

Micro Multi-Objective Evolutionary Algorithm with Piecewise Strategy for Embedded-processor-based Industrial Optimization: Supplementary Material

Hu Peng, Fanrong Kong, and Qingfu Zhang, *Fellow, IEEE*

I. SUPPLEMENT TO RELATED WORK

The difference in the weight update frequency of the decomposition-based algorithm is described in detail in the main text. In order to summarize, a summary table is given in the supplementary material, as shown in Table S - I below.

II. COMPUTATIONAL COMPLEXITY ANALYSIS

As for μ MOEA, the piecewise strategy and the weight vector update trigger mechanism govern the computational complexity. Therefore, this part analyzes these two major components respectively.

Initially, what must be prioritized is the key piecewise strategy, which costs $O(n\tau)$, where n denotes the size of the evolutionary population, τ is the time slice. Then, the weight update costs $O(pMN^2)$, where p is the probability of triggering the weight update, M is the number of objectives and N is the size of the archive. Therefore, the total time complexity of μ MOEA is $O(n\tau + pMN^2)$.

The space complexity of μ MOEA is also very worthy of discussion, which is mainly determined by the weight vectors, the neighborhood set, and the archive. Therefore, the total space complexity is $O(N(M + T + D))$.

III. SENSITIVITY ANALYSIS OF PARAMETERS

In μ MOEA, timeslice τ , theoretical convergence minimum ξ^* , the max flag of convergence δ , and tolerance rate γ are key parameters. To analyze the effect of these parameters on μ MOEA, μ MOEA was performed on the comprehensive experimental suites ZDT and DTZ. In this experiment, the archive size N , the population size n , and the neighborhood size T are set to 20, 5, and 5, respectively. In order to eliminate statistical errors, μ MOEAs with different parameter values are run independently for 30 times. Due to the limitation of the number of pages, the main text only shows the statistical data.

To analyze the sensitiveness of τ , we test μ MOEA with different values of parameter τ (e.g., $\tau=2$, $\tau=3$, $\tau=4$, $\tau=5$, $\tau=6$) and set ξ^* , δ , γ to 1.00E-4, 0.06, 0.4 respectively. The

experimental result is shown in the first row of Table S - II, where the rankings are obtained by the Friedman test. To make the results obvious, we highlight the best result in bold. As presented in the first row of Table S - II, μ MOEA with $\tau=4$ achieves the best. Therefore, for all subsequent experiments, τ is set to 4.

To explore the sensitiveness of ξ^* , we test μ MOEA with different values of parameter ξ^* (e.g., $\xi^*=8E-3$, $\xi^*=9E-3$, $\xi^*=1E-4$, $\xi^*=2E-4$, $\xi^*=3E-4$) and other parameters are consistent with the above part for parameter τ . The experimental result is shown in the second row of Table S - II and the best result is highlighted in bold. From the second row of Table S - II, it can be seen that μ MOEA with $\xi^*=1E-4$ gets the best performance. Therefore, for all subsequent experiments, ξ^* is set to 1E-4.

In addition, we experiment μ MOEA with different values of parameter δ (e.g., $\delta=0.04$, $\delta=0.05$, $\delta=0.06$, $\delta=0.07$, $\delta=0.08$) to analyze the sensitiveness of δ . Other parameters remain the same as the above experiments. The experimental results can be seen in the third row of Table S - II, from which it can be clearly found that μ MOEA with $\delta=0.06$ ranks best. Therefore, for all subsequent experiments, δ is set to 0.06.

Lastly, in order to study the sensitivity of γ , we test μ MOEA with different values of parameter γ (e.g., $\gamma=0.2$, $\gamma=0.3$, $\gamma=0.4$, $\gamma=0.5$, $\gamma=0.6$). Other parameters remain the same as the above experiments. The results are shown in the fourth row of Table S - II, where μ MOEA achieves the best performance when $\gamma=0.4$. Thus, for all subsequent experiments, γ is set to 0.4.

IV. SUMMARY OF COMPARISON ALGORITHMS

In this section, 12 excellent algorithms are selected as the comparison algorithms for the artificial optimization problems, and their specific parameter settings can be shown in Table S - III.

V. SUPPLEMENT TO EXPERIMENTS ON ARTIFICIAL TEST PROBLEMS

A. Performances on bi-objective and tri-objective problems

The more detailed results on DTLZ and ZDT problems are given in Table S - IV, Table S - V, Figure S - 1, Figure S - 2 and Figure S - 3. Table S - IV and Table S - V show the IGD of μ MOEA with other comparison algorithms. The value in parentheses in each table item indicates the standard deviation. Figure S - 1 and Figure S - 2 show the distribution of the final non-dominated solutions obtained by μ MOEA in several

This work was supported by the National Natural Science Foundation of China (62266024), the Science and Technology Foundation of Jiangxi Province, PR China (20202BABL202019). (Corresponding author: Hu Peng.)

H. Peng and F. Kong are with the School of Computer and Big Data Science, Jiujiang University, Jiujiang 332005, PR China (email: hu_peng@whu.edu.cn).

Q. Zhang is with the Department of Computer Science, City University of Hong Kong, Hong Kong (e-mail: qingfu.zhang@cityu.edu.hk).

Manuscript received June 1, 20XX; revised July 1, 20XX.

Table S - I. Summary of decomposition-based MOEA for weight vector update frequency

Algorithm	Update frequency		Description
MOEA/D-AWA [1]	periodically	every wag generation after $rate_evol * maxGen$ generation	
MOEA/D-URAW [2]	periodically	every wag generation after $rate_evol * maxGen$ generation	
MOEA/D-ABD [3]	periodically	every $0.1 * maxGen$ generation after $0.5 * maxGen$	
AdaW [4]	periodically	every 5% generation until the last 10% generation	
$pa\lambda$ -MOEA/D [5]	once		-
MOEA/D-TPN [6]	once		-
MOEA/D-RWV [7]	conditional	the degree of deviation of the solution	
μ MOEA	conditional	the degree of crowding after the solution converges	

Table S - II. Average ranks of μ MOEA with different value of τ , ξ^* , δ and γ

Parameters		Values					
τ	Value	2	3	4	5	6	
	Rank	2.58	3.63	2.13	3.25	3.42	
ξ^*	Value	8E-3	9E-3	1E-4	2E-4	3E-4	
	Rank	2.75	3.46	2.71	3.21	2.88	
δ	Value	0.04	0.05	0.06	0.07	0.08	
	Rank	3.33	2.75	2.71	3.04	3.17	
γ	Value	0.2	0.3	0.4	0.5	0.6	
	Rank	3.67	3.00	2.42	2.58	3.33	

Table S - III. Comparison algorithms and their descriptions and parameter settings.

Algorithm	Parameter setting
MOEA/D [8]	$T = 5$
MOEA/D-AWA [1]	$T=5$, $rate_evol=0.8$, $wag=100$, $rate_update_weight=0.05$
IDBEA [9]	
IBEA [10]	$\kappa = 0.05$
NSGAII [11]	
SPEAII [12]	
RVEA [13]	$\alpha=2$, $fr=0.1$
PESAII [14]	$div=10$
AMGA [15]	
AMGAII [16]	
μ GA [17]	$replacement_cycle=30$, $adaptive_grid = 25$, $convergence_criterion=3$
μ MOGA [18]	$convergence_criterion=3$
μ MOEA	$F = 0.5$, $CR = 0.5$, $T=5$, $\tau=4$, $\xi^*=1.00E-04$, $\delta=0.06$, $\gamma=0.4$

representative problems. Figure S - 3 illustrates convergence curves of μ MOEA with other comparison algorithms on DTLZ1, DTLZ4, ZDT2, and ZDT3.

B. Performances on many-objectives problems

Due to the page limit, only the statistical results are given in the original text. Due to the page limit, only statistical results are given in the original text. In order to comprehensively show the experimental results, this section supplements the parallel coordinate diagram and more detailed experimental results.

Parallel coordinates diagram are attached in Figure S - 4 and 5. Moreover, the more detailed results are shown in Table S - VI, VII, VIII and IX.

Based on the above experimental results, it can be found that most algorithms are difficult to converge to the Pareto front, and the quality of the solution set is not very good. However, μ MOEA has better distribution and convergence in DTL4 and MaF3 compared to other algorithms.

C. Performances on MOPs with large-scale sparse problems

In order to show the experimental results more clearly, Figure S - 6, Table S - X, and Table S - XI shows the comparison between other algorithms and μ MOEA in detail. Special description for Figure S - 6, the use of dark blue indicates the number of problems that μ MOEA is inferior compared with other algorithms. green indicates the merit quantity. grey means equal quantity. This bar chart clearly shows how μ MOEA compares with other algorithms. It is not difficult to find that the green bars are the most. Therefore, our algorithm performs well on SMOPs.

D. Validity on normal size population

The original text shows the overall statistics, which still show that μ MOEA is slightly better than other algorithms. The details of the experimental results, are also given in this section, which can be found in Table S - XII and Table XIII.

E. Validity on weight vectors trigger

In the validity verification of the weight vector update trigger in the original text, MOEA/D-AWA [1] is chosen as the comparison algorithm. The trigger was additionally added in

Table S - IV. IGD of μ MOEA with MOEA/D, MOEA/D-AWA, IDBEA, IBEA, NSGAI and SPEAI on ZDT and DTLZ Problems.

