Intro to Computer Science
CS-UH 1001, Spring 2022

Lecture 20: Recursion

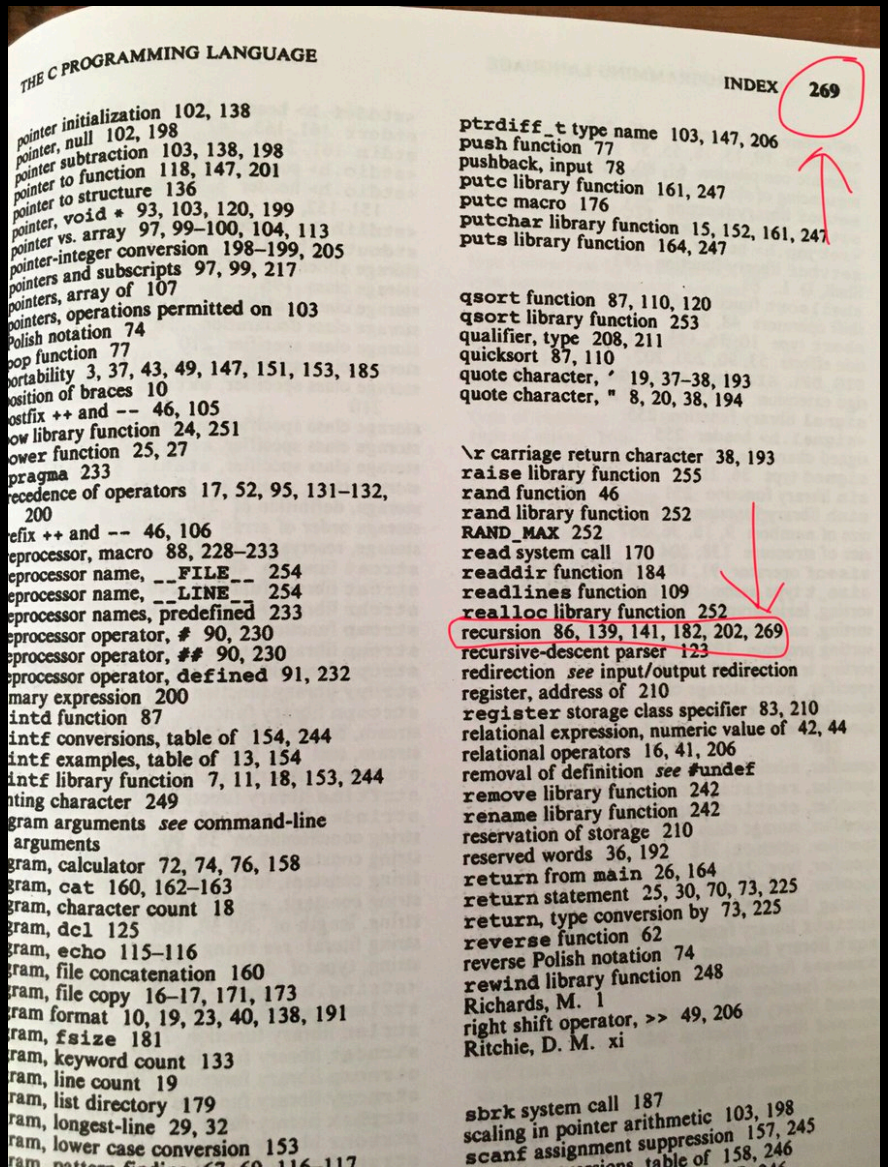
Recap Functions

- Function calls are like a detour in the flow of execution:
  - Instead of going to the next statement in the code, the flow jumps to the called function, executes all the statements there, and then comes back to pick up where it left off
- A return statement ends the execution of a function call
  - Functions can have multiple return statements

Recursion

- Recursion is more than a programming topic, it is
  - a method of problem solving
  - a different way of thinking of problems - that's the tricky part :)
- Recursion can solve some problems better than iterations (loops)
- Recursion can lead to elegant, simplistic and short code (when used well)
- Recursion is technically simply:
  - A function that calls itself (recursive function)

Recursion in Text Books



Recursion on google.com

Google

recursion

All Books Images News Videos More Settings Tools

About 16,000,000 results (0.48 seconds)

Did you mean: ***recursion***

Dictionary

Enter a word, e.g. "pie"

re·cur·sion

/rəˈkərZHən/

noun MATHEMATICS · LINGUISTICS

the repeated application of a recursive procedure or definition.

- a recursive definition.
  plural noun: **recursions**

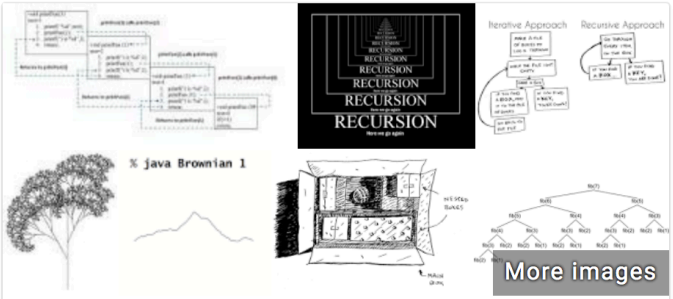
Translations, word origin, and more definitions

Feedback

Recursion - GeeksforGeeks

https://www.geeksforgeeks.org/recursion/

The process in which a function calls itself directly or indirectly is called **recursion** and the corresponding function is called as recursive function. Using recursive algorithm, certain problems can be solved quite easily.



