Geführtes Projekt.

Terms:

perihilion - Earth closest to sun

Aphelion - Earth furthest from sun

3 different vektor systems:

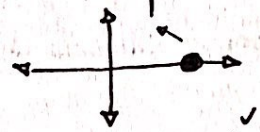
- Launch system => Moon @ origin
- Orbital Slingshot system => ~~Earth as origin~~ Origin: centre of Moons orbital path around Earth.
- Positional system => ~~Sun @ origin~~

Positional system: Sun is origin

- ~~Sun as origin centre of Earths Orbital course around the sun~~
- Plots positions of Earth and Mars @ ~~sertain~~ certain dates.
- 01.01 of any year, Earth will be in position: ~~(distance from sun~~

  ~~(distance from sun ; 0)~~

  $(r \cdot \cos v + x_0 ; r \cdot \sin v + y_0)$



r is the distance from the sun

v is the true Anomaly

$(x_0 ; y_0)$ is centre of orbit

- Earth Revolves around the sun in an anti-clockwise direction
- Need to calculate the equation for the orbital path ~~around~~ of Earth~~as~~ and Mars around the sun.

1 min = 60 s

- 0.0167 - eccentricity of earths orbit
- 0,0934 - eccentricity of mars' orbit.

1h = 60 m

1h = 60 · 60 s

h = 3600 s

~~graph~~-

- earths orbital speed: 107 000 km/h = 29,72 $\frac{km}{s}$
- mars orbital speed: 24 km/s

- ~~Create function to calculate centre of elipse~~
- earth orbital path:

  min. distance from sun [given] = 140×10^6 km → perihilion

  max. distance from sun [given] = 152×10^6 km → Aphelion

- create function to calculate orbital paths,

$$\cancel{\frac{x^2}{a^2} + \frac{y^2}{b^2}} = 1$$

- mars distances:

  min from sun: 206×10^6 km → perihelion

  max from sun: 249×10^6 km → aphelion.

$$\underbrace{\frac{(x+K)^2}{a^2} + \frac{(y+H)^2}{b^2}}_{\text{for Horizontal}}$$

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· Travel per day (~~xx~~ konstant in program)

~~Earth ≈ 2,8 × 10^6 km~~
~~Mars:~~
~~- planet speed: V_p ($\frac{km}{s}$)~~
~~- Time = 24h = ~~ ~~= 86400 s.~~
~~- distance = $V_p \cdot$ Time = $\frac{V_p}{86400 s}$~~

n-daily motion in degrees.

· Date of origin: 1. 1. 2020.

~~- any date entered before 1.1.2020 will have the position calculated with -distancePerDay × numberOfDays.~~
~~- Days after will be calculated with distancePerDay numberOfDays.~~

· Output will be ploted on a graph and the positions of the planets will be marked with circles. (Red for mars and blue for earth). The sun will be marked by a yellow star on the graph. The heading of the graph will be the date of the intended launch. On this date, Earth and mars will not have a minimum distance between them. The launch date shall be the day to launch the rocket from the moon, so that it reaches the location where Mars will be so that the rocket travels for a minimum distance and still reaches its destination: Mars.

· Assumptions made:

→ The earth and mars ~~xxxxx~~ both have their major axis on the y=ø ~~point~~. graph.

~~→ on 01.01.2020, Earth was at its perihelion.~~
~~→ on 08.04.2020~~

· All distance values are in AU- Astronomical Units

· All rotational units are in degrees.

~~Earths orbital~~

Orbital Slingshot system:

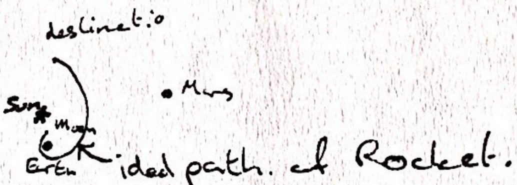
- ~~Origin will be the centre of the Rocket projektory around the Earth.~~ Origin at the sun.
- The projektory around the earth will be the same as the orbital path of the Moon around the Earth.
- Distances:

  min: 363,104 km → perijee

  max: 405 696 km → apogee
- Moons eccentricity:

  - 0,0549.
- ~~speed calculated using the Keplers 3rd Law~~
- Calculate Transfer orbit with sun as 1 Focus and another focus being ~~0,26~~ AU away from it.

