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COMPUTER GRAPHICS

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Chapter-4: Three Dimensional Geometric Transformation

3D

- In computer Graphics, 3-D (three dimensions or three-dimensional) describes an image that provides the perception of depth.
- 3D Graphics 3D computer graphics or three-dimensional computer graphics are graphics that use a three-dimensional representation of geometric data that is stored in the computer for the purposes of performing calculations and rendering 2D images.
- 2D is "flat", using the horizontal and vertical (X and Y) dimensions, the image has only two dimensions.
- 3D adds the depth (Z) dimension. This third dimension allows for rotation and visualization from multiple perspectives. It is essentially the difference between a photo and a sculpture.

What are the issue in 3D that makes it more complex than 2D?

When we model and display a three-dimensional scene, there are many more considerations we must take into account besides just including coordinate values as 2D, some of them are:

- Relatively more co-ordinate points are necessary in comparison with 2D.
- Object boundaries can be constructed with various combinations of plane and curved surfaces.
- Consideration of projection (dimension change with distance) and transparency.
- Many considerations on visible surface detection and remove the hidden surfaces

3D Geometric Transformation

- Methods for geometric transformations and object modeling in three dimensions are extended from two-dimensional methods by including considerations for the z coordinate.
- Translation of an object in 3D is obtained by specifying a three dimensional translation vector, which determines how much the object is to be moved in each of the three coordinate directions.

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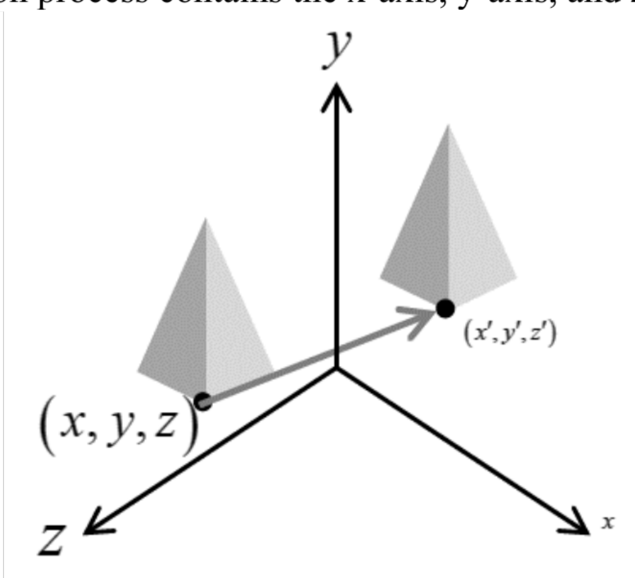
Homogeneous Representation in 3D

- 2D transformations can be represented by 3 x 3 matrices using homogeneous coordinates.
- Similarly, 3D transformations can be represented by 4 x 4 matrices, by using homogeneous coordinate representations of points in 2 spaces as well.
- Thus, instead of representing a point as (x, y, z), we represent it as (x, y, z, H),
  - Where two these quadruples represent the same point it one is a non-zero multiple of the other the quadruple (0, 0, 0, 0) is not allowed.
- A standard representation of a point (x, y, z, H) with H not zero is given by (x/H, y/H, z/H, 1).
- Transforming the point to (x, y, z, 1) form is called homogenizing.

Translation

- A translation moves all points in an object along the same straight line path to new positions.
- 3DTranslation process contains the x-axis, y-axis, and z-axis.



Let's translate a point from P (x, y, z) to Q (x' y', z') and Translation distance towards x, y and z axis are t_x, t_y, t_z then the path is represented by a vector, called the translation or shift vector. We can write the components as:

$x' = x + t_x$
$y' = y + t_y$
$z' = z + t_z$
$1=1$

We can also represent the 3D Translation in matrix form

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Example: A Point has coordinates P (1, 2, 3) in x, y, z-direction. Apply the translation with a distance of 2 towards x-axis, 3 towards y-axis, and 4 towards the z-axis. Find the new coordinates of the point?

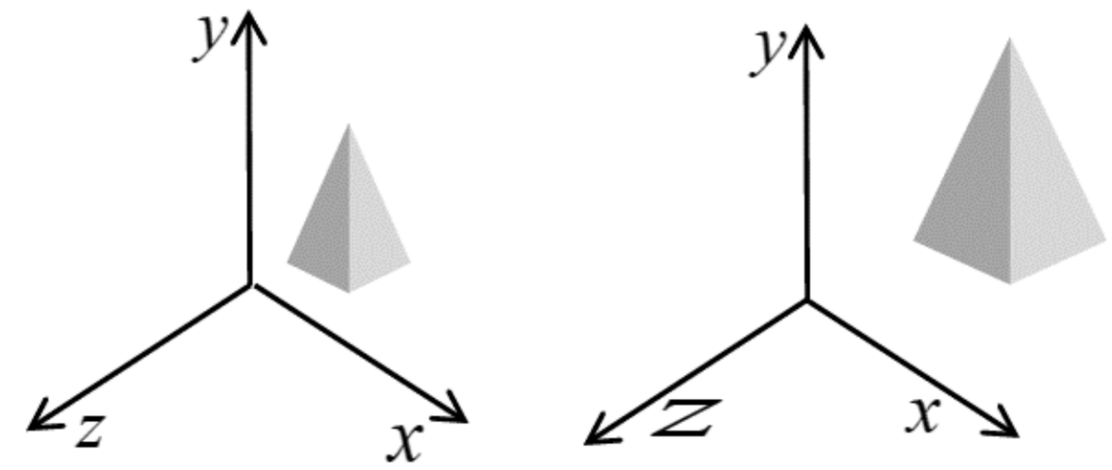
Solution:

We have,
Point P = (x_0, y_0, z_0) = (1, 2, 3)
Shift Vector = (T_x, T_y, T_z)
Let us assume the new coordinates of P = (x_1, y_1, z_1)
Now we are going to add translation vector and given coordinates, then

$X_1 = x_0 + T_x = (1 + 2) = 3$
$Y_1 = y_0 + T_y = (2 + 3) = 5$
$Z_1 = z_0 + T_z = (3 + 4) = 7$
Thus, the new coordinates are = (3, 5, 7)

Scaling

- Scaling changes the size of an object and involves the scale factors.
- The 2D and 3D scaling are similar, but the key difference is that the 3D plane also includes the z-axis along with the x and y-axis.
- The scaling factor towards x, y and z axis is denoted by **'S_x' 'S_y'** and **'S_z'** respectively.



The object before scaling

Object after scaling

The increment and decrement of an object is depends on two conditions. They are

i. If scaling factor $(S_x, S_y, S_z) > 1$, then the size of the object increased.
ii. If scaling factor $(S_x, S_y, S_z) < 1$, then the size of the object decreased.

Let us assume,
The initial coordinates of object = P (x, y, z)
Scaling factor for x-axis = S_x
Scaling factor for y-axis = S_y
Scaling factor for z-axis = S_z
The coordinates after Scaling = Q (x·, y·, z·)

- **We can represent the 3D Scaling in the form of equation-**

$$X_1 = x.\ S_x$$
$$Y_1 = y.\ S_y$$
$$Z_1 = z.\ S_z$$

- **Matrix representation of 3D Scaling**

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} \begin{matrix} = \\ = \\ = \\ = \end{matrix} \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

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Therefore, P' = S.P

Example: A 3D object that have coordinates points P(1, 4, 4), Q(4, 4, 6), R(4, 1, 2), T(1, 1, 1) and the scaling parameters are 3 along with x-axis, 4 along with y-axis and 4 along with z-axis. Apply scaling to find the new coordinates of the object?

Solution: We have,
The initial coordinates of object = P (1, 4, 4), Q (4, 4, 6), R (4, 1, 2), S (1, 1, 1)
Scaling factor along with x-axis (S_x) = 3
Scaling factor along with y-axis (S_y) = 4
Scaling factor along with z-axis (S_z) = 4
Let the new coordinates after scaling = **(x', y', z')**

For coordinate P:

X' = x. S_x = 1 x 3 = 3
Y' = y.S_y = 4 x 4 = 16
Z' = z. S_z = 4 x 4 = 16

The new coordinates = (3, 16, 16)

For coordinate Q:

X'= x.S_x = 4 x 3 = 12
Y'= y.S_y = 4 x 4 = 16
Z' = z.S_z = 6 x 4 = 24

The new coordinates = (12, 16, 24)

For coordinate R:

X' = x.S_x = 4 x 3 = 12
Y'= y.S_y = 1 x 4 = 4
Z'= z.S_z = 2 x 4 = 8

The new coordinates = (12, 4, 8)

For coordinate S:

X' = x.S_x = 1 x 3 = 3
Y' = y.S_y = 1 x 4 = 4
Z' = z.S_z = 1 x 4 = 4

The new coordinates = (3, 4, 4)

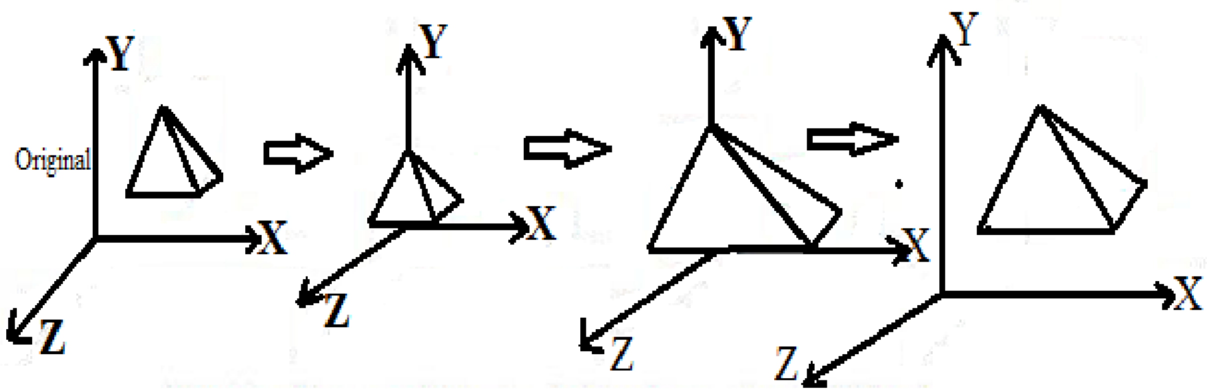
Thus, the new coordinates after scaling = P (3, 16, 16), Q (12, 16, 24), R (12, 4, 8), S (3, 4, 4).

