

# Interest Rate and Credit Models

## 15. Counterparty Credit Risk

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# Outline

- 1 Counterparty credit risk
- 2 Central counterparties
- 3 Credit exposure metrics
- 4 Quantifying credit exposure (XVAs)

# Financial risk

- Traditionally, financial risk has been broken up into several (partly overlapping) categories:
  - (i) Market risk
  - (ii) Credit risk
  - (iii) Liquidity risk
  - (iv) Operational risk
  - (v) Model risk
- The financial crisis of 2007-2008 heightened the importance of two other categories of risk:
  - (i) Counterparty risk
  - (ii) Systemic risk

# Definition of counterparty credit risk

- Counterparty credit risk is the risk that the counterparty on a financial transaction will fail to fulfill their contractual obligation.
- Counterparty risk arises from two categories of asset classes:
  - (i) OTC derivatives such as interest rate swaps (IRS) and swaptions, CDS and default swaptions, FX forwards.
  - (ii) Securities financing transactions such as repos and reverse repos, and securities borrowing and lending.

# Settlement and pre-settlement risk

- *Pre-settlement risk* is the risk that a counterparty will default prior to the final settlement of the transaction (at expiration).
- *Settlement risk* arises if there are timing differences between when the counterparties perform on their obligations.
- Exchange traded derivatives.
- OTC derivatives.
- ISDA Master Agreement.
- Repos and reverse repos.

# Netting

- *Payment netting* offers counterparties the ability to net cash flows occurring on the same day. For example, through floating leg compounding, payments on IRS are netted.
- *Closeout netting* allows the termination of all transactions between the solvent and insolvent counterparties by offsetting the values of the transactions. This may occur in a variety of situations:
  - (i) Trades may be parts of a hedged transaction.
  - (ii) A trade may be an unwind of another trade.
  - (iii) Trades are independent.
- Closeout netting allows the solvent institution to immediately recognize the gains vs losses in case of a counterparty default.

# Collateral

- Collateral is an asset supporting a risk in a legally enforceable way.
- The collateral receiver becomes the owner of the collateral only if the party posting the collateral defaults.
- The use of collateral in derivatives transactions is regulated by ISDA's credit support annex (CSA).

# Credit support annex (CSA)

- The ISDA Master Agreement is usually amended with a CSA.
- CSA specifies issues such as:
  - (i) Method and timing of the underlying valuation.
  - (ii) Amount of collateral to be posted.
  - (iii) The mechanics and timing of collateral transfer.
  - (iv) Eligible collateral.
  - (v) Dispute resolution.
  - (vi) Haircuts.
  - (vii) Possible rehypothecation of collateral.
  - (viii) Interest payments on collateral.

# Collateral terms

- Valuation agent.
- Types of collateral: cash, government securities, other (GSA securities, AAA MBS securities, corporate bonds, equities).
- Margin calls.
- Haircuts.
- As long as the counterparty posting collateral is not in default, it receives all the coupons, dividends and other cash flows. Interest is paid on cash collateral at the overnight indexed swap (OIS) rate.

# Collateral terms

- No CSA. Rarely applied.
- Two-way CSA. This is typical among similar counterparties.
- One-way CSA. This is a typical arrangement in bank / hedge fund relations.
- Collateral terms may be linked to the counterparties credit quality.

# Central clearing

- In 2010 both the US and EU published legislative proposals, according to which all standard OTC derivatives should be cleared through central counterparties (CCP).
- The key function of a CCP is contract *novation*.
- Novation is the replacement of a bilateral contract with several contracts, as a result of which the CCP interposes itself between the counterparties and acts as insurer of the counterparty risk.
- This is achieved through rigorous counterparty risk management such as variation margin and risk mutualization.

# Functions of a CCP

- As a result of contract novation, a CCP bears no net market risk, but it takes on counterparty risk.
- A CCP mitigates the counterparty risk by collecting financial resources from its members that cover potential losses in the event of a default (initial margin, variation margin, liquidity fund, default fund).
- With the collected funds, a CCP converts counterparty risk into other form of risks.

# Advantages of central clearing

- *Multilateral netting*: Increases the flexibility to enter new transactions and terminate existing ones.
- *Increased position size transparency*: The CCP is able to impose punitive measures on members with outsized positions.
- *Loss mutualization*: Part of the losses are distributed among clearing members.
- *Legal and operational efficiency*: A CCP working directly with regulators is likely to be more efficient than individual market participants.
- *Liquidity*: Frequent (at least daily) marking to market of the member positions will lead to more transparent valuation of cleared instruments.
- *Default management*: A CCP managing the liquidation of the positions of a defaulted member will be less disruptive to the market than uncoordinated actions by individual market participants.

# Multilateral netting

- Policymakers and regulators make the point that CCPs facilitate multilateral netting, which can alleviate systemic risk.
- The effect may be diminished by the presence of multiple CCPs (e.g. CME, ICE, LCH, ...), even though this fragmentation may be beneficial from the systemic risk point of view.
- This effect has been studied in a simple toy models setting by Duffie and Zhu.  
Among their findings are:
  - (i) The required number of members of a CCP to achieve netting reduction is a function of the correlation among the assets and the number of assets.
  - (ii) Increased netting benefit can only be achieved by a small number of CCPs clearing a large number of transactions. It may thus be inefficient to have multiple CCPs.

# Initial margin

- Initial margin is calculated using a VaR model. Occasionally, older systems (such as CME's SPAN) are used.
- Most CCPs use historical simulations to calculate VaR applied to 5-day portfolio returns.
- The 5-day period is referred to as the market period of risk (MPOR).
- For VaR calculation, a long look-back period (typically, between 1 and 5 years) is used.
- In some cases, for the sake of performance efficiency, parametric VaR models (such as Gaussian or Student t distribution based) are used.
- VaR level is calculated as the appropriate  $k$ -th worst return.
- VaR model's performance is subject to regulatory control based on daily back tests.

# Initial margin

- Portfolio returns tend to form a non-stationary time series.
- In order to bring this time series closer to stationarity, volatility scaling is applied to the time series of returns.
- Specifically, we define the exponentially weighted moving average (EWMA) model by

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_t^2,$$

where  $\lambda$  is a scaling parameter (typically 0.97), and  $r_t$  is the return on day  $t$ .

- Portfolio return on day  $t$  is scaled by  $\sigma_t$
- In order to account for fatter tails, *stressed VaR* is used. In addition to the scenarios from the look-back period, additional scenarios are used.

# Variation margin

- Variation margin is an adjustment to the initial margin calculated frequently (at least daily).
- A CCP can make an intraday margin call if large market moves threaten to deplete a clearing member's margin funds.
- Such practices are increasingly common, owing to the advancing technology.

# Default Fund and Additional charges

- Default Fund (DF) is a facility whose purpose is to mitigate potential losses in excess of the initial margin.
- As such, DF is often associated with tail risk events.
- The size of the DF is determined through the selection of stress scenarios which are applied to the clearing members' portfolios.
- The scenarios used for sizing of the DF consist of a number of historical and hypothetical scenarios.
- A CCP may also impose additional charges due to the specific nature of the cleared instruments.
- For example, a liquidity charge may be added to portfolios containing illiquid positions or outsized positions in liquid assets.

