Time Series Analysis of Global Temperature Anomalies

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Abstract: Global climate change is no longer a widely disputed topic, there are many institutes have given the analysis of it. Among of them, the Intergovernmental Panel on Climate Change (IPCC) is the leading international body for the assessment of climate change. The purpose of this project is to attempt to fit global climate anomalies (respect to the mean 57.2F) to a time series model, and to project out future global temperatures. The goal is to identify an ARMA(resp. ARIMA) model to accurately model the average global temperature change and forecast it forward.

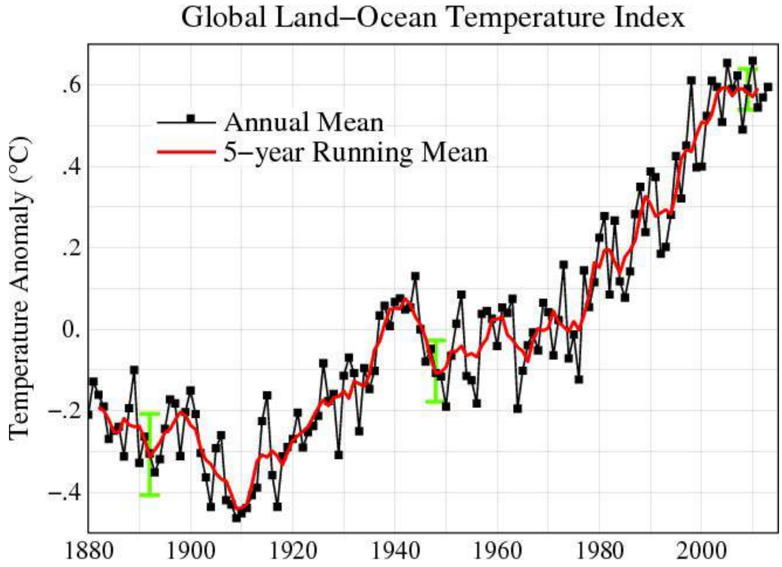
1. Introduction

The climate of planet Earth is unstable. Our evolutionary origins lie in the warm, relatively benign climate of equatorial Africa, but our ancestors battled the cold, harsh, and unforgiving climate of the last ice age in order to spread across the planet. One of the most significant accomplishments of our species is the discovery of fossil fuels and the means of turning the energy trapped within them into heat, transportation, and the basis for manufacturing and construction. NASA's GISS Surface Temperature Analysis (GISTEMP) has posted temperature deviations from the mean for the worldwide average temperature, where the mean is set to approximately 57.2F - the mean of temperatures for the period of 1951-1980. Their first published results (Hansen et al. 1981) showed that, contrary to impressions from northern latitudes, global cooling after 1940 was small, and there was net global warming of about 0.4°C between the 1880s and 1970s.

2. Description of Data

Our data is from National Oceanic and Atmospheric Administration, National Climatic Data Center (The Global Historical Climatology Network (GHCN-Monthly) data). The data is monthly global (From 90N to 90S) land and ocean temperature anomalies (with degrees ℃). The Global Historical Climatology Network (GHCN-Monthly) data base contains historical temperature, precipitation, and pressure data for thousands of land stations worldwide. A positive anomaly indicates that the observed temperature was warmer than the reference value, while a negative anomaly indicates that the observed temperature was cooler than the reference value.

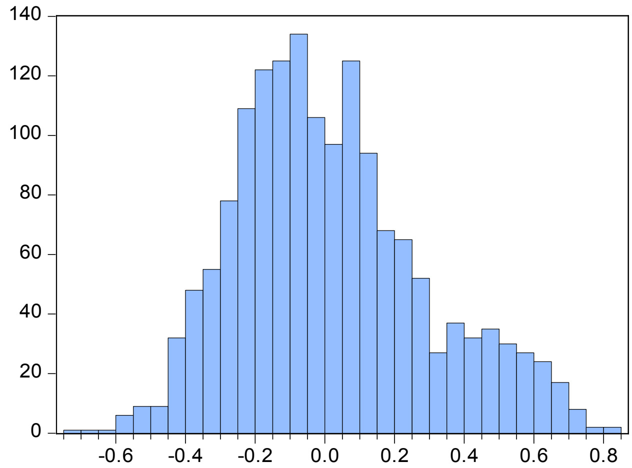
We first see a *Global Annual Mean Surface Air Temperature Change* in GISTEMP, line plot of global mean land-ocean temperature index, 1880 to present, with the base period 1951-1980. The dotted black line is the annual mean and the solid red line is the five-year mean. The green bars show uncertainty estimates.



