PHY566 Homework #5

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Random Numbers

1 Problem Description

a) Generate 1,000 and 1,000,000 random numbers evenly distributed between 0 and 1 respectively and plot the probability distribution of the generated random numbers with 10, 20, 50 and 100 subdivisions.

b) Generate 1,000 and 1,000,000 random numbers distributed according to a Gaussian distribution with width $\sigma = 1.0$ respectively. The Gaussian distribution function is given by

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

Then plot the probability distribution of the generated random numbers with 10, 20, 50 and 100 subdivisions and verify the result by overlaying a Gaussian onto the plots.

2 Numerical Methods

2.1 Random Number Generator

We generally give up on truly random numbers, and use "Pseudo-Random" sequences of numbers instead. One example of a pseudo-random generator is the linear congruential generator:

$$x_{n+1} = (ax_n + b) \mod c \tag{1}$$

This pseudo-random number generator takes a "seed" x_0 and generates a series of numbers which, depending on the choices of a, b, and c, can meet the necessary criteria for Monte Carlo techniques.

Python comes with a pseudo-random number generator called the "Merseinne Twister". This is a very fast generator, and it has been extensively tested by the mathematical community and given a clean bill of health. To access these random-number routines, import "random" package and **random.random()** will generate a random float between 0 and 1.

2.2 Marsaglia Algorithm

The Box-Muller algorithm has been improved by Marsaglia in a way that the use of trigonometric functions can be avoided. It is important, since computation of trigonometric functions is very time-consuming.

Algorithm: polar method (create $Z \sim \mathcal{N}(0,1)$):

1. repeat generate $U_1, U_2 \sim \mathcal{U}[0,1]$;
   $V_1 = 2U_1 - 1, V_2 = 2U_2 - 1$;
   until $W := V_1^2 + V_2^2 < 1$.
2. $Z_1 := V_1\sqrt{-2\log(W)/W}$
   $Z_2 := V_2\sqrt{-2\log(W)/W}$
   are both normal variates.

