SVOLGIMENTO APPELLO 11/06/2020 TRACCIA 1

n°1 $H_1 = \{$una vite viene prodotta dalla macchina 1$\}$

$H_2 = \{$una vite viene prodotta dalla macchina 2$\}$

$H_3 = \{$una vite viene prodotta dalla macchina 3$\}$

$E = \{$la vite è difettosa$\}$

$P(H_1) = \frac{50}{100} = 0.5$, $P(H_2) = \frac{30}{100} = 0.3$ $P(H_3) = \frac{20}{100} = 0.2$

$$P(E) \overset{TOTALE}{=} P(E|H_1)P(H_1) + P(E|H_2)P(H_2) + P(E|H_3)P(H_3) =$$

$$= 0.08 \times 0.5 + 0.06 \times 0.3 + 0.05 \times 0.2 = 0.068$$

$$P(H_2|E) \overset{BAYES}{=} \frac{P(E|H_2)P(H_2)}{P(E)} = \frac{0.06 \times 0.3}{0.068} = \boxed{0.2647}$$

$\boxed{n°2}$ $\quad X_1 \sim P(2)$, $\quad X_2 \sim P(3)$ $\quad$ indipendenti

a) $$P(X_1=3,\ X_2=5) \overset{\text{indipendenti}}{=} P(X_1=3)\, P(X_2=5) =$$

$$= e^{-2} \cdot \frac{2^3}{3!} \cdot e^{-3} \frac{3^5}{5!} = e^{-5} \cdot \frac{8 \cdot 243}{6 \cdot 120} = \boxed{0.01819}$$

$$= \frac{27}{10} e^{5}$$

b) $P(X_1 + X_2 = 1)$ $\qquad X_1 + X_2 \sim P(2+3)$ [Somma di due v.a. POISSON INDIPEND]

$$X_1 + X_2 \sim P(5)$$

$$P(X_1+X_2=1) = e^{-5} \cdot \frac{5^1}{1!} = 5 \times e^{-5} = \boxed{0.03369}$$

$\boxed{n°3}$ $\qquad f(x) = kx^2 \qquad 0<x<4$

$$f(x) \geq 0 \iff k \geq 0$$

$$\int_0^4 f(x)\,dx = 1 \implies k\int_0^4 x^2\,dx = 1 \implies k \cdot \left[\frac{x^3}{3}\right]_0^4 = 1$$

$$k \cdot \frac{64}{3} = 1 \implies \boxed{k = \frac{3}{64}}$$ accettabile

$$E(X) = \int_0^4 x\, f(x)\,dx = \frac{3}{64}\int_0^4 x^3\,dx = \frac{3}{64}\left[\frac{x^4}{4}\right]_0^4 = \frac{3}{64} \cdot \frac{256}{4} =$$

$$= \boxed{3}$$

$$V(X) = E(X^2) - E(X)^2$$

$$E(X^2) = \int_0^4 x^2 f(x)\,dx = \frac{3}{64}\int_0^4 x^4\,dx = \frac{3}{64}\left[\frac{X^5}{5}\right]_0^4 =$$

$$= \frac{3}{64}\cdot\frac{1024}{5} = \frac{48}{5}$$

$$V(X) = \frac{48}{5} - 9 = \boxed{\frac{3}{5}}$$

$\boxed{n°4}$ $X \sim N(\mu, \sigma^2)$ $\qquad n = 10$

a) $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{10}\left(3.1 + 3.3 + \dots + 3.3\right) = \boxed{3.58} = \bar{X}$

b) $1-\alpha = 0.95$; $\alpha = 0.05$; $\frac{\alpha}{2} = 0.025$; $1-\frac{\alpha}{2} = 0.975$

IF per μ con varianza incognita

$$\left[\bar{X} - \frac{t_{1-\frac{\alpha}{2}, n-1}\, S}{\sqrt{n}} ; \bar{x} + \frac{t_{1-\frac{\alpha}{2}, n-1}\, S}{\sqrt{n}}\right]$$

