Linear Programming: Homework 4

Homework to be submitted on Canvas by the start of class on Monday October 11th, 1pm.

Explain all your answers and show your working.

Question 1. Consider the following problem.

$$\begin{aligned} \max \quad & 4x_1 + 5x_2 - 3x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 = 10 \\ & x_1 - x_2 \geq 6 \\ & x_1 + 3x_2 + x_3 \leq 14 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(a) Solve the problem using the two-phase simplex method.

(b) Re-solve the problem using the Big-M simplex method.

Question 2.

Consider the following problem.

$$\begin{aligned} \min \quad & -10x_1 + 57x_2 + 9x_3 + 24x_4 \\ \text{s.t.} \quad & 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 \leq 0 \\ & 0.5x_1 - 1.5x_2 - 0.5x_3 + x_4 \leq 0 \\ & x_1 \leq 1 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

Adding slack variables $x_5, x_6, x_7 \geq 0$ to the constraints, the problem becomes

$$\begin{aligned} \min \quad & -10x_1 + 57x_2 + 9x_3 + 24x_4 \\ \text{s.t.} \quad & 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 + x_5 = 0 \\ & 0.5x_1 - 1.5x_2 - 0.5x_3 + x_4 + x_6 = 0 \\ & x_1 + x_7 = 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0. \end{aligned}$$

(a) Run the simplex method for this problem, starting with x_5, x_6, x_7 in the basis. Use the following pivoting rule. When there is a choice of many variables to bring into the basis, choose the one with smallest coefficient in the reduced cost vector (for example if the coefficients of x_1 and x_2 are -30 and -24 and no other variable has a negative coefficient, then choose x_1 to come into the basis). If there is a choice of many entries T_{ij} to pivot on, choose the one with the smallest i (that is, the highest in the tableau). What happens after you perform 6 pivots?

(b) Solve the problem, this time using Bland's pivoting rule.