

Question I

HWK4

(a) Original Objective $\max 4x_1 + 5x_2 - 3x_3$

$$\Rightarrow \min -4x_1 - 5x_2 + 3x_3$$

$$\text{s.t. } x_1 + 2x_2 + 3x_3 = 10$$

$$x_1 - x_2 - x_4 = 6$$

$$x_1 + 3x_2 + x_3 + x_5 = 14$$

$$x_1, \dots, x_5 \geq 0$$

Since constrain 1. already meet the standard formation I, we don't need to introduce a unnecessary slack variable into the system, because the coefficients of the slack variable are not a identical matrix, it's necessary to introduce artificial variables in to the system, which are x_6, x_7, x_8 .

PHASE I $\min -4x_1 - 5x_2 + 3x_3 \Rightarrow \min x_6 + x_7 + x_8$

$$\text{s.t. } x_1 + 2x_2 + x_3 + x_6 + x_7 + x_8 = 10$$

$$x_1 - x_2 - x_4 + x_7 = 6$$

$$x_1 + 3x_2 + x_3 + x_5 + x_8 = 14$$

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	0	0	0	-1	-1	-1	0	
0	1	2	1	0	0	1	0	0	10	
0	1	-1	0	-1	0	0	1	0	6	
0	1	3	1	0	1	0	0	1	14	

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	3	4	2	-1	1	0	0	0	30	
0	1	2	1	0	0	1	0	0	10	
0	1	-1	0	-1	0	0	1	0	6	
0	1	3	1	0	1	0	0	1	14	

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	1.666667	0	0.666667	-1	-0.333333	0	0	-1.333333	11.333333	
0	0.333333	0	0.333333	0	-0.666667	1	0	-0.666667	0.666667	2
0	1.333333	0	0.333333	-1	0.333333	0	1	0.333333	10.666667	8
0	0.333333	1	0.333333	0	0.333333	0	0	0.333333	4.666667	14

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	-1	-1	3	-5	0	2	8	
0	1	0	1	0	-2	3	0	-2	2	
0	0	0	-1	-1	3	-4	1	3	8	
0	0	1	0	0	1	-1	0	1	4	

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	0	0	0	-1	-1	-1	0	
0	1	0	0.333333	-0.666667	0	0.333333	0.666667	0	7.333333	
0	0	0	-0.333333	-0.333333	1	-1.333333	0.333333	1	2.666667	
0	0	1	0.333333	0.333333	0	0.333333	-0.333333	0	1.333333	

we have the BFS, $(x_1, x_2, x_3) = (6, \frac{4}{3}, \frac{20}{3})$

PHASE II

$$\min -4x_1 - 5x_2 + 3x_3$$

x0	x1	x2	x3	x4	x5	RHS
1	4	5	-3	0	0	0
0	1	0	0.333333	-0.666667	0	7.333333
0	0	0	-0.333333	-0.333333	1	2.666667
0	0	1	0.333333	0.333333	0	1.333333

x0	x1	x2	x3	x4	x5	RHS
1	0	0	-6	1	0	-36
0	1	0	0.333333	-0.666667	0	7.333333
0	0	0	-0.333333	-0.333333	1	2.666667
0	0	1	0.333333	0.333333	0	1.333333

x0	x1	x2	x3	x4	x5	RHS
1	0	-3	-7	0	0	-40
0	1	2	1	0	0	10
0	0	1	0	0	1	4
0	0	3	1	1	0	4

Since all the coefficients of R_0 are ≤ 0 , which indicates that with the variable increase the objective value will increase, so the original objective will drop, so we got the optimal solution where $(x_1, x_2, x_3) = (10, 0, 0)$, so the max value of objective function is 40

(b) Big-M simplex method.

$$\max 4x_1 + 5x_2 - 3x_3$$

$$\Rightarrow \min -4x_1 - 5x_2 + 3x_3$$

$$\Rightarrow \min -4x_1 - 5x_2 + 3x_3 - Mx_4 + Mx_5$$

$$\text{s.t.} \quad x_1 + 2x_2 + x_3 + Mx_6 = 10$$

$$x_1 - x_2 - x_4 + Mx_7 = 6$$

$$x_1 + 2x_2 + x_3 + x_5 + Mx_8 = 14$$

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	4	5	-3	0	0	-100	-100	-100	0	
0	1	2	1	0	0	1	0	0	10	
0	1	-1	0	-1	0	0	1	0	6	
0	1	3	1	0	1	0	0	1	14	
x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	304	405	197	-100	100	0	0	0	3000	
0	1	2	1	0	0	1	0	0	10	
0	1	-1	0	-1	0	0	1	0	6	
0	1	3	1	0	1	0	0	1	14	
x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	169	0	62	-100	-35	0	0	-135	1110	
0	0.333333	0	0.333333	0	-0.666667	1	0	-0.666667	0.666667	2
0	1.333333	0	0.333333	-1	0.333333	0	1	0.333333	10.666667	8
0	0.333333	1	0.333333	0	0.333333	0	0	0.333333	4.666667	14
x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	-107	-100	303	-507	0	203	772	
0	1	0	1	0	-2	3	0	-2	2	
0	0	0	-1	-1	3	-4	1	3	8	
0	0	1	0	0	1	-1	0	1	4	
x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	-6	1	0	-103	-101	-100	-36	
0	1	0	0.333333	-0.666667	0	0.333333	0.666667	0	7.333333	
0	0	0	-0.333333	-0.333333	1	-1.333333	0.333333	1	2.666667	
0	0	1	0.333333	0.333333	0	0.333333	-0.333333	0	1.333333	
x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	-3	-7	0	0	-104	-100	-100	-40	
0	1	2	1	0	0	1	0	0	10	
0	0	1	0	0	1	-1	0	1	4	
0	0	3	1	1	0	1	-1	0	4	

