5 **Phase 1**

$$\min \; x_6 + x_7 + x_8 \quad \text{s.t.}$$

$$
\begin{array}{rcl}
x_1 - x_2 + x_5 + x_6 & = & 2 \\
x_2 + x_3 - x_4 + x_5 + x_7 & = & 1 \\
-x_1 + x_2 - x_3 + x_4 - x_5 - x_8 & = & -1 \\
x_1, \ldots, x_8 \geq 0 & &
\end{array}
$$

6 **Example 2 (again)**

$$\min \; x_1 + 2x_2 + 3x_3 - x_4 - x_5 + Mx_6 + Mx_7 + Mx_8$$

s.t.

$$
\begin{aligned}
x_1 - x_2 \quad\quad\quad\quad + x_5 + x_6 \quad\quad\quad &= 2 \\
x_2 + x_3 - x_4 + x_5 \quad\quad + x_7 \quad\quad &= 1 \\
-x_1 + x_2 - x_3 + x_4 - x_5 \quad\quad\quad - x_8 &= -1 \\
x_1, \ldots, x_8 &\geq 0
\end{aligned}
$$

7) $\min \; -x_1 - 3x_2 + x_3$

$$
\begin{aligned}
\text{s.t.} \quad x_1 + x_2 + 2x_3 + x_4 \quad\quad\quad &= 4 \\
-x_1 \quad\quad + x_3 \quad\quad - x_5 \quad\quad &= 4 \\
x_3 \quad\quad\quad\quad - x_6 &= 3 \\
x_1, x_2, \ldots, x_6 &\geq 0
\end{aligned}
$$

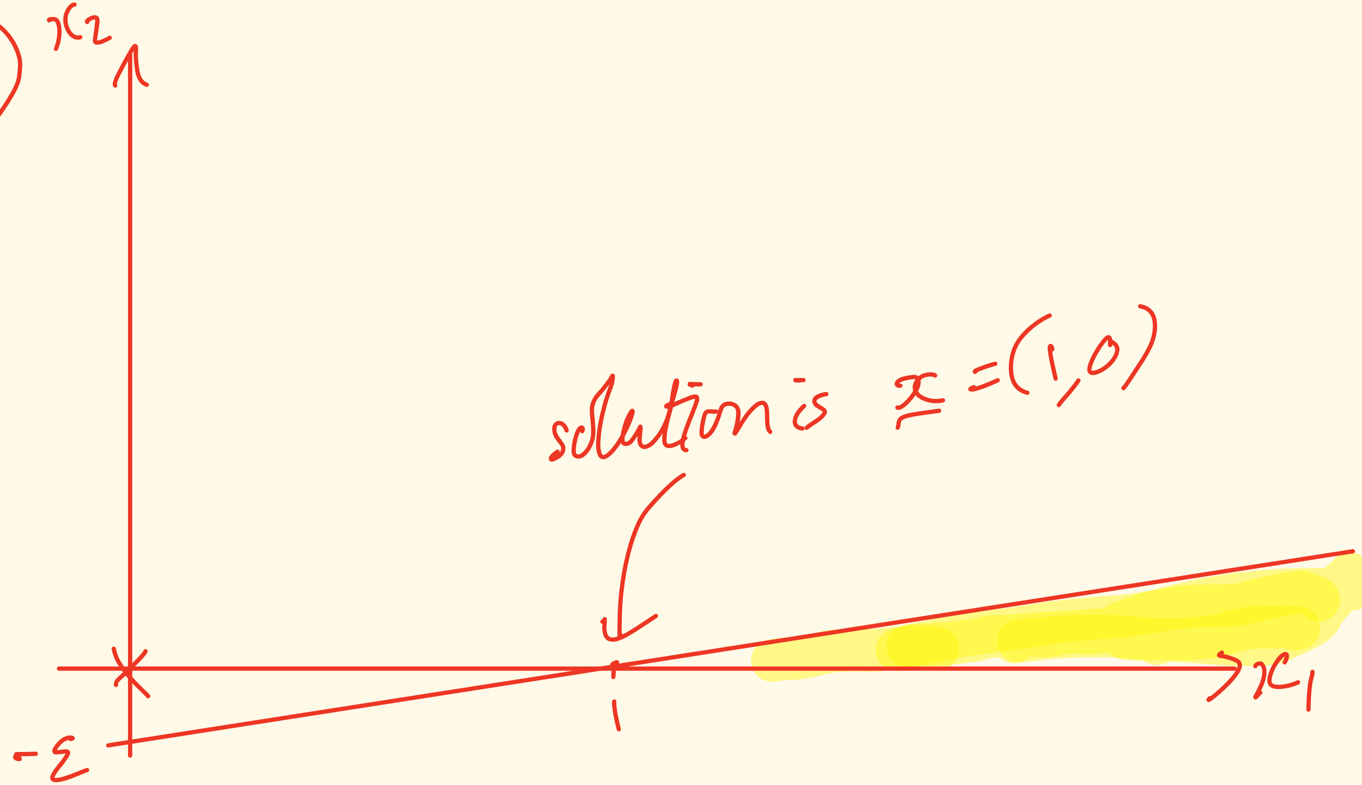
We will set $x_4 = 4$ in the initial bfs to save some time when using the big-M method.

