Linear Programming: Homework 1

Homework to be submitted on Canvas by the start of class on Monday September 20th, 1pm.

Please include AMPL model files for Questions 1 and 2 and an Excel file for Question 2.

Explain all your answers and show your working.

Question 1. Consider the constraints

$$\begin{aligned} 3x_1 + 2x_2 &\geq 60 \\ -x_1 + 2x_2 &\leq 20 \\ x_1 &\leq 30 \\ x_1 - x_2 &\leq 30 \\ x_1, x_2 &\geq 0. \end{aligned}$$

(a) Draw the constraints by hand and shade the feasible region.

(b) Use your diagram to identify the extreme points of the feasible region.

(c) Graphically solve the problem of minimizing $x_2 - 2x_1$ subject to the constraints above.

(d) Use AMPL to check that your solution is correct.

Question 2.

A firm of shoemakers can produce shoes for men, women and children. The output of women's shoes must be not less than 50% of the total. Each kind of shoe requires a certain type of machine and some skilled labor, and the number of machines and skilled workers available is fixed.

	Men's shoes	Women's shoes	Children's shoes
Machine time (hours per pair)	1.5	2	1
Skilled labor (hours per pair)	2.5	6	1.5
$ revenue ($ per pair)	26	56	14

The firm has 20 machines and the labor to operate them, and a staff of 50 skilled workers. Normal working is 40 hours per week.

(a) Formulate the problem of maximizing the firm's revenue algebraically as a linear program, clearly identifying decision variables, the objective function and constraints.

(b) Solve the LP using Excel.

(c) Solve the LP using AMPL and check you get the same answer!

Question 3.

Transform the following linear program into the Standard Form 1 (see lecture slides) by introducing new variables and constraints, if necessary.

$$\begin{aligned} \max \quad & x_1 + x_2 - 3x_3 + 2x_5 \\ \text{s.t.} \quad & x_1 + x_5 \geq 6 \\ & x_1 + x_2 + x_3 + x_4 + x_5 \leq 3 \\ & 2x_4 - 3x_2 = -4 \\ & x_5, x_4, x_3 \geq 0 \quad (x_1, x_2 \text{ unrestricted}) \end{aligned}$$

Note: you are not required to solve the LP.

Question 4.

Using Gauss-Jordan reduction, determine whether the following system has (a) no solutions, (b) a unique solution or (c) infinitely many solutions. In the case of (b) or (c), give all solutions.

$$\begin{array}{ccccccccc} x_1 & + & 3x_2 & + & x_3 & - & 4x_4 & = & 1 \\ & & 5x_2 & - & 6x_3 & + & x_4 & = & 0 \\ x_1 & - & 2x_2 & + & 4x_3 & & & = & 2 \end{array}$$