Fixed Point Scaling in 3D

Let fixed point (x_f, y_f, z_f).

Do:

1. Translate fixed point to the origin
2. Scale object relative to the coordinate origin
3. Translate fixed point back to its original position



CM = $\mathbf{T}_{(x,\,y,\,z)}$. $\mathbf{S}_{(x,\,y,\,z)}$. $\mathbf{T}_{(-x,\,-y,\,-z)}$

$$= \begin{bmatrix} 1 & 0 & 0 & x_f \\ 0 & 1 & 0 & y_f \\ 0 & 0 & 1 & z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_f \\ 0 & 1 & 0 & -y_f \\ 0 & 0 & 1 & -z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore,

P' = CM * P

Shearing

- Shearing transformations are used to modify object shapes.
- The basic difference between 2D and 3D Shearing is that the 3D plane also includes the z-axis.
- Let the point P (x, y,z) is obtained P' (x', y', z') after applying shear with shearing factor for x, y and z axis are **'Sh_x,' 'Sh_y,' and 'Sh_z.' respectively.**

Basically, Shearing in 3D Geometry are categorized into 3 different types

a) X- axis Shearing

- In this transformation alters Y and Z coordinate values by an amount that is proportional to the X value while leaving the X value unchanged i.e.

x'= x
y'= y + Shy * x
z'= z + shz * x

3D matrix representation,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ shy & 1 & 0 & 0 \\ shz & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

b) Y -axis shearing

This transformation alters Y and Z coordinate values by an amount that is proportional to the Y value while leaving the Y value unchanged i.e.

y'= y
x'= x + Shx * y
z'= z +Shz * y

3D Matrix Representation,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & shx & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & shz & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

a) Z -axis Shearing

This transformation alters x and y coordinate values by amount that is proportional to the z value while leaving z co-ordinate unchanged.

x’ = x + Shx * z
y’ = y + Shy * z
z’ = z

3D Matrix Representation,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & shx & 0 \\ 0 & 1 & shy & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Share Parameters Shx and Shy can be assigned any real values

Example.

Given a 3D triangle with points (0, 0, 0), (1, 1, 2) and (1, 1, 3). Apply shear parameter 2 on X axis, 2 on Y axis and 3 on Z axis and find out the new coordinates of the object.

Solution

Given that,

Old corner coordinates of the triangle = A (0, 0, 0), B(1, 1, 2), C(1, 1, 3)
Shearing parameter towards X direction (Sh_x) = 2
Shearing parameter towards Y direction (Sh_y) = 2
Shearing parameter towards Y direction (Sh_z) = 3

Shearing in X Axis

For Coordinates A (0, 0, 0)

Let the new coordinates of corner A after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X = 0$

$Y' = Y + Sh_y . X = 0 + 2 \times 0 = 0$

$Z' = Z + Sh_z . X = 0 + 3 \times 0 = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

For Coordinates B(1, 1, 2)

Let the new coordinates of corner B after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X = 1$

$Y' = Y + Sh_y \times X = 1 + 2 \times 1 = 3$

$Z' = Z + Sh_z \times X = 2 + 3 \times 1 = 5$

Thus, New coordinates of corner B after shearing = (1, 3, 5).

For Coordinates C (1, 1, 3)

Let the new coordinates of corner C after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X = 1$

$Y' = Y + Sh_y \times X = 1 + 2 \times 1 = 3$

$Z' = Z + Sh_z \times X = 3 + 3 \times 1 = 6$

Thus, New coordinates of corner C after shearing = (1, 3, 6).

Thus, New coordinates of the triangle after shearing in X axis = A (0, 0, 0), B(1, 3, 5), C(1, 3, 6).

Shearing in Y Axis

For Coordinates A (0, 0, 0)

Let the new coordinates of corner A after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X + Sh_x \times Y = 0 + 2 \times 0 = 0$

$Y' = Y = 0$
$Z' = Z + Sh_z \times Y = 0 + 3 \times 0 = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

For Coordinates B (1, 1, 2)

Let the new coordinates of corner B after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X + Sh_x \times Y = 1 + 2 \times 1 = 3$
$Y' = Y = 1$
$Z' = Z + Sh_z \times Y = 2 + 3 \times 1 = 5$

Thus, New coordinates of corner B after shearing = (3, 1, 5).

For Coordinates C (1, 1, 3)

Let the new coordinates of corner C after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X + Sh_x \times Y = 1 + 2 \times 1 = 3$
$Y' = Y = 1$
$Z' = Z + Sh_z \times Y = 3 + 3 \times 1 = 6$

Thus, New coordinates of corner C after shearing = (3, 1, and 6).

Thus, New coordinates of the triangle after shearing in Y axis = A (0, 0, 0), B (3, 1, 5), C (3, 1, 6).

Shearing in Z Axis

For Coordinates A(0, 0, 0)

Let the new coordinates of corner A after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X + Sh_x \times Z = 0 + 2 \times 0 = 0$
$Y' = Y + Sh_y \times Z = 0 + 2 \times 0 = 0$
$Z' = Z = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

For Coordinates B(1, 1, 2)

Let the new coordinates of corner B after shearing = (X', Y', Z').

Applying the shearing equations, we have,

$X' = X + Sh_x \times Z = 1 + 2 \times 2 = 5$

$Y' = Y + Sh_y \times Z = 1 + 2 \times 2 = 5$

$Z' = Z = 2$

Thus, New coordinates of corner B after shearing = (5, 5, 2).

<u>For Coordinates C(1, 1, 3)</u>

Let the new coordinates of corner C after shearing = (X', Y', Z').

Applying the shearing equations, we have-

$X' = X + Sh_x \times Z = 1 + 2 \times 3 = 7$

$Y' = Y + Sh_y \times Z = 1 + 2 \times 3 = 7$

$Z' = Z = 3$

Thus, New coordinates of corner C after shearing = (7, 7, 3).

Thus, New coordinates of the triangle after shearing in Z axis = A (0, 0, 0), B (5, 5, 2), C (7, 7, 3).

Reflection

- A Three-dimensional can be performed relative to a selected reflection axes or with respect to selected reflected plane.
- Three-dimensional reflection matrices are set up similar to those for two dimensional.
- Reflection relative to given axis are equivalent to 180^0 rotation about that axis.
- The reflection planes are either XY, XZ or YZ (i.e. 3 types)
- The reflected object is always formed on the other side of mirror.

Consider a point object O has to be reflected in a 3D plane.

Let, Initial coordinates of the object O = (X, Y, Z) and new coordinates of the reflected object O after reflection = (X', Y', Z')

Reflection Relative to XY Plane (i.e. Reflection along Z axis)

- X and y value same, z value flip

OR, this transformation changes the sign of the z coordinates, leaving the x and y coordinate values unchanged i.e.

$X' = X$

$Y' = Y$

Z' = -Z

In Matrix form, the above reflection equations may be represented as

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

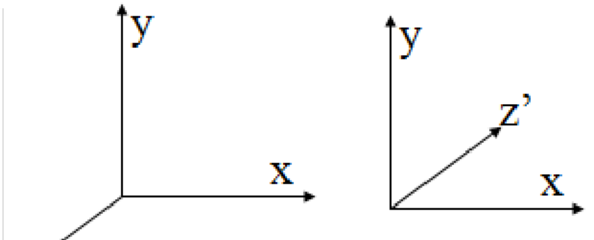


Fig. 3D reflection relative to the XY- Plane

Reflection Relative to YZ Plane (i.e. Reflection along X axis)

Y and Z value unchanged, X value flip

OR, this transformation changes the sign of the x coordinates, leaving the y and z coordinate values unchanged.

This reflection is achieved by using the following reflection equations

X' = -X
Y' = Y
Z' = Z

In Matrix form, the above reflection equations may be represented as

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

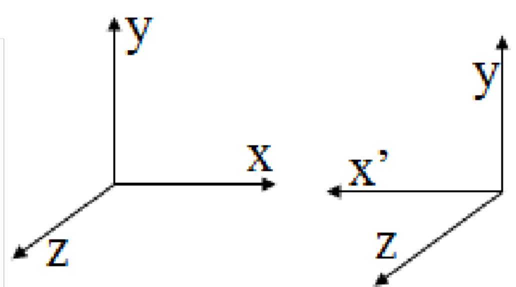


Fig. 3D reflection relative to the YZ- Plane

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Reflection Relative to XZ Plane (i.e. Reflection along Y axis)

❖ X and Z values are unchanged while Y value flip.

