

Introduction to Functional Programming

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Overview

- 1 Records
- 2 Arrays
- 3 Algebraic types - trees
- 4 Abstract Data Types
- 5 Bag as ADT



Records

```
:: Person = { name :: String
               , birthdate :: (Int,Int,Int)
               , fpprogramer :: Bool
             }
```

```
IsfpUser :: Person → String
IsfpUser {fpprogramer = True} = "Yes"
IsfpUser _                    = "No"
```

```
Start = IsfpUser { name = "Me"
                  , birthdate = (1,1,1999)
                  , fpprogramer = True}    // "Yes"
```



Records

```
:: Person = { name :: String
               , birthdate :: (Int,Int,Int)
               , fpprogramer :: Bool
             }
```

```
GetName :: Person → String
```

```
GetName p = p.name
```

```
GetName2 :: Person → String
```

```
GetName2 {name} = name
```

```
ChangeN :: Person String → Person
```

```
ChangeN p s = {p & name = s}
```

```
Start = ChangeN {name = "XY", birthdate = (1,1,2000),
                 fpprogramer = True} "Alex"
```



Records

```
:: Point = {  x      :: Real
              ,  y      :: Real
              , visible :: Bool
            }
```

```
:: Vector = { dx      :: Real
              , dy      :: Real
            }
```

```
Origo :: Point
```

```
Origo = {  x = 0.0
          ,  y = 0.0
          , visible = True
        }
```

```
Dist :: Vector
```

```
Dist = {  dx = 1.0
          , dy = 2.0
        }
```



Records

```
IsVisible :: Point → Bool
IsVisible {visible = True} = True
IsVisible _                 = False
```

```
xcoordinate :: Point → Real
xcoordinate p = p.x
```

```
hide :: Point → Point
hide p = { p & visible = False }
```

```
Move :: Point Vector → Point
Move p v = { p & x = p.x + v.dx, y = p.y + v.dy }
```

```
Start = Move (hide Origo) Dist
```



Records

```
:: Q = { nom :: Int
        , den :: Int
        }
```

```
QZero = { nom = 0, den = 1 }
```

```
QOne = { nom = 1, den = 1 }
```

```
simplify {nom=n,den=d}
  | d = 0 = abort " denominator is 0"
  | d < 0 = { nom = -n/g, den = -d/g }
  | otherwise = { nom = n/g, den = d/g }
where g = gcdm n d
```

```
gcdm x y = gcdnat (abs x) (abs y)
where gcdnat x 0 = x
      gcdnat x y = gcdnat y (x rem y)
```

```
mkQ n d = simplify { nom = n, den = d }
Start = mkQ 81 90
```



Arrays

```
MyArray :: {Int}
MyArray = {1,3,5,7,9}
```

```
Start = MyArray.[2] // 5
```

```
MapArray1 f a = {f e \\ e ←: a}
```

```
Start :: {Int}
Start = MapArray1 inc MyArray
```

```
// Comprehension transformations:
Array = {elem \\ elem ← List}
List = [elem \\ elem ←: Array]
```



Algebraic types

```
:: Day = Mon | Tue | Wed | Thu | Fri | Sat | Sun
```

```
:: Tree a = Node a (Tree a) (Tree a)
           | Leaf
```

```
sizeT :: (Tree a) → Int
```

```
sizeT Leaf = 0
```

```
sizeT (Node x l r) = 1 + sizeT l + sizeT r
```

```
Start = sizeT (Node 4 (Node 2 (Node 1 Leaf Leaf)
                             (Node 3 Leaf Leaf)) Leaf) // 4
```



Algebraic types

```
:: Tree a = Node a (Tree a) (Tree a)
      | Leaf
```

```
atree = Node 2 (Node 1 Leaf Leaf) (Node 3 Leaf Leaf)
```

```
depth :: (Tree a) → Int
```

```
depth Leaf = 0
```

```
depth (Node _ l r) = (max (depth l) (depth r)) + 1
```

```
Start = depth atree // 2
```