  ↳ Astronomical Unit



  $1 AU = 1{,}496 \times 10^8$

  major axis of Transfer orbit is distance between Earth and ~~...~~ where the Rocket will meet Mars. ✓

  can be done using simultaneous equations of the two orbits.
- calculate interception between Transfer orbit and projektory.
- using Keplers 3rd Law, calculate Period it will take for the Rocket to reach the Transfer orbit (at this point, engines will need to be used to change course to the Transfer orbit) and then how much longer it will take for the Rocket to reach ~~...~~ Mars. ✓

orbit for earth

$b^2 = a^2(1-e^2)$

a = perihilion + aphelion

$= (140+152) \times 10^6$

$= 292 \times 10^6$

$b = \sqrt{(292 \times 10^6)^2 (1 - 0{,}0167^2)}$

$= 291959279{,}2$ → will be saved in program

∴ equation is: $\dfrac{x^2}{(292 \times 10^6)^2} + \dfrac{y^2}{(291959279{,}2)^2} = 1$

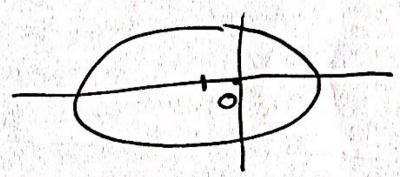
$60s = min$

$60(60s) = h$

$h = 3600\,s$

$s = \dfrac{h}{3600}$

Probe: $1 = \dfrac{x^2}{16} + \dfrac{y^2}{4}$



$a =$

$b =$

$x_0 =$ } centre

$y_0 =$

$e =$

$x = x0 + a \times \cos e$

$y = yo + b \times \sin e$

$km = 1000m$

$v = \sqrt{Gm \cdot \left(\frac{2}{r} + \frac{1}{a}\right)}$

circumference $p = \pi \left[3(a+b) - \sqrt{(3a+b)(a+3b)} \right]$

$AU = 149597871\,km$

$km = \dfrac{AU}{149597871}$

$e = \sqrt{1 - b^2/a^2}$

$e^2 = 1 - \dfrac{b^2}{a^2}$

$\dfrac{b^2}{a^2} = 1 - e^2$

$b^2 = a^2(1-e^2)$

$b = \sqrt{a^2(1-e^2)}$

$\dfrac{km}{s} = \dfrac{AU}{149597871\,s}$

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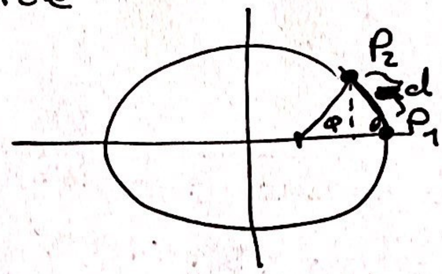
inteception

