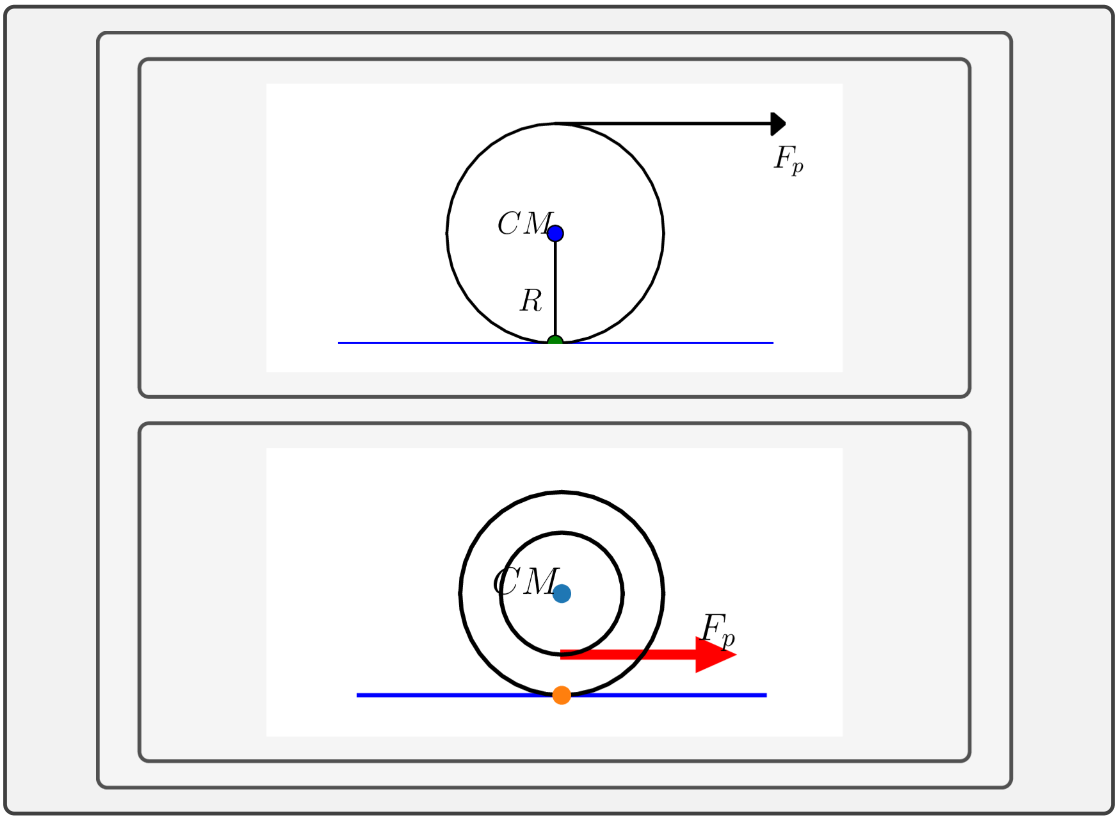
Easily mistaken problems on direction of friction in rolling motion (no slippage)

May 29, 2019



The direction of friction forces is easily mistaken in some interesting rolling motions. Surprisingly these cases are more commonly encountered in our life than we expect. This direction is essential to solving the dynamics of rolling motions. (Rolling means there is no slippage) A few easily-mistaken cases are reviewed here, and exercising problems are provided. This topic is commonly seen in university and grad school entry exam or qualification.

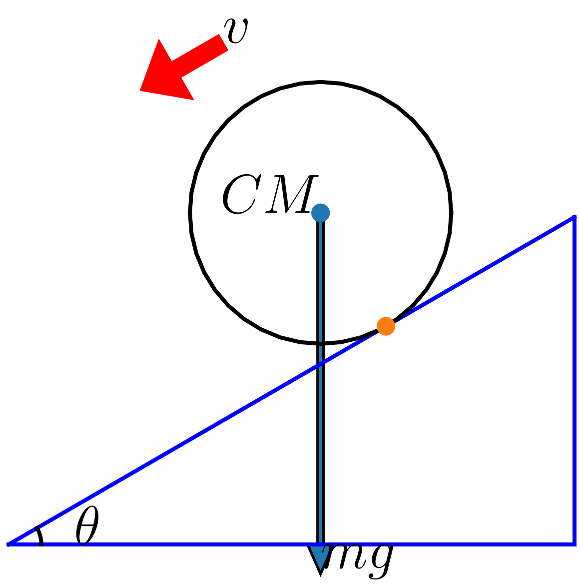
New examples (ensuring rolling on slipery surface and which way will a yoyo go on the ground) added.

