An indepth review of the direction of friction in rolling motion (no slippage)

May 1, 2019

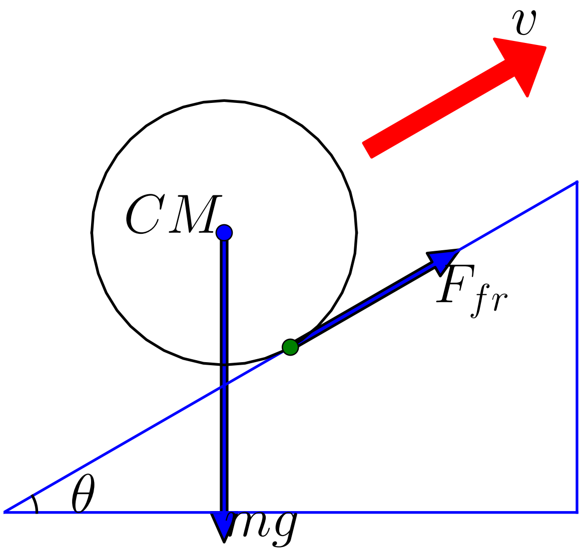
The direction of friction force is easily mistaken when considering a rolling motion. This is important and is ensential to solving the dynamics of rolling motions. (Rolling means there is no slippage) A few easily-mistaken cases are reviewed here, and excercising problems are provided.

1 determination of direction of friction acting on the rolling object

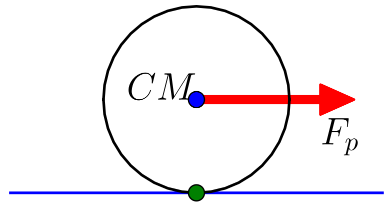
Rolling down hill Round object freely rolls down the hill. The only force that makes the object rotate is friction so friction has to go up the hill. This friction force is exerted on the wheel by the slope.

Pushing away uphill Object is forced to roll up the hill initially but external force is removed once the object is going upward. We are considering the later part of the motion when the external force is removed, so only gravitation is in place. The wheel is still rolling up the hill.

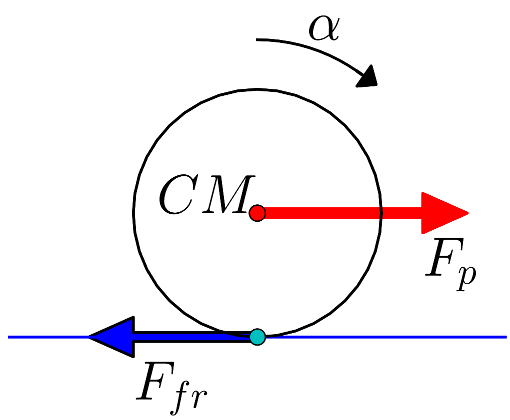
The rotation of the object slows down as it climbs up the hill. Friction is the only force that produces a torque to slow down the rotation. So it needs to go against the rotating direction. So the friction force acting on the wheel is up the hill.



External force F_p passing through CM point

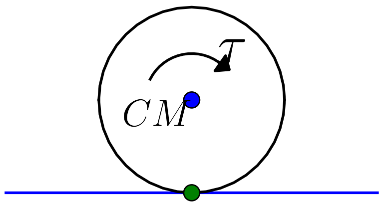


Friction is still the only force that produces an angular acceleration, so friction should have the same direction as angular acceleration. If the object is moving to the right and rotating faster, friction acting on the object goes towards left. If the object is moving to the left and rotation slow down because of the external force slowing it down, the friction force goes towards the left, to produce a torque and a angular acceleration to slow down the rotation.



$$\begin{cases} F_p - F_{fr} = ma \\ F_{fr} \cdot R = I\alpha = I\frac{a}{R} \end{cases}$$

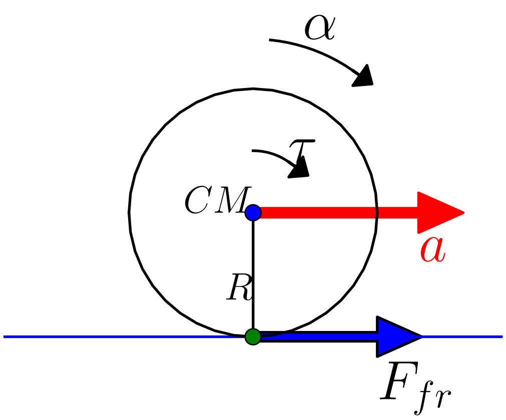
An external torque (not force) acting around the axial, like on car tire.



