Chapter 2

TESTING OF HYPOTHESIS

Introduction

One of the important application of statistical inference is the testing of hypothesis. Modern theory of probability plays an important role in decision making and the branch of statistics which helps us in arriving at the criterion for such decisions is called testing of hypothesis. It was initiated by J. Neyman and E.S. Pearson. It employees statistical techniques to arrive at decisions in certain situation where there is an element of uncertainty on the basis of sample whose size is fixed in advance. We draw conclusion about the population parameter on the basis of sample.

According to Webster hypothesis is defined as " A tentative theory or supposition provisionally adopted to explain certain facts and to guide in the investigation of others"

Hypothesis

A hypothesis is a tentative theory or supposition provisionally adopted to explain certain facts and to guide in the investigation of others.

A statistical hypothesis which is tentative statement or supposition about the estimated value of one or more parameter of the population is called parametric hypothesis .A statistical hypothesis about attributes is called non parametric hypothesis.

If a hypothesis completely determines the population, it is called a simple hypothesis, otherwise composite hypothesis.

In testing of hypothesis a statistic is computed from a sample drawn from the parent population and on the basis of the statistic it is observed whether the sample so drawn has come from the population with certain specified characteristic.

Terminology are the different terms used in testing of hypothesis. By the knowledge of terminology it becomes easier to know the terms during the numerical calculation.

Types of statistical hypothesis

Null hypothesis

The supposition about the population parameter is called null hypothesis. It is set up for testing a statistical hypothesis only to decide whether to accept or reject the null hypothesis. According to R.A. Fisher, null hypothesis is the hypothesis which is tested for possible rejection under the assumption that is true.

It is the hypothesis of no difference between sample statistic and parameter. It is hypothesis of no difference between parameters.

Null hypothesis is denoted by H_0 .It is set up as $H_0{:}\mu = \mu_0$

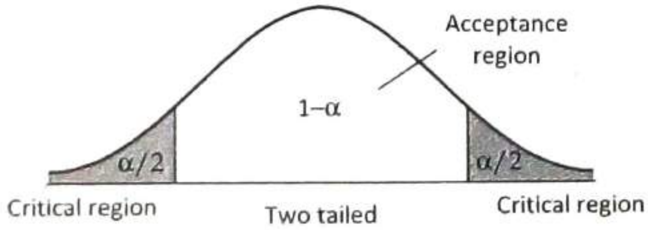
Suppose we want to test the average score of students in B.Sc. entrance exam is 55 then to start testing the hypothesis we assume the average score is 55.There is no difference between sample average and population average. Then the null hypothesis is $H_0{:}\mu = 55$

Alternative hypothesis

A hypothesis which is complementary to the null hypothesis is called an alternative hypothesis

Any hypothesis which is not null is also called alternative hypothesis. It is hypothesis of difference between sample statistic and parameter. It is hypothesis of difference between parameters.

Alternative hypothesis is denoted by H_1.



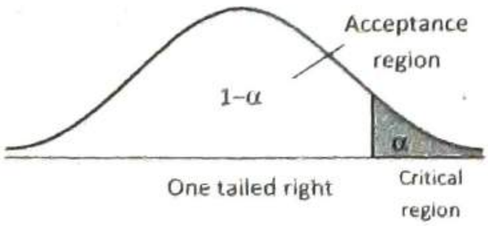
Alternative hypothesis are

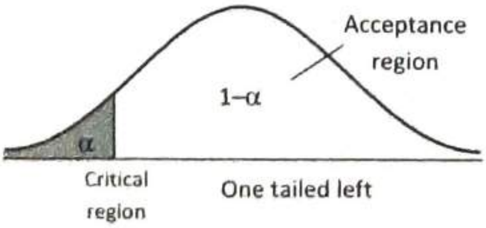
(i) two tailed

(ii) one tailed right

(iii) one tailed left

Alternative hypothesis is set up as H_1: $\mu \neq \mu_0$ for two tailed or H_1: $\mu > \mu_0$ for one tailed right or H_1: $\mu < \mu_0$ for one tail left.





The setting of alternative hypothesis is very important since it enable us to decide whether we have to use two tailed or one tailed test.

If the null hypothesis is rejected in the above example then average score of students in B.Sc. entrance exam is different from 55 or greater than 55 or less than 55. Among these only one situation arise. Then the alternative hypothesis is $H_1: \mu \neq 55$ or $H_1: \mu > 55$ or $H_1: \mu < 55$.

Errors in Hypothesis testing

The main objective of sampling theory is to draw valid inference about the population parameters on the basis of the sample. We decide to accept or reject the hypothesis after examining result from sample. In testing of hypothesis two types of errors are introduced.

Type I error

It is the error of rejecting null hypothesis H_0 when it is true. The probability of type I error is denoted by α, called the level of significance.

α = Probability (Type I error) = Probability (Rejecting H_0 / H_0 is true).

Type II error

It is error of accepting null hypothesis H_0 when it is false. It means it is error of accepting null hypothesis H_0 when alternative hypothesis H_1 is true. The probability of type II error is denoted by β.

β = Probability (Type II error) = Probability (Accepting H_0/ H_1 is true).

		Truth	
		H_0 is true	H_0 is false
Decision	Accept H_0	Correct decision	Type II error
	Reject H_0	Type I error	Correct decision

In testing of hypothesis, accepting H_0 when H_0 is true is the correct decision and also rejecting H_0 when H_0 is false is correct decision .The remaining conditions rejecting H_0 when H_0 is true and accepting H_0 when H_0 is false are errors.

The probability of type I error (α) and the probability of type II error (β) are called producer's risk and consumer's risk respectively.

Power of test (1-β)

β is the probability of accepting H_0 when H_0 is false. It is the probability of wrong decision. The probability of rejecting H_0 when H_0 is false is probability of correct decision is called power of test and is denoted by $1-\beta$. It is also called probability of accepting H_1 when H_1 is true. It is the probability of not making type II error.

Test statistic

The test statistic is the statistic based upon appropriate probability distribution. It is used to test whether the null hypothesis set up should be accepted or rejected .It helps us to decide whether to accept or reject the null hypothesis. Different probability distribution values are used in appropriate cases while testing hypothesis. The commonly used test statistic are

Z test: We use Z distribution under the normal curve for large sample(sample size n>30)

The Z test statistic is $Z = \dfrac{t - E(t)}{S.E.\ (t)} \sim N(0,1)$ as $n \to \infty$

t test :We use t distribution for small sample (sample size < 30)

The t test statistic is $t = \dfrac{\bar{x} - \mu}{S.E.\ (\bar{x})} \sim$ t distribution with n-1 degree of freedom.

Level of significance

In testing of hypothesis two types of errors are introduced namely type I error and type II erro The two errors are inversely related. Both errors can not be minimized at the same time. Usuall we fix type I error and minimize type II error. The maximum size of type I error prepared testing of hypothesis is called level of significance. It is denoted by α. Commonly used level significance are 5%, 1%.

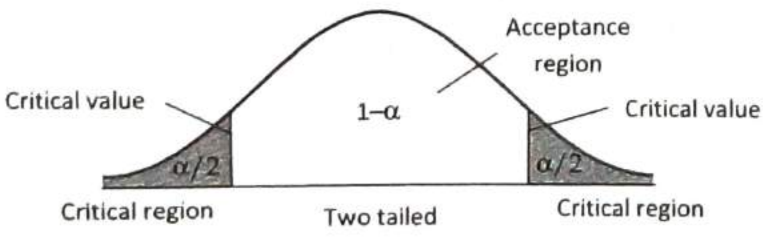
The level of significance should be chosen on the basis of power of test. If the power of test too low then high level of significance should be chosen. In such case level of significance ar 10% ,20%.

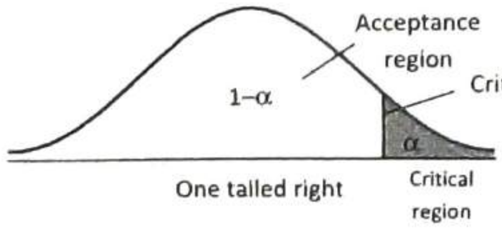
Critical region

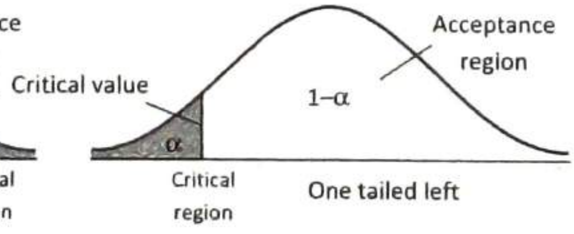
It is also called the rejection region. The set of all possible values of statistic is divided into tw regions one leading to the rejection of H_0 and other to the acceptance of H_0. The division made on the basis of level of significance and H_1. The region which leads to the rejection c H_0(null hypothesis) is called rejection region and is denoted by ω .While those region whic leads to the acceptance of H_0 is called acceptance region and is denoted by $\bar{\omega}$.

If the test statistic falls into the rejection region ,the null hypothesis is rejected. If the test statisti falls into the acceptance region, the null hypothesis is accepted.

The critical region is situated on both the tail or any one tail depending upon the alternativ hypothesis.







Critical value

It is also called significant value or tabulated value. The value of statistic which separates critical region and acceptance region is called critical value. It depend upon the level of significance and alternative hypothesis.

The critical value of Z for a singled tailed test at a level of significance α is the same as the critical value of Z for a two tailed test at a level of significance 2α.

For right tailed test $P(Z > Z_\alpha) = \alpha$

For left tailed test $P(Z < -Z_\alpha) = \alpha$

For two tailed test $P(|Z| > Z_\alpha) = \alpha$

or $\quad P(Z > Z_\alpha) + P(Z < -Z_\alpha) = \alpha$

or $\quad P(Z > Z_\alpha) + P(Z > Z_\alpha) = \alpha$

or $\quad 2P(Z > Z_\alpha) = \alpha$

or $\quad P(Z > Z_\alpha) = \frac{\alpha}{2}$

Area of each tail is $\frac{\alpha}{2}$ in two tailed test.

Degree of freedom

The number of independent variates which make up statistic is called degree of freedom. It is simply denoted by d.f.

One tailed test

A test of statistical hypothesis in which the alternative hypothesis H_1 looks for a definite increase(right tail) or definite decrease (left tail) in parameter is called one tailed test.

$H_0: \mu = \mu_0$ against $H_1: \mu > \mu_0$ (right tail)

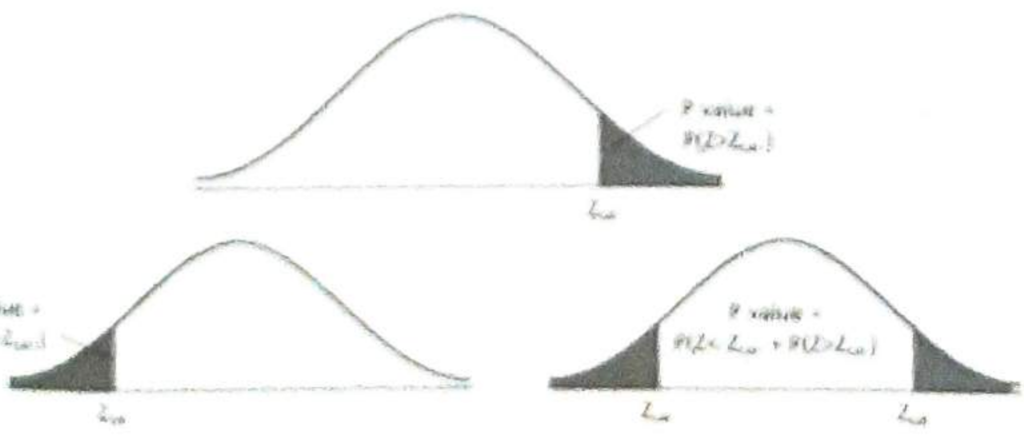
$H_0: \mu = \mu_0$ against $H_1: \mu < \mu_0$ (left tail)

Two tailed test

A test of statistical hypothesis in which the alternative hypothesis looks for a definite change in the parameter is called two tailed test.