Since (same as two-phase)

So the last tableau give the optimal solution, $(x_1, x_2, x_3) = (10, 0, 0)$, and the max value of the objective function is 40.

Question II

(a)

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	53	41	-204	-20	0	0	0	
0	1	-11	-5	18	2	0	0	0	
0	0	4	2	-8	-1	1	0	0	
0	0	11	5	-18	-2	0	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	0	14.5	-98	-6.75	-13.25	0	0	
0	1	0	0.5	-4	-0.75	2.75	0	0	
0	0	1	0.5	-2	-0.25	0.25	0	0	
0	0	0	-0.5	4	0.75	-2.75	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	-29	0	0	18	15	-93	0	0	
0	2	0	1	-8	-1.5	5.5	0	0	
0	-1	1	0	2	0.5	-2.5	0	0	
0	1	0	0	0	0	0	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	-20	-9	0	0	10.5	-70.5	0	0	
0	-2	4	1	0	0.5	-4.5	0	0	
0	-0.5	0.5	0	1	0.25	-1.25	0	0	
0	1	0	0	0	0	0	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	22	-93	-21	0	0	24	0	0	
0	-4	8	2	0	1	-9	0	0	
0	0.5	-1.5	-0.5	1	0	1	0	0	
0	1	0	0	0	0	0	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	7.5	-49.5	-6.5	-29	0	-5	0	0	
0	0.5	-5.5	-2.5	9	1	0	0	0	
0	0.5	-1.5	-0.5	1	0	1	0	0	
0	1	0	0	0	0	0	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	33	31	-164	-15	-5	0	0	
0	1	-11	-5	18	2	0	0	0	
0	0	4	2	-8	-1	1	0	0	
0	0	11	5	-18	-2	0	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	0	14.5	-98	-6.75	-13.25	0	0	
0	1	0	0.5	-4	-0.75	2.75	0	0	
0	0	1	0.5	-2	-0.25	0.25	0	0	
0	0	0	-0.5	4	0.75	-2.75	1	1	
x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	-29	0	0	18	15	-93	0	0	
0	2	0	1	-8	-1.5	5.5	0	0	
0	-1	1	0	2	0.5	-2.5	0	0	
0	1	0	0	0	0	0	1	1	

We can see that the matrix is the same when we do 3rd pivot so this pivot rule would cause cycling

(b) Since the coefficients of the slack variables are identical, and the RHS are ≥ 0 , so we could start with x_5, x_6, x_7 as bfs. without add artificial variables.

Starting x_5, x_6, x_7 , we can reach the optimal

solution that the coefficients are all ≥ 0

So that with the variable growing the objective

value will grow up, so the optimal solution minimize .

is $(x_1, x_2, x_3, x_4) = (1, 0, 1, 0)$, the object

value is -1 .

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	53	41	-204	-20	0	0	0	
0	1	-11	-5	18	2	0	0	0	
0	0	4	2	-8	-1	1	0	0	
0	0	11	5	-18	-2	0	1	1	

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	0	14.5	-98	-6.75	-13.25	0	0	
0	1	0	0.5	-4	-0.75	2.75	0	0	
0	0	1	0.5	-2	-0.25	0.25	0	0	
0	0	0	-0.5	4	0.75	-2.75	1	1	

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	-29	0	0	18	15	-93	0	0	
0	2	0	1	-8	-1.5	5.5	0	0	
0	-1	1	0	2	0.5	-2.5	0	0	
0	1	0	0	0	0	0	1	1	

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	-20	-9	0	0	10.5	-70.5	0	0	
0	-2	4	1	0	0.5	-4.5	0	0	
0	-0.5	0.5	0	1	0.25	-1.25	0	0	
0	1	0	0	0	0	0	1	1	

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	22	-93	-21	0	0	24	0	0	
0	-4	8	2	0	1	-9	0	0	
0	0.5	-1.5	-0.5	1	0	1	0	0	
0	1	0	0	0	0	0	1	1	

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	-27	1	-44	0	-20	0	0	
0	0	-4	-2	8	1	-1	0	0	
0	1	-3	-1	2	0	2	0	0	
0	0	3	1	-2	0	-2	1	1	

x0	x1	x2	x3	x4	x5	x6	x7	RHS	
1	0	-30	0	-42	0	-18	-1	-1	
0	0	2	0	4	1	-5	2	2	
0	1	0	0	0	0	0	1	1	
0	0	3	1	-2	0	-2	1	1	