OR, this transformation changes the sign of the y coordinates, leaving the x and z coordinate values unchanged.

This reflection is achieved by using the following reflection equations

X' = X
Y' = -Y
Z' = Z

In Matrix form, the above reflection equations may be represented as

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

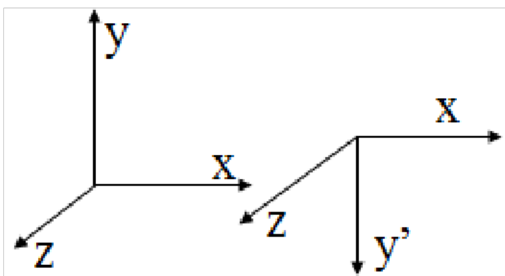


Fig. 3D reflection relative to the ZX- Plane

Example.

Given a 3D triangle with coordinate points A (3, 4, 1), B (6, 4, 2), C (5, 6, 3). Apply the reflection on the XY plane and find out the new coordinates of the object.

<u>Solution</u>

Given that,

Old corner coordinates of the triangle = A (3, 4, 1), B (6, 4, 2), C (5, 6, 3)
Reflection has to be taken on the XY plane

<u>For Coordinates A (3, 4, 1)</u>

Let the new coordinates of corner A after reflection = (X', Y', Z').

Applying the reflection equations, we have-

X' = X = 3
Y' = Y = 4
Z' = -Z= -1

Thus, New coordinates of corner A after reflection = (3, 4, -1).

For Coordinates B (6, 4, 2)

Let the new coordinates of corner B after reflection = (X', Y', Z')

Applying the reflection equations, we have-

X' = X = 6
Y' = Y = 4
Z' = -Z= -2

Thus, New coordinates of corner B after reflection = (6, 4, -2).

For Coordinates C (5, 6, 3)

Let the new coordinates of corner C after reflection = (X', Y', Z').

Applying the reflection equations, we have-

X' = X = 5
Y' = Y = 6
Z' = -Z= -3

Thus, New coordinates of corner C after reflection = (5, 6, -3).

Thus, New coordinates of the triangle after reflection = A (3, 4, -1), B (6, 4, -2), C(5, 6, -3).

Example

Given a 3D triangle with coordinate points A (3, 4, 1), B (6, 4, 2), C (5, 6, 3). Apply the reflection on the XZ plane and find out the new coordinates of the object.

Solution

Given that,

Old corner coordinates of the triangle = A (3, 4, 1), B (6, 4, 2), C(5, 6, 3)
Reflection has to be taken on the XZ plane

<u>For Coordinates A (3, 4, 1)</u>

Let the new coordinates of corner A after reflection = (X’, Y’, Z’).

Applying the reflection equations, we have-

X’ = X = 3
Y’ = -Y = -4
Z’ = Z= 1

Thus, New coordinates of corner A after reflection = (3, -4, 1).

<u>For Coordinates B (6, 4, 2)</u>

Let the new coordinates of corner B after reflection = (X’, Y’, Z’).

Applying the reflection equations, we have-

X’ = X = 6
Y’ = -Y = -4
Z’ = Z= 2

Thus, New coordinates of corner B after reflection = (6, -4, 2).

<u>For Coordinates C (5, 6, 3)</u>

Let the new coordinates of corner C after reflection = (X’, Y’, Z’).

Applying the reflection equations, we have-

X’ = X = 5
Y’ = -Y = -6
Z’ = Z= 3

Thus, New coordinates of corner C after reflection = (5, -6, 3).

Thus, New coordinates of the triangle after reflection = A (3, -4, 1), B (6, -4, 2), C (5, -6, 3).

Reflection about any axis parallel to one of the coordinate axes

<u>Do:</u>

1. Translate object so that reflection axis coincides with the parallel coordinate axis.
2. Perform specified reflection about that axis
3. Translate object back to its original Position

$$\mathbf{CM = T\ R.\ T^{-1}}$$

$$\mathbf{P' = CM.\ P}$$

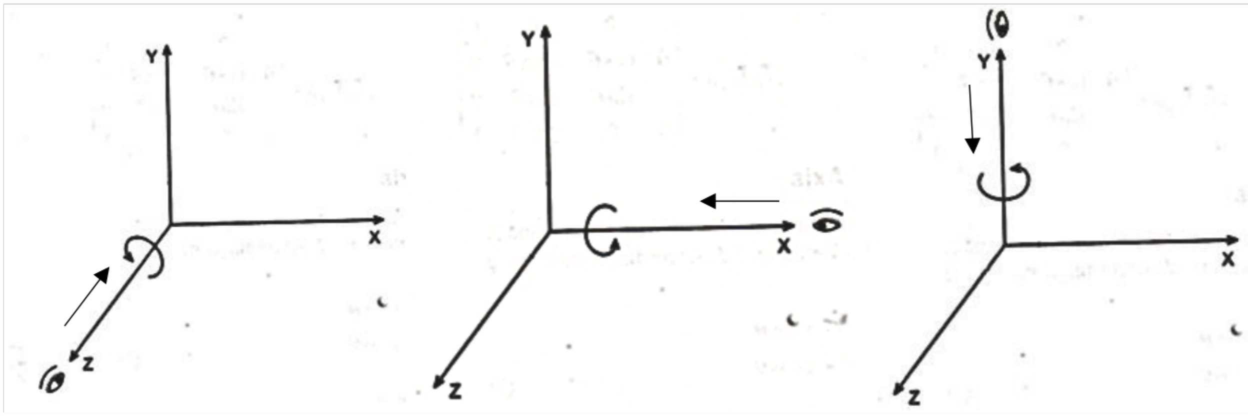
Reflection about any arbitrary plane in 3D Space

- Reflection about any arbitrary plane in 3D is similar to the reflection about any arbitrary line
- But the difference is that, we have to characterize the rotation by any normal vector 'N' in that plane.

Step 1: Translate the reflection plane to the origin of the coordinate system
Step 2: Perform appropriate rotations to make the normal vector of the reflection plane at the origin until it coincides with the z-axis.
Step 3: After that reflect the object through the z= 0 coordinate plane.
Step 4: Perform the inverse of the rotation transformation
Step 5: Perform the inverse of the translation

Rotation in 3D

- In CG, 3D rotation is a process of rotating an object to an angle in a three dimensional plane.
- Rotation is moving of an object about an angle.
- Movement can be anticlockwise or clockwise.
- 3D rotation is complex as compared to the 2D rotation. For 2D we describe the angle of rotation, but for a 3D angle of rotation and axis of rotation are required. The axis can be either x or y or z.
- To determine a rotation transformation for an object in 3D space, following information is required:
  - ✓ Angle of rotation.
  - ✓ Pivot point
  - ✓ Direction of rotation
  - ✓ Axis of rotation



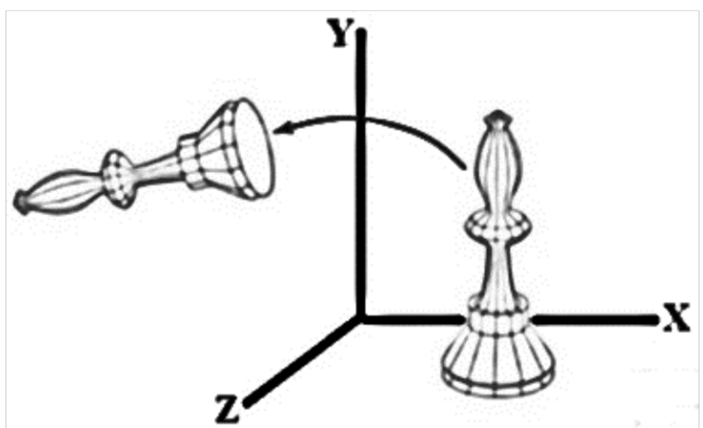
Figures: Coordinate axis Rotations

Let's consider a origin as the center of rotation and a point P(x, y, z) is rotated through an angle about any one of the axes to get the transformed point P'(x', y', z'), then the equation for each rotation can be obtained as follows.

3D Z-axis Rotation

- This rotation is achieved by using the following rotation equations
- Two dimension rotation equations can be easily convert into 3D z- axis rotation equation.
- Rotation about z axis we leave z coordinate unchanged.