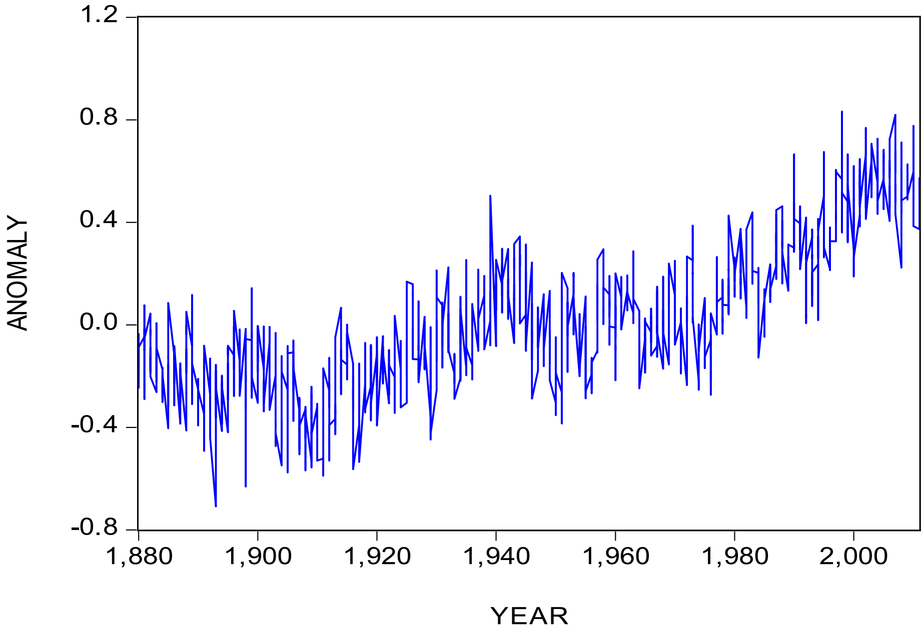
3. Model Specification With ARMA

3.1 Data Statistics

Below shows a plot of temperature anomalies in month from 1880, Jan to 2011,Jun from the data we obtain,



Series: ANOMALY	
Sample 1 1584	
Observations 1578	
Mean	0.015005
Median	-0.024500
Maximum	0.832300
Minimum	-0.708200
Std. Dev.	0.273153
Skewness	0.519255
Kurtosis	2.876294
Jarque-Bera	71.91766
Probability	0.000000



3.2 Unit Root Test

From the plot, the data are not so big, so we need not take logarithms. On the other hand, it would seem that the process is not stationary, so we first give the unit root tests as follows:

Null Hypothesis: ANOMALY has a unit root
Exogenous: None
Lag Length: 4 (Automatic based on SIC, MAXLAG=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.384688	0.0007
Test critical values:	1% level	-2.566433	
	5% level	-1.941025	
	10% level	-1.616564	

*MacKinnon (1996) one-sided p-values.

The Augmented Dickey-Fuller test with no test equation shows that there is no unit root (i.e. Reject the Null Hypothesis).

Similar results given by the Dickey-Fuller test with trend equation, PP test without trend as well as with trend:

Null Hypothesis: ANOMALY has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 3 (Automatic based on SIC, MAXLAG=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.158607	0.0000
Test critical values:	1% level	-3.963885	
	5% level	-3.412667	
	10% level	-3.128302	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: ANOMALY has a unit root
Exogenous: None
Bandwidth: 13 (Newey-West using Bartlett kernel)

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-6.268191	0.0000
Test critical values:	1% level	-2.566429	
	5% level	-1.941024	
	10% level	-1.616564	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.010687
HAC corrected variance (Bartlett kernel)	0.007054

Null Hypothesis: ANOMALY has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 23 (Newey-West using Bartlett kernel)

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-16.82442	0.0000
Test critical values:	1% level	-3.963873	
	5% level	-3.412662	
	10% level	-3.128299	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.009940
HAC corrected variance (Bartlett kernel)	0.017000

Even for the KPSS test, we will accept the null hypothesis that the anomaly of temperature is stationary.

