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实验摘要:

- 1、学习卷积，微分方程，高斯白噪声相关函数，傅里叶变换的拟合。
- 2、学习 `conv()`, `tf()`, `impulse()`, `lsim()`, `wgn()`, `xcorr()`, `autocorr()` 等函数用法。

实验题目

1. 利用MATLAB求下列函数的卷积，并绘制出图形
 - (1) $f_1(t) = \varepsilon(t) - \varepsilon(t-1)$, $f_2(t) = 2t[\varepsilon(t) - \varepsilon(t-1)]$
 - (2) $f_1(t) = \cos(30t)g_5(t)$, $f_2(t) = \varepsilon(t) - \varepsilon(t-4)$
 参考函数: `conv()`
2. 某系统满足的微分方程为

$$y''(t) + 4y'(t) + 3y(t) = 2f'(t) + f(t)$$
 - (1) 利用MATLAB求系统的单位冲击响应，并绘出图形
 - (2) 利用MATLAB求系统的单位阶跃响应，并绘出图形
 - (3) 利用MATLAB求系统对信号 $f(t) = 4\sin(2\pi t)\varepsilon(t)$ 的响应，并绘出图形
 参考函数: `tf()`, `impulse()`, `step()`, `lsim()`, `conv()`
3. 利用MATLAB产生高斯白噪声，绘出图形，并求其自相关函数，绘出图形。
参考函数: `randn()`, `wgn()`, `xcorr()`, `autocorr()`
4. 预习关于傅里叶级数的内容，用MATLAB或者Python进行以下实验，回答问题并给出实验过程中产生的结果图。
 - (1) 信号 $f(t)$ 的傅里叶级数为 $\sum_1^{\infty} \frac{\sin nt}{n}$ ，代入数字去逼近或者用解析法分析，估计 $f(t)$ 的形式。
 - (2) 写出你估计出的 $f(t)$ 的傅里叶级数，与上式对比，说明它的谐波和正余弦分量的情况。
 - (3) 取 $N = 50, 100, 200, \dots$ 画出 $f_N(t) = \sum_{n=1}^N \frac{\sin nt}{n}$ ，当 $N \rightarrow \infty$ 时，判断这个部分和与 $f(t)$ 的区别。
 - (4) 同样，取 $N = 50, 100, 200, \dots$ 画出 $F_N(t) = \frac{f_1(t) + f_2(t) + f_3(t) + \dots + f_N(t)}{N}$ ，和上面的图对比，分析他们之间的不同。

实验内容

- 1、(1)

$$1. (1). f_1(t) = \varepsilon(t) - \varepsilon(t-1), f_2(t) = 2t[\varepsilon(t) - \varepsilon(t-1)]$$

