Final Exam (part 1) - Computational Physics I

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Date: Monday 17 June 2024 **Duration:** 45 minutes
Credits: 10 points (5 questions) **Type of evaluation:** LAB

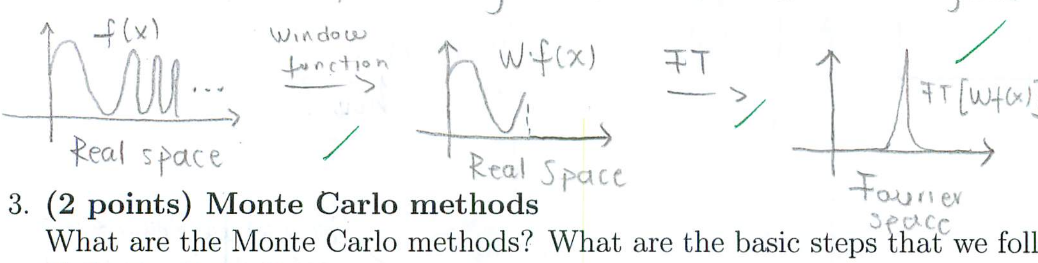
Part 1 is closed-book, in-class, and contains short-answer questions. Please provide short and concise answers to the following items:

1. **(2 points) Fourier transforms**
   Provide a concise definition of the Fourier Transform and explain its significance for physics. Illustrate your answer with the basic formula for the continuous Fourier Transform.

The continuous formula is $FT[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-i2\pi kx} f(x)\,dx$, where k is the wavenumber. What it allows to do is to go from real space (x) to Fourier space (k), where the frecuencies of the waves that added together give $f(x)$ are easily visible and can be manipulated (a thing that can't be done in the real space). It's important in physics because many measurements return signals that often may contain noise, and, with FTs, it's possible to get rid of that and extract the information we're interested in. Nice explanation!

2. **(2 points) Wavelet transforms**
   Explain the process of wavelet decomposition for a 1D signal. When do we use Fourier transforms and when wavelet transforms?

FTs work well for stationary signals, but, when the frecuencies of signals change in time or space, FTs won't be able to handle them.
→ Wavelet transforms will do the work in this case because they use something called window function to separate a portion of the signal where the frecuency holds. For a 1D signal:



and the same again for the other portions of the signal.

3. **(2 points) Monte Carlo methods**
   What are the Monte Carlo methods? What are the basic steps that we follow to carry out a Monte Carlo simulation?

Monte Carlo methods use randomness to solve a problem that might be, in principle, deterministic, but difficult to solve analitically. The basic steps are the following:

1) Identify the domain of possible inputs.
2) Choose an adequate statistical distribution to generate the random numbers.
3) Generate the random numbers, do the operations needed.
4) After performing the operations for a large set of random inputs, we should be close to the value we were looking for (the expectation value).

4. **(2 points) Fourier filtering**
Imagine you are given a 2D grayscale image with noise. Describe the steps you would take to remove the noise using a Fourier-based algorithm in python.

1) Perform the 2D FT of the image and shift the frecuencies to the center of the image in the Fourier space.

2) Since noise is translated into noise in the Fourier space as small amplitude ~~frequencies~~, I would manually set a new zero for the z-axis and apply the condition that all ~~frecuencies~~ below this new zero are zero.