Algebraic types

```
treesort :: ([a] → [a]) | Eq, Ord a  
treesort = collect o listtoTree
```

```
listtoTree :: [a] → Tree a | Ord, Eq a  
listtoTree [] = Leaf  
listtoTree [x:xs] = insertTree x (listtoTree xs)
```

```
insertTree :: a (Tree a) → Tree a | Ord a  
insertTree e Leaf = Node e Leaf Leaf  
insertTree e (Node x le ri)  
  | e ≤ x = Node x (insertTree e le) ri  
  | e > x = Node x le (insertTree e ri)
```

```
collect :: (Tree a) → [a]  
collect Leaf = []  
collect (Node x le ri) = collect le ++ [x] ++ collect ri
```

```
Start = treesort [3, 1, 5, 9, 2, 7, 0] // [0, 1, 2, 3, 5, 7, 9]
```

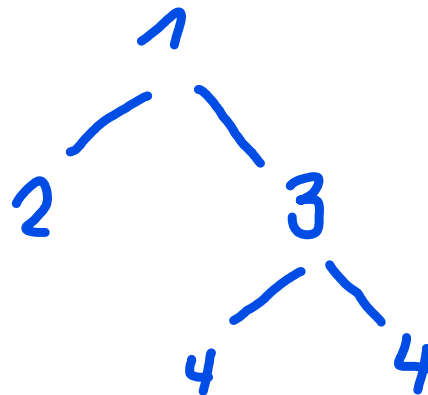


Algebraic types

```
:: Tree a = Node a (Tree a) (Tree a)
   | Leaf
```

```
atree = Node 2 (Node 1 Leaf Leaf) (Node 3 Leaf Leaf)
```

```
:: Tree2 a = Node2 a (Tree2 a) (Tree2 a)
   | Leaf2 a
```



More trees

```
nrNodes :: (Tree2 a) → Int
```

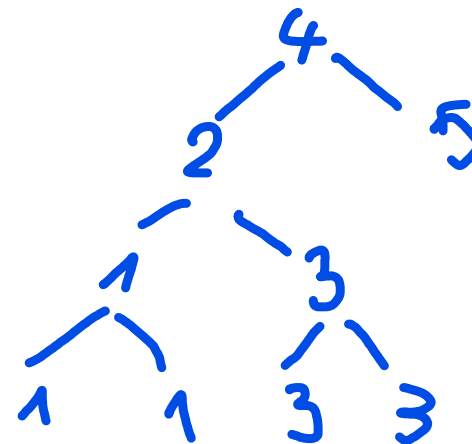
```
nrNodes (Leaf2 y) = 1
```

```
nrNodes (Node2 x l r) = 1 + nrNodes l + nrNodes r
```

```
aTree2 :: Tree2 Int
```

```
aTree2 = Node2 4 (Node2 2 (Node2 1 (Leaf2 1) (Leaf2 1))  
                        (Node2 3 (Leaf2 3) (Leaf2 3))) (Leaf2 5)
```

```
Start = nrNodes aTree2 // 9
```



More trees

```
:: Tree3 a b = Node3 a (Tree3 a b) (Tree3 a b)
   | Leaf3 b
```

```
aTree3 :: Tree3 Int Real
```

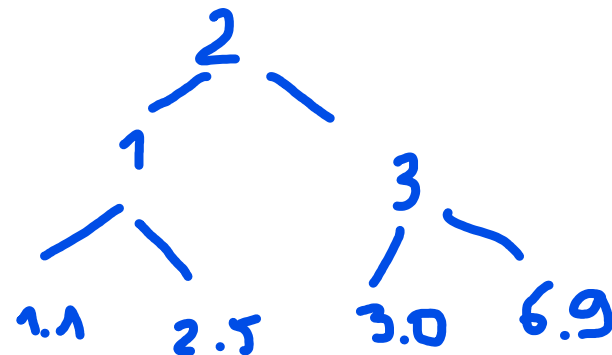
```
aTree3 = Node3 2 (Node3 1 (Leaf3 1.1) (Leaf3 2.5))
              (Node3 3 (Leaf3 3.0) (Leaf3 6.9))
```

```
sumLeaves :: (Tree3 Int Real) → Real
```

```
sumLeaves (Leaf3 y) = y
```

```
sumLeaves (Node3 x le ri) = sumLeaves le + sumLeaves ri
```