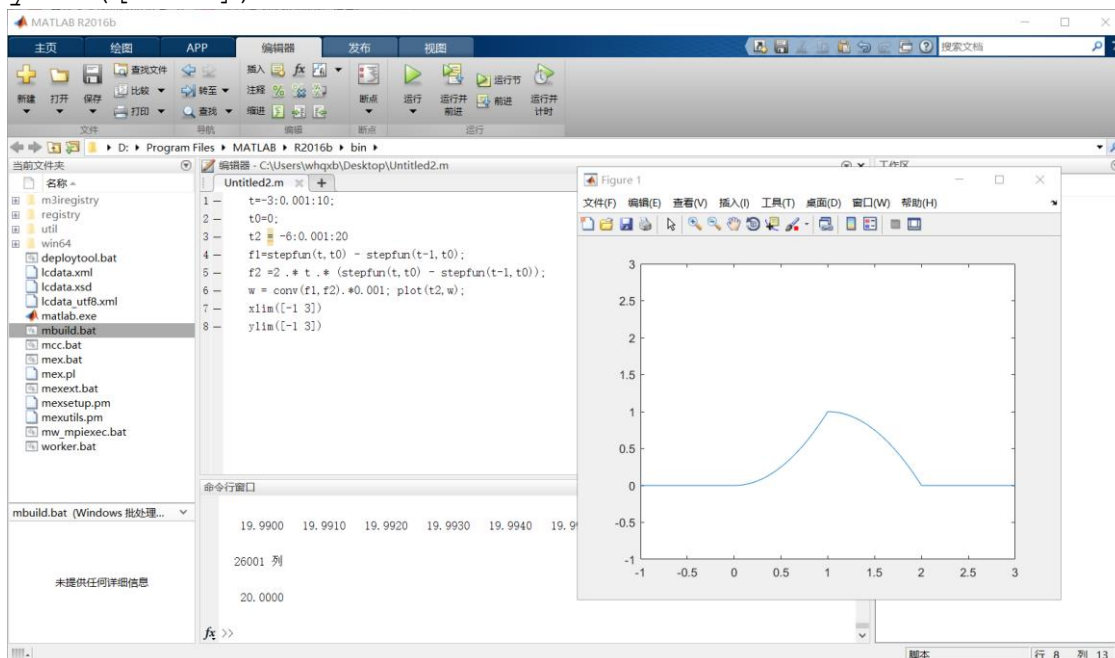
$$\text{设 } f(t) = f_1(t) * f_2(t)$$

$$\Rightarrow f(t) = \begin{cases} 0 & , t < 0, t > 2 \\ t & , 0 < t < 1 \\ -t^2 + 2 & , 1 < t < 2. \end{cases}$$

```

t=-3:0.001:10;
t0=0;
t2 = -6:0.001:20
f1=stepfun(t,t0) - stepfun(t-1,t0);
f2 = 2 .* t .* (stepfun(t,t0) - stepfun(t-1,t0));
w = conv(f1,f2).*0.001; plot(t2,w);
xlim([-1 3])
ylim([-1 3])

```



(2)

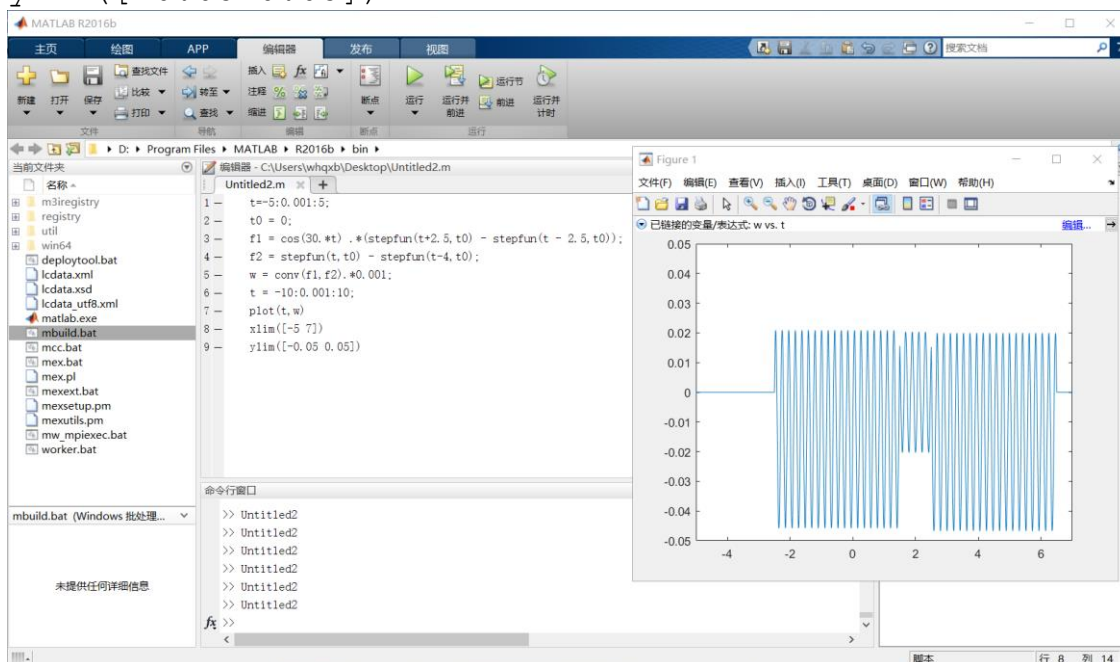
$$(2). f_1(t) = \cos(30t)g_f(t), f_2(t) = \varepsilon(t) - \varepsilon(t-4)$$

