2007 國際小學數學自然科學奧林匹亞 ISMO-數學基本題第一題 The 400-digit number 12345678901234567890…890 is given.

Step 1: Cross out all the digits in odd-numbered places.

Step 2: Cross out all the digits in odd-numbered places of the remaining digits.

…

Continue until no digits remain. What is the last digit to be crossed out?

Solution:

position	1st	2nd	3rd									13th								
total num of digits: 400	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0
200		2		4		6		8		0		2		4		6		8		0
100				4				8				2				6				0
50								8								6				
25																6				
12																				
6																				
3																				
1																				

After the first operation, 200 digits will remain and the positions of these remains are $2 \times N$, where N is an interger. After the second operation, 100 digits will remain and the position of them are $2^2 \times N$. 50 digits remain after the third operation and positions of them are $2^3 \times N$. So in the last operation only 1 digit remains and the position of this digit is $2^8 \times N$. But the position has to been smaller than 400, so N can only be 1. So the position is $2^8 \times 1 = 256$. Because the digit sequence has a repetive period of 10 digits. 256 position is $25 \times 10 + 6$, so it is the 6th position in the sequence, which has a number 6. QED