3 Results

3.1 Part a

3.1.1 $N = 1,000$

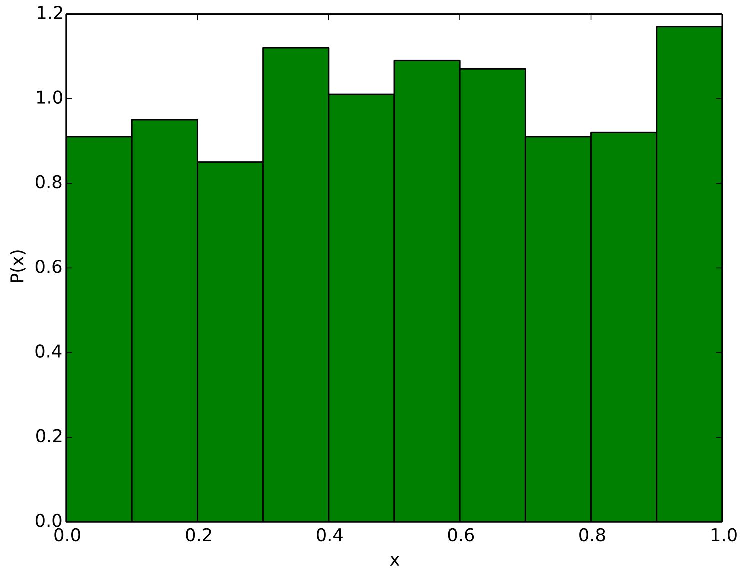

Figure 1: 10 bins

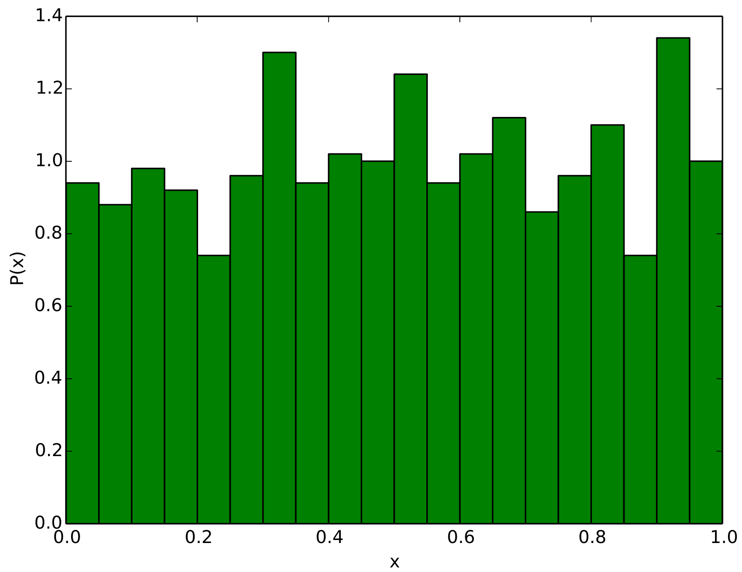

Figure 2: 20 bins

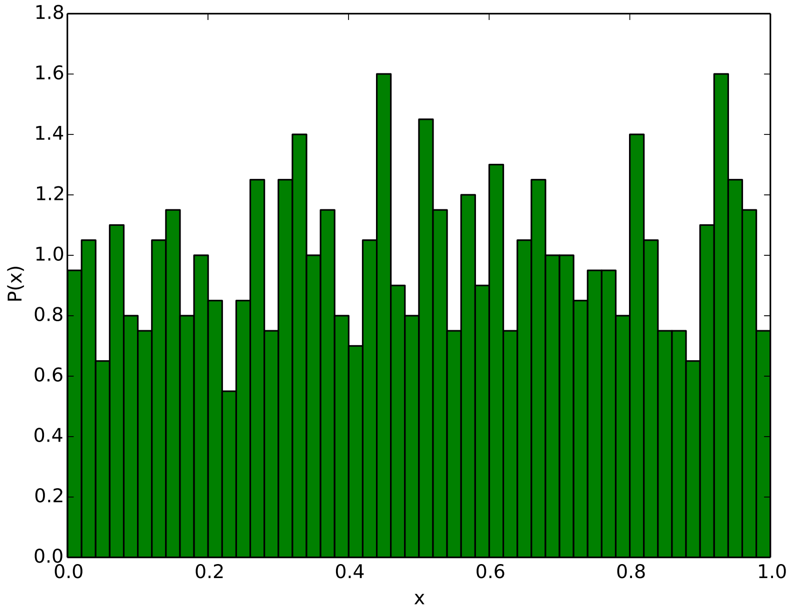

Figure 3: 50 bins

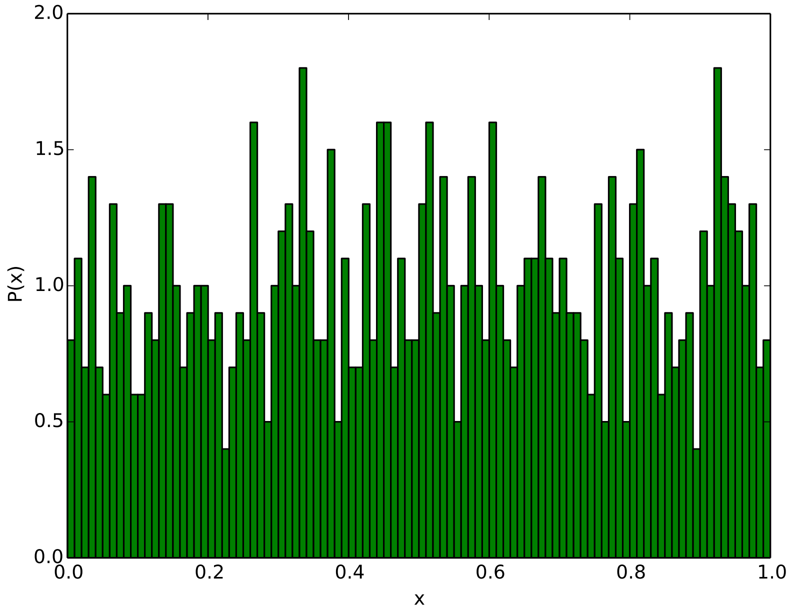