$$f(x) = g(x)$$

$g(x)$ = orbital path

$f(x)$ = circle



$$g(x,y) = \frac{(x - S_x)^2}{a^2} + \frac{y^2}{b^2} - 1$$

$$f(x,y) = \frac{(x+X)^2 + (y+Y)^2}{r^2} - 1$$

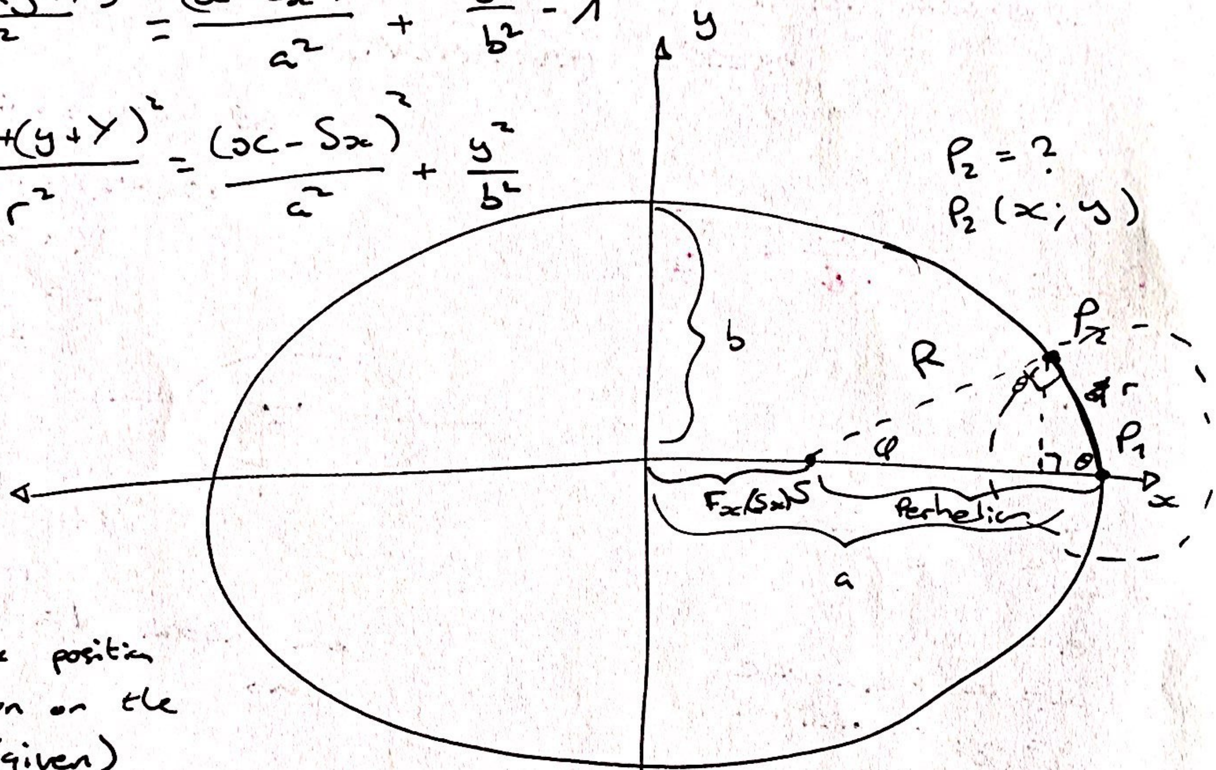
$$f(x,y) = g(x,y)$$

$$-1 + \frac{(x+X)^2 + (y+Y)^2}{r^2} = \frac{(x - S_x)^2}{a^2} + \frac{y^2}{b^2} - 1$$

$$\frac{(x+X)^2 + (y+Y)^2}{r^2} = \frac{(x - S_x)^2}{a^2} + \frac{y^2}{b^2}$$

$P_2 = ?$

$P_2\,(x;y)$



S_x is the position of the sun on the x axis (given)

$F_x = a -$ Perihelion

R is a tangent to circle around P_1

$$1 = \frac{x + S_x}{a} + \frac{y}{b}$$

$$r^2 = (x+X)^2 + (y+Y)^2$$

$$1 = \frac{(x+X)^2 + (y+Y)^2}{r^2}$$

$(X;Y)$
Position of centre of circle P_1

r is given

X is given

Y is given

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$$\frac{(x+X)^2+(y+Y)^2}{r^2} = \frac{(x+S_x)^2}{a^2} + \frac{y^2}{b^2}$$

$$\left((x+X)^2+(y+Y)^2\right)a^2b^2 = (x+S_x)^2 \cdot r^2 \cdot b^2 + y^2 \cdot a^2 \cdot r^2$$

$$a^2b^2(x+X)^2 + \cancel{2}a^2b^2(y+Y)^2 - (x+S_x)^2 r^2 b^2 - y^2 \cdot a^2 \cdot r^2 = 0$$

get Planet Position (r, X, Y, Fx, Fy, ...

