

# Introduction to Functional Programming

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# Overview

- 1 Records
- 2 Arrays
- 3 Algebraic types - trees
- 4 Abstract Data Types
- 5 Bag as ADT



# Records

```
:: Person = { name :: String
              , birthdate :: (Int,Int,Int)
              , fpprogramer :: Bool
              }
```

```
IsfpUser :: Person → String
IsfpUser {fpprogramer = True} = "Yes"
IsfpUser _                    = "No"
```

```
Start = IsfpUser { name = "Me"
                  , birthdate = (1,1,1999)
                  , fpprogramer = True}    // "Yes"
```



# Records

```
:: Person = { name :: String
              , birthdate :: (Int,Int,Int)
              , fpprogramer :: Bool
              }
```

```
GetName :: Person → String
```

```
GetName p = p.name
```

```
GetName2 :: Person → String
```

```
GetName2 {name} = name
```

```
ChangeN :: Person String → Person
```

```
ChangeN p s = {p & name = s}
```

```
Start = ChangeN {name = "XY", birthdate = (1,1,2000),
                 fpprogramer = True} "Alex"
```



# Records

```
:: Point = {  x      :: Real
              ,  y      :: Real
              , visible :: Bool
            }
```

```
:: Vector = { dx      :: Real
              , dy      :: Real
            }
```

```
Origo :: Point
```

```
Origo = {  x = 0.0
          ,  y = 0.0
          , visible = True
        }
```

```
Dist :: Vector
```

```
Dist = { dx = 1.0
        , dy = 2.0
        }
```



# Records

```
IsVisible :: Point → Bool
IsVisible {visible = True} = True
IsVisible _                 = False
```

```
xcoordinate :: Point → Real
xcoordinate p = p.x
```

```
hide :: Point → Point
hide p = { p & visible = False }
```

```
Move :: Point Vector → Point
Move p v = { p & x = p.x + v.dx, y = p.y + v.dy }
```

```
Start = Move (hide Origo) Dist
```



# Records

```

:: Q = { nom :: Int
        , den :: Int
        }
QZero = { nom = 0, den = 1 }
QOne  = { nom = 1, den = 1 }

simplify {nom=n,den=d}
  | d = 0 = abort " denominator is 0"
  | d < 0 = { nom = -n/g, den = -d/g}
  | otherwise = { nom = n/g, den = d/g}
  where g = gcdm n d

gcdm x y = gcdnat (abs x) (abs y)
  where gcdnat x 0 = x
        gcdnat x y = gcdnat y (x rem y)

mkQ n d = simplify { nom = n, den = d }
Start = mkQ 81 90

```



# Arrays

```
MyArray :: {Int}
MyArray = {1,3,5,7,9}
```

```
Start = MyArray.[2] // 5
```

```
MapArray1 f a = {f e \\ e ←: a}
```

```
Start :: {Int}
Start = MapArray1 inc MyArray
```

```
// Comprehension transformations:
Array = {elem \\ elem ← List}
List = [elem \\ elem ←: Array]
```





# Algebraic types

```
:: Day = Mon | Tue | Wed | Thu | Fri | Sat | Sun
```

```
:: Tree a = Node a (Tree a) (Tree a)
           | Leaf
```

```
sizeT :: (Tree a) → Int
```

```
sizeT Leaf = 0
```

```
sizeT (Node x l r) = 1 + sizeT l + sizeT r
```

```
Start = sizeT (Node 4 (Node 2 (Node 1 Leaf Leaf)
                              (Node 3 Leaf Leaf)) Leaf) // 4
```



# Algebraic types

```

:: Tree a = Node a (Tree a) (Tree a)
           | Leaf

```

```

atree = Node 2 (Node 1 Leaf Leaf) (Node 3 Leaf Leaf)

```

```

depth :: (Tree a) → Int

```

```

depth Leaf = 0

```

```

depth (Node _ l r) = (max (depth l) (depth r)) + 1

```

```

Start = depth atree // 2

```



# Algebraic types

```
treesort :: ([a] → [a]) | Eq, Ord a
treesort = collect o listtoTree
```

```
listtoTree :: [a] → Tree a | Ord, Eq a
listtoTree [] = Leaf
listtoTree [x:xs] = insertTree x (listtoTree xs)
```

```
insertTree :: a (Tree a) → Tree a | Ord a
insertTree e Leaf = Node e Leaf Leaf
insertTree e (Node x le ri)
  | e ≤ x = Node x (insertTree e le) ri
  | e > x = Node x le (insertTree e ri)
```

```
collect :: (Tree a) → [a]
collect Leaf = []
collect (Node x le ri) = collect le ++ [x] ++ collect ri
```

**Start** = treesort [3, 1, 5, 9, 2, 7, 0] // [0, 1, 2, 3, 5, 7, 9]



# Algebraic types

```
:: Tree a = Node a (Tree a) (Tree a)
    | Leaf
```

```
atree = Node 2 (Node 1 Leaf Leaf) (Node 3 Leaf Leaf)
```

```
:: Tree2 a = Node2 a (Tree2 a) (Tree2 a)
    | Leaf2 a
```



# More trees

```
nrNodes :: (Tree2 a) → Int
```

```
nrNodes (Leaf2 y) = 1
```

```
nrNodes (Node2 x l r) = 1 + nrNodes l + nrNodes r
```

```
aTree2 :: Tree2 Int
```

```
aTree2 = Node2 4 (Node2 2 (Node2 1 (Leaf2 1) (Leaf2 1))
                          (Node2 3 (Leaf2 3) (Leaf2 3))) (Leaf2 5)
```

```
Start = nrNodes aTree2      // 9
```