$X' = x \cos\theta - y \sin\theta$
$Y' = x \sin\theta + y \cos\theta$
$Z' = z$



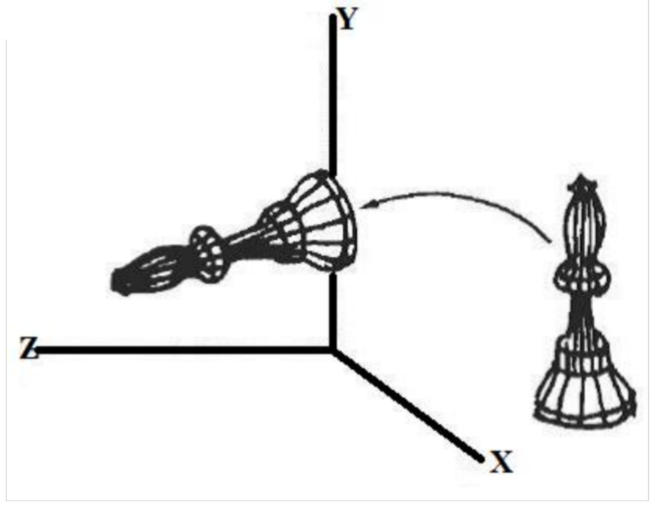
In matrix representation,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Therefore, $\mathbf{P' = R_{z(\theta)} . P}$

3D X-axis Rotation

This rotation is achieved by using the following rotation equations:



X' = x

Y' = y cos θ – z sin θ

Z' = y sin θ + z cos θ

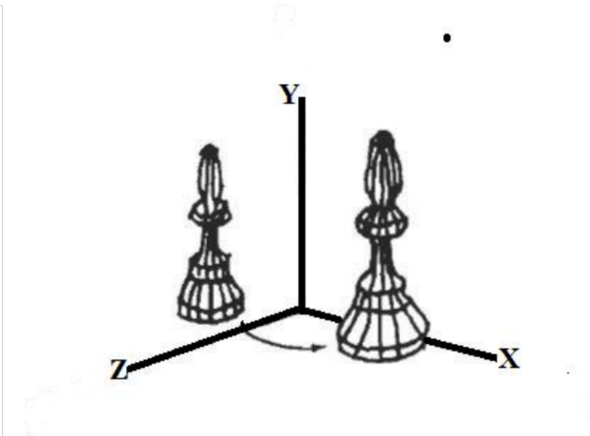
In matrix Representation,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta & -sin\theta & 0 \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Therefore, **P' = $\mathbf{R_{x\,(\theta)}}$. P**

3D Y-axis Rotation

Equations for this rotation are as:



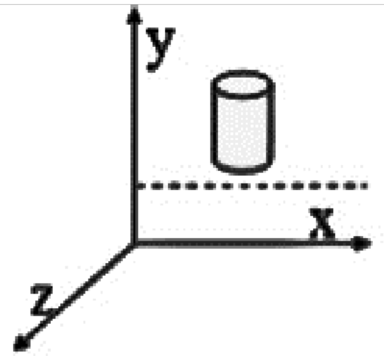
X' = z sin θ + x cos θ
Y' = y
Z' = z cos θ – x sin θ

Matrix representation of these equation,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} cos\,\theta & 0 & sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta & 0 & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

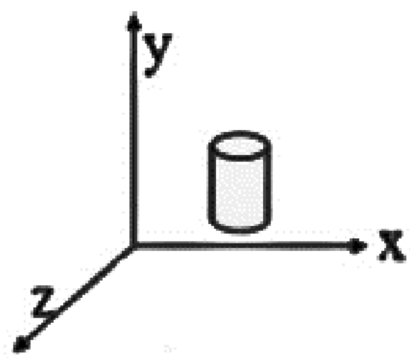
Therefore, **P' = $\mathbf{R_{y\,(\theta)}}$. P**

Rotation about an axis parallel to one of the coordinate axes

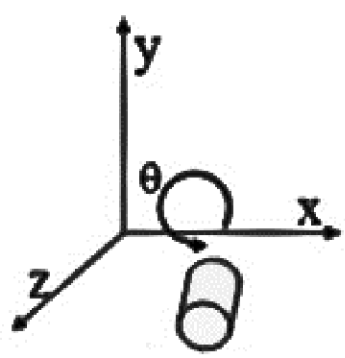


DO:

1. Translate object so that rotation axis coincides with the parallel coordinate axis.



2. Perform specified rotation about that axis



3. Translate object back to its original Position

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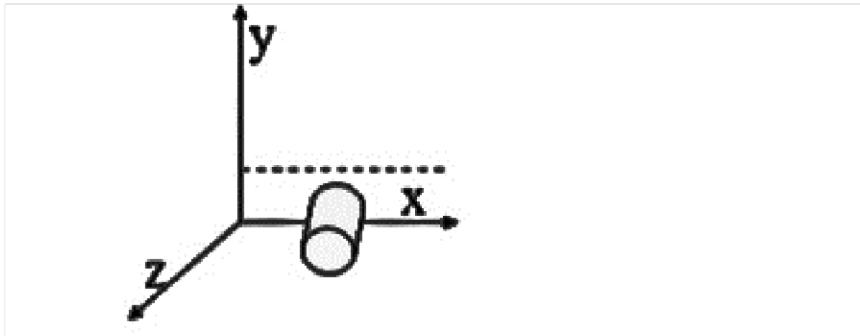
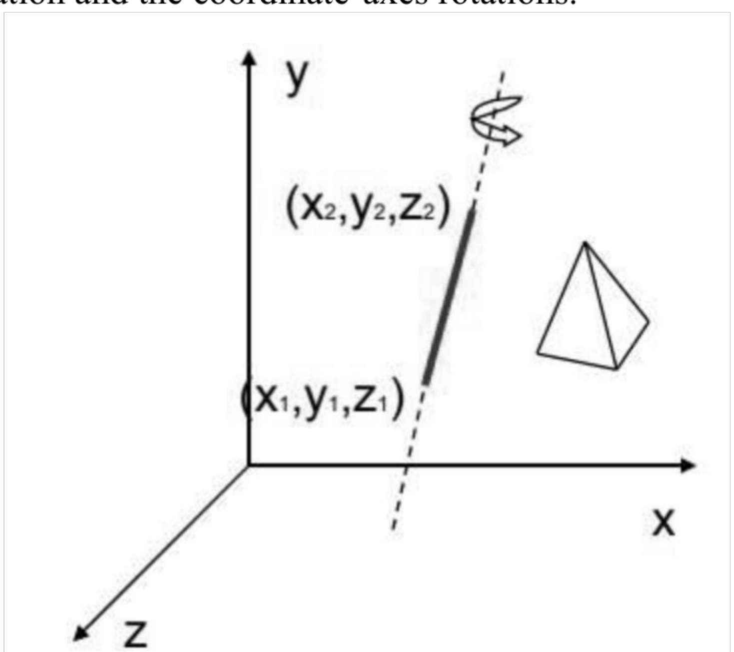


Therefore, CM= T R T^{-1}

P'= CM.P

Rotation about an arbitrary axis

- A rotation matrix for any axis that does not coincide with a coordinate axis can be set up as a composite Transformation involving combinations of translation and the coordinate-axes rotations.



<u>DO:</u>

Step-1: Translate the arbitrary axis so that it passes through origin.
Step-2: Align the arbitrary axis on any major co-ordinate axis (z-axis)
Step-3: Rotate the object about yz-plane or z-axis
Step-4: Perform inverse rotation about y-axis & then x-axis
Step-5: Perform inverse translation

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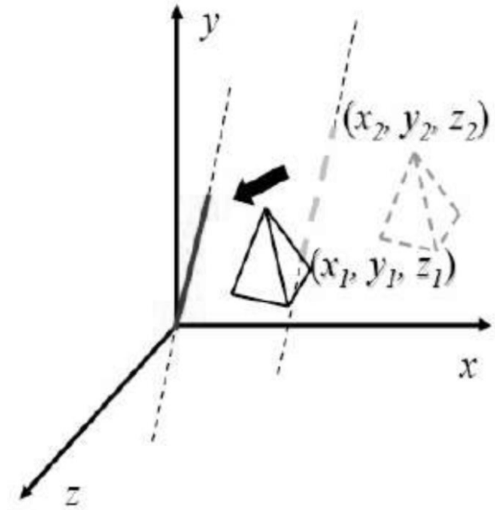
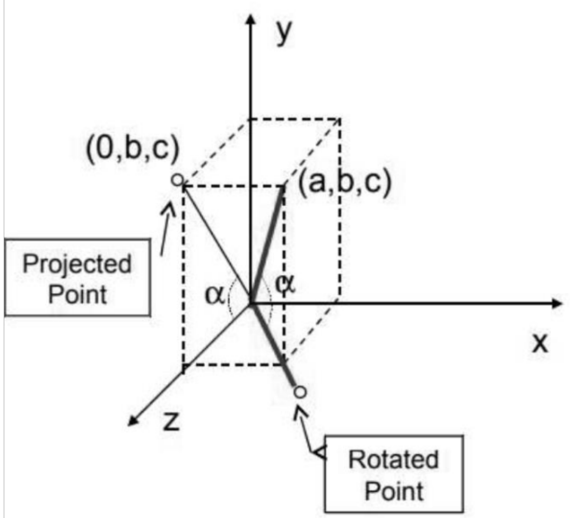
Step-1: Translate



$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-2: Rotate about X-axis by an angle α