$$f_1(t) * f_2(t) = \begin{cases} 0 & t < -2.5, t > 7.5 \\ \frac{1}{30} \sin(30t) \Big|_{-2.5}^t & -2.5 < t < 1.5 \\ \frac{1}{30} \sin(30t) \Big|_{t-4}^t & 1.5 < t < 2.5 \\ \frac{1}{30} \sin(30t) \Big|_{t-4}^{2.5} & 2.5 < t < 7.5 \end{cases}$$

```

t=-5:0.001:5;
t0 = 0;
f1 = cos(30.*t) .* (stepfun(t+2.5,t0) - stepfun(t -
2.5,t0));
f2 = stepfun(t,t0) - stepfun(t-4,t0);
w = conv(f1,f2).*0.001;
t = -10:0.001:10;
plot(t,w)
xlim([-5 7])
ylim([-0.05 0.05])

```



2、

$$2. y''(t) + 4y'(t) + 3y(t) = 2f(t) + f(t)$$

$$\Rightarrow h''(t) + 4h'(t) + 3h(t) = 2f'(t) + f(t)$$

$$\Rightarrow h_1''(t) + 4h_1'(t) + 3h_1(t) = f(t)$$

$$h_1'(0+) = h_1'(0-) + 1 = 1$$

$$h_1(0+) = h_1(0-) = 0$$

$$\text{则 } h_1(t) = (C_1 e^{-t} + C_2 e^{-3t}) \varepsilon(t)$$

$$C_1 + C_2 = 0, -C_1 - 3C_2 = 1$$

$$\Rightarrow C_1 = \frac{1}{2}, C_2 = -\frac{1}{2}$$

$$h_1(t) = (\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}) \varepsilon(t)$$

$$\Rightarrow h(t) = (-\frac{1}{2}e^{-t} + \frac{5}{2}e^{-3t}) \varepsilon(t)$$

$$g(t) = \int_0^+ h(\tau) d\tau = (\frac{1}{2}e^{-t} - \frac{5}{6}e^{-3t} + \frac{1}{3}) \varepsilon(t)$$

