APPELLO 13-07-2020 SVOLGIMENTO TRACCIA 2

$\boxed{n^\circ 1}$

$X \sim U(0,1)$

a) $Y = -\frac{1}{\lambda} \log X$ $\quad Y \sim ?$

b) $Z = \sqrt[3]{-\frac{1}{\lambda} \log X}$ $\quad Z \sim ?$ $\quad \lambda > 0$

a) Calcoliamo la F.d.D di Y

$$F_Y(t) = P(Y \leq t) = P\left(-\frac{1}{\lambda} \log X \leq t\right) =$$

$$= P\left(\frac{1}{\lambda} \log X \geq -t\right) = P(\log X \geq -\lambda t) =$$

$$= P\left(X \geq e^{-\lambda t}\right) = 1 - P\left(X \leq e^{-\lambda t}\right) =$$

$$= 1 - F_X\left(e^{-\lambda t}\right) = 1 - \left(\frac{e^{-\lambda t} - 0}{1-0}\right) =$$

$$= 1 - e^{-\lambda t}$$ perché $0 < e^{-\lambda t} < 1$, in quanto $\lambda > 0$

Quindi $Y \sim \exp(\lambda)$

b) $Z = \sqrt[3]{Y}$, perché Y assume valori positivi con probabilità 1

F1

Per $t \geq 0$, $F_Z(t) = P(Z \leq t) = P(\sqrt[3]{Y} \leq t) =$

$$= P(Y \leq t^3) = 1 - e^{-\lambda t^3}$$

Quindi la densità vale

$$f_Z(t) = -e^{-\lambda t^3} \cdot (-\lambda \cdot 3t^2) = 3\lambda t^2 e^{-\lambda t^3}$$

per $t > 0$ e $f_Z(t) = 0$ per $t < 0$.

e) $E(Y) = \dfrac{1}{\lambda}$; $V(Y) = \dfrac{1}{\lambda^2}$.

perché $Y \sim \exp(\lambda)$

n°2

$X \sim N(1,1)$ $\quad n=40 \quad \alpha = 0.05$

j	1 $(X\le 0)$	2 $(0<X\le 1)$	3 $(1<X\le 2)$	4 $(X>2)$
N_j	7	13	12	8

$$p_1 = P(\{X\le 0\}) = P\left(\left\{\underbrace{\frac{X-1}{1}}_{\sim N(0,1)} \le -1\right\}\right) =$$

$$= \Phi(-1) = 1-\Phi(1) = 1-0.841 = 0.159$$

$$p_2 = P(\{0 < X \le 1\}) = P(\{-1 < X-1 \le 0\}) =$$

$$= \Phi(0) - \Phi(1) = 0.341$$

$$p_3 = P(\{1 < X \le 2\}) = P(\{0 < X-1 \le 1\}) =$$

$$= \Phi(1) - \Phi(0) = 0.341$$

$$p_4 = P\{2 < X\} = P\{1 < X-1\} = 1 - P\{X-1 \le 1\}$$

$$= 1 - \Phi(1) \simeq 0.159$$

$n p_1 = 6.36$

$n p_2 = 13.64$

$n p_3 = 13.64$

$n p_4 = 6.36$

Quindi $n p_j > 5 \quad \forall j$

$$D_0 = \sum_{j=1}^{m} \frac{(N_j - np_j)^2}{np_j} \qquad m=4$$

$$D_0 = \frac{(7-6.36)^2}{6.36} + \frac{(13-13.64)^2}{13.64} + \frac{(12-13.64)^2}{13.64} +$$

$$+ \frac{(8-6.36)^2}{6.36} =$$

$$= 0.064 + 0.03 + 0.197 + 0.423 = 0.714$$

$$\begin{cases} \text{Se } D_0 \le \chi^2_{1-\alpha, m-1} & \text{si ACCETTA } H_0 \\ \text{Se } D_0 > \chi^2_{1-\alpha, m-1} & \text{si RIFIUTA } H_0 \end{cases}$$

$$\chi^2_{0.95,3} = 7.81$$

Poiché $D_0 = 0.714 < 7.81$ si accetta H_0

n°3 lezione 9 (8-4-2020)

file "Valore atteso, varianza e covarianza" pag 14 e 15.