Figure 4: 100 bins

3.1.2 $N = 1,000,000$

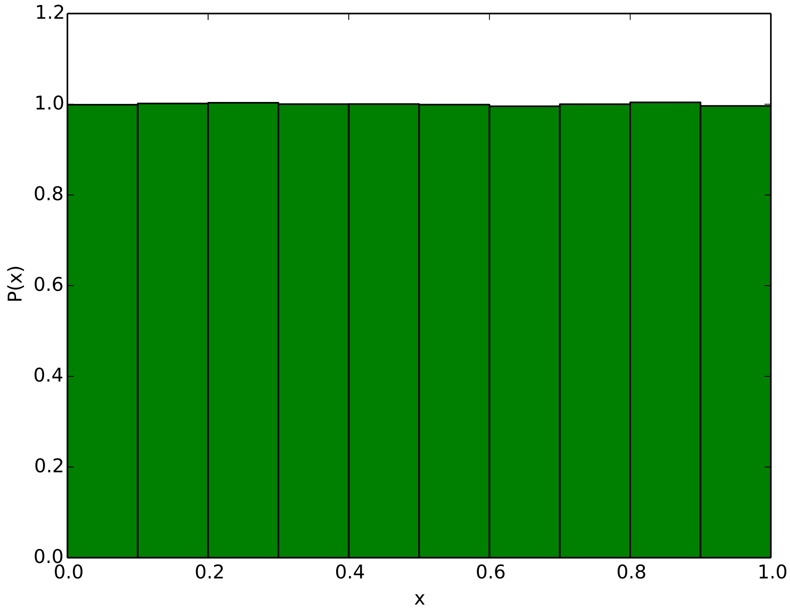

Figure 5: 10 bins

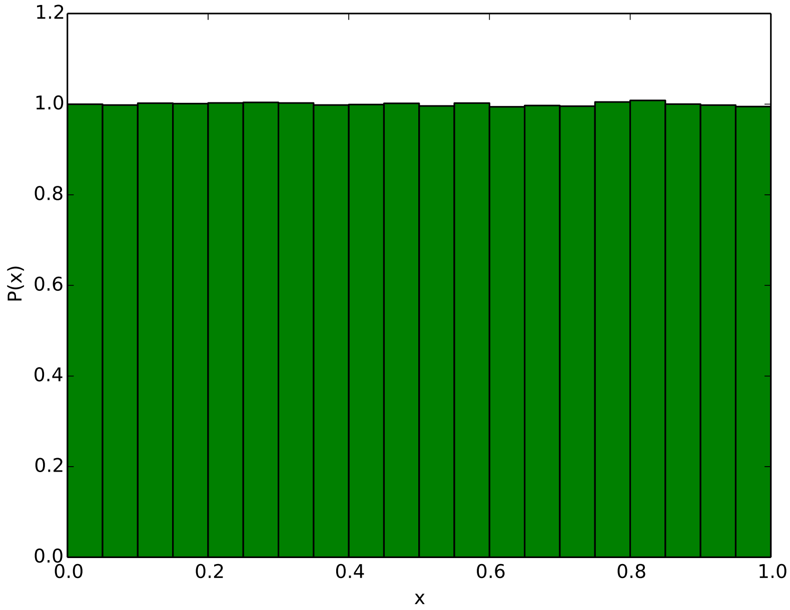

Figure 6: 20 bins

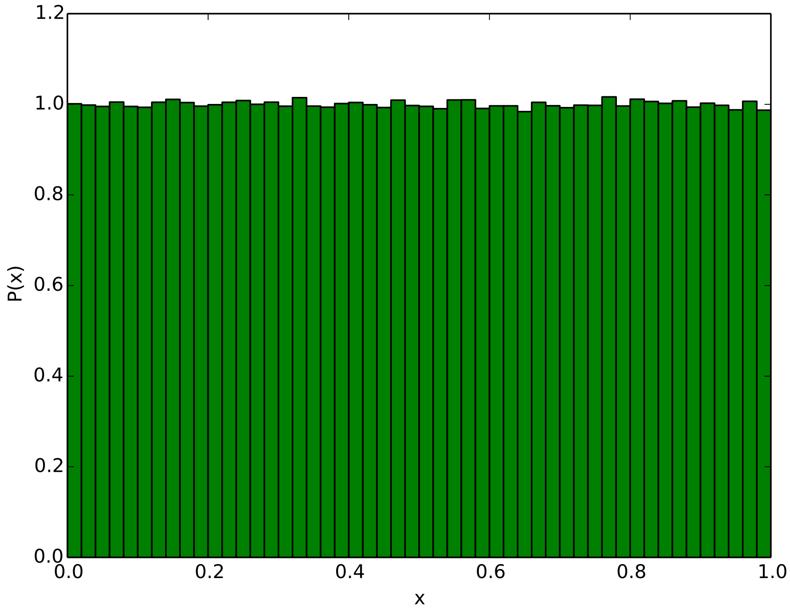

Figure 7: 50 bins

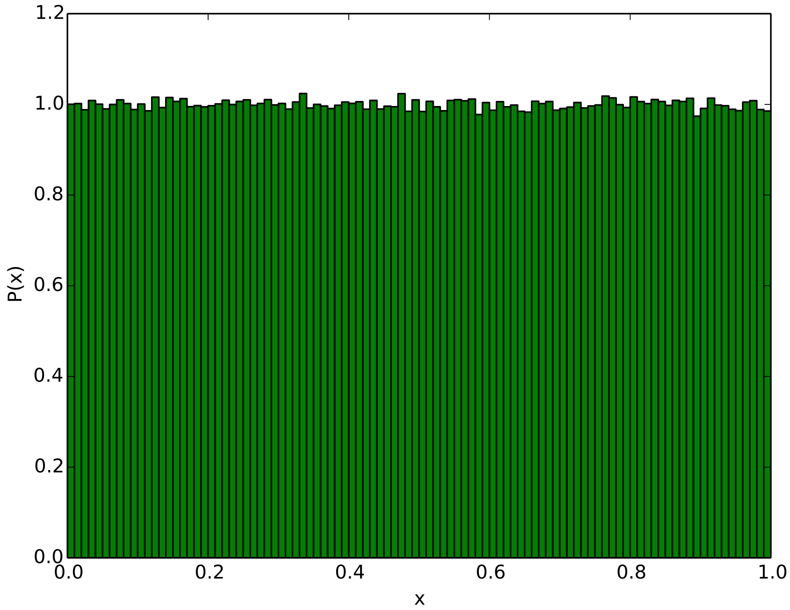