In this case, the rule of friction is not to cause rotation; the rotation about the center is done by the external torque. From 2nd law, in order to have no slipping, a translational force is needed to accelerate the CM of the object to move. Otherwise the object will slip. Friction in this case acts as this *force*, to give the CM of the object an acceleration to the right. One can judge from the relative movement between the object and the ground too. Friction prevents the object from skidding at the contact. So friction acting on the object goes to the right, while the mutual friction force acting on the ground goes to the left.

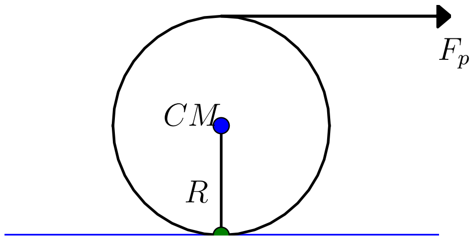
$$\begin{cases} F_{fr} = ma \\ \tau - F_{fr} \cdot R = I\alpha = I\frac{a}{R} \end{cases}$$

$$\Rightarrow \quad a = \frac{\tau}{mR + \frac{I}{R}}$$

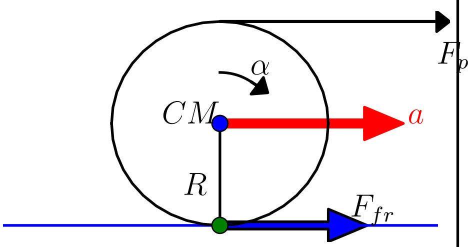


There is an interesting conclusion we can draw from the result of our calculation. In a real car race, the start of the race is very important. A car driver want to accelerate as fast as they can, but at the same time not to cause the tire to slip and loose grip. How to control the acceleration throttle and peddle is a hard problem. Well not so hard if you have studied this example. From the last equation we arrived, we know the acceleration of the tire a, which is essentially the same as the car's acceleration, is directly proportional to the torque τ. So before you may think you want to start with a smooth peddle control and put a little bit of throttle in the beginning so as not to loose grip. Wrong. Once you give the tire a torque τ, you acceleration a will be proportional to τ, what you want is the highest throttle right at the beginning! Of course not so high as to loose grip. So next time in a formula race, when the red light off and green light lid, put down the highest throttle that will not cause slip immediately. Give it a try. Easier to say than done! Ideally this is what you want to do. In reality, when you start off, the weight will tranfer from the front tires to the rear tires (because of the accleralation!), so there will be some delay before your rear tire get maximal down force and maximal grip. And drivers need to take this delay into account.

External forces not passing through CM pivot. (so there are both external force and torque.)



Let assume friction acting on the wheel goes right, then



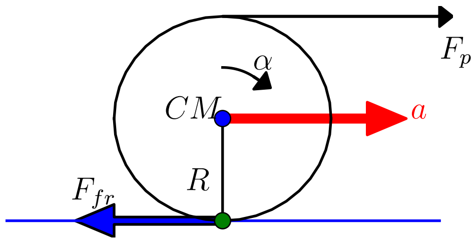
from 2nd law

$$\begin{cases} F_p + F_{fr} = ma \\ (F_p - F_{fr})\,R = I\alpha = I\frac{a}{R} \end{cases}$$

$$\Rightarrow \quad F_{fr} = \frac{1}{2}\left(m - \frac{I}{R^2}\right)a$$

So if $mR^2 > I$, then F_{fr} goes to the right!

If frction acting on the wheel goes to the left,



$$\begin{cases} F_p - F_{fr} = ma \\ (F_p + F_{fr})\,R = I\alpha = I\frac{a}{R} \end{cases}$$

$$\Rightarrow \quad F_{fr} = \frac{1}{2}\left(\frac{I}{R^2} - m\right)a$$

So if $mR^2 < I$, then F_{fr} goes to the left. But are there any objects that have $mR^2 < I$?

In this case the direction of friction depends on the condition $mR^2 > I$ or $mR^2 < I$. But in reality there is no object that can have $mR^2 < I$. So the second case is not physically meaningful. So we are left with the case of right-going friction. Friction always goes towards right. The only case the second senario will happen is when $mR^2 = I$, then friction force is zero. This is an interesting case worth meantioning. It means if we have a hoop object (all mass on the rim), then we don't need any frictional surface to cause the hoop to roll, if the force is applied on top of the hoop. This means we don't need to worry that the hoop will slip, even in the case when we can only provide low frictional coefficient surface (like on an ice surface). As long as your hand excert the force on top of the hoop, it will roll without slipping, even on ice.