$H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$

P value



It is the probability value of tail area under the curve of the test statistic used in hypothes Instead of critical value we can make decision on testing of hypothesis using p value.

The probability of obtaining a test statistic at least as extreme as the one that was actual observed assuming H_0 is true is called p value.

If p value is less than α then reject H_0.It is simply a measure of how likely the data were to ha occurred by chance assuming H_0 is true

For Z test, p value = Probability ($Z \geq | Z_{calculated} |$)

For right tailed test, p value is the area to the right of the computed value of the test statis under H_0

For left tailed test, p value is the area to the left of the computed value of test statistic under H_0

For two tailed test, p value is twice the area to the right of computed value of the test statistic twice the area to the left of computed value of the statistic under H_0.

If p value < α then reject H_0 at α level of significance, accept otherwise.

Steps use in testing of hypothesis

The following are various steps in testing of hypothesis in a systematic manner;

1: Set up null hypothesis H_0 .

2: Set up alternative hypothesis H_1. In H_1 decide to use whether one tailed or two tailed.

3: Choose the appropriate level of significance α depending upon the reliability of t estimates and permissible risk.

4: Identify the sample statistic to be computed and its sampling distribution.

5: Compute the test statistic under H_0.

6: Obtain the critical value of the test statistic from the appropriate table.

7: Compare the calculated value of test statistic with the critical value and then accordingly t decision to accept or reject H_0 is made.

For Z test, accept H_0 if $|Z| < Z_{tabulated}$, otherwise reject H_0.

Relationship between hypothesis testing and confidence interval

There is ordinarily close relationship between a test of hypothesis concerning a parameter parameters and the corresponding confidence interval. To illustrate this relationship we w examine the $(1 - \alpha)$ level confidence interval for μ, the mean of normal distribution with knov variance σ^2 and sample size n.

The interval limits are $\bar{X} \pm Z_{tabulated} \dfrac{\sigma}{\sqrt{n}} = \bar{X} \pm Z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$

This is acceptance region for testing $H_0 : \mu = \mu_0$ in two tailed test at α level of significance known σ^2 and sample size n . This indicates that the value μ lying within the (1-α) confiden interval corresponds to a value of test statistic $\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ that would lead to acceptance of the hypothesis. If we start with a value of μ lying outside the confidence interval, we find that corresponding value of the test statistic will fall into the rejection region.

$$\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu_0 \le \bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Here, $\bar{X} - Z\alpha_{/2}\frac{\sigma}{\sqrt{n}} \le \mu_0$

Or, $\bar{X} \le \mu_0 + Z\alpha_{/2}\frac{\sigma}{\sqrt{n}}$ .....(i)

Also, $\mu_0 \le \bar{X} + Z\alpha_{/2}\frac{\sigma}{\sqrt{n}}$

Or, $\mu_0 - Z\alpha_{/2}\frac{\sigma}{\sqrt{n}} \le \bar{X}$ .....(ii)

From i and ii

$$\mu_0 - Z\alpha_{/2}\frac{\sigma}{\sqrt{n}} \le \bar{X} \le \mu_0 + Z\alpha_{/2}\frac{\sigma}{\sqrt{n}}$$

The confidence limit for the statistic $\bar{X}$ of two tailed test is $\mu_0 \pm Z\alpha_{/2}\frac{\sigma}{\sqrt{n}}$. If $\bar{X}$ is found to be within the interval then H_0 is accepted, otherwise H_0 is rejected.

Large sample tests (Z test)

It is important parametric test based upon the normality assumption. When the samples are selected from population of known parameter with sample size more than 30 Z test is used. We consider that if sample size is more than 30 then sample selected from non normal population is also approximately normal distributed.

Z test is defined as the ratio of difference between t and E(t) to the S.E.(t)

$Z = \frac{t - E(t)}{S.E.\ (t)} \sim N(0, 1)$, where t = statistic , E(t) = Expected value of statistic and S.E.(t) = Standard error of the statistic.

Z test is used to test

- significance of single mean.
- significance of difference between two means.
- significance of single proportion.
- significance of difference between two proportions

One sample test for mean of normal population for known variance

Let us consider sample of size $n(n > 30)$ has been drawn from the normal population wi known variance $N(\mu,\sigma^2)$ then the sample mean $\bar{X} \sim N(\mu,\sigma^2/n)$.

Different steps in the test are;

Problem to test

$H_0: \mu = \mu_0$ (sample is drawn from population with mean μ_0)

$H_1: \mu \neq \mu_0$ (Two tailed test) or $H_1: \mu > \mu_0$ (One tailed right) or $H_1 : \mu < \mu_0$ (One tailed left)

Test statistic

For the sample selected from the population of unknown size

$$Z = \frac{\bar{X} - E(\bar{X})}{S.E.(\bar{X})} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad \text{for known variance}$$

$$= \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \quad \text{for unknown variance (for large sample size } \hat{\sigma} = s\text{)}$$

For the sample selected from the population of known size

$$Z = \frac{\bar{X} - E(\bar{X})}{S.E.(\bar{X})} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} \quad \text{for known variance}$$

$$= \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} \quad \text{for unknown variance}$$

Where $\bar{X}$ = sample mean, μ = population mean, σ = population s.d., s = sample s.d.,
N = population size, n = sample size

Level of significance

Let α be the level of significance. Usually we take $\alpha = .05$ unless we are given.

Critical value

Critical or tabulated value of Z is obtained from table according to the level of significance an alternative hypothesis.

Decision

Reject $|Z| > Z_{tabulated}$, accept otherwise.

Using p value approach

p value = Probability ($Z > |Zcal|$) (It can be obtained from standard normal table)

Decision

Reject H_0 at α level of significance if $p < \alpha$ for one tail

$2p < \alpha$ for two tail

Example 1

A sample of 400 students is found to have mean height of 170 cm. Can it be reasonably regarded as a sample from a large population with mean height 169.5 cm and standard deviation 3.5 cm?

Solution

Here

Sample size (n) = 400, Sample mean height ($\bar{X}$) = 170 ,

Population mean (μ) = 169.5, Population SD (σ) = 3.5

Problem to test

H_0: Mean height of students is 169.5 cm (μ = 169.5)

H_1: Mean height of students is not 169.5 ($\mu \neq 169.5$) (Two tailed)

Test statistic

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{170 - 169.5}{\frac{3.5}{\sqrt{400}}} = \frac{0.5 \times 20}{3.5} = 2.857$$

Critical value

Let α = 5% be the level of significance then critical value for two tailed test is $Z_{tab} = Z_{\alpha/2} = 1.96$.

Decision

Here $Z = 2.857 > Z_{tab} = 1.96$, reject H_0 at 5% level of significance.

Conclusion

The sample of 400 students can not be regarded as sample from large population with mean height 169.5 cm and standard deviation 3.5 cm.

Using p value approach

p = Prob (Z > 2.857) = 0.0021

2p = 2 x 0.0021 = 0.0042

At α = 5% = 0.05

2p = 0.0042 < α = 0.05

Reject H_0 5% level of significance.

Example 2

A manufacturer claim that their widget is more reliable than their main competitors. In order to verify this a sample of 40 widget from manufacturer's range was taken. The mean pass rate found to be 992 per 1000 with s.d. of 15 per 1000. Previous studies have shown that the mean pass rate of all widget on the market is 979 per 1000. Test the manufacturer's claim at 1% level of significance.

Solution

Here,

Sample size (n) = 40, Sample mean ($\bar{X}$) = 992, Sample Sd (s) = 15

Population mean (μ) = 979, Level of significance (α) = 1%

Problem to test

H_0: Average pass is 979 ($\mu = 979$)

H_1: Average pass is more than 979 ($\mu > 979$) (one tailed right)

Test statistic

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{992 - 989}{\frac{15}{\sqrt{40}}} = 5.48$$

Critical value

At α = 1% = 0.01 critical value for one tailed test is $Z_{tab} = Z_\alpha = 2.32$

Decision

Here $|Z| = 5.48 > Z_{tab} = 2.32$, reject H_0 at 1% level of significance.

Conclusion

The claim of manufacture is correct.

Example 3

A random sample of 100 pen drive selected from a batch of 2000 pen driveshows that the average thickness of the pen drive is 0.354 with a standard deviation 0.048 Are the sample from the lot having average thickness 0.35?

Solution

Here, Sample size (n) = 100 , Population size (N) = 2000 ,

Sample mean ($\bar{X}$) = 0.354, Sample SD (s) = 0.048, Population mean (μ) = 0.35

Problem to test

H_0 : Average thickness of pen drive is 0.35 ($\mu = 0.35$)

H_1:Average thickness of pen drive is not 0.35 ($\mu \neq 0.35$) (Two tailed)

Test statistic

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} = \frac{0.354 - 0.35}{\frac{0.048}{\sqrt{100}}\sqrt{\frac{2000-100}{2000-1}}} = \frac{0.004}{0.0047} = 0.854$$

Critical value

Let α =5% be the level of significance then critical value for two tailed test is $Z_{tab} = Z_{\alpha/2} = 1.96$.

Decision

Here $Z = 0.854 < Z_{tab} = 1.96$, accept H_0 at 5% level of significance.

Conclusion

Sample are from lot having average thickness of pen drive 0.35.

Example 4

If the mean breaking strength of a copper wire is 575 lbs with s.d. of 8.3 lbs. How many large sample must be used in order that there be one chance in 100 that the mean breaking strength of the sample is less than 572 lbs?

Solution

Here,

Population mean (μ) = 575, Population SD (σ) = 8.3, Sample size (n) = ?

P(mean breaking strength < 572) = $\frac{1}{100}$, Sample mean ($\bar{X}$) = 572, α = 1%

Problem to test

H_0: Mean breaking length of copper wire is 575 lbs ($\mu = 575$)

H_1: Mean breaking length of wire is less than 575 lbs ($\mu < 575$) (one tail left)

Test statistic

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{572 - 575}{\frac{8.3}{\sqrt{n}}} = -0.3614\sqrt{n}$$

$$|Z| = 0.3614\sqrt{n}$$

Critical value

At α = 1% = 0.01 critical value for one tailed test is Z tab = $Z_{\alpha/2}$ = 2.32.

Here, $0.3614\sqrt{n} = 2.32$

or $\sqrt{n} = \frac{2.32}{0.3614} = 6.419$

or $n = 41.2 \approx 41$.

Example 5

An ambulance service claims that it takes on the average 8.9 minutes to reach to its destination in emergency calls. To check the claim, the agency which licenses ambulance services has them timed on 50 emergency calls getting a mean of 9.3 minutes with s.d. of 1.6 minutes. What can they conclude at the level of significance α = 0.05? Use the confidence limit to make conclusion.

Solution

Here, Population mean (μ) = 8.9, Sample size (n) = 50 , Sample mean ($\bar{X}$) = 9.3,

Sample Sd (s) = 1.6, Level of significance (α) = 0.05

Problem to test

H_0: Average time taken by ambulance to reach destination is 8.9 min ($\mu = 8.9$)

H_1: Average time taken by ambulance to reach destination is not 8.9 min ($\mu \neq 8.9$) (Two tailed)

Critical value

At $\alpha = 0.05$ critical value for two tailed test is $Z_{tab} = Z_{\alpha/2} = 1.96$.