$$\therefore y(t) = f(t) * h(t)$$

$$= \frac{2}{4\pi^2 + 1} [\sin(2\pi t) - 2\pi \cos(2\pi t) + 2\pi e^{-t}]$$

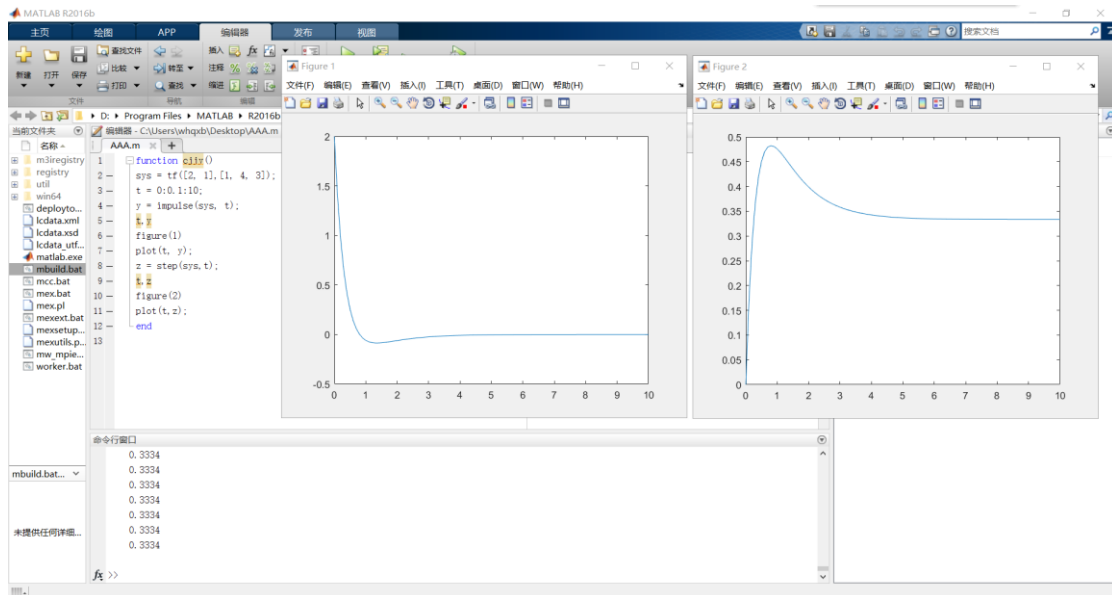
$$+ \frac{2}{4\pi^2 + 9} [3\sin(2\pi t) - 2\pi \cos(2\pi t) + 2\pi e^{-3t}]$$

```
(1)(2)function cjjy()
sys = tf([2, 1],[1, 4, 3]);
t = 0:0.1:10;
y = impulse(sys, t);
t,y
figure(1)
plot(t, y);
```

```

z = step(sys,t);
t,z
figure(2)
plot(t,z);
end

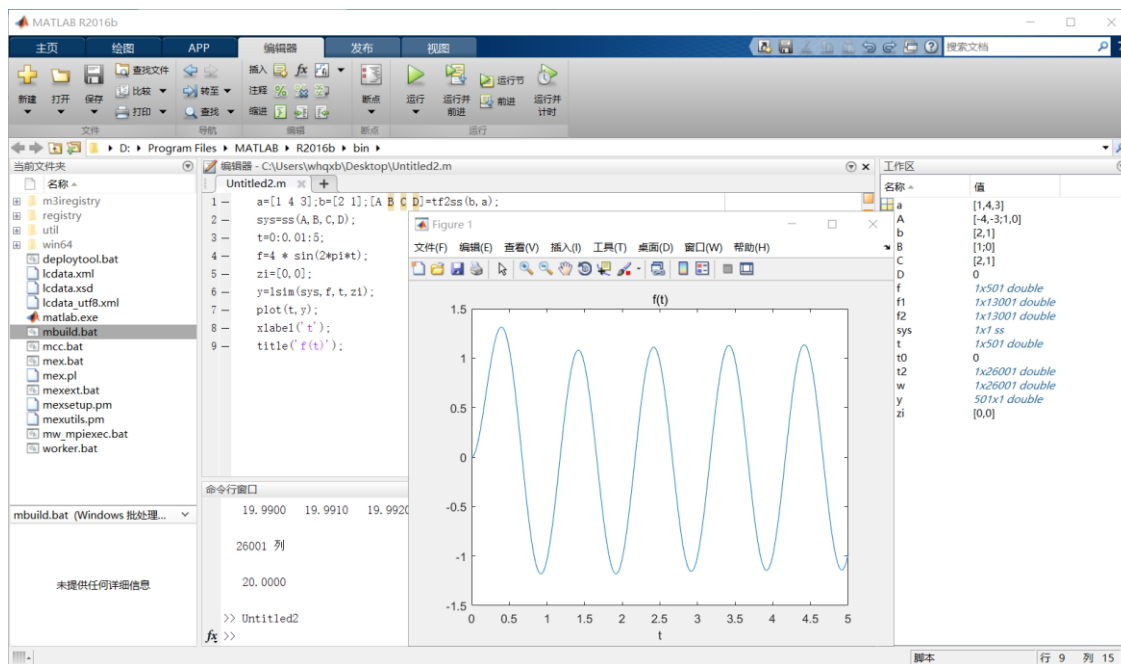
```



```

(3) a=[1 4 3];b=[2 1];[A B C D]=tf2ss(b,a);
sys=ss(A,B,C,D);
t=0:0.01:5;
f=4 * sin(2*pi*t);
zi=[0,0];
y=lsim(sys,f,t,zi);
plot(t,y);
xlabel('t');
title('f(t)');

