

HW1

1. Use induction to prove that

$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2, \quad n \in \mathbb{N}.$$

2. Is $\sqrt{6}$ irrational? Give the proof.
- 3*. Use induction to prove that $11^{n+1} + 12^{2n-1}$ is divisible by 133 for every $n \in \mathbb{N}$.
4. Reduce the following expressions for complex numbers to canonical forms

$$\frac{1-i}{1+i}, \quad (1+i\sqrt{2})^2, \quad \overline{\left(\frac{i}{1-i}\right)}.$$

5. Find the absolute value and the argument of the complex numbers:

$$-1 + i, \quad 2 - 5i$$

6. What points z on the complex plane \mathbb{C} satisfy the following condition:
 $\{z : |z - i| \leq 1\}$? Draw the picture.

Remark. Each problems is worth 1 point, the problems with the star are worth 2 points.