The limits of the acceptance region for $\bar{x} = \mu \pm Z\alpha_{/2} \frac{s}{\sqrt{n}} = 8.9 \pm 1.96 \times \frac{1.6}{\sqrt{50}} = 8.9 \pm \frac{3.136}{7.07}$

$$= 8.9 \pm 0.443$$

Taking + sign, $\bar{x} = 8.9 + 0.443 = 9.343$

Taking - sign, $\bar{x} = 8.9 - 0.443 = 8.457$

Decision

$\bar{X} = 9.3$ lies between 8.457 and 9.343, accept H_0 at 5% level of significance.

Conclusion: The claim of ambulance service that it takes on average 8.9 minutes to reach the destination is true.

Example 6

The quality control manager at light bulb factory needs to determine whether the mean life of a large shipment of light bulbs is equal to 375 hours. The population standard deviation is 100 hours. A random sample of 100 light bulbs indicate a sample mean 350 hours At the 0.01 level of significance is there evidence that the mean life is different from 375 hours? Use p value method to draw conclusion.

Solution

Here, Population mean (μ) = 375, population SD (σ) = 100, Sample size (n) = 100,

Sample mean ($\bar{x}$) = 350, level of significance (α) = 0.01

Problem to test

H_0 :Mean life of light bulb is 375 hours ($\mu = 375$)

H_1: Mean life of light bulb is not 375 hours ($\mu \neq 375$) (Two tailed)

Test statistic

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{350 - 375}{\frac{100}{\sqrt{100}}} = \frac{-25 \times 10}{100} = -2.5$$

Now, Prob($Z \geq |Z_{cal}|$) = Prob ($Z \geq 2.5$) = 0.5 - Prob ($0 \leq Z \leq 2.5$) =0.5 - 0.4938 =0.0062

For two tailed test p value = 2 Prob($Z \geq |Z_{cal}|$) = 2 × 0.0062 = 0.0124

Here $\alpha = 0.01$

Decision

P value = 0.0124 > α = 0.01, accept H_0 at 0.01 level of significance.

Conclusion

Mean life time of light bulbs is not different from 375 hours.

Test of significance difference between two means

Let us consider two independent samples of size n_1 and n_2 be drawn from population having means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively. Let $\bar{X}_1$ and $\bar{X}_1$ be the sample means.

For large n_1 and n_2,

$\bar{X}_1 \sim N(\mu_1, \sigma_1^2/n_1)$

$\bar{X}_2 \sim N(\mu_1, \sigma_2^2/n_2)$ then

$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \sigma_1^2/n_1 + \sigma_2^2/n_2)$

Different steps in the test are

Problem to test

$H_0 : \mu_1 = \mu_2$ Two means are not significantly different.

$H_1: \mu_1 \neq \mu_2$ (two tailed) or $H_1 : \mu_1 < \mu_2$ (one tailed left) or $H_1 : \mu_1 > \mu_2$ (one tailed right)

Test statistic

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - E(\bar{X}_1 - \bar{X}_2)}{S.E.(\bar{X}_1 - \bar{X}_2)} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \text{ when population means and variances are known}$$

$$= \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \text{ when population variances are known (Under } H_0, \mu_1 = \mu_2)$$

$$= \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \text{ when population variances are unknown}$$

for large sample size $\hat{\sigma}_1^2 = s_1^2$ and $\hat{\sigma}_2^2 = s_2^2$

where $\bar{x}_1$ = sample mean of size n_1, $\bar{x}_2$ = sample mean of size n_2

σ_1^2 = population variance of first population, σ_2^2 = population variance of second population

s_1^2 = sample variance of first sample, s_2^2 = sample variance of second sample.

Level of significance

Let α be the level of significance. Usually we take α = .05 unless we are given.

Critical value

Critical or tabulated value of Z is obtained from table according to the level of significance an alternative hypothesis.

Decision

Reject H_0 at a level of significance if $|Z| > Z_{tabulated}$, accept otherwise.

Example 7

In a random sample of 500 the mean is found to be 20. In another independent sample of 400 the mean is 15. Could the samples have been drawn from the same population with S.D. 4?

Solution

Here, First sample size $(n_1) = 500$, First sample mean $(\bar{x}_1) = 20$,

Second sample size $(n_2) = 400$, Second sample mean $(\bar{x}_2) = 15$, Population SD $(\sigma_1 = \sigma_2) = 4$

Let, first population mean $= \mu_1$ and second population mean $= \mu_2$

Problem to test

H_0: There is no significant difference between two population $(\mu_1 = \mu_2)$

H_1: There is significant difference between two population $(\mu_1 \neq \mu_2)$ (Two tailed)

Test statistic

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(20 - 15)}{\sqrt{\frac{4^2}{5000} + \frac{4^2}{400}}} = \frac{5}{\sqrt{0.032 + 0.04}} = \frac{5}{0.27} = 18.51$$

Critical value

Let $\alpha = 5\%$ be the level of significance the critical value for two tailed test is

$Z_{tabulated} = Z_{\alpha/2} = 1.96$

Decision

$Z = 18.51 > Z_{tabulated} = 1.96$, reject H_0 at 5% level of significance.

Conclusion

Samples are not drawn from same population with s.d.4.

Example 8

A random sample of 1000 workers from Pokhara show that their mean wages of \$47 per week with a standard deviation of \$ 28. A random sample of 1500 workers from Kathmandu show that their mean wage of \$ 49 per week with a standard deviation of \$ 40. Is there any significant difference between their mean level of wages?

Solution

Here

Sample workers from Pokhara $(n_1) = 1000$

Sample mean wage from Pokhara ($\bar{X}_1$) = 47, Sample Sd of wage from Pokhara (s_1) = 28

Sample workers from Kathmandu (n_2) = 1500,

Sample mean wage from Kathmandu ($\bar{X}_2$) = 49, Sample Sd of wage from Kathmandu (s_2) =40

Let μ_1 = Population mean wage of workers from Pokhara and μ_2 = Population mean wage of workers from Kathmandu.

Problem to test

H_0 : There is no significant difference in mean wages of workers between Pokhara and Kathmandu ($\mu_1 = \mu_2$)

H_1: There is significant difference in mean wage of workers between Pokhara and Kathmandu ($\mu_1 \neq \mu_2$) (Two tailed)

Test statistic

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(47 - 49)}{\sqrt{\frac{28^2}{1000} + \frac{40^2}{1500}}} = \frac{-2}{\sqrt{0.748 + 1.0666}} = \frac{-2}{\sqrt{1.8506}} = -1.47$$

$Z| = 1.47$

Critical value

Let $\alpha = 0.05$ be the level of significance then the critical value for two tailed test is

$Z_{tabulated} = Z_{\alpha/2} = 1.96$

Decision

$|Z| = 1.47 < Z_{tabulated} = 1.96$, accept H_0 at 0.05 level of significance

Conclusion

There is no significant difference between mean wage of workers in Pokhara and Kathmandu.

Example 9

A sample survey is conducted to compare the weights of two makes of laser printer. A random sample of weight of 50 canon laser printer showed a mean weight 3.3 kg and s.d.1 kg. Another random sample of weight of 40 brother lazer printer showed a mean weight 2.81 kg and s.d. 0.5 kg. Do the data support the research hypothesis that the weight of canon laser printer on average heavier than the weight of brother laser printer? Use p value method at 1% level of significance.

Solution

Here,

Sample size of canon birth (n_1) = 50, Sample mean weight of canon birth ($\bar{X}_1$) = 3.3,

Sample SD of weight of canon birth (s_1) = 1, Sample size of brother birth (n_2) = 40,

Sample mean weight of brother birth ($\bar{X}_2$) = 2.81, Sample SD of weight of brother birth (s_2) =
Level of significance (α) = 1%.

Let μ_1 = population mean weight of canon, μ_2 = population mean weight of brother

Problem to test

H_0: There is no difference in weight of canon and brother ($\mu_1 = \mu_2$)

H_1: Weight of canon is more than weight of brother ($\mu_1 > \mu_2$)

Test statistic

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{3.3 - 2.81}{\sqrt{\frac{1^2}{50} + \frac{0.5^2}{40}}} = \frac{0.49}{\sqrt{0.02 + 0.00625}} = \frac{0.49}{0.162} = 3.024$$

Now, Prob($Z \geq Z_{calculated}$) = Prob ($Z \geq 3.024$) = 0.5 – Prob($0 \leq Z \leq 3.024$) = 0.5 – 0.49874 = 0.00126

Here α = 1% = 0.01

It is one tail test, hence p value = Prob ($Z \geq Z_{calculated}$) = 0.00126

Decision

P value = 0.00126 < α = 0.01, reject H_0 at 1% level of significance.

Conclusion

The weight of canon laser printer on average heavier than the weight of brother laser printer.

Test for Single Proportion

Let P be the population proportion i.e. proportion of units possessing a certain characteristic the population. Let a random sample of size n has been drawn from the population. Let X be t number of units possessing the characteristic in the sample then sample proportion is p = X .For large n binomial distribution can be approximated by normal distribution.

$x \sim N(nP, nPQ)$

$$p = \frac{X}{n} \sim N(P, \frac{PQ}{n}), \; Q = 1 - P$$

Different steps in the test are;

Problem to test

$H_0: P = P_0$

$H_1: p \neq P_0$ (Two tailed test) or H_1: $P > P_0$ (One tailed right) or H_1 : $P < P_0$ (One tailed left)

Test statistic

For the sample selected from the population of unknown size

$$Z = \frac{p - E(p)}{s.e.\ (p)} = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

For the sample selected from the population of known size

$$Z = \frac{p - E(p)}{s.e.(p)} = \frac{p - P}{\sqrt{\frac{(N-n)}{(N-1)}\frac{PQ}{n}}}$$

Where, p = sample proportion, P = population proportion, Q = 1 - P, N = population size, n = sample size

Level of significance

Let α be the level of significance. Usually we take $\alpha = .05$ unless we are given.

Critical value

Critical or tabulated value of Z is obtained from table according to the level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $|z| > Z_{tabulated}$, accept otherwise.

Example 10

A coin is tossed 400 times and it turns up head 216 times. Discuss whether the coin is unbiased?

Solution

Here, Sample size (n) = 400, No of head turns up (x) = 216

Sample proportion of head(p) $= \frac{x}{n} = \frac{216}{400} = 0.54$

Population proportion of head (P) = 0.5

Problem to test

H_0 : Coin is unbiased (P = 0.5)

H_1 : Coin is biased (P$\neq$ 0.5) (two tailed)

Test statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{400}}} = \frac{0.04 \times 20}{0.5} = 1.6$$

Critical value

Let α = 5% be the level of significance the critical value for two tailed test is $Z_{tabulated} = Z_{\alpha/2} = 1.96$

Decision

$Z = 1.6 < Z_{tabulated} = 1.96$, accept H_0 at 5% level of significance.

Conclusion

The coin is unbiased.

Example 11

In a random sample of 400 persons from a large population 120 are females. Can it be said the males and females are in the ratio 5:3 in the population? Use 10% level of significance.

Solution

Here,

Sample size (n) = 400, no of females(x) = 120

Sample proportion of female (p) $= \frac{x}{n} = \frac{120}{400} = 0.3$

Population proportion of female (P) $= \frac{3}{3+5} = \frac{3}{8} = 0.375$, $Q = 1-P = 0.625$

Problem to test

H_0 : Male and female are in ratio 5:3 (P = 0.375)

H_1 : Male and female are not in ratio 5:3 ($P \neq 0.375$) (Two tailed)

Test statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.3 - 0.375}{\sqrt{\frac{0.375 \times 0.625}{400}}} = \frac{-0.075 \times 20}{\sqrt{0.23475}} = -3.099$$

Critical value

At $\alpha = 10\% = 0.1$ level of significance, then critical value for two tailed test is $Z_{\text{tabulated}} = Z_{\alpha}$ 1.645

Decision

$|Z| = 3.099 > Z_{\text{tabulated}} = 1.645$, reject H_0 at 10 % level of significance.

