HW4

Question I

(a) Original Objective $\max 4x_1+5x_2-3x_3$

$\Rightarrow \min -4x_1-5x_2+3x_3$

$$\begin{aligned} \text{S.t.}\quad & x_1+2x_2+3x_3 && = 10 \\ & x_1-x_2 \quad - x_4 && = 6 \\ & x_1+3x_2+x_3 \quad + x_5 && = 14 \end{aligned}$$

$x_1, \cdots x_5 \geq 0$

Since constrain 1. already meet the standard formation I, we don't need to introduce a uncessary slack varaible into the system, Because the coefficients of the slack variable are not a identical matrix, it's necessary to introduce artifical variables in to the system, which are x_6, x_7, x_8.

PHASE I $\min -4x_1-5x_2+3x_3 \Rightarrow \min x_6+x_7+x_8$

$$\begin{aligned} \text{S.t.}\quad & x_1+2x_2+x_3+ \quad +x_6+x_7+x_8 && =10 \\ & x_1-x_2 \quad -x_4 \quad +x_7 && =6 \\ & x_1+3x_2+x_3 \quad +x_5 \quad +x_8 && =14 \end{aligned}$$

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	0	0	0	-1	-1	-1	0	
0	1	2	1	0	0	1	0	0	10	
0	1	-1	0	-1	0	0	1	0	6	
0	1	3	1	0	1	0	0	1	14	

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	3	4	2	-1	1	0	0	0	30	
0	1	2	1	0	0	1	0	0	10	
0	1	-1	0	-1	0	0	1	0	6	
0	1	3	1	0	1	0	0	1	14	

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	1.666667	0	0.666667	-1	-0.33333	0	0	-1.33333	11.33333	
0	0.333333	0	0.333333	0	-0.66667	1	0	-0.66667	0.666667	2
0	1.333333	0	0.333333	-1	0.333333	0	1	0.333333	10.66667	8
0	0.333333	1	0.333333	0	0.333333	0	0	0.333333	4.666667	14

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	-1	-1	3	-5	0	2	8	
0	1	0	1	0	-2	3	0	-2	2	
0	0	0	-1	-1	3	-4	1	3	8	
0	0	1	0	0	1	-1	0	1	4	

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	0	0	0	0	0	-1	-1	-1	0	
0	1	0	0.333333	-0.66667	0	0.333333	0.666667	0	7.333333	
0	0	0	-0.33333	-0.33333	1	-1.33333	0.333333	1	2.666667	
0	0	1	0.333333	0.333333	0	0.333333	-0.33333	0	1.333333	

we have the BFS, $(x_1, x_3, x_5) = (6, \frac{4}{3}, \frac{20}{3})$

PHASE II

min $-4x_1 - 5x_2 + 3x_3$

x0	x1	x2	x3	x4	x5	RHS
1	4	5	-3	0	0	0
0	1	0	0.333333	-0.66667	0	7.333333
0	0	0	-0.33333	-0.33333	1	2.666667
0	0	1	0.333333	0.333333	0	1.333333

x0	x1	x2	x3	x4	x5	RHS
1	0	0	-6	1	0	-36
0	1	0	0.333333	-0.66667	0	7.333333
0	0	0	-0.33333	-0.33333	1	2.666667
0	0	1	0.333333	0.333333	0	1.333333

x0	x1	x2	x3	x4	x5	RHS
1	0	-3	-7	0	0	-40
0	1	2	1	0	0	10
0	0	1	0	0	1	4
0	0	3	1	1	0	4

Since all the coefficences of R_0 are ≤ 0, which indicates that with the variable increase the objective value will increase, so the original objective will drop, so we got the optional solution where $(x_1, x_2, x_3) = (10, 0, 0)$, so the max value of objective function is 40

(b) Big-M simplex method.

max $\quad 4x_1 + 5x_2 - 3x_3$

$\Rightarrow$ min $\quad -4x_1 - 5x_2 + 3x_3$

$\Rightarrow$ min $\quad -4x_1 - 5x_2 + 3x_3 - Mx_4 + Mx_5$

s.t. $\quad x_1 + 2x_2 + x_3 \quad + Mx_6 \quad = 10$

$\quad x_1 - x_2 \quad - x_4 \quad + Mx_7 \quad = 6$

$\quad x_1 + 3x_2 + x_3 \quad + x_5. \quad + Mx_8 = 14$

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS
1	4	5	-3	0	0	-100	-100	-100	0
0	1	2	1	0	0	1	0	0	10
0	1	-1	0	-1	0	0	1	0	6
0	1	3	1	0	1	0	0	1	14