Null Hypothesis: ANOMALY is stationary
Exogenous: Constant
Bandwidth: 31 (Newey-West using Bartlett kernel)

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		3.864153
Asymptotic critical values*:	1% level	0.739000
	5% level	0.463000
	10% level	0.347000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Residual variance (no correction)	0.074565
HAC corrected variance (Bartlett kernel)	1.970362

To sum up, from the unit root tests we given above, we can make a conclusion that there is no unit root in temperature anomalies data.

3.3 Establish Models

We now obtain the SACF, SPACF of the raw data from EViews:

Date: 07/06/14 Time: 17:32
Sample: 1 1584
Included observations: 1578

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.924	0.924	1351.0	0.000
		2	0.900	0.312	2632.1	0.000
		3	0.884	0.175	3868.3	0.000
		4	0.872	0.123	5072.2	0.000
		5	0.859	0.071	6242.6	0.000
		6	0.849	0.056	7384.6	0.000
		7	0.843	0.076	8511.5	0.000
		8	0.833	0.028	9613.0	0.000
		9	0.822	0.011	10687.	0.000
		10	0.815	0.035	11744.	0.000
		11	0.807	0.016	12780.	0.000
		12	0.799	0.019	13797.	0.000
		13	0.789	-0.007	14790.	0.000
		14	0.780	-0.003	15760.	0.000
		15	0.772	0.011	16712.	0.000
		16	0.766	0.016	17647.	0.000
		17	0.762	0.041	18576.	0.000
		18	0.758	0.030	19495.	0.000
		19	0.759	0.063	20417.	0.000
		20	0.757	0.037	21335.	0.000
		21	0.760	0.068	22260.	0.000
		22	0.757	0.018	23178.	0.000
		23	0.759	0.047	24101.	0.000
		24	0.760	0.044	25028.	0.000
		25	0.750	-0.060	25930.	0.000
		26	0.745	-0.010	26821.	0.000
		27	0.738	-0.030	27696.	0.000
		28	0.736	0.016	28567.	0.000
		29	0.732	0.006	29430.	0.000
		30	0.732	0.027	30293.	0.000
		31	0.729	0.005	31150.	0.000
		32	0.730	0.041	32009.	0.000
		33	0.732	0.050	32874.	0.000
		34	0.732	0.028	33738.	0.000
		35	0.733	0.038	34606.	0.000
		36	0.731	0.015	35470.	0.000

From the plot above, it seem that the raw data is not stationary since Autocorrelations don't decrease in a finite time, but this has been test from the previous unit root test. So we just follow the result. As for PACFs, the lags to drop off to near zero within a finite steps. So we can raise an ARMA model. Now we should compute the EACF of the raw data. I get help from TAOLUO to compute EACF in R, and get the following results:

AR/MA

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	x	x	x	x	x	x	x	x	x	x
1	x	x	o	o	o	o	o	o	o	o	o	o	o	o
2	x	x	o	o	o	o	o	o	o	o	o	o	o	o
3	x	x	o	o	o	o	o	o	o	o	o	o	o	o
4	x	x	o	o	o	o	o	o	o	o	o	o	o	o
5	x	x	x	x	x	o	o	o	o	o	o	o	o	o
6	x	x	x	x	x	x	o	o	o	o	o	o	o	o
7	x	x	x	x	x	x	o	o	o	o	o	o	o	o

From the above results, we can found an ARMA(1,2) model for temperature anomalies' data. Here, since the original data has been modified by the mean of temperature, we just need to establish a model without mean, or if we get a model with mean, the value is very small (I have tried to do that and get a constant term in 10^-4 level.) Then, we can get an ARMA(1,2) model with zero mean from Eviews,

Dependent Variable: ANOMALY
Method: Least Squares
Date: 07/06/14 Time: 17:46
Sample (adjusted): 2 1578
Included observations: 1577 after adjustments
Convergence achieved after 9 iterations
MA Backcast: 0 1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.989626	0.004072	243.0188	0.0000
MA(1)	-0.466351	0.025440	-18.33106	0.0000
MA(2)	-0.114514	0.025414	-4.505858	0.0000