$$\frac{(x+X)^2}{a^2} + \cancel{\frac{(y+Y)^2}{b^2}} = 1 \qquad y = \varnothing$$

$$x + X = a$$

$$x = a - X$$

$$\frac{(x-1)^2}{2} + y^2 = 2$$

$$y = \sqrt{2 - \frac{(x-1)^2}{2}}$$

~~$a^2b^2(x^2+2xX+X$~~

$$x = \frac{1}{b^2(a^2-r^2)} \cdot \sqrt{(-a^2b^2(a^2b^2X^2 + 2a^2b^2Yy + a^2b^2y^2 - a^2r^2y^2 - b^2X^2r^2 - 2b^2Yr^2y - b^2k^2r^2 - 2b^2kr^2S_x - b^2r^2S^2 - b^2r^2y^2 + r^4y^2)) - a^2b^2k - b^2r^2s)}$$

$$1000\,m = km$$

$$m = \frac{km}{1000}$$

$$\frac{m}{s} = \frac{km}{1000\,s}$$

$60\,s = min$

$60 \cdot 60 = 1h$

$24h = 1\,day$

$24 \cdot 3600$

$$\cancel{M =} \frac{kg\,m}{s^2} \cdot \frac{m^2}{kg} = \frac{m^3}{s^2}$$

$$\frac{m}{s} = \frac{m^3}{s^2} \cdot kg : m$$

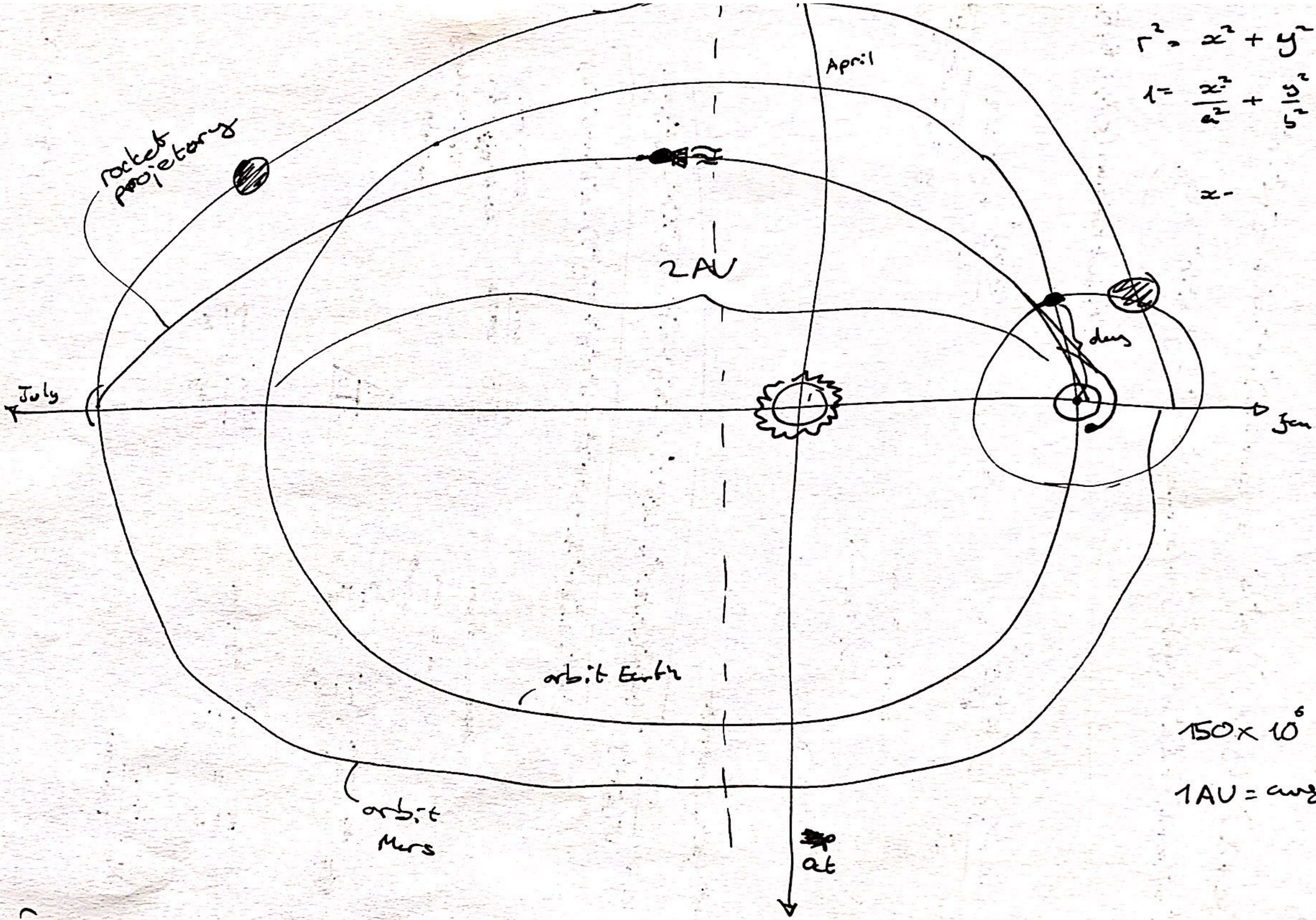
$$\frac{N m^2}{kg} \cdot \frac{kg}{m}$$

$$N \cdot \frac{km^2}{1000000}$$

$$r^2 = x^2 + y^2$$

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$x-$



150×10^6 km

1AU = avg

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position of day $\frac{2}{365}$: $\pm(-0{,}98265 \mid -0{,}01748)$

calculating position: $f(x,y) = g(x,y)$

- work with funktions $\Rightarrow$ half of the ~~semicircle~~ circle/ellipse

- $y = \sqrt{r^2 - (x - X)^2} + Y$

(X, Y) being the centre of circle

- $y = \sqrt{b^2 - \frac{(x-k)^2}{a^2}} + h$

(k, h) being the centre of the elipse, a the semi major axis and b the semi minor axis.