Recursion

Computer science

Recursion in computer science is a method of solving a problem where the solution depends on solutions to smaller instances of the same problem. The approach can be applied to many types of problems, and recursion is one of the central ideas of computer science. Wikipedia

Feedback

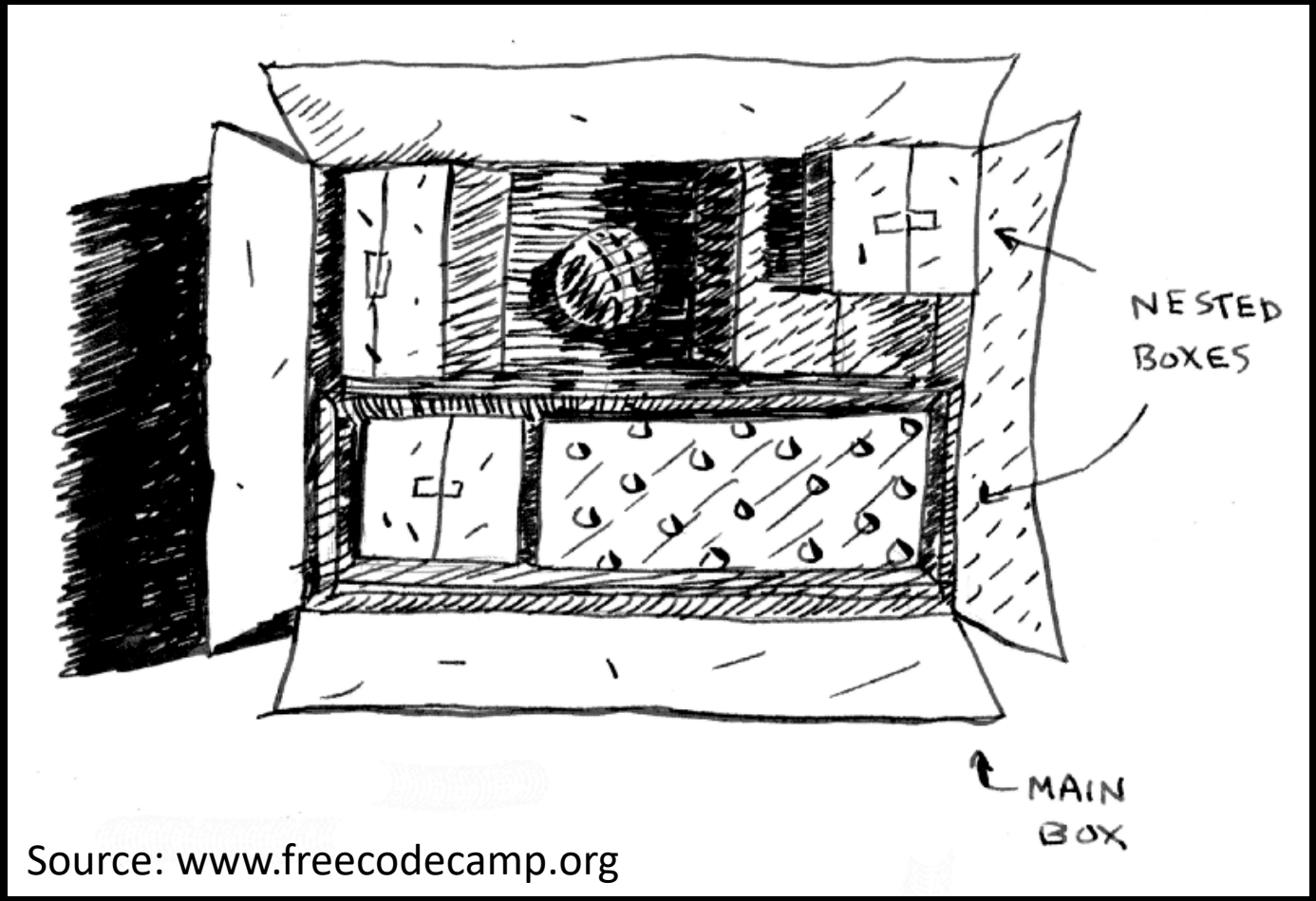
See results about

Recursion

Recursion occurs when a thing is defined in terms of itself or of its type. Recursion is used in a variety of ...

