HW4

Question I

(a) Original Objective $\max 4x_1+5x_2-3x_3$

$\Rightarrow \min -4x_1-5x_2+3x_3$

$$
\begin{aligned}
\text{s.t. } & x_1+2x_2+3x_3 && = 10 \\
& x_1-x_2 \quad - x_4 && = 6 \\
& x_1+3x_2+x_3 \quad + x_5 && = 14
\end{aligned}
$$

$x_1, \cdots x_5 \geq 0$

Since constrain 1. already meet the standard formation I, we don't need to introduce a ~~unecessary~~ slack varaible into the system, Because the coefficients of the slack variable are not a identical matrix, it's necessary to introduce artifical variables in to the system, which are x_6, x_7, x_8.

PHASE I $\min -4x_1-5x_2+3x_3 \Rightarrow \min x_6+x_7+x_5$

$$
\begin{aligned}
\text{s.t. } & x_1+2x_2+x_3+ \quad +x_6 && =10 \\
& x_1-x_2 \quad - x_4 \quad +x_7 && =6 \\
& x_1+3x_2+x_3 \quad +x_5 && =14
\end{aligned}
$$

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	0	0	-1	-1	-1	0
0	1	2	1	0	0	1	0	10
0	1	-1	0	-1	0	0	1	6
0	1	3	1	0	1	0	0	14

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	3	4	2	-1	0	0	0	30
0	1	2	1	0	0	1	0	10
0	1	-1	0	-1	0	0	1	6
0	1	3	1	0	1	0	0	14

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	1.6667	0	0.6667	-1	-1.333	0	0	11.333
0	0.3333	0	0.3333	0	-0.667	1	0	0.6667
0	1.3333	0	0.3333	-1	0.3333	0	1	10.667
0	0.3333	1	0.3333	0	0.3333	0	0	4.6667

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	-1	-1	2	-5	0	8
0	1	0	1	0	-2	3	0	2
0	0	0	-1	-1	3	-4	1	8
0	0	1	0	0	1	-1	0	4

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	-0.333	-0.333	0	-2.333	-0.667	2.6667
0	1	0	0.3333	-0.667	0	0.3333	0.6667	7.3333
0	0	0	-0.333	-0.333	1	-1.333	0.3333	2.6667
0	0	1	0.3333	0.3333	0	0.3333	-0.333	1.3333

we have the BFS, (x_1, x_2, x_5)

PHASE II

min $-4x_1 - 5x_2 + 3x_3$

x0	x1	x2	x3	x4	x5	RHS
1	4	5	-3	0	0	0
0	1	0	0.3333	-0.667	0	7.3333
0	0	0	-0.333	-0.333	1	2.6667
0	0	1	0.3333	0.3333	0	1.3333

x0	x1	x2	x3	x4	x5	RHS
1	0	0	-6	1	0	-36
0	1	0	0.3333	-0.667	0	7.3333
0	0	0	-0.333	-0.333	1	2.6667
0	0	1	0.3333	0.3333	0	1.3333

x0	x1	x2	x3	x4	x5	RHS
1	0	-3	-7	0	0	-40
0	1	2	1	0	0	10
0	0	1	0	0	1	4
0	0	3	1	1	0	4

Since all the coefficences of R_0 are ≤ 0, which indicates that with the variable increase the objective value will increase, so the original objective will drop, so we got the optional solution where $(x_1, x_2, x_3) = (10, 0, 0)$, so the max value of objective function is 40

(b) Big-M simplex method.

max $\quad 4x_1 + 5x_2 - 3x_3$

$\Rightarrow$ min $\; -4x_1 - 5x_2 + 3x_3$

$\Rightarrow$ min $\; -4x_1 - 5x_2 + 3x_3 - Mx_6 - Mx_7$

$$\text{s.t.} \quad \begin{aligned} x_1 + 2x_2 + x_3 \quad\quad\quad\quad + Mx_6 \quad\quad &= 10 \\ x_1 - x_2 \quad\quad - x_4 \quad\quad\quad\quad + Mx_7 &= 6 \\ x_1 + 3x_2 + x_3 \quad\quad + x_5 \quad\quad\quad\quad &= 14 \end{aligned}$$

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	4	5	-3	0	0	-100	-100	0
0	1	2	1	0	0	1	0	10
0	1	-1	0	-1	0	0	1	6
0	1	3	1	0	1	0	0	14

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	204	105	97	-100	0	0	0	1600
0	1	2	1	0	0	1	0	10
0	1	-1	0	-1	0	0	1	6
0	1	3	1	0	1	0	0	14