$r = 0{,}0175 \rightarrow A$

$X = 0{,}9833 \rightarrow B$

$Y = 0 \rightarrow C$

$k = 0{,}0167 \rightarrow D$

$h = 0 \rightarrow E$

$a = 1{,}\cancel{820} \rightarrow F$

$b = 0{,}9999 \rightarrow X$

saved pos. in Calculator

solving for y:

ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{y^2}{b^2} = \frac{a^2}{a^2} - \frac{x^2}{a^2}$

$\frac{y}{b} = \frac{\sqrt{a^2 - x^2}}{a}$

$y = \frac{b}{a}\sqrt{a^2 - x^2}$

circle: $\frac{x^2 + y^2}{a^2} = 1$

$x^2 + y^2 = a^2$

$y^2 = a^2 - x^2$

$y = \sqrt{a^2 - x}$

$360^\circ = 2\pi$ rad

$180^\circ = \pi$ rad

$1^\circ = \frac{\pi}{180}$ rad.

$b = \frac{ya}{\sqrt{a^2 - x^2}}$

$\frac{y^2}{b^2} = 1 \cdot \frac{-x^2}{a^2}$

$b^2 = y^2 / \frac{-x^2}{a^2}$

$= \frac{y^2 a^2}{x^2}$

$b = \frac{ya}{x - x_0}$

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Mit CamScanner gescannt

Raket projectory.

1. ~~ea~~ from earth to mars
2. from moon to ~~orbital~~ projektory.

1. using Hohmann transfer
   - leaves earth at the projektory's perihilion
   - arive @ Mars at the orbits aphelian

- ~~Keplers second Law: the Law of Areas~~
- Assumption: Earth and Mars' orbits are on the same plane
- Find projektory A (semi-major axis) problem
- Find location of second Focus (1. Focus is the sun)
  - calculate focal distance.