1 Determination of direction of friction acting on the rolling object

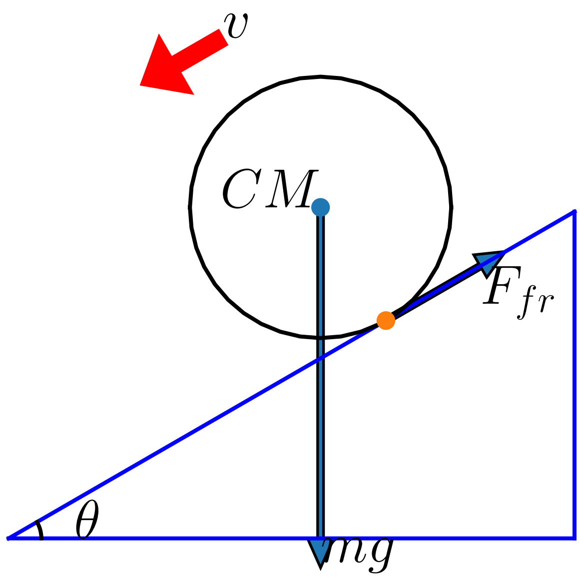
The rule of thumb is to separate the rotation motion about CM and translational motion and check for consistency. But there are always exceptions. For more advanced (usually the interesting ones are more advanced) cases not only do we need that, but also we need to make assumptions (can be multiple) and use elimination methods to find the real answer.

Another vital skill is to correctly write down the positive and negative signs of the variables when writing down the 2nd laws. You will see it's importance in the examples (especially the yoyo one where things get intriguing and peculiar). And several layers of elimination have to be made in some cases. You have to know to correctly set up the positive coordinate and arrive from vector equation to variable equation (where only the magnitude is concerned), both for linear and angular 2nd laws, and the linkage between them.

Rolling downhill Round object freely rolls down the hill.

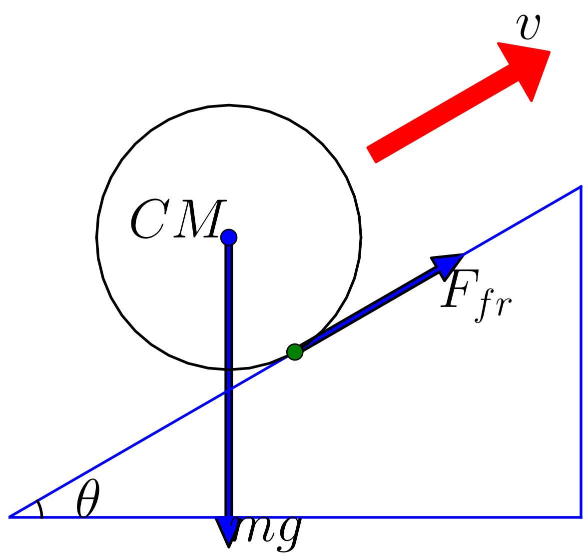


The only force that makes the object rotate counter-clockwise is friction so friction has to go up the hill. This friction force is exerted on the wheel by the slope.

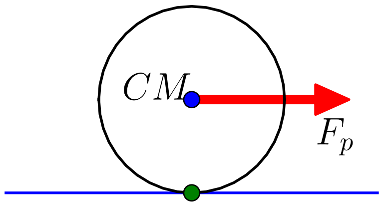

v
CM
F_{fr}
θ
mg

Kicked away uphill Object is forced to roll up the hill initially but external force is removed once the object is going upward. We are considering the later part of the motion when the external force is removed, so only gravitation is in place. The wheel is still rolling up the hill.