```



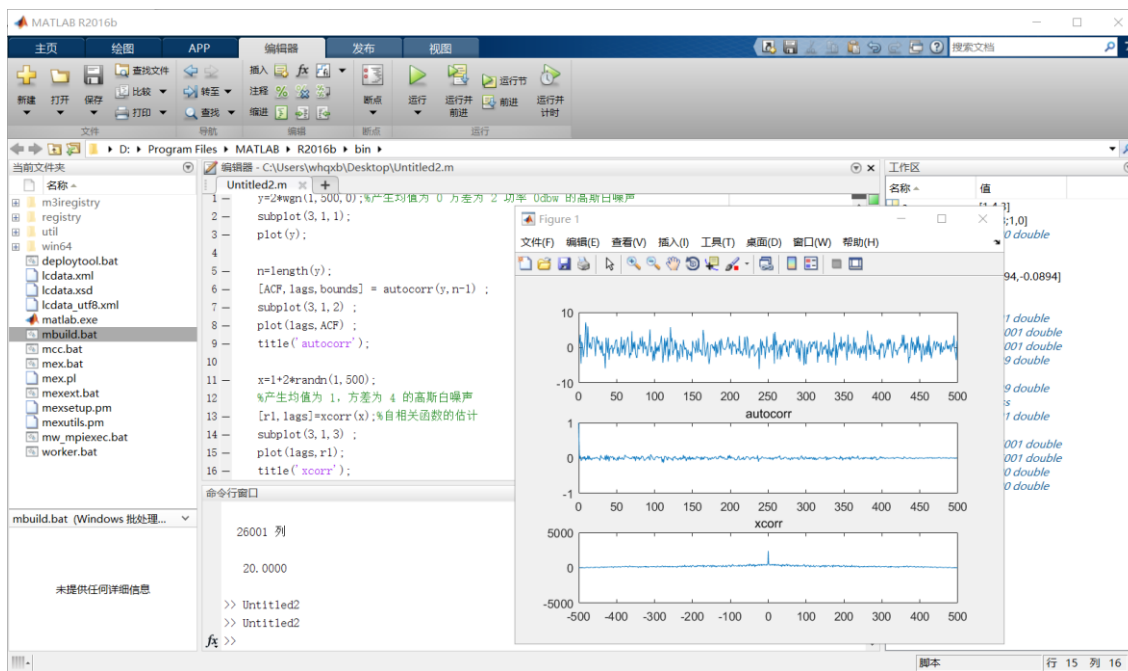
3、 $y=2*\text{wgn}(1,500,0);$ %产生均值为 0 方差为 2 功率 0dbw 的高斯白噪声

```
subplot(3,1,1);
plot(y);
```

```
n=length(y);
[ACF,lags,bounds] = autocorr(y,n-1) ;
subplot(3,1,2) ;
plot(lags,ACF) ;
title('autocorr');
```

$x=1+2*\text{randn}(1,500);$ %产生均值为 1，方差为 4 的高斯白噪声

```
[r1,lags]=xcorr(x); %自相关函数的估计
subplot(3,1,3) ;
plot(lags,r1);
title('xcorr');
```

