# Take-home exam 1

*Dynamical Systems*

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**Problem 1: Classification of fixed points**

A particle of mass $m = 1$ is moving in the potential $V(x) = -(1/2)x^2 + (1/4)x^4$. Find and classify the fixed points (node, saddle, focus) according to their stability.

Solution goes here.

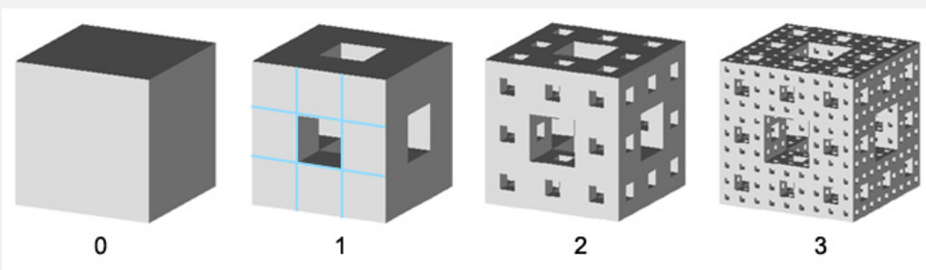
**Problem 2: Hopf bifurcation**

Consider the system $\ddot{x} + \lambda(x^2 - 1)\dot{x} + x - a = 0$. Find the curves on the space of parameters $(\lambda, a)$ where a Hopf bifurcation occurs.

The second solution goes here.

**Problem 3: Fractal dimension**

Calculate the fractal dimension of the following object shown at three successive levels of construction.




Solution goes here.

**Problem 4: Sensitivity and analytical solution**

Consider the map $x_{n+1} = f(x_n) = (2x_n - 1)^3$ , for $x_n \in [-1, 1]$.

(a) Show, by iterating two close initial conditions, that this map is chaotic.

(b) Show that $x_n = \cos^3(2^n \cos^{-1}(x_0))$ is a solution $\forall n$.

Solution goes here.

### Problem 5: Bifurcation diagram and Lyapunov exponent

Consider the map $x_{n+1} = f(x_n) = \sin^2(r \arcsin \sqrt{x_n})$, for $x_n \in [0, 1]$.

(a) Obtain the bifurcation diagram of $x_n$ as a function of $r$, for $r \in [1, 4]$.

(b) Calculate the Lyapunov exponent as a function of $r$, for $r \in [1, 4]$.

Solution goes here.

### Problem 6: Phase space

The evolution of a system is described by the following equation:

$$\dddot{x} + a\ddot{x} + \dot{x} - |x| + 1 = 0, \text{ for } a > 0.$$

(a) Find the fixed points of this system.

(b) Plot the attractor of this system in its phase space for $a = 0.6$. Is it strange?

(c) Show that this system is not chaotic for $a = 0.68$.

Solution goes here.

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