0.0.1 Inference

Testing models Consider the linear model

$$\mathbb{Y} = \mathbb{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad \cdots \quad \text{full model}$$

where $\gamma(\mathbb{X}) = \gamma \leq p$, $E(\boldsymbol{\varepsilon}) = 0$, $Var(\boldsymbol{\varepsilon}) = \sigma^2 I$

Consider the linear model

$$\boldsymbol{y} = \mathbb{W}\boldsymbol{\gamma} + \boldsymbol{\varepsilon} \quad \cdots \quad \text{redued model}$$

where $C(\mathbb{W}) \subset C(\mathbb{X})$, $E(\boldsymbol{\varepsilon}) = 0$, $Var(\boldsymbol{\varepsilon}) = \sigma^2 I$

note:

1. the estimation space of R.M. is smaller than in the F.M.
2. The goal is to test whether or not the reduced model is also correct.
   - If R.M. is correct, there is no reason not to use it.
   - Smaller models are easier to interpret.
3. Let $P_{\mathbb{X}}$ and $P_{\mathbb{W}}$ denote the ppm onto $C(\mathbb{X})$ and $C(\mathbb{W})$. Because $C(\mathbb{W}) \subset C(\mathbb{X})$, we know that $P_{\mathbb{X}} - P_{\mathbb{W}}$ is the ppm onto $C(P_{\mathbb{X}} - P_{\mathbb{W}}) = \underbrace{C(\mathbb{W})^{\perp} C(\mathbb{X})}_{\maltese}$

Lemma 1 *Assume $P_{\mathbb{X}}$ and $P_{\mathbb{W}}$ are projection matries and $P_2 - P_1$ is P.D., then*

1. $P_1 P_2 = P_2 P_1 = P_2$
2. $P_1 - P_2$ *is a p.m.*

note:

1. P_1 is a p.m onto $C(\mathbb{X})$
2. P_2 is a p.m onto $C(\mathbb{W}) \subset C(\mathbb{X})$
3. $P_1 - P_2$ is a p.m onto the orthogonal complement of $\mathbb{W}$ with in $\mathbb{X}$

$\boxed{\text{figure}}$