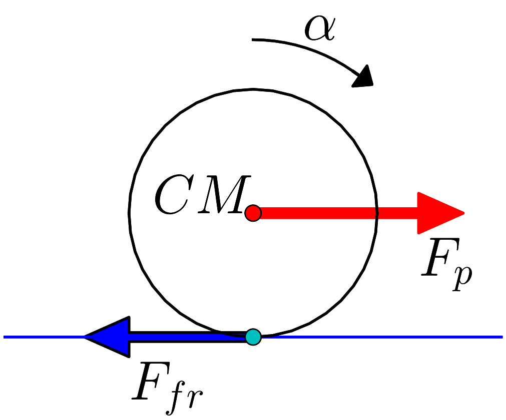
The rotation of the object slows down as it climbs up the hill. Friction is the only force that produces a torque to slow down the rotation. So it needs to go against the rotating direction. So the friction force acting on the wheel is up the hill.



External force F_p passing through CM point

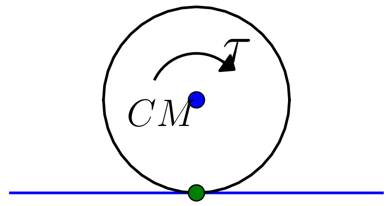


Friction is still the only force that produces an angular acceleration, so friction should have the same direction as angular acceleration. If the object is moving to the right and rotating faster, friction acting on the object goes towards left. If the object is moving to the left and rotation slow down because of the external force slowing it down, the friction force goes towards the left, to produce a torque and a angular acceleration to slow down the rotation.



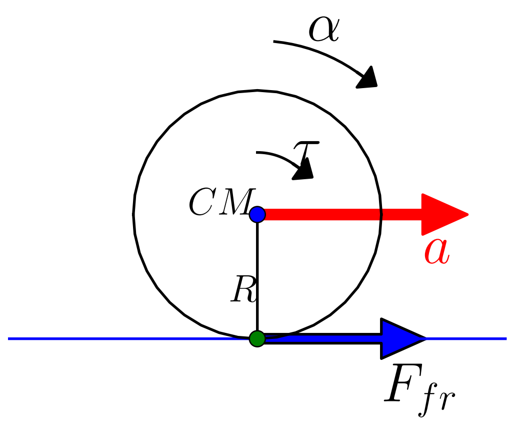
$$\begin{cases} F_p - F_{fr} = ma \\ F_{fr} \cdot R = I\alpha = I\frac{a}{R} \end{cases}$$

Car tire An external torque (not force) acting around the axial, like on car tire.



In this case, the rule of friction is not to cause rotation; the rotation about the center is done by the external torque. From 2nd law, in order to have no slipping, a translational force is needed to accelerate the CM of the object to move. Otherwise the object will slip. Friction in this case acts as this *force*, to give the CM of the object an acceleration to the right. One can judge from the relative movement between the object and the ground too. Friction prevents the object from skidding at the contact. So friction acting on the object goes to the right, while the mutual friction force acting on the ground goes to the left.

$$
\begin{cases} F_{fr} = ma \\ \tau - F_{fr}R = I\alpha = I\dfrac{a}{R} \end{cases}
$$
$$
\Rightarrow a = \frac{\tau}{mR + \frac{I}{R}}
$$

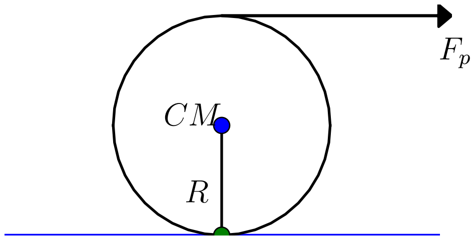


Application Together with the previous example, we now arrive at an exciting and somewhat surprising (to some outsiders maybe) conclusion. In a front-wheel-drive car (that's almost all cars), the front tires are the drive tires, meaning they are the tires that are exerted on by engine's torque. According to the example we just showed, these tires experience forward friction. The rear tires, however, are pulled forward by a force at the CM (the axile). They don't have the torque from the engine. So according to the previous example, these tires will experience backward friction. So the friction forces on the front and rear tires are opposite. At first sight, it may seem reasonable to say all four tires share the same direction of friction. But that's not correct.

