Micro Multiobjective Evolutionary Algorithm with Piecewise Strategy for Embedded-processor-based Industrial Optimization: Supplementary Material

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I. Supplement to related work

The difference in the weight update frequency of the decomposition-based algorithm is described in detail in the main text. In order to summarize, a summary table is given in the supplementary material, as shown in Table S - I below.

II. Computational Complexity Analysis

As for μMOEA, the piecewise strategy and the weight vector update trigger mechanism govern the computational complexity. Therefore, this part analyzes these two major components respectively.

Initially, what must be prioritized is the key piecewise strategy, which costs $O(n\tau)$, where n denotes the size of the evolutionary population, τ is the time slice. Then, the weight update costs $O(pMN^2)$, where p is the probability of triggering the weight update, M is the number of objectives and N is the size of the archive. Therefore, the total time complexity of μMOEA is $O(n\tau + pMN^2)$.

The space complexity of μMOEA is also very worthy of discussion, which is mainly determined by the weight vectors, the neighborhood set, and the archive. Therefore, the total space complexity is $O(N(M+T+D))$.

III. Sensitivity analysis of parameters

In μMOEA, timeslice τ, theoretical convergence minimum ξ^*, the max flag of convergence δ, and tolerance rate γ are key parameters. To analyze the effect of these parameters on μMOEA, μMOEA was performed on the comprehensive experimental suites ZDT and DTZ. In this experiment, the archive size N, the population size n, and the neighborhood size T are set to 20, 5, and 5, respectively. In order to eliminate statistical errors, μMOEAs with different parameter values are run independently for 30 times. Due to the limitation of the number of pages, the main text only shows the statistical data.

To analyze the sensitiveness of τ, we test μMOEA with different values of parameter τ (e.g., τ=2, τ=3, τ=4, τ=5, τ=6) and set ξ^*, δ, γ to 1.00E-4, 0.06, 0.4 respectively. The experimental result is shown in the first row of Table S - II, where the rankings are obtained by the Friedman test. To make the results obvious, we highlight the best result in bold. As presented in the first row of Table S - II, μMOEA with τ=4 achieves the best. Therefore, for all subsequent experiments, τ is set to 4.

To explore the sensitiveness of ξ^*, we test μMOEA with different values of parameter ξ^* (e.g., ξ^*=8E-3, ξ^*=9E-3, ξ^*=1E-4, ξ^*=2E-4, ξ^*=3E-4) and other parameters are consistent with the above part for parameter τ. The experimental result is shown in the second row of Table S - II and the best result is highlighted in bold. From the second row of Table S - II, it can be seen that μMOEA with ξ^*=1E-4 gets the best performance. Therefore, for all subsequent experiments, ξ^* is set to 1E-4.

In addition, we experiment μMOEA with different values of parameter δ (e.g., δ=0.04, δ=0.05, δ=0.06, δ=0.07, δ=0.08) to analyze the sensitiveness of δ. Other parameters remain the same as the above experiments. The experimental results can be seen in the third row of Table S - II, from which it can be clearly found that μMOEA with δ=0.06 ranks best. Therefore, for all subsequent experiments, δ is set to 0.06.

Lastly, in order to study the sensitivity of γ, we test μMOEA with different values of parameter γ (e.g., γ=0.2, γ=0.3, γ=0.4, γ=0.5, γ=0.6). Other parameters remain the same as the above experiments. The results are shown in the fourth row of Table S - II, where μMOEA achieves the best performance when γ=0.4. Thus, for all subsequent experiments, γ is set to 0.4.

IV. Summary of Comparison Algorithms

In this section, 12 excellent algorithms are selected as the comparison algorithms for the artificial optimization problems, and their specific parameter settings can be shown in Table S - III.

V. Supplement to experiments on artificial test problems

A. Performances on bi-objective and tri-objective problems

The more detailed results on DTLZ and ZDT problems are given in Table S - IV, Table S - V, Figure S - 1, Figure S - 2 and Figure S - 3. Table S - IV and Table S - V show the IGD of μMOEA with other comparison algorithms. The value in parentheses in each table item indicates the standard deviation. Figure S - 1 and Figure S - 2 show the distribution of the

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Table S - I. Summary of decomposition-based MOEA for weight vector update frequency

Algorithm	Update frequency	Description
MOEA/D-AWA [1]	periodically	every *wag* generation after $rate\_evol * maxGen$ generation
MOEA/D-URAW [2]	periodically	every *wag* generation after $rate\_evol * maxGen$ generation
MOEA/D-ABD [3]	periodically	every 0.1**maxGen* generation after $0.5 * maxGen$
AdaW [4]	periodically	every 5% generation until the last 10% generation
$pa\lambda$-MOEA/D [5]	once	-
MOEA/D-TPN [6]	once	-
MOEA/D-RWV [7]	conditional	the degree of deviation of the solution
μMOEA	conditional	the degree of crowding after the solution converges

Table S - II. Average ranks of μMOEA with different value of τ, ξ^*, δ and γ

Parameters		Values				
τ	Value	2	3	4	5	6
	Rank	2.58	3.63	**2.13**	3.25	3.42
ξ^*	Value	8E-3	9E-3	1E-4	2E-4	3E-4
	Rank	2.75	3.46	**2.71**	3.21	2.88
δ	Value	0.04	0.05	0.06	0.07	0.08
	Rank	3.33	2.75	**2.71**	3.04	3.17
γ	Value	0.2	0.3	0.4	0.5	0.6
	Rank	3.67	3.00	**2.42**	2.58	3.33

Table S - III. Comparison algorithms and their descriptions and parameter settings.

Algorithm	Parameter setting
MOEA/D [8]	$T = 5$
MOEA/D-AWA [1]	T=5, *rate_evol*=0.8, *wag*=100, *rate_update_weight*=0.05
IDBEA [9]	
IBEA [10]	$\kappa = 0.05$
NSGAII [11]	
SPEAII [12]	
RVEA [13]	α=2, fr=0.1
PESAII [14]	*div*=10
AMGA [15]	
AMGAII [16]	
μGA [17]	*replacement_cycle*=30, *adaptive_grid* = 25, *convergence_criterion*=3
μMOGA [18]	*convergence_criterion*=3
μMOEA	$F = 0.5$, $CR = 0.5$, T=5, τ=4, ξ^*=1.00E-04, δ=0.06, γ=0.4

final non-dominated solutions obtained by μMOEA in several representative problems. Figure S - 3 illustrates convergence curves of μMOEA with other comparison algorithms on DTLZ1, DTLZ4, ZDT2, and ZDT3.

B. Performances on many-objectives problems

Due to the page limit, only the statistical results are given in the original text.Due to the page limit, only statistical results are given in the original text. In order to comprehensively show the experimental results, this section supplements the parallel coordinate diagram and more detailed experimental results.

Parallel coordinates diagram are attached in Figure S - 4 and 5. Moreover, the more detailed results are shown in Table S - VI, VII, VIII and IX.

Based on the above experimental results, it can be found that most algorithms are difficult to converge to the Pareto front, and the quality of the solution set is not very good. However, μMOEA has better distribution and convergence in DTL4 and MaF3 compared to other algorithms.

C. Performances on MOPs with large-scale sparse problems

In order to show the experimental results more clearly, Figure S - 6, Table S - X, and Table S - XI shows the comparison between other algorithms and μMOEA in detail. Special description for Figure S - 6, the use of dark blue indicates the number of problems that μMOEA is inferior compared with other algorithms. green indicates the merit quantity. grey means equal quantity. This bar chart clearly shows how μMOEA compares with other algorithms. It is not difficult to find that the green bars are the most. Therefore, our algorithm performs well on SMOPs.

D. Validity on normal size population

The original text shows the overall statistics, which still show that μMOEA is slightly better than other algorithms. The details of the experimental results, are also given in this section, which can be found in Table S - XII and Table XIII.

E. Validity on weight vectors trigger

In the validity verification of the weight vector update trigger in the original text, MOEA/D-AWA [1] is chosen as the

Table S - IV. IGD of μMOEA with MOEA/D, MOEA/D-AWA, IDBEA, IBEA, NSGAII and SPEAII on ZDT and DTLZ Problems.