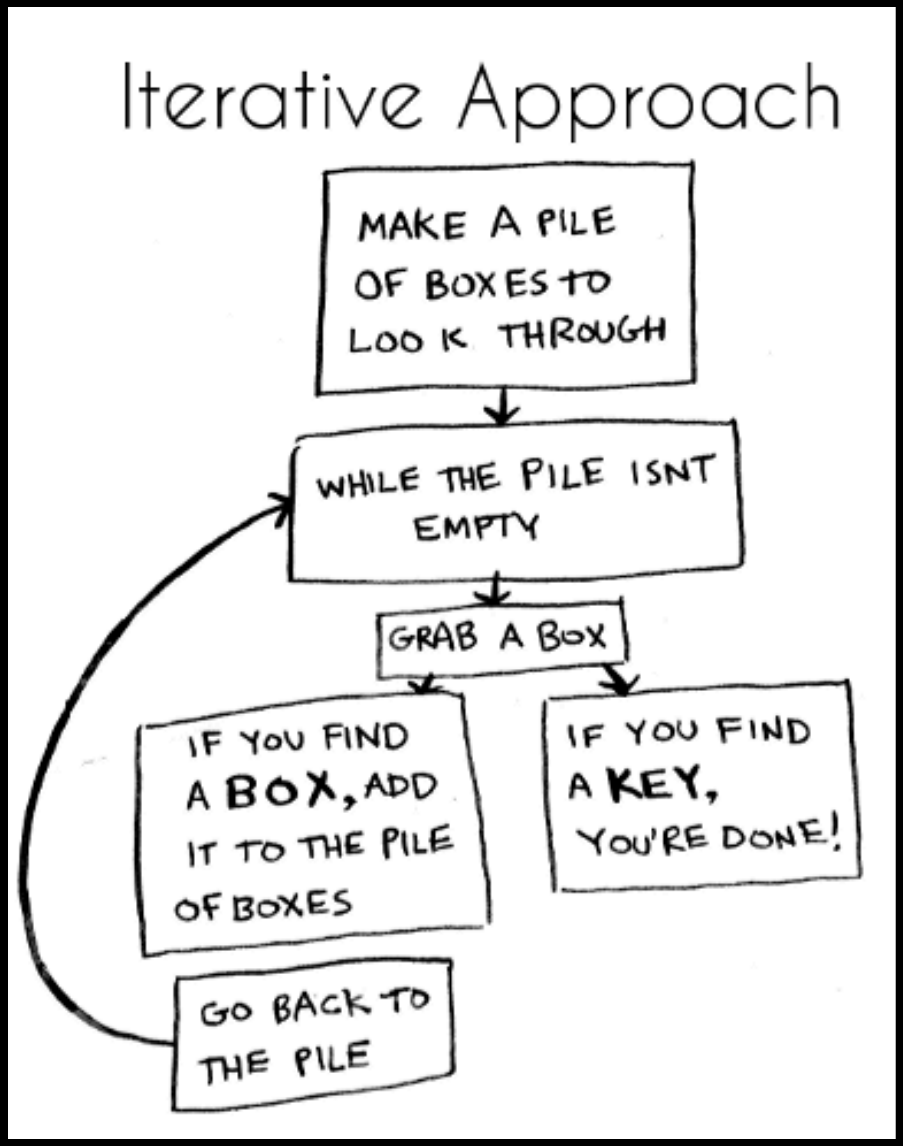
Motivating Example

- A key in a box in a box in a box in a box…

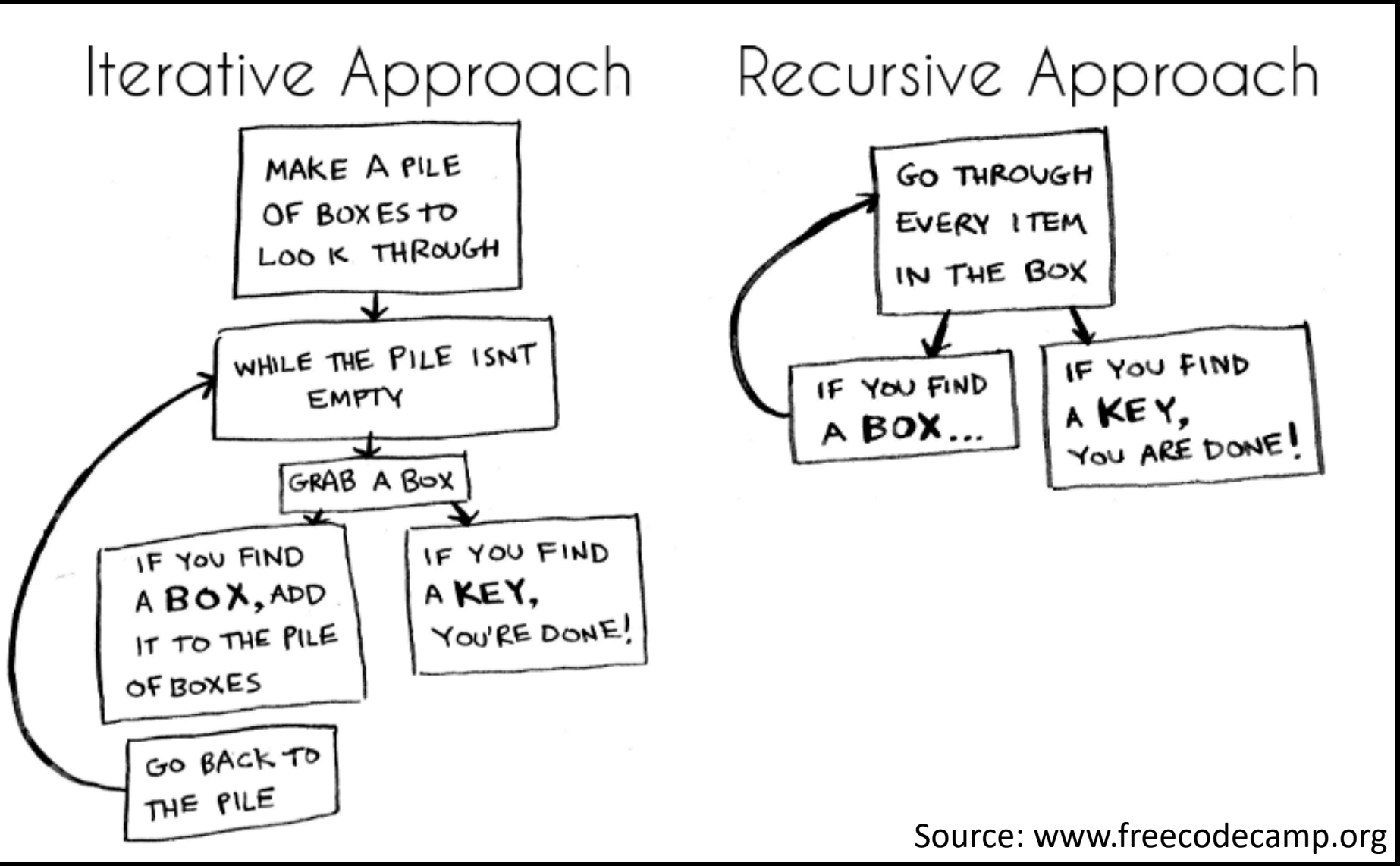


Source: www.freecodecamp.org

Motivating Example



Motivating Example



Source: www.freecodecamp.org

Another Example

- How to find out how many people are in the queue in front of you?
- Assumptions:
  - Your vision is poor. You can not look far
  - You are not allowed to move
  - You are only allowed to speak to the person in front of you or behind you

Another Example

- Recursion is all about breaking a big problem into smaller instances of the same problem
- Each person can solve a small part of the problem

- Solution:
  - If there is someone in front of you, ask him/her how many people are in front of him/her
    - When he/she responds with a value N, then you will answer N+1
  - If there is nobody in front of you, you will answer 0

Dry/Run Solution

- Suppose you are 4th in line, and you are determining how many people are in front to you
- Person 4: Tap person 3 shoulder, Ask people in front and wait
  - Person 3: Tap and ask person 2 and wait
    - Person 2: Tap and ask person 1 and wait
      - Person 1 : As person 1 is in front and no one is there to tap their shoulder. Tell person behind it is 0
    - Person 2: Tell person behind it is 0+1 = 1
  - Person 3: Tell person behind it is 1+1 = 2
- Person 4: Just use the answer of person 3 to find the final answer, which is 2 +1 = 3

Components of Recursion

- If we analyze our solution, there were two main things that were happening:
  - Simplifying/Reducing the problem in terms of itself and then solving it using same logic (Recursive Case)
  - Simplification doesn't go on forever, (Base Case)
    - It breaks/stop when encounter a problem version which couldn’t be simplified further

Recursion algorithm

- Recursive Case: The set of instructions that will be used over and over
  - **Divide**: Split the problem into one or more simpler or smaller versions of the problem
  - **Call**: Recursive call to solve a simpler version of a problem
  - **Combine**: Combining the solutions of the versions into a solution for the problem/complex version

- Base Case: The point where you stop applying the recursive case, the problem is simple enough to be solved directly
- In both cases, we return whatever answer we arrived on

- In the Queue Problem:
  - Recursive case is: Tap person in front of you. Ask how many people are in front of them. Wait for their answer and add 1
    - Tap person in front of you (Divide)
    - Ask how many people are in front of them (Call)
    - Wait for their answer and add 1 (Combine)
  - Base case is: Person 1. You do not execute the above
  - If someone asked, tell them how many people are in front of you (return)

Recursion is Simple

Recursion is a function that calls itself

Example:

```
def greeting():
    print("Hello World")
    greeting()

greeting()
```

Hello World
Hello World
Hello World
Hello World
.
.
.

Infinite loop
There is not way for the function to stop executing

Recursion with Base Case

Every recursion must have a base case!