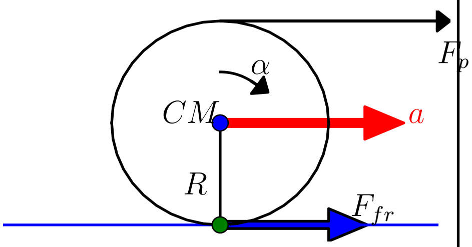
There is another interesting conclusion we can draw from the result of our calculation. In a car race, the start of the race is significant. A car driver wants to accelerate as fast as they can, but at the same time not to cause the tire to slip and lose grip. How to control the acceleration throttle and peddle is a hard problem. Well not so hard if you have studied this example. From the last equation we arrived, we know the acceleration of the tire a, which is essentially the same as the car's acceleration, is directly proportional to the torque τ. So before you may think you want to start with a smooth peddle control and put a little bit of throttle in the beginning so as not to lose grip. Wrong. Once you give the tire a torque τ, you acceleration a will be proportional to τ, what you want is the highest throttle right at the beginning (of course not so high that you will lose grip). It's hard. So next time in a formula race, when the red light off and green light lid, put down the highest throttle that will not cause slip immediately. Give it a try. Easier to say than done! Ideally, this is what you want to do. In reality, when you start, the weight will transfer from the front tires to the rear tires (because of the acceleration!), so there will be some delay before your rear tire get maximal downforce and maximal grip. And this is why you want to smoothen your throttle input. And drivers need to take this delay into account. But if your mechanics made your car real rigid, then this delay will be very short.

Yes even in a front-wheel-drive car there is weight transfer. As long as the CM of the car is not on the same level as the car axile, the car itself will tend to rotate and thus give rise to a weight transfer.

Rolling with hands on top External forces not passing through CM pivot. (so there are both external force and torque.)



Let assume friction acting on the wheel goes right, then



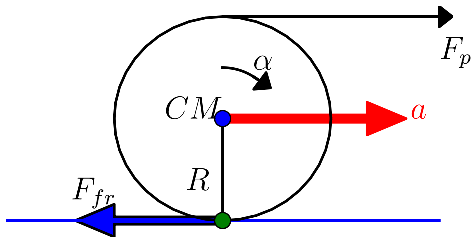
from 2nd law

$$\begin{cases} F_p + F_{fr} = ma \\ (F_p - F_{fr})\,R = I\alpha = I\dfrac{a}{R} \end{cases}$$

$$\Rightarrow F_{fr} = \frac{1}{2}\left(m - \frac{I}{R^2}\right)a$$

So if $mR^2 > I$, then F_{fr} goes to the right!

If frction acting on the wheel goes to the left,



$$\begin{cases} F_p - F_{fr} = ma \\ (F_p + F_{fr})\,R = I\alpha = I\dfrac{a}{R} \end{cases}$$

$$\Rightarrow F_{fr} = \frac{1}{2}\left(\frac{I}{R^2} - m\right)a$$

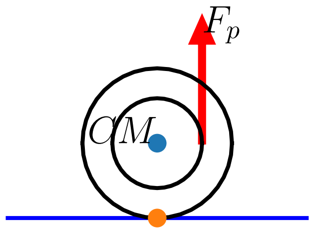
So if $mR^2 < I$, then F_{fr} goes to the left. But are there any objects that have $mR^2 < I$?

In this case the direction of friction depends on the condition $mR^2 > I$ or $mR^2 < I$. But in reality there is no object that can have $mR^2 < I$. So the second case is not physically meaningful. So we are left with the case of right-going friction. Friction always goes towards right. The only case the second senario will happen is when $mR^2 = I$, then friction force is zero. This is an interesting case worth meantioning. It means if we have a hoop object (all mass on the rim), then we don't need any frictional surface to cause the hoop to roll, if the force is applied on top of the hoop. This means we don't need to worry that the hoop will slip, even in the case when we can only provide low frictional coefficient surface (like on an ice surface). As long as your hand excert the force on top of the hoop, it will roll without slipping, even on ice.

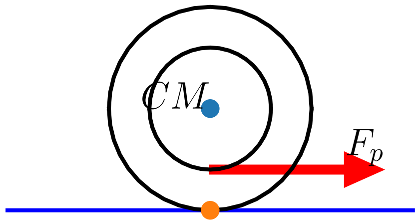
We can also recgonize in the first senario if $mR^2 < I$ then F_{fr} is negative, so the friction should be opposite to the positive direction we assume (to the right). This way we can save the time of working on the 2nd case.

