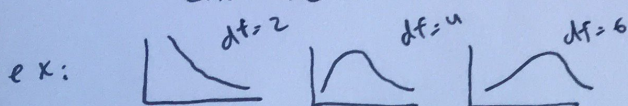


## CHI SQUARE DISTRIBUTION

- Distribution of SS of  $k$  ind. normal variables  
- use w/ Chi. 2 test



$$df = (\text{levels in 1}) (\text{levels in 2})$$

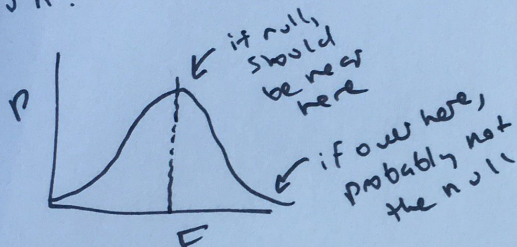
- only need 1 parameter,  $m$ , to describe it - error for 1 obs.

$$\chi^2 = \chi^2(m) \quad \chi^2 = \sum_{i=1}^m Z_i^2$$

$$Z_i = \frac{x_i - \bar{x}}{s} \leftarrow \begin{array}{l} \text{sample mean} \\ \text{sample s.d.} \end{array}$$

## F-DISTRIBUTION

- test statistic in F-tests follow this distribution - under the null.





# LIKELIHOOD

$L = P(\text{DATA} | \text{MODEL})$  Probability of data given the model

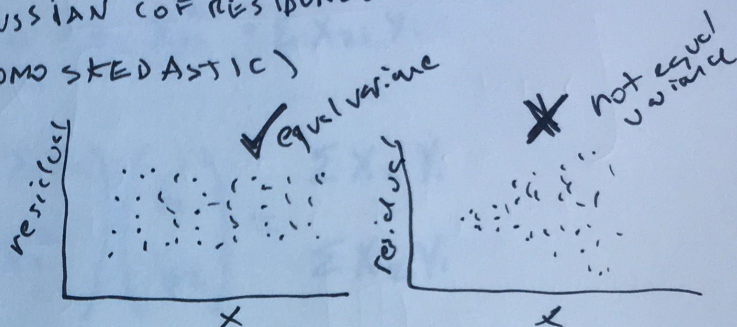
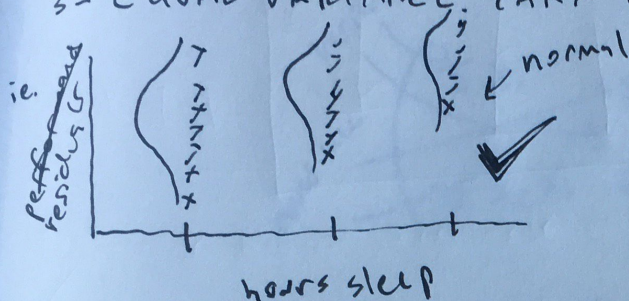
$$L = \prod_{i=1}^n P(y_i | x_i)$$

## LM ASSUMPTIONS

1- OBSERVATIONS ARE INDEPENDENT

2- DISTRIBUTION IS NORMAL / GAUSSIAN COF RESIDUALS

3- EQUAL VARIANCE (AKA HOMO SKEDASTIC)



## MODEL COMPARISON

$$SSE = \sum_{i=1}^n (\hat{y}_i - y_i)^2$$

$$F = \frac{\frac{(SS_1 - SS_2)}{(k_2 - k_1)}}{\frac{SS_2}{n - k_2}}$$

SS = sum of squares  
k = # parameters  
n/m = sample size

$$R^2 = \frac{\frac{SS_1}{n} - \frac{SS_2}{n}}{\frac{SS_1}{n}} = 1 - \frac{SS_2}{SS_0} \quad (\text{for null})$$

$$R^2_{adj} = 1 - \frac{\frac{SS_2}{(n - k_2)}}{\frac{SS_1}{(n - k_1)}} = 1 - \frac{\frac{SS_2}{n - k_2}}{\frac{SS_0}{(n - 1)}} \quad (\text{for null})$$

$$R^2_{cv} = \frac{SS_1 - SS_2}{SS_1} \quad \left. \vphantom{R^2_{cv}} \right\} \text{calculated on hold-out set of test data the model was } \phi \text{ trained on}$$

SSE = Way to quantify errors absolutely

$R^2$  = % of variance model explains for vs. another baseline

$R^2_{adj}$  = like  $R^2$ , but penalizes extra parameters

$R^2_{cv}$  = like  $R^2$ , but controls for overfit using test data set

F-stat = usually for testing ~~into~~ if  $\neq$  null (independent distributions)  
test stat has an F distribution under the null



# NORMAL EQUATION FOR BI-MULTIVARIATE (if means = 0)

$$\hat{y} = b_0 + b_1 x_1$$

$$b_1 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{\sum_{i=1}^n x_{1i} y_{1i}}{\sum_{i=1}^n x_{1i}^2}$$

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_j x_j$$

$$\frac{dL}{db_1} = 0 \quad \frac{dL}{db_2} = 0 \Rightarrow \begin{cases} b_1 \sum x_{1i}^2 + b_2 \sum x_{1i} x_{2i} = \sum x_{1i} y_i \\ b_2 \sum x_{1i} x_{2i} + b_2 \sum x_{2i}^2 = \sum x_{2i} y_i \end{cases}$$

$$\begin{pmatrix} \sum x_{1i}^2 & \sum x_{1i} x_{2i} \\ \sum x_{2i} x_{1i} & \sum x_{2i}^2 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \sum x_{1i} y_i \\ \sum x_{2i} y_i \end{pmatrix}$$

$$X^T X \vec{b} = X^T y$$

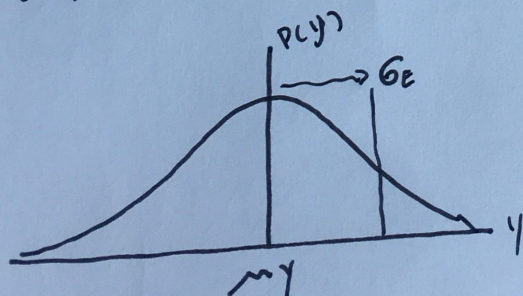
## PEARSON'S CORRELATION COEFF. (if means = 0)

$$\text{Cor}(x, y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}} = \frac{\sum x y}{\sqrt{\sum x^2 \sum y^2}}$$

## NORMAL DISTRIBUTION (AKA GAUSSIAN)

$$PDF = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{1}{2\sigma^2} (y - \mu)^2}$$

$$L = \prod_{i=1}^n \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{1}{2\sigma^2} (y_i - \hat{y}_i)^2} \quad \left. \begin{matrix} \end{matrix} \right\} \text{when you assume = variance}$$



• defined by  $\mu + \sigma$  (mean and std.)

$$e.g. N(\mu, \sigma)$$