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS
1	304	405	197	-100	100	0	0	0	3000
0	1	2	1	0	0	1	0	0	10
0	1	-1	0	-1	0	0	1	0	6
0	1	3	1	0	1	0	0	1	14

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS	
1	169	0	62	-100	-35	0	0	-135	1110	
0	0.333333	0	0.333333	0	-0.66667	1	0	-0.66667	0.666667	2
0	1.333333	0	0.333333	-1	0.333333	0	1	0.333333	10.66667	8
0	0.333333	1	0.333333	0	0.333333	0	0	0.333333	4.666667	14

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS
1	0	0	-107	-100	303	-507	0	203	772
0	1	0	1	0	-2	3	0	-2	2
0	0	0	-1	-1	3	-4	1	3	8
0	0	1	0	0	1	-1	0	1	4

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS
1	0	0	-6	1	0	-103	-101	-100	-36
0	1	0	0.333333	-0.66667	0	0.333333	0.666667	0	7.333333
0	0	0	-0.33333	-0.33333	1	-1.33333	0.333333	1	2.666667
0	0	1	0.333333	0.333333	0	0.333333	-0.33333	0	1.333333

x0	x1	x2	x3	x4	x5	x6	x7	x8	RHS
1	0	-3	-7	0	0	-104	-100	-100	-40
0	1	2	1	0	0	1	0	0	10
0	0	1	0	0	1	-1	0	1	4
0	0	3	1	1	0	1	-1	0	4

Since ..... (same as two-phase) ......

So the last tableau give the optional solution, $(x_1, x_2, x_3) = (10, 0, 0)$, and the max value of the objective function is 40.

Question II

(a)

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	10	-57	-9	-24	0	0	0	0
0	0.5	-5.5	-2.5	9	1	0	0	0
0	0.5	-1.5	-0.5	1	0	1	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	53	41	-204	-20	0	0	0
0	1	-11	-5	18	2	0	0	0
0	0	4	2	-8	-1	1	0	0
0	0	11	5	-18	-2	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	-29	0	-40	0.5	-20.5	0	0
0	1	-1	0	-2	-0.5	2.5	0	0
0	0	2	1	-4	-0.5	0.5	0	0
0	0	1	0	2	0.5	-2.5	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	1	-30	0	-42	0	-18	0	0
0	-2	2	0	4	1	-5	0	0
0	-1	3	1	-2	0	-2	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	-30	0	-42	0	-18	-1	-1
0	0	2	0	4	1	-5	2	2
0	0	3	1	-2	0	-2	1	1
0	1	0	0	0	0	0	1	1

So the optional solution is $(x_1, x_2, x_3, x_4) = (1, 0, 1, 0)$

So the minimize objective value is -1

(b) Since the coefficients of the slack variables are indentical, and the RHS are ≥ 0, so we could start with x_5, x_6, x_7 as bfs. without add artificial variables.

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	10	-57	-9	-24	0	0	0	0
0	0.5	-5.5	-2.5	9	1	0	0	0
0	0.5	-1.5	-0.5	1	0	1	0	0
0	1	0	0	0	0	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	53	41	-204	-20	0	0	0
0	1	-11	-5	18	2	0	0	0
0	0	4	2	-8	-1	1	0	0
0	0	11	5	-18	-2	0	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	0	14.5	-98	-6.75	-13.25	0	0
0	1	0	0.5	-4	-0.75	2.75	0	0
0	0	1	0.5	-2	-0.25	0.25	0	0
0	0	0	-0.5	4	0.75	-2.75	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	-29	0	-40	0.5	-20.5	0	0
0	1	-1	0	-2	-0.5	2.5	0	0
0	0	2	1	-4	-0.5	0.5	0	0
0	0	1	0	2	0.5	-2.5	1	1

x0	x1	x2	x3	x4	x5	x6	x7	RHS
1	0	-30	0	-42	0	-18	-1	-1
0	1	0	0	0	0	0	1	1
0	0	3	1	-2	0	-2	1	1
0	0	2	0	4	1	-5	2	2

Starting x_5, x_6, x_7, we can reach the opitional solution that the coiffcients are all ≥ 0 so that with the variable growing the objective value will grow up, so the optional solution is $(x_1, x_2, x_3, x_4) = (1, 0, 1, 0)$, the minimise object value is -1.