So new LP is:

$$\begin{aligned}
\min\quad & -x_1 - 3x_2 + x_3 + Mx_7 + Mx_8 \\
\text{s.t.}\quad & x_1 + x_2 + 2x_3 + x_4 = 4 \\
& -x_1 + x_3 - x_5 + x_7 = 4 \\
& x_3 - x_6 + x_8 = 3
\end{aligned}$$

initial bfs is $x_4 = 4,\ x_7 = 4,\ x_8 = 3$.

8



Use big-M method: first add slack variable x_3 and artificial variable x_4:

min $\underline{x_1 + Mx_4}$ s.t. $\varepsilon x_1 - x_2 - x_3 + x_4 = \varepsilon$

$x_1, x_2, x_3, x_4 \geq 0.$

initial bfs is $\underline{x} = (0, 0, 0, \varepsilon)$

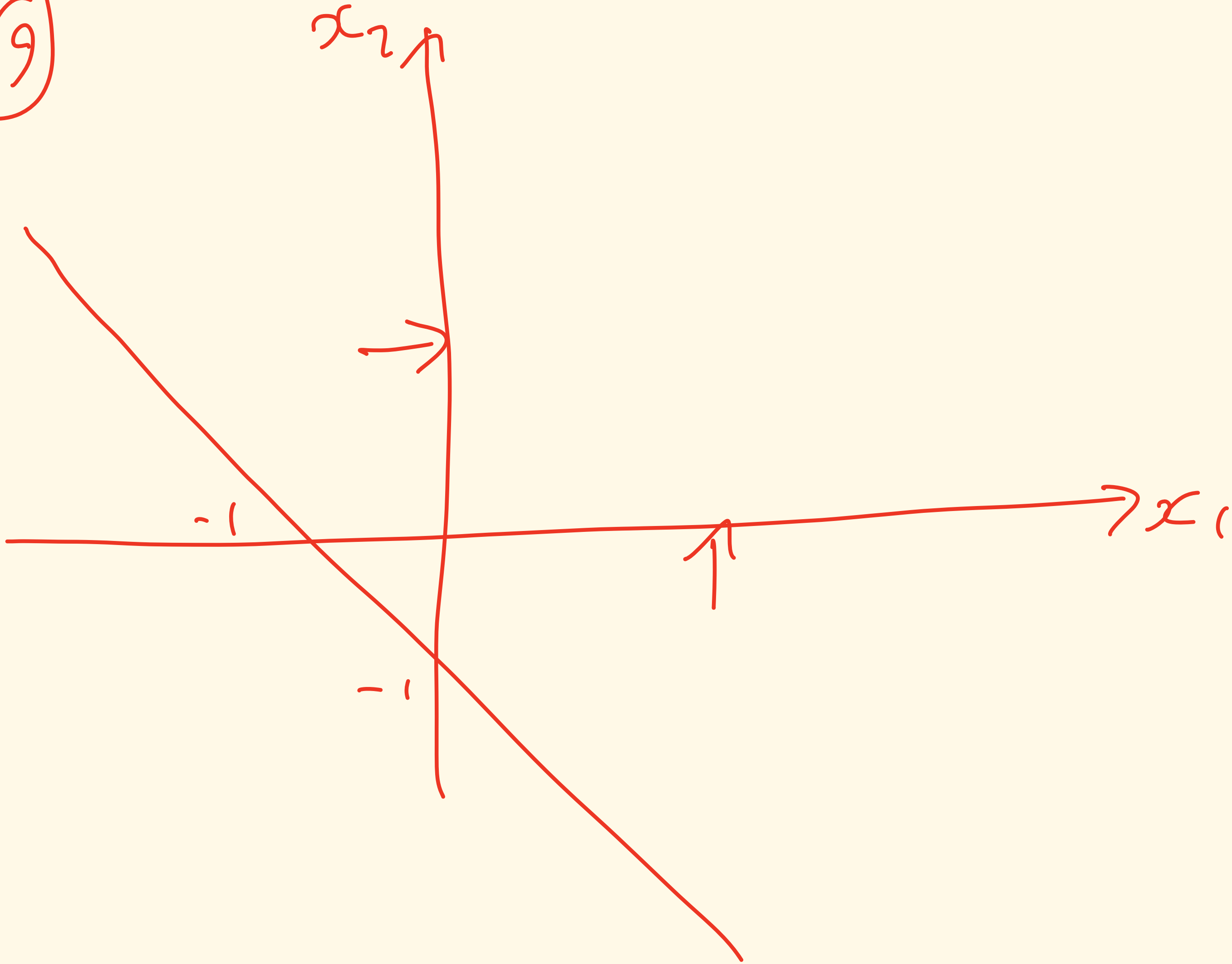
corresponds to a basic, <u>non-feasible</u> solution $\underline{x} = (0, 0)$ to original L.P.