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAII	SPEAII	μMOEA
DTLZ1	3	7	3.3629e-1 (1.99e-1) -	3.0217e+0 (1.43e+1) -	3.6875e-2 (1.86e-1) -	4.3851e-1 (2.77e-2) -	5.4047e-2 (6.15e-3) -	3.9458e-2 (3.64e-3) -	**3.8681e-5 (2.07e-4)**
DTLZ2	3	12	3.6158e-2 (4.41e-2) =	3.2061e-1 (1.75e-1) -	**3.8071e-5 (1.22e-5) +**	1.3633e-1 (1.67e-2) -	1.5406e-1 (1.97e-2) -	8.5383e-2 (1.14e-2) -	7.7364e-4 (3.50e-4)
DTLZ3	3	12	6.9504e+0 (2.89e+0) -	3.1690e+1 (1.24e+2) -	8.4141e+0 (1.98e+1) -	8.6752e-1 (1.50e-1) -	2.8622e-1 (3.06e-1) -	2.4317e-1 (5.57e-1) -	**1.7173e-1 (4.56e-1)**
DTLZ4	3	12	2.0906e-1 (1.11e-1) -	2.8593e-1 (3.43e-1) -	7.6959e-1 (2.62e-1) -	3.9292e-1 (3.46e-1) -	4.6927e-1 (4.06e-1) -	6.6335e-1 (2.91e-1) -	**2.3656e-2 (2.41e-2)**
DTLZ5	3	12	1.2890e-1 (8.36e-5) -	3.2805e-2 (9.08e-3) -	8.0160e-2 (6.85e-3) -	4.2841e-2 (5.48e-3) -	2.9998e-2 (3.06e-3) -	**2.0354e-2 (3.26e-3) +**	2.7196e-2 (2.37e-3)
DTLZ6	3	12	1.4696e-1 (1.02e-2) -	3.6431e-2 (1.19e-2) -	6.2332e-2 (1.24e-2) -	4.8818e-2 (7.47e-3) -	3.2266e-2 (3.83e-3) -	**1.8944e-2 (2.27e-3) +**	2.6933e-2 (2.25e-3)
DTLZ7	3	22	3.5820e-1 (1.15e-1) -	6.4368e-1 (1.55e-1) -	1.7176e+0 (2.42e+0) -	4.1332e-1 (2.53e-1) -	2.7081e-1 (1.12e-1) -	2.3385e-1 (1.24e-1) =	**1.8261e-1 (2.69e-2)**
ZDT1	2	30	3.0348e-2 (5.24e-3) -	2.1218e-2 (9.28e-4) -	2.3364e-2 (5.06e-3) -	2.8664e-2 (3.78e-3) -	2.4106e-2 (3.22e-3) -	1.9740e-2 (2.15e-3) =	2.0050e-2 (4.99e-4)
ZDT2	2	30	2.0332e-2 (2.85e-3) -	1.7408e-2 (6.70e-4) -	4.7182e-1 (5.31e-1) -	2.7872e-2 (3.67e-3) -	2.6707e-2 (3.05e-3) -	1.9047e-2 (2.34e-3) -	**1.6419e-2 (1.31e-4)**
ZDT3	2	30	7.5772e-2 (3.92e-2) -	4.4165e-2 (4.84e-2) =	7.1334e-2 (1.86e-2) -	4.7143e-2 (1.25e-2) -	3.7696e-2 (2.03e-2) =	3.1106e-2 (9.63e-3) =	**2.9499e-2 (1.89e-3)**
ZDT4	2	10	2.6429e-1 (9.12e-2) -	2.9635e-2 (3.38e-2) -	8.3270e-2 (1.76e-1) -	5.3316e-1 (8.90e-2) -	2.3575e-2 (3.15e-3) -	**1.8752e-2 (2.11e-3) =**	1.8816e-2 (1.56e-4)
ZDT6	2	10	2.8657e-2 (1.50e-2) -	1.7758e-2 (8.70e-4) -	2.2958e-1 (2.98e-1) =	2.0613e-2 (2.96e-3) -	2.3837e-2 (3.45e-3) -	**1.4610e-2 (2.40e-3) +**	1.7198e-2 (1.11e-3)
Average Ranking			8.42	7.46	9.00	8.25	6.50	2.96	**2.04**
+/-/=			0/11/1	0/11/1	1/10/1	0/12/0	0/11/1	3/5/4	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.

Table S - V. IGD of μMOEA with RVEA, PESAII, AMGA, μGA and μMOGA on ZDT and DTLZ Problems

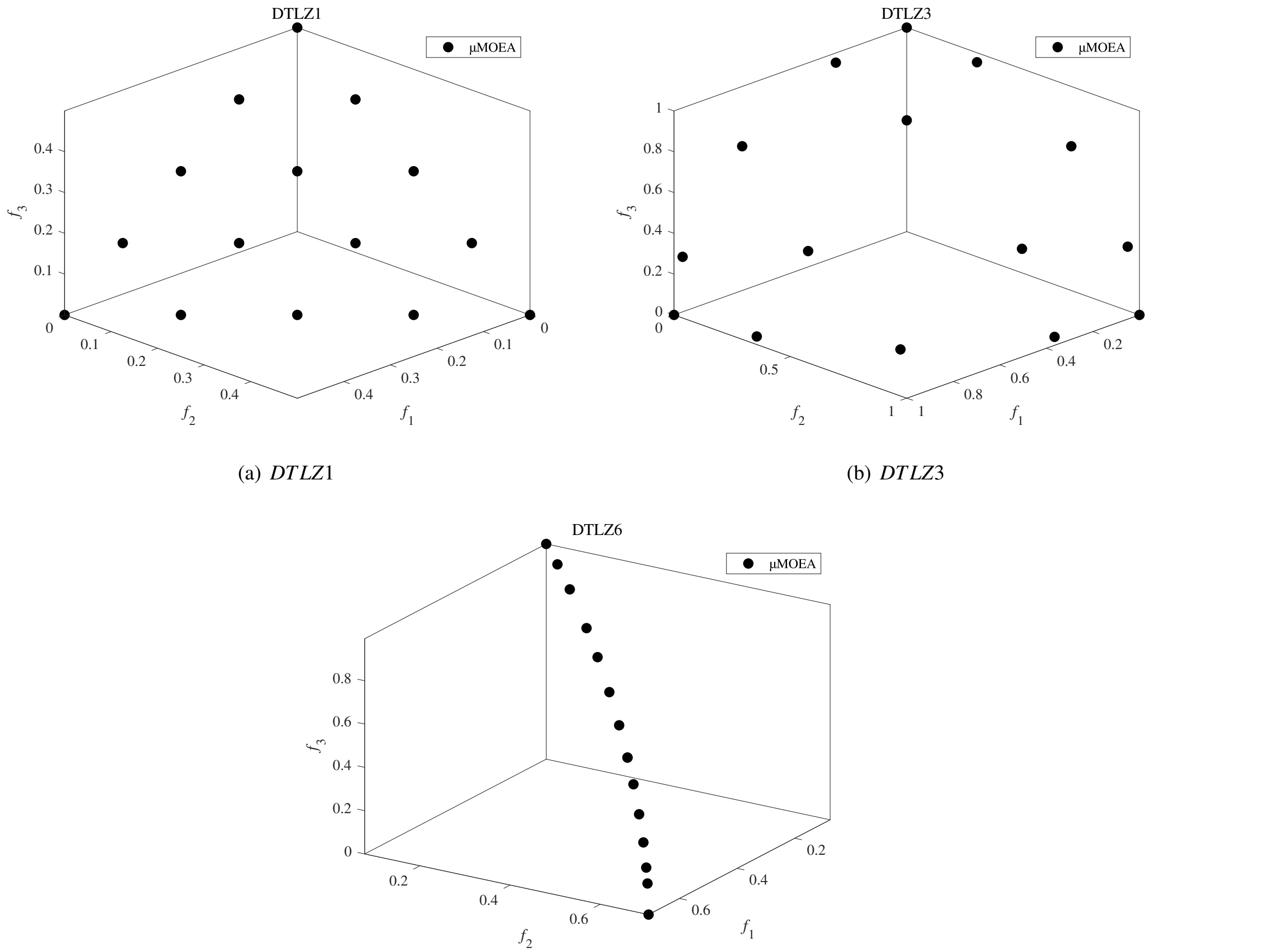
Problem	M	D	RVEA	PESAII	AMGA	AMGAII	μGA	μMOGA	μMOEA
DTLZ1	3	7	6.8678e-4 (6.65e-4) -	1.4672e-1 (3.20e-2) -	5.4166e-2 (5.89e-3) -	5.9763e-1 (6.42e-1) -	5.8720e-1 (7.99e-1) -	5.5530e-2 (4.73e-3) -	**3.8681e-5 (2.07e-4)**
DTLZ2	3	12	5.6687e-5 (2.83e-5) +	3.8008e-1 (6.35e-2) -	1.5181e-1 (1.25e-2) -	1.6033e-1 (1.57e-2) -	5.1857e-1 (8.81e-2) -	1.5132e-1 (1.58e-2) -	7.7364e-4 (3.50e-4)
DTLZ3	3	12	1.8779e-1 (4.71e-1) -	6.1763e-1 (3.59e-1) -	8.7104e-1 (1.09e+0) -	2.1239e+1 (2.31e+1) -	1.1841e+1 (9.24e+0) -	7.1327e-1 (7.86e-1) -	**1.7173e-1 (4.56e-1)**
DTLZ4	3	12	3.0650e-1 (3.13e-1) =	5.1668e-1 (3.93e-1) -	8.4874e-1 (2.82e-1) -	3.9589e-1 (3.56e-1) -	8.2259e-1 (2.39e-1) -	7.9401e-1 (3.33e-1) -	**2.3656e-2 (2.41e-2)**
DTLZ5	3	12	1.6549e-1 (4.99e-2) -	6.1080e-2 (3.47e-2) -	2.6495e-2 (3.42e-3) =	2.6428e-2 (3.54e-3) =	1.1039e-1 (5.41e-2) -	2.8107e-2 (3.08e-3) =	2.7196e-2 (2.37e-3)
DTLZ6	3	12	3.0030e-1 (1.57e-1) -	8.9336e-2 (6.82e-2) -	2.6406e-2 (3.41e-3) =	2.3715e-2 (3.28e-3) +	6.8609e-1 (9.97e-1) -	2.8581e-2 (3.00e-3) -	2.6933e-2 (2.25e-3)
DTLZ7	3	22	2.7501e-1 (8.90e-4) -	5.6877e-1 (2.35e-1) -	7.8522e-1 (1.77e-1) -	7.9859e-1 (1.80e-1) -	9.5215e-1 (1.93e-1) -	3.9248e-1 (2.36e-1) -	**1.8261e-1 (2.69e-2)**
ZDT1	2	30	**1.8597e-2 (3.81e-4) +**	1.4555e-1 (7.72e-2) -	2.3582e-2 (2.93e-3) -	2.2811e-2 (2.12e-3) -	2.0557e-1 (1.04e-1) -	2.4239e-2 (1.80e-3) -	2.0050e-2 (4.99e-4)
ZDT2	2	30	1.6536e-2 (3.59e-5) -	7.7986e-2 (5.58e-2) -	2.5105e-2 (3.39e-3) -	2.3315e-2 (2.88e-3) -	9.6861e-2 (4.15e-2) -	2.5094e-2 (3.20e-3) -	**1.6419e-2 (1.31e-4)**
ZDT3	2	30	3.9232e-2 (4.61e-3) -	7.6556e-2 (2.41e-2) -	4.3499e-2 (2.12e-2) -	5.6039e-2 (5.08e-2) -	1.5926e-1 (6.80e-2) -	3.6243e-2 (1.09e-2) -	**2.9499e-2 (1.89e-3)**
ZDT4	2	10	2.0096e-2 (1.52e-3) -	7.8296e-2 (4.71e-2) -	2.1890e-2 (2.86e-3) -	1.9841e-2 (2.68e-3) -	3.0877e-1 (8.08e-2) -	2.3003e-2 (2.55e-3) -	1.8816e-2 (1.56e-4)
ZDT6	2	10	1.4728e-2 (4.91e-5) +	2.9075e-2 (5.10e-3) -	1.7981e-2 (3.22e-3) =	1.7839e-2 (3.42e-3) =	2.2485e-2 (4.25e-3) -	2.2286e-2 (2.14e-3) -	1.7198e-2 (1.11e-3)
Average Ranking			4.42	9.67	6.88	6.88	11.83	6.71	**2.04**
+/-/=			3/8/1	0/12/0	0/9/3	1/9/2	0/12/0	0/11/1	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.

