n° 1

n° 1 $X = \{$v.a. che rappresenta il numero di chiamate sul telefono personale$\}$

$Y = \{$v.a. che rappresenta il numero di chiamate sul telefono di lavoro$\}$

$X \sim P(3) \qquad Y \sim P(6)$

a) $P(Y=0) = e^{-6} \cdot \frac{6^0}{0!} = \boxed{e^{-6}}$

b) $P(X+Y \leq 3) = \; ?$

$X+Y =$ n° di telefonate ricevute dai due telefoni.

$X+Y \sim P(3+6) \Rightarrow X+Y \sim P(9)$

$$P(X+Y\leq 3) = P(\{X+Y=0\} \cup \{X+Y=1\} \cup \{X+Y=2\} \cup \{X+Y=3\}) =$$

$$= P(\{X+Y=0\}) + P(\{X+Y=1\}) + P(\{X+Y=2\}) + P(\{X+Y=3\})$$

$$= e^{-9}\cdot\frac{9^0}{0!} + e^{-9}\cdot\frac{9}{1!} + e^{-9}\cdot\frac{9^2}{2!} + e^{-9}\cdot\frac{9^3}{3!} =$$

$$= e^{-9}\left(1+9+\frac{81}{2}+\frac{243}{2}\right) =$$

$$= e^{-9}\cdot\frac{344}{2} = \boxed{172\cdot e^{-9}}$$

c) Sia $p_{X|X+Y}(\cdot|n)$ la probabilità di X dato $X+Y=n$.

Se $0 \leq k \leq n$

$$p_{X|X+Y}(k|n) = \frac{P(X=k, X+Y=n)}{P(X+Y=n)} =$$

$$= \frac{P(X=k, Y=n-k)}{P(X+Y=n)} =$$

$$= \frac{P(X=k)\,P(Y=n-k)}{P(X+Y=n)} =$$

$$= \frac{e^{-3} \cdot \frac{3^k}{k!} \cdot e^{-6} \cdot \frac{6^{n-k}}{(n-k)!}}{e^{-9} \cdot \frac{9^n}{n!}} =$$

$$= \frac{3^k \cdot 6^{n-k}}{9^n} \cdot \frac{n!}{k!(n-k)!} = \binom{n}{k} \left(\frac{3}{9}\right)^k \cdot \left(\frac{6}{9}\right)^{n-k}$$

$$= \binom{n}{k} \left(\frac{3}{9}\right)^k \left(1 - \frac{3}{9}\right)^{n-k}$$

binomiale $b\left(n, \frac{1}{3}\right)$

La media vale $n \cdot \frac{1}{3}$;

$\boxed{n^{\circ}2}$

$$f(x) = \begin{cases} \frac{x+2}{2} & x \in [-2,-1] \\ 1/2 & x \in [-1,1] \\ \frac{(2-x)}{2} & x \in [1,2] \\ 0 & \text{altrimenti.} \end{cases}$$

a) $E(x) = \int_{-\infty}^{+\infty} x f(x)\, dx =$

$$= \int_{-2}^{-1} x \frac{(x+2)}{2}\, dx + \int_{-1}^{1} \frac{1}{2} x\, dx + \int_{1}^{2} x \frac{(2-x)}{2}\, dx =$$

$$= \int_{-2}^{-1} \left(\frac{x^2}{2} + x\right) dx + \frac{1}{2}\left[\frac{x^2}{2}\right]_{-1}^{1} + \int_{1}^{2} \left(x - \frac{x^2}{2}\right) dx =$$

$$= \left[\frac{x^3}{6} + \frac{x^2}{2}\right]_{-2}^{1} + \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\right) + \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_{1}^{2} =$$

$$= \frac{1}{6} + \frac{1}{2} + \frac{8}{6} - 2 + 2 - \frac{8}{6} - \frac{1}{2} - \frac{1}{6} = 0$$

b) $V(X) = E[(X-E(X))^2] =$

$$= \int_{-\infty}^{+\infty} x^2 f(x)\,dx =$$

$$= \int_{-2}^{-1} x^2 \frac{(x+2)}{2}\,dx + \int_{-1}^{1} \frac{1}{2} \cdot x^2\,dx +$$

$$+ \int_{1}^{2} x^2 \frac{(2-x)}{2}\,dx =$$

$$= \left[\frac{x^4}{8} + \frac{x^3}{3}\right]_{-2}^{-1} + \frac{1}{2}\left[\frac{x^3}{3}\right]_{-1}^{1} +$$

$$+ \left[\frac{x^3}{3} - \frac{x^4}{8}\right]_{1}^{2} =$$

$$= \frac{1}{8} - \frac{1}{3} - 2 + \frac{8}{3} + \frac{1}{6} + \frac{1}{6} + \frac{8}{3} - 2$$

$$- \frac{1}{3} + \frac{1}{8} =$$

$$= \frac{3-8-48+64+ - +64-48-8+3}{24} = \frac{30}{24} = \boxed{\frac{5}{4}}$$

$\boxed{n^\circ 3}$ $n=10$ $X \sim N(\mu, 4)$

Intervallo di fiducia per la media

VARIANZA NOTA

$$\left[\bar{X} - \frac{z_{1-\frac{\alpha}{2}} \cdot \sigma}{\sqrt{n}} ; \bar{X} + \frac{z_{1-\frac{\alpha}{2}} \cdot \sigma}{\sqrt{n}} \right]$$

$1-\alpha = 0.95$ $\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$ $1 - \frac{\alpha}{2} = 0.975$

$z_{0,975} = 1.96$; $\sigma = 2$

$\bar{X} = 56,8$

I.F.: $[56,8 - 1,2396 ; 56,8 + 1,2396] =$

$= [55,5604 ; 58,0396]$

n°4

$$f(x;\theta) = \theta x^{\theta-1}, \quad 0<x<1, \quad \theta>0$$

FUNZIONE DI VEROSIMIGLIANZA:

$$L(\theta) = f_\theta(x_1) \cdots f_\theta(x_n) =$$

$$= \theta x_1^{\theta-1} \cdots \theta x_n^{\theta-1} =$$

$$= \theta^n x_1^{\theta-1} \cdots x_n^{\theta-1} =$$

$$= \theta^n (x_1 \cdots x_n)^{\theta-1}$$

$$\ln L(\theta) = \ln\left(\theta^n (x_1 \cdots x_n)^{\theta-1}\right) =$$

$$= \ln \theta^n + \ln (x_1 \cdots x_n)^{\theta-1} =$$

$$= n \ln \theta + (\theta-1) \ln(x_1 \cdots x_n) =$$

$$= n \ln \theta + (\theta-1) \sum_{i=1}^{n} \ln(x_i) =$$

$$= n \ln \theta + \theta \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \ln(x_i),$$

$$\frac{d}{d\theta} \ln L(\theta) \geq 0$$

$$\frac{d}{d\theta}\left(n \ln \theta + \theta \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \ln x_i\right) =$$

$$= \frac{n}{\theta} + \sum_{i=1}^{n} \ln x_i \geq 0$$

$$n + \theta \sum_{i=1}^{n} \ln x_i \geq 0$$

$$\hat{\theta} = \frac{-n}{\sum_{i=1}^{n} \ln x_i} = \frac{-n}{\ln\left(\prod_{i=1}^{n} x_i\right)}$$