HW5

Question 1

(a) min $3x_1+5x_2-3x_3+4x_4$

St. $x_1+5x_2-2x_3-2x_4=21$

$2x_1 \quad -5x_3+4x_4=2$

$x_1, x_2, x_3, x_4 \geq 0$

Since there are no equality in the constraints so there is no need to introduce slack variables we just add artificial variables A_1 and A_2 and apply big-M method.

y0	y1	y2	y3	y4	RHS
1	-3	-5	3	-4	0
0	-1	-5	2	2	-21
0	2	0	-5	4	2

x0	x1	x2	x3	x4	A1	A2	RHS
1	-3	-5	3	-4	-100	-100	0
0	1	5	-2	-2	1	0	21
0	2	0	-5	4	0	1	2

x0	x1	x2	x3	x4	A1	A2	RHS
1	297	495	-697	196	0	0	2300
0	1	5	-2	-2	1	0	21
0	2	0	-5	4	0	1	2

Q1a

x0	x1	x2	x3	x4	A1	A2	RHS
1	198	0	-499	394	-99	0	221
0	0.2	1	-0.4	-0.4	0.2	0	4.2
0	2	0	-5	4	0	1	2

x0	x1	x2	x3	x4	A1	A2	RHS
1	1	0	-6.5	0	-99	-98.5	24
0	0.4	1	-0.9	0	0.2	0.1	4.4
0	0.5	0	-1.25	1	0	0.25	0.5

x0	x1	x2	x3	x4	A1	A2	RHS
1	0	0	-4	-2	-99	-99	23
0	0	1	0.1	-0.8	0.2	-0.1	4
0	1	0	-2.5	2	0	0.5	1

So we got $(x_1, x_2, x_3, x_4)=(1,4,0,0)$

1b) So currently the problem change to

min $3x_1+5x_2-3x_3+4x_4$

St. $-x_1-5x_2+2x_3+2x_4=-21$

$2x_1 \quad -5x_3+4x_4=-1$

$x_1, x_2, x_3, x_4 \geq 0$

change the RHS of the primal wouldn't change the constraints of the dual but will change the objective function. The current dual and the original dual are

current : Max $-21y_1 - y_2$ original= Max $-21y_1+21y_2$

St. $-y_1+2y_2 \le 3$ St. $-y_1+2y_2 \le 3$

$-5y_1 \le 5$ $-5y_1 \le 5$

$2y_1-5y_2 \le -3$ $2y_1-5y_2 \le -3$

$2y_1+4y_2 \le 4$ $2y_1+4y_2 \le 4$

y_1, y_2 unrestricted y_1, y_2 unrestricted

After using $y_1''-y_1'=y_1$, $y_2''-y_2'=y_2$, $y_1'', y_1', y_2'', y_2' \ge 0$, the current optimal solution is $(y_1, y_2) = (-1, +0.2)$ optimal value 20.8, the original problem's are $(y_1, y_2)=(1,1)$, whose current objective value are 23 the optimal value not the same, but still feasible since the constraints are not change, since $20.8 < 23$ so it's not necessary optimal, which depend on the varified RHS

x1	x2	x3	x4	RHS
-3	-5	3	-4	0
1	5	-2	-2	21
-2	0	5	-4	1

x1	x2	x3	x4	RHS
0	-5	-4.5	2	-1.5
0	5	0.5	-4	21.5
1	0	-2.5	2	-0.5

x1	x2	x3	x4	RHS
0	0	-4	-2	20
0	1	0.1	-0.8	4.3
1	0	-2.5	2	-0.5

x1	x2	x3	x4	RHS
-1.6	0	0	-5.2	20.8
0.04	1	0	-0.72	4.28
-0.4	0	1	-0.8	0.2

Change the objective function to max and the constraints X -1

Set up the tableau and make x_1 and x_2 bs through ERO, only find that the r_2 has negative RHS, plus x_3's coefficients are negative so we pivot on this one

and got $x_2 = 4.28$ and $x_3 = 0.2$ and the optimal value match the optimal value of the dual problem.

Question 2

(a) primal: min $\underline{c}^T \underline{x}$

St. $\underline{x} = \underline{s}$

$\underline{x} = \underline{d}$

$x \geq 0$

dual: Max: $\underline{s}^T \underline{y} + \underline{d}^T \underline{z}$

St. $\underline{y} + \underline{z} \leq c$.

y, z unrestricted

(b) An entrepreneur who will buy the commodity at a supply node and Sell it on the demand node, and has to decide how much profit he will make y_i at the supply center i and how much profit he will make z_j at the demand center d_j.

Since s_i of product was offered in supply center i, and d_i of product was needed in demand center j, the revenue (which the entrepreneur wish to maximize is

$$s_1 y_1 + s_2 y_2 + \cdots + s_i y_i + d_1 z_1 + d_2 z_2 + \cdots + d_j z_j$$

the price has to make, so the buy & resell process's profict must less equal to the direct shipment cost, otherwise their is no need for the demand center to buy the entrepreneur's product, they co. just order from the supply center.

Question 3

(b) $\min \ 2x_2 - 5x_3 + MA_1$

$$\text{s.t.} \quad 2x_1 + x_2 + x_3 + s_1 = 15$$
$$2x_1 + 3x_2 - 4x_3 - s_2 + A_1 = 10$$
$$x_1 - x_2 + x_3 + s_3 = 9$$

x0	x1	x2	x3	s1	s2	s3	A1	rhs
1	0	-2	5	0	0	0	-100	0
0	2	1	1	1	0	0	0	15
0	2	3	-4	0	-1	0	1	10
0	1	-1	1	0	0	1	0	9

x0	x1	x2	x3	s1	s2	s3	A1	rhs
1	200	298	-395	0	-100	0	0	1000
0	2	1	1	1	0	0	0	15
0	2	3	-4	0	-1	0	1	10
0	1	-1	1	0	0	1	0	9

x0	x1	x2	x3	s1	s2	s3	A1	rhs
1	595	-97	0	0	-100	395	0	4555
0	1	2	0	1	0	-1	0	6
0	6	-1	0	0	-1	4	1	46
0	1	-1	1	0	0	1	0	9

x0	x1	x2	x3	s1	s2	s3	A1	rhs
1	0	-1287	0	-595	-100	990	0	985
0	1	2	0	1	0	-1	0	6
0	0	-13	0	-6	-1	10	1	10
0	0	-3	1	-1	0	2	0	3

x0	x1	x2	x3	s1	s2	s3	A1	rhs
1	0	0	0	-1	-1	0	-99	-5
0	1	0.7	0	0.4	-0.1	0	0.1	7
0	0	-1.3	0	-0.6	-0.1	1	0.1	1
0	0	-0.4	1	0.2	0.2	0	-0.2	1

the same in the AMPL $(x_1, x_2, x_3) = (7, 0, 1)$

the constrain is tight at 1, 2, and $x_2 \geq 0$,

so $\min \ (15 - 2x_1 - x_2 - x_3) + (10 - 2x_1 - 3x_2 + 4x_3) - x_2$

$$\text{s.t.} \quad 2x_1 + x_2 + x_3 \leq 15$$
$$2x_1 + 3x_2 - 4x_3 \geq 10$$
$$x_1 - x_2 + x_3 \leq 9$$
$$2x_2 - 5x_3 = -5$$

the result is a different optimal value -10 with $(x_1, x_2, x_3) = (0, 10, 5)$