History of Option Pricing Theory After Black-Scholes' Work

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12 Apr 2014

It is widely agreed that the modern theory of option pricing began in 1973[1] with the publication of the Black–Scholes–Merton model. Except for the early years (pre-1973), this history is restricted to papers that use the no arbitrage and complete markets technology to priceoptions. Equilibrium option pricing models are not discussed herein. In particular, this excludes the consideration of option pricing in incomplete markets.

1. Equity Derivatives: Black–Scholes–Merton model And More

The original Black–Scholes–Merton model is based on the following five assumptions: (i) competitive markets, (ii) frictionless markets, (iii) geometric Brownian motion, (iv) deterministic interest rates, and (v) no credit risk. The first two assumptions — competitive and frictionless markets —are the mainstay of finance. Competitive markets means that all traders act as price takers, believing their trades have no impact on the market price. Frictionless markets imply that there are no transaction costs nor trade restrictions, for example, no short sale constraints. Geometric Brownian motion implies that the stock price is lognormally distributed with a constant volatility. Deterministic interest rates are selfexplanatory. No credit risk means that the investors(all counterparties) who trade financial securities will not default on their obligations. Today the formula is more commonly derived using the Ito formula and the option replication idea introduced by Merton[16].

Extensions of the Black–Scholes–Merton model that relaxed assumptions (i)–(iii) quickly flourished. Significant papers relaxing the geometric Brownianmotion assumption include those by Merton[18] and Cox and Ross[5], who studied jump and jump-diffusion processes. Merton's paper also included the insight that if unhedgeable jump risk is diversifiable, then it carries no risk premium. Under this assumption, one can value jump risk using the statistical probability measure, enabling the simple pricing of options in an incomplete market. This insight was subsequently invoked in the context of stochastic volatility option pricing and in the context of pricing credit risk derivatives.

Option pricing with stochastic volatility in incomplete markets was subsequently studied by Hull and White[11] and Heston[9].

A new class of Levy processes was introduced by Madan and Milne[15] into option pricing and generalized by Carr et al.[3] Levy processes have the nice property that their characteristic function is known, and it can be shown that an option's price can be represented in terms of the stock price's characteristic function. This leads to some alternative numerical procedures for computing option values using fast Fourier transforms.

The relaxation of the frictionless market assumption has received less attention in the literature. The inclusion of transaction costs into option pricing was originally studied by Leland[14], while Heath and Jarrow[7] studied the imposition of margin requirements. A more recent investigation into the impact of transaction costs on option pricing, using the martingale pricing technology.

The relaxation of the competitive market assumption was first studied by Jarrow[12] via the consideration of a large trader whose trades change the price. Jarrow's approach maintains the no arbitrage assumption, or a no market manipulation assumption.

The Black–Scholes–Merton model has been applied to foreign currency options[6] and to all types of exotic options on both equities and foreign currencies.

2. Interest Rate Derivatives: Heath–Jarrow–Morton model And More

Recall that a defining characteristic of the Black–Scholes–Merton model is that it assumes deterministic interest rates. This assumption limits its usefulness in two ways. First, it cannot be used for long-dated contracts. Indeed, for long-dated contracts (greater than a year or two), interest rates cannot be approximated as being deterministic. Second, for short dated contracts, if the underlying asset's price process is highly correlated with interest rate movements, then interest rate risk will affect hedging, and therefore valuation. The extreme cases, of course, are interest rate derivatives where the underlyings are the interest rates themselves.

A class of interest rate pricing models were developed by Vasicek[20], Brennan and Schwartz[2], and Cox et al.(CIR)[4]. This class, called the spot rate models, had two limitations. First, they depended on the market price of interest rate risk, or equivalently, the expected return on default free bonds. This dependence, just as with the option pricing models pre-Black–Scholes–Merton, made their imple mentation problematic. Second, these models could not easily match the initial yield curve. This calibration is essential for the accurate pricing and hedging of interest rate derivatives because any discrepancies in yield curve matching may indicate "false" arbitrage opportunities in the priced derivatives.

