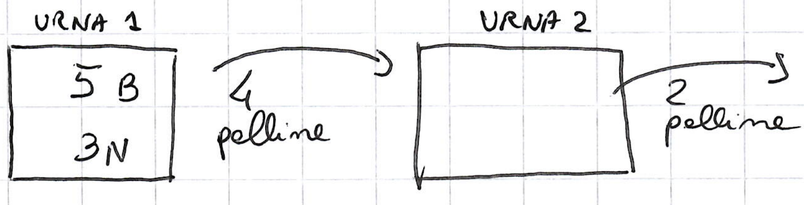
Appello di Calcolo delle Probabilità e Statistica

11/05/2020

n°1 URNA 1: 5 B 3 N

URNA 2: VUOTA



a) $A := \{$ le due palline estratte dalla 2ª urna sono nere $\}$

$P(A)$?

$E_j = \{$ ci sono j palline bianche tra le 4 estratte dall'urna 1 $\}$ $\quad j = 1, 2, 3, 4$

Estraggo 4 palline dall'urna 1:

$$\Omega = \{(B,B,B,B), (B,B,B,N), (B,B,N,N), (B,N,N,N)\}$$

$$\Omega = \bigcup_{j=1}^{4} E_j. \qquad E_i \cap E_j = \emptyset \quad \forall i \neq j, \quad i,j = 1 \dots 4$$

$$P(E_j) > 0 \quad \forall j$$

$$P(A) \overset{\text{TOTALE}}{=} \sum_{j=1}^{4} P(A|E_j)\, P(E_j)$$

$$P(E_1) = \frac{\binom{5}{1}\binom{3}{3}}{\binom{8}{4}} = \frac{5}{70} = \frac{1}{14}$$

$$P(E_2) = \frac{\binom{5}{2}\binom{3}{2}}{\binom{8}{4}} = \frac{10 \cdot 3}{70} = \frac{3}{7}$$

$$P(E_3) = \frac{\binom{5}{3}\binom{3}{1}}{\binom{8}{4}} = \frac{10 \cdot 3}{70} = \frac{3}{7}$$

$$P(E_4) = \frac{\binom{5}{4}\binom{3}{0}}{\binom{8}{4}} = \frac{5}{70} = \frac{1}{14}$$

URNA 2 DATO E_1

1 B
3 N

$$P(A|E_1) = \frac{\binom{3}{2}\binom{1}{0}}{\binom{4}{2}} = \frac{3}{6} = \frac{1}{2}$$

URNA 2 DATO E_2

2 B
2 N

$$P(A|E_2) = \frac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}} = \frac{1}{6}$$

URNA 2 DATO E_3

3 B
1 N

$$P(A|E_3) = 0$$

URNA 2 DATO E_4

4 B

$$P(A|E_4) = 0$$

$$P(A) = \sum_{j=1}^{4} P(A|E_j)\,P(E_j) =$$

$$= \frac{1}{2} \cdot \frac{1}{14} + \frac{1}{6} \cdot \frac{3}{7} = \frac{1}{28} + \frac{1}{14} = \frac{3}{28} = 0{,}107$$

b) $B = \{$le due palline estratte dalla II urna sono bianche$\}$

$$P(B) = \sum_{j=1}^{4} P(B|E_j)\,P(E_j)$$

$$P(B|E_1) = 0$$

$$P(B|E_2) = \frac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}} = \frac{1}{6}$$

$$P(B|E_3) = \frac{\binom{3}{2}\binom{1}{0}}{\binom{4}{2}} = \frac{3}{6} = \frac{1}{2}$$

$$P(B|E_4) = 1$$

$$P(B) = \frac{1}{6}\cdot\frac{3}{7} + \frac{1}{2}\cdot\frac{3}{7} + 1\cdot\frac{1}{14} = \frac{1}{14} + \frac{3}{14} + \frac{1}{14} = \boxed{\frac{5}{14}}$$

c) $C = \{$una pallina è nera e l'altra è bianca$\}$

$$P(C) = 1 - P(A) - P(B) = 1 - \frac{3}{28} - \frac{5}{14} = \frac{28-3-10}{28} = \boxed{\frac{15}{28}}$$