Table S - VI. IGD of μMOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAII and SPEAII on 5-objective problems DTLZ

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAII	SPEAII	μMOEA
DTLZ1	5	9	3.0167e-1 (8.17e-2) -	**8.2887e-2 (6.21e-2) =**	4.7394e+0 (4.24e+0) -	5.1382e-1 (1.25e-2) -	1.6823e-1 (1.32e-1) -	1.5230e-1 (1.49e-1) -	1.0543e-1 (1.82e-1)
DTLZ2	5	14	4.8339e-1 (1.13e-1) -	1.1154e-1 (1.45e-1) =	**3.8034e-2 (2.07e-1) +**	4.3014e-1 (1.16e-1) -	3.0980e-1 (1.82e-2) -	4.7155e-1 (1.55e-1) -	1.1037e-1 (7.41e-2)
DTLZ3	5	14	5.1701e+0 (2.61e+0) -	5.2382e+0 (5.81e+0) -	6.8623e+0 (2.15e+1) -	1.3004e+0 (3.58e-1) -	1.2948e+1 (6.11e+0) -	5.8478e+1 (1.88e+1) -	1.0806e+0 (1.03e+0)
DTLZ4	5	14	4.1045e-1 (7.16e-2) -	1.3750e-1 (1.34e-1) -	9.9475e-1 (2.15e-1) -	8.2703e-1 (2.05e-1) -	3.2687e-1 (2.27e-2) -	8.8347e-1 (5.18e-1) -	**1.3431e-1 (4.01e-2)**
DTLZ5	5	14	**9.6405e-2 (5.68e-2) +**	1.6860e-1 (6.73e-2) =	7.4035e-1 (7.82e-5) -	2.7614e-1 (1.24e-1) -	4.1741e-1 (1.12e-1) -	2.3840e+0 (7.28e-2) -	1.8094e-1 (4.12e-2)
DTLZ6	5	14	**1.3789e-1 (6.21e-2) +**	1.9417e-1 (6.46e-2) =	3.2814e-1 (1.79e-1) -	3.2099e-1 (9.90e-2) -	4.9048e+0 (8.05e-1) -	9.9950e+0 (3.20e-2) -	2.0130e-1 (1.11e-1)
DTLZ7	5	24	7.4932e-1 (1.42e-1) +	9.7811e-1 (9.08e-2) -	4.1178e+0 (3.57e+0) -	1.4565e+0 (5.99e-1) -	6.9203e-1 (3.18e-2) +	**6.1580e-1 (4.60e-2) +**	8.5605e-1 (1.09e-1)
Average Ranking			4.57	3.29	8.71	6.57	5.43	8.57	**2.71**
+/-/=			3/4/0	0/3/4	1/6/0	0/7/0	1/6/0	1/6/0	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.



(a) $DTLZ1$

(b) $DTLZ3$

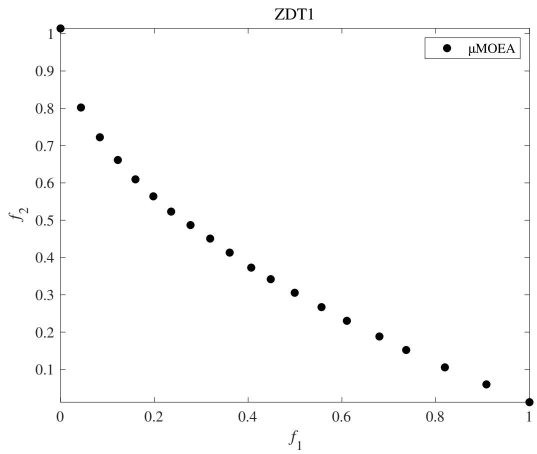
(c) $DTLZ6$

Figure S - 1. The distribution of the final non-dominated solutions by μMOEA on DTLZ. (a) DTLZ1. (b) DTLZ3. (c) DTLZ6.

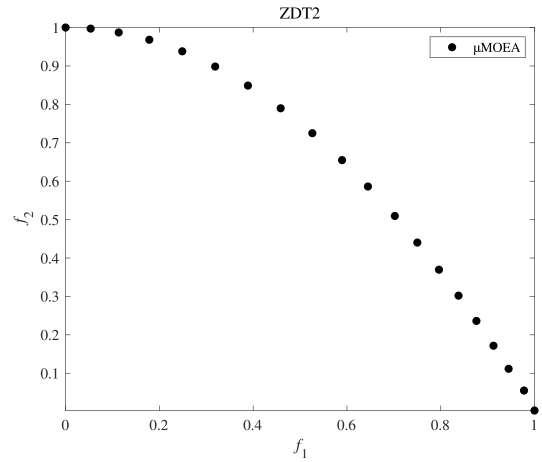
Table S - VII. IGD of μMOEA with RVEA, PESAII, AMGA, AMGAII, μGA and μMOGA on 5-objective problems DTLZ

Problem	M	D	RVEA	PESAII	AMGA	AMGAII	μGA	μMOGA	μMOEA
DTLZ1	5	9	1.7127e-1 (8.65e-2) -	1.1823e+0 (1.17e+0) -	1.9947e-1 (1.23e-1) -	5.9121e+0 (2.20e+0) -	1.0941e+0 (1.26e+0) -	3.4354e-1 (2.70e-1) -	1.0543e-1 (1.82e-1)
DTLZ2	5	14	5.6381e-1 (3.70e-1) -	8.1259e-1 (5.01e-2) -	3.0686e-1 (1.94e-2) -	5.0384e-1 (2.90e-2) -	8.5216e-1 (9.93e-2) -	3.1140e-1 (2.52e-2) -	1.1037e-1 (7.41e-2)
DTLZ3	5	14	**1.0743e+0 (1.86e+0) =**	1.4342e+1 (1.47e+1) -	1.7186e+1 (1.41e+1) -	1.2263e+2 (1.87e+1) -	2.4819e+1 (1.49e+1) -	2.2751e+1 (1.28e+1) -	1.0806e+0 (1.03e+0)
DTLZ4	5	14	3.9854e-1 (2.56e-1) -	5.3132e-1 (6.12e-2) -	7.1955e-1 (4.24e-1) -	9.5360e-1 (2.23e-1) -	9.7962e-1 (1.57e-1) -	4.8344e-1 (3.33e-1) -	**1.3431e-1 (4.01e-2)**
DTLZ5	5	14	5.1119e-1 (2.44e-1) -	8.6620e-1 (2.37e-1) -	4.0998e-1 (1.19e-1) -	5.9841e-1 (1.51e-1) -	5.6735e-1 (3.03e-1) -	4.9949e-1 (1.78e-1) -	1.8094e-1 (4.12e-2)
DTLZ6	5	14	4.4436e-1 (1.84e-1) -	6.6955e+0 (1.41e+0) -	3.9385e+0 (1.35e+0) -	1.3011e+0 (4.55e-1) -	6.5046e+0 (1.64e+0) -	5.1305e+0 (8.48e-1) -	2.0130e-1 (1.11e-1)
DTLZ7	5	24	9.0465e-1 (1.05e-1) =	2.5523e+0 (3.07e-1) -	6.4978e-1 (2.82e-2) +	1.2229e+0 (3.98e-1) -	2.6841e+0 (3.44e-1) -	6.8590e-1 (3.59e-2) +	8.5605e-1 (1.09e-1)
Average Ranking			6.00	10.43	6.00	10.43	11.14	7.14	**2.71**
+/-/=			0/5/2	0/7/0	1/6/0	0/7/0	0/7/0	1/6/0	

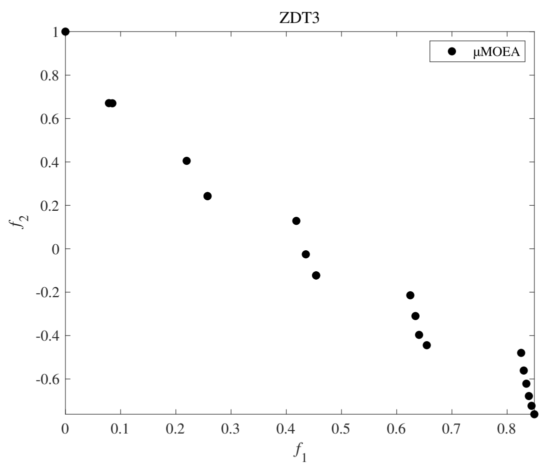
M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.