$c = \sqrt{a^2 - b^2}$

$e = \frac{\sqrt{a^2-b^2}}{a}$

$e = \frac{c}{a} \Rightarrow c = e \cdot a$ $\quad a^2 = b^2 + c^2$ $\quad b = \frac{ay}{\sqrt{a^2 - x^2}}$

c - Focal distance $\quad$ e - eccentricity.
a - semi-major axis
b - semi-minor axis

- Keplers 3rd Law to calculate period of Transfer and Mars travel time along its orbit.
  - $p^2 = k \cdot a^3$ $\quad$ p - Period
    k - Proportionality konstant
    a - semi-major axis

- use motions of earth and mars to calculate the ideal relective position of Earth and Mars.
  - calculate degrees of movement for every day (earth and mars) save it in a vektor. φ = degree of movement per 24h.
  - calculate position of Mars at the time of launch, subtract the amount of its motion during the spacecrafts travel
  - ~~assumption: the spacecraft will take 259 earth days to travel the to the aphelian of its projektory.~~

- ~~Assumption: earth is at its perihelion on Jan 1.~~
- rocket speed = $10000 \frac{km}{h}$ $\quad$ h = 3600 s $\quad$ (b)

2. from moon to projektary

- calculate position of moon ✓
- calculate intersection of moons orbit and the ✓
  projektary to mars.
- Assumption: moons orbit is on the same plane as
  the earths, and Mars'. ✓
- find singshot A (semi-major axis) ✓
- find location of second focus (1. is the earth) ✓
- use Keplers Third law to calculate Period ✓
  of slingshot, and the period of time to get
  to the point of interception without the slingshot
  and add it to the period of the Transfer
- find directional ~~kan~~ vektor for launch from moon.

3. Finishing Touches:
- add legend to display: → journy Period, ~~earth date of launch~~
  ~~and~~ → earth date of launch & arrival.

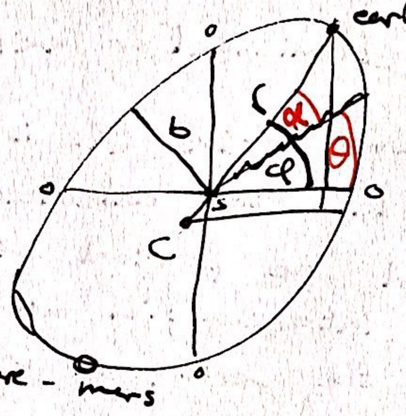
	Day	Launch	Land
earth			
mars			

Matrix days.

$$1 = \frac{(x+x_0)^2}{a} + \frac{(y+y_0)^2}{b}$$

$$y + y_0 = \frac{b}{a}\sqrt{a^2-(x-x_0)^2}$$

$$y_0 = \frac{b}{a}\sqrt{a^2-(x-x_0)^2} - y$$



$\varphi = \alpha + \theta$

$r = a$

α - True anomaly

S - $(0;0)$ θ - rotation.

↳ Focus.

C - centre of ellipse.

φ = True anomaly

$_cX = r\cos\varphi$ - earth$(x,0)$

$_cY = r\sin\varphi$ - earth$(0,y)$

↳ = earth$(0,\varphi)$

↳ earth$(r;\varphi)$ + proj C

daily motion of rocket in degrees

d. Launch Day = day $\emptyset$

l - mean longitude

p - perihelion.