Conclusion

Males and females are not in ratio 5:3 in the population.

Example 12

A developer has claimed that at least 98% of the software which he supplied to a tribhuvan university conformed to specifications an examination of a sample of 500 software revealed that 30 were defective. Test the claim at a significance level of 0.01

Solution

Here,

Population proportion of software conforming specification (P) = 0.98

Sample size (n) = 500

No of software conforming specification in sample (x) = 500 – 30 = 470

Level of significance (α) = 0.01

Sample proportion of software conforming specification (p) $= \frac{x}{n} = \frac{470}{500} = 0.94$

Problem to test

H_0 : Atleast 98% software confirmed specification ($P \geq 0.98$)

H_1 : Less than 98% software confirmed specification ($P < 0.98$)

(one tailed left)

Test statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.94 - 0.98}{\sqrt{\frac{0.98 \times 0.02}{500}}} = \frac{-0.04}{\sqrt{0.0000392}} = -6.45$$

Critical value

At $\alpha = 0.01$ level of significance, critical value for one tailed test is

$Z_{tabhlated} = Z_\alpha = 2.326.$

Decision

$|Z| = 6.45 > Z_{tabulated} = 2.326$, Reject H_0 at 0.01 level of significance.

Conclusion

The claim of the manufacturer that at least 98% software supplied to a Tribhuvan University conformed to specification is not correct.

Example 13

In a random sample of 600 cars making a right turn at a certain intersection, 157 pulled out into a wrong lane. Test the hypothesis that actually 30% of all drivers make this mistake at the given intersection at $\alpha = 0.05$ using p value.

Solution

Here, Sample size of car (n) = 600, Car in wrong lane (x) = 157,

Sample proportion of car in wrong lane (p) $= \frac{x}{n} = \frac{157}{600} = 0.2616$

Population proportion (P) = 0.3, $Q = 1 - P = 0.7$, $\alpha = 0.05$

Problem to test

H_0 : 30% of drivers make mistake ($P = 0.3$)

H_1 : 30% of drivers do not make mistake ($P \neq 0.3$) (Two tailed)

Test statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.2616 - 0.3}{\sqrt{\frac{0.7 \times 0.3}{600}}} = \frac{-0.0384}{\sqrt{0.00035}} = -2.13$$

Now, $|Z| = 2.13$

$\text{Prob}(Z \geq |Z_{calculated}|) = \text{Prob}(Z \geq 2.13) = 0.5 - \text{Prob}(0 \leq Z \leq 2.13)$

$= 0.5 - 0.4834 = 0.0166$

For two tailed test, p value = 2 $\text{Prob}(Z \geq |Z_{calculated}|)$

$$= 2\times0.0166 = 0.0322$$

Here $\alpha = 0.05$

Decision

P value = $0.0322 < \alpha = 0.05$, reject H_0 at 0.05 level of significance.

Conclusion

30% of all the drivers do not make mistake at the given intersection.

Test of difference between two proportions

Let P_1 and P_2 be the two population proportions possessing a certain characteristic. Let two independent samples of sizes n_1 and n_2 be drawn from the two population. Also p_1 and p_2 be the proportion of units possessing certain characteristic in the two samples.

For large sample size

$$p_1 \sim N(P_1, \frac{P_1Q_1}{n_1})$$

$$p_2 \sim N(P_2, \frac{P_2Q_2}{n_2}) \text{ then}$$

$$P_1 - P_2 \sim N\left(P_1 - P_2, \frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}\right)$$

Different steps in the test are;

Problem to test

$H_0: P_1 = P_2$

$H_1: P_1 \neq P_2$ (Two tailed test) or $H_1: P_1 > P_2$ (One tailed right)

or $H_1 : P_1 < P_2$ (One tailed left)

Test statistic

$$Z = \frac{p_1 - p_2 - E(p_1 - p_2)}{S.E.\ (p_1 - p_2)} = \frac{p_1 - p_2 - (p_1 - p_2)}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}}$$

$$= \frac{p_1 - p_2}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}} \text{ if population proportions are given}$$

$$= \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \text{ if population proportions are not given}$$

Where, P_1 = population proportion of first population, P_2 = Population proportion of second population, p_1 = sample proportion of first sample of size n_1, p_2 = sample proportion of second sample of size n_2, $P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$, $Q = 1 - P$

Level of significance

Let α be the level of significance. Usually we take α = .05 unless we are given.

Critical value

Critical or tabulated value of Z is obtained from table according to the level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $|Z| > Z_{tabulated}$, accept otherwise.

Example 14

There are two software developer in incubation officer. First developer faces 21 box while applying 500 logics. Similarly another develper faces 3 box in 100 logics. Are the two developer differ significant in their performance?

Solution

Here,

No. of bugs by first developer (x_1) = 21

No. of logic of first developer (n_1) = 500

No. of bugs by second developer (x_2) = 3

No. of logic of second developer (n_2) = 100

Sample proportion of bugs by first developer (p_1) = $\frac{x_1}{n_1} = \frac{21}{500} = 0.042$

No. of logic of second developer bugs by second developer (p_2) = $\frac{x_2}{n_2} = \frac{3}{100} = 0.03$.

Let P_1 = Population proportion of bugs by first developer (P_2) = Population proportion of bugs by second developer

$$P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2} = \frac{500 \times 0.042 + 100 \times 0.03}{5000 + 100} = \frac{24}{600} = 0.04,$$

$Q = 1 - P = 1 - 0.04 = 0.96$

Problem to test

H_0: There is no significant difference in performance of two developer ($P_1 = P_2$)

H_1: There is significant difference in performance of two developer ($P_1 \neq P_2$) (Two tailed)

Test statistic

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.042 - 0.03}{\sqrt{0.04 \times 0.96\left(\frac{1}{500} + \frac{1}{100}\right)}} = \frac{0.012}{\sqrt{0.0004608}} = 0.571$$

Now, Prob($Z \geq Z_{calculated}$) = Prob ($Z \geq 0.571$) = 0.5 – Prob ($0 \leq Z \leq 0.571$)

= 0.5 – 0.2157 = 0.284

For two tailed test, p value = 2Prob($Z \geq Z_{calculated}$) = 2 × 0.284 = 0.568

Here $\alpha = 1\% = 0.01$

Decision

P value = 0.568 > $\alpha = 0.01$, accept H_0at 1 % level of significance.

Conclusion

There is no significant difference in performance of two developer.

Example 15

A machine puts out 16 imperfect articles in a sample of 500. After the machine is overhauled, puts 3 imperfect articles in a batch of 100. Has the machine improved?

Solution

Here,

Imperfect articles by machine before overhauled (x_1) = 16, Sample size of articles before machine is overhauled (n_1) = 500, Imperfect articles by machine after overhauled (x_2) = Sample size of articles after machine is overhauled (n_2) = 100, Sample proportion of imperfect articles before overhauled (p_1) = $\frac{x_1}{n_1} = \frac{16}{500} = 0.032$, Sample proportion of imperfect articles after overhauled (p_2) = $\frac{x_2}{n_2} = \frac{3}{100} = 0.03$.

Let $P_1$and P_2be the population proportion of imperfect articles before and after overhauled respectively,

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{16 + 3}{500 + 100} = \frac{19}{600} = 0.03166,\ Q = 1\text{-}P = 1\text{-}\ 0.03166 = 0.96833$$

Problem to test

H_0: $P_1 = P_2$

H_1: $P_1 < P_2$ (one tailed left)

Test statistic

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.032 - 0.03}{\sqrt{0.03166 \times 0.96833\left(\frac{1}{500} + \frac{1}{100}\right)}}$$

$$= \frac{0.002}{\sqrt{0.00036789}} = \frac{0.002}{0.019} = 0.105$$

Critical value

Let $\alpha = 5\% = 0.05$ be the level of significance, then critical value for one tailed test is $Z_{tabulated}$ $Z_\alpha = 1.645$.

Decision

$Z = 0.105 < Z_{tabulated} = 1.645$, accept H_0 at 5 % level of significance.

Conclusion

Machine has not improved after overhauled.

Example 16

1000 apples kept under one type of storage were found to show rotting of the extent of 4%. 1500 apples kept under another kind of storage showed 3% rotting. Can it be reasonably concluded that the second type of storage is superior to the first?

Solution

Here, sample size of apples in first storage (n_1) = 1000. Sample proportion of rotting apples in first storage (p_1) = 4% = 0.04, Sample size of apples in second storage (n_2) = 1500, Sample proportion of rotting apples in second storage (p_2) = 3% = 0.03

Let P_1 = Population proportion of rotting apples kept in one storage, P_2 = Population proportion of rotting apples kept in second storage.

$$P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2} = \frac{1000 \times 0.04 + 1500 \times 0.03}{100 + 1500} = \frac{85}{2500} = 0.034.$$

$Q = 1 - P = 1 - 0.034 = 0.966$

Problem to test

$H_0 : P_1 = P_2$

$H_1 : P_1 > P_2$ (One tailed right)

Test statistic

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.04 - 0.03}{\sqrt{0.034 \times 0.966\left(\frac{1}{1000} + \frac{1}{1500}\right)}} = \frac{0.01}{\sqrt{0.00005474}} = \frac{0.01}{0.0074} = 1.351$$

Critical value

Let $\alpha = 5\% = 0.05$ be the level of significance then critical value for one tailed test is $Z_{tabulated} = Z_\alpha = 1.645$.

Decision

$Z = 1.351 < Z_{tabulated} = 1.645$, accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between first storage and second storage.

Example 17

In city A, there are 856 births in a year of which 51% were males. In cities A and B combined, the proportion of male births in a total of 1360 was 47%. Is there any significant difference in the proportion of male births in the two cities?

Solution

Here,

Sample size in city A(n_1) = 856, Sample proportion of male birth in city A (p_1) = 51% = 0.51

Total proportion of male birth (P) = 47% = 0.47

$Q = 1 - P = 0.53$, $n_1 + n_2 = 1360$, $n_2 = 1360 - 856 = 504$

$$P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2}$$

or $$0.47 = \frac{856 \times 0.51 + 504 \times p^2}{1360}$$

or $$p_2 = 0.40$$

Sample proportion of male birth in city B (p_2) = 0.4

Problem to test

H_0 : There is no significant difference in proportion of male birth in city A and city B ($P_1 = P_2$)

H_1 : There is significant difference in proportion of male birth in city A and city B ($P_1 \neq P_2$)

Test statistic

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.51 - 0.40}{\sqrt{0.47 \times 0.53\left(\frac{1}{856} + \frac{1}{504}\right)}} = \frac{0.11}{0.027} = 4.07$$

Critical value

Let α = 5% be the level of significance then the critical value for two tailed test is $Z_{tabulated}$ = $Z_{\alpha/2} = 1.96$.

Decision

$Z = 4.07 > Z_{tabulated} = 1.96$, reject H_0 at 5% level of significance.

Conclusion

There is significant difference in the proportions of male births in two cities.

Example 18

A computer manufacturing firm claims that its brand A computer outsells its brand B by 8%. it is found that 42 out of a sample of 200 users prefer brand A and 18 out of another sample 100 user prefer brand B, test whether the 8% is a valid claim. Use 5% level of significance.

Solution

Here

First sample user (n_1) = 200, Brand A prefer from first sample (x_1) = 42

Second sample user (n_2) = 100, Brand B prefer from second sample (x_2) = 18

Sample proportion of brand A prefer user (p_1) $= \frac{x_1}{n_1} = \frac{42}{200} = 0.21$

Sample proportion of brand B prefer user (p_2) $= \frac{x_2}{n_2} = \frac{18}{100} = 0.18$

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{42 + 18}{200 + 100} = \frac{60}{300} = 0.2, Q = 1 - P = 1 - 0.2 = 0.8$$

Let P_1 = Population proportion of sells of brand A computer, P_2 = Population proportion of sel of brand B computer.