# Default management

- In the case of a clearing member default, the CCP takes the following actions.
- Auctions the positions of the defaulted member. Depending on liquidity conditions, some assets will require warehousing.
- May transfer some positions to surviving clearing members.
- Allocate the losses associated with portfolio liquidation according to the following loss waterfall (this may vary depending on the CCP):
  - (i) Initial margin
  - (ii) Member Default Fund
  - (iii) CCP equity (“skin in the game”)
  - (iv) Default Fund (of surviving members)
  - (v) Right of reassessment
  - (vi) Remaining CCP capital
  - (vii) Liquidity support or failure

# Have CCPs ever failed?

- CCP failures have been infrequent. Notable examples include:
  - (i) In 1974 French *Caisse de Liquidation* failed as a result of a sharp drop in sugar prices and the inability of a large member to post additional margin.
  - (ii) In 1983 *Commodity Clearinghouse* of Kuala Lumpur failed as a result of a crash in oil futures and the inability of several brokers to follow up on margin calls.
  - (iii) In 1987 *Hong Kong Futures Exchange Clearing Corporation* had to be bailed out as a result of margin calls following the global stock market crash.
- Note that the recent failures of Lehman Brothers, MF Global, and Knight Capital did not cause losses for CCPs (let alone failures...).

# Counterparty exposure

- A defining feature of counterparty risk is the asymmetry of potential losses.
- In the event of a counterparty default, the institution determines the value of the transactions:
  - *Negative value*. The institution is in debt to the defaulted counterparty, and is still obliged to settle the amount, but from the valuation perspective the position is essentially unchanged.
  - *Positive value*. The defaulted counterparty is incapable to fulfill its commitments, and the institution will have a claim on the assets of the defaulted party. It can expect to recover a certain fraction of its claim.
- We can thus define the exposure of the institution to the counterparty as

$$\text{Exposure} = \max(\text{Value}, 0).$$

- This asymmetry is similar to being short an option.

# Counterparty exposure

- Counterparty risk is bilateral since both counterparties can default.
- This can be captured by defining

$$\text{NegativeExposure} = \min(\text{Value}, 0).$$

- While the current exposure is known with certainty, the future exposure can be defined only probabilistically.
- Quantifying future exposure is difficult as it may involve long periods.
- This bears some similarities to the VaR methodology. However, unlike VaR, future exposure has to be understood in multi-period terms.
- Consequently, future exposure depends not only on volatility but also on the drift.

# Metrics of future exposure

- *Expected future value* (EFV) is the expected value (forward) of the netted positions at some point in the future.
- Note that EFV may significantly differ from the current value of the positions.
- *Potential future exposure* (PFE) is, roughly, the worst exposure at a certain time point in the future, given a confidence level  $\alpha$  (say 99%).
- Its definition is analogous to the definition of  $VaR_\alpha$ .
- *Expected exposure* (EE) is defined as the expected value of exposure values.

# Metrics of future exposure

- In the case of the normal distribution  $N(\mu, \sigma^2)$ , the future value of the portfolio is

$$E = \mu + \sigma Z,$$

where  $Z \sim N(0, 1)$ .

- PFE is given by the same formula as VaR:

$$PFE_\alpha = \mu + \sigma N^{-1}(\alpha).$$

- Exposure is given by

$$\begin{aligned} E &= \max(V, 0) \\ &= \max(\mu + \sigma Z, 0), \end{aligned}$$

and thus

$$\begin{aligned} EE &= \int_{-\mu/\sigma}^{\infty} (\mu + \sigma x) \varphi(x) dx \\ &= \mu N(\mu/\sigma) + \sigma \varphi(\mu/\sigma), \end{aligned}$$

where  $\varphi(x)$  is the density of the normal distribution.

# Metrics of future exposure

- Maximum PFE is the highest value of PFE over a given time horizon.
- Expected positive exposure* (EPE) is the average exposure over a given time horizon. For example, in the normal model with  $V(t) = \sigma\sqrt{t}Z$  where  $\sigma$  is the annualized volatility, the EPE is given by:

$$\begin{aligned} EPE &= \frac{1}{T} \int_0^T \sigma\sqrt{t}\varphi(0)dt \\ &= \frac{2}{3\sqrt{2\pi}} \sigma T^{1/2} \\ &= 0.27\sigma T^{1/2}. \end{aligned}$$

- Negative expected exposure* (NEE) is the average negative exposure over a given time horizon.
- Effective expected positive exposure* (EEPE) takes into account rollover risk of short dated transactions.

# Factors driving credit exposure

- Loans and bonds.
- Future uncertainty associated with FRAs and FX forwards. It is characterized by the square root law:

$$\text{Exposure} \propto t^{1/2}.$$

- In order to see it, suppose we wish to calculate the exposure on a forward contract and assume the normal model for its future value:

$$dV(t) = \mu dt + \sigma dW(t).$$

- Then  $V(t) \sim N(\mu t, \sigma^2 t)$ , and thus, by the previous calculations,

$$PFE_\alpha(t) = \mu t + \sigma \sqrt{t} N^{-1}(\alpha),$$

$$EE_\alpha(t) = \mu t N(\mu/\sigma\sqrt{t}) + \sigma \sqrt{t} \varphi(\mu/\sigma\sqrt{t}).$$

# Factors driving credit exposure

- Future uncertainty associated with periodic cash flows such as swaps is approximately given by

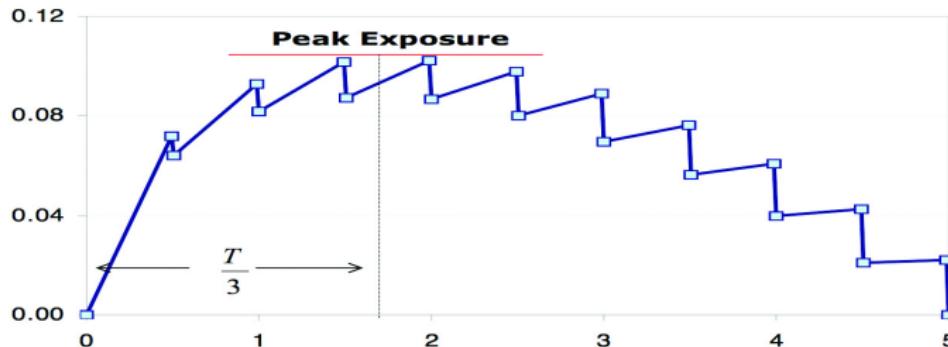
$$\text{Exposure} \propto (T - t)t^{1/2},$$

where  $T$  is the tenor of the swap. This is a consequence of the balance between future uncertainties over payments, and the roll-off of coupon payments over time.

- In order to see it, we model the future value of a swap as  $V(t) \sim N(0, \sigma t(T-t)^2)$ , where  $T - t$  is the approximate duration of the swap at time  $t$  (this is valid for a flat forward curve). The maximum exposure is at  $t = T/3$ .

# Factors driving credit exposure

- The graph below shows the exposure profile of an interest rate swap.



# Factors driving credit exposure

- The exposure of a payer swap is higher than the receiver swap due to the expectation to net pay initially, and net receive in later stages of the swap.
- This effect is even more pronounced in cross-currency swaps. The overall high interest rates paid are expected to be offset by the gain on the notional exchange at the maturity of the contract.
- This expected gain on exchange of notional leads to a significant exposure for the payer of the high interest rate.
- Options.
- CDSs.

