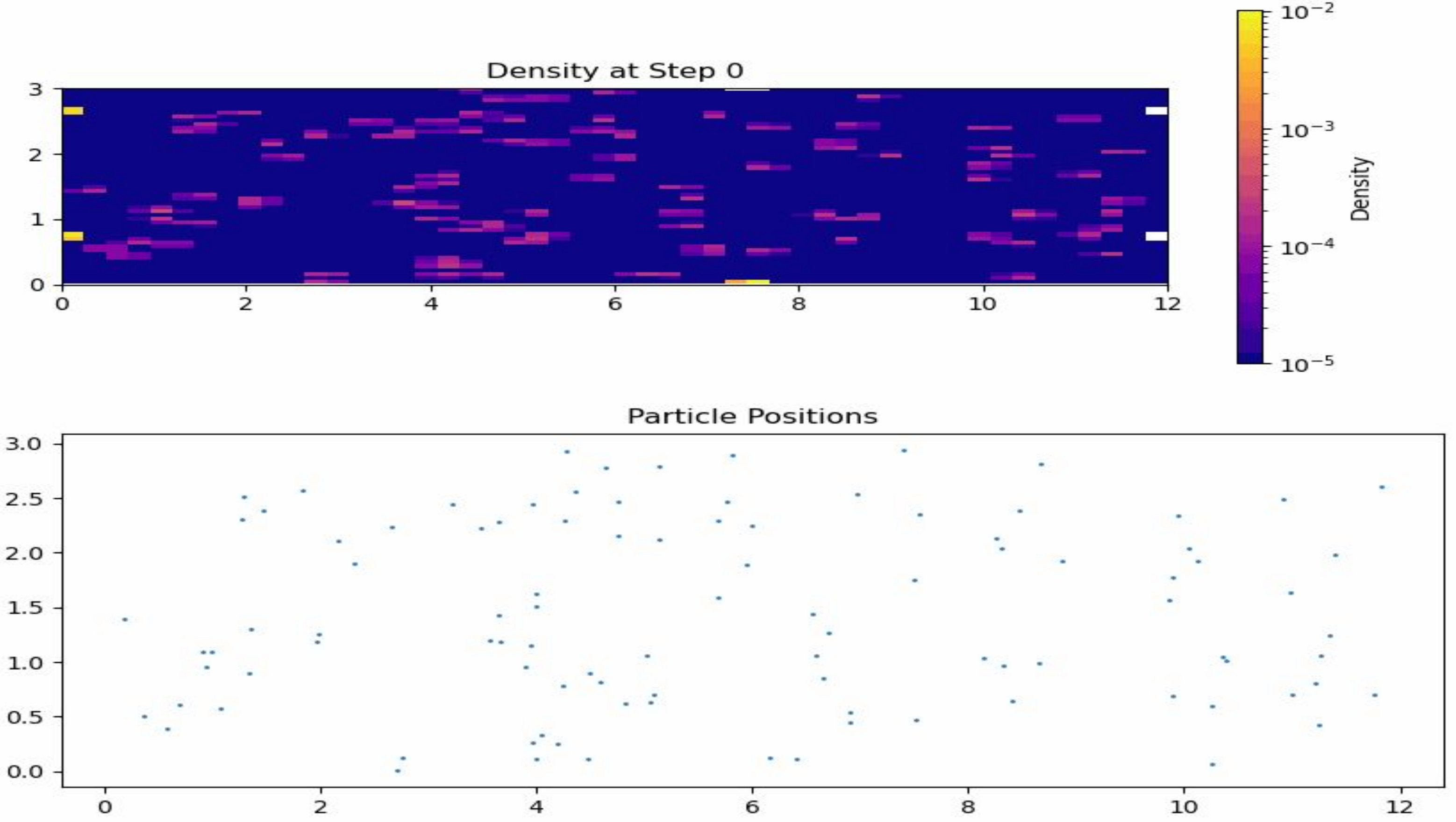
Update particle velocities and positions based on forces using integration methods (LeapFrog, Runge Kutta) step 6

Boundary Conditions (Sticky, Periodic, Reflective)