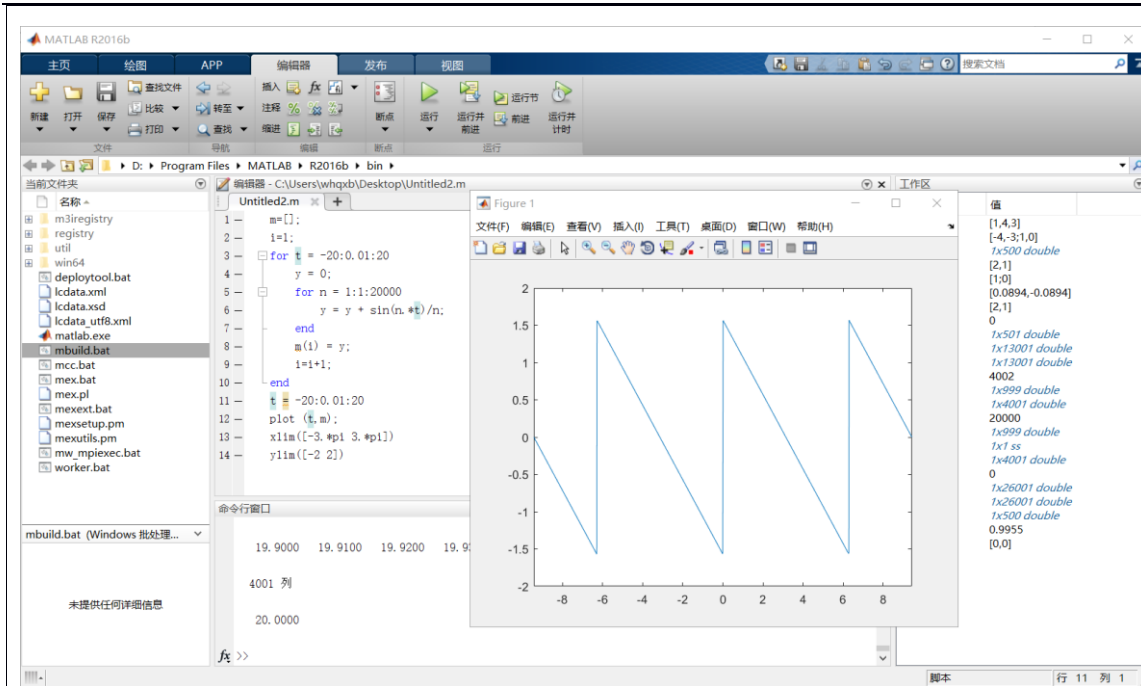
4、(1)

$$4.(1). T=2\pi, f(t) = \frac{\pi-t}{2} \quad (0 < t < T).$$

```

m=[];
i=1;
for t = -20:0.01:20
    y = 0;
    for n = 1:1:20000
        y = y + sin(n.*t)/n;
    end
    m(i) = y;
    i=i+1;
end
t = -20:0.01:20
plot (t,m);
xlim([-3.*pi 3.*pi])
ylim([-2 2])

```



(2)

4.(2).

$$f(t) = \frac{\pi-t}{2} \quad 0 < t < 2\pi \quad \text{周期 } T=2\pi$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos nt dt$$

$$= \frac{1}{\pi} \int_0^{2\pi} \frac{\pi-t}{2} \cos nt dt$$

$$= \frac{1}{\pi n} \left[\frac{\pi-t}{2} \sin nt \Big|_0^{2\pi} - \int_0^{2\pi} \sin nt \cdot \left(-\frac{1}{2}\right) dt \right]$$

$$= 0$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} \frac{\pi-t}{2} \sin nt dt$$

$$= -\frac{1}{\pi n} \left[\frac{\pi-t}{2} \cos nt \Big|_0^{2\pi} - \int_0^{2\pi} \cos nt \left(-\frac{1}{2}\right) dt \right]$$

