Linear Programming: Homework 5

Homework to be submitted on Canvas by the start of class on Monday October 25th, 1pm.

Explain all your answers and show your working.

Question 1.

Consider the following linear program

$$\begin{aligned} \min \quad & 3x_1 + 5x_2 - 3x_3 + 4x_4 \\ \text{s.t.} \quad & -x_1 - 5x_2 + 2x_3 + 2x_4 = -21 \\ & 2x_1 - 5x_3 + 4x_4 = 2 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

Suppose we have run the primal simplex method and found variables x_1 and x_2 are basic at the optimum.

(i) Use row operations to verify that the solution with x_1 and x_2 in the basis is optimal.

(ii) Suppose now you are told that the conditions are changed, and as a result the right-hand side of the second inequality is now -1, making the current solution infeasible. Verify that nevertheless, the current dual solution is still feasible, but not necessarily optimal. Set up the tableau for the dual simplex method, starting at the current solution that was optimal in the original problem, and find the new optimal solution.

Question 2.

Consider the typical transportation problem: there are n supply centers $\{1, 2 \ldots, n\}$, where the ith center has a supply capacity of s_i, and m demand centers $\{1, \ldots, m\}$, where the jth center has a demand of d_j. Furthermore it costs c_{ij} to ship from supply center i to demand center j. The goal is to decide a transportation schedule x_{ij} from supply i to demand j such that:

- the total amount shipped from each supply center i is exactly equal to its supply capacity s_i,
- the total amount received by demand center j is exactly d_j,
- the total cost of shipment $\sum_{ij} c_{ij}x_{ij}$ is as small as possible.

(i) Write the linear programming formulation of this problem and also write its dual.

(ii) Give an economic interpretation of the dual. Pattern your interpretation after the diet problem from Class 6. Start with an entrepreneur who will buy the commodity at a supply node and sells it at a demand node, and follow up to see what will happen at the optimal solution. **Be brief and precise**.

Question 3.

Consider the following linear programming:

$$\begin{aligned} &\min\ 2x_2 - 5x_3 \\ \text{subject to } &2x_1 + x_2 + x_3 \leq 15 \\ &-2x_1 - 3x_2 + 4x_3 \leq -10 \\ &x_1 - x_2 + x_3 \leq 9 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

(i) Solve the problem using AMPL.

(ii) Decide whether it has multiple optimal solutions, and if so give another solution.