(5) At the optimum, $\underline{c}^T \underline{x} = \underline{b}^T \underline{y}$

If we add ε to RHS of inequality j in the primal, then the new RHS of the primal constraints is $\underline{b} + \varepsilon \underline{e}_j$.

So new objective function for dual is

$$(\underline{b} + \varepsilon \underline{e}_j)^T \underline{y} = \underline{b}^T \underline{y} + \varepsilon y_j$$

So as long as ε is small enough, and $\underline{y}$ is still optimal, the objective increases by εy_j.

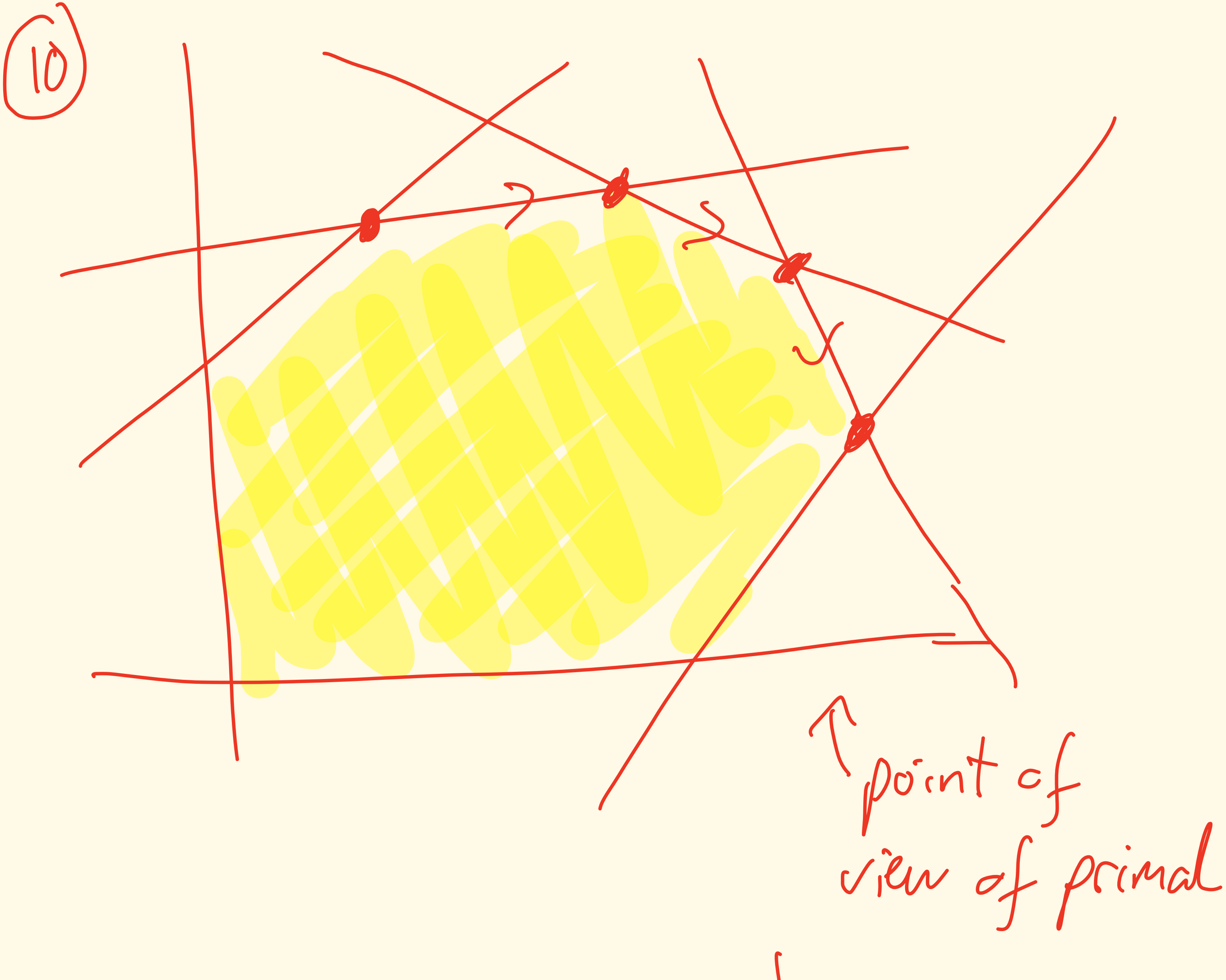
(8) Primal: $\min\ \underline{c}^T \underline{x}$ s.t. $A\underline{x} = \underline{b}$,
$\underline{x} \geq 0$.