# More trees

```
:: Tree3 a b = Node3 a (Tree3 a b) (Tree3 a b)
               | Leaf3 b
```

```
aTree3 :: Tree3 Int Real
```

```
aTree3 = Node3 2 (Node3 1 (Leaf3 1.1) (Leaf3 2.5))
               (Node3 3 (Leaf3 3.0) (Leaf3 6.9))
```

```
sumLeaves :: (Tree3 Int Real) → Real
```

```
sumLeaves (Leaf3 y) = y
```

```
sumLeaves (Node3 x le ri) = sumLeaves le + sumLeaves ri
```

```
Start = sumLeaves aTree3 // 13.5
```



# Algebraic types

*// Triple branches*

```
:: Tree4 a = Node4 a (Tree4 a) (Tree4 a) (Tree4 a)
    | Leaf4
```

*// Rose-tree - tree with variable multiple branches*

*// No leaf constructor, node with no branches*

```
:: Tree5 a = Node5 a [Tree5 a]
```

*// Every node has one branch = list*

```
:: Tree6 a = Node6 a (Tree6 a)
    | Leaf6
```

*// Tree with different types*

```
:: Tree7 a b = Node7a Int (Tree7 a b) (Tree7 a b)
    | Node7b b (Tree7 a b)
    | Leaf7a b
    | Leaf7b Int
```



# Map, foldr on trees

```

:: BTree a                = Bin (BTree a) (BTree a)
                           | Tip a

```

```

mapbtree                :: (a → b) (BTree a) → BTree b
mapbtree f (Tip x)      = Tip (f x)
mapbtree f (Bin t1 t2) = Bin (mapbtree f t1) (mapbtree f t2)

```

```

foldbtree                :: (a a → a) (BTree a) → a
foldbtree f (Tip x)      = x
foldbtree f (Bin t1 t2) = f (foldbtree f t1) (foldbtree f t2)

```

```

aBTree = Bin (Bin (Bin (Tip 1) (Tip 1))
               (Bin (Tip 3) (Tip 3))) (Tip 5)

```

```

Start = mapbtree inc aBTree
Start = foldbtree (+) aBTree // 13

```





# Abstract Data Types

**definition module** Stack

**::** Stack **a**

**newStack** **::** (Stack **a**)      *// Creates empty stack*

**empty**    **::** (Stack **a**)  $\rightarrow$  Bool    *// Checks if a stack is empty*

**push**     **::** **a** (Stack **a**)  $\rightarrow$  Stack **a**    *// push new element on top of the stack*

**pop**       **::** (Stack **a**)  $\rightarrow$  Stack **a**      *// Remove the top element from the stack*

**top**       **::** (Stack **a**)  $\rightarrow$  **a**              *// Return the top element from the stack*



# Abstract Data Types

**implementation module** Stack

**import** StdEnv

**::** Stack **a** **:=** [**a**]

**newStack** **::** Stack **a**

**newStack** = []

**empty** **::** (Stack **a**)  $\rightarrow$  Bool

**empty** [] = True

**empty** **x** = False

**push** **::** **a** (Stack **a**)  $\rightarrow$  Stack **a**

**push** **e** **s** = [**e** : **s**]

**pop** **::** (Stack **a**)  $\rightarrow$  Stack **a**

**pop** [**e** : **s**] = **s**

**top** **::** (Stack **a**)  $\rightarrow$  **a**

**top** [**e** : **s**] = **e**



# Bag

```
definition module Bag
import StdEnv
```

```
:: Bag a
```

```
newB      ::      (Bag a)                // empty bag
isEmpty ::      (Bag a) → Bool
insertB ::  a (Bag a) → Bag a  | Eq a  // insert an element
removeB ::  a (Bag a) → Bag a  | Eq a  // remove an element
sizeB    ::      (Bag a) → Int          // return all nr elements
```



# Bag

```
implementation module Bag  
import StdEnv
```

```
:: Bag a := [(Int, a)]
```

```
newB :: Bag a  
newB = []
```

```
isempty :: (Bag a) → Bool  
isempty [] = True  
isempty x = False
```



# Bag

```
insertB :: a (Bag a) → Bag a | Eq a
insertB e [] = [(1,e)]
insertB e [(m,x):t]
| e == x = [(m+1,x):t]
= [(m,x)] ++ insertB e t
```

```
removeB :: a (Bag a) → Bag a | Eq a
removeB e [] = []
removeB e [(m,x):t]
| e == x && (m-1) == 0 = t
| e == x = [(m-1,x):t]
= [(m,x)] ++ removeB e t
```



# Bag

```
sizeB    :: (Bag a) → Int
sizeB [] = 0
sizeB [(m,x):t] = m + sizeB t
```

*// tests of implementations:*

```
Start      = ( "s0 = newB = ",          s0, '\n'
               , "s1 = insertB 1 s0 = ", s1, '\n'
               , "s2 = insertB 1 s1 = ", s2, '\n'
               , "s3 = insertB 2 s2 = ", s3, '\n'
               , "s4 = removeB 1 s3 = ", s4, '\n'
               , "s5 = sizeB      s3 = ", s5, '\n'
               , "test = isempty s3 = ", test, '\n')
```



## Bag

where

```
s0          = newB
s1          = insertB 1      s0
s2          = insertB 1      s1
s3          = insertB 2      s2
s4          = removeB 1      s3
s5          = sizeB          s3
test        = isempty       s3
```

```
/* ("s0 = newB = ", [], '
', "s1 = insertB 1 s0 = ", [(1,1)], '
', "s2 = insertB 1 s1 = ", [(2,1)], '
', "s3 = insertB 2 s2 = ", [(2,1), (1,2)], '
', "s4 = removeB 1 s3 = ", [(1,1), (1,2)], '
', "s5 = sizeB s3 = ", 3, '
', "test = isempty s3 = ", False, '
') */
```