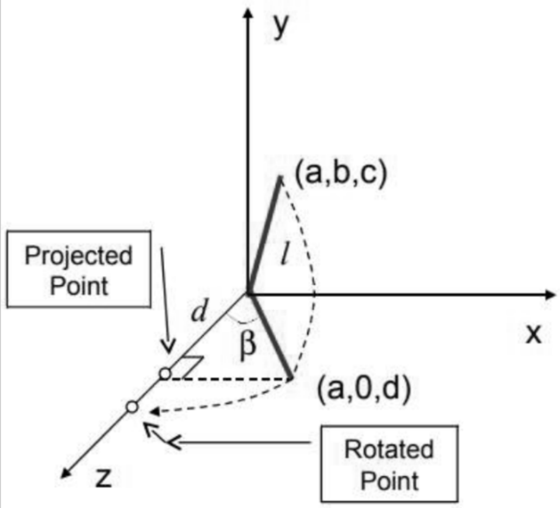


$$\sin\alpha = \frac{b}{\sqrt{b^2+c^2}} = \frac{b}{d}$$

$$\cos\alpha = \frac{c}{\sqrt{b^2+c^2}} = \frac{c}{d}$$

$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-3: Rotate about Y-axis by an angle β



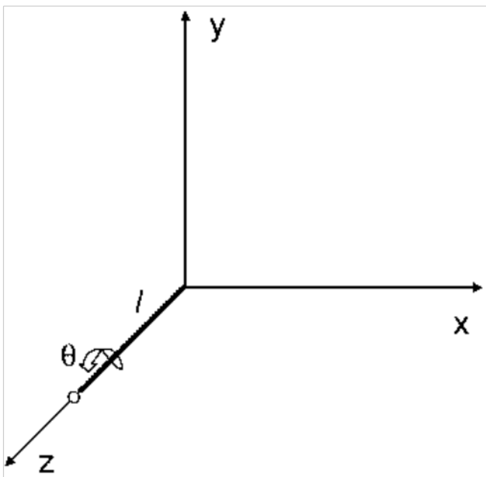
$$\sin\beta = \frac{a}{l}, \qquad \cos\beta = \frac{d}{l}$$

$$l^2 = a^2 + b^2 + c^2 = a^2 + d^2$$

$$d = \sqrt{b^2 + c^2}$$

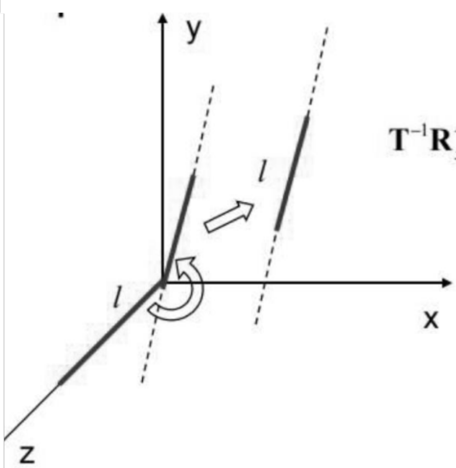
$$\mathbf{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} d/l & 0 & -a/l & 0 \\ 0 & 1 & 0 & 0 \\ a/l & 0 & d/l & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-4: Rotate about Z-axis by an angle θ



$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-5 Apply the inverse transformation



$$\mathbf{T}^{-1}\mathbf{R}_x^{-1}(\alpha)\mathbf{R}_y^{-1}(\beta) = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence,

Composite matrix (CM) = $\mathbf{T^{-1} R_x^{-1} R_y^{-1} R_z R_y R_x T}$

$\mathbf{P' = CM * P}$

Rotation about any arbitrary plane in 3D Space

- The rotation about any arbitrary plane perform same operation as the rotation about any arbitrary line, the only difference is that we have to characterize the rotation by any normal vector 'N' in that plane.

DO:

Step 1: Translate the rotation plane to the origin of the coordinate system
Step 2: Perform appropriate rotations to make the normal vector of the rotation plane at the origin until it coincides with the z-axis.
Step 3: After that rotate the object through the z= 0 coordinate plane.
Step 4: Perform the inverse of the rotation transformation
Step 5: Perform the inverse of the translation

3-D Viewing

- The basic idea of the 3D viewing transformation is similar to the 2D viewing transformation.
- Viewing window is defined in world space that specifies how the viewer is viewing the scene.
- A corresponding view port is defined in screen space, and a mapping is defined to transform points from world space to screen space based on these specifications.
- The view port portion of the transformation is the same as the 2D case.
- Specification of the window, however, requires additional information and results in a more complex mapping to be defined.
- Defining a viewing window in world space coordinates is exactly like it sounds; sufficient information needs to be provided to define a rectangular window at some location and orientation.

The usual viewing parameters that are specified are:

a) **Eye Point**- The position of the viewer in world space
b) **Look Point**- The point that the eye is looking at

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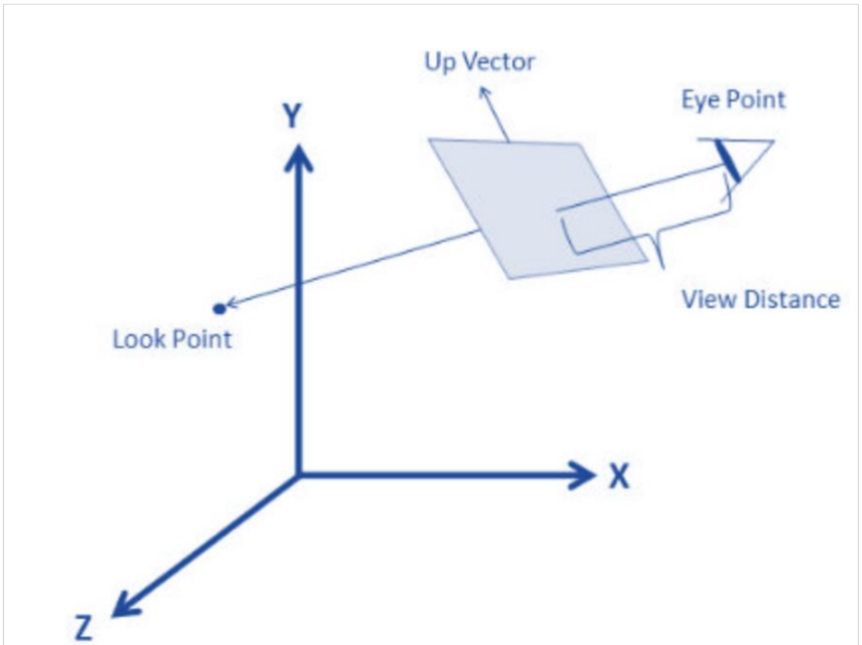
c) **View Distance** -The distance that the window is from the eye
d) **Window Size -** The height and width of the window in world space coordinates Up Vector which direction represents "up" to the viewer, this parameter is sometimes specified as an angle of rotation about the viewing axis



3D Viewing pipeline

The steps for computer generation of a view of 3D scene are analogous to the process of taking photograph by a camera. For a snapshot, we need to position the camera at a particular point in space and then need to decide camera orientation. Finally when we snap the shutter, the scene is cropped to the size of window of the camera and the light from the visible surfaces is projected into the camera film.

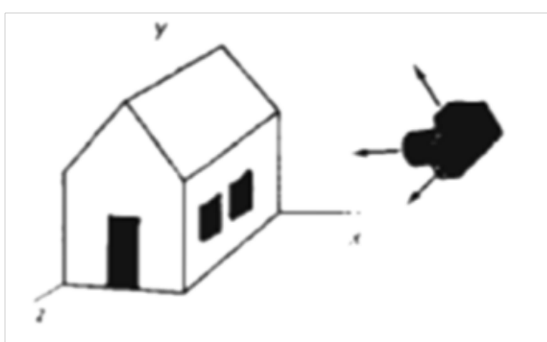


Fig: Photographing a scene involves selection of a camera position and orientation

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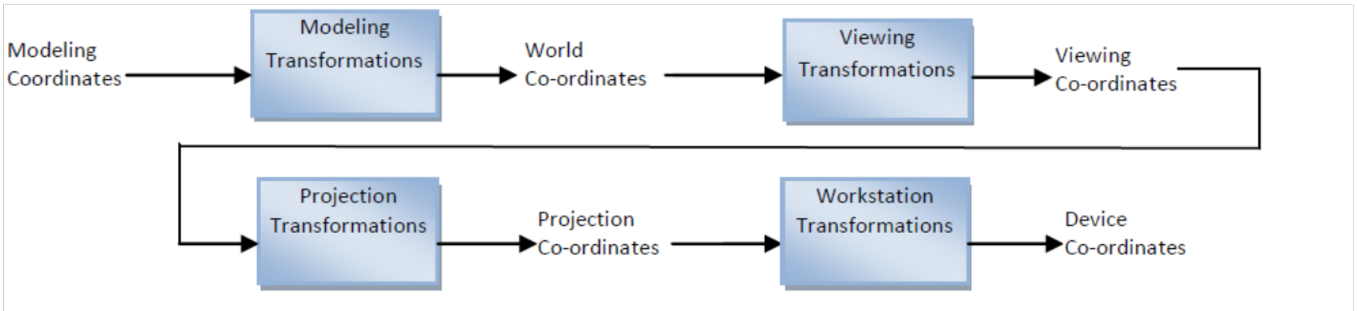


Fig: General three-dimensional transformation pipeline from modeling coordinates to final device coordinates

- 3D view pipeline is the processing steps for modeling and converting a world-coordinate description of a scene to device coordinates.
- Once the scene has been modeled, world-coordinate positions are converted to viewing coordinates.
- The viewing-coordinate system is used in graphics packages as a reference for specifying the observer viewing position and the position of the projection plane, which we can think of in analogy with the camera film plane.
- Next, projection operations are performed to convert the viewing-coordinate description of the scene to coordinate positions on the projection plane, which will then be mapped to the output device.
- Objects outside the specified viewing limits are clipped from further consideration, and the remaining objects are processed through visible-surface identification and surface rendering procedures to produce the display within the device view port.

