Neural Network

$$v_i(x, w) = \sigma(w_i^{(1)} x + b_i^{(1)})$$

$$f_{NN}(x, w) = v_1(x, w) \cdot w_1^{(2)} + v_2(x, w) w_2^{(2)} + \dots + v_n(x, w) w_n^{(2)} + b^{(2)}$$

$$v_1(x, w) = \sigma(w^{(1)}, x + b_1^{(1)})$$

$$\sigma'(w_i^{(1)} x + b_i^{(1)})'$$

$$l(w) = (y - f_{NN}(x, w))^2$$

$$\frac{\partial l}{\partial b^{(1)}} = \sum_{j=1}^{k} 2\left(y_j - f_{NN}(x_j, w)\right)$$

$$\frac{\partial l}{\partial w_i^{(2)}} = \sum_{j=1}^{k} 2\left(y_j - f_{NN}(x_j, w)\, v_i(x_j, w)\right)$$

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(1) $$\frac{\partial f_{NN}}{\partial b^{(2)}} = 1$$

(2) $$\frac{\partial f_{NN}}{\partial w_i^{(2)}} = v_i(x, w)$$

(3) $$\frac{\partial f_{NN}}{\partial b_i^{(1)}} = \sigma'\left(w_i^{(1)} x + b_i^{(1)}\right) \cdot w_i^{(2)}$$

(4) $$\frac{\partial f_{NN}}{\partial w_i^{(1)}} = \sigma'\left(w_i^{(1)} x + b_i^{(1)}\right) x$$

$$\frac{\partial \ell}{\partial b^{(2)}} = 2(y - f_{NN}(x, w))$$

$$\frac{\partial \ell}{\partial w_i^{(2)}} = 2(y - f_{NN}(x, w))\, v_i(x, w)$$

$$\frac{\partial \ell}{\partial b_i^{(1)}} = 2(y - f_{NN}(x, w))\, \sigma'(w_i^{(1)} x + b_i^{(1)})\, w_i^{(2)}$$

$$\frac{\partial \ell}{\partial w_i^{(1)}} = 2(y - f_{NN}(x, w))\, \sigma'(w_i^{(1)} x + b)\, x$$

$$\left.\begin{aligned}&(x_1, y_1)\\&(x_2, y_2)\\&(x_k, y_k)\end{aligned}\right\} l(w) = (y_1 - fun(x_1, w))^2$$

$$+ \dots + (y_k - fun(x_k, w))^2$$