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAI	SPEAI	μ MOEA
DTLZ1	3	7	3.3629e-1 (1.99e-1) -	3.0217e+0 (1.43e+1) -	3.6875e-2 (1.86e-1) -	4.3851e-1 (2.77e-2) -	5.4047e-2 (6.15e-3) -	3.9458e-2 (3.64e-3) -	3.8681e-5 (2.07e-4)
DTLZ2	3	12	3.6158e-2 (4.41e-2) =	3.2061e-1 (1.75e-1) -	3.8071e-5 (1.22e-5) +	1.3633e-1 (1.67e-2) -	1.5406e-1 (1.97e-2) -	8.5383e-2 (1.14e-2) -	7.7364e-4 (3.50e-4)
DTLZ3	3	12	6.9504e+0 (2.89e+0) -	3.1690e+1 (1.24e+2) -	8.4141e+0 (1.98e+1) -	8.6752e-1 (1.50e-1) -	2.8622e-1 (3.06e-1) -	2.4317e-1 (5.57e-1) -	1.7173e-1 (4.56e-1)
DTLZ4	3	12	2.0906e-1 (1.11e-1) -	2.8593e-1 (3.43e-1) -	7.6959e-1 (2.62e-1) -	3.9292e-1 (3.46e-1) -	4.6927e-1 (4.06e-1) -	6.6335e-1 (2.91e-1) -	2.3656e-2 (2.41e-2)
DTLZ5	3	12	1.2890e-1 (8.36e-5) -	3.2805e-2 (9.08e-3) -	8.0160e-2 (6.85e-3) -	4.2841e-2 (5.48e-3) -	2.9998e-2 (3.06e-3) -	2.0354e-2 (3.26e-3) +	2.7196e-2 (2.37e-3)
DTLZ6	3	12	1.4696e-1 (1.02e-2) -	3.6431e-2 (1.19e-2) -	6.2332e-2 (1.24e-2) -	4.8818e-2 (7.47e-3) -	3.2266e-2 (3.83e-3) -	1.8944e-2 (2.27e-3) +	2.6933e-2 (2.25e-3)
DTLZ7	3	22	3.5820e-1 (1.15e-1) -	6.4368e-1 (1.55e-1) -	1.7176e+0 (2.42e+0) -	4.1332e-1 (2.53e-1) -	2.7081e-1 (1.12e-1) -	2.3385e-1 (1.24e-1) =	1.8261e-1 (2.69e-2)
ZDT1	2	30	3.0348e-2 (5.24e-3) -	2.1218e-2 (9.28e-4) -	2.3364e-2 (5.06e-3) -	2.8664e-2 (3.78e-3) -	2.4106e-2 (3.22e-3) -	1.9740e-2 (2.15e-3) =	2.0050e-2 (4.99e-4)
ZDT2	2	30	2.0332e-2 (2.85e-3) -	1.7408e-2 (6.70e-4) -	4.7182e-1 (5.31e-1) -	2.7872e-2 (3.67e-3) -	2.6707e-2 (3.05e-3) -	1.9047e-2 (2.34e-3) -	1.6419e-2 (1.31e-4)
ZDT3	2	30	7.5772e-2 (3.92e-2) -	4.4165e-2 (4.84e-2) =	7.1334e-2 (1.86e-2) -	4.7143e-2 (1.25e-2) -	3.7696e-2 (2.03e-2) =	3.1106e-2 (9.63e-3) =	2.9499e-2 (1.89e-3)
ZDT4	2	10	2.6429e-1 (9.12e-2) -	2.9635e-2 (3.38e-2) -	8.3270e-2 (1.76e-1) -	5.3316e-1 (8.90e-2) -	2.3575e-2 (3.15e-3) -	1.8752e-2 (2.11e-3) =	1.8816e-2 (1.56e-4)
ZDT6	2	10	2.8657e-2 (1.50e-2) -	1.7758e-2 (8.70e-4) -	2.2958e-1 (2.98e-1) =	2.0613e-2 (2.96e-3) -	2.3837e-2 (3.45e-3) -	1.4610e-2 (2.40e-3) +	1.7198e-2 (1.11e-3)
Average Ranking			8.42	7.46	9.00	8.25	6.50	2.96	2.04
+/-/=			0/11/1	0/11/1	1/10/1	0/12/0	0/11/1	3/5/4	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

Table S - V. IGD of μ MOEA with RVEA, PESAI, AMGA, μ GA and μ MOGA on ZDT and DTLZ Problems

Problem	M	D	RVEA	PESAI	AMGA	AMGAI	μ GA	μ MOGA	μ MOEA
DTLZ1	3	7	6.8678e-4 (6.65e-4) -	1.4672e-1 (3.20e-2) -	5.4166e-2 (5.89e-3) -	5.9763e-1 (6.42e-1) -	5.8720e-1 (7.99e-1) -	5.5530e-2 (4.73e-3) -	3.8681e-5 (2.07e-4)
DTLZ2	3	12	5.6687e-5 (2.83e-5) +	3.8008e-1 (6.35e-2) -	1.5181e-1 (1.25e-2) -	1.6033e-1 (1.57e-2) -	5.1857e-1 (8.81e-2) -	1.5132e-1 (1.58e-2) -	7.7364e-4 (3.50e-4)
DTLZ3	3	12	1.8779e-1 (4.71e-1) -	6.1763e-1 (3.59e-1) -	8.7104e-1 (1.09e+0) -	2.1239e+1 (2.31e+1) -	1.1841e+1 (9.24e+0) -	7.1327e-1 (7.86e-1) -	1.7173e-1 (4.56e-1)
DTLZ4	3	12	3.0650e-1 (3.13e-1) =	5.1668e-1 (3.93e-1) -	8.4874e-1 (2.82e-1) -	3.9589e-1 (3.56e-1) -	8.2259e-1 (2.39e-1) -	7.9401e-1 (3.33e-1) -	2.3656e-2 (2.41e-2)
DTLZ5	3	12	1.6549e-1 (4.99e-2) -	6.1080e-2 (3.47e-2) -	2.6495e-2 (3.42e-3) =	2.6428e-2 (3.54e-3) =	1.1039e-1 (5.41e-2) -	2.8107e-2 (3.08e-3) =	2.7196e-2 (2.37e-3)
DTLZ6	3	12	3.0030e-1 (1.57e-1) -	8.9336e-2 (6.82e-2) -	2.6406e-2 (3.41e-3) =	2.3715e-2 (3.28e-3) +	6.8609e-1 (9.97e-1) -	2.8581e-2 (3.00e-3) -	2.6933e-2 (2.25e-3)
DTLZ7	3	22	2.7501e-1 (8.90e-4) -	5.6877e-1 (2.35e-1) -	7.8522e-1 (1.77e-1) -	7.9859e-1 (1.80e-1) -	9.5215e-1 (1.93e-1) -	3.9248e-1 (2.36e-1) -	1.8261e-1 (2.69e-2)
ZDT1	2	30	1.8597e-2 (3.81e-4) +	1.4555e-1 (7.72e-2) -	2.3582e-2 (2.93e-3) -	2.2811e-2 (2.12e-3) -	2.0557e-1 (1.04e-1) -	2.4239e-2 (1.80e-3) -	2.0050e-2 (4.99e-4)
ZDT2	2	30	1.6536e-2 (3.59e-5) -	7.7986e-2 (5.58e-2) -	2.5105e-2 (3.39e-3) -	2.3315e-2 (2.88e-3) -	9.6861e-2 (4.15e-2) -	2.5094e-2 (3.20e-3) -	1.6419e-2 (1.31e-4)
ZDT3	2	30	3.9232e-2 (4.61e-3) -	7.6556e-2 (2.41e-2) -	4.3499e-2 (2.12e-2) -	5.6039e-2 (5.08e-2) -	1.5926e-1 (6.80e-2) -	3.6243e-2 (1.09e-2) -	2.9499e-2 (1.89e-3)
ZDT4	2	10	2.0096e-2 (1.52e-3) -	7.8296e-2 (4.71e-2) -	2.1890e-2 (2.86e-3) -	1.9841e-2 (2.68e-3) -	3.0877e-1 (8.08e-2) -	2.3003e-2 (2.55e-3) -	1.8816e-2 (1.56e-4)
ZDT6	2	10	1.4728e-2 (4.91e-5) +	2.9075e-2 (5.10e-3) -	1.7981e-2 (3.22e-3) =	1.7839e-2 (3.42e-3) =	2.2485e-2 (4.25e-3) -	2.2286e-2 (2.14e-3) -	1.7198e-2 (1.11e-3)
Average Ranking			4.42	9.67	6.88	6.88	11.83	6.71	2.04
+/-/=			3/8/1	0/12/0	0/9/3	1/9/2	0/12/0	0/1/1	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

Table S - VI. IGD of μ MOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAI and SPEAI on 5-objective problems DTLZ

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAI	SPEAI	μ MOEA
DTLZ1	5	9	3.0167e-1 (8.17e-2) -	8.2887e-2 (6.21e-2) =	4.7394e+0 (4.24e+0) -	5.1382e-1 (1.25e-2) -	1.6823e-1 (1.32e-1) -	1.5230e-1 (1.49e-1) -	1.0543e-1 (1.82e-1)
DTLZ2	5	14	4.8339e-1 (1.13e-1) -	1.1154e-1 (1.45e-1) =	3.8034e-2 (2.07e-1) +	4.3014e-1 (1.16e-1) -	3.0980e-1 (1.82e-2) -	4.7155e-1 (1.55e-1) -	1.1037e-1 (7.41e-2)
DTLZ3	5	14	5.1701e+0 (2.61e+0) -	5.2382e+0 (5.81e+0) -	6.8623e+0 (2.15e+1) -	1.3004e+0 (3.58e-1) -	1.2948e+1 (6.11e+0) -	5.8478e+1 (1.88e+1) -	1.0806e+0 (1.03e+0)
DTLZ4	5	14	4.1045e-1 (7.16e-2) -	1.3750e-1 (1.34e-1) -	9.9475e-1 (2.15e-1) -	8.2703e-1 (2.05e-1) -	3.2687e-1 (2.27e-2) -	8.8347e-1 (5.18e-1) -	1.3431e-1 (4.01e-2)
DTLZ5	5	14	9.6405e-2 (5.68e-2) +	1.6860e-1 (6.73e-2) =	7.4035e-1 (7.82e-5) -	2.7614e-1 (1.24e-1) -	4.1741e-1 (1.12e-1) -	2.3840e+0 (7.28e-2) -	1.8094e-1 (4.12e-2)
DTLZ6	5	14	1.3789e-1 (6.21e-2) +	1.9417e-1 (6.46e-2) =	3.2814e-1 (1.79e-1) -	3.2099e-1 (9.90e-2) -	4.9048e+0 (8.05e-1) -	9.9950e+0 (3.20e-2) -	2.0130e-1 (1.11e-1)
DTLZ7	5	24	7.4932e-1 (1.42e-1) +	9.7811e-1 (9.08e-2) -	4.1178e+0 (3.57e+0) -	1.4565e+0 (5.99e-1) -	6.9203e-1 (3.18e-2) +	6.1580e-1 (4.60e-2) +	8.5605e-1 (1.09e-1)
Average Ranking			4.57	3.29	8.71	6.57	5.43	8.57	2.71
+/-/=			3/4/0	0/3/4	1/6/0	0/7/0	1/6/0	1/6/0	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