Infine:

$$P(E_4|B) = ?$$

$$P(E_4|B) = (\text{BAYES}) = \frac{P(B|E_4)\cdot P(E_4)}{P(B)} = \frac{1\cdot\frac{1}{14}}{\frac{5}{14}} = \boxed{\frac{1}{5}}$$

$\boxed{n°2}$ $X \sim N(1,1)$ $Y \sim N(2,3)$ indip.

a) $Z = 2X+1$

$$E(Z) = E(2X+1) = 2E(X)+1 = 2\cdot 1+1 = 3$$

$$V(Z) = V(2X+1) = 4V(X) = 4\cdot 1 = 4$$

$$\boxed{Z \sim N(3,4)}$$

b) $E((Z-Y)^2) = E(Z^2 - 2ZY + Y^2) =$

$$= E(Z^2) - 2E(ZY) + E(Y^2) = \left\{ \begin{array}{c} Z \text{ e } Y \text{ sono} \\ \text{indipendenti} \end{array} \right\}$$

$$= V(Z) + E(Z)^2 - 2E(Z)E(Y) + V(Y) + E(Y)^2 =$$

$$= 4 + 9 - 2\cdot 3\cdot 2 + 3 + 4 = \boxed{8};$$

c) $P(Z<2) = ?$

$$P(Z<2) = P\left(\frac{Z-3}{2} < \frac{2-3}{2}\right) = P\left(\frac{Z-3}{2} < -\frac{1}{2}\right)$$

$$= \left\{ \frac{Z-3}{2} \sim N(0,1) \right\} = \Phi\left(-\frac{1}{2}\right) =$$

$$= 1 - \Phi\left(\frac{1}{2}\right) = 1 - 0,69146 = \boxed{0,3085}$$

$\boxed{n°3}$ $\quad n=10 \qquad X \sim N(\mu,\sigma^2)$

a) IF al 99% per σ:

$1-\alpha = 0.99 \quad ; \quad \alpha = 0.01 \quad ; \quad \frac{\alpha}{2} = 0{,}005$

$1-\frac{\alpha}{2} = 0.995$

IF per σ^2 di livello $1-\alpha = 0.99$:

$$\left[\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2},n-1}} ; \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2},n-1}}\right]$$

$n-1=9$

$$S^2 = \frac{1}{9}\sum_{i=1}^{10}(x_i-\bar{x})^2$$

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = 144.3$$

$$S^2 = \frac{1}{9}\left[(140-144.3)^2+(136-144.3)^2+\dots+(151-144.3)^2\right]$$

$$= \frac{1}{9} \times 290{,}1 = 32{,}233$$

$(n-1)S^2 = 9 \cdot S^2 = 290{,}1$

$\chi^2_{0{,}995,9} = 23.589$

$\chi^2_{0.005,9} = 1.735$

$$\underset{\text{per } \sigma^2}{IF} = \left[\frac{230,1}{23.589}, \frac{230,1}{1.735}\right] : \left[12.238\ ;\ 167.205\right]\ ;$$

IF per σ = $\boxed{[3.507;\ 12.931]}$

b) $\alpha = 0.05$

$H_0: \mu = 143 \quad \vee \quad H_1: \mu \neq 143$

Test bilaterale sulla media nel caso di varianza non nota

$$\begin{cases} \text{se} \left|\frac{(\bar{X}-\mu_0)\sqrt{n}}{S}\right| \leq t_{1-\frac{\alpha}{2}, n-1} & \text{si accetta } H_0 \\ \text{se} \left|\frac{(\bar{X}-\mu_0)\sqrt{n}}{S}\right| > t_{1-\frac{\alpha}{2}, n-1} & \text{si rifiuta } H_0 \end{cases}$$

$$\frac{(\bar{x}-\mu_0)\sqrt{n}}{S} = \frac{(144.3-143)\sqrt{10}}{\sqrt{32,233}} = 0.724$$

$\alpha = 0.05 \qquad \frac{\alpha}{2} = 0.025 \qquad 1-\frac{\alpha}{2} = 0.975$

$t_{1-\frac{\alpha}{2}, n-1} = t_{0.975, 9} = 2.262$

Poiché $0.724 < 2.262$ si accetta H_0.