This solution may be optimal if

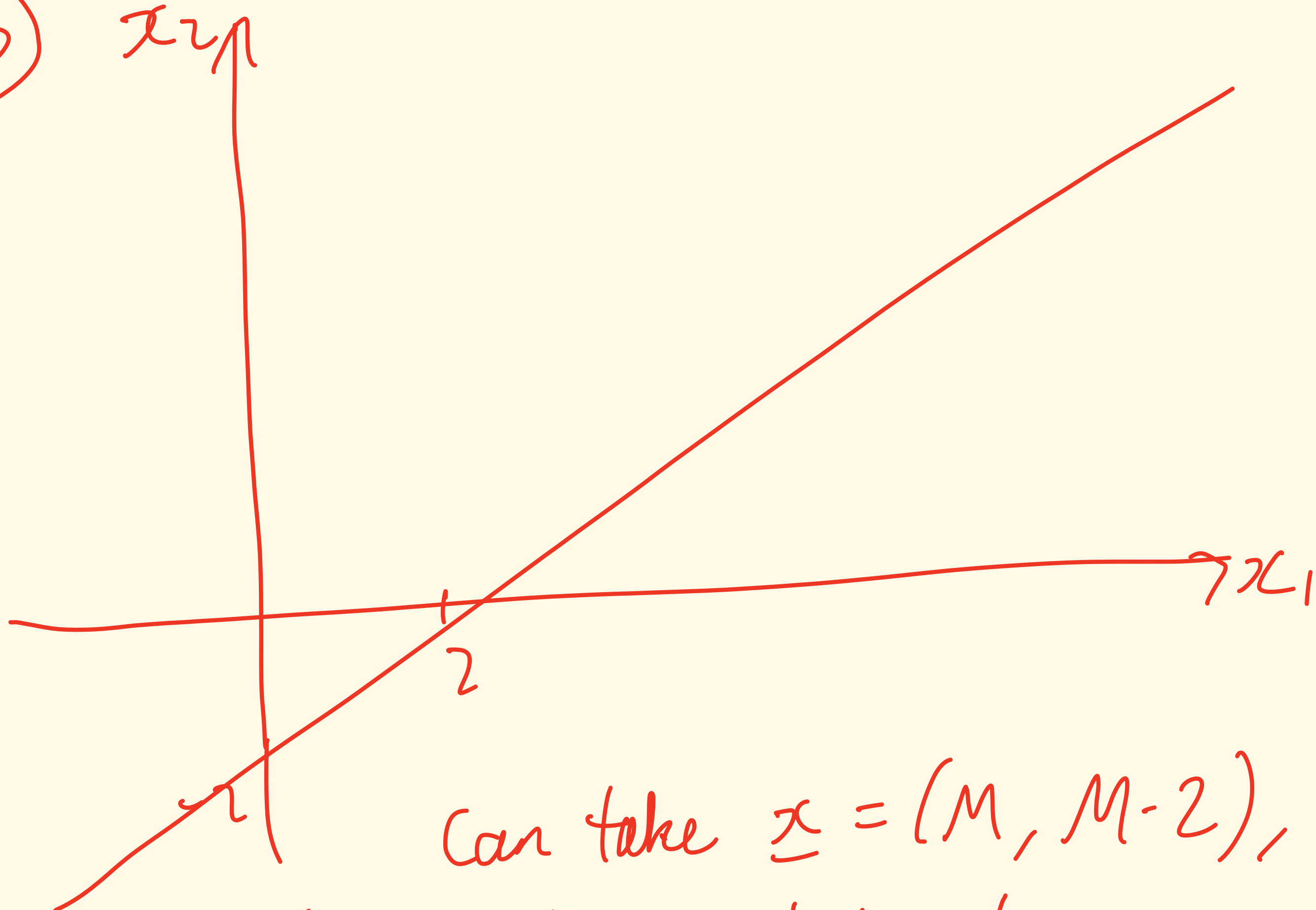
$M(\varepsilon) < 1.$

So we have to make sure M is big enough so that $M\varepsilon > 1$, so we end up with the correct solution.