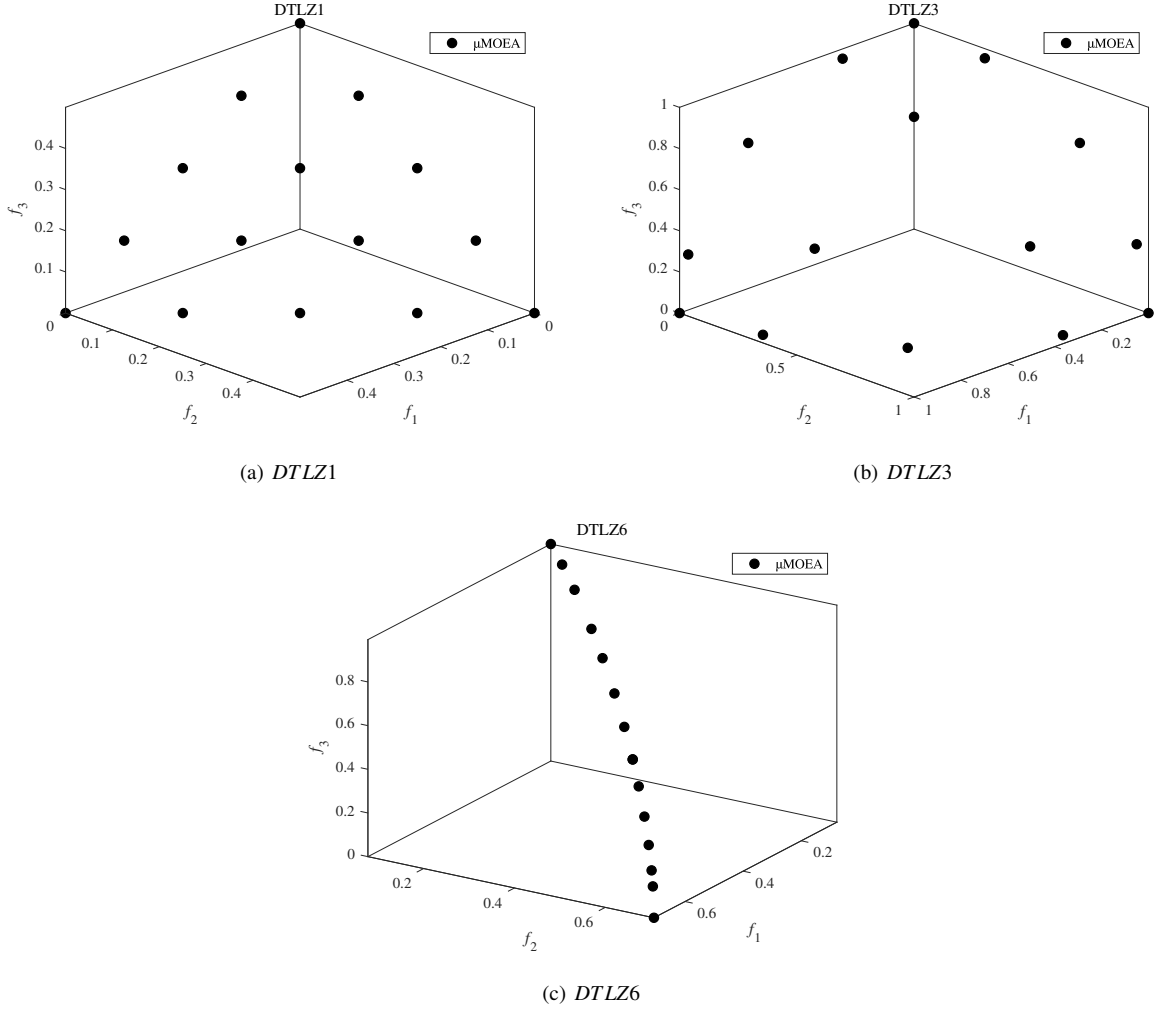


Figure S - 1. The distribution of the final non-dominated solutions by μ MOEA on DTLZ. (a) DTLZ1. (b) DTLZ3. (c) DTLZ6.

Table S - VII. IGD of μ MOEA with RVEA, PESAI, AMGA, AMGAI, μ GA and μ MOGA on 5-objective problems DTLZ

Problem	M	D	RVEA	PESAI	AMGA	AMGAI	μ GA	μ MOGA	μ MOEA
DTLZ1	5	9	1.7127e-1 (8.65e-2)	1.1823e+0 (1.17e+0)	1.9947e-1 (1.23e-1)	5.9121e+0 (2.20e+0)	1.0941e+0 (1.26e+0)	3.4354e-1 (2.70e-1)	1.0543e-1 (1.82e-1)
DTLZ2	5	14	5.6381e-1 (3.70e-1)	8.1259e-1 (5.01e-2)	3.0686e-1 (1.94e-2)	5.0384e-1 (2.90e-2)	8.5216e-1 (9.93e-2)	3.1140e-1 (2.52e-2)	1.1037e-1 (7.41e-2)
DTLZ3	5	14	1.0743e+0 (1.86e+0)	1.4342e+1 (1.47e+1)	1.7186e+1 (1.41e+1)	1.2263e+2 (1.87e+1)	2.4819e+1 (1.49e+1)	2.2751e+1 (1.28e+1)	1.0806e+0 (1.03e+0)
DTLZ4	5	14	3.9854e-1 (2.56e-1)	5.3132e-1 (6.12e-2)	7.1955e-1 (4.24e-1)	9.5360e-1 (2.23e-1)	9.7962e-1 (1.57e-1)	4.8344e-1 (3.33e-1)	1.3431e-1 (4.01e-2)
DTLZ5	5	14	5.1119e-1 (2.44e-1)	8.6620e-1 (2.37e-1)	4.0998e-1 (1.19e-1)	5.9841e-1 (1.51e-1)	5.6735e-1 (3.03e-1)	4.9949e-1 (1.78e-1)	1.8094e-1 (4.12e-2)
DTLZ6	5	14	4.4436e-1 (1.84e-1)	6.6955e+0 (1.41e+0)	3.9385e+0 (1.35e+0)	1.3011e+0 (4.55e-1)	6.5046e+0 (1.64e+0)	5.1305e+0 (8.48e-1)	2.0130e-1 (1.11e-1)
DTLZ7	5	24	9.0465e-1 (1.05e-1)	2.5523e+0 (3.07e-1)	6.4978e-1 (2.82e-2)	1.2229e+0 (3.98e-1)	2.6841e+0 (3.44e-1)	6.8590e-1 (3.59e-2)	8.5605e-1 (1.09e-1)
Average Ranking			6.00	10.43	6.00	10.43	11.14	7.14	2.71
+/-/=			0/5/2	0/7/0	1/6/0	0/7/0	0/7/0	1/6/0	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

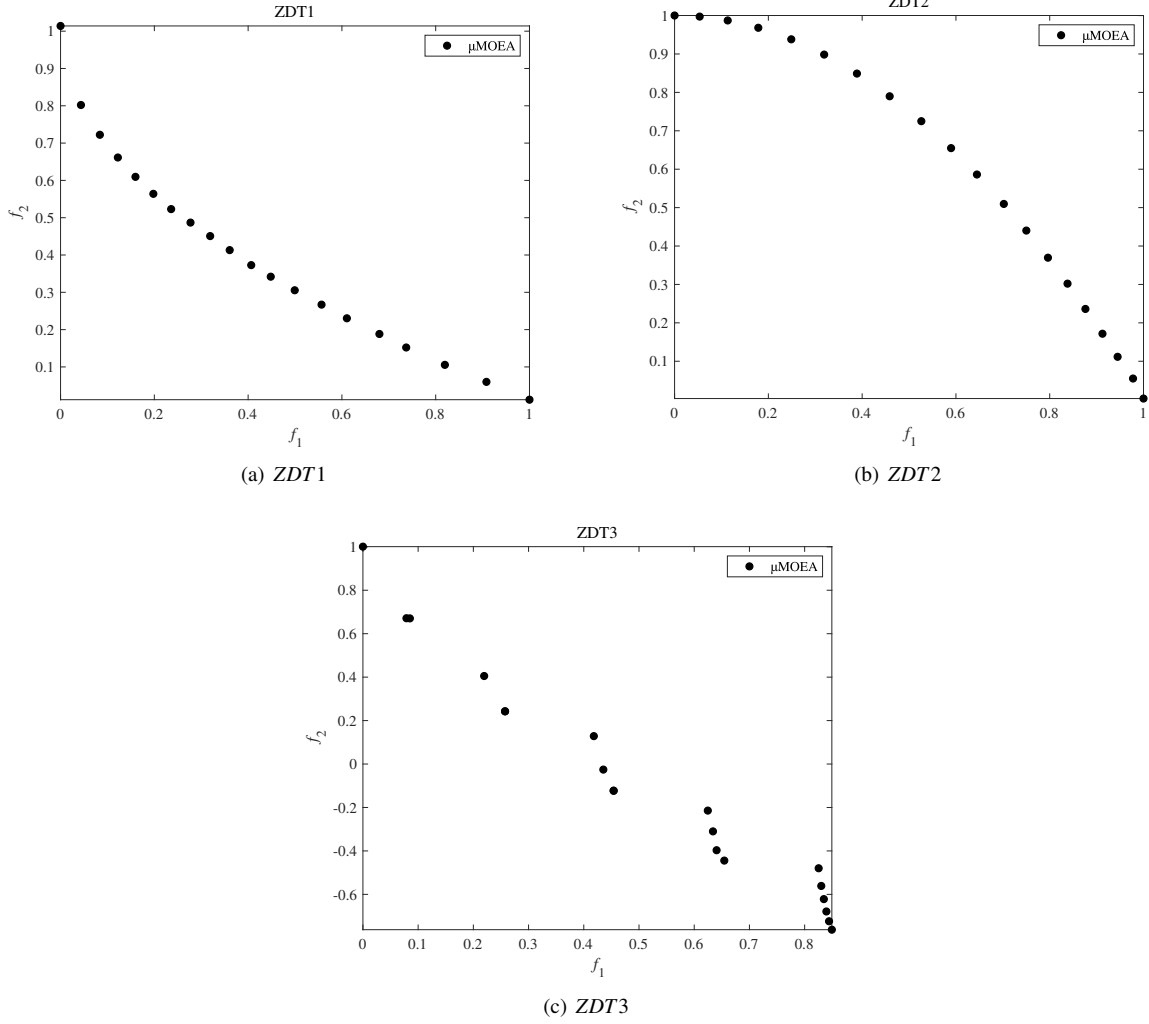


Figure S - 2. The distribution of the final non-dominated solutions by μ MOEA on ZDT. (a) ZDT1. (b) ZDT2. (c) ZDT3.