(a) $ZDT1$



(b) $ZDT2$



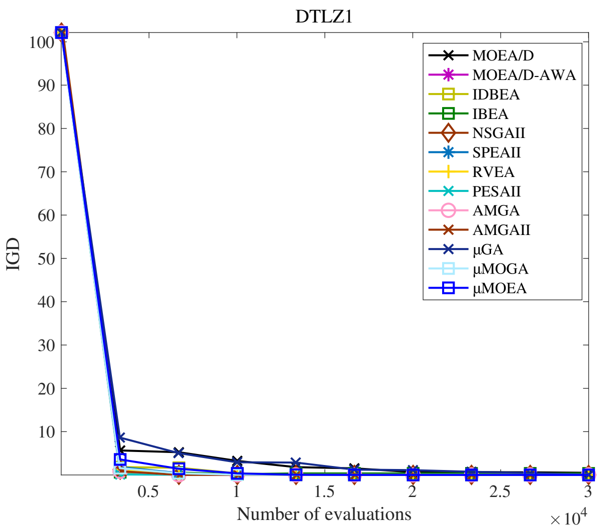
(c) $ZDT3$

Figure S - 2. The distribution of the final non-dominated solutions by μMOEA on ZDT. (a) ZDT1. (b) ZDT2. (c) ZDT3.

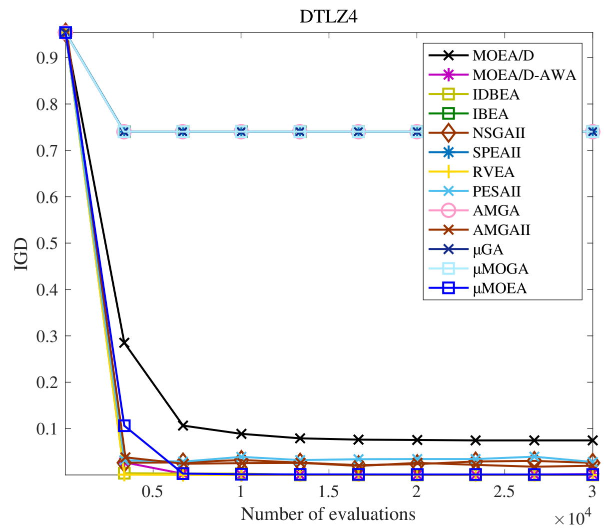
Table S - VIII. IGD of μMOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAII and SPEAII on 5-objective problems MaF

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAII	SPEAII	μMOEA
MaF1	5	14	4.5787e-1 (2.23e-3) +	7.5095e-1 (3.40e-1) -	4.8791e-1 (2.14e-2) =	3.8307e-1 (5.15e-2) +	2.2816e-1 (1.45e-2) +	**1.6302e-1 (2.22e-2) +**	4.9144e-1 (2.01e-2)
MaF3	5	14	3.8624e-1 (5.43e-2) -	5.3497e+10 (1.46e+11) -	4.6599e+7 (2.09e+8) -	1.0126e+0 (2.13e-3) -	1.1887e+0 (1.88e+0) -	3.7929e+1 (1.27e+2) -	**8.2859e-2 (9.36e-3)**
MaF4	5	14	5.8607e+0 (2.17e+0) -	2.6071e+1 (7.11e+1) -	5.7435e+0 (7.84e-2) -	1.3683e+1 (6.44e-1) -	**2.8588e+0 (3.16e-1) +**	3.0651e+0 (3.09e-1) +	3.5622e+0 (5.42e-1)
MaF5	5	14	4.2334e+0 (1.07e+0) -	3.3127e+0 (1.22e+0) =	1.4651e+1 (9.86e+0) -	8.3215e+0 (6.75e+0) -	2.9846e+0 (2.41e-1) =	6.3281e+0 (4.06e+0) -	**2.9020e+0 (2.40e-1)**
MaF7	5	24	7.8407e-1 (4.36e-2) -	1.4449e+0 (5.60e-1) -	4.1320e+0 (3.48e+0) -	9.8021e-1 (3.29e-1) -	6.8218e-1 (4.53e-2) =	**6.0252e-1 (3.55e-2) +**	7.1298e-1 (8.20e-2)
MaF8	5	2	5.8957e-1 (7.90e-2) -	4.5132e-1 (1.26e-1) =	3.7276e-1 (1.03e-2) +	1.0037e+0 (1.16e-1) -	4.1072e-1 (6.19e-2) +	**2.6945e-1 (5.02e-2) +**	4.4988e-1 (7.35e-2)
MaF9	5	2	7.4034e-1 (1.70e-1) -	1.6539e+0 (2.74e+0) -	**3.2684e-1 (2.65e-2) +**	1.2005e+0 (1.70e-1) -	1.0024e+2 (1.35e+2) -	1.5166e+0 (6.85e-1) -	4.9157e-1 (1.70e-1)
MaF10	5	14	1.9875e+0 (8.17e-2) -	2.4653e+0 (1.12e+0) -	2.3013e+0 (8.22e-1) -	1.8278e+0 (5.33e-1) -	1.2613e+0 (1.46e-1) =	1.1526e+0 (2.90e-1) +	1.2606e+0 (2.01e-1)
MaF11	5	14	1.0692e+0 (1.10e-1) =	1.0608e+0 (2.34e-1) =	**2.3110e-1 (1.99e-1) +**	1.8639e+0 (1.31e-1) -	1.2625e+0 (1.34e-1) -	1.3381e+0 (1.28e-1) -	1.0741e+0 (1.93e-1)
MaF12	5	14	2.3412e+0 (3.68e-1) -	1.3975e+0 (1.22e-1) +	3.5502e-1 (1.25e-1) +	1.0942e+0 (1.08e-1) +	1.9135e+0 (1.46e-1) -	1.7822e+0 (1.65e-1) -	1.5568e+0 (1.93e-1)
MaF13	5	5	5.0035e-1 (3.50e-2) -	7.1846e-1 (2.67e-1) -	6.0581e-1 (2.78e-1) =	5.4509e-1 (1.07e-1) -	5.8747e-1 (1.75e-1) -	1.0653e+0 (1.36e-1) -	4.4558e-1 (7.93e-2)
MaF14	5	100	9.3763e-1 (1.17e-1) +	6.0517e+3 (1.03e+4) -	1.2754e+3 (3.47e+3) -	1.5978e+0 (8.19e-1) =	1.2416e+0 (2.40e-1) -	3.2763e+1 (8.06e+1) -	1.0142e+0 (1.12e-1)
MaF15	5	100	9.3922e-1 (5.98e-2) -	1.0729e+0 (3.07e-1) -	1.0066e+0 (4.89e-2) -	1.1166e+0 (5.99e-2) -	1.2836e+1 (4.39e+0) -	4.9979e+1 (1.85e+1) -	8.5853e-1 (5.09e-2)
Average Ranking			5.73	8.65	7.19	7.23	5.69	6.31	**3.92**
+/-/=			2/10/1	1/9/3	4/7/2	2/10/1	3/7/3	5/8/0	

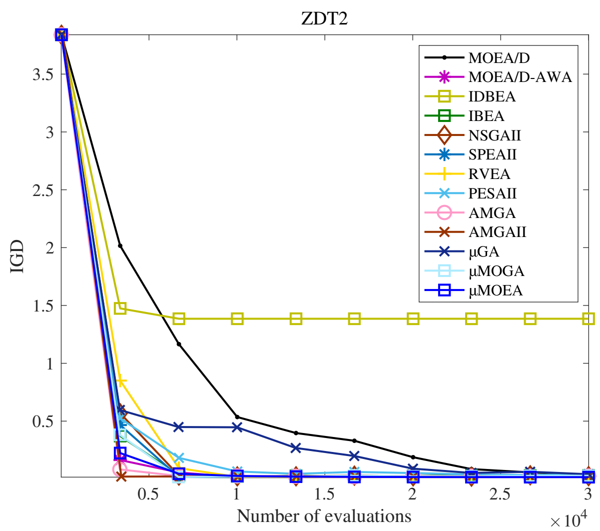
M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.