$$= -\frac{1}{n\pi} \left[\frac{\pi-2\pi}{2} - \frac{\pi}{2} \right]$$

$$= \frac{1}{n}.$$

$$\Rightarrow f(t) = \sum_{n=1}^{\infty} \frac{\sin(nt)}{n} \quad \text{符合(1)}.$$

$$A_0 = a_0 = 0 \quad A_n = \sqrt{a_n^2 + b_n^2} = \frac{1}{n}.$$

$$\varphi_n = \frac{\pi}{2}.$$

$$f(t) = 0 + \sum_{n=1}^{\infty} \frac{1}{n} \cos\left(nt + \frac{\pi}{2}\right)$$

$$n \text{ 次谐波为 } \frac{1}{n} \cos\left(nt + \frac{\pi}{2}\right).$$

$$\text{正弦分量为 } \sum_{n=1}^{\infty} \frac{1}{n} \sin(nt), \text{ 无余弦分量.}$$

```
t = -1:0.01:1;
```

```
syms n
```

```
w1 = symsum(sin(n*t)/n,n,1,50);
```

```
w2 = symsum(sin(n*t)/n,n,1,100);
```

```
w3 = symsum(sin(n*t)/n,n,1,200);
```

```

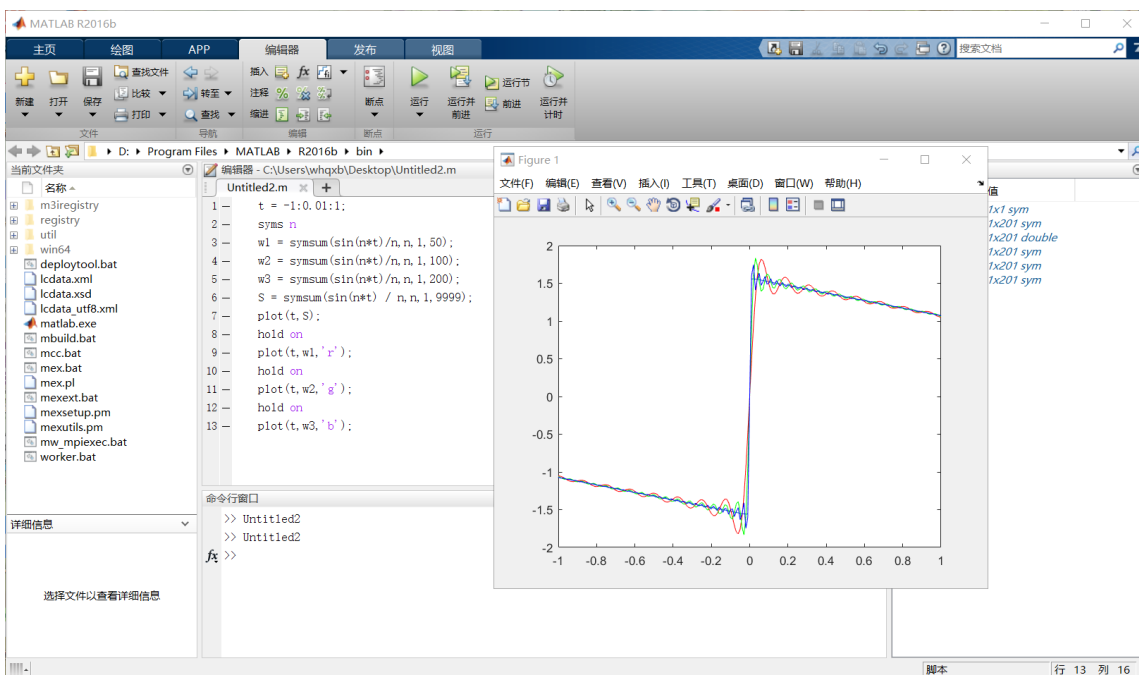
S = symsum(sin(n*t) / n,n,1,9999);
plot(t,S);
hold on
plot(t,w1,'r');
hold on
plot(t,w2,'g');
hold on
plot(t,w3,'b');