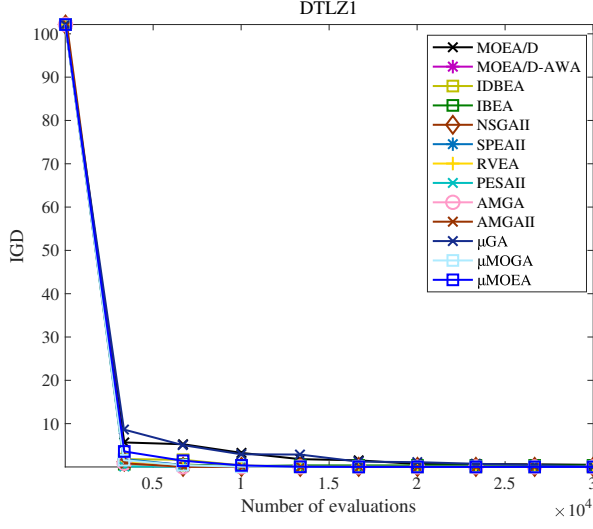
Table S - VIII. IGD of μ MOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAII and SPEAII on 5-objective problems MaF

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAII	SPEAII	μ MOEA
MaF1	5	14	4.5787e-1 (2.23e-3) +	7.5095e-1 (3.40e-1) -	4.8791e-1 (2.14e-2) =	3.8307e-1 (5.15e-2) +	2.2816e-1 (1.45e-2) +	1.6302e-1 (2.22e-2) +	4.9144e-1 (2.01e-2)
MaF3	5	14	3.8624e-1 (5.43e-2) -	5.3497e+10 (1.46e+11) -	4.6599e+7 (2.09e+8) -	1.0126e+0 (2.13e-3) -	1.1887e+0 (1.88e+0) -	3.7929e+1 (1.27e+2) -	8.2859e-2 (9.36e-3)
MaF4	5	14	5.8607e+0 (2.17e+0) -	2.6071e+1 (7.11e+1) -	5.7435e+0 (7.84e-2) -	1.3683e+1 (6.44e-1) -	2.8588e+0 (3.16e-1) +	3.0651e+0 (3.09e-1) +	3.5622e+0 (5.42e-1)
MaF5	5	14	4.2334e+0 (1.07e+0) -	3.3127e+0 (1.22e+0) =	1.4651e+1 (9.86e+0) -	8.3215e+0 (6.75e+0) -	2.9846e+0 (2.41e-1) =	6.3281e+0 (4.06e+0) -	2.9020e+0 (2.40e-1)
MaF7	5	24	7.8407e-1 (4.36e-2) -	1.4449e+0 (5.60e-1) -	4.1320e+0 (3.48e+0) -	9.8021e-1 (3.29e-1) -	6.8218e-1 (4.53e-2) =	6.0252e-1 (3.55e-2) +	7.1298e-1 (8.20e-2)
MaF8	5	2	5.8957e-1 (7.90e-2) -	4.5132e-1 (1.26e-1) =	3.7276e-1 (1.03e-2) +	1.0037e+0 (1.16e-1) -	4.1072e-1 (6.19e-2) +	2.6945e-1 (5.02e-2) +	4.4988e-1 (7.35e-2)
MaF9	5	2	7.4034e-1 (1.70e-1) -	1.6539e+0 (2.74e+0) -	3.2684e-1 (2.65e-2) +	1.2005e+0 (1.70e-1) -	1.0024e+2 (1.35e+2) -	1.5166e+0 (6.85e-1) -	4.9157e-1 (1.70e-1)
MaF10	5	14	1.9875e+0 (8.17e-2) -	2.4653e+0 (1.12e+0) -	2.3013e+0 (8.22e-1) -	1.8278e+0 (5.33e-1) -	1.2613e+0 (1.46e-1) =	1.1526e+0 (2.90e-1) +	1.2606e+0 (2.01e-1)
MaF11	5	14	1.0692e+0 (1.10e-1) =	1.0608e+0 (2.34e-1) =	2.3110e-1 (1.99e-1) +	1.8639e+0 (1.31e-1) -	1.2625e+0 (1.34e-1) -	1.3381e+0 (1.28e-1) -	1.0741e+0 (1.93e-1)
MaF12	5	14	2.3412e+0 (3.68e-1) -	1.3975e+0 (1.22e-1) +	3.5502e-1 (1.25e-1) +	1.0942e+0 (1.08e-1) +	1.9135e+0 (1.46e-1) -	1.7822e+0 (1.65e-1) -	1.5568e+0 (1.93e-1)
MaF13	5	5	5.0035e-1 (3.50e-2) -	7.1846e-1 (2.67e-1) -	6.0581e-1 (2.78e-1) =	5.4509e-1 (1.07e-1) -	5.8747e-1 (1.75e-1) -	1.0653e+0 (1.36e-1) -	4.4558e-1 (7.93e-2)
MaF14	5	100	9.3763e-1 (1.17e-1) +	6.0517e+3 (1.03e+4) -	1.2754e+3 (3.47e+3) -	1.5978e+0 (8.19e-1) =	1.2416e+0 (2.40e-1) -	3.2763e+1 (8.06e+1) -	1.0142e+0 (1.12e-1)
MaF15	5	100	9.3922e-1 (5.98e-2) -	1.0729e+0 (3.07e-1) -	1.0066e+0 (4.89e-2) -	1.1166e+0 (5.99e-2) -	1.2836e+1 (4.39e+0) -	4.9979e+1 (1.85e+1) -	8.5853e-1 (5.09e-2)
Average Ranking			5.73	8.65	7.19	7.23	5.69	6.31	3.92
+/-/=			2/10/1	1/9/3	4/7/2	2/10/1	3/7/3	5/8/0	

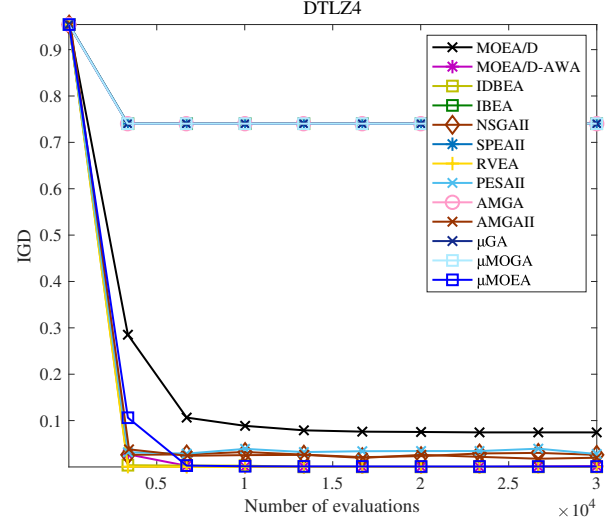
M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

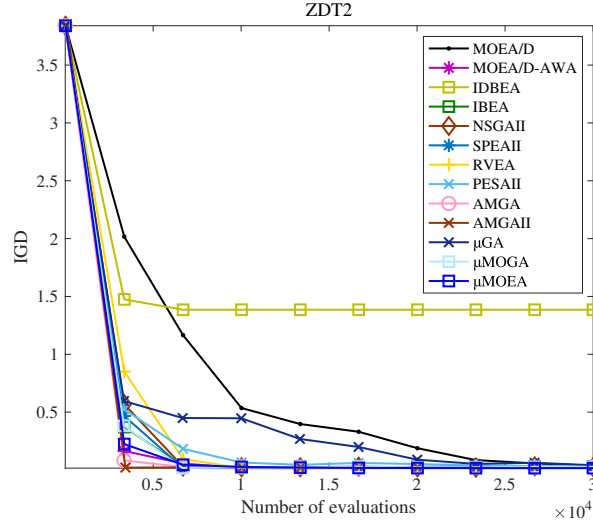
“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.



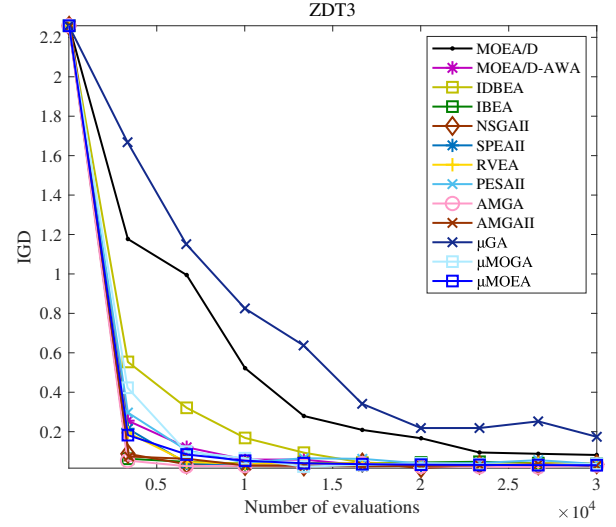
(a) DTLZ1



(b) DTLZ4

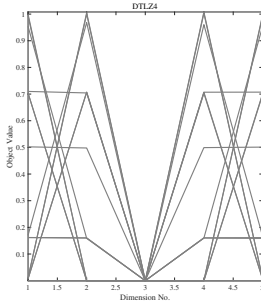


(c) ZDT2

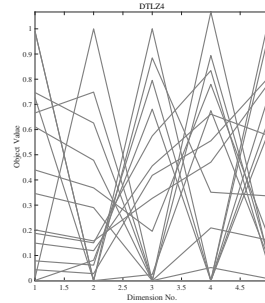


(d) ZDT3

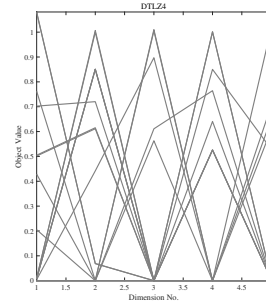
Figure S - 3. Convergence curves of μ MOEA with other comparison algorithms. (a) DTLZ1. (b) DTLZ4. (c) ZDT2. (d) ZDT3.