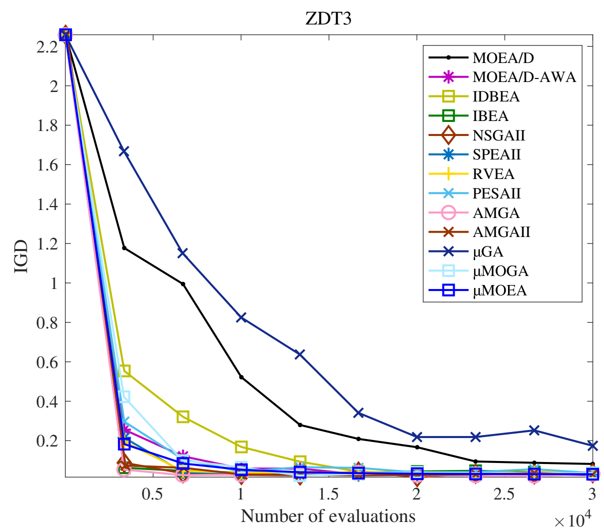
(a) $DTLZ1$



(b) $DTLZ4$

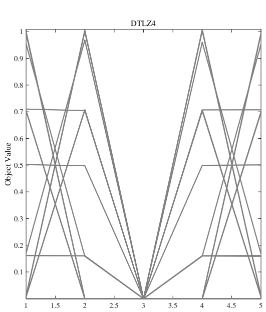


(c) $ZDT2$

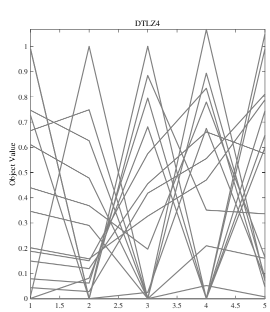


(d) $ZDT3$

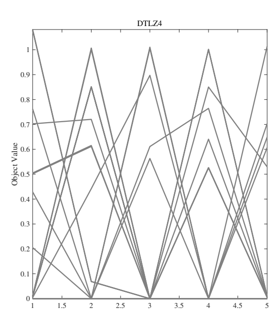
Figure S - 3. Convergence curves of μMOEA with other comparison algorithms. (a) DTLZ1. (b) DTLZ4. (c) ZDT2. (d) ZDT3.



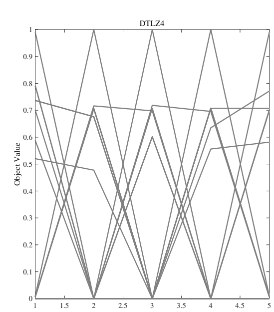
(a) RVEA



(b) NSGAII

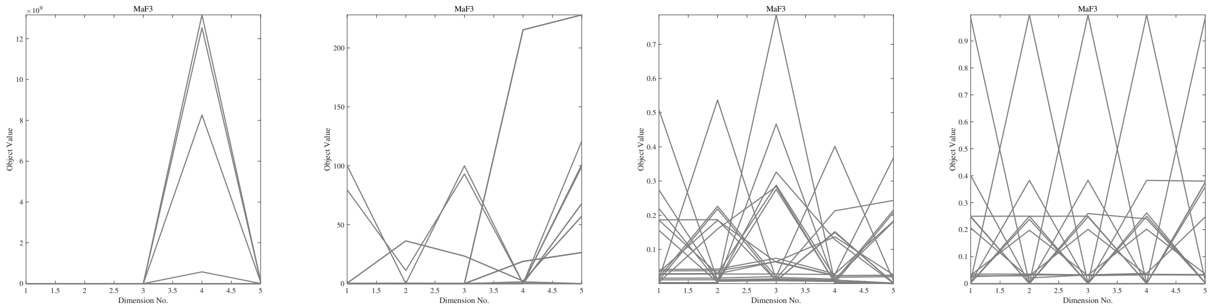


(c) MOEA/D-AWA



(d) μMOEA

Figure S - 4. The final solution set of the best four algorithms on the 5-objective DTLZ4. (a) RVEA. (b) NSGAII. (c) MOEA/D-AWA. (d) μMOEA.



(a) AMGA (b) MOEA/D (c) RVEA (d) μMOEA

Figure S - 5. The final solution set of the best four algorithms on the 5-objective MaF3. (a) AMGA. (b) MOEA/D. (c) RVEA. (d) μMOEA.

Table S - IX. IGD of μMOEA with RVEA, PESAII, AMGA, AMGAII, μGA and μMOGA on 5-objective problems MaF

Problem	M	D	RVEA	PESAII	AMGA	AMGAII	μGA	μMOGA	μMOEA
MaF1	5	14	6.7383e-1 (9.24e-2) -	5.5800e-1 (4.03e-2) -	2.1884e-1 (1.25e-2) +	2.7309e-1 (2.21e-2) +	6.4768e-1 (1.00e-1) -	2.2282e-1 (1.26e-2) +	4.9144e-1 (2.01e-2)
MaF3	5	14	2.2012e-1 (9.64e-3) -	4.9196e+5 (2.45e+6) -	4.6583e-1 (6.38e-1) -	1.1013e+4 (3.75e+3) -	3.4849e+3 (1.37e+4) -	2.8719e+0 (3.25e+0) -	**8.2859e-2 (9.36e-3)**
MaF4	5	14	1.1240e+1 (3.67e+0) -	1.0529e+1 (1.61e+0) -	2.9044e+0 (3.07e-1) +	4.4153e+1 (6.91e+1) -	1.5693e+1 (1.08e+1) -	2.8636e+0 (2.93e-1) +	3.5622e+0 (5.42e-1)
MaF5	5	14	3.3123e+0 (4.60e+0) =	5.6212e+0 (1.26e+0) -	1.4148e+1 (1.11e+1) -	9.3303e+0 (3.21e+0) -	1.5370e+1 (8.19e+0) -	5.1114e+0 (6.54e+0) =	**2.9020e+0 (2.40e-1)**
MaF7	5	24	1.0719e+0 (5.55e-2) -	2.5349e+0 (2.23e-1) -	6.7944e-1 (4.24e-2) =	1.2311e+0 (3.45e-1) -	2.8085e+0 (3.29e-1) -	6.8864e-1 (4.44e-2) =	7.1298e-1 (8.20e-2)
MaF8	5	2	6.3534e-1 (1.19e-1) -	5.3955e-1 (1.52e-1) -	3.4179e-1 (3.29e-2) +	3.5882e-1 (5.63e-2) +	5.3582e-1 (1.04e-1) -	3.7262e-1 (5.56e-2) +	4.4988e-1 (7.35e-2)
MaF9	5	2	5.4332e-1 (1.82e-1) =	6.2116e+1 (7.45e+1) -	1.3801e+2 (1.86e+2) -	1.4798e+1 (1.75e+1) -	1.8420e+1 (1.97e+1) -	1.4462e+1 (2.27e+1) -	4.9157e-1 (1.70e-1)
MaF10	5	14	**8.1580e-1 (1.88e-1) +**	2.5974e+0 (2.69e-1) -	1.5912e+0 (1.57e-1) -	2.2721e+0 (7.87e-2) -	3.0924e+0 (3.45e-1) -	1.3982e+0 (2.26e-1) -	1.2606e+0 (2.01e-1)
MaF11	5	14	1.4357e+0 (2.69e-2) -	3.3221e+0 (8.17e-1) -	1.2873e+0 (1.55e-1) -	1.2345e+0 (1.57e-1) -	4.3811e+0 (1.03e+0) -	1.2780e+0 (1.88e-1) -	1.0741e+0 (1.93e-1)
MaF12	5	14	**1.9567e-1 (1.63e-1) +**	4.0085e+0 (7.25e-1) -	1.8358e+0 (1.50e-1) -	2.2398e+0 (1.85e-1) -	5.5253e+0 (8.89e-1) -	1.9270e+0 (1.50e-1) -	1.5568e+0 (1.93e-1)
MaF13	5	5	9.1752e-1 (1.23e-1) -	7.2866e-1 (1.90e-1) -	5.4162e-1 (1.56e-1) -	**4.0629e-1 (4.82e-2) +**	6.9209e-1 (1.71e-1) -	6.6576e-1 (2.13e-1) -	4.4558e-1 (7.93e-2)
MaF14	5	100	**8.6088e-1 (1.81e-1) +**	1.0658e+1 (1.25e+1) -	9.8463e-1 (7.62e-2) =	2.3502e+0 (1.53e+0) -	2.0499e+2 (5.43e+2) -	2.3835e+0 (1.34e+0) -	1.0142e+0 (1.12e-1)
MaF15	5	100	8.3860e-1 (6.17e-2) =	1.2319e+0 (6.39e-2) -	**8.2411e-1 (5.23e-1) +**	1.9874e+0 (1.65e+0) -	2.3109e+0 (2.62e+0) -	1.7578e+1 (1.05e+1) -	8.5853e-1 (5.09e-2)
Average Ranking			5.88	10.15	5.08	7.69	11.08	6.38	**3.92**
+/-/=			3/7/3	0/13/0	4/7/2	3/10/0	0/13/0	3/8/2	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.