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	169	0	62	-100	-35	0	0	1110
0	0.3333	0	0.3333	0	-0.667	1	0	0.6667
0	1.3333	0	0.3333	-1	0.3333	0	1	10.667
0	0.3333	1	0.3333	0	0.3333	0	0	4.6667

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	-107	-100	303	-507	0	772
0	1	0	1	0	-2	3	0	2
0	0	0	-1	-1	3	-4	1	8
0	0	1	0	0	1	-1	0	4

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	-6	1	0	-103	-101	-36
0	1	0	0.3333	-0.667	0	0.3333	0.6667	7.3333
0	0	0	-0.333	-0.333	1	-1.333	0.3333	2.6667
0	0	1	0.3333	0.3333	0	0.3333	-0.333	1.3333

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	-3	-7	0	0	-104	-100	-40
0	1	2	1	0	0	1	0	10
0	0	1	0	0	1	-1	0	4
0	0	3	1	1	0	1	-1	4

Since ......(same as two-phase) ....

So the last tableau give the optional solution, $(x_1, x_2, x_3) = (10, 0, 0)$, and the max value of the objective function is 40.

Question II

(a)

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	53	41	-204	-20	0	0	0
0	1	-11	-5	18	2	0	0	0
0	0	4	2	-8	-1	1	0	0
0	0	11	5	-18	-2	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	14.5	-98	-6.75	-13.25	0	0
0	1	0	0.5	-4	-0.75	2.75	0	0
0	0	1	0.5	-2	-0.25	0.25	0	0
0	0	0	-0.5	4	0.75	-2.75	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	-29	0	0	18	15	-93	0	0
0	2	0	1	-8	-1.5	5.5	0	0
0	-1	1	0	2	0.5	-2.5	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	-20	-9	0	0	10.5	-70.5	0	0
0	-2	4	1	0	0.5	-4.5	0	0
0	-0.5	0.5	0	1	0.25	-1.25	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	22	-93	-21	0	0	24	0	0
0	-4	8	2	0	1	-9	0	0
0	0.5	-1.5	-0.5	1	0	1	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	7.5	-49.5	-6.5	-29	0	-5	0	0
0	0.5	-5.5	-2.5	9	1	0	0	0
0	0.5	-1.5	-0.5	1	0	1	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	33	31	-164	-15	-5	0	0
0	1	-11	-5	18	2	0	0	0
0	0	4	2	-8	-1	1	0	0
0	0	11	5	-18	-2	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	14.5	-98	-6.75	-13.25	0	0
0	1	0	0.5	-4	-0.75	2.75	0	0
0	0	1	0.5	-2	-0.25	0.25	0	0
0	0	0	-0.5	4	0.75	-2.75	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	-29	0	0	18	15	-93	0	0
0	2	0	1	-8	-1.5	5.5	0	0
0	-1	1	0	~~2~~	0.5	-2.5	0	0
0	1	0	0	0	0	0	1	1

We can see that the matrix is the same when we do 3rd pivot so this pivot rule would cause cycling

(b) Since the coefficients of the slack variables are indentical, and the RHS are ≥ 0, so we could start with x_5, x_6, x_7 as bfs. without add artificial variables.

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	53	41	-204	-20	0	0	0
0	1	-11	-5	18	2	0	0	0
0	0	4	2	-8	-1	1	0	0
0	0	11	5	-18	-2	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	14.5	-98	-6.75	-13.25	0	0
0	1	0	0.5	-4	-0.75	2.75	0	0
0	0	1	0.5	-2	-0.25	0.25	0	0
0	0	0	-0.5	4	0.75	-2.75	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	-29	0	0	18	15	-93	0	0
0	2	0	1	-8	-1.5	5.5	0	0
0	-1	1	0	2	0.5	-2.5	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	-20	-9	0	0	10.5	-70.5	0	0
0	-2	4	1	0	0.5	-4.5	0	0
0	-0.5	0.5	0	1	0.25	-1.25	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	22	-93	-21	0	0	24	0	0
0	-4	8	2	0	1	-9	0	0
0	0.5	-1.5	-0.5	1	0	1	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	-27	1	-44	0	-20	0	0
0	0	-4	-2	8	1	-1	0	0
0	1	-3	-1	2	0	2	0	0
0	0	3	1	-2	0	-2	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	-30	0	-42	0	-18	-1	-1
0	0	2	0	4	1	-5	2	2
0	1	0	0	0	0	0	1	1
0	0	3	1	-2	0	-2	1	1

Starting x_5, x_6, x_7, we can reach the opitional solution that the coiffcients are all ≥ 0 so that with the variable growing the objectiv value will grow up, so the optional solution is $(x_1, x_2, x_3, x_4) = (1, 0, 1, 0)$, the minimise object value is -1.