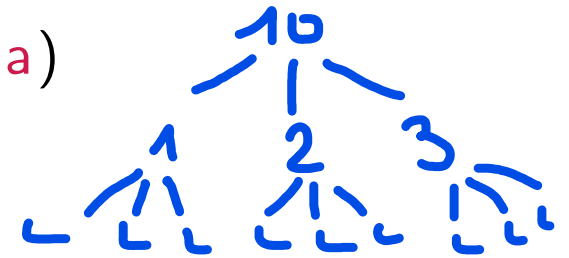
```
Start = sumLeaves aTree3 // 13.5
```



Algebraic types

// Triple branches

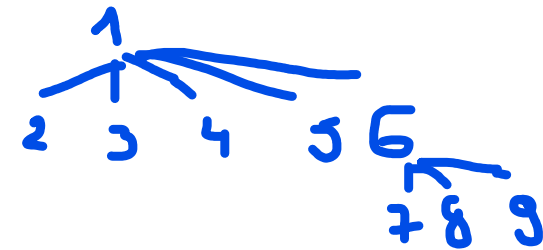
```
:: Tree4 a = Node4 a (Tree4 a) (Tree4 a) (Tree4 a)
   | Leaf4
```



// Rose-tree - tree with variable multiple branches

// No leaf constructor, node with no branches

```
:: Tree5 a = Node5 a [Tree5 a]
```



// Every node has one branch = list

```
:: Tree6 a = Node6 a (Tree6 a)
   | Leaf6
```



// Tree with different types

```
:: Tree7 a b = Node7a Int (Tree7 a b) (Tree7 a b)
   | Node7b b (Tree7 a b)
   | Leaf7a b
   | Leaf7b Int
```

Map, foldr on trees

```

:: BTree a                = Bin (BTree a) (BTree a)
                           | Tip a

```

```

mapbtree                :: (a → b) (BTree a) → BTree b
mapbtree f (Tip x)      = Tip (f x)
mapbtree f (Bin t1 t2) = Bin (mapbtree f t1) (mapbtree f t2)

```

```

foldbtree                :: (a a → a) (BTree a) → a
foldbtree f (Tip x)      = x
foldbtree f (Bin t1 t2) = f (foldbtree f t1) (foldbtree f t2)

```

```

aBTree = Bin (Bin (Bin (Tip 1) (Tip 1))
                (Bin (Tip 3) (Tip 3))) (Tip 5)

```



```

Start = mapbtree inc aBTree
Start = foldbtree (+) aBTree // 13

```


Abstract Data Types

definition module Stack

:: Stack **a**

newStack **::** (Stack **a**) *// Creates empty stack*

empty **::** (Stack **a**) \rightarrow Bool *// Checks if a stack is empty*

push **::** **a** (Stack **a**) \rightarrow Stack **a** *// push new element on top of the stack*

pop **::** (Stack **a**) \rightarrow Stack **a** *// Remove the top element from the stack*

top **::** (Stack **a**) \rightarrow **a** *// Return the top element from the stack*



Abstract Data Types

implementation module Stack

import StdEnv

$:: \text{Stack } a := [a]$

$\text{newStack} :: \text{Stack } a$

$\text{newStack} = []$

$\text{empty} :: (\text{Stack } a) \rightarrow \text{Bool}$

$\text{empty } [] = \text{True}$

$\text{empty } x = \text{False}$

$\text{push} :: a (\text{Stack } a) \rightarrow \text{Stack } a$

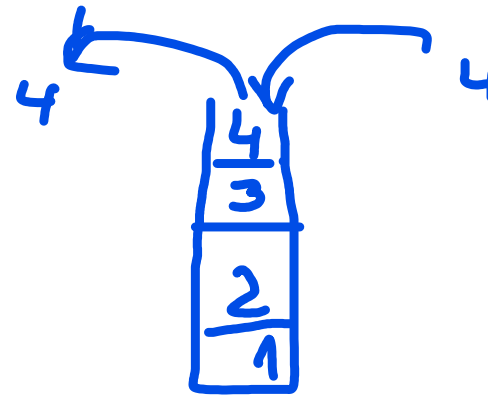
$\text{push } e \ s = [e : s]$

$\text{pop} :: (\text{Stack } a) \rightarrow \text{Stack } a$

$\text{pop } [e : s] = s$

$\text{top} :: (\text{Stack } a) \rightarrow a$

$\text{top } [e : s] = e$



Bag

```
definition module Bag
import StdEnv
```

```
:: Bag a
```

```
newB      ::      (Bag a)           // empty bag
isempty   ::      (Bag a) → Bool
insertB   ::  a    (Bag a) → Bag a  | Eq a  // insert an element
removeB   ::  a    (Bag a) → Bag a  | Eq a  // remove an element
sizeB     ::      (Bag a) → Int      // return all nr elements
```