(a) RVEA



(b) NSGAI



(c) MOEA/D-AWA

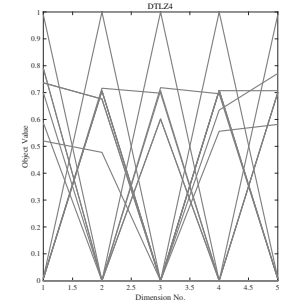
(d) μ MOEA

Figure S - 4. The final solution set of the best four algorithms on the 5-objective DTLZ4. (a) RVEA. (b) NSGAI. (c) MOEA/D-AWA. (d) μ MOEA.

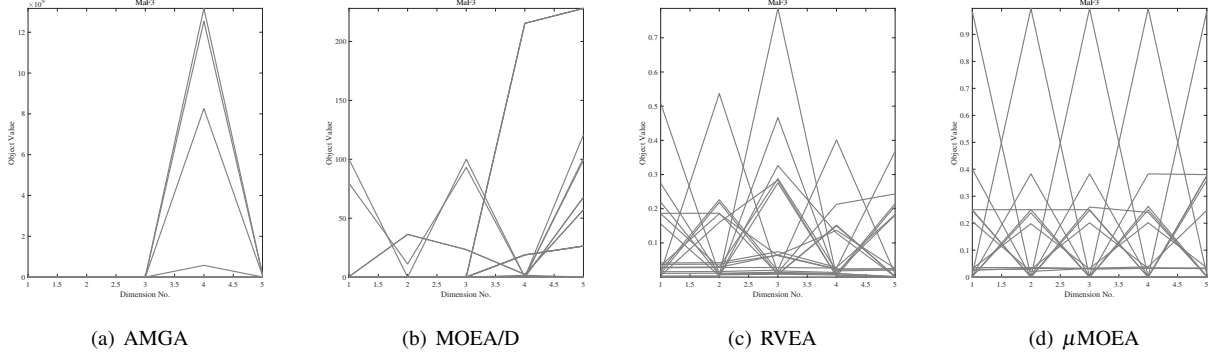


Figure S - 5. The final solution set of the best four algorithms on the 5-objective MaF3. (a) AMGA. (b) MOEA/D. (c) RVEA. (d) μ MOEA.

Table S - IX. IGD of μ MOEA with RVEA, PESAI, AMGA, AMGAI, μ GA and μ MOGA on 5-objective problems MaF

Problem	M	D	RVEA	PESAI	AMGA	AMGAI	μ GA	μ MOGA	μ MOEA
MaF1	5	14	6.7383e-1 (9.24e-2) -	5.5800e-1 (4.03e-2) -	2.1884e-1 (1.25e-2) +	2.7309e-1 (2.21e-2) +	6.4768e-1 (1.00e-1) -	2.2282e-1 (1.26e-2) +	4.9144e-1 (2.01e-2)
MaF3	5	14	2.2012e-1 (9.64e-3) -	4.9196e+5 (2.45e+6) -	4.6583e-1 (6.38e-1) -	1.1013e+4 (3.75e+3) -	3.4849e+3 (1.37e+4) -	2.8719e+0 (3.25e+0) -	8.2859e-2 (9.36e-3)
MaF4	5	14	1.1240e+1 (3.67e+0) -	1.0529e+1 (1.61e+0) -	2.9044e+0 (3.07e-1) +	4.4153e+1 (6.91e+1) -	1.5693e+1 (1.08e+1) -	2.8636e+0 (2.93e-1) +	3.5622e+0 (5.42e-1)
MaF5	5	14	3.3123e+0 (4.60e+0) =	5.6212e+0 (1.26e+0) -	1.4148e+1 (1.11e+1) -	9.3303e+0 (3.21e+0) -	1.5370e+1 (8.19e+0) -	5.1114e+0 (6.54e+0) =	2.9020e+0 (2.40e-1)
MaF7	5	24	1.0719e+0 (5.55e-2) -	2.5349e+0 (2.23e-1) -	6.7944e-1 (4.24e-2) =	1.2311e+0 (3.45e-1) -	2.8085e+0 (3.29e-1) -	6.8864e-1 (4.44e-2) =	7.1298e-1 (8.20e-2)
MaF8	5	2	6.3534e-1 (1.19e-1) -	5.3955e-1 (1.52e-1) -	3.4179e-1 (3.29e-2) +	3.5882e-1 (5.63e-2) +	5.3582e-1 (1.04e-1) -	3.7262e-1 (5.56e-2) +	4.4988e-1 (7.35e-2)
MaF9	5	2	5.4332e-1 (1.82e-1) =	6.2116e+1 (7.45e+1) -	1.3801e+2 (1.86e+2) -	1.4798e+1 (1.75e+1) -	1.8420e+1 (1.97e+1) -	1.4462e+1 (2.27e+1) -	4.9157e-1 (1.70e-1)
MaF10	5	14	8.1580e-1 (1.88e-1) +	2.5974e+0 (2.69e-1) -	1.5912e+0 (1.57e-1) -	2.2721e+0 (7.87e-2) -	3.0924e+0 (3.45e-1) -	1.3982e+0 (2.26e-1) -	1.2606e+0 (2.01e-1)
MaF11	5	14	1.4357e+0 (2.69e-2) -	3.3221e+0 (8.17e-1) -	1.2873e+0 (1.55e-1) -	1.2345e+0 (1.57e-1) -	4.3811e+0 (1.03e+0) -	1.2780e+0 (1.88e-1) -	1.0741e+0 (1.93e-1)
MaF12	5	14	1.9567e-1 (1.63e-1) +	4.0085e+0 (7.25e-1) -	1.8358e+0 (1.50e-1) -	2.2398e+0 (1.85e-1) -	5.5253e+0 (8.89e-1) -	1.9270e+0 (1.50e-1) -	1.5568e+0 (1.93e-1)
MaF13	5	5	9.1752e-1 (1.23e-1) -	7.2866e-1 (1.90e-1) -	5.4162e-1 (1.56e-1) -	4.0629e-1 (4.82e-2) +	6.9209e-1 (1.71e-1) -	6.6576e-1 (2.13e-1) -	4.4558e-1 (7.93e-2)
MaF14	5	100	8.6088e-1 (1.81e-1) +	1.0658e+1 (1.25e+1) -	9.8463e-1 (7.62e-2) =	2.3502e+0 (1.53e+0) -	2.0499e+2 (5.43e+2) -	2.3835e+0 (1.34e+0) -	1.0142e+0 (1.12e-1)
MaF15	5	100	8.3860e-1 (6.17e-2) =	1.2319e+0 (6.39e-2) -	8.2411e-1 (5.23e-1) +	1.9874e+0 (1.65e+0) -	2.3109e+0 (2.62e+0) -	1.7578e+1 (1.05e+1) -	8.5853e-1 (5.09e-2)
Average Ranking			5.88	10.15	5.08	7.69	11.08	6.38	3.92
+/-/=			3/7/3	0/13/0	4/7/2	3/10/0	0/13/0	3/8/2	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

Table S - X. IGD of μ MOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAI and SPEAI on sparse MOP

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAI	SPEAI	μ MOEA
SMOP1	2	100	3.1428e-1 (1.56e-2) -	1.3862e-2 (6.32e-3) +	3.1464e-2 (9.40e-3) -	2.1135e-2 (2.54e-3) =	3.7273e-2 (6.63e-3) -	2.8398e-2 (6.12e-3) -	1.9873e-2 (7.69e-3)
SMOP2	2	100	9.4525e-1 (3.24e-2) -	6.7078e-1 (7.91e-2) -	7.4212e-1 (5.43e-2) -	7.4594e-1 (7.26e-2) -	7.3258e-1 (7.08e-2) -	7.1297e-1 (6.97e-2) -	1.6854e-1 (5.54e-2)
SMOP3	2	100	1.5530e+0 (5.61e-2) -	9.6450e-1 (6.67e-2) -	9.2103e-1 (6.15e-2) -	8.8756e-1 (8.14e-2) -	8.7101e-1 (5.27e-2) -	8.4655e-1 (5.90e-2) -	7.8533e-1 (1.31e-2)
SMOP4	2	100	4.5293e-1 (1.69e-2) -	3.3726e-1 (6.84e-2) -	3.4205e-1 (3.50e-2) -	3.2744e-1 (3.70e-2) -	3.2580e-1 (2.97e-2) -	3.0369e-1 (3.73e-2) -	2.6627e-2 (1.96e-2)
SMOP5	2	100	3.8406e-1 (5.55e-3) -	3.4337e-1 (1.09e-2) -	3.3846e-1 (2.78e-4) +	3.3855e-1 (1.14e-3) +	3.4559e-1 (2.79e-3) -	3.4208e-1 (8.82e-4) -	3.3886e-1 (6.14e-4)
SMOP6	2	100	1.0930e-1 (5.48e-3) -	2.5024e-2 (1.96e-2) +	1.3972e-2 (2.07e-3) +	5.7219e-2 (1.06e-2) +	3.0016e-2 (3.13e-3) +	2.6500e-2 (1.33e-3) +	6.3074e-2 (8.19e-3)
SMOP7	2	100	4.6937e-1 (2.19e-2) -	1.8740e-1 (2.27e-2) -	2.4948e-1 (3.71e-2) -	1.9870e-1 (1.75e-2) -	2.0726e-1 (2.52e-2) -	2.0680e-1 (2.25e-2) -	1.6663e-1 (2.01e-2)
SMOP8	2	100	2.0316e+0 (7.95e-2) -	1.4710e+0 (1.07e-1) -	1.5029e+0 (1.51e-1) -	1.3667e+0 (9.43e-2) -	1.3731e+0 (1.21e-1) -	1.4041e+0 (1.21e-1) -	1.0619e+0 (7.79e-2)
Average Ranking			12.00	5.44	6.50	5.56	6.13	5.13	3.00
+/-/=			0/8/0	2/6/0	2/6/0	2/5/1	1/7/0	1/7/0	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