Figure 8: 100 bins

3.2 Part b

3.2.1 $N = 1,000$

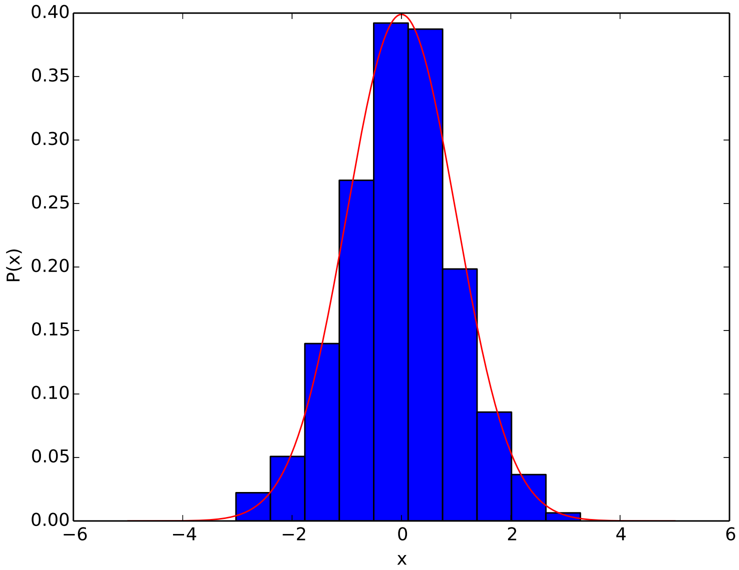

Figure 9: 10 bins

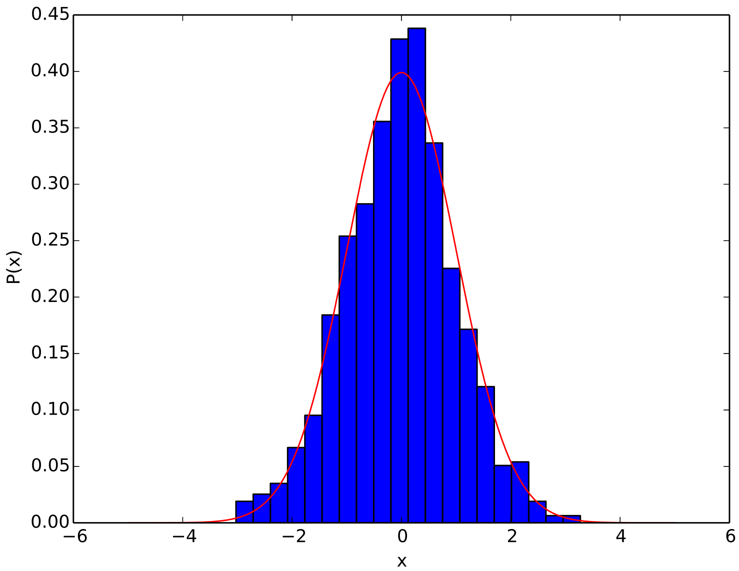

Figure 10: 20 bins

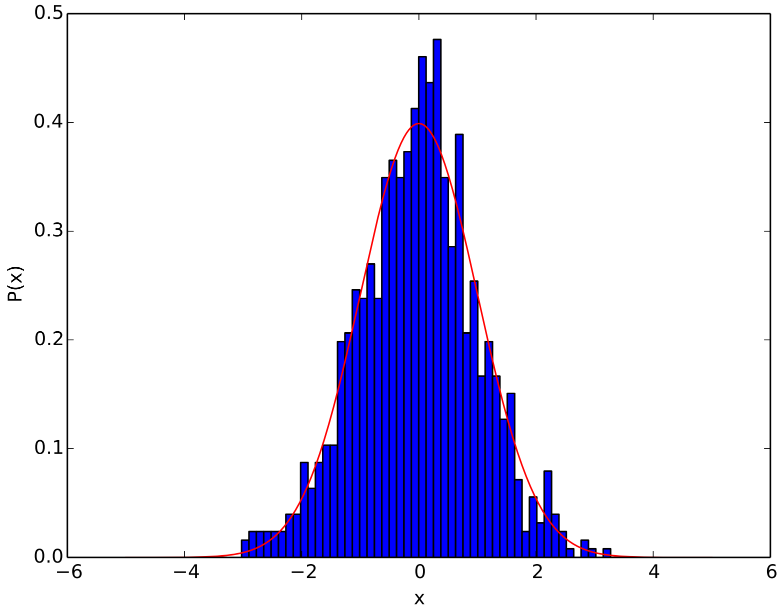

Figure 11: 50 bins

