FH Vorarlberg
University of Applied Sciences

Orbital Dynamics:

Modelling of a Slingshot around Earth towards Mars in MATLAB

Work Type: Project

Course Title: Guided Project

University of Applied Science Vorarlberg

Bachelor Mechatronics Fulltime

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Introduction:

Motivation:

The National Aeronautic and Space Administration (NASA) in the United States of America released a video of their plans to build a Luna Orbital Platform Gateway (LOP-G) to help with Mars mission (https://www.youtube.com/watch?v=Uiy67s8zqHU). This video planted the idea of using the minimal amount of fuel to get to Mars by using the gravity of Earth and the sun to create and elliptical trajectory towards Mars. Due to the moon low gravity, launching a rocket from its surface will cost a lot less in fuel (the comparison between launching a rocket from Earth and the moon has been done in the program).

Problem:

Due to the differences in orbital periods between Earth and Mars, they move closer and further apart. The problem would be calculating the ideal position of Mars at which the Hohmann transfer (a manoeuvre to transfer a satellite or spacecraft to another orbit) will end and calculate the transit time as well as the required delta-V (fuel) to complete the transfer.

Theory:

Kepler's Laws of Orbital Dynamics (https://www.britannica.com/science/Keplers-laws-of-planetary-motion#ref214571):

First Law: the law of ellipses

All planets move about the sun in elliptical orbits, having the sun as one of the foci.

Second Law: the law of equal areas

A line segment joining a planet and the sun sweeps out equal areas during equal intervals of time.

Third Law: the law of harmony

The squares of the sidereal periods (of revolution) of the planets are directly proportional to the cubes of their mean distances from the sun.

The Rocket Equation:

The rocket equation is a mathematical equation predicting the velocity of a rocket, a rocket being any vehicle that preserves momentum by discarding some of its weight (fuel) while accelerating. The derivation in the Appendix.

Assumptions:

- The orbits for Earth, the moon, and Mars share a plane, i.e. there is no inclination to any of their obits, and so the trajectories calculated will also be on the same plane. (The orbits for all celestial bodies are more accurately plotted with altering inclination to Earth's orbit.).
- On the first of January 2020, Mars was 5 days behind Earth on its orbit. (Mars was much further ahead on its orbit in January of 2020. Mars's close approach is in October in the year 2020).
- Both Earth and Mars have 24 hour days (neither of the days last exactly 24 hours).
- The Luna Orbital Platform Gateway is fully functional (although the LOPG is only due to be operational in the mid to late 2020's).

Main Chapter:

Method:

1: Initialise the crucial values of the Celestial Bodies:

The crucial values being the current year of Earth and Mars, the aphelion, the perihelion, the eccentricity, the mean daily motion, the mean celestial longitude, and the celestial longitude at the Perihelion for Mars, Earth and the moon.

2: Find the Axes (semi-major and semi-minor) of the orbital paths for Earth, the moon and Mars:

The axes can be calculated by using the values initialised.

3: Find orbital paths of Earth and Mars:

The orbital paths can be calculated by using the axes from the previous step and entering them into the equations for an ellipse in polar format.

4: Find the position of Earth and Mars:

The positions for Earth and Mars can be calculated with the true anomaly and the distance between the sun and the celestial body (The formulas for the true anomaly are in the appendix).

5: Determine the launch day and the landing day for Earth, Mars and the moon:

All respective to their own orbital revolutions.

6: Determine the moon's orbital path around Earth:

The moon's orbital path was calculated in the same way was the orbital paths for the celestial bodies in the previous steps.

7: Calculate the trajectory of the rocket and the transit period using Kepler's laws:

These laws will result in an elliptical path for the rocket to travel. This elliptical path is also known as a Hohmann Transfer. The same methods used to calculate the elliptical paths of the celestial bodies was used to calculate the Hohmann Transfer.

8: Calculate the trajectory of the slingshot orbit around Earth:

This was done using the same methods determining the trajectory of the Hohmann Transfer to Mars and calculating where the two paths intersected. The path would then result in the trajectory around Earth and into the Hohmann Transfer to Mars.

9: Calculate delta-V using the rocket equation:

The values for the total mass of the rocket and the separate mass of just the spacecraft were initialised and used to calculate the delta-V from Earth and from the moon for the comparison of required fuel needed to escape each of their gravitational forces.

10: Plot the all orbits, positions of celestial bodies, and trajectories on a graph.

11: Label the celestial bodies on the graph an give the graph a title.