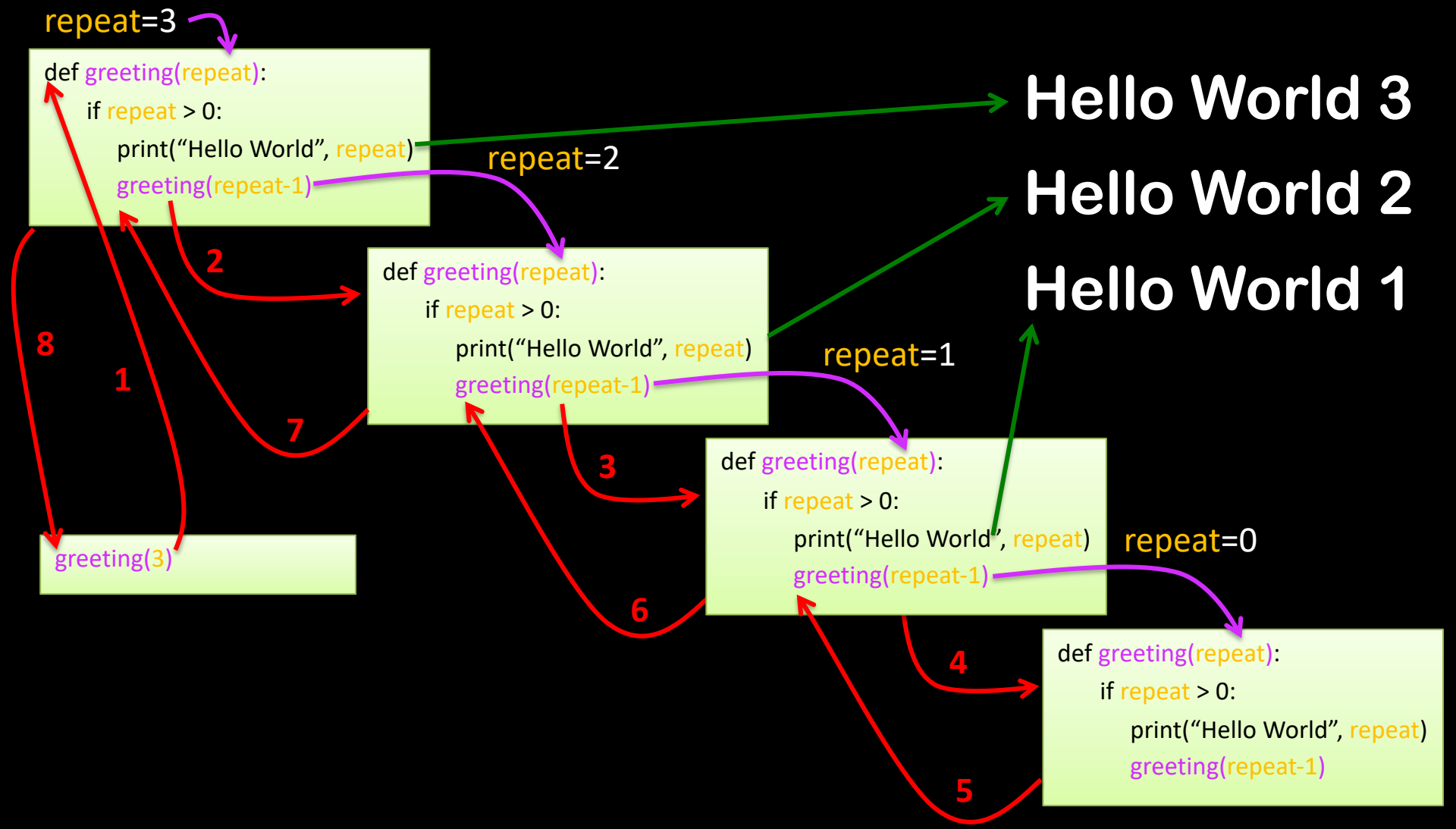
```
def greeting(repeat):
    if repeat > 0:
        print("Hello World", repeat)
        greeting(repeat - 1)

greeting(3)
```

```
Hello World 3
Hello World 2
Hello World 1
```