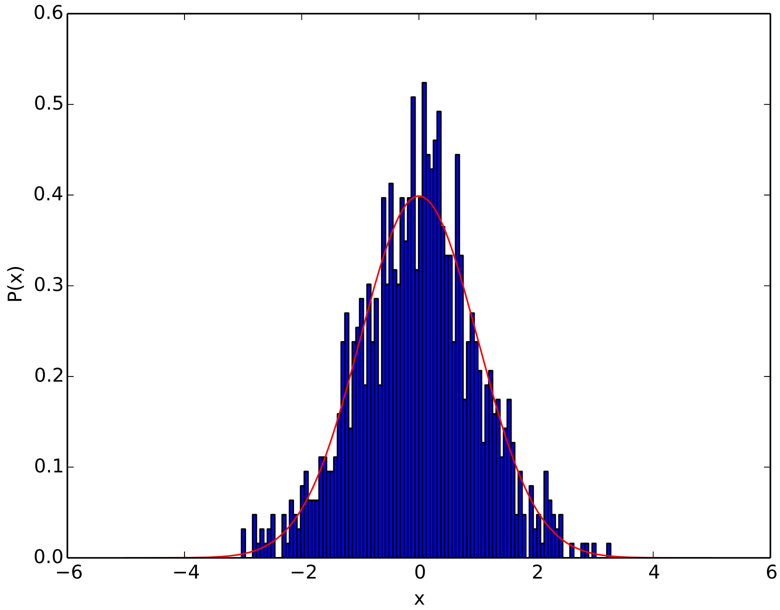

Figure 12: 100 bins

3.2.2 $N = 1,000,000$

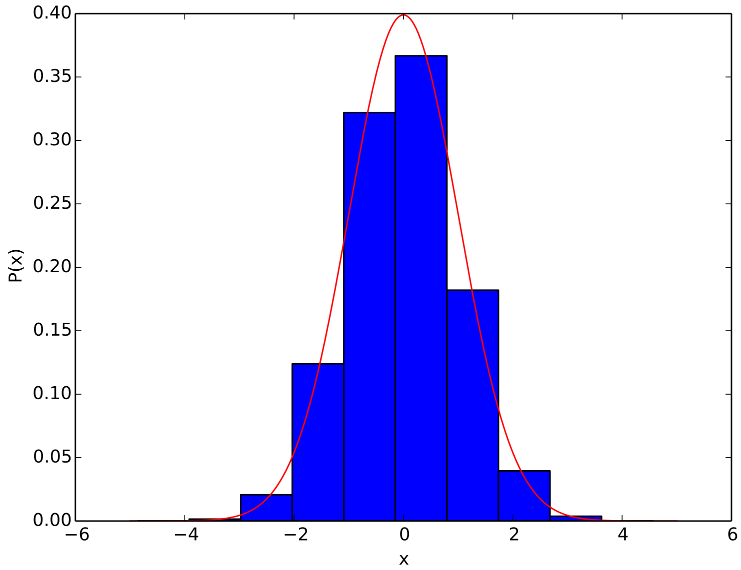

Figure 13: 10 bins

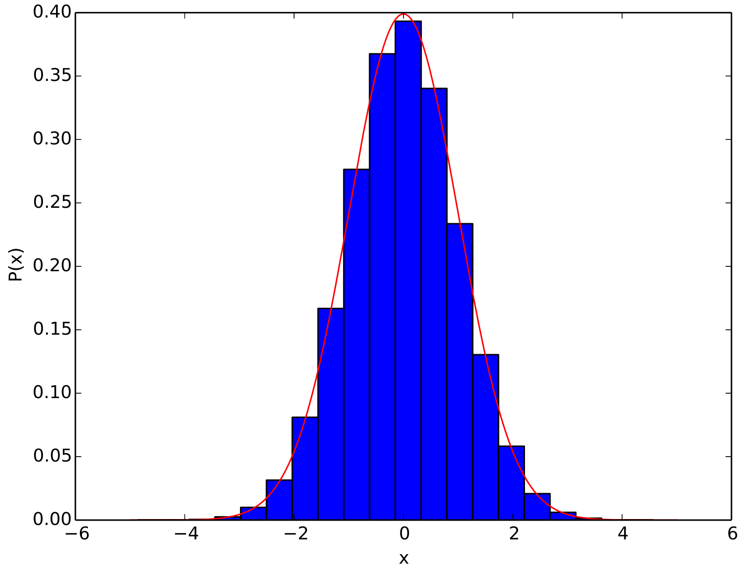

Figure 14: 20 bins

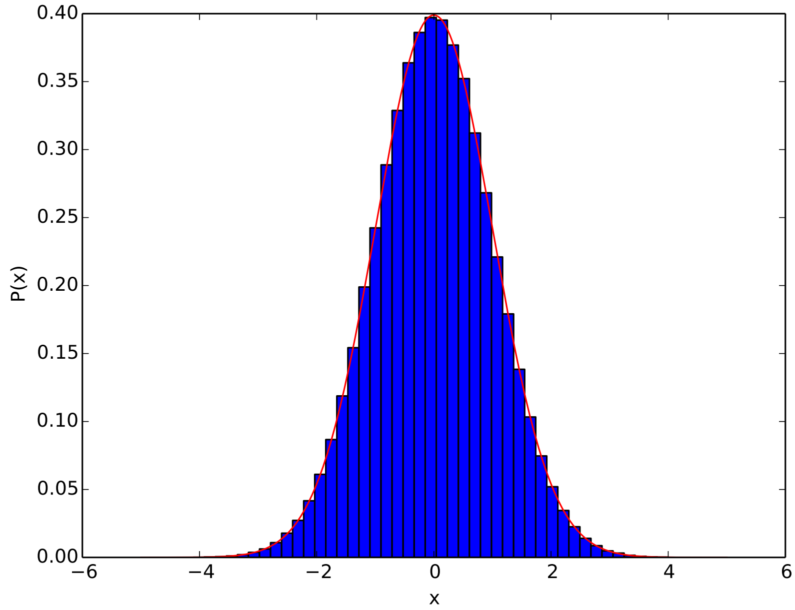