R-squared	0.879259	Mean dependent var	0.015038
Adjusted R-squared	0.879106	S.D. dependent var	0.273236
S.E. of regression	0.095004	Akaike info criterion	-1.867900
Sum squared resid	14.20648	Schwarz criterion	-1.857697
Log likelihood	1475.839	Hannan-Quinn criter.	-1.864108
Durbin-Watson stat	1.985235		

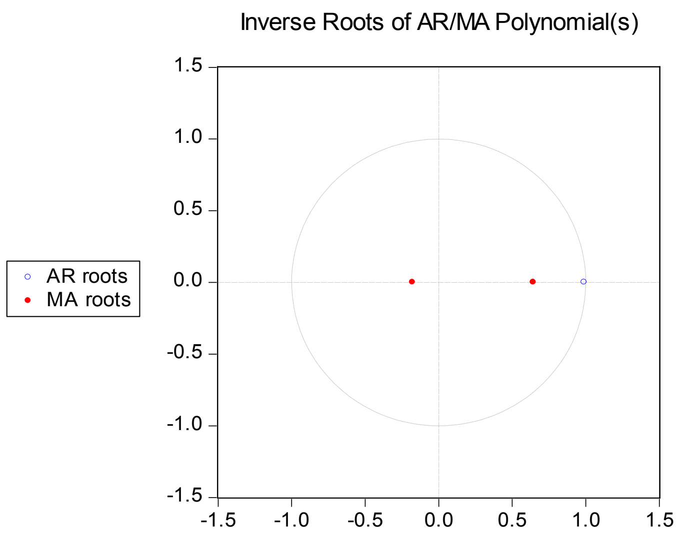
Inverted AR Roots	.99	
Inverted MA Roots	.64	-.18

i.e. We get a model as follows:

$$X = 0.989626X_{t-1} + a_t - 0.466351a_{t-1} - 0.114514a_{t-2}$$

The negative coefficients indicate that the current temperature is negatively correlated with prior temperatures. The P-Values of the coefficients are low and the R^2 of the model is (adjusted R-squared) 0.8791 and standard error 0.0095. The Durbin-Watson value is closer to 2. This model seems to be a good one. Then we consider some structures of this ARMA model. The meaning of this model is that the temperature anomaly relies on the previous month's data as well as the fluctuation for previous two month.

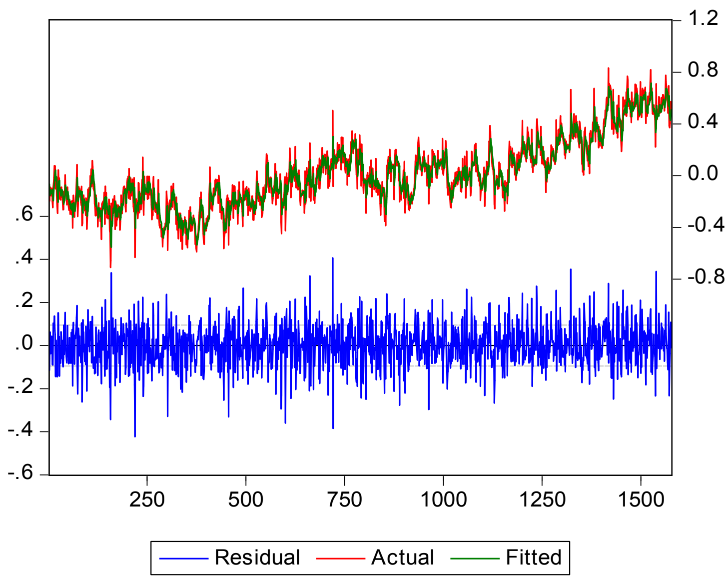
What's more, for the root of stationary and invertible equations, we have that the root of AR(1) is 0.989626, which is very closed to 1.