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2} = \sqrt{\frac{1}{9}\sum_{i=1}^{n}(X_i - \bar{X})^2} = 0.5116$$

$t_{0.975,9} = 2.26216$

$$IF\left[3.58 - \frac{2.26 \times 0.512}{\sqrt{10}} ; 3.58 + \frac{2.26 \times 0.512}{\sqrt{10}}\right] =$$

$$= \left[3.58 - 0.366 ; 3.58 + 0.366\right] =$$

$$= \left[3.214 ; 3.946\right]$$

$\boxed{n°5}$ $X = (X_1, \dots, X_n)$; $X \sim N(\mu, \sigma^2)$ μ, σ^2 incog

$\alpha \in (0,1)$ fissato $H_0: \sigma^2 = \sigma_0^2$ v $H_1: \sigma^2 \neq \sigma_0^2$

$(n-1)\frac{S^2}{\sigma^2} \sim \chi^2_{n-1}$; se H_0 è vera $\frac{S^2}{\sigma_0^2}(n-1) \sim \chi^2_{n-1}$

E' ragionevole accettare H_0 quando $\left|\frac{S^2}{\sigma_0^2}\right| \le c$ per c opportuno

Quindi rifiuto H_0 se $\left|\frac{S^2}{\sigma_0^2}\right| > c$.

$$\alpha = P(H_0 \text{ si rifiuta} \mid H_0 \text{ vera}) = P\left(\left|\frac{S^2}{\sigma_0^2}\right| > c \,\middle|\, \sigma^2 = \sigma_0^2\right)$$

$$= P_{H_0}\left(\left|(n-1)\frac{S^2}{\sigma_0^2}\right| > c(n-1)\right)$$

Per una proprietà dei quantili χ^2 si ha:

$\exists!$ $\chi^2_{\frac{\alpha}{2}, n-1}$; $\chi^2_{1-\frac{\alpha}{2}, n-1}$ t.c.

$$1-\alpha = P\left(\chi^2_{\frac{\alpha}{2}, n-1} \le (n-1)\frac{S^2}{\sigma_0^2} \le \chi^2_{1-\frac{\alpha}{2}, n-1}\right)$$

Fk

passando al complementare

$$\alpha = P\left(\left\{(n-1)\frac{S^2}{\sigma_0^2} < \chi^2_{\frac{\alpha}{2}, n-1}\right\} \cup \left\{(n-1)\frac{S^2}{\sigma_0^2} > \chi^2_{1-\frac{\alpha}{2}, n-1}\right\}\right)$$

$$= P\left((n-1)\frac{S^2}{\sigma_0^2} < \chi^2_{\frac{\alpha}{2}, n-1}\right) + P\left\{(n-1)\frac{S^2}{\sigma_0^2} > \chi^2_{1-\frac{\alpha}{2}, n-1}\right\}$$

Le regioni critiche di rifiuto di H_0 sono:

$$\left\{(n-1)\frac{S^2}{\sigma_0^2} > \chi^2_{1-\frac{\alpha}{2}, n-1}\right\} \cup \left\{(n-1)\frac{S^2}{\sigma_0^2} < \chi^2_{\frac{\alpha}{2}, n-1}\right\}$$

Quindi:

$$\begin{cases} \text{Se } \chi^2_{\frac{\alpha}{2}, n-1} \leq (n-1)\dfrac{S^2}{\sigma_0^2} \leq \chi^2_{1-\frac{\alpha}{2}, n-1} & \text{SI ACCETTA } H_0 \\ \text{Se } (n-1)\dfrac{S^2}{\sigma_0^2} > \chi^2_{1-\frac{\alpha}{2}, n-1} & \text{SI RIFIUTA } H_0 \\ \text{oppure } (n-1)\dfrac{S^2}{\sigma_0^2} < \chi^2_{\frac{\alpha}{2}, n-1} & \end{cases}$$