Problems Encountered and Solutions:

While writing this project, four major problems were encountered: 1) finding the position of the celestial bodies, 2) the rotating of the orbital paths, 3) finding the perfect Hohmann transfer, and 4) plotting the ellipses in MATLAB.

1. The problem with regards to the positioning of the planets was due to the poor initial idea by which Pythagoras's theorem was used to calculate the total distance that the celestial body would travel over 24 hours. This was a problem because the total distance would form a straight line and completely disregard the small arc that the planet would have travelled along. Over the first few days it did not seem to be a problem. However, the small arcs that were discarded accumulate to a rather large distance after a few months and thus would cause the rocket to completely miss Mars, rendering the entire transfer pointless. This problem was fixed by calculating the mean and the true anomaly. By calculating the anomalies, it would find the position of the celestial body at a given degree around the body at its orbits first focus (in the case of Mars and Earth, it was around the sun).

2. The next problem encountered was just a maths problem of calculating the rotation of an ellipse. This problem was solved on page 6 b on the Hand-written notes. (see further on).

3. A problem came up while finding the exact day that the perfect Hohmann Transfer would take place. Due to the inconsistent nature of the movement of Earth and Mars, the program ran into an infinite loop trying to calculate the exact day that the transfer would take place. This problem was fixed by calculating the ideal location of Mars on a given launch day.

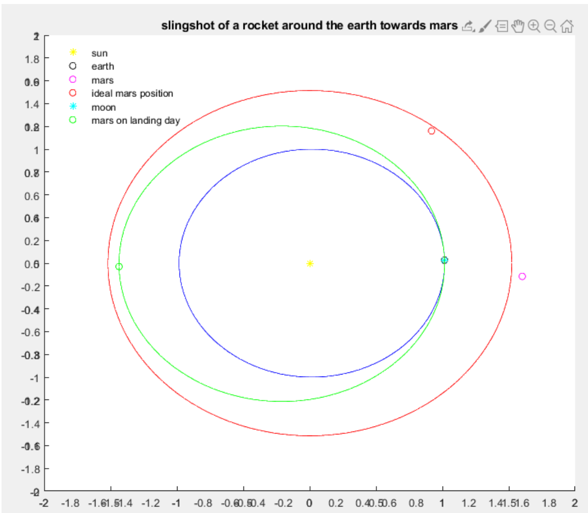
4. Plotting the ellipses was difficult on MATLAB because the compiler did not allow for variables that were not initialised or stated that they were symbols. This problem was solved due to trial and error of many different variations of the mathematical equation of an ellipse.

Conclusion:

Results:

Day 1 (2020.01.01 as launch date):

Planetary orbits and trajectory to Mars:



Results calculated:

```
periodOfJourneyToMars =

   248

periodOfSlingShot =

    14

totalPeriodOfJourney =

   262

earthDeltaV =

   62.5591

moonDeltaV =

   13.2938
```

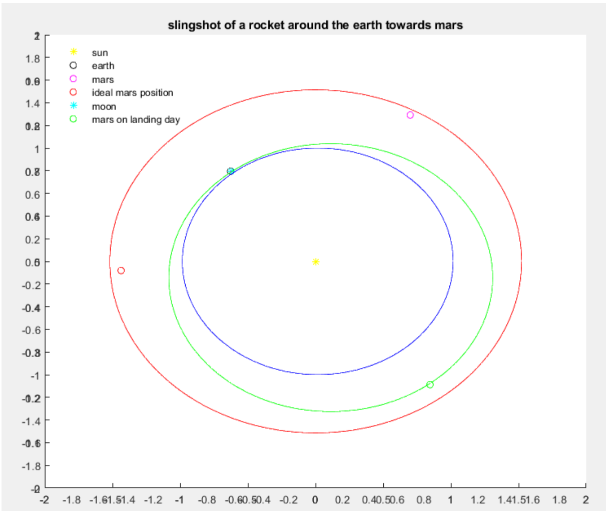
Green ellipse: Hohmann transfer

Red ellipse: Mars's orbital path

Blue ellipse: Earth's orbital path

Day 130 (2020.05.09. as the launch day):

Planetary orbits and trajectory to Mars:



Green ellipse: Hohmann transfer

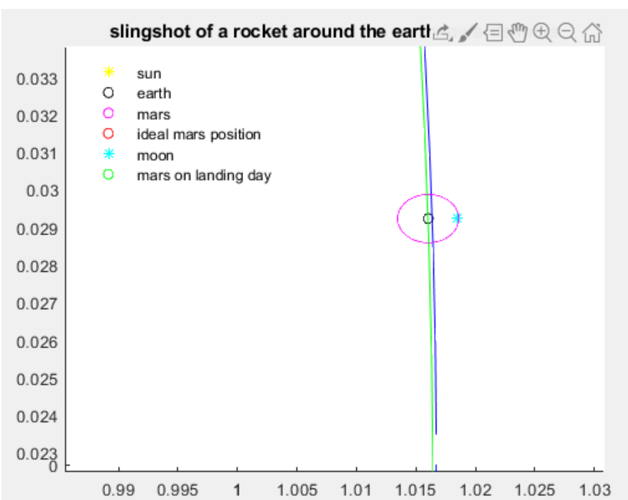
Red ellipse: Mars's orbital path

Blue ellipse: Earth's orbital path

Results calculated:

```
periodOfJourneyToMars =

   239


periodOfSlingShot =

    14


totalPeriodOfJourney =

   253


earthDeltaV =

   62.5591


moonDeltaV =

   13.2938
```

Zoomed in to the Orbit of the Moon:



Green ellipse: Hohmann transfer

Red ellipse: Mars's orbital path

Blue ellipse: Earth's orbital path

Discussion:

A MATLAB program was written that calculated the transfer trajectory from the Luna Orbital Platform Gateway around Earth an on the trajectory from Earth to Mars. Due to the uncertainty of the exact position of Mars, the ideal position of Mars on the day of the launch was calculated.

The Program Produced a model where the transfer orbit as well as the planetary orbits are visible. The program also plotted the positions of Earth, the moon, current position of Mars, the position of Mars at the end of the transfer, and the ideal position of Mars on the day of the launch. The output of the program also consists of the period of the transfer with and without the slingshot around Earth and the required delta-V for the rocket to be launched from Mars and from the moon for the comparison of the required delta-v from Earth and from the moon.

The expectations for this project have been met. The program calculates the required delta-V to get to Mars and the period of the total Journey. The program also calculates the delta-V and the period of the Journey if the rocket was launched from Earth for an extra comparison.

Content Goals:

- Model the orbits of Earth, Mars, and the moon.
- Model the Hohmann Transfer from Earth to Mars on any given day.
- Plot the position of Earth, the moon, and Mars.
- Plot the ideal position of Mars and the day that the rocket land Mars.
- Calculate the transit time and required delta-V.

Learning Goals:

- Understand of how to plot complex graph in MATLAB.
- Understand how to find intersections of two functions in MATLAB.

Summary:

Providing a general model that allows for a gravitational slingshot from the Luna Orbital Platform Gateway around Earth can provide an optimal travel window for missions to Mars. Using Kepler's Laws and the rocket equation we can model the orbital path of the rocket to Mars and calculate the required delta-V (fuel) and the resulting transit time.

Possible Points for Continuation and Improvements:

This program could be improved by:

- Implementing an angle of incline to the orbits.
- Positioning Mars and Earth more accurately to the date – for example plotting the closest position between Earth and Mars in October, where it is going to be this year instead of in January.
- Producing a date which the rocket with land on Mars.
- Producing a better visualisation of the whole Transfer by using high quality images generated by MATLAB's image processing applications.
- Plotting the transfer around the moon on a separate plot to make the transfer more visible.
- Calculating total fuel costs.
- Visualising the trajectory of the rocket around the moon.

Glossary:

Velocity:	The change of distance over time (speed).
Delta-V:	The change in velocity over time and is directly proportional to the consumption of fuel.
Aphelion:	The position of a planet when it is furthest away from its star.
Perihelion:	The position of a planet when it is closest to its star.
Celestial Longitude:	The angular distance along a planets orbital plane from the primary direction.
Orbit:	The path a body revolves around a much larger celestial body.
LOP-G:	The Luna Orbital Platform Gateway (LOP-G) is a space station that will orbit around the moon and is due for construction in the mid 2020's.
Mean Anomaly:	The angle between a celestial body's original position and its current position if the orbit was a perfect circle.
True Anomaly:	The true angle between a celestial body's original position and its current position.