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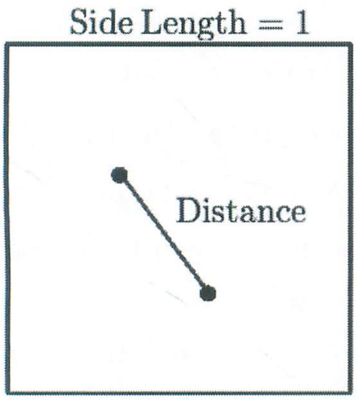
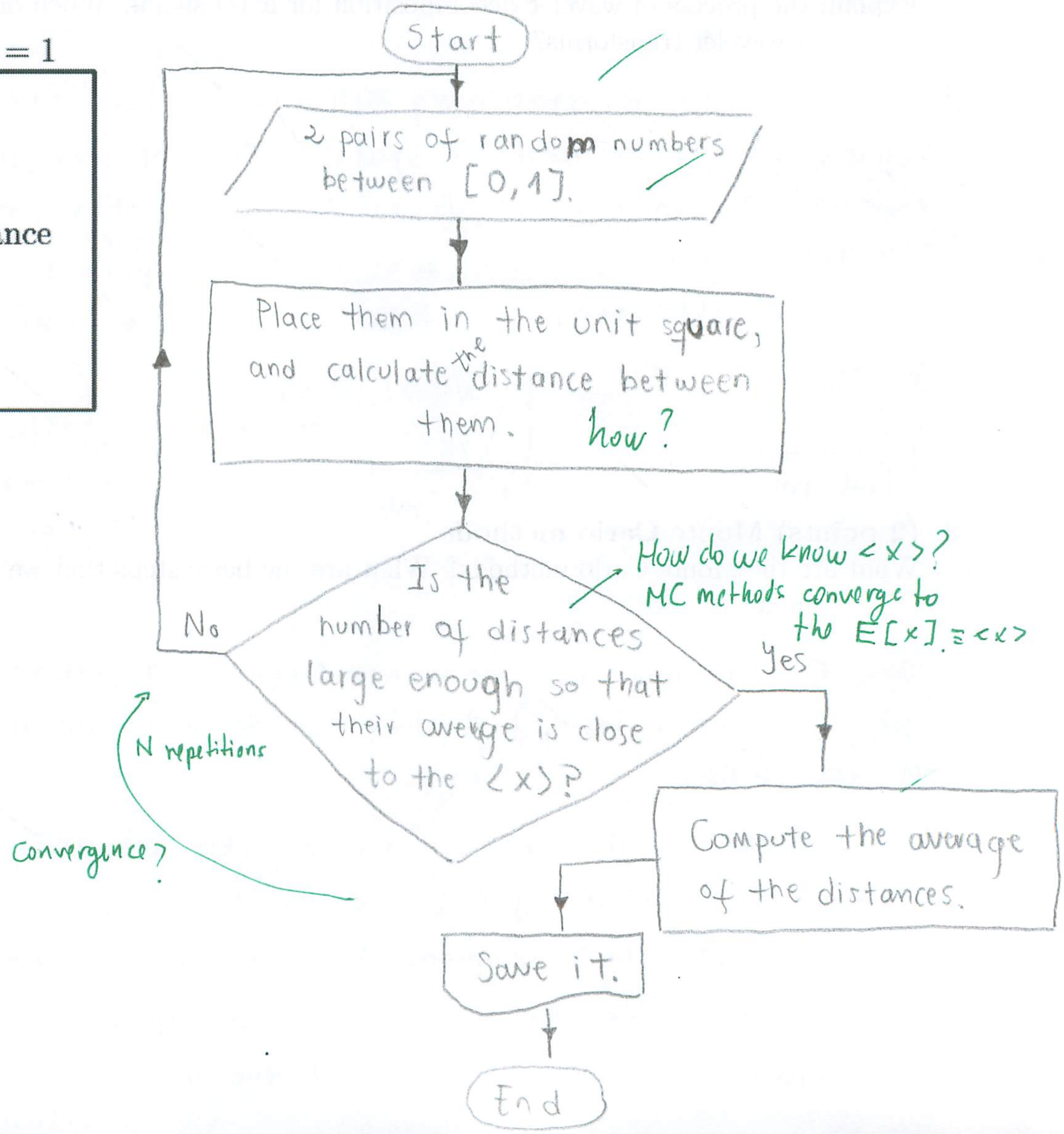
amplitudes not frequencies.

3) Then, perform the inverse 2D FT to this new image without the small-amplitude ~~frequencies~~ signal and hopefully we would now have an image without most, if not all, the noise.

* Of course, we are in risk of loosing details of the image, but overall the result would be good.

5. **(2 points) Monte Carlo simulation**
Imagine you are asked to estimate the average distance between randomly placed points within a unit square (see figure below). Describe the python workflow that you would use to estimate the average distance between two randomly placed points within the unit square.





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