Figure 15: 50 bins

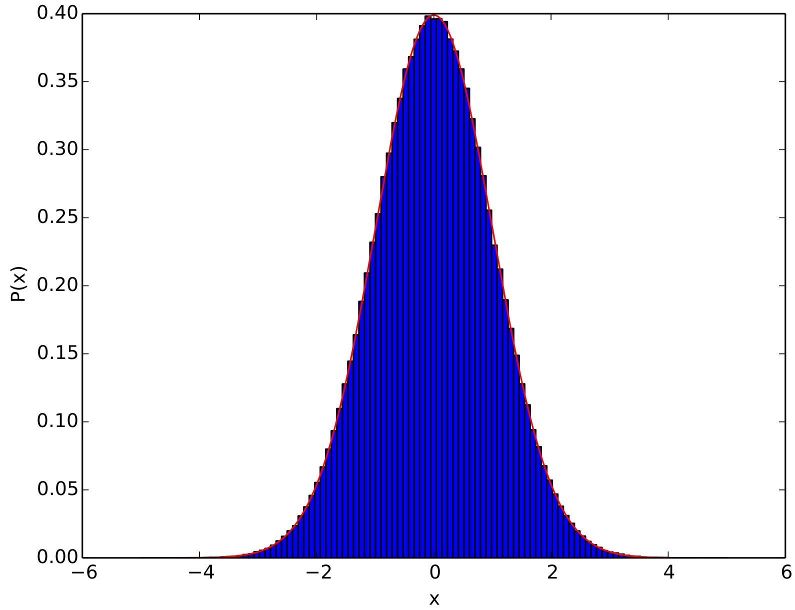

Figure 16: 100 bins

4 Discussion

As can be seen from the above figures, the probability distribution of the generated random numbers always fluctuates around the distribution function, thus our results are reliable. And for a fixed number of subdivisions, the fluctuation of $1,000,000$ random numbers is smaller than that of $1,000$ random numbers. For a fixed number of random numbers, the fluctuation is more obvious as the number of subdivisions increases.