# Impact of netting and correlation

- If netting agreements are absent, exposures are considered additive.
- With netting allowable, one can offset at the netting set levels, and the exposures are zero.
- A high positive correlation between two instruments.
- Netting factor is defined as

$$\text{NettingFactor} = \frac{\sqrt{n + n(n - 1)\bar{\rho}}}{n},$$

where  $n$  is the number of exposures, and  $\bar{\rho}$  is the average correlation.

- The netting factor is the ratio of net exposure to gross exposure, and is 1 if there is no netting benefit ( $\bar{\rho} = 1$ ) and 0 if there is maximum netting benefit  $\bar{\rho} = -\frac{1}{n-1}$ .
- The netting benefit improves (lower netting value) for large number of exposures and low correlations.
- For  $\bar{\rho} = 0$ , the netting factor is  $\frac{1}{\sqrt{n}}$ , a significant reduction of the exposure.

# Impact of netting and correlation

- In order to understand the meaning of the netting factor, we assume that each asset  $X_i, i = 1, \dots, n$  within a netting set is modeled as  $X_i \sim N(\mu_i, \sigma_i^2)$ . Then the total mean and standard deviation are given by

$$\mu = \sum_{i=1}^n \mu_i,$$

$$\sigma^2 = \sum_{i,j=1}^n \rho_{ij} \sigma_i \sigma_j,$$

where  $\rho_{ij}$  is the correlation between assets  $i$  and  $j$ .

- Assuming average  $\mu_i$  to be zero, and average  $\sigma_i$  to be  $\bar{\sigma}$ , this gives:

$$\mu = 0,$$

$$\sigma^2 = (n + n(n-1)\bar{\rho})\bar{\sigma}^2,$$

where  $\bar{\rho}$  is the average correlation.

# Impact of netting and correlation

- Therefore, the EE of the netting set is

$$EE_{NS} = \frac{\bar{\sigma}}{\sqrt{2\pi}} \sqrt{n + n(n-1)\bar{\rho}}.$$

- The gross (no netting) EE is

$$EE_{NN} = \frac{\bar{\sigma}}{\sqrt{2\pi}} n.$$

- The netting benefit is thus

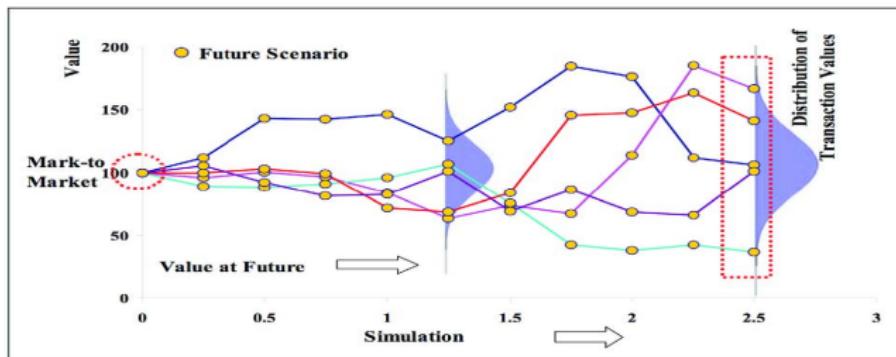
$$\frac{EE_{NS}}{EE_{NN}} = \frac{\sqrt{n + n(n-1)\bar{\rho}}}{n}.$$

# Monte Carlo methodology

- *Monte Carlo simulations* is a universal approach to quantifying credit exposure, and is considered state of the art. Its complexity can be high.
- The first task is to identify the relevant risk factors and specify their dynamics.
- A compromise has to be struck between the model comprehensiveness and parsimony. For example, one can settle on using a one-factor interest rate model rather than three- or four-factor.
- Adding a new risk factor also requires understanding its dependence on the other risk factors. This leads to additional complexity of the model.
- Calibration decisions have to be made: historical or cross-sectional data (or a mix of both).
- Generally, risk models tend to be less detailed and complex than front office models.

# Monte Carlo methodology

The graph below illustrates the Monte Carlo approach to credit exposure.



# Monte Carlo methodology

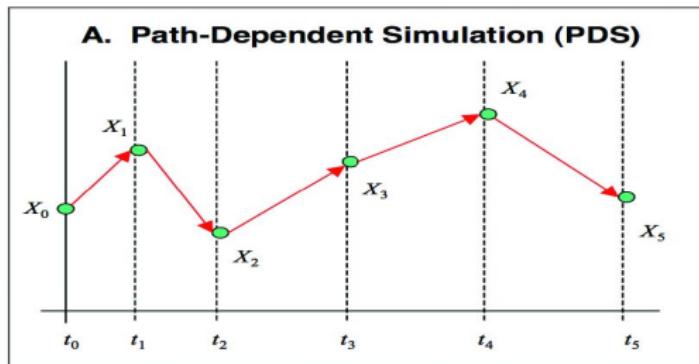
- *Scenario generation:*
  - (i) Time grid selection.
  - (ii) Path dependent simulation vs long jump.
- *Revaluation.* This step requires using efficient valuation model. If need be, approximations have to be made as in the Longstaff-Schwartz approach to valuation of American option.
- *Aggregation.* Next, let  $V(k, s, t)$  denote the value of instrument  $k$  under scenario  $s$  at future time  $t$ . We now compute the aggregates over each netting set (NS):

$$V_{NS}(s, t) = \sum_{k \in NS} V(k, s, t).$$

- *Postprocessing.* This step requires applying the logic corresponding to risk mitigants such as collateral, additional termination events, etc.
- *Extraction.* We can now collapse the scenarios to metrics such as EE, other metrics can be computed out of scenario level data.

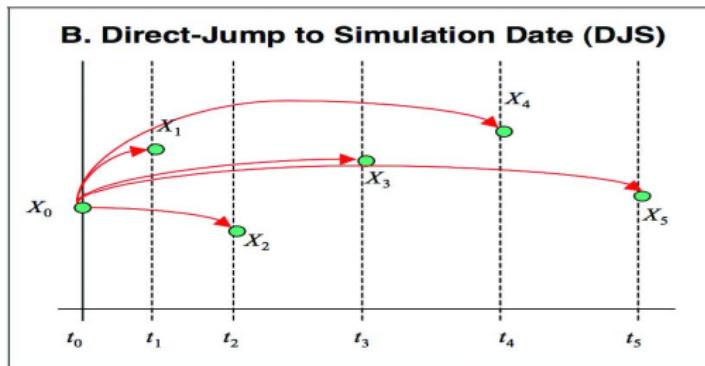
# Monte Carlo methodology

The graph below illustrates the path dependent simulation method.



# Monte Carlo methodology

The graph below illustrates the long jump simulation method.



# CVA formula

- We will now combine the concepts introduced earlier in the course (default probability and recovery rate) with the concept of credit exposure - this is important for accurate pricing of counterparty risk.
- We will first make three simplifying assumptions:
  - (i) The institution (bank, denoted B in the following) is default free, while the counterparty (denoted C) is credit risky. We are ignoring here the debt valuation adjustment (DVA).
  - (ii) It is possible to perform risk free valuation. We are thus assuming the existence of a discount rate.
  - (iii) The credit exposure and default probability are independent. This ignores the wrong way risk.
- Later we will relax these assumptions.
- We first derive the formula for CVA and then discuss its use within an institution.