Table S - X. IGD of μMOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAII and SPEAII on sparse MOP

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAII	SPEAII	μMOEA
SMOP1	2	100	3.1428e-1 (1.56e-2) -	**1.3862e-2 (6.32e-3) +**	3.1464e-2 (9.40e-3) -	2.1135e-2 (2.54e-3) =	3.7273e-2 (6.63e-3) -	2.8398e-2 (6.12e-3) -	1.9873e-2 (7.69e-3)
SMOP2	2	100	9.4525e-1 (3.24e-2) -	6.7078e-1 (7.91e-2) -	7.4212e-1 (5.43e-2) -	7.4594e-1 (7.26e-2) -	7.3258e-1 (7.08e-2) -	7.1297e-1 (6.97e-2) -	**1.6854e-1 (5.54e-2)**
SMOP3	2	100	1.5530e+0 (5.61e-2) -	9.6450e-1 (6.67e-2) -	9.2103e-1 (6.15e-2) -	8.8756e-1 (8.14e-2) -	8.7101e-1 (5.27e-2) -	8.4655e-1 (5.90e-2) -	7.8533e-1 (1.31e-2)
SMOP4	2	100	4.5293e-1 (1.69e-2) -	3.3726e-1 (6.84e-2) -	3.4205e-1 (3.50e-2) -	3.2744e-1 (3.70e-2) -	3.2580e-1 (2.97e-2) -	3.0369e-1 (3.73e-2) -	**2.6627e-2 (1.96e-2)**
SMOP5	2	100	3.8406e-1 (5.55e-3) -	3.4337e-1 (1.09e-2) -	**3.3846e-1 (2.78e-4) +**	3.3855e-1 (1.14e-3) +	3.4559e-1 (2.79e-3) -	3.4208e-1 (8.82e-4) -	3.3886e-1 (6.14e-4)
SMOP6	2	100	1.0930e-1 (5.48e-3) -	2.5024e-2 (1.96e-2) +	1.3972e-2 (2.07e-3) +	5.7219e-2 (1.06e-2) +	3.0016e-2 (3.13e-3) +	2.6500e-2 (1.33e-3) +	6.3074e-2 (8.19e-3)
SMOP7	2	100	4.6937e-1 (2.19e-2) -	1.8740e-1 (2.27e-2) -	2.4948e-1 (3.71e-2) -	1.9870e-1 (1.75e-2) -	2.0726e-1 (2.52e-2) -	2.0680e-1 (2.25e-2) -	1.6663e-1 (2.01e-2)
SMOP8	2	100	2.0316e+0 (7.95e-2) -	1.4710e+0 (1.07e-1) -	1.5029e+0 (1.51e-1) -	1.3667e+0 (9.43e-2) -	1.3731e+0 (1.21e-1) -	1.4041e+0 (1.21e-1) -	1.0619e+0 (7.79e-2)
Average Ranking			12.00	5.44	6.50	5.56	6.13	5.13	3.00
+/-/=			0/8/0	2/6/0	2/6/0	2/5/1	1/7/0	1/7/0	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.

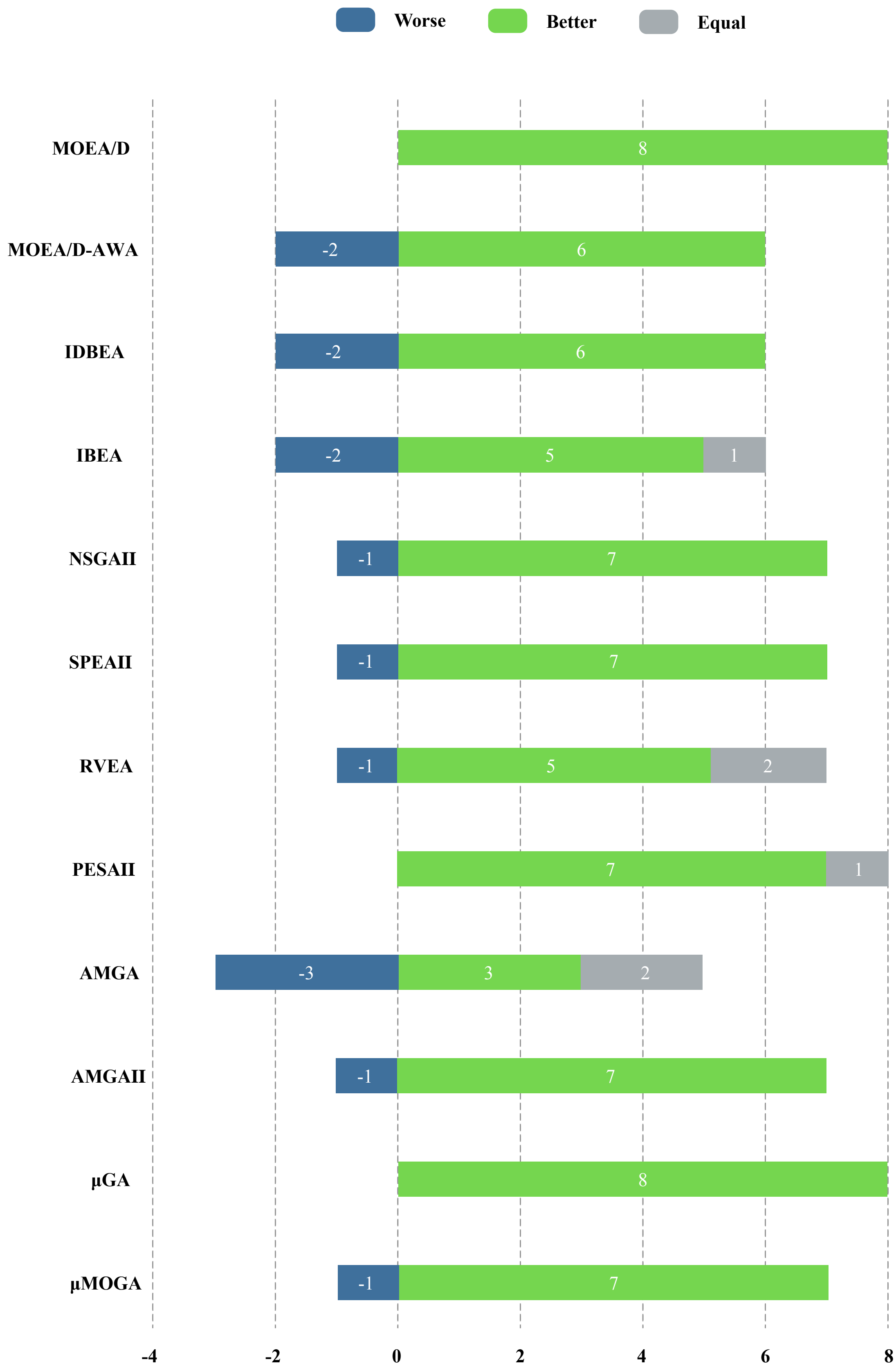


Figure S - 6. Columnar chart of comparison among μMOEA and other algorithms on large-scale spare MOP. Each row represents the result of comparison with a competitor algorithm, where the green denotes the better number of test problems that μMOEA performs better that this competitor algorithm, the blue shows the worse number, the grey refers to the equal number.

Table S - XI. IGD of μMOEA with RVEA, PESAII, AMGA, AMGAII, μGA and μMOGA on sparse MOP

Problem	M	D	RVEA	PESAII	AMGA	AMGAII	μGA	μMOGA	μMOEA
SMOP1	2	100	1.9453e-2 (8.84e-3) =	1.5058e-1 (7.56e-2) -	2.0994e-2 (2.70e-3) =	1.8123e-1 (4.07e-2) -	3.1726e-1 (4.72e-2) -	5.7681e-2 (7.59e-3) -	1.9873e-2 (7.69e-3)
SMOP2	2	100	7.5561e-1 (6.23e-2) -	8.0021e-1 (6.13e-2) -	6.5815e-1 (3.13e-1) -	4.6328e-1 (4.56e-2) -	1.0982e+0 (8.06e-2) -	7.5288e-1 (6.35e-2) -	**1.6854e-1 (5.54e-2)**
SMOP3	2	100	9.1181e-1 (6.59e-2) -	9.2736e-1 (8.09e-2) -	**6.8240e-1 (1.80e-2) +**	1.1723e+0 (1.44e-1) -	1.5930e+0 (9.63e-2) -	1.0611e+0 (6.38e-2) -	7.8533e-1 (1.31e-2)
SMOP4	2	100	3.4625e-1 (3.72e-2) -	3.7079e-1 (5.13e-2) -	2.2549e-1 (1.74e-1) -	1.4242e-1 (4.88e-2) -	5.7719e-1 (6.01e-2) -	3.6806e-1 (4.36e-2) -	**2.6627e-2 (1.96e-2)**
SMOP5	2	100	3.3868e-1 (3.24e-4) =	3.5261e-1 (7.03e-3) -	3.4225e-1 (1.98e-3) -	3.7004e-1 (1.28e-2) -	4.2204e-1 (2.64e-2) -	3.4512e-1 (1.65e-3) -	3.3886e-1 (6.14e-4)
SMOP6	2	100	**1.1171e-2 (1.48e-3) +**	6.4885e-2 (1.99e-2) =	2.2438e-2 (1.95e-3) +	9.5829e-2 (1.84e-2) -	1.4640e-1 (3.19e-2) -	3.3083e-2 (3.60e-3) +	6.3074e-2 (8.19e-3)
SMOP7	2	100	1.9186e-1 (2.03e-2) -	2.3099e-1 (2.47e-2) -	**1.2898e-1 (4.87e-2) +**	4.1538e-1 (2.48e-2) -	5.0472e-1 (6.71e-2) -	2.1489e-1 (2.50e-2) -	1.6663e-1 (2.01e-2)
SMOP8	2	100	1.4322e+0 (1.03e-1) -	1.4722e+0 (1.04e-1) -	9.8243e-1 (2.76e-1) =	**8.6958e-1 (1.31e-2) +**	2.1032e+0 (1.32e-1) -	1.5545e+0 (1.40e-1) -	1.0619e+0 (7.79e-2)
Average Ranking			5.25	9.69	**2.81**	7.50	13.00	9.00	3.00
+/-/=			1/5/2	0/7/1	3/3/2	1/7/0	0/8/0	1/7/0	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.