```

```

(3) t = -1:0.01:1;
syms n
w1 = symsum(sin(n*t)/n,n,1,50);
w2 = symsum(sin(n*t)/n,n,1,100);
w3 = symsum(sin(n*t)/n,n,1,200);
S = symsum(sin(n*t) / n,n,1,9999);
plot(t,S);
hold on
plot(t,w1,'r');
hold on
plot(t,w2,'g');
hold on
plot(t,w3,'b');

```

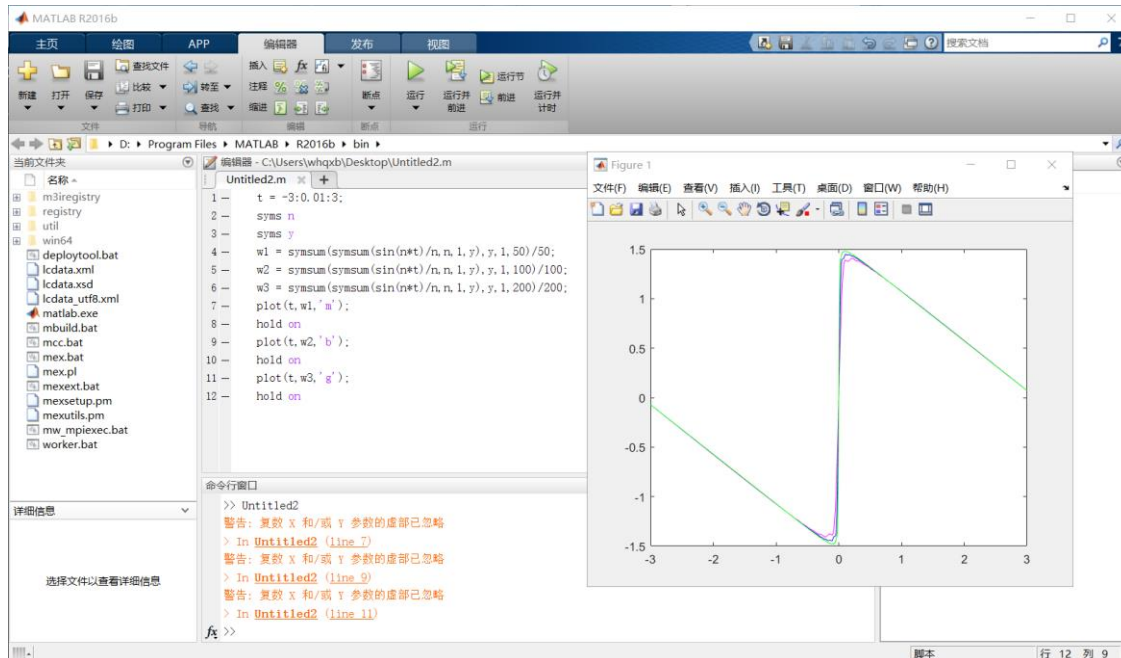


```

(4) t = -3:0.01:3;
syms n
syms y
w1 = symsum(symsum(sin(n*t)/n,n,1,y),y,1,50)/50;
w2 = symsum(symsum(sin(n*t)/n,n,1,y),y,1,100)/100;
w3 = symsum(symsum(sin(n*t)/n,n,1,y),y,1,200)/200;

```

```
plot(t,w1,'m');  
hold on  
plot(t,w2,'b');  
hold on  
plot(t,w3,'g');  
hold on
```



实验总结

最后一题跑的时间太久了，风扇狂转，大概是电脑配置不太行，再加上 **matlab** 对多线程的支持不太行。

参考文献

<https://zhidao.baidu.com/question/1050287455988787939.html>

<https://zhidao.baidu.com/question/159436424.html>

<https://blog.csdn.net/lfdanding/article/details/50726678>