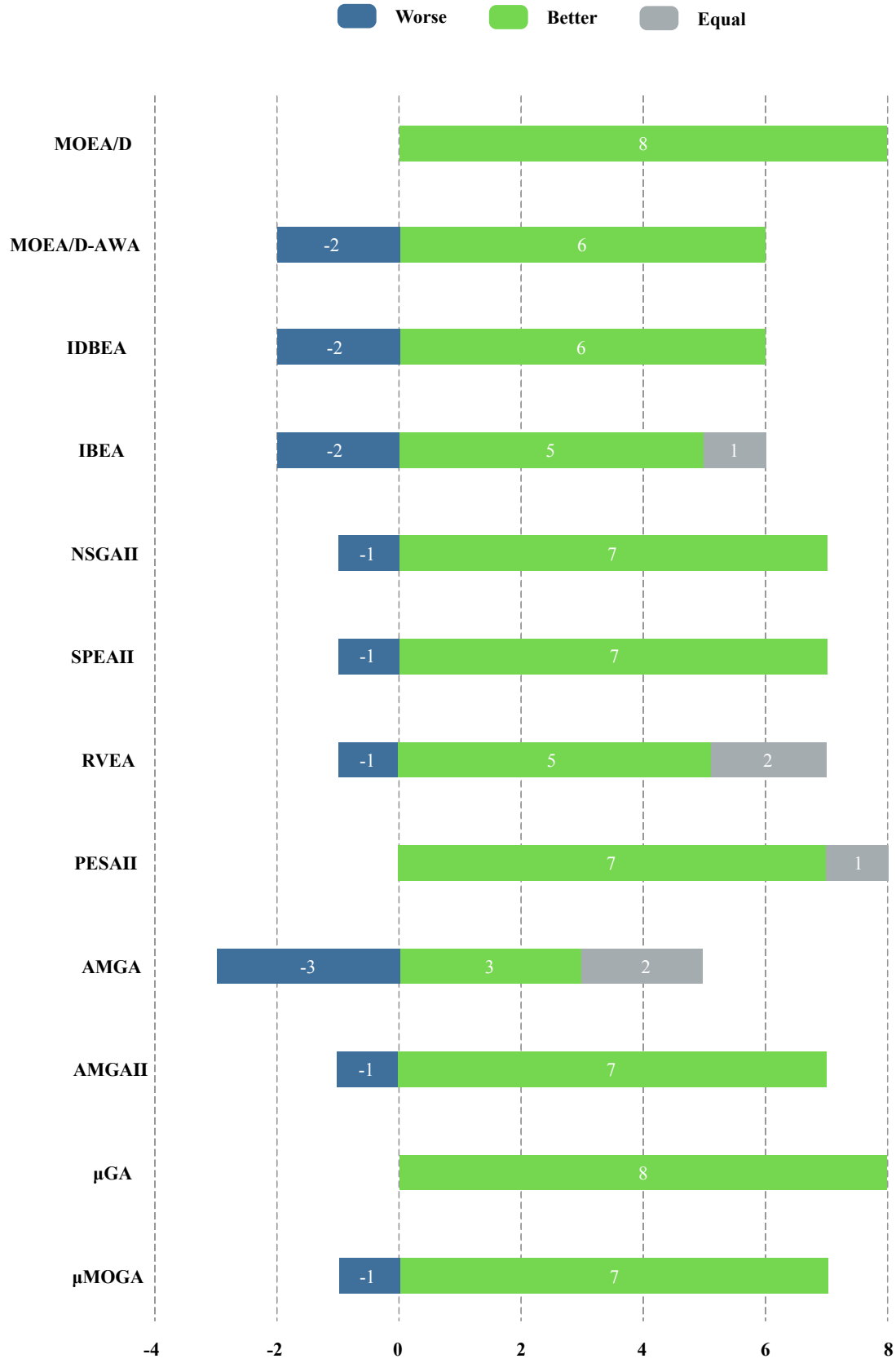


Figure S - 6. Columnar chart of comparison among μ MOEA and other algorithms on large-scale sparse MOP. Each row represents the result of comparison with a competitor algorithm, where the green denotes the better number of test problems that μ MOEA performs better than this competitor algorithm, the blue shows the worse number, the grey refers to the equal number.

Table S - XI. IGD of μ MOEA with RVEA, PESAI, AMGA, AMGAI, μ GA and μ MOGA on sparse MOP

Problem	M	D	RVEA	PESAI	AMGA	AMGAI	μ GA	μ MOGA	μ MOEA
SMOP1	2	100	1.9453e-2 (8.84e-3) =	1.5058e-1 (7.56e-2) -	2.0994e-2 (2.70e-3) =	1.8123e-1 (4.07e-2) -	3.1726e-1 (4.72e-2) -	5.7681e-2 (7.59e-3) -	1.9873e-2 (7.69e-3)
SMOP2	2	100	7.5561e-1 (6.23e-2) -	8.0021e-1 (6.13e-2) -	6.5815e-1 (3.13e-1) -	4.6328e-1 (4.56e-2) -	1.0982e+0 (8.06e-2) -	7.5288e-1 (6.35e-2) -	1.6854e-1 (5.54e-2)
SMOP3	2	100	9.1181e-1 (6.59e-2) -	9.2736e-1 (8.09e-2) -	6.8240e-1 (1.80e-2) +	1.1723e+0 (1.44e-1) -	1.5930e+0 (9.63e-2) -	1.0611e+0 (6.38e-2) -	7.8533e-1 (1.31e-2)
SMOP4	2	100	3.4625e-1 (3.72e-2) -	3.7079e-1 (5.13e-2) -	2.2549e-1 (1.74e-1) -	1.4242e-1 (4.88e-2) -	5.7719e-1 (6.01e-2) -	3.6806e-1 (4.36e-2) -	2.6627e-2 (1.96e-2)
SMOP5	2	100	3.3868e-1 (3.24e-4) =	3.5261e-1 (7.03e-3) -	3.4225e-1 (1.98e-3) -	3.7004e-1 (1.28e-2) -	4.2204e-1 (2.64e-2) -	3.4512e-1 (1.65e-3) -	3.3886e-1 (6.14e-4)
SMOP6	2	100	1.1171e-2 (1.48e-3) +	6.4885e-2 (1.99e-2) =	2.2438e-2 (1.95e-3) +	9.5829e-2 (1.84e-2) -	1.4640e-1 (3.19e-2) -	3.3083e-2 (3.60e-3) +	6.3074e-2 (8.19e-3)
SMOP7	2	100	1.9186e-1 (2.03e-2) -	2.3099e-1 (2.47e-2) -	1.2898e-1 (4.87e-2) +	4.1538e-1 (2.48e-2) -	5.0472e-1 (6.71e-2) -	2.1489e-1 (2.50e-2) -	1.6663e-1 (2.01e-2)
SMOP8	2	100	1.4322e+0 (1.03e-1) -	1.4722e+0 (1.04e-1) -	9.8243e-1 (2.76e-1) =	8.6958e-1 (1.31e-2) +	2.1032e+0 (1.32e-1) -	1.5545e+0 (1.40e-1) -	1.0619e+0 (7.79e-2)
Average Ranking			5.25	9.69	2.81	7.50	13.00	9.00	3.00
+/-/=			1/5/2	0/7/1	3/3/2	1/7/0	0/8/0	1/7/0	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

Table S - XII. IGD of μ MOEA with MOEA/D, MOEA/D-AWA, IBEA, IDBEA, NSGAI and SPEAI in normal population size

Problem	M	D	MOEA/D	MOEA/D-AWA	IDBEA	IBEA	NSGAI	SPEAI	μ MOEA
DTLZ1	3	7	3.3122e-2 (2.48e-4) -	7.4641e-3 (2.43e-3) -	3.3965e-2 (8.08e-2) -	1.5029e-1 (2.41e-2) -	3.1878e-2 (2.50e-2) -	1.7165e-2 (1.41e-3) -	3.0654e-3 (1.65e-3)
DTLZ2	3	12	7.5989e-2 (6.44e-4) -	9.8647e-3 (1.38e-3) +	1.0184e-3 (5.58e-4) +	7.5617e-2 (2.78e-3) -	7.1734e-2 (3.16e-3) -	4.9092e-2 (2.33e-3) -	2.5972e-2 (6.42e-3)
DTLZ3	3	12	1.8849e-1 (2.86e-1) -	2.6809e-1 (3.52e-1) -	3.3015e+0 (1.83e+0) -	4.7721e-1 (1.39e-1) -	3.2791e-1 (6.14e-1) -	2.4162e-1 (4.94e-1) -	5.7476e-2 (1.12e-1)
DTLZ4	3	12	4.6368e-1 (3.01e-1) -	9.8242e-2 (1.97e-1) -	2.8600e-1 (2.99e-1) =	7.5133e-2 (4.15e-3) -	6.9768e-2 (2.82e-3) -	2.2215e-1 (2.85e-1) -	3.0958e-2 (3.30e-3)
DTLZ5	3	12	1.4549e-2 (2.43e-5) -	1.1459e-2 (3.34e-4) -	1.6699e-2 (4.22e-3) -	1.5730e-2 (1.35e-3) -	5.6999e-3 (3.40e-4) -	4.3124e-3 (2.21e-4) +	4.6688e-3 (3.21e-4)
DTLZ6	3	12	1.4560e-2 (7.15e-6) -	1.1439e-2 (4.15e-4) -	2.0050e-2 (1.26e-2) -	2.5799e-2 (4.13e-3) -	5.8603e-3 (3.13e-4) -	3.9330e-3 (3.15e-4) +	4.5486e-3 (2.99e-4)
DTLZ7	3	22	2.3037e-1 (7.44e-2) -	1.4559e-1 (4.70e-2) -	2.1108e-1 (5.30e-1) -	7.8502e-2 (3.89e-3) -	8.7673e-2 (5.52e-2) -	6.9679e-2 (5.64e-2) +	7.2336e-2 (4.74e-3)
ZDT1	2	30	4.3503e-3 (3.01e-3) +	3.7275e-3 (1.40e-4) +	8.9363e-3 (1.84e-3) -	4.4443e-3 (2.58e-4) +	4.6658e-3 (2.44e-4) +	3.9355e-3 (1.70e-4) +	8.2224e-3 (9.71e-4)
ZDT2	2	30	3.7118e-3 (6.38e-4) +	3.6890e-3 (6.36e-4) +	7.3848e-2 (1.64e-1) -	8.1970e-3 (7.59e-4) -	4.8126e-3 (2.70e-4) -	3.8232e-3 (2.33e-4) +	4.4173e-3 (5.87e-4)
ZDT3	2	30	1.1115e-2 (8.30e-3) -	6.4048e-3 (1.81e-3) -	2.1833e-2 (6.29e-3) -	1.4184e-2 (6.03e-3) -	5.0664e-3 (7.57e-4) =	4.8123e-3 (5.02e-4) =	4.9569e-3 (6.28e-4)
ZDT4	2	10	7.2802e-3 (2.05e-3) -	5.7079e-3 (1.19e-3) -	3.3268e+0 (2.64e+0) -	9.1160e-2 (7.25e-2) -	5.7978e-3 (1.30e-3) -	5.0226e-3 (9.83e-4) -	3.7070e-3 (1.37e-4)
ZDT6	2	10	3.0740e-3 (6.81e-5) =	3.0490e-3 (7.59e-5) =	8.3807e-2 (2.83e-1) -	4.3584e-3 (2.01e-4) -	3.6175e-3 (2.36e-4) -	3.1083e-3 (1.94e-4) =	3.0525e-3 (1.36e-4)
Average Ranking			6.74	3.96	9.33	8.42	5.79	2.92	2.88
+/-/=			2/9/1	3/8/1	1/10/1	1/11/0	1/10/1	5/5/2	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