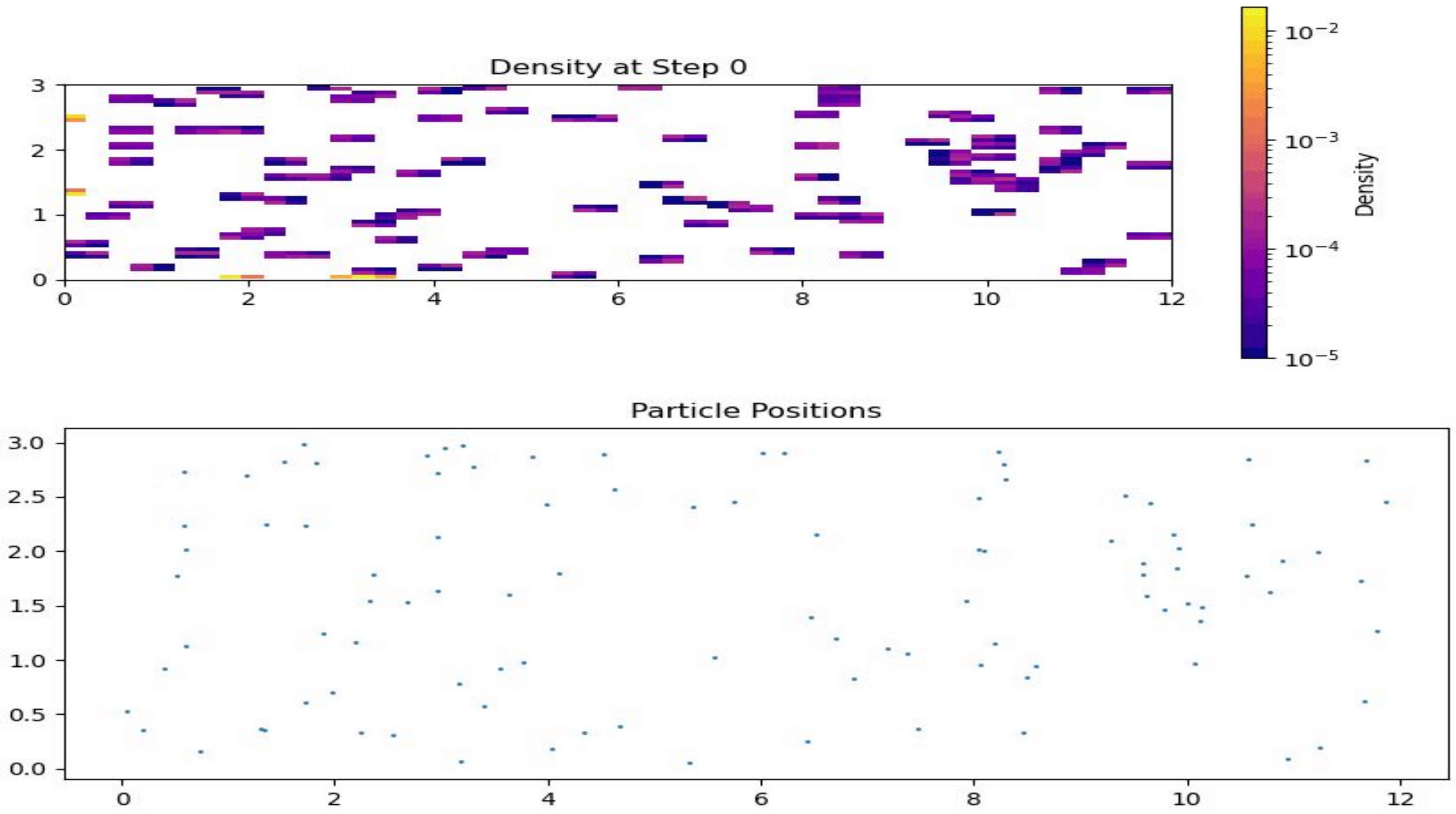
Particle In Cell Method



Density at Step 0
0
1
2
3
0
2
4
6
8
10
12
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Density
Particle Positions
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10
12



Density at Step 0
Density
Particle Positions


Density at Step 0
Density
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Particle Positions

step 7

Update the charge and current density by mapping particle positions back on to the grid

Particle In Cell Method

03 the simulator

Users simply have to open the Jupyter Notebook and in a call call either:

run_simulation_1d(parameter)
or
run_simulation_2d(parameters)

All functions are modularized

Allows users to see the steps that are occuring

thank you