From this token, *we may consider that the raw date may have a unit root which is not significant, but we can try it for ARIMA model after that.*

3.3 Residual Test

Firstly, we have the residual plot (respect to original data) as follow:

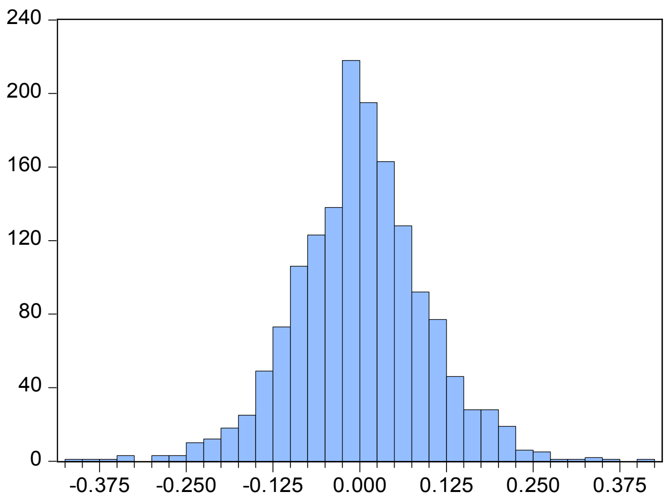


Since we have analyzed the DW statistics for this ARMA(1,2) model that it is very closed to 2, so we have that there is almost no autocorrelations between the residual, while we can get more information as follows from the Ljung – Box test (Q) statistics:

Date: 07/06/14 Time: 19:39
Sample: 2 1578
Included observations: 1577
Q-statistic probabilities adjusted for 3 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.007	0.007	0.0668	
		2	0.030	0.030	1.4550	
		3	-0.021	-0.022	2.1782	
		4	-0.013	-0.013	2.4383	0.118
		5	-0.032	-0.030	4.0243	0.134
		6	-0.046	-0.045	7.3810	0.061
		7	-0.004	-0.003	7.4114	0.116
		8	-0.005	-0.004	7.4483	0.189
		9	-0.029	-0.032	8.7944	0.185
		10	-0.006	-0.007	8.8457	0.264
		11	-0.013	-0.014	9.1120	0.333
		12	0.003	-0.000	9.1267	0.426
		13	-0.023	-0.024	9.9630	0.444
		14	-0.037	-0.040	12.172	0.351
		15	-0.041	-0.043	14.799	0.253
		16	-0.052	-0.052	19.036	0.122
		17	-0.033	-0.035	20.758	0.108
		18	-0.051	-0.054	24.864	0.052
		19	-0.012	-0.020	25.104	0.068
		20	-0.028	-0.037	26.354	0.068
		21	0.027	0.016	27.551	0.069
		22	0.001	-0.010	27.554	0.092
		23	0.036	0.022	29.604	0.077

The fluctuations of the ACF and PACF of residuals are small and the p-value are significant for the previous. Furthermore, the chi-square statistic applied to the first 23 autocorrelation is Q=29.604< $\chi^2_{23-3,0.05}$ =31.41. So we can claim that the residual is a white noise.



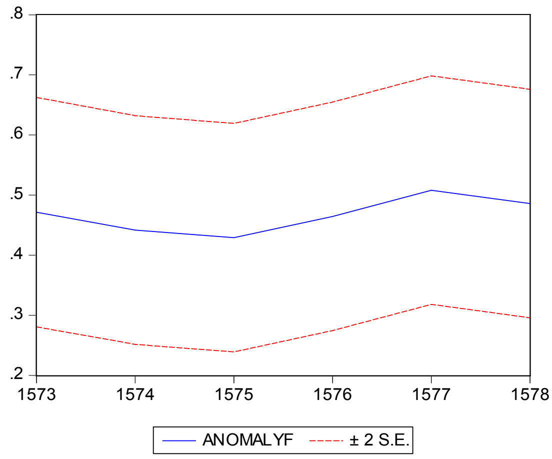
Series: Residuals
Sample 2 1578
Observations 1577

Mean	0.001265
Median	0.000333
Maximum	0.406166
Minimum	-0.421849
Std. Dev.	0.094935
Skewness	-0.036103
Kurtosis	4.179317
Jarque-Bera	91.72896
Probability	0.000000

As for the normalization test of residuals, we can see that the skewness is almost zero while kurtosis is bigger that 3 and also from the Jarque-Bera statistics, we can get the conclusion that the residual is not normal distributed.

3.4 Forecast

Compare the actual data of January 2011 to June and 95% prediction confidence intervals. The actual data are included in the prediction confidence intervals. It means that the forecasting results are good, so we conclude the model adequately describes the land temperature time series.