Projections

- Theory of Projections In engineering, 3-dimensonal objects and structures are represented graphically on a 2-dimensional media.
- The act of obtaining the image of an object is termed projection.
- The image obtained by projection is known as a view.
- All projection theory are based on two variables
    - Line of sight
    - Plane of projection
- A **plane of projection** (i.e., an image or picture plane) is an imaginary flat plane upon which the image created by the line of sight is projected. The image is produced by connecting the points where the lines of sight pierce

the projection plane. In effect, 3-D object is transformed into a 2-D representation, also called projections. The paper or computer screen on which a drawing is created is a plane of projection

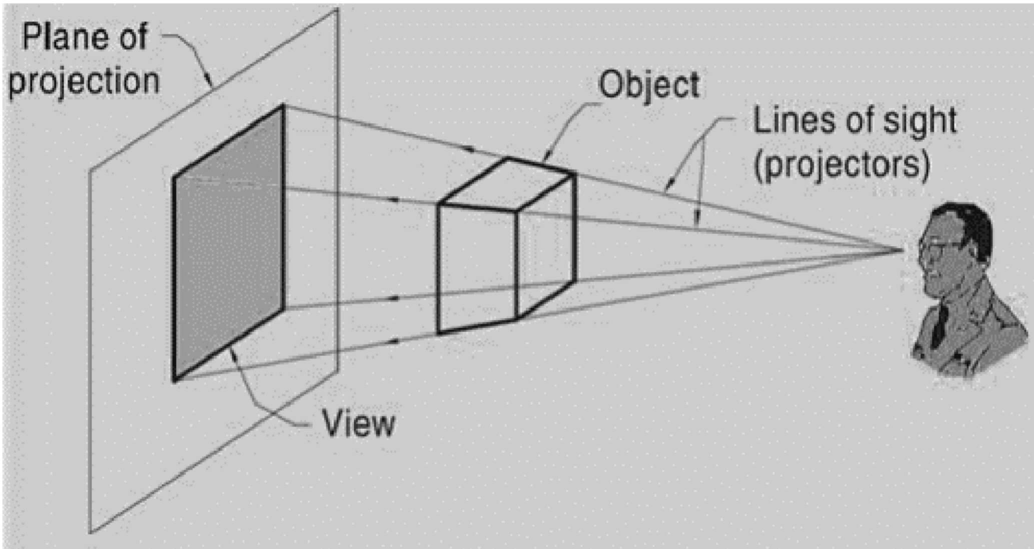


Figure : A simple Projection system

- ❖ Transform objects or points in a coordinate system from dimension m into a coordinate system of dimension n where m < n.
- ❖ Projection means the image or act of obtaining an image of the object.

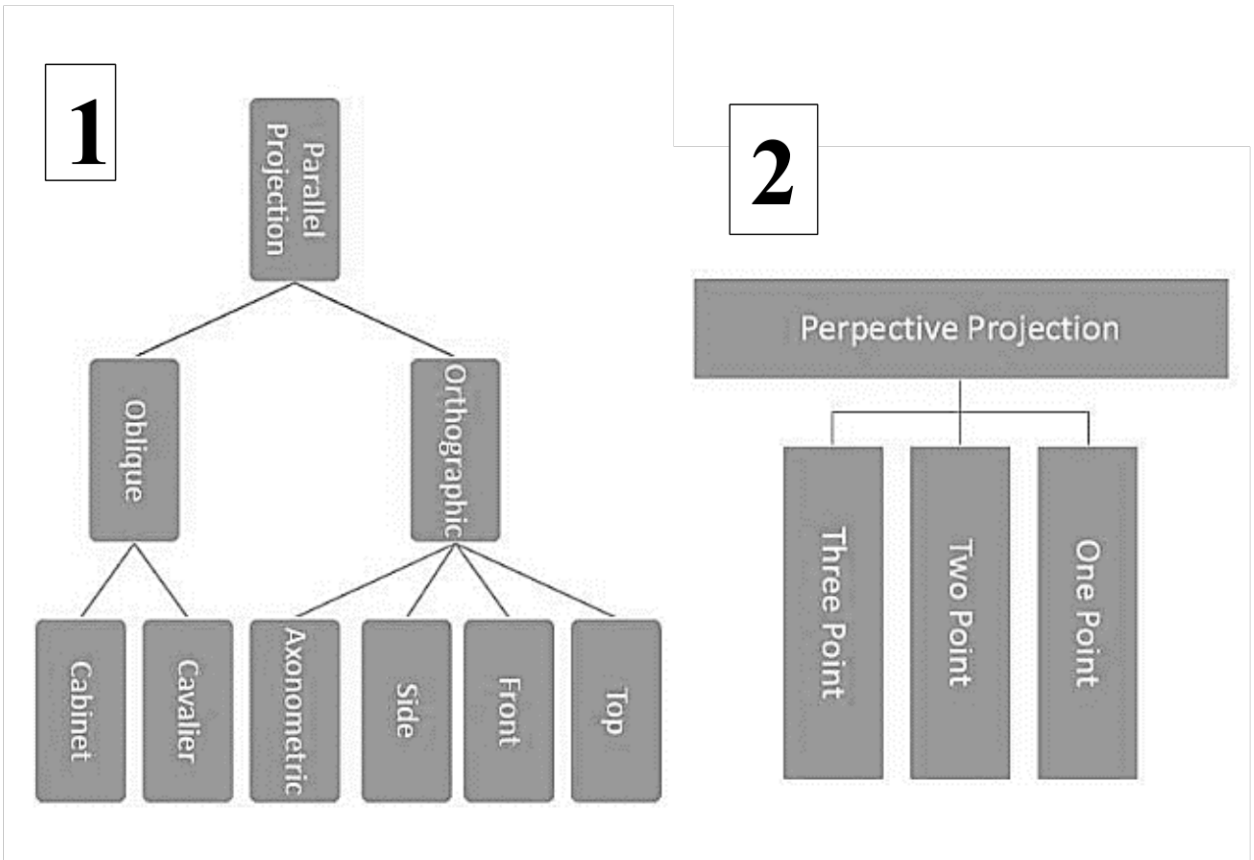
- ❖ **3D projection** is method of mapping three-dimensional points to a two-dimensional plane.
  - In general, a projection transforms an N-dimension points to N-1 dimensions.
  - Ideally, an object is projected by projecting each of its endpoints. But an object has infinite number of points. So we cannot project all those points. What we do is that we project only the corner points of an object on a 2D plane and we will join these projected points by a straight line in a 2D plane.
- ❖ Once world co-ordinate description of the objects in a scene are converted to viewing co-ordinates, we can project the three dimensional objects onto the two dimensional view plane.

<u>Basic points in projection:</u>

- ❖ **Center of projection**: Point from where projection is taken.

- **Projection / view plane**: the plane on which the projection of the object is formed.
- **Projectors**: Lines emerging from the center of projection and hitting the projection plane.

There are two basic projection methods:



1. **Parallel projection**

- In this projection, the co-ordinate positions are transformed to the view plane along parallel lines.
- It preserves relative proportions of objects so that accurate views of various sides of an object are obtained but doesn't give realistic representation of the 3D object.
- Can be used for exact measurements so parallel lines remain parallel

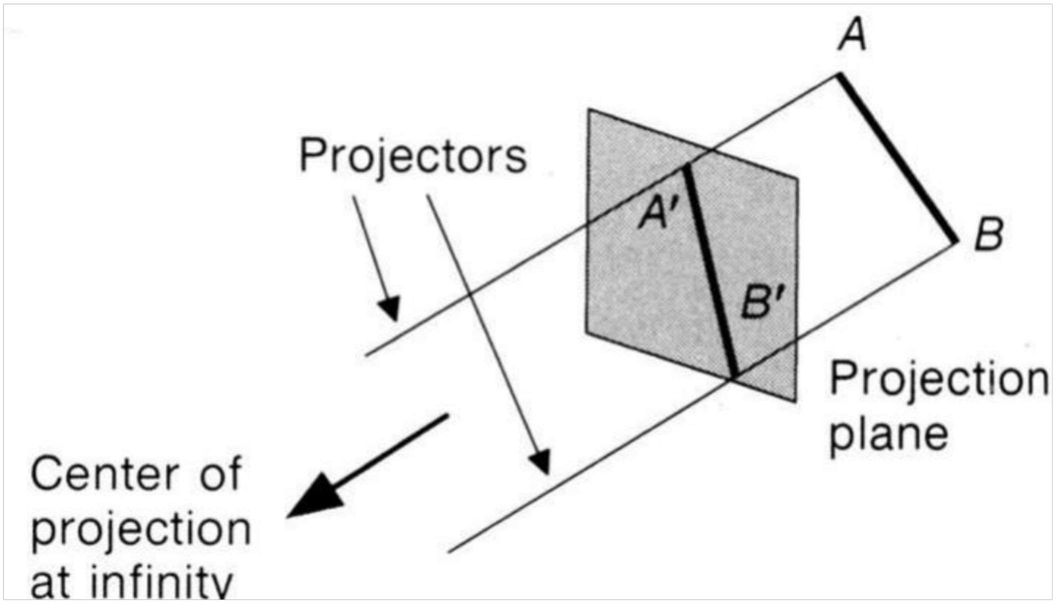


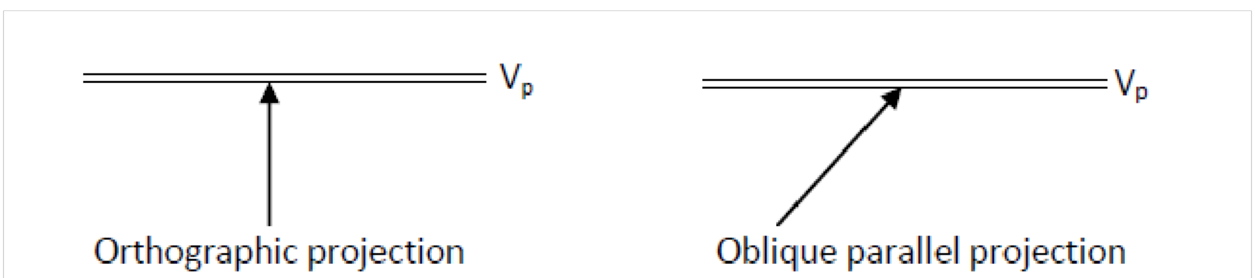
Fig: Parallel projection of an object to the view plane

