Localization using encoder and GPS data

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The system model of nonholonomic mobile robot

- The kinematics of nonholonomic mobile robot is given by

$$q_k = f(q_{k-1}, u_{k-1}, \gamma_{k-1}) = q_{k-1} + \begin{bmatrix} \cos\phi_{k-1} & 0 \\ \sin\phi_{k-1} & 0 \\ 0 & \Delta t \end{bmatrix} u_{k-1} + \gamma_{k-1} \tag{1}$$

  with $\phi_{k-1} = \theta_{k-1} + \frac{1}{2}\omega_{k-1}\Delta t$ where $q_{k-1} = \begin{bmatrix} x_{k-1} & y_{k-1} & \theta_{k-1} \end{bmatrix}^T$, $u_{k-1} = [\rho_{k-1}, \omega_{k-1}]^T$, γ_k is the process noise with $\gamma_{k-1} \sim \mathcal{N}(0, Q_{k-1})$, and Δt is the time interval.

- The measurement is given by

$$z_k = h(q_k, v_k) = q_k + v_k \tag{2}$$

  where v_k is the measurement noise with $v_k \sim \mathcal{N}(0, R_k)$.

Prediction by Odometry data

- The linear and angular velocities can be derived from the wheel velocities as

$$\rho_{k-1} = \frac{(\overline{W}_r v_r + \overline{W}_l v_l)}{2} \text{ and } \omega_{k-1} = \frac{(\overline{W}_r v_r - \overline{W}_l v_l)}{\overline{WB}} \quad (3)$$

  where $\overline{W}_r$ and $\overline{W}_l$ are right and left wheel radius, $\overline{WB}$ is the wheel baseline of the robot, and v_r and v_l are the right and left wheel velocities.

- Kalman filter is initialized as follows:

$$\begin{aligned} \hat{q}_0^+ &= E(q_0) \\ P_0^+ &= E((q_0 - \hat{q}_0^+)(q_0 - \hat{q}_0^+)^T) \end{aligned}$$

Prediction by Odometry data

- A priori state estimate is given by

$$\hat{q}_k^- = q_k = f(q_{k-1}, u_{k-1}), \quad k \in \{1, \cdots, n\} \tag{4}$$

where $\hat{q}_k^-$ is the mean of the predicted state before updating.

- Covariance prediction is given by

$$P_k^- = A_{k-1} P_{k-1}^+ A_{k-1}^T + B_{k-1} Q_{k-1} B_{k-1}^T, \quad k \in \{1, \cdots, n\} \tag{5}$$

where the Jacobian

$$A_{k-1} = \frac{\partial f}{\partial q_{k-1}} = \begin{bmatrix} 1 & 0 & -\rho_{k-1}\sin(\phi_{k-1}) \\ 0 & 1 & \rho_{k-1}\cos(\phi_{k-1}) \\ 0 & 0 & 1 \end{bmatrix} \text{ and}$$

$$B_{k-1} = \frac{\partial f}{\partial u_{k-1}} = \begin{bmatrix} \cos(\phi_{k-1}) & -\frac{1}{2}\rho_{k-1}\sin(\phi_{k-1}) \\ \sin(\phi_{k-1}) & \frac{1}{2}\rho_{k-1}\cos(\phi_{k-1}) \\ 0 & 1 \end{bmatrix}.$$

Update by GPS data

- Update for each time step $k \in \{1, \cdots, n\}$:

$$
\begin{aligned}
R_k &= \begin{bmatrix} \sigma_{gps}^2 & 0 & 0 \\ 0 & \sigma_{gps}^2 & 0 \\ 0 & 0 & \sigma_\theta^2 \end{bmatrix} \\
\Psi_k &= H_k P_k^- H_k^T + R_k && (6) \\
K_k &= P_k^- H_k^T \Psi_k^{-1} && (7) \\
z_k &= \begin{bmatrix} x_{h_k} & y_{h_k} & \tan^{-1}\left(\frac{y_{h_k} - y_{h_{k-1}}}{x_{h_k} - x_{h_{k-1}}}\right) \end{bmatrix}^T \\
\hat{z}_k &= \hat{q}_k^- \\
q_k^+ &= \hat{q}_k^- + K_k(z_k - \hat{z}_k) && (8) \\
P_k^+ &= [I - K_k H_k] P_k^- && (9)
\end{aligned}
$$

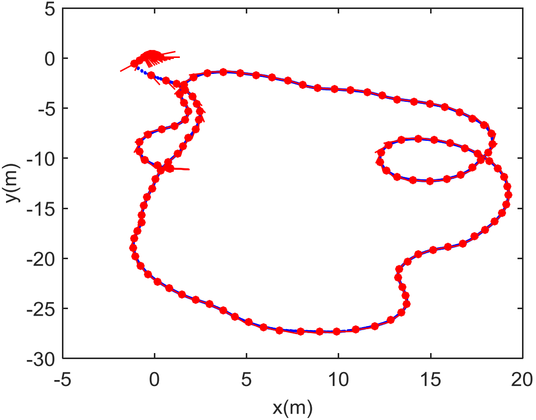

Figure: Localization result using encoder and GPS data