Dual: $\max\ \underline{b}^T \underline{y}$ s.t. $A^T \underline{y} + \underline{s} = \underline{c}$
$\underline{s} \geq 0$.

We know that $\underline{x}^*$ is optimal for the primal and $(\underline{y}^*, \underline{s}^*)$ is optimal for the dual if

$$\left.\begin{aligned} \underline{c}^T \underline{x}^* - \underline{b}^T \underline{y}^* &= 0 \\ A\underline{x} &= \underline{b} \\ A^T \underline{y} + \underline{s} &= \underline{c} \\ \underline{x}, \underline{s} &\geq 0 \end{aligned}\right\} \text{a feasibility problem.}$$

So if we can solve the feasibility problem, then we can solve both the primal and dual problems.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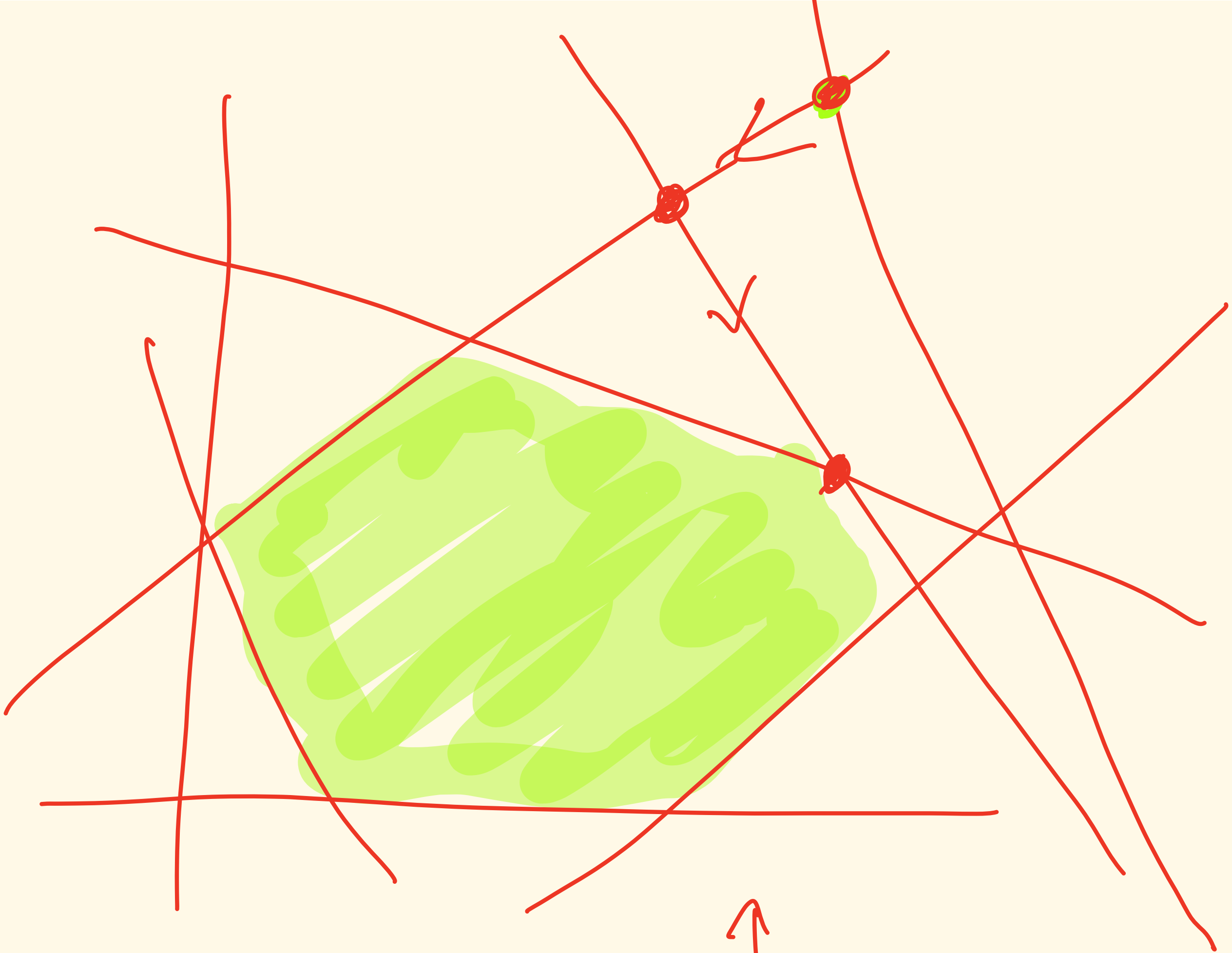
point of
view of primal