There are two different types of parallel projections

1. **<u>Orthographic parallel projection</u>**

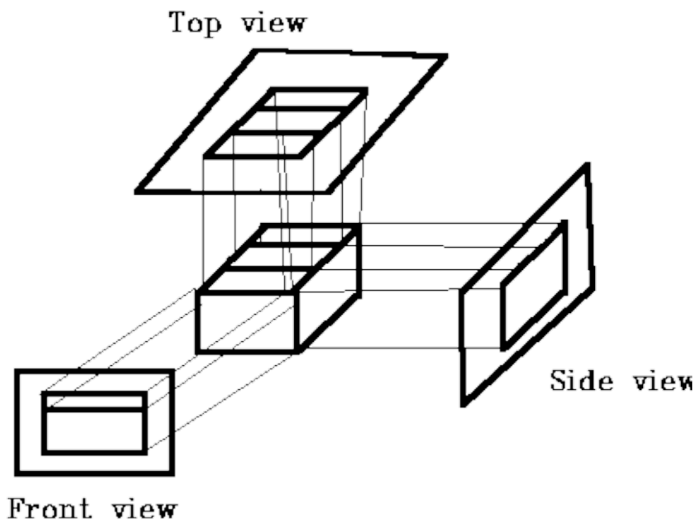
- ❖ When the projection lines are perpendicular to view plane, the projection is **orthographic parallel projection.**

- ❖ Otherwise it is **oblique parallel projection**.

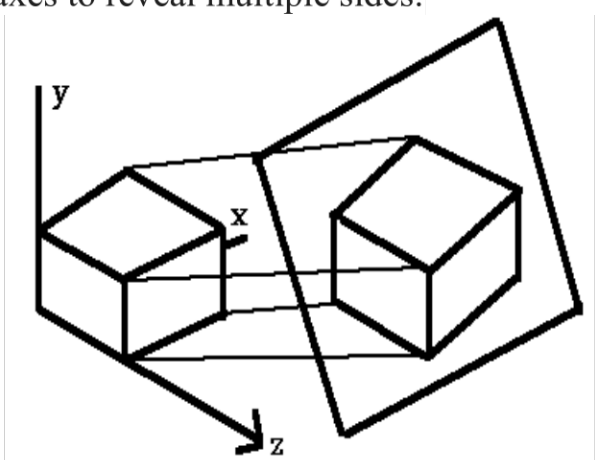


- ❖ Orthographic projections are most often used to produce the front, side, and top views of an object.
- ❖ Front, side, and rear orthographic projections of an object are called ***elevations*** and
- ❖ A top orthographic projection is called a ***plain view.***

- Engineering and Architectural drawings commonly employ these orthographic projections.



- Orthographic projections that show more than one face of an object. Such views are called **axonometric orthographic projections**.
- The most commonly used axonometric projection is the **isometric projection**. Where the projection plane intersects each coordinate axis in the model coordinate system at an equal distance.
- **Axonometric projection** is a type of **orthographic projection** used for creating a pictorial drawing of an object, where the lines of sight are perpendicular to the plane of **projection**, and the object is rotated around one or more of its axes to reveal multiple sides.



- In orthographic projection, the center of projection lies at infinity and the projections are perpendicular to the view plane. So, a true size and shape of

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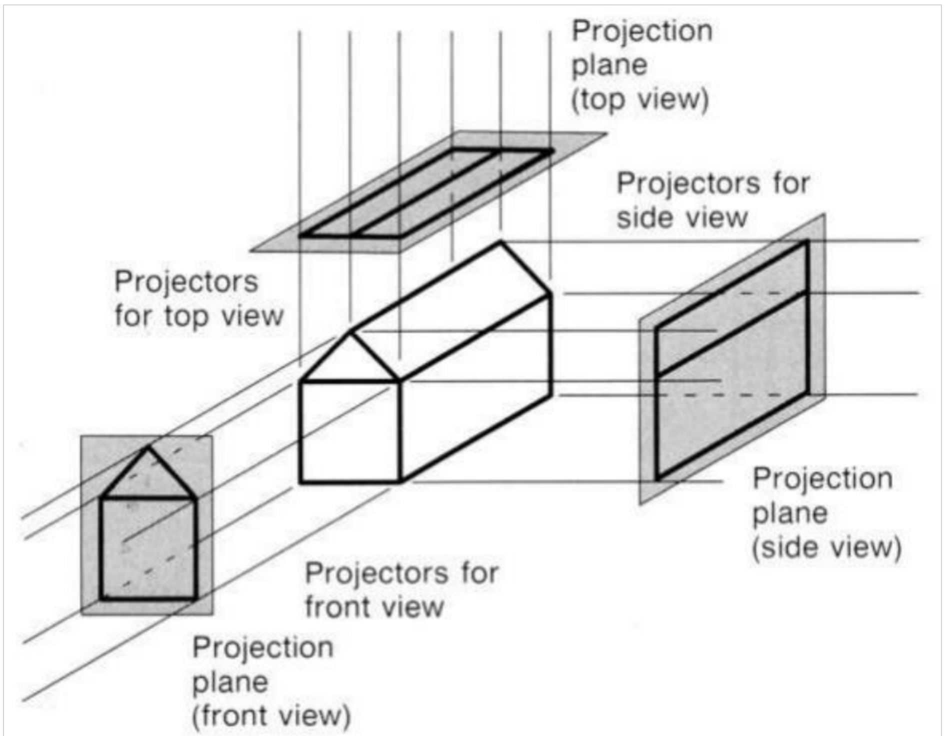
a single face of object is obtained. In orthographic projection the direction of projection is normal to the projection of the plane.

There are **three types of orthographic** projections

- Front Projection
- Top Projection
- Side Projection



2. **Oblique parallel Projection**

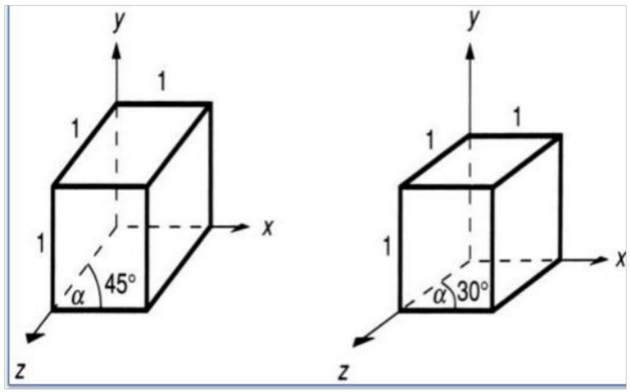
- A projection in which the angle between the projectors and the plane of plane of projection is not equal to 90 is called oblique projection.
- An oblique projection is formed by parallel projections from a center of projection at infinity intersects the plane of projection at an oblique angle.

There are **two types of oblique** projections

1. **Cavalier** and
2. **Cabinet**.

- The **Cavalier projection** makes 45° angle with the projection plane. The projection of a line perpendicular to the view plane has the same length as the line itself in Cavalier projection. In a cavalier projection, the foreshortening factors for all three principal directions are equal.

- The **Cabinet projection** makes 63.4° angle with the projection plane. In Cabinet projection, lines perpendicular to the viewing surface are projected at ½ their actual length.