Table S - XIII. IGD of μ MOEA with RVEA, PESAI, AMGA, AMGAI, μ GA and μ MOGA in normal population size

Problem	M	D	RVEA	PESAI	AMGA	AMGAI	μ GA	μ MOGA	μ MOEA
DTLZ1	3	7	3.4354e-3 (2.75e-3) =	2.7317e-2 (2.19e-3) -	4.3451e-2 (5.54e-2) -	9.8155e-1 (6.89e-1) -	1.1528e+1 (5.80e+0) -	5.1640e-2 (1.00e-1) -	3.0654e-3 (1.65e-3)
DTLZ2	3	12	5.8894e-4 (2.96e-4) +	7.1230e-2 (5.34e-3) -	7.4608e-2 (4.12e-3) -	7.2352e-2 (3.11e-3) -	2.3487e-1 (8.36e-2) -	7.1233e-2 (3.73e-3) -	2.5972e-2 (6.42e-3)
DTLZ3	3	12	1.1856e+0 (1.41e+0) -	3.7660e-1 (5.91e-1) -	6.8491e+0 (4.20e+0) -	2.9122e+1 (9.74e+0) -	8.1363e+1 (2.50e+1) -	8.8830e-1 (1.01e+0) -	5.7476e-2 (1.12e-1)
DTLZ4	3	12	8.4028e-4 (1.03e-3) +	6.3613e-2 (3.03e-3) -	6.7634e-1 (4.08e-1) -	4.0748e-1 (4.29e-1) -	8.9458e-1 (1.45e-1) -	8.0348e-1 (3.34e-1) -	3.0958e-2 (3.30e-3)
DTLZ5	3	12	7.8271e-2 (1.07e-2) -	1.1652e-2 (1.43e-3) -	4.6750e-3 (3.12e-4) =	4.7327e-3 (3.17e-4) =	7.5945e-2 (7.19e-2) -	5.1951e-3 (3.18e-4) -	4.6688e-3 (3.21e-4)
DTLZ6	3	12	8.6033e-2 (1.93e-2) -	1.3459e-2 (2.17e-3) -	4.6459e-3 (2.18e-4) =	4.5032e-3 (2.70e-4) =	4.3599e+0 (1.18e+0) -	5.4106e-3 (3.25e-4) -	4.5486e-3 (2.99e-4)
DTLZ7	3	22	1.0661e-1 (2.58e-3) -	2.0772e-1 (1.94e-1) -	5.3358e-1 (2.98e-1) -	3.3225e-1 (3.36e-1) -	1.9010e+0 (1.35e+0) -	3.4930e-1 (2.88e-1) -	7.2336e-2 (4.74e-3)
ZDT1	2	30	1.9885e-2 (2.99e-3) -	1.1589e-2 (1.65e-3) -	3.9614e-3 (1.85e-4) +	3.9236e-3 (2.33e-4) +	8.5164e-1 (2.89e-1) -	4.3687e-3 (2.41e-4) +	8.2224e-3 (9.71e-4)
ZDT2	2	30	2.9768e-2 (3.98e-3) -	1.1257e-2 (1.87e-3) -	4.2012e-3 (2.28e-4) =	4.1865e-3 (6.01e-4) +	5.2043e-1 (5.65e-2) -	4.5161e-3 (2.63e-4) =	4.4173e-3 (5.87e-4)
ZDT3	2	30	3.2039e-2 (6.16e-3) -	2.3495e-2 (2.30e-2) -	1.5258e-2 (4.84e-2) =	9.2831e-3 (1.09e-2) =	8.6642e-1 (3.29e-1) -	1.3151e-2 (1.36e-2) =	4.9569e-3 (6.28e-4)
ZDT4	2	10	5.7260e-2 (4.19e-2) -	1.2981e-2 (2.95e-3) -	5.4106e-3 (1.49e-3) -	5.0914e-2 (6.93e-2) -	2.1237e+1 (9.20e+0) -	5.9052e-3 (1.29e-3) -	3.7070e-3 (1.37e-4)
ZDT6	2	10	2.7566e-2 (5.47e-3) -	7.3764e-3 (8.06e-4) -	3.3039e-3 (2.45e-4) -	1.4390e-1 (2.48e-1) -	1.3640e-1 (1.51e-1) -	3.6210e-3 (1.92e-4) -	3.0525e-3 (1.36e-4)
Average Ranking			8.17	7.71	7.17	7.58	12.83	7.50	2.88
+/-/=			2/9/1	0/12/0	1/7/4	2/7/3	0/12/0	1/9/2	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The bottom part records the average ranking of these algorithms in the Friedman test.

“+”, “-”, “=” denote comparison algorithms are respectively better than, worse than, or close to μ MOEA through Wilcoxon rank test.

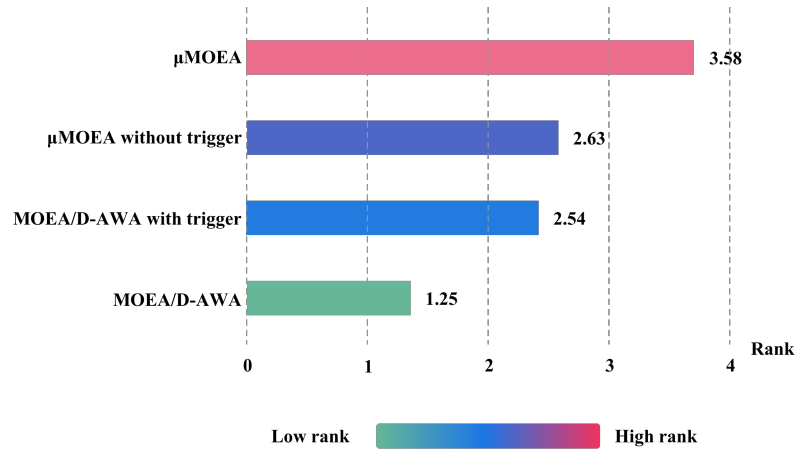


Figure S - 7. Ranking histogram of four types of algorithms for small population. The bottom shows the transition color from low to high ranking. The higher the ranking algorithm, the color tends to be red, and the ranking score value is larger.

MOEA/D-AWA [1] and experimental comparisons were made. In order to reduce the number of pages in the experimental part of the original text, the original text only shows the effective improvement rate. The detailed results are placed in Table S - XIV, and the average ranking obtained in descending order was plotted as histogram Figure S - 7, where it can be seen that μ MOEA performs best, and MOEA/D-AWA [1] with trigger mechanism performs much better than MOEA/D-AWA [1] without trigger mechanism.

VI. SUPPLEMENT TO SIMULATION BASE ON LOW-POWER EMBEDDED PROCESSOR

A. Semi-autogenous grinding optimization problem type-A

According to the obtained mathematical model [19], the power model f_1 , mill output ore model f_2 and steel ball wear model f_3 are selected as optimization objectives. That is:

$$\begin{aligned}
 &\text{minimize } P = c_1 D^3 \sin \alpha (x_1 + x_2 + x_3) (3.2 - 3v) N (1 - \frac{0.1}{2^{9-10N}}) \\
 &\text{minimize } \frac{1}{m_{out}} = \frac{1}{lx_1} \\
 &\text{minimize } m_{ball} = \beta (x_1 + x_3) \\
 &\text{subject to } x_1 + x_2 + x_3 - 355.55 \geq 0 \\
 &\quad 389.90 - x_1 \geq 0 \\
 &\quad 143.15 - x_2 \geq 0 \\
 &\quad 166.62 - x_3 \geq 0
 \end{aligned} \tag{1}$$

where $\min P$ is to ensure that the power consumption of the SAG is as low as possible to achieve energy saving and reduce consumption. $\min \frac{1}{m_{out}}$ is to ensure that the output of the SAG is as high as possible to achieve efficient output. $\min m_{ball}$ is to ensure that the amount of steel balls consumed by the SAG operation is as small as possible, and the loss is reduced while the output is efficient. The x_1 , x_2 , and x_3 in the objective function definition are decision variables, which respectively represent the quality of minerals, water, and steel balls retained in the SAG. The rest in the objective function definition are

some parameters, whose values and meanings can be seen in Table S - XV.