Table S - XII. IGD of μMOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAII and SPEAII in normal population size

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAII	SPEAII	μMOEA
DTLZ1	3	7	3.3122e-2 (2.48e-4) -	7.4641e-3 (2.43e-3) -	3.3965e-2 (8.08e-2) -	1.5029e-1 (2.41e-2) -	3.1878e-2 (2.50e-2) -	1.7165e-2 (1.41e-3) -	**3.0654e-3 (1.65e-3)**
DTLZ2	3	12	7.5989e-2 (6.44e-4) -	9.8647e-3 (1.38e-3) +	1.0184e-3 (5.58e-4) +	7.5617e-2 (2.78e-3) -	7.1734e-2 (3.16e-3) -	4.9092e-2 (2.33e-3) -	2.5972e-2 (6.42e-3)
DTLZ3	3	12	1.8849e-1 (2.86e-1) -	2.6809e-1 (3.52e-1) -	3.3015e+0 (1.83e+0) -	4.7721e-1 (1.39e-1) -	3.2791e-1 (6.14e-1) -	2.4162e-1 (4.94e-1) -	**5.7476e-2 (1.12e-1)**
DTLZ4	3	12	4.6368e-1 (3.01e-1) -	9.8242e-2 (1.97e-1) -	2.8600e-1 (2.99e-1) =	7.5133e-2 (4.15e-3) -	6.9768e-2 (2.82e-3) -	2.2215e-1 (2.85e-1) -	3.0958e-2 (3.30e-3)
DTLZ5	3	12	1.4549e-2 (2.43e-5) -	1.1459e-2 (3.34e-4) -	1.6699e-2 (4.22e-3) -	1.5730e-2 (1.35e-3) -	5.6999e-3 (3.40e-4) -	**4.3124e-3 (2.21e-4) +**	4.6688e-3 (3.21e-4)
DTLZ6	3	12	1.4560e-2 (7.15e-6) -	1.1439e-2 (4.15e-4) -	2.0050e-2 (1.26e-2) -	2.5799e-2 (4.13e-3) -	5.8603e-3 (3.13e-4) -	**3.9330e-3 (3.15e-4) +**	4.5486e-3 (2.99e-4)
DTLZ7	3	22	2.3037e-1 (7.44e-2) -	1.4559e-1 (4.70e-2) -	2.1108e-1 (5.30e-1) -	7.8502e-2 (3.89e-3) -	8.7673e-2 (5.52e-2) -	**6.9679e-2 (5.64e-2) +**	7.2336e-2 (4.74e-3)
ZDT1	2	30	4.3503e-3 (3.01e-3) +	**3.7275e-3 (1.40e-4) +**	8.9363e-3 (1.84e-3) -	4.4443e-3 (2.58e-4) +	4.6658e-3 (2.44e-4) +	3.9355e-3 (1.70e-4) +	8.2224e-3 (9.71e-4)
ZDT2	2	30	3.7118e-3 (6.38e-4) +	**3.6890e-3 (6.36e-4) +**	7.3848e-2 (1.64e-1) -	8.1970e-3 (7.59e-4) -	4.8126e-3 (2.70e-4) -	3.8232e-3 (2.33e-4) +	4.4173e-3 (5.87e-4)
ZDT3	2	30	1.1115e-2 (8.30e-3) -	6.4048e-3 (1.81e-3) -	2.1833e-2 (6.29e-3) -	1.4184e-2 (6.03e-3) -	5.0664e-3 (7.57e-4) =	**4.8123e-3 (5.02e-4) =**	4.9569e-3 (6.28e-4)
ZDT4	2	10	7.2802e-3 (2.05e-3) -	5.7079e-3 (1.19e-3) -	3.3268e+0 (2.64e+0) -	9.1160e-2 (7.25e-2) -	5.7978e-3 (1.30e-3) -	5.0226e-3 (9.83e-4) -	**3.7070e-3 (1.37e-4)**
ZDT6	2	10	3.0740e-3 (6.81e-5) =	**3.0490e-3 (7.59e-5) =**	8.3807e-2 (2.83e-1) -	4.3584e-3 (2.01e-4) -	3.6175e-3 (2.36e-4) -	3.1083e-3 (1.94e-4) =	3.0525e-3 (1.36e-4)
Average Ranking			6.74	3.96	9.33	8.42	5.79	2.92	**2.88**
+/-/=			2/9/1	3/8/1	1/10/1	1/11/0	1/10/1	5/5/2	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.

Table S - XIII. IGD of μMOEA with RVEA, PESAII, AMGA, AMGAII, μGA and μMOGA in normal population size

Problem	M	D	RVEA	PESAII	AMGA	AMGAII	μGA	μMOGA	μMOEA
DTLZ1	3	7	3.4354e-3 (2.75e-3) =	2.7317e-2 (2.19e-3) -	4.3451e-2 (5.54e-2) -	9.8155e-1 (6.89e-1) -	1.1528e+1 (5.80e+0) -	5.1640e-2 (1.00e-1) -	**3.0654e-3 (1.65e-3)**
DTLZ2	3	12	**5.8894e-4 (2.96e-4) +**	7.1230e-2 (5.34e-3) -	7.4608e-2 (4.12e-3) -	7.2352e-2 (3.11e-3) -	2.3487e-1 (8.36e-2) -	7.1233e-2 (3.73e-3) -	2.5972e-2 (6.42e-3)
DTLZ3	3	12	1.1856e+0 (1.41e+0) -	3.7660e-1 (5.91e-1) -	6.8491e+0 (4.20e+0) -	2.9122e+1 (9.74e+0) -	8.1363e+1 (2.50e+1) -	8.8830e-1 (1.01e+0) -	**5.7476e-2 (1.12e-1)**
DTLZ4	3	12	**8.4028e-4 (1.03e-3) +**	6.3613e-2 (3.03e-3) -	6.7634e-1 (4.08e-1) -	4.0748e-1 (4.29e-1) -	8.9458e-1 (1.45e-1) -	8.0348e-1 (3.34e-1) -	3.0958e-2 (3.30e-3)
DTLZ5	3	12	7.8271e-2 (1.07e-2) -	1.1652e-2 (1.43e-3) -	4.6750e-3 (3.12e-4) =	4.7327e-3 (3.17e-4) =	7.5945e-2 (7.19e-2) -	5.1951e-3 (3.18e-4) -	4.6688e-3 (3.21e-4)
DTLZ6	3	12	8.6033e-2 (1.93e-2) -	1.3459e-2 (2.17e-3) -	4.6459e-3 (2.18e-4) =	4.5032e-3 (2.70e-4) =	4.3599e+0 (1.18e+0) -	5.4106e-3 (3.25e-4) -	4.5486e-3 (2.99e-4)
DTLZ7	3	22	1.0661e-1 (2.58e-3) -	2.0772e-1 (1.94e-1) -	5.3358e-1 (2.98e-1) -	3.3225e-1 (3.36e-1) -	1.9010e+0 (1.35e+0) -	3.4930e-1 (2.88e-1) -	7.2336e-2 (4.74e-3)
ZDT1	2	30	1.9885e-2 (2.99e-3) -	1.1589e-2 (1.65e-3) -	3.9614e-3 (1.85e-4) +	3.9236e-3 (2.33e-4) +	8.5164e-1 (2.89e-1) -	4.3687e-3 (2.41e-4) +	8.2224e-3 (9.71e-4)
ZDT2	2	30	2.9768e-2 (3.98e-3) -	1.1257e-2 (1.87e-3) -	4.2012e-3 (2.28e-4) =	4.1865e-3 (6.01e-4) +	5.2043e-1 (5.65e-2) -	4.5161e-3 (2.63e-4) =	4.4173e-3 (5.87e-4)
ZDT3	2	30	3.2039e-2 (6.16e-3) -	2.3495e-2 (2.30e-2) -	1.5258e-2 (4.84e-2) =	9.2831e-3 (1.09e-2) =	8.6642e-1 (3.29e-1) -	1.3151e-2 (1.36e-2) =	4.9569e-3 (6.28e-4)
ZDT4	2	10	5.7260e-2 (4.19e-2) -	1.2981e-2 (2.95e-3) -	5.4106e-3 (1.49e-3) -	5.0914e-2 (6.93e-2) -	2.1237e+1 (9.20e+0) -	5.9052e-3 (1.29e-3) -	**3.7070e-3 (1.37e-4)**
ZDT6	2	10	2.7566e-2 (5.47e-3) -	7.3764e-3 (8.06e-4) -	3.3039e-3 (2.45e-4) -	1.4390e-1 (2.48e-1) -	1.3640e-1 (1.51e-1) -	3.6210e-3 (1.92e-4) -	3.0525e-3 (1.36e-4)
Average Ranking			8.17	7.71	7.17	7.58	12.83	7.50	**2.88**
+/-/=			2/9/1	0/12/0	1/7/4	2/7/3	0/12/0	1/9/2	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The bottom part records the average ranking of these algorithms in the Friedman test.
"+","-","=" denote comparison algorithms are respectively better than, worse than, or close to μMOEA through Wilcoxon rank test.