Also in the first case, let's say if a is fixed, then the smaller the I, the bigger the F_{fr}, and the smaller the $F_p = \frac{1}{2}ma + \frac{I}{2R^2}a$. In other words, the smaller the inertia is, the lessor force we need to apply, which means its easier for us to make it roll. And vice versa.

Which way will the yoyo go? If we have a yoyo on the ground, and we pull upward on the string wrapped around the inner thud of the yoyo, it is apparent that the yoyo unrolls itself, and it rolls away from the place you pull them.

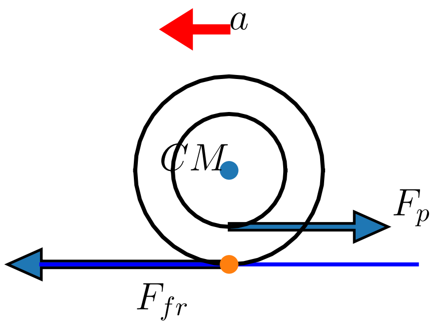


If we have a yoyo on the ground, and we pull horizontally on the string that's wrapped counter-clockwise around the inner thud of the yoyo, emerging from the bottom side, like the graph shown below, which way will the yoyo go?



This is counter intuitive at first sight. Because if the yoyo goes to the right, it should roll clockwise about its CM. But the torque from F_p about CM is counter-clockwise, which means rotation about CM is c.c. But rolling to right can't be c.c. It seems the yoyo will slip. Or the yoyo should go left? But there is no external force pointing to the left side. What's going on?

We can use elimination method to solve this problem. First we assume the yoyo will accelerate towards left,



Assume positive coordinate goes to the left, we can write down

$$F_{fr} - F_p = ma$$

(If you assume positive coordinate goes to the right, from vector force diagram this will give you $-F_{fr} + F_p = m(-a)$,which is the same as the above.)

This means F_{fr} must be larger than F_p. This is important because we will use it immediately.

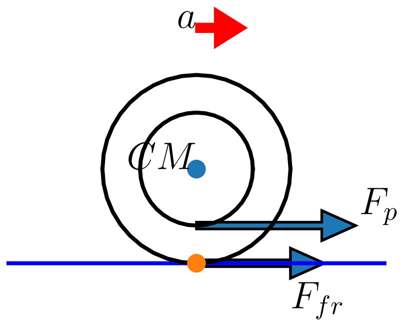
Now lets look at the rotational motion. Because $F_{fr} > F_p$, we bound to have $F_{fr} \times R > F_p \times r$, so that

$$F_{fr}R - F_p r = I\alpha > 0$$

This means the angular acceleration will have the same direction as the torque $F_{fr}R$, that means clockwise rotation about CM. However from the diagram for the yoyo to roll to the left, it needs counter-clockwise rotation. So this is not going to happen. Contradiction means our first assumption is not correct.

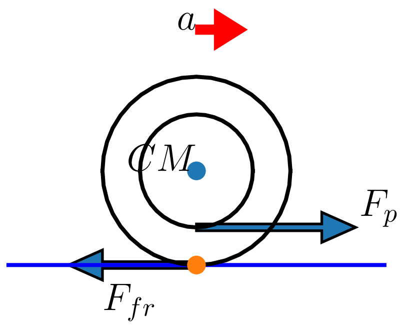
So the yoyo has to accelerate towards right. But we still aren't sure where the friction goes. So we need to make more assumptions and check consistency.

Let's say the friction goes to the right,



But in this case both forces create a torque that gives counter-clockwise rotation about CM, which will contradict the clockwise right-going rolling motion. So this is still not correct.

So the friction has to go towards the left.



So this is the only plausible way the reality can happen. Now we can setup the 2nd laws. Assume positive coordinate goes right. Because the roller goes to the right, $F_p - F_{fr} = ma > 0$. F_p is greater than F_{fr}. Although $F_p > F_{fr}$, we can't be sure that $F_p r > F_{fr}R$. So more assumptions and checkings are needed. If $F_p r > F_{fr}R$, then $F_p r - F_{fr}R > 0 = I\alpha$ will give a counter-clockwise rotation. This will contradict the right rolling motion with clockwise rotation. So the roller will slip if $F_p r > F_{fr}R$. So we are left with a quite stringent condition, that is $F_p r < F_{fr}R$ while $F_p > F_{fr}$. This is the only way the yoyo can successfully roll to the right without any slipage. Our 2nd laws are,

$$\begin{cases} F_p - F_{fr} = ma \\ (F_p \cdot r - F_{fr} \cdot R) = I\alpha = I\dfrac{a}{R} \end{cases}$$

