Newton's Method for Optimization

Gradient

∇f

Newton

$\nabla f, \nabla^2 f$

$$f(x, y) = a_0 x^2 + a_1 xy + a_2 y^2 + a_3 x + a_4 y + a_5$$

$$\nabla f(x, y) = (2a_0 x + a_1 y + a_3, a_1 x + 2a_2 y + a_4)$$

$$\nabla^2 f = \begin{bmatrix} 2a_0 & a_1 \\ a_1 & 2a_2 \end{bmatrix}$$

$$\begin{bmatrix} 2a_0 & a_1 \\ a_1 & 2a_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -a_3 \\ -a_4 \end{bmatrix}$$

$$\begin{cases} 2a_0 x + a_1 y = -a_3 \\ a_1 x + 2a_2 y = -a_4 \end{cases}$$