9



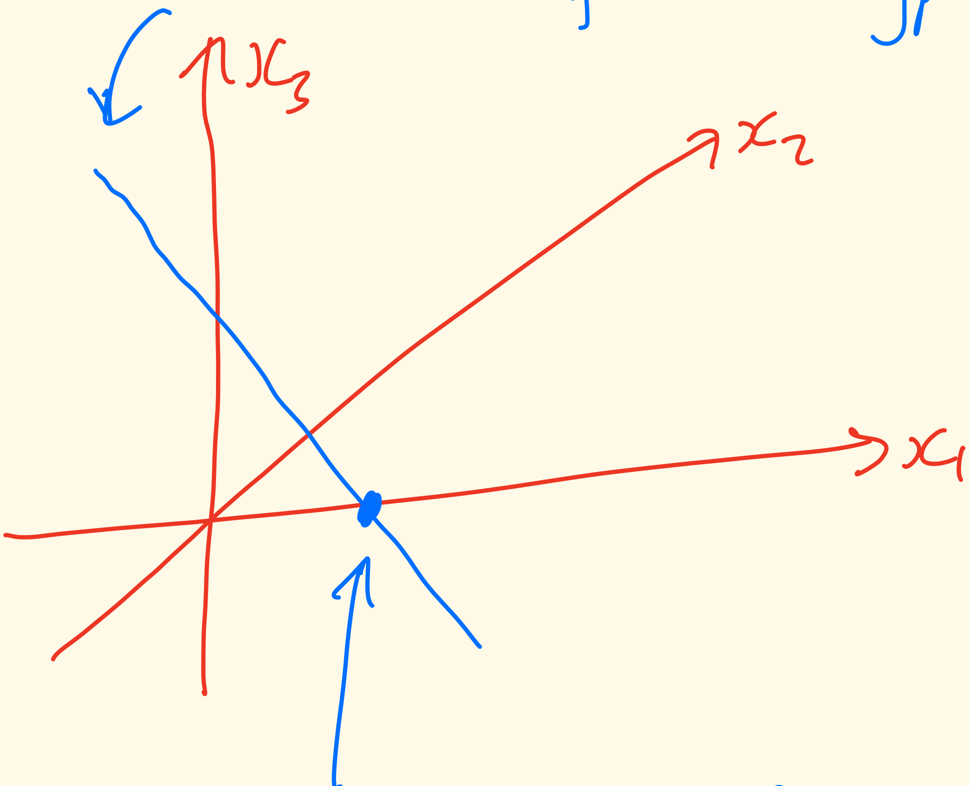
10



Can take $\underline{x} = (M, M-2)$,

where M is arbitrarily large.

intersection of two hyperplanes



here, $x_2 = x_3 = 0$, so degenerate

14