We can also recgonize in the first senario if $mR^2 < I$ then F_{fr} is negative, so the friction should be opposite to the positive direction we assume (to the right). This way we can save the time of working on the 2nd case.

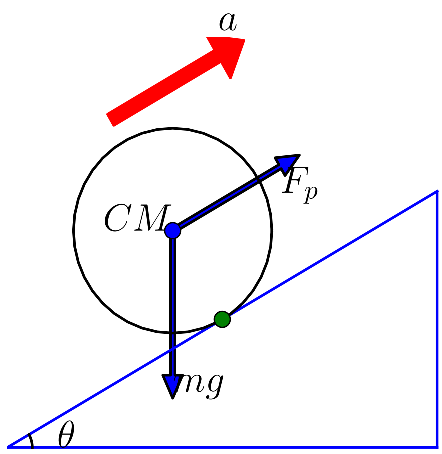
Also in the first case, let's say if a is fixed, then the smaller the I, the bigger the F_{fr}, and the smaller the $F_p = \frac{1}{2}ma + \frac{I}{2R^2}a$. In other words, the smaller the inertia is, the lessor force we need to apply, which means its easier for us to make it roll. And vice versa.

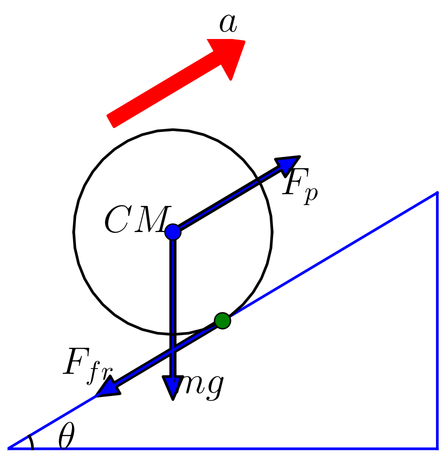
A yoyo.

2 Study Cases

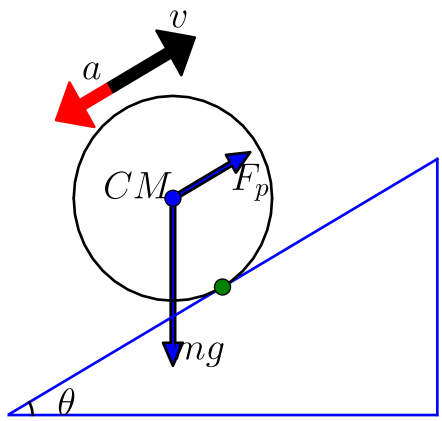
Determine the direction of friction force acting on the roller. (Assume rolling without slipping.)

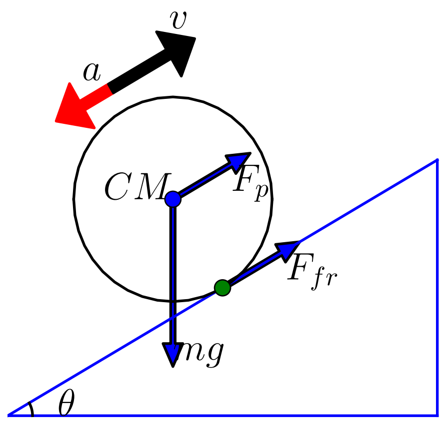
An applied force F_p passing through CM acts on the roller. The magnitude of F_p is larger than $mg \sin\theta$, so the roller is accelerating up the hill.



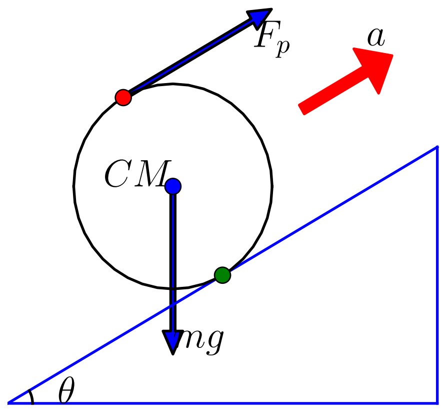


An applied force F_p passing through CM acts on the roller. The magnitude of F_p is smaller than $mg\sin\theta$. The roller is on its way to the highest point it can reach, and it is already slowing down.





An applied force F_p passing through the top edge of the roller, up the hill. The roller is accelerating up the hill. Determine the direction of friction acting on the roller.



Friction can go up or down the hill, depending on the condition. Similar to case 5.