B. Semi-autogenous grinding optimization problem type-B

The specific definition of SAGOP-B is as follows:

$$\begin{aligned}
 &\text{minimize } P = c_2 D^3 \sin \alpha (x_1 + x_2 + x_3) (3.2 - 3v) N (1 - \frac{0.1}{2^{9-10N}}) \\
 &\text{minimize } -m_{out} = -lx_1 \\
 &\text{minimize } m_{ball} = \beta (x_1 + x_3) \\
 &\text{subject to } 50 \leq x_1 \leq 958.5 \\
 &\quad 12.5 \leq x_2 \leq 355 \\
 &\quad 98.45 \leq x_3 \leq 415.35
 \end{aligned} \tag{2}$$

where the definitions of objective function 1 and objective function 2 have been changed, the value range of decision variables has been adjusted, and the definitions of other parameters can also be found in Table S - XV.

C. Micro-grid energy optimization problem

The two objectives of Micro-grid energy optimization problem are operating cost f_1 and emission treatment cost f_2 respectively. The definition can be described as the following mathematical formula:

$$\begin{aligned}
 &\text{minimize } f_1 = C_F + C_D + C_M + C_g \\
 &\text{minimize } f_2 = \sum_{t=1}^T \{ \sum_{i=1}^M \sum_{j=1}^J [\xi_j a_{ij} P_i(t)] + \sum_{j=1}^J [\xi_j b_j P_{buy(t)}] \}
 \end{aligned} \tag{3}$$

where C_F , C_D , C_M , and C_g are fuel, depreciation, maintenance, and transaction costs, respectively. M and J denote the number of DGs and pollutant gas emissions. ξ_j is the pollution coefficient of the j -th gas, a_{ij} , and b_j are the emission coefficients of the j -th gas during the operation of the i -th DG and smart grid. Lastly, $P_i(t)$ and $P_{buy(t)}$ are the power generation of the i -th distributed power source and electricity purchased at time t . For more detailed information can be found in our previous work [20].

Table S - XIV. IGD of μ MOEA and MOEA/D-AWA in weight vectors trigger validity experiment for small population

Problem	M	D	MOEA/D-AWA		μ MOEA	
DTLZ1	3	7	no trigger with trigger	4.0201e-1 (6.97e-2) 1.0678e-2 (1.82e-2)	no trigger with trigger	4.3477e-3 (2.31e-3) 3.8681e-5 (2.07e-4)
DTLZ2	3	12	no trigger with trigger	4.7753e-1 (1.61e-1) 3.5249e-2 (7.41e-2)	no trigger with trigger	1.2175e-2 (1.44e-2) 8.2142e-4 (3.87e-4)
DTLZ3	3	12	no trigger with trigger	1.3837e+1 (1.84e+1) 1.2113e+1 (1.96e+1)	no trigger with trigger	2.2709e-1 (4.45e-1) 1.0989e-1 (3.02e-1)
DTLZ4	3	12	no trigger with trigger	1.0387e-1 (1.10e-1) 1.0832e-1 (2.28e-1)	no trigger with trigger	4.4215e-2 (1.67e-2) 1.7276e-2 (2.06e-2)
DTLZ5	3	12	no trigger with trigger	2.9399e-2 (4.87e-3) 2.8127e-2 (3.25e-3)	no trigger with trigger	2.7340e-2 (1.73e-3) 2.6731e-2 (2.17e-3)
DTLZ6	3	12	no trigger with trigger	3.1013e-2 (6.99e-3) 2.8662e-2 (3.04e-3)	no trigger with trigger	2.7587e-2 (2.30e-3) 2.7083e-2 (2.26e-3)
DTLZ7	3	22	no trigger with trigger	7.2846e-1 (3.17e-1) 3.6315e-1 (1.12e-1)	no trigger with trigger	2.0758e-1 (3.66e-2) 1.8479e-1 (2.06e-2)
ZDT1	2	30	no trigger with trigger	2.1398e-2 (1.13e-3) 1.8855e-2 (1.00e-4)	no trigger with trigger	2.1948e-2 (1.44e-3) 2.0027e-2 (5.11e-4)
ZDT2	2	30	no trigger with trigger	1.6863e-2 (5.81e-4) 1.6467e-2 (1.22e-4)	no trigger with trigger	1.7223e-2 (8.67e-4) 1.6414e-2 (1.44e-4)
ZDT3	2	30	no trigger with trigger	5.0975e-2 (3.31e-2) 2.6952e-2 (1.90e-3)	no trigger with trigger	2.8418e-2 (4.26e-3) 2.9390e-2 (2.09e-3)
ZDT4	2	10	no trigger with trigger	2.6893e-2 (1.64e-2) 1.8991e-2 (3.37e-4)	no trigger with trigger	2.1310e-2 (2.25e-3) 1.8816e-2 (1.56e-4)
ZDT6	2	10	no trigger with trigger	1.9052e-2 (5.32e-3) 1.7809e-2 (7.64e-4)	no trigger with trigger	1.7751e-2 (9.31e-4) 1.7879e-2 (1.45e-3)
Effective improvement rate			91.67%		83.33%	

M is the number of objective and D is the dimensions of decision variables. Each row lists the IGD value(the value in parentheses is the standard deviation) of the algorithm under the problem and the best value highlights in bold.

The results of each algorithm are divided into two categories, namely without triggers and with triggers.

The effective improvement rate is shown at the bottom, which is the ratio of the number of successfully improved problems to the total number of problems.

REFERENCES

- [1] Y. Qi, X. Ma, F. Liu, L. Jiao, J. Sun, J. Wu, Moea/d with adaptive weight adjustment, *Evolutionary Computation* 22 (2) (2014) 231–264.
- [2] L. R. de Farias, P. H. Braga, H. F. Bassani, A. F. Araújo, Moea/d with uniformly randomly adaptive weights, in: *Proceedings of the Genetic and Evolutionary Computation Conference*, 2018, pp. 641–648.
- [3] C. Zhang, K. C. Tan, L. H. Lee, L. Gao, Adjust weight vectors in moea/d for bi-objective optimization problems with discontinuous pareto fronts, *Soft Computing* 22 (12) (2018) 3997–4012.
- [4] M. Li, X. Yao, What weights work for you? adapting weights for any pareto front shape in decomposition-based evolutionary multiobjective optimisation, *Evolutionary Computation* 28 (2) (2020) 227–253.
- [5] S. Jiang, Z. Cai, Z. Jie, Multiobjective optimization by decomposition with pareto-adaptive weight vectors, in: *Seventh International Conference on Natural Computation, ICNC 2011, Shanghai, China, 26–28 July, 2011*, 2011.
- [6] S. Jiang, S. Yang, An improved multiobjective optimization evolutionary algorithm based on decomposition for complex pareto fronts, *IEEE transactions on cybernetics* 46 (2) (2015) 421–437.
- [7] C. Zhang, L. Gao, X. Li, W. Shen, J. Zhou, K. C. Tan, Resetting weight vectors in moea/d for multiobjective optimization problems with discontinuous pareto front, *IEEE Transactions on Cybernetics* (2021).
- [8] Q. Zhang, H. Li, Moea/d: A multiobjective evolutionary algorithm based on decomposition, *IEEE Transactions on Evolutionary Computation* 11 (6) (2007) 712–731.
- [9] M. Asafuddoula, T. Ray, R. Sarker, A decomposition-based evolutionary algorithm for many objective optimization, *IEEE Transactions on Evolutionary Computation* 19 (3) (2014) 445–460.
- [10] E. Zitzler, S. Künzli, Indicator-based selection in multiobjective search, in: *International Conference on Parallel Problem Solving from Nature*, Springer, 2004, pp. 832–842.
- [11] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: Nsga-ii, *IEEE Transactions on Evolutionary Computation* 6 (2) (2002) 182–197.

Table S - XV. The meaning and value of the parameters in the definition of SAGOP

Parameter	Description	Value
c_1	System parameter	0.028
c_2	System parameter	0.15
D	Mill diameter	9.8 m
α	Angle of inclination of mill	30°
v	The ratio of mineral to water in the mill	0.3
N	The ratio of the actual speed of the motor to the rated speed	0.7
l	Mineral grinding parameter	20 h^{-1}
β	Steel ball wear parameter	0.5 h^{-1}

- [12] E. Zitzler, M. Laumanns, L. Thiele, Spea2: Improving the strength pareto evolutionary algorithm, TIK-report 103 (2001).
- [13] R. Cheng, Y. Jin, M. Olhofer, B. Sendhoff, A reference vector guided evolutionary algorithm for many-objective optimization, IEEE Transactions on Evolutionary Computation 20 (5) (2016) 773–791.
- [14] D. W. Corne, N. R. Jerram, J. D. Knowles, M. J. Oates, M. J. Pesa-ii, Region-based selection in evolutionary multiobjective optimization, in: Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2001), pp. 283–290.
- [15] S. Tiwari, P. Koch, G. Fadel, K. Deb, Amga: an archive-based micro genetic algorithm for multi-objective optimization, in: Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation, 2008, pp. 729–736.
- [16] S. Tiwari, G. Fadel, K. Deb, Amga2: improving the performance of the archive-based micro-genetic algorithm for multi-objective optimization, Engineering Optimization 43 (4) (2011) 377–401.
- [17] C. Coello, G. T. Pulido, A micro-genetic algorithm for multiobjective optimization, in: International Conference on Evolutionary Multi-criterion Optimization, Springer, 2001, pp. 126–140.
- [18] G. P. Liu, X. Han, C. Jiang, An efficient multi-objective optimization approach based on the micro genetic algorithm and its application, International Journal of Mechanics & Materials in Design 8 (1) (2012) 37–49.
- [19] Z. Junyang, Multi-objective optimization and control method for semi-autogenous grinding process operation, Changsha University of Technology 07 (2020).
- [20] H. Peng, C. Wang, Y. Han, W. Xiao, X. Zhou, Z. Wu, Micro multi-strategy multi-objective artificial bee colony algorithm for microgrid energy optimization, Future Generation Computer Systems (2022).