Appello del 13-07-2020 Traccia 3

N°1

$$f(t) = \begin{cases} \vartheta\, t^{\vartheta-1} & \text{se } 0 < t \le 1 \\ 0 & \text{altrimenti} \end{cases}$$

$\vartheta > 0$

a) FdD = ?

$F_X(x) = P(X \le x) = \vartheta \int_0^x t^{\vartheta-1} dt = \left[t^\vartheta\right]_0^x = x^\vartheta$ se $0 < x < 1$

$F_X(x) = 0$ se $x \le 0$

$F_X(x) = 1$ se $x \ge 1$

b) $P(X \ge 3)$? e $P(X \le \frac{1}{3})$?

$P(X \ge 3) = 1 - P(X < 3) = 1 - F_X(3) =$

$= 1 - 1 = 0$

$P(X \le \frac{1}{3}) = F_X\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^\vartheta$

c) $Y = -\log X$ $\quad Y \sim ?$

Calcoliamo la FdD di Y:

$F_Y(y) = P(Y \le y) = P(-\log X \le y) =$

$$P(\log \cancel{X} \geq -y) = P(X \geq e^{-y}) = 1 - P(X \leq e^{-y}) =$$

$$= 1 - F_X(e^{-y}) = 1 - e^{-\theta y} \quad \underline{\text{se} \quad y > 0}$$

$$F_Y(y) = 0 \quad \text{se} \quad y \leq 0$$

Quindi $Y \sim exp(\theta)$

d) $$E(X) = \theta \int_0^1 t \cdot t^{\theta - 1} dt = \theta \int_0^1 t^{\theta} dt = \left[\frac{\theta t^{\theta+1}}{\theta + 1} \right]_0^1 =$$

$$= \frac{\theta}{\theta + 1};$$

$$E(X^2) = \theta \int_0^1 t^2 \cdot t^{\theta - 1} dt = \theta \int_0^1 t^{\theta+1} dt = \left[\theta \cdot \frac{t^{\theta+2}}{\theta + 2} \right]_0^1 =$$

$$= \frac{\theta}{\theta + 2}$$

$$V(X) = E(X^2) - E(X)^2 = \frac{\theta}{\theta+2} - \left(\frac{\theta}{\theta+1} \right)^2 =$$

$$= \frac{\theta}{\theta+2} - \frac{\theta^2}{(\theta+1)^2} = \frac{\theta(\theta^2 + 2\theta + 1) - \theta^2(\theta+2)}{(\theta+1)^2(\theta+2)} =$$

$$= \frac{\theta^3 + 2\theta^2 + \theta - \theta^3 - 2\theta^2}{(\theta+1)^2(\theta+2)} = \boxed{\frac{\theta}{(\theta+1)^2(\theta+2)}}$$

$\boxed{n^\circ 2}$ $n=500$

difetti	0	1	2	3	4
pezzi	225	183	64	23	5

$m=5$

$\alpha=0.05$ $\qquad H_0: X \sim P(\lambda)$

a) Uno stimatore (MV) per λ è dato da:

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{500}\sum_{i=1}^{5} x_i f_i = \frac{400}{500} = 0.8$$

dove f_i sono le frequenze;

b) $p_X(k) = P_X(k) = \dfrac{e^{-0.8}\cdot 0.8^k}{k!}$

$p_0 = P(X=0) = e^{-0.8} = 0.4493$; $np_0 = 224,65$

$p_1 = P(X=1) = e^{-0.8}\times 0.8 = 0.3594$; $np_1 = 179.7$

$p_2 = P(X=2) = e^{-0.8}\times\dfrac{0.8^2}{2} = 0.1438$; $np_2 = 71.9$

$p_3 = P(X=3) = e^{-0.8}\times\dfrac{0.8^3}{6} = 0.0383$; $np_3 = 19.15$

$p_4 = P(X=4) = e^{-0.8}\times\dfrac{0.8^4}{24} = 0.0072$; $np_4 = 3.6 < 5$

Devo unificare le ultime due classi.

$\tilde{p}_3 = p_3 + p_4 = 0.046$

$n\tilde{p}_3 = 500 \times 0.046 = 23 > 5$

$\tilde{N}_3 = N_3 + N_4 = 28$

$\bar{m} = 4$

$$D_0 = \sum_{j=1}^{4} \frac{(N_j - np_j)^2}{np_j} =$$

$$= \frac{(225 - 224,65)^2}{224,65} + \frac{(183 - 178.7)^2}{178.7} +$$

$$+ \frac{(64 - 71.9)^2}{71.9} + \frac{(28 - 23)^2}{23} =$$

$$= 0,00055 + 0.061 + 0.87 + 1.087 =$$

$$= 2.02$$

$q = 1 =$ n° parametri stimati

$$\begin{cases} \text{se } D_0 \le \chi^2_{1-\alpha, \bar{m}-q-1} & \text{si accetta } H_0 \\ \text{se } D_0 > \chi^2_{1-\alpha, \bar{m}-q-1} & \text{si rifiuta } H_0 \end{cases}$$

$\chi^2_{0.95, 2} = 5.99$

$D_0 = 2.02 < 5.99 \Rightarrow$ si accetta H_0

n°3 $X \sim U(a, b)$ con $a, b \in \mathbb{R}$, $a < b$

Lezione 9 (8-4-2020)

file "Valore atteso, varianza e covarianza"

pag 17-18.