Problem to test

$H_0 : P_1 - P_2 = 0.08$

$H_1 : P_1 - P_2 \neq 0.08$ (Two tailed)

Test statistic

$$Z = \frac{p_1 - p_2 - (p_1 - p_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.21 - 0.18 - 0.08}{\sqrt{0.2 \times 0.8\left(\frac{1}{200} + \frac{1}{100}\right)}} = \frac{-0.05}{\sqrt{0.0024}} = -1.02$$

Critical value

At $\alpha = 5\%$ level of significance, critical value for two tailed test is Z tabulated = $Z_{\alpha/2} = 1.96$.

Decision

$|Z| = 1.02 < Z_{tabulated} = 1.96$, accept H_0 at 5% level of significance.

Conclusion

We can conclude that a difference of 8% in the sale of two brands of computer is a valid claim.

Small Sample Tests

When sample selected from population is less than or equal to 30 is called small sample size. In such cases sampling distribution of statistic is not approximately normally distributed .For small samples the statistic value estimated vary from sample to sample and also far from population parameter .So that the inference drawn from small sample is less precise in comparison to the inference from large sample. Hence modification in the hypothesis testing is made and are called exact sample test or small sample test .Different small sample test are based upon exact sampling distribution i.e. t distribution , F distribution ,Fisher's Z distribution, χ^2 distribution.

t test

When the sample size is less than or equal to 30 then the sampling distribution of the sample mean follows student's t distribution. The t distribution is also similar to normal distribution having shape as in normal distribution but little bit flatter. As the sample size is more than 30 then shape of t distribution is more likely to normal curve.

t test is

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim \text{t distribution with n-1 degree of freedom.}$$

where $\bar{X} = \frac{\Sigma X}{n}$, $S = \sqrt{\frac{\Sigma(X - \bar{X})^2}{n - 1}}$ is the unbiased estimate of the population standard deviation.

t test is based upon the assumption that

- sample size $n \leq 30$
- sample is selected from normal population.

- population standard deviation is not known.
- samples are independent.

It is used to test

- significance of single mean.
- significance of difference between means.
- significance of correlation coefficient.
- significance of regression coefficient.

One sample test for mean of normal population with unknown variance

Consider a random sample of size $n(n \leq 30)$ selected from normal population having mean and unknown variance. Let $x_1, x_2, x_3, \ldots\ldots x_n$ be sample of size n.

It is based upon the assumption that samples are selected from normal population with unknown variance and the samples observations are independent.

Different steps in the test are;

Problem to test

H_0: $\mu = \mu_0$ (sample is drawn from population with mean μ_0)

H_1 $\mu \neq \mu_0$ (Two tailed test) or H_1: $\mu > \mu_0$ (One tailed right) or $H_1 : \mu < \mu_0$ (One tailed left)

Test statistic

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim \text{ t distribution with n-1 degree of freedom.}$$

where, $\bar{X} = \frac{\Sigma X}{n}$, S^2 (sample mean square) $= \frac{\Sigma(X - \bar{X})^2}{n - 1} = \frac{1}{n - 1}\{\Sigma X^2 - n\bar{X}^2\}$

$$= \frac{1}{n - 1}\{\Sigma d^2 - n\bar{d}^2\}, \text{ where } d = x - A$$

If sample variance $s^2 = \frac{\Sigma(X - \bar{X})^2}{n - 1}$ then

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n - 1}}}$$

Level of significance

Let α be the level of significance. Usually we take $\alpha = .05$ unless we are given.

Critical value

Critical or tabulated value of t is obtained from table according to the level of significance degree of freedom and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $|t| > t_{tabulated}$, accept otherwise.

Confidence limit of population mean for small sample size

At α% level of significance for n-1 degree of freedom the critical value of t is $t_{\alpha/2\,(n-1)}$, then $(100 - \alpha\%)$ confidence or fudicial limits for population mean μ is given by

$$\bar{X} \pm t_{\alpha/2\,(n-1)} \frac{S}{\sqrt{n}}, \text{ where } S^2 = \frac{\Sigma(X - \bar{X})^2}{n - 1} = \frac{1}{n - 1}\{\Sigma X^2 - n\bar{X}^2\}$$

If the sample standard deviation is given then confidence limits for population mean μ is given by $\bar{X} \pm t_{\alpha/2\,(n-1)} \dfrac{s}{\sqrt{n - 1}}$, where $s^2 = \dfrac{\Sigma(X - \bar{X})^2}{n}$

Example 19

A machine is designed to produce insulating washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024 cm with a standard deviation of 0.002 cm. Test the significance of the deviation of mean.

Solution

Here

Population mean (μ) = 0.025, Sample size (n) = 10, Sample mean ($\bar{X}$) = 0.024

Sample SD (s) = 0.002

Problem to test

H_0 : Average thickness is 0.025cm ($\mu = 0.025$)

H_1 : Average thickness is not 0.025 ($\mu \neq 0.025$) (Two tailed)

Test statistic

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n - 1}}} = \frac{0.024 - 0.025}{\frac{0.002}{\sqrt{10 - 1}}} = \frac{-0.001 \times 3}{0.002} = -1.5$$

Critical value

Let α =5% be the level of significance then critical value for two tailed test is

$t_{tabulate} = t_{\alpha/2(n-1)} = 2.262$.

Decision

$|t| = 1.5 < t_{tabulated} = 2.262$, accept H_0 at 5% level of significance.

Conclusion

There is no significant deviation between the sample mean and population mean.

Example 20

A university librarian suspects that the average number of books checked out to each student per visit has changed recently. In the past average of 3.4 books were checked out. However recently sample of 23 students averaged 4.3 books per visit with a s.d. of 1.5 books. At 0.01 level of significance has the average check out changed? Use confidence limit to draw conclusion.

Solution

Here, Average books checked out (μ) = 3.4, Sample size (n) = 23 .

Sample average books checked out ($\bar{X}$) = 4.3, Sample Sd (s) = 1.5

Problem to test

H_0 : Average books checked out is 3.4 (μ = 3.4)

H_1 : Average books checked out is not 3.4 ($\mu \neq 3.4$) (Two tailed)

Critical value

Here

$\alpha = 0.01$ then critical value $t_{\alpha/2\,(n-1)} = 2.819$

Now

Confidence limit for $\bar{X} = \mu \pm t_{\alpha/2\,(n-1)} \dfrac{s}{\sqrt{n-1}} = 3.4 \pm 2.819 \times \dfrac{1.5}{\sqrt{23-1}}$

$$= 3.4 \pm \frac{4.2285}{4.69} = 3.4 \pm 0.901$$

Taking + sign, $\bar{X} = 3.4 + 0.901 = 4.301$

Taking – sign, $\bar{X} = 3.4 - 0.901 = 2.499$

$\bar{X} = 4.3$ lies between 2.499 and 4.301, accept H_0 at 5% level of significance.

Conclusion

The average number of books checked out to each student per visit has not changed

Example 21

A random sample download speed of 10 network points of Subisu ISP give the following data in mbps?

70 ,120 ,110 ,101 ,88 , 83 ,95 ,107 ,100 ,98

Do these data support the assumption of population mean of 100 mbps?

Solution

Here,

Population mean (μ) = 100

Sample size (n) = 10

Download speed (X)	d= X - A (A=90)	d^2
70	-20	400
120	30	900
110	20	400
101	11	121
88	-2	4
83	-7	49
95	5	25
107	17	289
100	10	100
98	8	64
	$\Sigma d = 72$	$\Sigma d^2 = 2352$

Now, $\bar{X} = A + \frac{\Sigma d}{n} = 90 + \frac{72}{10} = 97.2$

$$S^2 = \frac{1}{n-1}\{\Sigma d^2 - n\bar{d}^2\} = \frac{1}{10-1}\{2352 - 10 \times 7.2^2\} = \frac{1833.6}{9} = 203.74$$

$$S = 14.27$$

Problem to test

H_0: Download speed level in population is 100 mg/dl ($\mu = 100$)

H_1: Download speed in population is not 100 mbps ($\mu \neq 100$) (Two tailed)

Test statistic

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{97.2 - 100}{\frac{14.27}{\sqrt{10}}} = -0.621$$

Critical value

Let $\alpha = 0.05$ be the level of significance then critical value for two tailed test is

$t_{tabulated} = t_{\alpha/2(n-1)} = 2.262.$

Decision

$|t| = 0.621 < t_{tabulated} = 2.262$, accept H_0 at 0.05 level of significance.

Conclusion

The data support the assumption of mean download speed of 100 mbps in the population.

Example 22

The heights of 10 males of a given locality are found to be 70 , 67 ,62 , 68 , 61 , 68 , 70 , 64, 64 ,66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level.

Solution

Here,

Sample size (n) = 10, Population mean (μ) = 64, Level of significance (α) = 5% = 0.05

Height (X)	d = X – A (A = 66)	d^2
70	4	16
67	1	1
62	-4	16
68	2	4
61	-5	25
68	2	4
70	4	16
64	-2	4
64	-2	4
66	0	0
	$\Sigma d = 0$	$\Sigma d^2 = 90$

$$\bar{X} = A + \frac{\Sigma d}{n} = 66 + 0 = 66$$

$$S^2 = \frac{1}{n-1}\{\Sigma d^2 - n\bar{d}^2\} = \frac{1}{9}\{90 - 10 \times 0^2\} = 10$$

$$S = 3.16$$

Problem to test

H_0 : Average height of male is 64 inches (μ = 6.4)

H_1 : Average height of male is more than 64 inches ($\mu > 64$) (One tailed right)

Test statistic

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{66 - 64}{\frac{3.16}{\sqrt{10}}} = 2$$

Critical value

Tabulated value of t for α = 5% level of significance for one tailed test is $t_{tabulated} = t_{\alpha(n-1)} = 1.833$

Decision

$t = 2 > t_{tabulated} = 1.833$, reject H_0 at 5% level of significance.

Conclusion

We conclude that average height is greater than 64 inches.

Example 23

The heights of 10 children selected at random from a given locality had a mean 63.2 cms and variance 6.25 cms. Test at 5% level of significance the hypothesis that the children of the given locality are on the average less than 65 cms in all.

Solution

Here,

Sample size (n) = 10, Sample mean ($\bar{X}$) = 63.2, Sample variance (s^2) = 6.25

$s = \sqrt{6.25} = 2.5$, Level of significance (α) = 5%, Population mean (μ) = 65

Problem to test

H_0: Mean height of children is 65 cms ($\mu = 65$)

H_1 : Mean height of children is less than 65 cms ($\mu < 65$) (one tailed left)

Test statistic

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{63.2 - 65}{\frac{2.5}{\sqrt{10-1}}} = \frac{-1.8 \times 3}{2.5} = -2.158$$

Critical value

The tabulated value of t at $\alpha = 0.05$ level of significance for one tailed test is $t_{tabulated} = t_{\alpha(n-1)} = 1.83$

Decision

$|t| = 2.158 > t_{tabulated} = 1.83$, reject H_0 at 5% level of significance.

Conclusion

The mean height of children is less than 65 cms.

Example 24

A random sample of size 25 showed a mean of 65 inches with a standard deviation of 25 inches. Determine 98% confidence intervals for the mean of the population.