projA =

proj Focus 1 = sun $(0;0)$

~~proj focus 2~~ = $(_cX, _cY)$

centre Proj

$x = r \cdot \cos(\varphi) + X_0$

$y = r \cdot \sin(\varphi) + Y_0$

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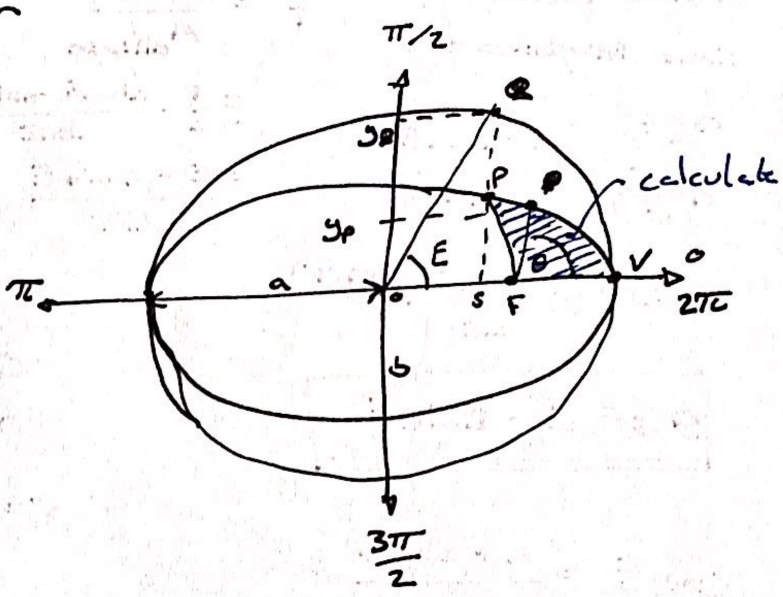
Keplers Equation Derivation



time = t

orbital period = T

P is position at t

F - Focus

θ - true anomaly

$\theta(T) = 2\pi = \phi$

E - eccentric anomaly

ε - eccentricity

- calculate area of QSV
- subtract area of Δ OQS
- Subtract area of QPV Δ
- Subtract area of Δ SPF

equation for ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y = \frac{b}{a}\sqrt{a^2 - x^2}$

equation for circle: $\frac{x^2 + y^2}{a^2} = 1 \Rightarrow y = \sqrt{a^2 - x}$

$y_Q = \sqrt{a^2 - x^2}$

$y_P = \frac{b}{a}\sqrt{a^2 - x^2} = \frac{b}{a} y_Q$

area of ~~QSV~~ QOS $= \frac{1}{2} a^2 \cos E \cdot a \sin E$

$A_{QSV} = A_{QOV} - A_{QOS}$

$= \frac{1}{2} a^2 E - \frac{1}{2} a^2 \cos E \sin E$

$= \frac{1}{2} a^2 (E - \cos E \sin E)$

$A_{PSV} = \frac{1}{2} ab\, (E - \cos E \sin E)$

$A_{PSF} = \frac{1}{2} (a(\varepsilon - \cos E)\, b \sin E$

$= \frac{1}{2} ab (\varepsilon - \cos E)(\sin E)$

$= \frac{1}{2} ab(\varepsilon \sin E - \cos E s$

$A_{PFV} = A_{PSV} - A_{PSF}$

$= \frac{1}{2} ab(E - \cos E \sin E) - \frac{1}{2} ab(\varepsilon \sin E - \cos E \sin E)$

$= \frac{1}{2} ab\,(E - \varepsilon \sin E)$

orbital period $= T$

time ~~length~~ $= t$

$\frac{t}{T} = \frac{A_{FVP}}{A_{ellipse}}$

$= \frac{1}{2} \cdot \frac{ab(E - \varepsilon \sin E)}{ab\pi}$

$= \frac{E - \varepsilon \sin E}{2\pi}$

$M = \theta(t)$

$= E - \varepsilon \sin E$

$M = \frac{t}{T} 2\pi$

$\frac{\theta(t)}{\theta(T)} = \frac{E - \varepsilon \sin E}{2\pi}$

$\theta(t) = \left(\frac{E - \varepsilon \sin E}{2\pi}\right)\theta(T)$

$\boxed{\theta(t) = E - \varepsilon \sin E}$

$\overline{OS} = a \cos E$

$OF = a\varepsilon$

$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta}$

$\cos E = \frac{\varepsilon + \cos\theta}{1+\varepsilon\cos\theta}$

$OF = OS - SF$

$a\varepsilon = a\cos E - r\cos\theta$

$\cos\theta = \frac{\varepsilon - \cos E}{\varepsilon\cos E - 1}$

The Rocket Equation:

$p = mv = \underbrace{dm}_{\text{exhaust mass}}(v - \underbrace{v_e}_{\text{exhaust velocity}}) = (\overbrace{v + dv}^{\text{new velocity}})(\overbrace{m + dm}^{\text{new mass}})$

$= dm\vec{v} - dm\vec{v_e} = \vec{v}m + dm\vec{v} + md\vec{v} + \underbrace{d\vec{v}dm}_{\text{infinitesimally small}}$

$\vec{p}(t+dt) = \vec{v}m + dm\vec{v}_e + md\vec{v}$

$\vec{p}(t) = m\vec{v}$

$d\vec{p} = p(t+dt) - p(t) = dm\vec{v}_e + md\vec{v} = 0$

$\rightarrow d\vec{v}m = dm\vec{v_e}$

$\underbrace{m\frac{d\vec{v}}{dt}}_{ma} = \underbrace{-\frac{dm}{dt}v_e}_{\text{thrust}}$

$\vec{v} - \underbrace{\vec{v}_0}_{\text{initial } v} = \vec{v}_e \ln\left(\frac{\overbrace{m_0}^{\text{initial mass}}}{m}\right)$

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Calculating Position:

v - true anomaly (deg)

M - mean anomaly (deg)

e - eccentricity

a - semi-major axis

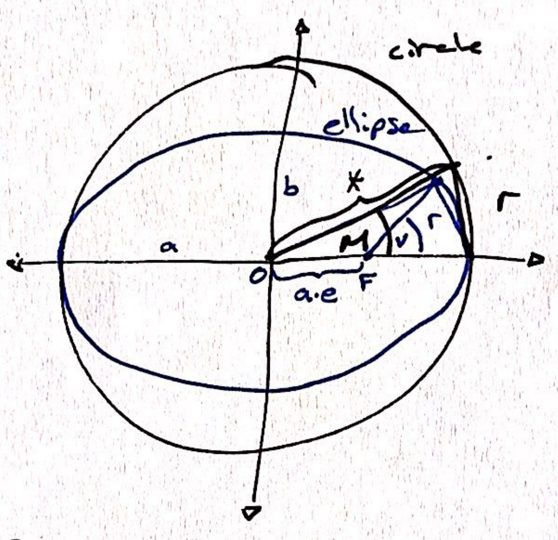
b - semi-minor axis.

$$v = M + \frac{180}{\pi} \cdot \left(\left(2 \cdot e - \frac{e^3}{4} \right) \sin(M) + \frac{5}{4} \times e^2 \cdot \sin(2M) + \frac{13}{12} \times e^3 \sin(3M) \right)$$

* avg distance from sun. = 1AU

the idea:

~~Mars M~~ = 25



$M = n \times d + L - p$

n - daily motion

d - days sine date of elements (on. 20.08.1997) ~~(Now 21st June 1997)~~

$y = \emptyset : \quad \frac{(x^* - x_0)^2}{a^2} = 1$

$x_2 = a + x_0$

$x_0 = x - a$

$x_0 = 0{,}1 = +1{,}016 - a$

$a = -0{,}1 + 1{,}016$

$xC = (r + 0{,}21) \times \cos(v)$

$yC = (r + 0{,}21) \times \sin(v)$

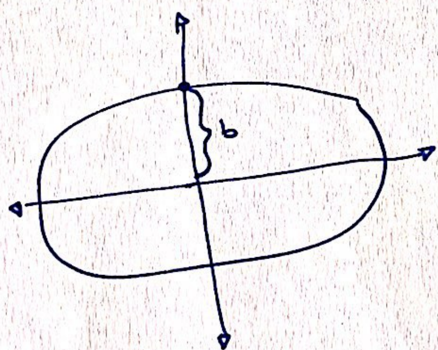
projCentere = $[xC; xY]$

$x_0 = 0{,}26 = 1{,}016 - a$

$a = 1{,}016 - 0{,}26$

$b \sin(t) = y + y_0$

$b = \frac{y + y_0}{\sin(t)}$



$r \times \sin 90°$

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