point of view
of dual problem

x_1	x_2	x_3	x_4	x_5		
1	2	3	-1	0	5	$\times -1$
2	2	1	0	-1	6	$\times -1$
3	4	5	0	0		

x_1	x_2	x_3	x_4	x_5	
-1	-2	-3	1	0	-5
-2	-2	-1	0	1	-6
3	4	5	0	0	

$\underline{x} = (0,0,0,-5,-6)$ is a basic solution

$\underline{x} \geq 0$

Choose x_5 to leave basis.

If we chose x_1 to enter the basis, we'd have to add $\frac{3}{2} \times R2$ to $R3$, to get $s_1 = 0$.

If we chose x_2 to enter the basis, we'd have to add $\frac{4}{2} = 2 \times R2$ to $R3$, but then we would get $s_1 = -1$ (not allowed)

Also can't add x_3 to basis.

So pivot on T_{21}

x_1	x_2	x_3	x_4	x_5		
0	-1	$-5/2$	1	$-1/2$	-2	$R1 - \frac{1}{2} \times R2$
1	1	$1/2$	0	$-1/2$	3	
0	1	$7/2$	0	$3/2$	-9	

still have $\underline{s} \geq 0$, but $\underline{x}$ still not feasible.

To choose which T_{ij} to pivot on, for $i=1$, we find the j that minimizes $\dfrac{(s_v)_j}{-T_{ij}}$ for $T_{ij} < 0$

x_1	x_2	x_3	x_4	x_5	
0	1	$5/2$	-1	$1/2$	2
1	0	-2	1	-1	1
0	0	1	1	1	-11

R1+R2

R1+R3

$s \geq 0$, $x \geq 0$, so optimal!

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- $$\underline{c}^T(\lambda \underline{x}_1 + (1-\lambda)\underline{x}_2) = \lambda \underline{c}^T \underline{x}_1 + (1-\lambda)\underline{c}^T \underline{x}_2$$
$$= \lambda \underline{c}^T \underline{x}_1 + (1-\lambda)\underline{c}^T \underline{x}_1$$
$$= \underline{c}^T \underline{x}_1 = \underline{c}^T \underline{x}_2$$

- $$A(\lambda \underline{x}_1 + (1-\lambda)\underline{x}_2) = \lambda A \underline{x}_1 + (1-\lambda) A \underline{x}_2$$
$$\geq \lambda \underline{b} + (1-\lambda)\underline{b}$$
$$= \underline{b}$$