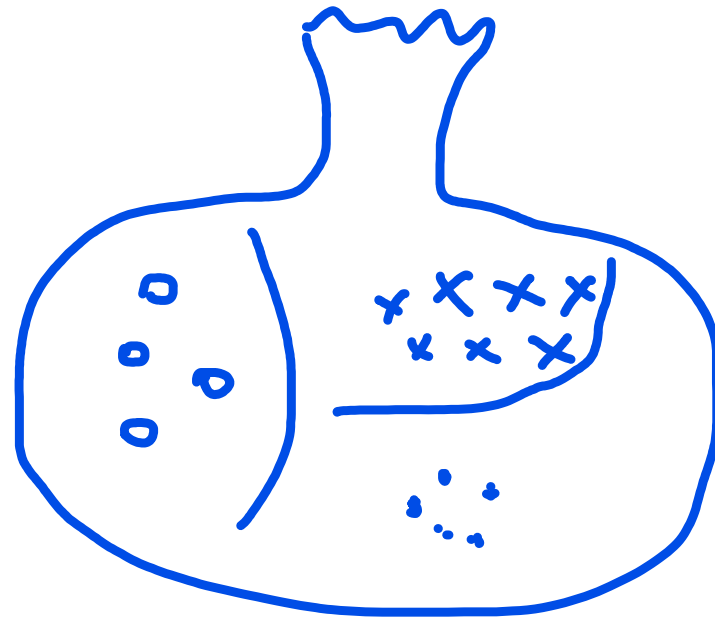
Bag

```
implementation module Bag
import StdEnv
```

```
:: Bag a := [(Int, a)]
```

```
newB :: Bag a
newB = []
```

```
isempty :: (Bag a) → Bool
isempty [] = True
isempty x = False
```



4 0
1 x
5 ·

Bag

```
insertB :: a (Bag a) → Bag a | Eq a
insertB e [] = [(1,e)]
insertB e [(m,x):t]
  | e == x = [(m+1,x):t]
  = [(m,x)] ++ insertB e t
```

```
removeB :: a (Bag a) → Bag a | Eq a
removeB e [] = []
removeB e [(m,x):t]
  | e == x && (m-1) == 0 = t
  | e == x = [(m-1,x):t]
  = [(m,x)] ++ removeB e t
```



Bag

```
sizeB    :: (Bag a) → Int
sizeB [] = 0
sizeB [(m,x):t] = m + sizeB t
```

// tests of implementations:

```
Start      = ( "s0 = newB = ",          s0, '\n'
               , "s1 = insertB 1 s0 = ", s1, '\n'
               , "s2 = insertB 1 s1 = ", s2, '\n'
               , "s3 = insertB 2 s2 = ", s3, '\n'
               , "s4 = removeB 1 s3 = ", s4, '\n'
               , "s5 = sizeB      s3 = ", s5, '\n'
               , "test = isempty s3 = ", test, '\n')
```



Bag

where

```
s0      = newB
s1      = insertB 1 s0
s2      = insertB 1 s1
s3      = insertB 2 s2
s4      = removeB 1 s3
s5      = sizeB s3
test    = isempty s3
```

```
/* ("s0 = newB = ", [], '
', "s1 = insertB 1 s0 = ", [(1,1)], '
', "s2 = insertB 1 s1 = ", [(2,1)], '
', "s3 = insertB 2 s2 = ", [(2,1), (1,2)], '
', "s4 = removeB 1 s3 = ", [(1,1), (1,2)], '
', "s5 = sizeB s3 = ", 3, '
', "test = isempty s3 = ", False, '
') */
```