Solution

Here

Sample size (n) = 25, sample mean ($\bar{X}$) = 65, Sample SD (s) = 25

Confidence limit $(1 - \alpha) = 98\% = 0.98$, Level of significance (α) = 0.02

Confidence limit for μ is $\bar{X} \pm t_{\alpha/2\,(n-1)} \frac{s}{\sqrt{n-1}} = 65 \pm 2.492 \times \frac{25}{\sqrt{25-1}} = 65 \pm 12.717$.

Taking – sign

$65 - 12.717 = 52.283$

Taking + sign

$65 + 12.717 = 77.717$

Hence confidence limit is 52.283 inches to 77.717 inches.

Example 23

The heights of 10 children selected at random from a given locality had a mean 63.2 cms and variance 6.25 cms. Test at 5% level of significance the hypothesis that the children of the given locality are on the average less than 65 cms in all.

Solution

Here,

Sample size (n) = 10, Sample mean ($\bar{X}$) = 63.2, Sample variance (s^2) = 6.25

$s = \sqrt{6.25} = 2.5$, Level of significance (α) = 5%, Population mean (μ) = 65

Problem to test

H_0: Mean height of children is 65 cms ($\mu = 65$)

H_1 : Mean height of children is less than 65 cms ($\mu < 65$) (one tailed left)

Test statistic

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{63.2 - 65}{\frac{2.5}{\sqrt{10-1}}} = \frac{-1.8 \times 3}{2.5} = -2.158$$

Critical value

The tabulated value of t at $\alpha = 0.05$ level of significance for one tailed test is $t_{tabulated} = t_{\alpha(n-1)} = 1.83$

Decision

$|t| = 2.158 > t_{tabulated} = 1.83$, reject H_0 at 5% level of significance.

Conclusion

The mean height of children is less than 65 cms.

Example 24

A random sample of size 25 showed a mean of 65 inches with a standard deviation of 25 inches. Determine 98% confidence intervals for the mean of the population.

Solution

Here

Sample size (n) = 25, sample mean ($\bar{X}$) = 65, Sample SD (s) = 25

Confidence limit $(1 - \alpha) = 98\% = 0.98$, Level of significance (α) = 0.02

Confidence limit for μ is $\bar{X} \pm t_{\alpha/2\,(n-1)} \frac{s}{\sqrt{n-1}} = 65 \pm 2.492 \times \frac{25}{\sqrt{25-1}} = 65 \pm 12.717.$

Taking - sign

$65 - 12.717 = 52.283$

Taking + sign

$65 + 12.717 = 77.717$

Hence confidence limit is 52.283 inches to 77.717 inches.

Example 25

A city health department wishes to determine the mean bacteria count per volume of water at lake beach. Researchers have collected 10 water sample of unit volume and have found the bacteria counts to be 175, 190, 215, 198, 184, 207, 210, 193, 196, 180. Obtain the 95% confidence limit for the mean bacteria per count per unit volume of water at the lake beach.

Solution

Here

Sample size (n) = 10, Confidence limit $(1 - \alpha) = 95\%$, Level of significance $(\alpha) = 0.05$

Number of bacteria(X)	d= X- 196	d^2
175	-21	441
190	-6	36
215	19	361
198	2	4
184	-12	144
207	11	121
210	14	196
193	-3	9
196	0	0
180	-16	256
	$\Sigma d = -12$	$\Sigma d^2 = 1568$

$$\bar{X} = A + \frac{\Sigma d}{n} = 196 + \frac{-12}{10} = 194.8$$

$$S^2 = \frac{1}{n-1}\{\Sigma d^2 - n\bar{d}^2\} = \frac{1}{9}\{1568 - 10 \times (-1.2)^2\} = 172.62$$

$$S = \sqrt{172.62} = 13.139$$

Now 95% confidence limit for mean is $\bar{x} \pm t_{\alpha/2(n-1)} \frac{S}{\sqrt{n}}$

$$= 194.8 \pm 2.262 \times \frac{13.139}{\sqrt{10}} = 194.8 \pm 9.398$$

Taking - sign

194.8-9.398 = 185.402

Taking + sign

194.8+ 9.398 = 204.198

Hence the 95% confidence limit for the mean bacteria per count per unit volume of water at lake beach lies between 185.402 to 204.198.

Test for difference between two means (small sample)

Let us consider two independent samples of sizes n_1 ($n_1 \leq 30$) and n_2 ($n_2 \leq 30$) be drawn from two normal populations of means μ_1 and μ_2 with unknown variances respectively. Let $x_1, x_2, x_3 \ldots\ldots\ldots x_{n_1}$ be sample of size n1 and $y_1, y_2\, y_3, \ldots\ldots y_{n_2}$ be sample of size n_2 respectively.

Different steps in the test are;

Problem to test

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ (Two tailed test) or H1: $\mu_1 > \mu_2$ (One tailed right) or $H_1: \mu_1 < \mu_2$ (One tailed left)

Test statistic

$$t = \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{S_2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim \text{t distribution with } n_1 + n_2 - 2 \text{ degree of freedom.}$$

$$= \frac{\bar{X} - Y}{\sqrt{S_2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where, $\bar{X} = \frac{\Sigma X}{n_1}$, $\bar{Y} = \frac{\Sigma Y}{n_2}$, $s^2 = \frac{1}{n_1 + n_2 - 2}[\Sigma(X - \bar{X})^2 + \Sigma(Y - \bar{Y})^2]$

$$= \frac{(n_1 - 1) S_{12} + (n_2 - 1) S_{22}}{n_1 + n_2 - 2}, \; S_1^2 = \frac{\Sigma(X - \bar{X})^2}{n_1 - 1}, \; S_2^2 = \frac{\Sigma(Y - \bar{Y})^2}{n_2 - 1}$$

When sample variances are given then, $s^2 = \frac{1}{n_1 + n_2 - 2}[\Sigma(X - \bar{X})^2 + \Sigma(Y - \bar{Y})^2]$

$$= \frac{n_1 s_{12} + n_2 s_{22}}{n_1 + n_2 - 2}, \; s_1^2 = \frac{\Sigma(X - \bar{X})^2}{n_1}, \; s_2^2 = \frac{\Sigma(Y - \bar{Y})^2}{n_2}$$

Level of significance

Let α be the level of significance. Usually we take $\alpha = .05$ unless we are given.

Critical value

Critical or tabulated value of t is obtained from table according to the level of significance, degree of freedom and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $|t| > t_{tabulated}$, accept otherwise.

Example 26

The mean life of a sample of 10 lamps of projector was found to be 1456 hours with a standard deviation of 423 hours. A second sample of 17 lamps of projector chosen from a different batch showed a mean life of 1280 hours with a standard deviation of 398 hours. Is there significant difference between the means of the two batches?

Solution

Here, First sample size (n_1) = 10, First sample mean ($\bar{X}$) = 1456, First sample SD (s_1) = 423,

Second sample size (n_2) = 17, Second sample mean ($\bar{Y}$) = 1280, Second sample SD (s_2) = 398

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{10 \times 423^2 + 17 \times 398^2}{10 + 17 - 2} = \frac{1789290 + 2692869}{25} = \frac{4482159}{25} = 179286.36$$

Let μ_1 and μ_2 be population mean life of first type of projector lamp and second type of projector lamp respectively.

Problem to test

H_0 : There is no significant difference between mean life of two types of projector lamps ($\mu_1 = \mu_2$)

H_1: There is significant difference between mean life of two types of projector lamps ($\mu_1 \neq \mu_2$) (two tailed)

Test statistic

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{S_2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{1456 - 1280}{\sqrt{179286.36\left[\frac{1}{10} + \frac{1}{17}\right]}} = \frac{176}{168.52} = 1.04$$

Critical value

Let $\alpha = 5\%$ be the level of significance then critical value for two tailed test is

$t_{tabulated} = t_{\alpha/2\,(n_1 + n_2 - 2)} = 2.06$

Decision

$t = 1.04 < t_{tabulated} = 2.06$, accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between mean of two type of projector lamps.

Example 27

A study was conducted to IT students to complete a certain maze. The following data reveal the time taken in second to complete the maze by eight students each of BCA and BSc CSIT Can't be conclude that B.Sc. CSIT students more efficients that of BCA students. Use 1% level of significant.

D_1	8	12	13	9	3	8	10	9
D_2	10	8	12	15	6	11	12	12

Solution

Let μ_1 and μ_2 be the average time to complete maze by BCA and BSc. CSIT students.

Here, $n_1 = n_2 = 8$, $\alpha = 1\% = 0.01$

D_1 (X)	D_2 (Y)	X^2	Y^2
8	10	64	100
12	8	144	64

13	12	169	144
9	15	81	225
3	6	9	36
8	11	64	121
10	12	100	144
9	12	81	144
$\Sigma x = 72$	$\Sigma y = 86$	$\Sigma x^2 = 712$	$\Sigma y^2 = 978$

$$\bar{X} = \frac{\Sigma X}{n_1} = \frac{72}{8} = 9 \qquad \bar{y} = \frac{\Sigma Y}{n_2} = \frac{86}{8} = 10.75$$

$$S_1^2 = \frac{\Sigma(X - \bar{X})^2}{n_1 - 1} = \frac{\Sigma X^2 - n\bar{X}^2}{n_1 - 1} = \frac{1}{7}[712 - 8 \times 9^2] = \frac{64}{7}$$

$$S_2^2 = \frac{\Sigma(Y - \bar{Y})^2}{n_2 - 1} = \frac{\Sigma Y^2 - n\bar{Y}^2}{n_2 - 1} = \frac{1}{7}[978 - 8 \times 10.75^2] = \frac{53.5}{7}$$

$$S_2 = \frac{(n_1 - 1) S_{12} + (n_2 - 1) S_{22}}{n_1 + n_2 - 2} = \frac{7 \times \frac{64}{7} + 7 \times \frac{53.5}{7}}{8 + 8 - 2} = \frac{117.5}{14} = 8.392$$

Problem to test

H_0 : BCA and BSc CSIT students are equally efficient ($\mu_1 = \mu_2$).

H_1 : BSc CSIT students one more efficient than BCA students.

Test statistic

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{S_2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{9 - 10.75}{\sqrt{8.392\left[\frac{1}{8} + \frac{1}{8}\right]}} = \frac{-1.75}{\sqrt{2.098}} = -1.208$$

Critical value

At $\alpha = 0.01$ level of significance for one tailed test the critical value is $t_{tabulated} = t_{\alpha(n_1 + n_2 - 2)} = 2.624$

Decision

$|t| = 1.208 < t_{tabulated} = 2.624$, accept H_0 at 1% level of significance.

Conclusion

BCA and BSc CSIT students are equally efficient.

Example 28

Measuring specimen of nylon yarn taken from two machines, it was found that 8 specimens from the first machine had a mean denier of 9.67 with a standard deviation 1.81 while 10 specimens from the second machine had a mean denier of 7.43 with a standard deviation of 1.48. Assuming that the population sampled are normal and have same variance, test the hypothesis $\mu_1 - \mu_2 = 1.5$ against the alternative hypothesis $\mu_1 - \mu_2 > 1.5$ at 0.05 level of significance.

Solution

Here, Sample from first machine $(n_1) = 8$, Sample mean from first machine $(\bar{X}) = 9.67$,

Sample Sd from first machine $(s_1) = 1.81$, Sample size from second machine $(n_2) = 10$,

Sample mean from second machine $(\bar{Y}) = 7.43$, Sample Sd from second machine $(s_2) = 1.48$,

Level of significance $(\alpha) = 0.05$

Let μ_1 and μ_2 be the population mean of first machine and second machine respectively.

$$S^2 = \frac{n_1 s_{12} + n_2 s_{22}}{n_1 + n_2 - 2} = \frac{8 \times 1.81^2 + 10 \times 1.48^2}{8 + 10 - 2} = \frac{26.2088 + 21.904}{16} = \frac{48.1128}{16} = 3.00705$$

Problem to test

$H_0 : \mu_1 - \mu_2 = 1.5$

$H_1 : \mu_1 - \mu_2 > 1.5$

Test statistic

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{S_2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(9.67 - 7.43) - 1.5}{\sqrt{3.00705\left[\frac{1}{8} + \frac{1}{10}\right]}} = \frac{0.74}{\sqrt{0.6765}} = 0.902$$

Critical value

At $\alpha = 0.05$ level of significance, the critical value for one tailed test is $t_{tabulated} = t_{\alpha(n_1+n_2-2)} = 1.746$

Decision

$t = 0.902 < t_{tabulated} = 1.746$, accept H_0 at 0.05 level of significance.