# CVA formula

- When valuing a financial transaction, it is possible to separate the counterparty risk as follows:

$$\text{Risky Value} = \text{Riskless Value} - \text{CVA}.$$

To see this, we proceed as follows.

- Let  $\hat{V}(t, T)$  denote the (counterparty credit) risky value of a netted set of derivatives positions with a maximum maturity  $T$ , and let  $V(t, T)$  denote its risk-free value of the positions. We assume that the mark to market of the positions is given by  $V(t, T)$
- Then  $V(s, T)$ , where  $t \leq s \leq T$ , denotes the future (uncertain) MtM. It is related to  $V(t, T)$  by  $V(t, T) = P(t, s)V(s, T)$ , where  $P(t, u)$  is the riskless discount factor (zero coupon bond).
- As usual, the default time of the counterparty is denoted by  $\tau$ .
- We consider two cases:

# CVA formula

- Case 1. *Counterparty does not default before T.* In this case, the risky position is equivalent to the risk-free position and we write the corresponding payoff as

$$\mathbf{1}_{\tau > T} V(t, T).$$

- Case 2. *Counterparty defaults before T.* In this case, the payoff consists of two terms: the value of the position that would be paid before the default time (all cash flows before default will still be paid by the counterparty)

$$\mathbf{1}_{\tau \leq T} V(t, \tau),$$

and the payoff at default calculated as follows.

- Here, if the MtM of the trade at the default time,  $V(\tau, T)$ , is positive, then the institution will receive a recovery rate  $R$  of the risk-free value of the derivatives positions. If it is negative then they will still have to settle this amount. Hence, the default payoff at time  $\tau$  is:

$$\mathbf{1}_{\tau \leq T} (RV(\tau, T)^+ + V(\tau, T)^-).$$

# CVA formula

- Putting these payoffs together, we have the following expression for the value of the risky position under the risk-neutral measure:

$$\widehat{V}(t, T) = \mathbb{E}^Q [1_{\tau > T} V(t, T) + 1_{\tau \leq T} (V(t, \tau) + RV(\tau, T)^+ + V(\tau, T)^-)].$$

- Using the relation  $x^- = x - x^+$ , we rearrange the terms as follows:

$$\begin{aligned}\widehat{V}(t, T) &= \mathbb{E}^Q [1_{\tau > T} V(t, T) + 1_{\tau \leq T} (V(t, \tau) + RV(\tau, T)^+ + V(\tau, T) - V(\tau, T)^+)] \\ &= \mathbb{E}^Q [1_{\tau > T} V(t, T) + 1_{\tau \leq T} (V(t, \tau) + (R - 1)V(\tau, T)^+ + V(\tau, T))].\end{aligned}$$

- Since  $V(t, \tau) + V(\tau, T) = V(t, T)$ , we can rewrite this as

$$\widehat{V}(t, T) = \mathbb{E}^Q [1_{\tau > T} V(t, T) + 1_{\tau \leq T} (V(t, T) + (R - 1)V(\tau, T)^+)].$$

# CVA formula

- Finally, using  $1_{\tau > T} V(t, T) + 1_{\tau \leq T} V(t, T) = V(t, T)$ , we can write:

$$\widehat{V}(t, T) = V(t, T) - E^Q[1_{\tau \leq T}(1 - R)V(\tau, T)^+].$$

The above equation defines the risky value of a netting set of derivatives positions with respect to the risk-free value.

- We write it as

$$\widehat{V}(t, T) = V(t, T) - CVA(t, T),$$

$$CVA(t, T) = E^Q[1_{\tau \leq T}(1 - R)V(\tau, T)^+],$$

where the term  $CVA(t, T)$  is called the credit value adjustment (CVA).

# CVA formula

- The CVA formula is very useful because it allows us to value a transaction and computing counterparty risk separately.
- In principle, the CVA components can be handled centrally.
- Within a bank, one desk is responsible for risk free valuation (front office) and one is responsible for counterparty risk valuation (CVA desk).
- In reality, things are a bit more complicated. Due to risk mitigants such as netting and collateral, CVA is not truly linear.
- This means that the risky value of a transaction is defined within the netting set of other transactions, and cannot be calculated individually.

# Standard CVA formula

- The standard CVA formula is an approximation to the CVA formula above, and is used throughout the industry. To derive it, we write:

$$CVA(t, T) = (1 - \bar{R})E^Q[1_{u \leq T} V^*(u, T)^+],$$

where  $\bar{R}$  is the expected recovery rate, and

$$V^*(u, T) \triangleq V(u, T) | \tau = u.$$

- This is a key point in the analysis, as the above statement requires the exposure at a future date  $V(u, T)$ , conditioned on the counterparty default at  $\tau = u$ . Ignoring wrong-way risk means that  $V^*(u, T) = V(u, T)$ .
- Since the expected value above is taken over all times before the final maturity, we represent it as an integral over all possible default times  $u$ :

$$CVA(t, T) = (1 - \bar{R})E^Q \left[ \int_t^T P(t, u) V(u, T)^+ dQ(t, u) \right],$$

where  $Q(t, u)$  is the cumulative counterparty default probability.

## Standard CVA formula

- We denote by

$$EE_t(u, T) \triangleq E^Q [P(t, u) V(u, T)^+]$$

the discounted expected exposure calculated under the risk neutral measure.

- Assuming for simplicity that the default probabilities are deterministic, we thus have:

$$CVA(t, T) = (1 - \bar{R}) \int_t^T EE_t(u, T) dQ(t, u).$$

- Finally, we compute approximately the above expression using:

$$CVA(t, T) \approx (1 - \bar{R}) \sum_{i=1}^m EE_t(t, T_i) (Q(t, T_i) - Q(t, T_{i-1})),$$

where  $t = T_0 < T_1 < \dots < T_m = T$  (using the trapezoid rule would, of course, lead to a better accuracy).

## Standard CVA formula

- The last formula is the *standard CVA formula*. We will write it schematically as

$$\text{CVA} \approx (1 - \text{Rec}) \sum_{i=1}^m \text{DiscFact}(T_i) \text{EE}(T_i) \text{DefProb}(T_{i-1}, T_i).$$

- It has four components:
  - LGD represented by  $1 - \text{Rec}$ , where Rec stands for recovery rate.
  - Discount factors to the event dates.
  - Expected exposure to the event dates.
  - Default probabilities between to the event dates.

## CVA as a spread

- With further simplifying assumptions, we can obtain a simple expression for CVA linking it to the credit spread of the counterparty. To do this, we work with the undiscounted expected exposure and write the CVA formula as:

$$CVA(t, T) = (1 - \bar{R}) \int_t^T P(t, u) EE(u, T) dQ(t, u).$$

- Suppose that we can approximate the undiscounted expected exposure term,  $EE(u, T)$ , as a fixed known amount, namely the EPE. The fixed EPE could be approximately computed as the EE averaged over time:

$$\begin{aligned} EPE &= \frac{1}{T-t} \int_t^T EE(t, u) du \\ &\approx \frac{1}{m} \sum_{i=1}^m EE(t, T_i). \end{aligned}$$

- Clearly, this approximation can be made if the relationship between EPE, default probability, and discount factors can be assumed homogeneous through time.