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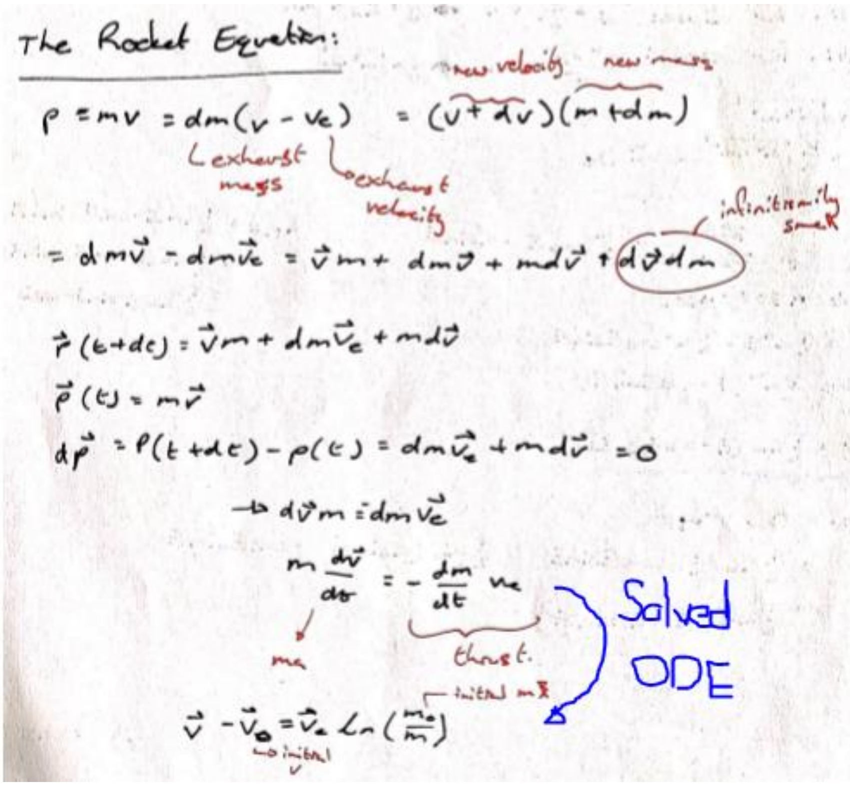
13. Dr. David R. Williams, Mars Fact Sheet, Nasa (Last Updated: September 27, 2018, DRW), accessed on 27 May 2020, <https://nssdc.gsfc.nasa.gov/planetary/factsheet/Marsfact.html>

14. Dr. David R. Williams, Moon Fact Sheet, Nasa (Last Updated: January 13, 2020, DRW), accessed on 27 May 2020, <https://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html>

Appendix:

The Rocket Equation:



The Rocket Equation:

$$p = mv = \underbrace{dm}_{\text{exhaust mass}}(v - \underbrace{v_e}_{\text{exhaust velocity}}) = (\overbrace{v + dv}^{\text{new velocity}})(\overbrace{m + dm}^{\text{new mass}})$$

$$= dm\vec{v} - dm\vec{v}_e = \vec{v}m + dm\vec{v} + m\,d\vec{v} + \underbrace{d\vec{v}\,dm}_{\text{infinitesimally small}}$$

$$\vec{p}(t+dt) = \vec{v}m + dm\vec{v}_e + m\,d\vec{v}$$

$$\vec{p}(t) = m\vec{v}$$

$$d\vec{p} = p(t+dt) - p(t) = dm\vec{v}_e + m\,d\vec{v} = 0$$

$$\rightarrow d\vec{v}m = dm\vec{v}_e$$

$$\underbrace{m\frac{d\vec{v}}{dt}}_{ma} = \underbrace{-\frac{dm}{dt}v_e}_{\text{thrust}}$$

Solved ODE

$$\vec{v} - \underbrace{\vec{v}_0}_{\text{initial } v} = \vec{v}_e \ln\left(\frac{m_0}{m}\right) \quad m_0\text{: initial mass}$$

Code to get the True Anomaly

```
function vr = getTrueAnomaly(n, d, l, p, e, a)

    M = n * d + l - p;                                          %Calculate Mean Anomaly
    M = M - floor(M/360) * 360;                                 %Ensure that the Mean Anomaly doesn't exceed 360 degrees

    v = M + 180/pi * ((2 *e - e^3 / 4) * sin(M) + ...          %Calculate True Anomaly
        5 / 4 * e^2 * sin(2*M) + 13 / 12 * e^2 * sin(3*M));

    r = a * (1 - e^2) / (1 + e * cos(v));                       %Calculating the Distance from the first focus (commonly the sun)

    if v > 360
        v = v - 360;
    end

    vr = [r, v];                                                %output the distance and the true anomaly

end
```