Conclusion

$\mu_1 - \mu_2 = 1.5$

Example 29

Samples of two types of electric bulbs were tested for length of life and the following data were obtained:

	Type I	Type II
Number in the sample	8	7
Mean of the sample (in hours)	1134	1024
Standard deviation of sample(in hrs)	35	40

Is mean life of first type of bulbs more than second types of bulbs? Test at 5% level of significance.

Solution

Here

$$n_1 = 8, n_2 = 7, \bar{x} = 1134, \bar{Y} = 1024, s_1 = 35, s_2 = 40, \alpha = 5\% = 0.05$$

$$S^2 = \frac{n_1 s_{12} + n_2 s_{22}}{n_1 + n_2 - 2} = \frac{8 \times 35^2 + 7 \times 40^2}{8 + 7 - 2} = \frac{9800 + 11200}{13} = 1615.38$$

Let μ_1 and μ_2 be the mean life of first type of electric bulbs and second type of electric bulbs respectively.

Problem to test

H_0 : Mean life of type I bulb and type II bulb is same ($\mu_1 = \mu_2$)

H_1 : Mean life of type I bulb is more than type II bulb ($\mu_1 > \mu_2$)

Test statistic

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{S_2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{1134 - 1024}{\sqrt{1615.38\left[\frac{1}{8} + \frac{1}{7}\right]}} = \frac{110}{\sqrt{432.69}} = \frac{110}{20.8} = 5.28$$

Critical value

At $\alpha = 0.05$ level of significance critical value for one tailed test is

$$t_{\text{tabulated}} = t_{\alpha(n_1 + n_2 - 2)} = 1.771$$

Decision

$t = 5.28 > t_{\text{tabulated}} = 1.771$, reject H_0 at 5% level of significance.

Conclusion

Mean life of first type of bulbs is more than that of second type of bulbs.

Test of difference between two means of dependent samples (Paired t test)

Let us consider two dependent (related) samples of sizes $n(n \leq 30)$ be drawn from two normal populations of means μ_1 and μ_2 with unknown variances respectively. Let $x_1, x_2, x_3 \ldots\ldots x_n$ and $y_1, y_2\, y_3, \ldots\ldots y_n$ be samples of size n. Let $d_i = x_i - y_i$ be difference between the observations in the i^{th} sample.

Different steps in the test are;

Problem to test

$H_0 : \mu_d = 0 (\mu_1 - \mu_2 = 0)$

$H_1: \mu_d \neq 0$ (Two tailed test) or $H_1 : \mu_d > 0$(One tailed right) or $H_1 : \mu_d < 0$ (One tailed left)

Test statistic

$$t = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} \sim \text{t distribution with n-1 degree of freedom.}$$

Where, $d = X - Y$, $\bar{d} = \frac{\Sigma d}{n}$, $S_d^2 = \frac{\Sigma(d - \bar{d})^2}{n - 1} = \frac{1}{n - 1}\{\Sigma d^2 - n\bar{d}^2\}$

Level of significance

Let α be the level of significance. Usually we take $\alpha = .05$ unless we are given.

Critical value

Critical or tabulated value of t is obtained from table according to the level of significa degree of freedom and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $|t| > t_{tabulated}$, accept otherwise.

Example 30

A certain stimulus administered to each of the 12 patients resulted in the following increase blood pressure:

5 , 2 , 8 , -1 , 3 , 0 , -2 , 1 , 5 , 0 , 4 and 6

Can it be concluded that the stimulus will in general be accompanied by an increase in blo pressure.

Solution

Here

$d = X - Y$, $n = 12$

d	5	2	8	-1	3	0	-2	1	5	0	4	6	$\Sigma d = 31$
d^2	25	4	64	1	9	0	4	1	25	0	16	36	$\Sigma d^2 = 18$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{31}{12} = 2.58$$

$$S_d^2 = \frac{1}{n - 1}\{\Sigma d^2 - n\bar{d}^2\} = \frac{1}{11}\{185 - 12 \times 2.58^2\} = 9.5382, S_d = \sqrt{9.5382} = 3.08$$

Problem to test

$H_0 : \mu_d = 0 \ (\mu_1 = \mu_2)$

$H_1 : \mu_d < 0 \ (\mu_1 < \mu_2)$ (one tailed)

Test statistic

$$t = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} = \frac{2.58}{\frac{3.08}{\sqrt{12}}} = \frac{2.58 \times \sqrt{12}}{3.08} = \frac{2.58 \times 3.46}{3.08} = 2.89$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then critical value for one tailed test is

$t_{tabulated} = t_{\alpha(n-1)} = 1.8$

Decision

$t = 2.89 > t_{tabulated} = 1.8$, reject H_0 at 5% level of significance.

Conclusion

The stimulus in general be accompanied by an increase in blood pressure.

Example 31

Ten students were given a test in SPSS. Then they were given a month's training and another test was held. The marks obtained by the 10 students in the two tests are given below:

S N of students	1	2	3	4	5	6	7	8	9	10
Test I	12	15	10	13	18	10	8	17	9	7
Test II	12	17	12	12	14	12	16	16	18	12

Test whether the students have benefited by the training or not.

Solution

Here

Sample size (n) = 10

Students	Marks in test I(X)	Marks in test II(Y)	d= X-Y	d^2
1	12	12	0	0
2	15	17	-2	4
3	10	12	-2	4
4	13	12	1	1
5	18	14	4	16
6	10	12	-2	4
7	8	16	-8	64
8	17	16	1	1
9	9	18	-9	81
10	7	12	-5	25
			$\Sigma d = -22$	$\Sigma d^2 = 200$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-22}{10} = -2.2$$

$$S_d^2 = \frac{1}{n-1}\{\Sigma d^2 - n\bar{d}^2\} = \frac{1}{9}\{200 - 10 \times (-2.2)^2\} = 16.84,\ S_d = \sqrt{16.84} = 4.1$$

Problem to test

H_0 :Students have not benefited by training ($\mu_d = 0$)

H_1 : Students have benefited by training ($\mu_d < 0$) (one tailed)

Test statistic

$$t = \frac{\bar{d}}{\frac{Sd}{\sqrt{n}}} = \frac{-2.2}{\frac{4.1}{\sqrt{12}}} = -1.69$$

Critical value

Let α = 5% be the level of significance, then critical value for one tail test is

$t_{tabulated} = t_{\alpha(n-1)} = 1.83$

Decision

$|t| = 1.69 < t_{tabulated} = 1.83$, accept H_0 at 5% level of significance.

Conclusion

The students have not benefited by the training.

Example 32

A training was done to increase the efficiency of worker for dissembling of computer. Th decrease in assembly time of 10 workers after training were as follows:

Solution

Here

Sample size (n) = 10, Level of significance (α) = 5% = 0.05.

Patient	1	2	3	4	5	6	7	8	9	10	
Decrease in assembly time	6	3	-2	4	-3	4	6	0	0	2	$\Sigma d = 20$
d^2	36	9	4	16	9	16	36	0	0	4	$\Sigma d^2 = 130$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{20}{10} = 2$$

$$S_d^2 = \frac{1}{n-1}\{\Sigma d^2 - n\bar{d}^2\} = \frac{1}{9}\{130 - 10 \times 2^2\} = 10, S_d = \sqrt{10}$$

Problem to test

H_0 : The training has no effect on decreasing the assembly time. ($\mu_d = 0$)

H_1 : The training significantly decreases the assembly time. ($\mu_d \neq 0$) (Two tailed)

Test statistic

$$t = \frac{\bar{d}}{\frac{Sd}{\sqrt{n}}} = \frac{2}{\frac{\sqrt{10}}{\sqrt{10}}} = 2$$

Critical value

At α = 0.05 level of significance, the critical value for two tailed test is $t_{tabulated} = t_{\alpha/2(n-1)} = 2.26$

Decision

$t = 2 < t_{tabulated} = 2.26$, accept H_0 at 5% level of significance.

Conclusion

The drug has no effect on change of blood pressure.

Example 33

An I.Q. test was administered to 5 persons before and after they were given the nourishing food Horlicks. The results are given below.

Candidates	I	II	III	IV	V
I.Q. before Horlicks	110	120	123	132	125
I.Q. after Horlicks	120	118	125	136	121

Test whether there is any change in I.Q. after the Horlicks at 1% level of significance.

Solution

Here

Sample size (n) = 5, Level of significance (α) = 1%.

Candidates	I.Q. before(X)	I.Q. after (Y)	d=X-Y	d^2
I	110	120	-10	100
II	120	118	2	4
III	123	125	-2	4
IV	132	136	-4	16
V	125	121	4	16
			$\Sigma d = -10$	$\Sigma d^2 = 140$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-10}{5} = -2$$

$$S_d^2 = \frac{1}{n-1}\{\Sigma d^2 - n\bar{d}^2\} = \frac{1}{4}\{140 - 5 \times (-2)^2\} = \frac{120}{4} = 30,\ S_d = \sqrt{30}$$

Problem to test

H_0 : There is no significant difference in IQ before and after Horlicks ($\mu_d = 0$)

H_1 : There is significant difference in IQ before and after Horlicks ($\mu_d \neq 0$) (two tailed)

Test statistic

$$t = \frac{\bar{d}}{\frac{Sd}{\sqrt{n}}} = \frac{-2}{\frac{\sqrt{30}}{\sqrt{5}}} = \frac{-2}{\sqrt{6}} = \frac{-2}{2.45} = -0.816$$

Critical value

At α = 1% level of significance for two tailed test critical value is $t_{tabulated} = t_{\alpha/2(n-1)} = 4.6$

Decision

$|t| = 0.816 < t_{tabulated} = 4.6$, accept H_0 at 1% level of significance.

Conclusion

There is no significant difference in I.Q. after Horlicks was given.

EXERCISE 2

1. What do you mean by hypothesis? Describe null hypothesis and alternative hypothesis.
2. Differentiate between acceptance region and rejection region.
3. Define with examples i) Level of significance ii) Degree of freedom iii) Critical value iv) Type II error v) Type I error
4. Explain the procedure for testing significance of mean for large sample.
5. Explain the procedure for testing significance of difference between two means for large samples.
6. Write down different steps in testing of hypothesis.
7. Describe one tailed and two tailed test
8. What is Z test? Write down it's uses.
9. Explain the procedure for testing significance of mean for small sample.
10. Explain the procedure for testing significance of difference between two means for small samples.
11. Explain the procedure for testing significant of difference between two means for small dependent samples.
12. Explain the procedure for testing significance of proportion for large sample.
13. Explain the procedure for testing significance of difference between two proportions for large samples.
14. A sample of size 400 was drawn and the sample mean was found to be 99. Test whether this sample could have come from a normal population with mean 100 and standard deviation 8 at 5% level of significance. Ans: Z = -2.5
15. The mean life time of 400 laptop cells produced by a company is found to be 1570 hours with a standard deviation of 150 hours. Test the hypothesis that the mean life time of the laptop cells produced by the company is 16000 hours against the alternative hypothesis that it is greater than 1600 hours at 1% level of significance.