## CVA as a spread

- Using this approach, the CVA can be expressed as

$$CVA(t, T) = (1 - \bar{R})E^Q \left[ \int_t^T P(t, u) dQ(t, u) \right] EPE.$$

- We see that this is simply the value of CDS protection on a notional equal to the EPE. Hence we have the following approximation giving a running CVA (i.e. expressed as a spread):

$$CVA \approx EPE \times \text{Spread}.$$

- The approximate formula, which is a variant of the credit triangle studied earlier in the course, is often useful for intuitive understanding of the drivers of CVA.
- We can use the corresponding risky annuity to convert the spread into an upfront value.

# CVA calibration issues

- For CVA calibration, we should be calculating the exposure under the risk neutral measure rather than the physical measure.
- The drift terms associated with the physical measure may lead to price distortions.
- This requires calibration to the cross-sectional market data rather than historical market data.
- For example, in the case of interest rate derivatives, we should be using currently observed swap and money market rates, cap / floor and swaption volatilities, smile data, etc, rather than historical time series of swap rates.

# Semi-analytical methods for CVA

- In case of specific instruments it is possible to evaluate CVA in closed form.
- Such explicit formulas are of limited use, as they apply only to stand-alone positions without accounting for netting or collateral.
- These formulas are, however, of interest as they can be used for quick calculations and intuitive understanding of CVA.
- Below we discuss two examples of such formulas, namely an option and an interest rate swap.

# CVA for an option

- In this case we have a simplification since the exposure of the long option position can never be negative:

$$\begin{aligned} CVA_{\text{opt}}(t, T) &= (1 - \bar{R}) E^Q [1_{\tau < T}] E^Q [P(t, \tau) V_{\text{opt}}(\tau, T)] \\ &= (1 - \bar{R}) Q(t, T) V_{\text{opt}}(t, T), \end{aligned}$$

where  $V(t, T)_{\text{opt}}$  is the option premium.

- This means that the value of the risky option can be calculated as:

$$\begin{aligned} \hat{V}_{\text{opt}}(t, T) &= V_{\text{opt}}(t, T) - CVA_{\text{opt}}(t, T) \\ &= V_{\text{opt}}(t, T) - (1 - \bar{R}) Q(t, T) V_{\text{opt}}(t, T) \\ &= V_{\text{opt}}(t, T) S(t, T) + \bar{R} Q(t, T) V_{\text{opt}}(t, T). \end{aligned}$$

- We see that, with zero recovery, the risky premium is the risk-free value multiplied by the survival probability over the life of the option.

# CVA for a swap

- In the case of a swap, the exposure can be positive or negative.
- Consider a receiver swap (the calculation is analogous for a payer); its CVA is

$$\begin{aligned} CVA_{\text{rec swap}}(t, T) &= (1 - \bar{R}) E^Q \left[ \int_t^T P(t, u) V_{\text{rec swap}}(u, T)^+ dQ(t, u) \right] \\ &\approx (1 - \bar{R}) \sum_{i=1}^m E^Q [P(t, T_i) V_{\text{rec swap}}(T_i, T)^+ (Q(t, T_i) - Q(t, T_{i-1}))], \end{aligned}$$

where  $T_i$ ,  $i = 1, \dots, m$  are the fixed coupon payment days.

- Assuming that rates are independent of the credit of C, we write it as

$$CVA_{\text{rec swap}}(t, T) \approx (1 - \bar{R}) \sum_{i=1}^m E^Q [P(t, T_i) V_{\text{rec swap}}(T_i, T)^+] (Q(t, T_i) - Q(t, T_{i-1})).$$

# CVA for a swap

- We note that

$$[P(t, T_i) V_{\text{rec swap}}(T_i, T)^+]$$

represents the time  $t$  premium on a payer swaption expiring on  $T_i$ .

- Therefore, the CVA of a receiver swap can be written as:

$$CVA_{\text{rec swap}}(t, T) = (1 - \bar{R}) \sum_{i=1}^m (Q(t, T_i) - Q(t, T_{i-1})) V_{\text{pay swaption}}(t, T_i, T).$$

- The intuition is that the counterparty has the “option” to default at any point in the future and therefore cancel the trade (execute the reverse position).
- The values of these swaptions are weighted by the relevant default probabilities and recovery is taken into account.
- This formula was obtained by Sorensen and Bolliger in 1994.

# CVA for a swap

- The Sorensen and Bollier gives us a useful insight on CVA calculations, namely that a CVA calculation is at least as complex as pricing the underlying instrument.
- To price the swap CVA we need to know swaption volatility across expirations and strikes. The price of the swap does not depend on volatility and yet its CVA does.
- The asymmetry between payer and receiver swaps is captured naturally. The receiver (payer) swaptions corresponding to the payer (receiver) swaps are ITM (OTM).
- In the latter case, the strike of the swaptions moves significantly out of the money when the bank receives a quarterly cash flow while not having yet made a semiannual payment.
- The above analysis can be extended to other products where any transaction can be represented as a basket of options.

## Bilateral CVA formula

- One of the assumptions used in the derivation of the CVA formula was that the bank could not default. Historically, banks have charged their corporate counterparties CVA linked to their credit quality and bank's exposure.
- In the post-Lehman world it became painfully clear that banks themselves are credit risky entities.
- Since credit exposure has a liability component (the negative exposure defined earlier), this could be included in the counterparty credit valuation model.
- This leads us to the concept of the *debt valuation adjustment* (DVA).
- DVA is a somewhat controversial concept. While it resolves some theoretical issues with CVA, it leads to some unintuitive consequences that we will discuss later.

# Bilateral CVA formula

- We shall now find an expression for the risky value  $\hat{V}(t, T)$  of a netted set of derivatives positions with a maximum maturity date  $T$ , where, unlike above, we assume that both institution B may and its counterparty C may default.
- Denote the default times of the institution and its counterparty by  $\tau_B$ , and  $\tau_C$ , respectively, and their recovery rates by as  $R_B$  and  $R_C$ , respectively.
- Let  $\tau^1 = \min(\tau_B, \tau_C)$  denote the first-to-default time of the institution and counterparty.
- Following the notation and logic of the previous section, we consider the following cases:

## Bilateral CVA formula

- 1. *Neither counterparty nor bank defaults before T.* In this case, the risky position is equivalent to the risk-free position and we write the corresponding payoff as:

$$1_{\tau^1 > T} V(t, T).$$

- 2. *Counterparty defaults first and also before time T.* This is the default payoff as in the previous section

$$1_{\tau^1 \leq T} 1_{\tau^1 = \tau_C} (R_C V(\tau^1, T)^+ + V(\tau^1, T)^-).$$

- 3. *Bank defaults first and also before time T.* This is an additional term compared with the unilateral CVA case and corresponds to the institution itself defaulting. If B owes money to C (negative MtM) then it will pay only a recovery fraction of this. If the C owes B money (positive MtM) then it will still receive this. Hence, the payoff is the opposite of case 2 above:

$$1_{\tau^1 \leq T} 1_{\tau^1 = \tau_B} (R_B V(\tau^1, T)^- + V(\tau^1, T)^+).$$

# Bilateral CVA formula

- 4. If either the bank or counterparty does default then all cash flows prior to the first-to-default date will be paid. The payoff is

$$\mathbf{1}_{\tau^1 \leq T} V(t, \tau).$$

- Putting the above payoffs together, we obtain the following expression for the value of the risky position:

$$\begin{aligned}\widehat{V}(t, T) = & \mathbb{E}^Q [\mathbf{1}_{\tau^1 > T} V(t, T) + \mathbf{1}_{\tau^1 \leq T} \mathbf{1}_{\tau^1 = \tau_C} (R_C V(\tau^1, T)^+ + V(\tau^1, T)^-) \\ & + \mathbf{1}_{\tau^1 \leq T} \mathbf{1}_{\tau^1 = \tau_B} (R_B V(\tau^1, T)^- + V(\tau^1, T)^+) + \mathbf{1}_{\tau^1 \leq T} V(t, \tau)].\end{aligned}$$