Forecast: ANOMALYF
Actual: ANOMALY
Forecast sample: 1573 1578
Included observations: 6

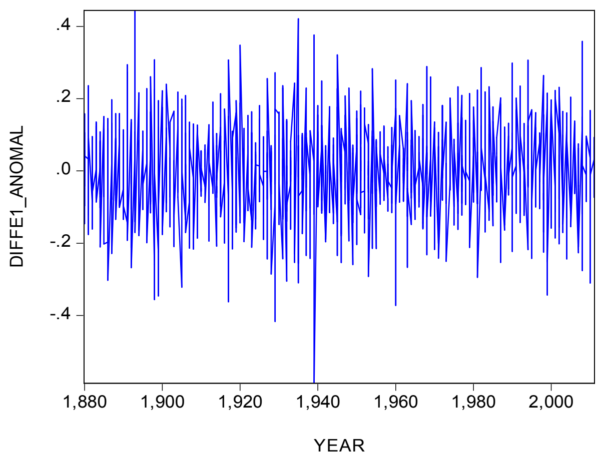
Root Mean Squared Error	0.075328
Mean Absolute Error	0.067000
Mean Abs. Percent Error	14.04319
Theil Inequality Coefficient	0.078556
Bias Proportion	0.062840
Variance Proportion	0.417318
Covariance Proportion	0.519843

Forecast in Jan 2011 to Jun 2011

Forecast	Real
0. 4701	0. 4731
0. 4326	0. 4046
0. 4102	0. 4373
0. 4873	0. 572
0. 5004	0. 4992
0. 4913	0. 5677

4. Alternative Model ARIMA

We have talked about the stationary of the ARMA model before. Since our invert root for AR is very closed to 1, hence, we try to do this model through ARIMA. Taking First Differences produces a plot that appears more stationary as shown below.



To determine what type of ARIMA model is most appropriate, it is first necessary to get sample autocorrelations. Below is a Correlogram for the first 36 lags of the 1st difference.

Date: 07/06/14 Time: 14:50
Sample: 1 1584
Included observations: 1577

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.342	-0.342	184.62	0.000
		2	-0.059	-0.199	190.11	0.000
		3	-0.026	-0.139	191.15	0.000
		4	0.006	-0.081	191.21	0.000
		5	-0.010	-0.064	191.37	0.000
		6	-0.033	-0.084	193.15	0.000
		7	0.017	-0.047	193.61	0.000
		8	0.011	-0.022	193.80	0.000
		9	-0.025	-0.044	194.77	0.000
		10	0.007	-0.027	194.86	0.000
		11	-0.008	-0.033	194.97	0.000
		12	0.018	-0.006	195.51	0.000
		13	-0.008	-0.010	195.60	0.000
		14	-0.012	-0.023	195.84	0.000
		15	-0.007	-0.030	195.91	0.000
		16	-0.019	-0.050	196.50	0.000
		17	0.006	-0.038	196.55	0.000
		18	-0.029	-0.068	197.88	0.000
		19	0.014	-0.045	198.20	0.000
		20	-0.029	-0.077	199.57	0.000
		21	0.034	-0.029	201.38	0.000
		22	-0.027	-0.059	202.57	0.000
		23	0.001	-0.053	202.58	0.000
		24	0.081	0.052	213.04	0.000
		25	-0.038	0.004	215.31	0.000
		26	0.015	0.023	215.66	0.000
		27	-0.035	-0.022	217.64	0.000
		28	0.013	-0.011	217.89	0.000
		29	-0.019	-0.035	218.46	0.000
		30	0.013	-0.011	218.72	0.000
		31	-0.026	-0.051	219.83	0.000
		32	-0.008	-0.062	219.93	0.000
		33	0.019	-0.035	220.51	0.000
		34	-0.007	-0.045	220.60	0.000
		35	0.013	-0.023	220.86	0.000
		36	0.006	-0.019	220.92	0.000

As hoped, the sample autocorrelation approaches zero fairly quickly then oscillates about zero as lag increases. Since the lags of PACF to drop off to near zero, it does seem to support a moving average model.

