History of Option Pricing Theory After Black-Scholes' Work

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B-S-M Model

- The modern theory of option pricing: Black–Scholes–Merton model;
- Merton's improved work[Mer73] (rational BS model) has been discussed by Tao Luo before;
- BS equation setablished in 1973[BS73].

Five Assumptions of B-S-M Model

1. Competitive markets (Trades have no impact on the market price)
2. Frictionless markets (No transaction costs nor trade restrictions)
3. Geometric Brownian motion (The stock price is lognormally distributed with a constant volatility)
4. Deterministic Interest Rates
5. No Credit Risk

The first two assumptions are the mainstay of finance.

Extensions of the Black–Scholes–Merton model

- Relaxing GBM assumption: Jump and Jump-diffusion processes [CR76][Mer76]
- Option pricing with stochastic volatility in incomplete markets[HW87][Hes93]
- Levy processes was introduced by Madan and Milne [MM91] into option pricing and generalized by Carr et al.[CGMY03]
  i. Known characteristic function;
  ii. Using fast Fourier transforms.
- The relaxation of the FM assumption [Lel85][HJ87]
- The relaxation of the CM assumption[Jar92]
- Exotic Options[GK83] (Ammerican, Asian, Russian, Binary, Digital, Barrier, Parisian)

Interest Rate Derivatives

This deterministic interest rates assumption limits its usefulness in two ways.

1. It cannot be used for long-dated contracts;

2. For short dated contracts, if the underlying asset's price process is highly correlated with interest rate movements, then interest rate risk will affect hedging, and therefore valuation.

Interest Rate Derivatives

Spot rate models

- A class of interest rate pricing models were developed by Vasicek[Vas77], Brennan and Schwartz[BS79], and Cox et al.(CIR)[CIR85]

- Depended on the market price of interest rate risk.

- These models could not easily match the initial yield curve.

HJM model[HJM92]

- Key Tchniques: no drift estimation is needed.
- Based on modeling the forward rates.

Instantaneous forward rate $f(t,T), t \leqslant T$ is the continuous compounding rate defined by

$$f(t,T) = -\frac{1}{P(t,T)}\frac{\partial}{\partial T}P(t,T) = -\frac{\partial \mathrm{logP}(t,T)}{\partial T}$$

Basic relation between the rates and the bond prices:

$$P(t,T) = e^{-\int_0^T f(t,s)\,\mathrm{d}s}$$

HJM model

The assumption of the HJM model is that the forward rates $f(t,T)$ satisfy for any T:

$$\mathrm{d}f(t,T) = \mu(t,T)\mathrm{d}t + \sigma(t,T)\mathrm{d}W_t$$

To be compatible with assumption of the existence of martingale measures,

$$\frac{\mathrm{d}P(t,T)}{P(t,T)} = [r(t) - \alpha(t,T)\theta(t)]\mathrm{d}t + \alpha(t,T)\mathrm{d}W_t$$

No-arbitrage condition in the HJM model:

$$\mu(t,T) = \sigma(t,T)\left(\int_t^T \sigma(t,s)\mathrm{d}s - \theta(t)\right)$$

Under the martingale probability measure, $\theta = 0$

$$d\,f(t,T) = \sigma(t,T)\left(\int_t^T \sigma(t,s)\ d\,s\right)d\,t + \sigma(t,T)\,d\,\tilde{W}$$

BUT instantaneous forward rate is not observed directly.

The Application of HJM Model is not good.

The solution[MSS97] was to use a simple interest rate, compounded discretely, for the London Interbank Offer Rate (LIBOR).

Credit risk derivative pricing models

Structural approach (first model for studying credit risk)[Mer74]

- that default occurs smoothly
- the firm's assets are neither traded nor observable

Address the absence of a jump at default include that by Zhou[Zho01]

Jarrow and Turnbull[JT95] developed an alternative credit risk model that overcame the second shortcoming.

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Thank You!