# Bilateral CVA formula

- We can simplify the above expression as:

$$\begin{aligned}\widehat{V}(t, T) = & \mathbb{E}^Q [1_{\tau^1 > T} V(t, T) + 1_{\tau^1 \leq T} V(t, \tau^1) + 1_{\tau^1 \leq T} V(\tau^1, T) \\ & + 1_{\tau^1 \leq T} 1_{\tau^1 = \tau_C} (R_C V(\tau^1, T)^+ - V(\tau^1, T)^+) \\ & + 1_{\tau^1 \leq T} 1_{\tau^1 = \tau_B} (R_B V(\tau^1, T)^- - V(\tau^1, T)^-)].\end{aligned}$$

- This can finally be written as

$$\begin{aligned}\widehat{V}(t, T) = & V(t, T) + \mathbb{E}^Q [1_{\tau^1 \leq T} 1_{\tau^1 = \tau_C} (R_C V(\tau^1, T)^+ - V(\tau^1, T)^+) \\ & + 1_{\tau^1 \leq T} 1_{\tau^1 = \tau_B} (R_B V(\tau^1, T)^- - V(\tau^1, T)^-)].\end{aligned}$$

- After rearranging the terms, we write this as

$$\begin{aligned}\widehat{V}(t, T) = & V(t, T) - \mathbb{E}^Q [1_{\tau^1 \leq T} 1_{\tau^1 = \tau_C} (1 - R_C) V(\tau^1, T)^+ \\ & + 1_{\tau^1 \leq T} 1_{\tau^1 = \tau_B} (1 - R_B) V(\tau^1, T)^-].\end{aligned}$$

# Bilateral CVA formula

- We can thus identify the *bilateral CVA* (BCVA) as

$$\begin{aligned} BCVA(t, T) = & \mathbb{E}^Q [1_{\tau^1 \leq T} 1_{\tau^1 = \tau_C} (1 - R_C) V(\tau^1, T)^+ \\ & + 1_{\tau^1 \leq T} 1_{\tau^1 = \tau_B} (1 - R_B) V(\tau^1, T)^-]. \end{aligned}$$

- Finally, under the assumptions of no wrong-way risk and no simultaneous default between the default of the institution and its counterparty, we obtain a formula analogous to that derived in the case of unilateral CVA:

$$\begin{aligned} BCVA(t, T) = & (1 - \bar{R}_C) \mathbb{E}^Q \left[ \int_t^T P(t, u) V(u, T)^+ S_B(t, u) dQ_C(t, u) \right] \\ & + (1 - \bar{R}_B) \mathbb{E}^Q \left[ \int_t^T P(t, u) V(u, T)^- S_C(t, u) dQ_B(t, u) \right], \end{aligned}$$

where  $S(t, u)$  denotes survival probability.

## Bilateral CVA formula

- Proceeding as in the calculation above for CVA, we can approximate this formula by

$$\begin{aligned} BCVA(t, T) &\approx (1 - \bar{R}_C) \sum_{i=1}^m EE_t(t, T_i) S_B(t, T_{i-1}) (Q_C(t, T_i) - Q_C(t, T_{i-1})) \\ &+ (1 - \bar{R}_B) \sum_{i=1}^m NEE_t(t, T_i) S_C(t, T_{i-1}) (Q_B(t, T_i) - Q_B(t, T_{i-1})) \end{aligned}$$

where  $t = T_0 < T_1 < \dots < T_m = T$  is a suitable time grid.

- This approximation is a basis for the standard BCVA formula.

# Standard BCVA formula

- We will state the standard BCVA formula as

$$\begin{aligned} \text{BCVA} = & (1 - \text{Rec}_C) \sum_{i=1}^m \text{DiscFact}(T_i) \text{EE}(T_i) \text{SurvProb}_B(0, T_{i-1}) \text{DefProb}_C(T_{i-1}, T_i) \\ & + (1 - \text{Rec}_B) \sum_{i=1}^m \text{DiscFact}(T_i) \text{NEE}(T_i) \text{SurvProb}_C(0, T_{i-1}) \text{DefProb}_B(T_{i-1}, T_i). \end{aligned}$$

- The first term is essentially the CVA formula (adjusted for the bank's own survival).
- The second term represents a negative contribution (since NEE is negative) and is called the *debt valuation adjustment* (DVA).

## Standard BCVA formula

- In practice, the survival probability of B is not included in the CVA formula, and the survival probability of C is not included in the DVA formula.
- Also, the correlation of defaults between B and C is ignored. Such a correlation, if positive, would impact CVA and DVA.
- Finally, the definitions of EE and ENE rely on standard valuation methodologies, rather than the actual close-out values that may be realized in case of a default.

## BCVA as a spread

- To gain understanding the meaning of BCVA we can carry out the same approximations as in the case of CVA to derive:

$$\text{BCVA} \approx \text{EE} \times \text{Spread}_C + \text{ENE} \times \text{Spread}_B.$$

- This can be interpreted as the fact that the institution should subtract from CVA the component accounting for its own credit.
- In particular, if we can assume that  $\text{ENE} \approx -\text{EE}$ , then

$$\text{BCVA} \approx \text{EE} \times (\text{Spread}_C - \text{Spread}_B),$$

and the credit charge is proportional to the difference in the spreads.

## Properties of DVA and the DVA controversy

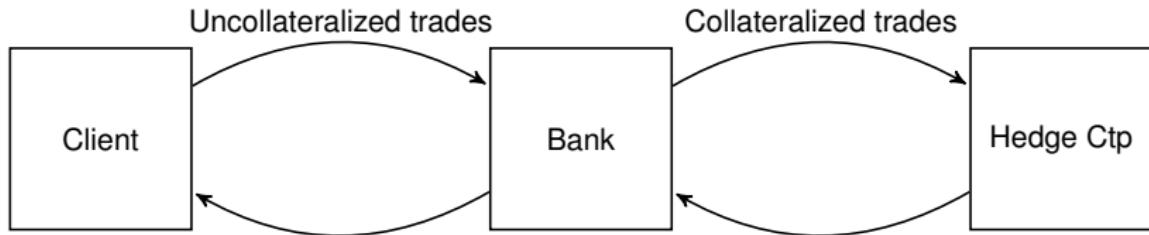
- A bank using DVA attaches value to its own potential default. This may have consequences that may (or may not) be reasonable.
- A credit risky derivative can be worth more than a risk-free derivative: the BCVA can be negative (unlike CVA).
- If all counterparties agree on using BCVA, pricing counterparty is a zero sum game.
- Risk mitigants can increase BCVA. For example, netting (which increases CVA) may increase DVA.
- A bank can show accounting profits as a result of widening (own) credit spreads.
- DVA is largely disregarded in pricing and replaced as a funding benefit adjustment (FBA). We shall discuss this relationship later.
- Basel III requirements ignore DVA.