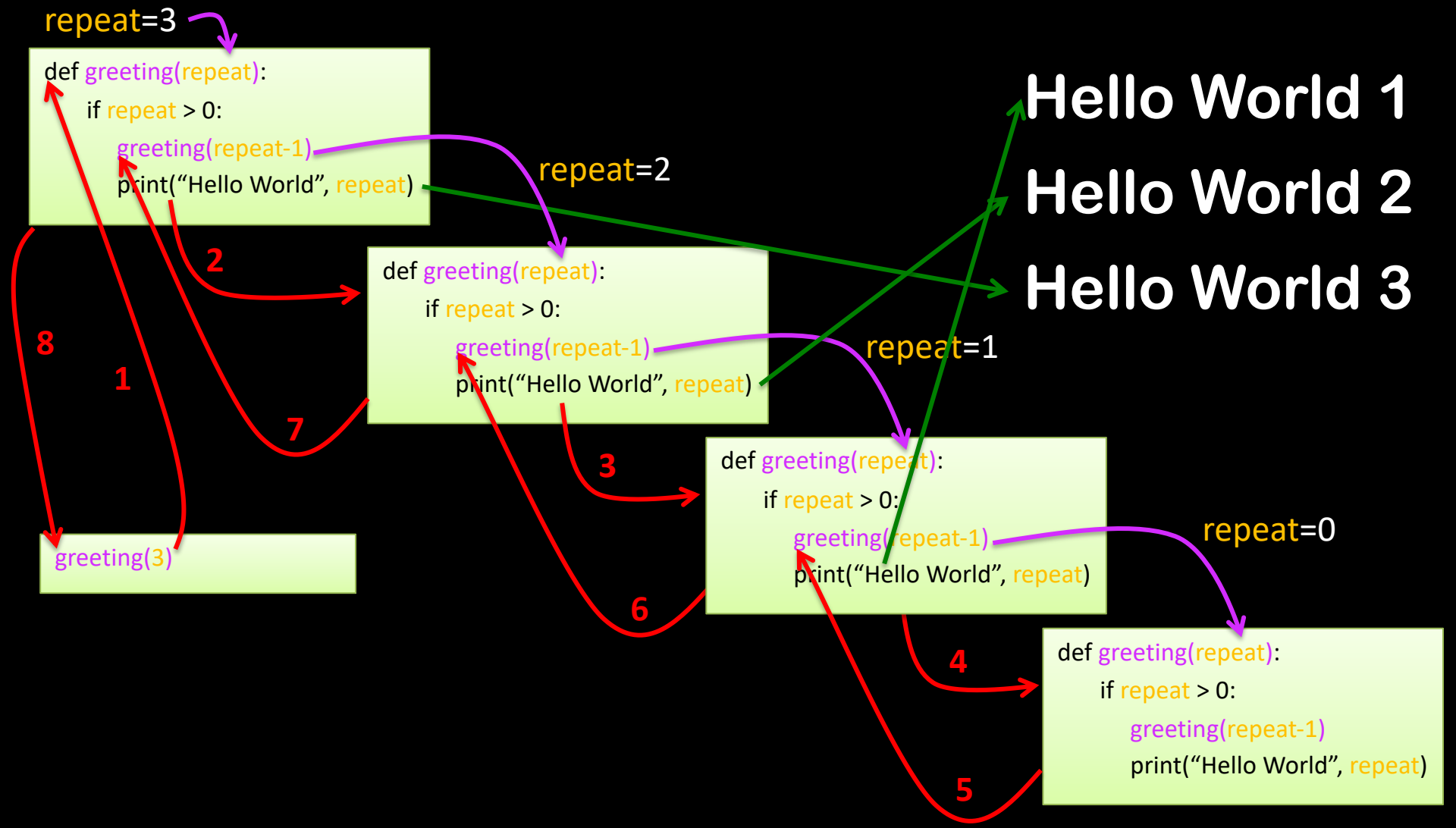
General case (recursive function call)

Base case or Exit condition

Simple Recursion (Illustration)



Simple Recursion (Illustration)



Solving a Simple Problem with Recursion

- A classic example to solve using recursion is factorial (n!)

- Factorial is the product of all positive integers less-than or equal-to a given integer

- Factorial of n! is defined as:
  - If n > 0 then n! = 1 x 2 x 3 x 4 x 5 x …. x n
  - If n = 0 then 0! = 1

Factorial: The Iterative Approach

- 4! = 4 x 3 x 2 x 1

- Iterative approach:

```
n = 4
factorial = 1
for i in range(1, n+1):
    factorial = factorial * i
```

Factorial: The Recursive Approach

- 4! = 4 x 3 x 2 x 1

- Factorial is recursive by nature
  - n! = n x (n -1)!
  - 0! = 1

4! = 4 x 3!
= 4 x 3 x 2!
= 4 x 3 x 2 x 1!
= 4 x 3 x 2 x 1 x 0!

Getting closer to the base case…

Base case

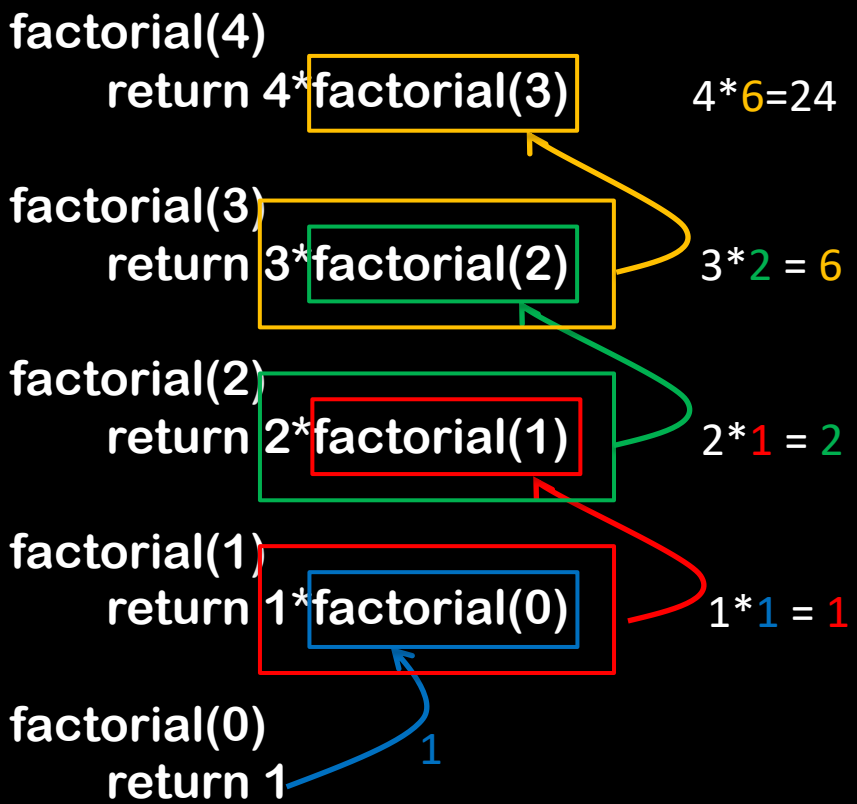
Factorial using Recursion

```
def factorial(n):
    if n == 0:
        return 1

    return n * factorial(n - 1)
```