    Ans: Z=-4, sig, $p < 0.01$
16. The mean breaking strength of cables supplied by a manufacturer is 1800 with a standard deviation 100. By a new technique in manufacturing process it is claimed that the breaking strength of the cables have increased. In order to test this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 0.01 level of significance?

    Ans: Z = 3.535, $p < 0.01$, sig

17. A sample of 400 male students is found to have a mean height of 171.38 cm. Can it be reasonably regarded as a sample from a large population with mean 171.17 cm and standard deviation 3.30 cm. Use confidence limit to draw conclusion. Ans: Z=1.27, insig.

18. For a sample of 100 women taken from a large population enrolled in a weight reducing program the sample mean diastolic blood pressure is 101 and the s.d. is 42. Can you conclude that on the average the women enrolled in the program have diastolic blood pressure exceed the value of 75 recommended by medical societies?

Ans: Z=6.19, sig.

19. Suppose that chest circumference of presumably normal newborn baby girls is normally distributed with mean 13 inch and s.d. 0.7 inch. A group of 49 newborn baby girls from a population group living in a remote region and thought perhaps to constitute a genetic isolate are studied and found to have an average chest circumference of 12.6 inch. In this evidence that the group of 49 came from a population with mean different from 13 inch.

Ans: Z=-4, sig

20. A new variety of potato grown in 250 plots gave rise to a mean yield of 82. Quintals per hectare with a s.d. of 14.6 quintals per hectare. Is it reasonable to assert that the new variety is superior in yield to the standard variety with an estimated yield of 80.2 quintals per hectare?

Ans: Z=1.94, insig.

21. In a certain factory there are two independent processes for manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 gms with a standard deviation of 12 gms, while the other process are 124 and 14 in a sample of 400 items. Is the difference between the mean weights significant at 10% level of significance. Use p value method.

Ans: Z=-3.874, p<0.001, sig

22. The nicotine content of two brands of cigarettes has to be compared. A sample of 50 cigarettes of brand A has a mean nicotine content 20.5 with a s.d. of 2.5 . A sample of 50 cigarettes of brand B has a mean of 17.5 with a s.d. of 2.1. Is there any reason to believe that two brands are different so far as the nicotine content is considered? Use α =5%

Ans: Z=-6.49, sig.

23. A random sample of 300 families showed that the average birth rate is 30. Another sample of 400 families showed that the average birth rate is 28. Could the samples reasonably be regarded as the samples drawn from the same population with standard deviation 4?

Ans: Z=6.54, sig.

24. A random sample of size 35 taken from a normal population with standard deviation 5.2 has a mean 81. A second sample of size 36 taken from other normal population with a standard deviation 3.4 has a mean 76. Test whether the two sample means do not differ significantly at 5% level of significance.

Ans: Z=4.78

25. Two research laboratories have independently produced drugs that provide relief to arthritis suffers. The first drug was tested on a group of 90 arthritis victims and produced

an average of 8.5 hours relief with a standard deviation of 1.8 hours. The second drug was tested on 80 arthritis victims producing an average of 7.9 hours of relief with a standard deviation of 2.1 hours. At 5% level of significance does the second drug provide a significantly shorter period of relief?
Ans: Z=1.98, sig

26. Two random samples of Nepalese people taken from rural and urban region gave the following data of income :

Sample	Size	Average monthly income	s.d.
I from rural region	150	800	50
II from urban region	100	1250	30

Test whether the average monthly income of rural people is significantly less than that of the urban people.
Ans: Z = – 88.82, sig

27. An investigation of two kinds of photocopying equipment showed that 71 failures of first kind of equipment took on average 83.2 minutes to repair with a standard deviation of 19.3 minutes, while 75 failures of the second kind of equipment took on average 90.8 minutes to repair with a standard deviation of 21.4 minutes. Test the hypothesis that on the average it takes an equal amount of time to repair either kind of equipment.

Ans: Z=2.25, sig

28. A dice was thrown 9000 times and of these 3220 yielded a 3 or 4. Is this consistent with the hypothesis that the dice is unbiased?

Ans: Z = 4.97, Reject H_0

29. A sample of size 600 persons selected at random from a large city shows that the percentage of male in the sample is 53%. It is believed that male to the total population ratio in the city is ½. Test whether this belief is confirmed by the observation.

Ans: Z = 1.47, Accept H_0

30. The coordinator in a college claimed that at least 98% of the students submit their assignment on time. Taking the sample of 250 students, 15 were not submitting assignment in whole semester. Test his claim at 10% level of significance.

Ans: Z=–4.5, Reject H_0

31. To check on an ambulance service's claim that at least 40% of its calls are life threatening emergencies, a random sample was taken from its files and it was found that only 49 out of 150 calls were life threatening emergencies. Test the claim at 1% level of significance using p value method.
Ans: p=0.032, Accept H_0

32. It is claimed that both tea and coffee are equally popular in Kathmandu district. If in random sample of 600 persons, 310 were regular consumer of tea. Is the claim justified at 1% level of significance?
Ans: z=0.78, Accept H_0

33. A certain process produces 10% defective articles. A supplier of new raw material claims that the use of his material would reduce the proportion of defectives. A random sample of 400 units using this new material was taken out of which 34 were defective units. Can the supplier's claim be accepted? Test at 1% level of significance.
Ans: z=–1, Accept H_0

34. A machine produces 20 defective articles in a batch of 400. After overhauling it produces 10 defectives in a batch of 300. Has the machine improved. Ans: z=1.1, Accept H_0

35. At a certain date in a large city 400 out of a random sample of 500 men were found to be smokers. After the tax on tobacco had been heavily increased, another random sample of 600 men in the same city included 400 smokers. Was the observed decrease in the proportion of smokers significant? Test at 5% level of significance. Ans: z=4.95, Reject H_0

36. In a random samples of 600 and 1000 men from two cities 400 and 600 men are found to be literate. Do the data indicate that the population are significantly different in the percentage of literacy? Ans: z=2.4, Reject H_0

37. Random samples of 250 bolts manufactured by machine A and 200 bolts manufactured by machine B showed 24 and 10 defective bolts respectively .Do the machines showing same quality of performance? Use 5% level of significance.

Ans: z=1.84, Accept H_0

38. In a sample of 600 students of a certain college 400 are found to use dot pens. In another college from a sample of 900students 450 were found to use dot pens. Test whether the two colleges are significantly different with respect to the habit of using the dot pens at 1% level of significance? Use p value method.

Ans: p=0, Reject H_0

39. Rainfall records of a particular place for last 12 years for the month of July showed that average rainfall was 50 mm and standard deviation of 30 mm . Do you agree that the average rainfall at the place was less than 512 mm? Use 10% level of significance.

Ans: t = 1.38

40. Ten patients are selected at random from a population of patients and their blood pressure recorded are as follows;

125 ,147 ,118 , 145 ,140 , 128 , 155 , 150 , 160 ,149.

Do the data support the hypothesis that the population average blood pressure of patients is 135?Use 5% level of significance. Ans: (t=0.0618)

41 A fertilizer mixing machine is set to give 12 kg of nitrate for every quintal bag of fertilizer. Ten 100 kg bags are examined . The percentage of nitrate are as follows.

11.8, 13, 12.2, 14, 13.2, 11.9, 12.1, 12.2, 11.95, 12.1

Is there reason to believe that the machine is defective? Use confidence limit to draw conclusion. Ans: t = 0.024

42 A random sample of size 16 has the sample mean 53. The sum of the square of deviation taken from the mean value is 150. Can this sample be regarded as taken from the population having 56 as its mean at 99% confidence limit?

Ans: t = 3.74, sig.

43. In the past a machine has produced washers having a mean thickness of 0.05 cm. To determine whether the machine is in proper order , a sample of 10 washers is taken of

which mean thickness is 0.053 cm and s.d. is 0.003. Test the hypothesis that the machine working in proper order.

Ans: Ans: t = 3, sig

44. The time (in minutes) spent by 10 randomly selected customers using internet in a cyber cafe are as follows;

35, 20, 30, 45, 60, 40, 65, 40, 25, 50

Can you say average time spent by customers is more than 30 minutes at 5% level of significance?

Ans: t = 2.4, sig

45. A random sample of 10 bulbs has the following life in months; 24, 26, 32, 28, 20, 20, 23, 3 30 and 43.Obtain the 95% confidence limit for the population mean life of bulbs.

Ans: 22.9, 33.0

46. A random sample of size 10 showed a mean life of 28 years with the standard deviation of 7.1 years. Determine the fiducial limits for population mean.

47 The average number of articles produced by two machines per day are 200 and 250 with standard deviations 20 and 25 respectively on the basis of records of 25 days production. Can you regard both the machines equally efficient at 1% level of significance.

Ans: t=7.8, sig

48. Two kinds of manure were applied to sixteen one hectare plot, other condition remaining the same. The yields in quintals are given below:

Manure I	18	20	36	50	49	36	34	49	41
Manure II	29	28	26	35	30	44	46		

Is there any significant difference between the mean yields? Use 5% level of significance.

Ans: t=0.57, not sig

49. To test the effect of a fertilizer on rice production, 24 plots of land having equal areas were chosen. Half of these plots were treated with fertilizer and the other half were untreated. Other condition were the same. The mean yield of rice on untreated plots was 4.8 quintals with a standard deviation of 0.4 quintal, while the mean yield on the treated plots was 5 quintals with a standard deviation of 0.36 quintal. Can we conclude that there is significant improvement in rice production because of fertilizer at 5% level of significance.

Ans: t=2.7

50. Two new drugs A and B are given to two independent groups of 10 and 12 patients with heart disease respectively. The reduction of blood pressure due to the two new drugs A and B are given below;

Drug A	7	16	14	9	10	11	6	8	10	9		
Drug B	10	12	16	14	11	12	13	8	12	15	9	12

Would you conclude that the drug A is less effective than the drug B in reducing the blood pressure of patients with heart disease at 5% level of significance? Ans: t=0.0485

51. The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sum of square of the deviations from the mean are 26.94 and 18.73 respectively. Can the sample be considered to have been drawn from the same normal population?

Ans: t=-1.3288

52. A group of five patients treated with medicine A weigh 42,39,48,60 and 41 kgs. A second group of 7 patients from the same hospital treated with medicine B weigh 38,42,56,64,68,69 and 62 kgs. Do you agree with the claim the medicine B increase the weight significantly?

Ans: t=0.0596

53. A drug was given to 10 patients . The increment in their blood pressure were recorded to be 8, 10,-2 ,0, 5, -1 ,9 ,12 ,6 and 5. Is it reasonable to believe that the drug has no increase on change of blood pressure?Ans: (t=0.0003)

54. Following data gave the yields of rice in 10 experimental plots in two successive years;

Serial no of plots	1	2	3	4	5	6	7	8	9	10
Yield in 1st year	23	20	19	21	18	20	18	22	16	18
Yield in 2nd year	24	19	22	18	20	22	20	20	18	17

Test whether the mean of difference between the yields of two successive years is zero or not at 5% level of significance. Ans: t = 0.3039

55. Ten students were given intensive training on python for a month. The scores obtained in tests I and V given below.

S No of students	1	2	3	4	5	6	7	8	9	10
Marks in 1st test	50	52	53	60	65	67	48	69	72	80
Marks in 5th test	65	55	65	65	60	67	49	82	74	86

Does the score from test I to test V show an improvement? Test at 5% level of significance.

Ans: t = 0.153

56. Memory capacity of 10 students was tested before and after training, state whether the training was effective or not from the following scores;

Roll No.	1	2	3	4	5	6	7	8	9	10
Before training	12	14	11	8	7	10	3	0	5	6
After training	15	16	10	7	5	12	10	2	3	8

Ans: t = 0.280

□□□