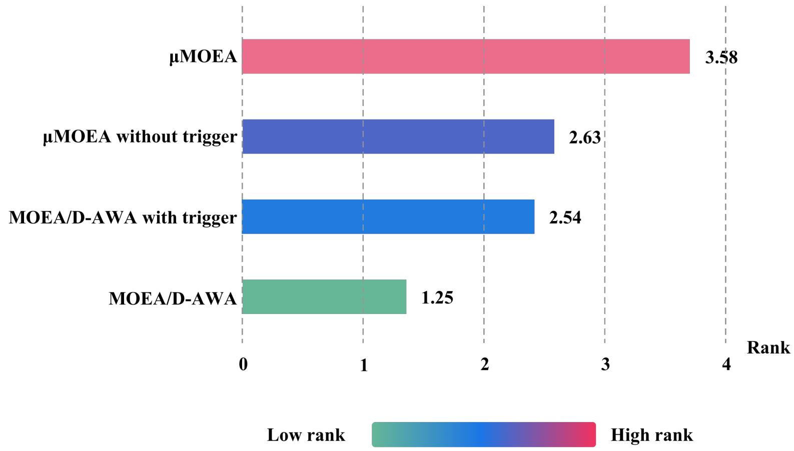


Figure S - 7. Ranking histogram of four types of algorithms for small population. The bottom shows the transition color from low to high ranking. The higher the ranking algorithm, the color tends to be red, and the ranking score value is larger.

comparison algorithm. The trigger was additionally added in MOEA/D-AWA [1] and experimental comparisons were made. In order to reduce the number of pages in the experimental part of the original text, the original text only shows the effective improvement rate. The detailed results are placed in Table S - XIV, and the average ranking obtained in descending order was plotted as histogram Figure S - 7, where it can be seen that μMOEA performs best, and MOEA/D-AWA [1] with trigger mechanism performs much better than MOEA/D-AWA [1] without trigger mechanism.

VI. Supplement to Simulation Base on Low-Power Embedded Processor

A. Semi-autogenous grinding optimization problem type-A

According to the obtained mathematical model [19], the power model f_1, mill output ore model f_2 and steel ball wear model f_3 are selected as optimization objectives. That is:

$$\begin{aligned}
\text{minimize} \quad & P = c_1 D^3 sin\alpha(x_1+x_2+x_3)(3.2-3v)N(1-\frac{0.1}{2^{9-10N}}) \\
\text{minimize} \quad & \frac{1}{m_{out}} = \frac{1}{lx_1} \\
\text{minimize} \quad & m_{ball} = \beta(x_1+x_3) \\
\text{subject to} \quad & x_1+x_2+x_3-355.55 \geq 0 \\
& 389.90 - x_1 \geq 0 \\
& 143.15 - x_2 \geq 0 \\
& 166.62 - x_3 \geq 0
\end{aligned} \tag{1}$$

where $min\, P$ is to ensure that the power consumption of the SAG is as low as possible to achieve energy saving and reduce consumption. $min\, \frac{1}{m_{out}}$ is to ensure that the output of the SAG is as high as possible to achieve efficient output. $min\, m_{ball}$ is to ensure that the amount of steel balls consumed by the SAG operation is as small as possible, and the loss is reduced while the output is efficient. The x_1, x_2, and x_3 in the objective function definition are decision variables, which respectively represent the quality of minerals, water, and steel balls retained in the SAG. The rest in the objective function definition are some parameters, whose values and meanings can be seen in Table S - XV.

B. Semi-autogenous grinding optimization problem type-B

The specific definition of SAGOP-B is as follows:

$$\begin{aligned}
\text{minimize} \quad & P = c_2 D^3 sin\alpha(x_1+x_2+x_3)(3.2-3v)N(1-\frac{0.1}{2^{9-10N}}) \\
\text{minimize} \quad & -m_{out} = -lx_1 \\
\text{minimize} \quad & m_{ball} = \beta(x_1+x_3) \\
\text{subject to} \quad & 50 \leq x_1 \leq 958.5 \\
& 12.5 \leq x_2 \leq 355 \\
& 98.45 \leq x_3 \leq 415.35
\end{aligned} \tag{2}$$

where the definitions of objective function 1 and objective function 2 have been changed, the value range of decision variables has been adjusted, and the definitions of other parameters can also be found in Table S - XV.

C. Micro-grid energy optimization problem

The two objectives of Micro-grid energy optimization problem are operating cost f_1 and emission treatment cost f_2 respectively. The definition can be described as the following mathematical formula:

$$\begin{aligned}
\text{minimize} \quad & f_1 = C_F + C_D + C_M + C_g \\
\text{minimize} \quad & f_2 = \sum_{t=1}^{T}\{\sum_{i=1}^{M}\sum_{j=1}^{J}[\xi_j a_{ij} P_i(t)] + \sum_{j=1}^{J}[\xi_j b_j P_{buy(t)}]\}
\end{aligned} \tag{3}$$

where C_F, C_D, C_M, and C_g are fuel, depreciation, maintenance, and transaction costs, respectively. M and J denote the number of DGs and pollutant gas emissions. ξ_j is the pollution coefficient of the j-th gas, $a_{i,j}$, and b_j are the emission coefficients of the j-th gas during the operation of the i-th DG and smart grid. Lastly, $P_i(t)$ and $P_{buy(t)}$ are the power generation of the i-th distributed power source and electricity purchased at time t. For more detailed information can be found in our previous work [20].

Table S - XIV. IGD of μMOEA and MOEA/D-AWA in weight vectors trigger validity experiment for small population

Problem	M	D	MOEA/D-AWA		μMOEA	
DTLZ1	3	7	no trigger	4.0201e-1 (6.97e-2)	no trigger	4.3477e-3 (2.31e-3)
			with trigger	**1.0678e-2 (1.82e-2)**	with trigger	**3.8681e-5 (2.07e-4)**
DTLZ2	3	12	no trigger	4.7753e-1 (1.61e-1)	no trigger	1.2175e-2 (1.44e-2)
			with trigger	**3.5249e-2 (7.41e-2)**	with trigger	**8.2142e-4 (3.87e-4)**
DTLZ3	3	12	no trigger	1.3837e+1 (1.84e+1)	no trigger	2.2709e-1 (4.45e-1)
			with trigger	**1.2113e+1 (1.96e+1)**	with trigger	**1.0989e-1 (3.02e-1)**
DTLZ4	3	12	no trigger	**1.0387e-1 (1.10e-1)**	no trigger	4.4215e-2 (1.67e-2)
			with trigger	1.0832e-1 (2.28e-1)	with trigger	**1.7276e-2 (2.06e-2)**
DTLZ5	3	12	no trigger	2.9399e-2 (4.87e-3)	no trigger	2.7340e-2 (1.73e-3)
			with trigger	**2.8127e-2 (3.25e-3)**	with trigger	**2.6731e-2 (2.17e-3)**
DTLZ6	3	12	no trigger	3.1013e-2 (6.99e-3)	no trigger	2.7587e-2 (2.30e-3)
			with trigger	**2.8662e-2 (3.04e-3)**	with trigger	**2.7083e-2 (2.26e-3)**
DTLZ7	3	22	no trigger	7.2846e-1 (3.17e-1)	no trigger	2.0758e-1 (3.66e-2)
			with trigger	**3.6315e-1 (1.12e-1)**	with trigger	**1.8479e-1 (2.06e-2)**
ZDT1	2	30	no trigger	2.1398e-2 (1.13e-3)	no trigger	2.1948e-2 (1.44e-3)
			with trigger	**1.8855e-2 (1.00e-4)**	with trigger	**2.0027e-2 (5.11e-4)**
ZDT2	2	30	no trigger	1.6863e-2 (5.81e-4)	no trigger	1.7223e-2 (8.67e-4)
			with trigger	**1.6467e-2 (1.22e-4)**	with trigger	**1.6414e-2 (1.44e-4)**
ZDT3	2	30	no trigger	5.0975e-2 (3.31e-2)	no trigger	2.8418e-2 (4.26e-3)
			with trigger	**2.6952e-2 (1.90e-3)**	with trigger	2.9390e-2 (2.09e-3)
ZDT4	2	10	no trigger	2.6893e-2 (1.64e-2)	no trigger	2.1310e-2 (2.25e-3)
			with trigger	**1.8991e-2 (3.37e-4)**	with trigger	**1.8816e-2 (1.56e-4)**
ZDT6	2	10	no trigger	1.9052e-2 (5.32e-3)	no trigger	**1.7751e-2 (9.31e-4)**
			with trigger	**1.7809e-2 (7.64e-4)**	with trigger	1.7879e-2 (1.45e-3)
Effective improvement rate			91.67%		83.33%	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.
The results of each algorithm are divided into two categories, namely without triggers and with triggers.
The effective improvement rate is shown at the bottom, which is the ratio of the number of successfully improved problems to the total number of problems.

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Table S - XV. The meaning and value of the parameters in the definition of SAGOP

Parameter	Description	Value
c_1	System parameter	0.028
c_2	System parameter	0.15
D	Mill diameter	9.8 m
α	Angle of inclination of mill	30°
v	The ratio of mineral to water in the mill	0.3
N	The ratio of the actual speed of the motor to the rated speed	0.7
l	Mineral grinding parameter	20 h^{-1}
β	Steel ball wear parameter	0.5 h^{-1}

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