General case
(decrement n towards 0)

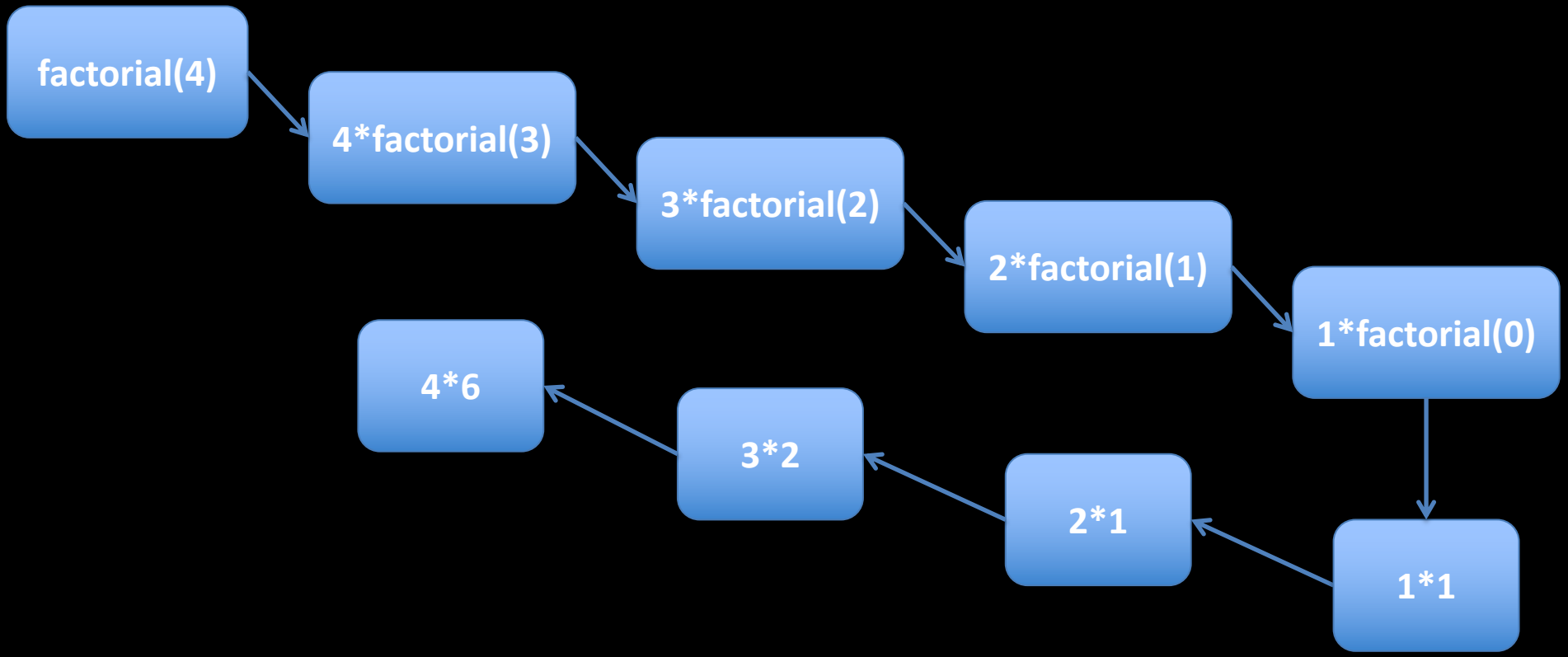
Base case

Factorial using Recursion




```
def factorial(n):
    if n == 0:
        return 1
    return n * factorial(n-1)
```

Factorial using Recursion



This is called Linear recursion

Fibonacci Series

- The Fibonacci series is named after the Italian mathematician Leonardo Fibonacci