To address these problems, Ho and Lee[10] applied the binomial model to interest rate derivatives with a twist. Instead of imposing an evolution on the spot rate, they had the zero coupon bond price curve that evolved in a binomial tree. Motivated by this paper, Heath–Jarrow–Morton[8] generalized this idea in the context of a continuous time and multifactor model to price interest rate derivatives. The key step in the derivation of the HJM model was determined as the necessary and sufficient conditions for an arbitrage free evolution of the term structure of interest rates.

The defining characteristic of the HJM model is that there is a continuum of underlying assets, a term structure, whose correlated evolution needs to be considered when pricing and hedging options. For interest rate derivatives, this term structure is the term structure of interest rates. To be specific, it is the term structure of default free interest rates. But there are other term structures of relevance, including foreign interest rates, commodity futures prices, convenience yields on commodities, and equity forward volatilities. The key to these techniques is the recognition that the drifts of the no-arbitrage evolution of certain variables can be expressed as functions of their volatilities and the correlations among themselves. In other words, no drift estimation is needed.Models developed according to the HJM framework are different from the so-called short-rate models in the sense that HJM-type models capture the full dynamics of the entire forward rate curve, while the short-rate models only capture the dynamics of a point on the curve (the short rate).

However, models developed according to the general HJM framework are often non-Markovian and can even have infinite dimensions. A number of researchers have made great contributions to tackle this problem. They show that if the volatility structure of the forward rates satisfy certain conditions, then an HJM model can be expressed entirely by a finite state Markovian system, making it computationally feasible.

The original HJM paper showed that instantaneous forward rates being lognormally distributed is inconsistent with no arbitrage. Hence, geometric Brownian motion was excluded as an acceptable forward rate process. This was unfortunate because it implies that caplets, options on forward rates, will not satisfy Black's formula. This inconsistency between theory and practice lead to a search for a theoretical justification for using Black's formula with caplets.

This problem was resolved by Sandmann et al.[19], Miltersen et al. , and Brace et al.. The solution was to use a simple interest rate, compounded discretely, for the London Interbank Offer Rate (LIBOR). Of course, simple rates better match practice. And it was shown that the evolution of a simple LIBOR could evolve as a geometric Brownian motion in an arbitrage free setting. Subsequently, the lognormal evolution has been extended to jump diffusions, Levy processes, and stochastic volatilities.

3. Credit risk derivative pricing models

The previously discussed models excluded the consideration of default when trading financial securities. The first model for studying credit risk, called the structural approach, was introduced by Merton[17]. This risk-controlling ability enabled firms to seek out arbitrage opportunities, and in the process, lever up on the remaining financial risks, which are credit/counterparty, liquidity, and operational risk. This greater risk exposure by financial institutions to both credit and liquidity risk (as evidenced by the events surrounding the failure of Long Term Capital Management)spurred the more rapid development of credit risk modeling.

As the first serious contribution to credit risk modeling, Merton's original model was purposely simple. Merton considered credit risk in the context of a firm issuing only a single zero coupon bond. As such, risky debt could be decomposed into riskless debt plus a short put option on the assets of the firm. Shortly thereafter, extensions to address this simple liability structure were quickly discovered by Black and Cox Jones et al. and Leland among others.

The structural approach to credit risk modeling has two well-known empirical shortcomings: (i) that default occurs smoothly, implying that bond prices do not jump at default and (ii) that the firm's assets are neither traded nor observable. The first shortcoming means that for short maturity bonds, credit spreads as implied by the structural model are smaller than those observed in practice. Extensions of the structural approach that address the absence of a jump at default include that by Zhou[21]. These extensions, however, did not overcome the second shortcoming.

Almost 20 years after Merton's original paper, Jarrow and Turnbull[13] developed an alternative credit risk model that overcame the second shortcoming. As a corollary, this approach also overcame the first shortcoming. This alternative approach has become known as the reduced form model. An important contribution to the credit risk model literature was the integration of structural and reduced form models.

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