Transformation Matrix for oblique parallel projection v. imp

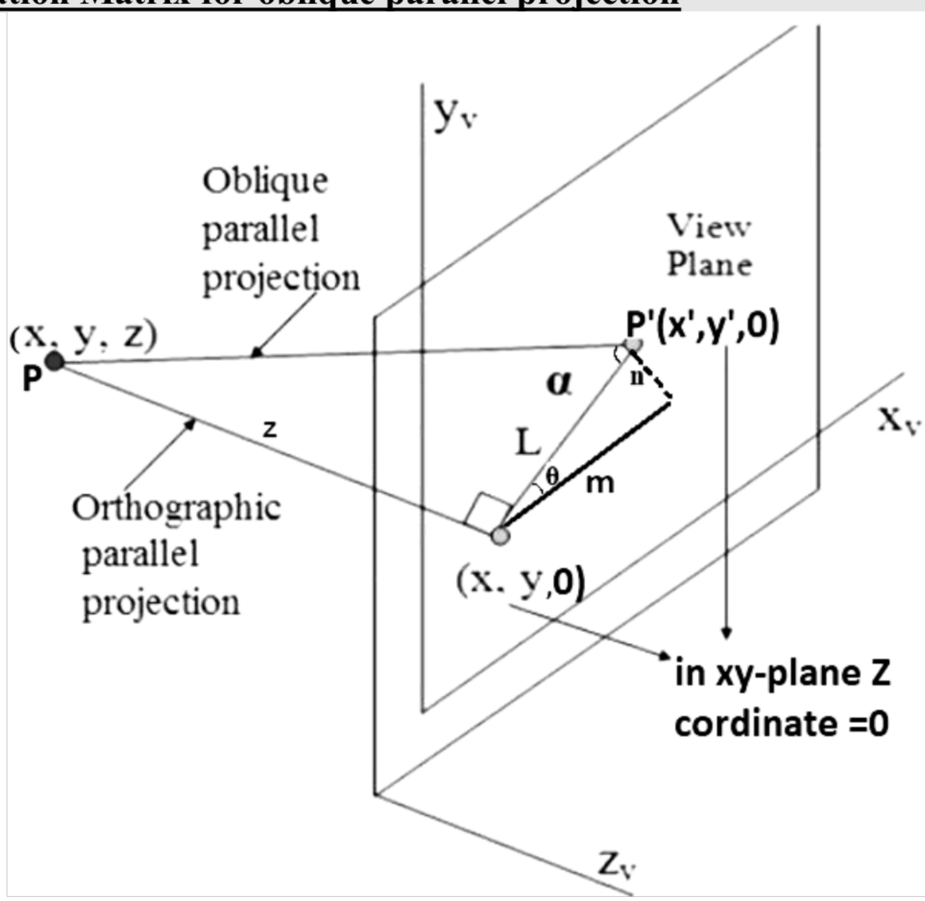


Figure: Oblique Parallel Projection

Consider a Point P (x, y, z) is projected to position P' (x', y', 0) on the view plane as parallel projection. If the P is projected orthographically then orthographic projection coordinated on the plane are (x, y, 0). Oblique projection line from (x, y, z) to (x', y', 0) makes an angle with the line on the projection plane that joins (x', y', 0) and (x, y, 0).

Consider the line of length L is at an angle θ with the horizontal direction in the projection plane.

Now, calculating projection coordinates

$$X' = x + m$$

Find value of m, cos θ = m/L then m = L cos θ
Similarly,

$$Y' = y + n$$

For the value of n, $\sin\theta = \frac{n}{L}$ then n = L sin θ

Therefore,

$$x' = x + L\cos\theta \text{ and } y' = y + L\sin\theta$$

Since, L depends on the angle α and z coordinate of point to be projected then

$$\tan\alpha = \frac{z}{L}$$

Thus,

$$L = \frac{z}{\tan\alpha}$$

$$= z L_1, \text{ where } L_1 \text{ is the inverse of } \tan\alpha \text{ (i.e. } L_1 = \frac{1}{\tan\alpha} = \cot\alpha)$$

So the oblique projection equations are:

$$X' = x + z (L_1 \cos\theta)$$
$$Y' = y + z (L_1 \sin\theta)$$

Finally, the homogeneous transformation matrix for producing any parallel projection onto the x_v, y_v plane can be written as:

$$M_{Parallel} = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1 \cos\theta & 0 \\ 0 & 1 & L_1 \sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Orthographic projection is obtained when $L_1 = 0$ (occurs at projection angle α of 90 degree) Oblique projection is obtained with non-zero values for L_1.

b) Perspective Projection

- When an object is viewed from different directions and at different distances, the appearance of the object will be different. Such view is called perspective view.

- In this projection, the co-ordinate positions are transformed to the view plane along lines that converges to a point called as **center of projection**.

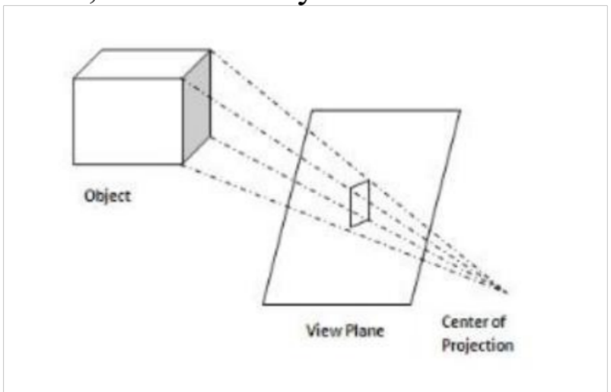
- The projection is perspective if the distance is finite or has fixed **vanishing point** (All lines appear to meet at some point in the view plane.) or the incoming rays converge at a point. Thus the viewing dimension of object is not exact i.e. seems large or small according as the movement of the view plane from the object.

- The perspective projection perform the operation as the scaling (i.e. zoom in & out) but here the object size is only maximize to the size of real present object i.e. only size reduces not enlarge. This operation is possible here only the movement of the view plane towards or far from the object position.

- In perspective projection, all lines of sight start at a single point. Distance from the observer to the object is finite and the object is viewed from a single point – projectors are not parallel.

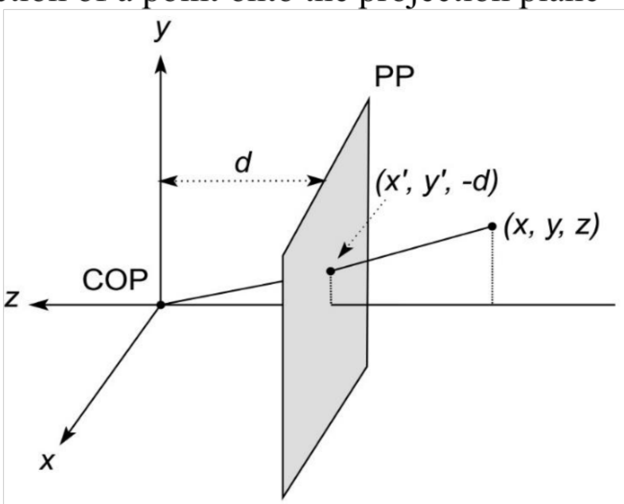
Conclusion:

A perspective projection can be described as the projector lines (lines of sight) that converge at the center of projection, which results in many visual effects of an object. Perspective projection depends on the relative position of the *eye* and the *viewplane*. A perspective projection of an object is often considered more realistic than a parallel projection, since it nearly resembles human vision and photography.



Transformation matrix for perspective projection (Derivation) **V**$^{\text{imp}}$

Consider the projection of a point onto the projection plane



By similar triangles, we can compute how much the *x* and *y* coordinates are scaled as

$$\frac{x'}{x} = -\frac{d}{z}$$

$$\frac{y'}{y} = -\frac{d}{z}$$

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- ✓ Remember how transformations work with the last coordinate always set to one.
- ✓ What happens if the coordinate is not one?
- ✓ We divide all the coordinates by *h*
- ✓ If *h*= *1*, then nothing changes.

$$\begin{bmatrix} X/\mathbf{h} \\ Y/\mathbf{h} \\ Z/\mathbf{h} \\ \mathbf{h}/\mathbf{h} \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

These points are to recall the homogeneous coordinate system, no need to write these ticked line inside the box in exam or answer for this

- ✓ Sometimes we call this division step the "perspective divide."

Now we can re-write the perspective projection as a matrix equation

$$\begin{bmatrix} X \\ Y \\ \mathbf{h} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix}$$

After division by *h,* we get

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{x}{y}d \\ -\frac{y}{z}d \\ 1 \end{bmatrix}$$

Again, projection implies dropping the *z* coordinate to give a 2D image, but we usually keep it around a little while longer.

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Difference between Perspective and Parallel Projection VVImp

BASIS OF DIFFERENCE	PERSPECTIVE PROJECTION	PARALLEL PROJECTION
Description	A perspective projection can be described as the projector lines (lines of sight) that converge at the center of projection, which results in many visual effects of an object.	A parallel projection is a projection of an object in three-dimensional space onto a fixed plane referred as the projection plane or image plane, where the rays, known as lines of sight or projection lines are parallel to each other.
Types	One-point perspective Projection. Two-point perspective projection. Three-point perspective projection.	Orthographic parallel projection Oblique parallel projection.
Accurate View Of Object	Perspective projection cannot give the accurate view of object.	Parallel projection can give the accurate view of object.
Object Representation	Perspective projection represents the object in three dimensional way.	Parallel projection represents the object in a different way like telescope.
Realistic View of Object	Perspective projection forms a realistic picture of object.	Parallel projection does not form realistic view of object.
Distance Of The Object From The Center Of Projection	The distance of the object from the center of projection is finite.	In parallel projection, the distance of the object from the center of projection is infinite.
Projector	Projector in perspective projection is not parallel.	Projector in parallel projection is parallel.
Preservation Of Relative Portion Of An Object	Perspective projection cannot preserve the relative proportion of an object.	Parallel projection can preserve the relative proportion of an object.
Lines Of Projection	The lines of perspective projection are not parallel.	The lines of parallel projection are parallel.

End of Chapter