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …

- Each number is the sum of the previous two numbers

Fibonacci Series

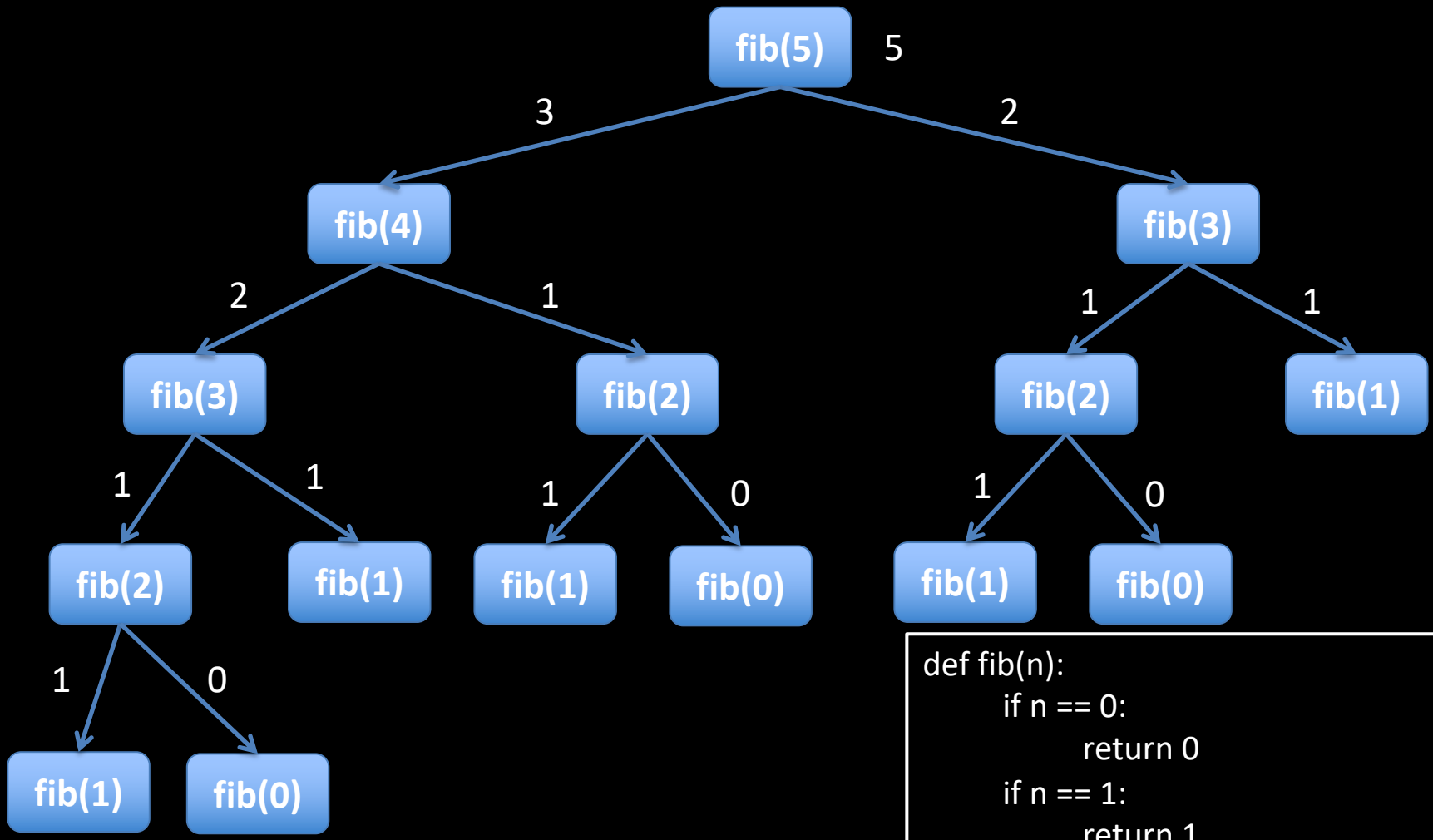
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …

- Mathematically, the sequence fib(n) can be defined as:
  - If n = 0 then fib(n) = 0 ← Base case #1
  - If n = 1 then fib(n) = 1 ← Base case #2
  - If n > 1 then fib(n) = fib(n-1) + fib(n-2)

  where n is the nth term of the sequence

- The Fibonacci sequence definition by default has a recursion
- Exercise (ex_21.1): Write a recursive function for fib(n) e.g. fib(6) => 8

Example: Fibonacci Series fib(5)




```
def fib(n):
        if n == 0:
                return 0
        if n == 1:
                return 1

        return fib(n-1) + fib(n-2)
```

This is called Binary recursion

Efficiency of Recursion

- The execution takes long because it computes the same values over and over
- Computation of fib(5) calls fib(2) three times!
- Keeping previously calculated values, such as fib(2), to avoid computing the values more than once improves algorithm performance
- This technique is called Memoization

To understand recursion, you must first understand recursion!

Exercise: Fibonacci with Memoization

- Modify the recursive function for fib(n) so that it uses memoization

- Compare the execution time by using

```
import time
start = time.time()
print(fib(30))
end = time.time()
print(end - start)
```

Common Errors

- Infinite recursion:
  - A function calling itself over and over with no end in sight
  - The computer needs some amount of memory for book keeping (stack call) during each call
  - After some number of calls, all available memory for this purpose is exhausted
  - Your program shuts down and reports a “stack overflow”
- Causes:
  - The arguments don’t get simpler or because a special terminating case is missing

Breakout session I:

Sum of n

- Write a recursive function that sums n numbers

  sum(n) = 1 + 2 + 3 + … + (n-1) + n

Power of n

- Write a recursive function that calculates x^n
- Mathematically, the power function can be described as:

$$x^n = \begin{cases} 1 & if\ n = 0 \\ x * x^{n-1} & if\ n > 0 \end{cases}$$

Assume x and n are both positive integers

Reverse List

- Write a recursive function that reverses a list
- For example:

```
print(recursive_reverse([1,2,3,4]))
[4,3,2,1]
```


PROBLEMS WITH RECURSION ?
↓Please take one ↓
WITH
ECURSIO
WHAT THE-
PROBLEMS WITH RECURSION ?
↓Please take one ↓