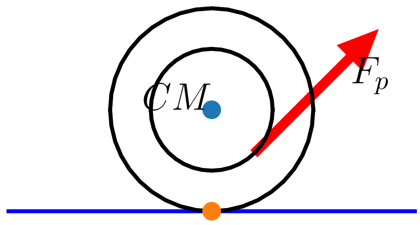
$$\Rightarrow F_{fr} = a\left[\frac{m + \frac{I}{Rr}}{\frac{R}{r} - 1}\right]$$

If $I = \frac{1}{2}mR^2$ and $r = \frac{R}{2}$, then $F_{fr} = ma\left(\frac{1+\frac{1}{2}\frac{R^2}{Rr}}{\frac{R}{r}-1}\right) = 2ma$, $F_p = ma + F_{fr} = 3ma$, this means when comparing with the previous case of F_p passing through the center of mass where the solution is $F_p = \frac{3}{2}ma$, we now need even larger external force to move the yoyo to the right.

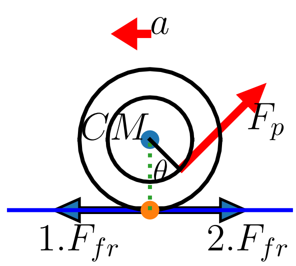
The stringent condition is not to be worried. If we can solve these equations successfully, the requirement of $F_p r < F_{fr} R$ while $F_p > F_{fr}$ will be automatically satisfied. It is interesting to note that when r gets bigger, the relative magnitude of F_{fr} of the solution when compared to F_p will gets bigger too, to satisfy the condition. It means more of you external force will be given to fight the friction, so it will be harder (more force needed) to pull to yoyo. In reality, you will feel strange why is it so hard to pull this yoyo towards you. Just ask a yoyo player! Because a huge amount of force is needed, compared to its mass times acceleration. It is like the thing(yoyo) weight several time than it appeared to be (eg the 3ma right above).

In summary, yoyo will always go to the right. So it is possible to horizontally retracting a yoyo on the ground. The force needed will surprise you.

Now let's go bunker. What if the setup is pretty much the same as the last question, but we change the angle of the string from horizontal to a slant angle that is θ degree from the vertical? This is a case between the first and the second problem setups. The two problems have opposite outcomes. What will happen inbetween?



First we setup our graph and force diagram. Let's assume the yoyo go towards left.



We will use the same logic to check consistency, and this time we will simply right the the formula. It should be clear by looking at the equations. We futher check wether friction goes left or right.

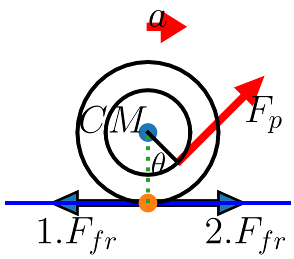
1. first case, if friction goes left

$$F_{fr} - F_p \cos\theta = ma > 0$$
$$F_p r - F_{fr} R = I \frac{a}{R} > 0$$

   for both equations to be greater than zero, $r - R\cos\theta > 0$. So if this condition holds, friction goes left.

2. second case, if friction goes right. This will not be possible because there is no net force going to the left.

Similarly, assume if yoyo goes right,



1. in the case of a left friction,

$$F_p \cos\theta - F_{fr} = ma > 0$$
$$F_{fr} R - F_p r = I \frac{a}{R} > 0$$

   so in this case it requires $r - R\cos\theta < 0$ for a left friction.

2. in the case of a right friction. This is not possible becuase there is no c.w. torque.

So the conclusion is firction always goes left!!! But if $r - R\cos\theta > 0$, the yoyo itself goes left. If $r - R\cos\theta < 0$, the yoyo goes right!

So you can do some magic trick now by varying the angle θ!

Differential gearing will not move? Require friction? Not directly related to our yoyo problem, but it's still surprising.