# Funding value adjustment (FVA)

- Despite the increased use of collateral, a significant portion of OTC derivatives remain uncollateralized or undercollateralized.
- Funding costs (or benefits) arise in the following situations:
  - (i) Undercollateralized transactions give rise to both costs and benefits. For example, a non-CSA counterparty creates a funding requirement for a bank trading with it (this relates to the need to hedge the transaction with a CSA counterparty).
  - (ii) Even if collateral is posted, it may not be usable. Collateral that cannot be rehypothecated is useless from a funding perspective.

## Funding value adjustment (FVA)

- The funding cost / benefits can be illustrated by means of Figure 1. A bank enters into uncollateralized trades with a counterparty which are hedged through collateralized trades with another bank.
- If the trades have a positive value, than the hedges have a negative (off-setting) value and the bank needs to post a collateral.
- The return on the collateral is (typically) OIS, and so there is an associated *cost*, if the bank cannot fund at OIS.



**Figure:** 1. Uncollateralized trade hedged with a collateralized counterparty

## Funding value adjustment (FVA)

- On the other hand, if the trades have a negative value, the counterparty on the hedges posts collateral, which results in a funding *benefit* for the bank.
- Note that the benefit can only arise if the collateral can be reused, i.e. rehypothecation is allowed.
- The above analogy does not apply if:
  - (i) The bank does not hedge its trades. There should still be an FVA without exchange of collateral.
  - (ii) The profit (spread) on the trades is not part of its value, but the FVA may be charged.
  - (iii) ...
- In general, funding costs arise from the uncollateralized positive value of a portfolio.

## FVA formula (FVA)

- The FVA formula below has a structure similar to the BCVA formula discussed earlier. We shall state this formula without justification and will derive it later.
- Let  $\text{Spr}_b(T_{i-1}, T_i)$  and  $\text{Spr}_l(T_{i-1}, T_i)$  denote the funding spread for borrowing and lending, respectively. Then

$$\begin{aligned} \text{FVA} &= \sum_{i=1}^m \text{DiscFact}(T_i) \text{EE}(T_i) \text{SurvProb}_B(0, T_{i-1}) \text{SurvProb}_C(0, T_{i-1}) \text{Spr}_b(T_{i-1}, T_i) \delta_i \\ &\quad + \sum_{i=1}^m \text{DiscFact}(T_i) \text{NEE}(T_i) \text{SurvProb}_C(0, T_{i-1}) \text{SurvProb}_B(0, T_{i-1}) \text{Spr}_l(T_{i-1}, T_i) \delta_i \\ &= \text{FCA} + \text{FBA}. \end{aligned}$$

- The first term is called the *funding cost adjustment* (FCA), and the second term represents the *funding benefit adjustment* (FBA).

## FVA formula (FVA)

- Note that we have the following (approximate) credit triangle formula for FVA:

$$FVA \approx Spr_b \times EE + Spr_l \times NEE.$$

Here, as usual, we assume that the funding spreads and exposures are constant.

- In case if  $Spr_b(T_{i-1}, T_i) \approx Spr_l(T_{i-1}, T_i)$ , we can use the identity

$$EE + NEE = EFV$$

to conclude that

$$FVA \approx \sum_{i=1}^m \text{DiscFact}(T_i) \text{EE}(T_i) \text{SurvProb}_B(0, T_{i-1}) \text{SurvProb}_C(0, T_{i-1}) \text{Spr}(T_{i-1}, T_i) \delta_i.$$

# FVA formula (FVA)

- This is analogous to BCVA which is a sum of CVA and DVA: CVA and FCA are related to the exposure, while DVA and FBA are related to the negative exposure.
- There are, however, important differences between FVA and BCVA.
- The FVA references a bank's own spread (both for borrowing and lending), while BCVA references counterparty's spread in the CVA term and the bank's spread in the DVA term.
- Funding spread should not be confused with the CDS spread (used in the BCVA formula). This spread reflects funding cost in excess of OIS.

# FVA in practice

- We mentioned above that DVA is generally viewed as a funding benefit.
- Although DVA and FBA are mathematically similar, there are some differences between them:
  - (i) DVA requires risk neutral default probability calibrated to the CDS market, while FBA requires a bank's own funding spread.
  - (ii) DVA and FBA are applied at the level of a netting set. In fact, FBA is relevant to the entire portfolio of all counterparties.

# Margin value adjustment (MVA)

- Margin valuation adjustment arises from the initial margin (IM) requirements imposed by CCPs on their clearing members.
- It is appropriate to consider it separately with MVA as it quantifies the cost of posting IM as well as other funds such as default fund or liquidity fund discussed above.
- It is significant to separate it from FVA, as the methodologies to determine IM are conceptually quite complicated.

# Margin value adjustment (MVA)

- Similarly to the xVA's discussed thus far, the MVA is an integral of the expected initial margin profile.
- Specifically, the MVA is given by the following formula:

$$\text{MVA} = \sum_{i=1}^m \text{DiscFact}(T_i) \text{EIM}(T_i) \text{SurvProb}(0, T_{i-1}) \text{Spr}_{IM}(T_{i-1}, T_i) \delta_i.$$

Here EIM is the expected initial margin, and  $\text{Spr}_{IM}$  is the spread between the funding cost of posting the initial margin and the remuneration rate.

- The remuneration rate on initial margin may be lower than OIS.
- IM is calculated via Monte Carlo simulations.
- Since this has to be done for each time slice going forward inside a Monte Carlo simulation, the problem of calculating MVA is computationally expensive.
- One way around it is to use a Longstaff-Schwarz style algorithm.

# CME / LCH basis

- The impact of MVA has recently manifested itself as the *CME-LCH basis* for interest rate swaps.
- Until April 2015 the cost of clearing a swap had been steady and was about 0.15 bp.
- In May 2015, this basis started widening dramatically, affecting primarily longer dated swaps.
- At the peak, the cost of clearing of 10Y swap at CME was higher by about 2.5 bp than at LCH.
- For a swap with a notional value of \$1 bn, that represents a difference of around \$2.5 mm.

## CME / LCH basis

- Figure 2 plots the historical time series of the CME-LCH basis, and is taken from the Clarus Technology blog.

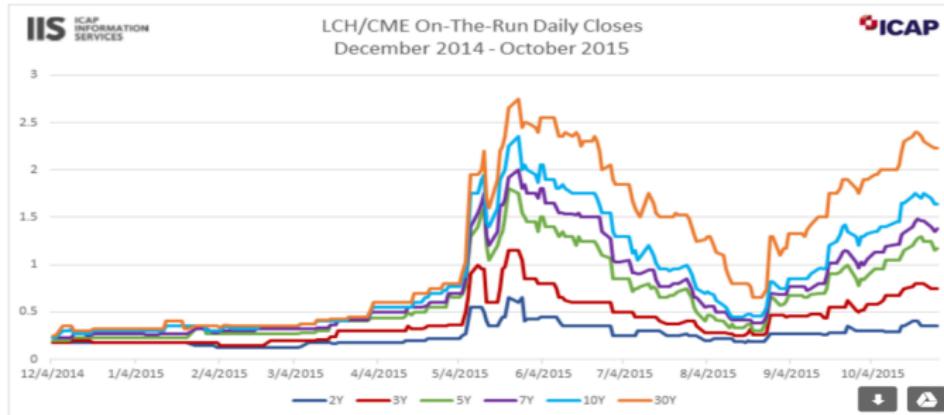


Figure: 2. CME-LCH spread: historical time series

# CME / LCH basis

- Figure 3 shows a market snapshot on October 28, 2015, and is also taken from the Clarus Technology blog.

