3 $A\underline{x} = \underline{b} \iff (B\ N)\begin{pmatrix}\underline{x}_B \\ \underline{x}_N\end{pmatrix} = \underline{b}$

$$B\underline{x}_B + N\underline{x}_N = \underline{b}$$

$$\underline{x}_B = B^{-1}\underline{b} - B^{-1}N\underline{x}_N$$

4 Suppose $\underline{x}$ is a bfs, so

$\underline{x} = (\underline{x}_B\ \underline{x}_N)$ where $\underline{x}_B = B^{-1}\underline{b}$,

$\underline{x}_N = \underline{0}$

m equalities from $A\underline{x} = \underline{b}$

n-m equalities from $\underline{x} \geq 0$.

So $\underline{x}$ is on the intersection of $m+(n-m)$ $= n$ L.I. hyperplanes that define the feasible region.

So $\underline{x}$ is an extreme point.

Similarly, if $\underline{x}$ is an extreme pt., it is a bfs.

7 $\quad \underline{x}_B + B^{-1}N\underline{x}_N = B^{-1}\underline{b}$

$$\begin{array}{ccc|c|c}
\hline
1 & 0 & 0 & & \\
0 & 1 & 0 & \left(B^{-1}N \right) & B^{-1}\underline{b} \\
0 & 0 & 1 & & \\
\hline
 & & c^T & &
\end{array}$$

$$\text{obj} = \underline{c}^T\underline{x} = \left(\underline{c}_B \; \underline{c}_N\right)^T \begin{pmatrix} \underline{x}_B \\ \underline{x}_N \end{pmatrix}$$

$$= \underline{c}_B^T \underline{x}_B + \underline{c}_N^T \underline{x}_N$$

$$= \underline{c}_B^T\left(B^{-1}\underline{b} - B^{-1}N\underline{x}_N\right) + \underline{c}_N^T \underline{x}_N$$

$$= \underline{c}_B^T B^{-1} \underline{b} + (\underline{c}_N^T - \underline{c}_B^T B^{-1} N) \underline{x}_N$$

⑧ We have $\underline{x}_B = B^{-1}\underline{b} - B^{-1}N\underline{x}_N$

New $\underline{x}_N$ is $\alpha \underline{e}_j$. How big can α be?

Write $\underline{x}_B^{old}$ for the old bfs

$\underline{x}_B^{new}$ for the new bfs.

Then $\underline{x}_B^{old} = B^{-1}\underline{b}$

and $\underline{x}_B^{new} = B^{-1}\underline{b} - B^{-1}N(\alpha \underline{e}_j)$

$$= \underline{x}_B^{old} - B^{-1}N\alpha\underline{e}_j$$

ith component:

$$(\underline{x}_B^{new})_i = (\underline{x}_B^{old})_i - \underline{e}_i^T B^{-1}N\alpha\underline{e}_j$$

$$= (\underline{x}_B^{old})_i - \alpha(B^{-1}N)_{ij}$$

$$= (\underline{x}_B^{old})_i - \alpha T_{ij}$$

Need $(\underline{x}_B^{new})_i \geq 0$, so

$$(\underline{x}_B^{old})_i - \alpha T_{ij} \geq 0$$

$$\Rightarrow \alpha \leq \frac{(x_B^{old})_i}{T_{ij}} \quad \text{if } T_{ij} > 0$$

(11) $\min \ x_1 + 2x_2 - 3x_3 - x_4 - 5x_5$

s.t.

$$\begin{aligned} x_1 - x_2 \qquad\qquad\qquad + x_5 &= 2 \\ x_2 + x_3 - x_4 + x_5 &= 1 \\ -x_1 + x_2 - x_3 + x_4 - x_5 &= -1 \end{aligned}$$

$$x_i \geq 0 \quad \forall i$$

Auxillary LP:

$$\min \quad x_6 + x_7 + x_8$$

$$\text{s.t.} \quad \begin{aligned} x_1 - x_2 \quad\quad\quad + x_5 + x_6 \quad\quad &= 2 \\ x_2 + x_3 - x_4 + x_5 \quad + x_7 \quad &= 1 \\ -x_1 + x_2 - x_3 + x_4 - x_5 \quad\quad - x_8 &= -1 \\ x_i \geq 0 \ \forall i \end{aligned}$$

$\{x_6 = 2, x_7 = 1, x_8 = 1, x_1 = x_2 = x_3 = x_4 = x_5 = 0\}$

is a bfs