Application

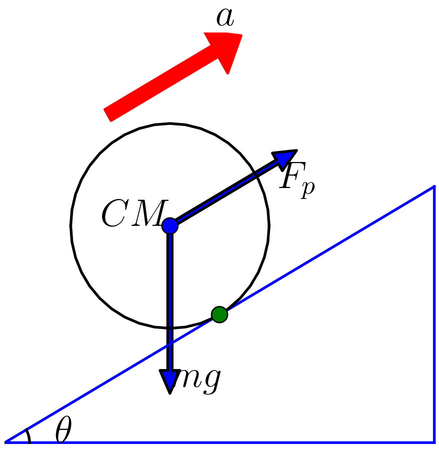
Next time if you want to reel your yoyo or reel a cable to a cable spool, you can try this method. With this method (cable coming out from under the thud), the spool will always follow you (cuz you are pulling it), as oppose to going away from you. If you use the method in the third example (where we push with hand on top) to reel a spool, you will need to drag the cable along with you (extra weight). So if you don't have a motor to reel the cable, you can try this.

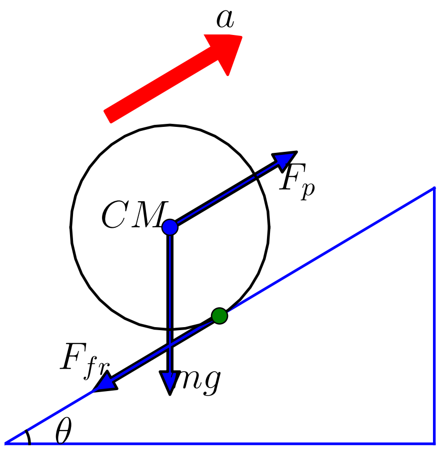
So yoyo player has this trick to retract a yoyo too.

2 Study Cases

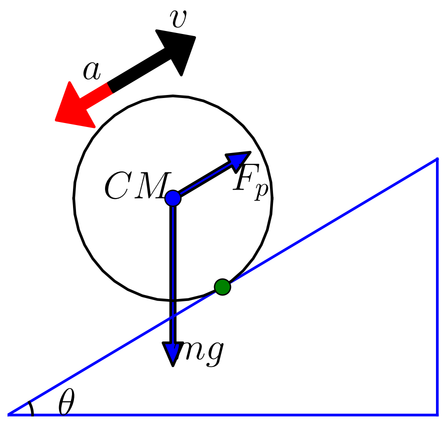
Determine the direction of friction force acting on the roller. (Assume rolling without slipping.)

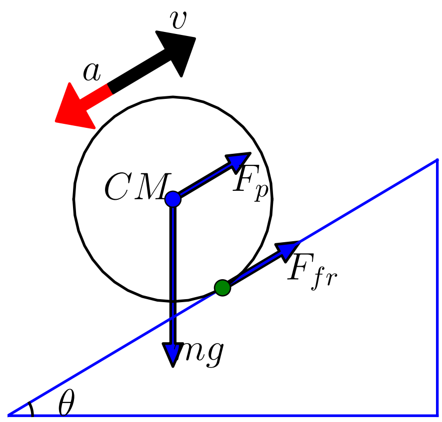
An applied force F_p passing through CM acts on the roller. The magnitude of F_p is larger than $mg\sin\theta$, so the roller is accelerating up the hill.



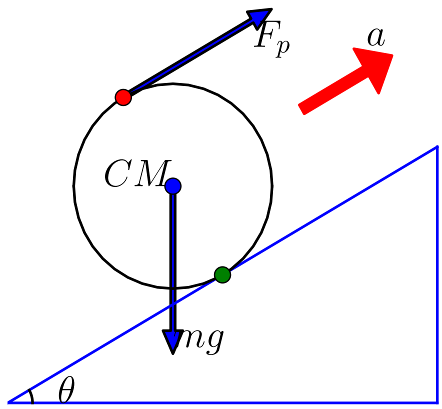


An applied force F_p passing through CM acts on the roller. The magnitude of F_p is smaller than $mg\sin\theta$. The roller is on its way to the highest point it can reach, and it is already slowing down.





An applied force F_p passing through the top edge of the roller, up the hill. The roller is accelerating up the hill. Determine the direction of friction acting on the roller.



Friction can go up or down the hill, depending on the condition. Similar to case 5.