The screenshot shows a web browser window with the title "19981". The URL bar contains "19981". The page content is a market snapshot titled "10/28 17:00 GMT [ REUTERS CAPITAL MARKETS SOURCE:TCAP ]". It includes a subtitle "USD INDICATIVE IRS SEMI BOND LCH / CME BASIS". Below this is a table with columns: SB, Term, LCH MID, CME(bps), and CME MID. The data rows show the spread for various maturities from 1Y to 30Y.

SB	Term	LCH MID	CME(bps)	CME MID
	1Y	0.49685	+0.1500	0.49835
	2Y	0.74477	+0.3500	0.74827
	3Y	0.98391	+0.7500	0.99141
	4Y	1.19722	+1.0000	1.20722
	5Y	1.38652	+1.1700	1.39822
	6Y	1.55191	+1.3000	1.56491
	7Y	1.69400	+1.3800	1.70780
	8Y	1.81150	+1.4600	1.82610
	9Y	1.90895	+1.5500	1.92445
	10Y	1.99148	+1.6400	2.00788
	12Y	2.12866	+1.7000	2.14566
	15Y	2.26651	+1.7800	2.28431
	20Y	2.40151	+1.9300	2.42081
	25Y	2.46493	+2.0800	2.48573
	30Y	2.50155	+2.2300	2.52385

Figure: 3. Term structure of the CME-LCH spread

# Mechanics of the CME / LCH basis

- CME cleared volume is mostly driven by fixed income asset managers and other end users of swaps. They generally look to swap the fixed coupons on their bond holdings into floating coupons, and thus they tend to pay fixed on swaps.
- As a consequence, on the client-dealer swap trade, the dealer is receiving fixed. To hedge their risk, the dealers pay fixed in the interdealer swap market or sell ED futures.
- Traditionally, the dealers have been clearing their swap trades at LCH.
- The result is that dealer books at CME are largely directional, with little offsetting exposure, and the resulting margin and funding costs are thought to be the source of the price difference.

# Capital value adjustment (KVA)

- Banks have to hold considerable resources, *regulatory capital*, against their OTC derivatives positions.
- Capital requirements against OTC derivatives consist of three components:
  - (i) *CVA charge*. It reflects the MTM volatility of the counterparty risk due to credit spread volatility.
  - (ii) *Default risk capital charge*. It is a liquidity reserve for the case of a counterparty default.
  - (iii) *Market risk capital charge*. Banks are expected to hedge their market risk, this charge is usually not assessed.

# Capital value adjustment (KVA)

- The KVA is given by the following formula:

$$\text{KVA} = \sum_{i=1}^m \text{DiscFact}(T_i) \text{EC}(T_i) \text{SurvProb}(0, T_{i-1}) \text{CC}(T_{i-1}, T_i) \delta_i.$$

Here EC is the expected capital profile, and CC is the cost of capital.

- In order to compute KVA, one needs a methodology for generating future capital scenarios.

# Wrong way risk

- We showed earlier that, under the assumption of independence of counterparty credit and exposure, the CVA can be expressed as a product of the counterparty credit spread and the exposure.
- If the independence assumption does not hold, the CVA can
  - (i) increase relative to the standard value, in which case we talk about the *wrong way risk* (WWR),
  - (ii) or decrease, which is termed the *right way risk* (RWR).
- In the following, WWR means both WWR and RWR.

## Examples of wrong way risk

- Buying a put option on an underlying positively correlated with the counterparty.
- Trading FX forwards or cross currency interest rate swaps with a sovereign.
- Interest rate swaps: high interest rates may trigger defaults.
- Buying protection on a CDS on a name correlated with the counterparty.
- Commodity forwards / swaps with counterparties exposed to the underlying.

# Quantifying WWR

- A natural way of incorporating WWR into the standard CVA formula is through the conditional EE.
- We can rewrite the general expression for CVA in the form

$$\begin{aligned} CVA(t, T) &= (1 - \bar{R}) E^Q [1_{\tau \leq T} V(\tau, T)^+] \\ &= (1 - \bar{R}) \int_t^T E^Q [V(\tau, T)^+ | \tau = u] dQ(t, u). \end{aligned}$$

- Replacing, as usual, the integral with its discrete approximation, we can write this as

$$CVA(t, T) \approx (1 - \bar{R}) \sum_{i=1}^m EE_t(t, T_i | \tau = T_i) (Q(t, T_i) - Q(t, T_{i-1})),$$

where  $EE_t(t, T_i | \tau = T_i)$  denotes the expected exposure conditioned on the time  $T_i$  being the default time of the counterparty.

# Quantifying WWR

- Earlier in these Notes we derived an expression for the expected exposure in a simple normal model where the asset value follows the process, namely

$$V(t) = \mu t + \sigma \sqrt{t} Y,$$

where  $Y$  is a standard normal variable.

- Then

$$EE(s) = s\mu N(\mu\sqrt{s}/\sigma) + \sqrt{s}\sigma\varphi(\mu\sqrt{s}/\sigma),$$

where  $s$  denotes the time horizon.

- We can derive an analogous expression for EE in the presence of WWR assuming a constant default intensity:

$$Q(t) = 1 - e^{-\lambda t}.$$

- Assume that the time to default is given by:

$$\tau = Q^{-1}(N(Z)),$$

where  $Z$  is standard normal.

# Quantifying WWR

- Finally (we are mimicking here the Gaussian copula model), we set

$$Y = \rho Z + \sqrt{1 - \rho^2} \varepsilon,$$

with standard normal  $\varepsilon$ .

- Then,

$$\begin{aligned} EE[s|\tau = s] &= E[V(s)^+ | \tau = s] \\ &= E[V(s)^+ | Z = N^{-1}(Q(\tau))] \\ &= \int_{-\mu(s)/\sigma(s)}^{\infty} (\tilde{\mu}(u) + \tilde{\sigma}(u)x) \varphi(x) dx, \end{aligned}$$

where

$$\begin{aligned} \tilde{\mu}(s) &= \mu(s) + \rho\sigma(s)N^{-1}(Q(\tau)) \\ \tilde{\sigma}(s) &= \sigma(s)\sqrt{1 - \rho^2}. \end{aligned}$$

# Quantifying WWR

- Carrying out the integral we find that the conditional EE is given by

$$EE[s|\tau = s] = \tilde{\mu}(s) N(\tilde{\mu}(s)/\tilde{\sigma}(s)) + \tilde{\sigma}(s) \varphi(\tilde{\mu}(s)/\tilde{\sigma}(s)).$$

- Note that this expression reduces to the previous formula if  $\rho = 0$ .

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