More precisely, we compute the EACF of the first difference of anomalies, and get the following results:

AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	o	o	o	o	o	o	o	o	o	o	o	o
1	x	x	o	o	o	o	o	o	o	o	o	o	o	o
2	x	x	o	o	o	o	o	o	o	o	o	o	o	o
3	x	x	o	o	o	o	o	o	o	o	o	o	o	o
4	x	x	x	x	x	o	o	o	o	o	o	o	o	o
5	x	x	x	x	x	x	o	o	o	o	o	o	o	o
6	x	x	x	x	o	x	o	o	o	o	o	o	o	o
7	x	x	x	x	x	x	o	o	o	o	o	o	o	o

I.e. We obtain an ARIMA(0,1,2), we can get an ARIMA(0,1,2) model with zero mean from Eviews, and we obtain:

Dependent Variable: DIFFE1_ANOMAL
Method: Least Squares
Date: 07/06/14 Time: 20:58
Sample: 1 1572
Included observations: 1572
Convergence achieved after 7 iterations
MA Backcast: -1 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.474025	0.025052	-18.92157	0.0000
MA(2)	-0.122621	0.025086	-4.888033	0.0000

R-squared	0.183585	Mean dependent var	0.000260
Adjusted R-squared	0.183065	S.D. dependent var	0.105405
S.E. of regression	0.095269	Akaike info criterion	-1.862946
Sum squared resid	14.24971	Schwarz criterion	-1.856126
Log likelihood	1466.275	Hannan-Quinn criter.	-1.860411
Durbin-Watson stat	1.982669		
Inverted MA Roots	.66	-.19	

Also, we can do the unit root test of first difference, and of course get no unit root "as the same to" raw data, what's more, we can see that the inverted MA root is smaller than 1. So In this form, we have the formula for ARIMA:

$$X - X_{t-1} = a_t - 0.4740251a_{t-1} - 0.122621a_{t-2}$$

the negative coefficients indicate that the current temperature is negatively correlated with prior temperatures. Although the P-Values of the coefficients are low, *the R^2 of the model is only 0.18*, showing only approximate 50% correlation between the prior values and the predicted value. This seems to indicate that there are explanatory variables present other than prior years.

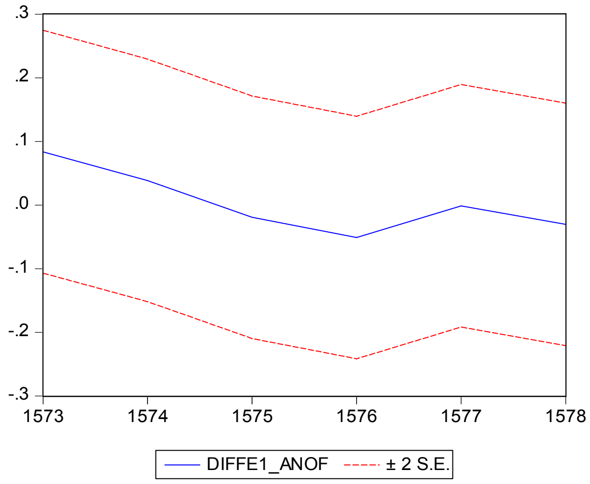
As a diagnostic test, it is important to look at the residuals to see if the error terms are normally distributed. Below is a histogram of the residuals.

Date: 07/06/14 Time: 21:06
Sample: 1 1572
Included observations: 1572
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.008	0.008	0.0927	
		2	0.034	0.034	1.9676	
		3	-0.020	-0.020	2.5703	0.109
		4	-0.014	-0.015	2.8699	0.238
		5	-0.037	-0.035	4.9811	0.173
		6	-0.049	-0.048	8.8177	0.066
		7	-0.010	-0.008	8.9811	0.110
		8	-0.011	-0.009	9.1621	0.165
		9	-0.035	-0.037	11.093	0.135
		10	-0.012	-0.014	11.313	0.185
		11	-0.020	-0.021	11.929	0.217
		12	-0.005	-0.009	11.971	0.287
		13	-0.032	-0.034	13.571	0.258
		14	-0.046	-0.051	16.916	0.153
		15	-0.049	-0.053	20.776	0.077
		16	-0.059	-0.062	26.340	0.023
		17	-0.041	-0.045	28.955	0.016
		18	-0.057	-0.065	34.149	0.005
		19	-0.019	-0.032	34.747	0.007
		20	-0.034	-0.049	36.601	0.006
		21	0.021	0.003	37.339	0.007
		22	-0.004	-0.023	37.363	0.011
		23	0.031	0.008	38.886	0.010

The fluctuations of the ACF and PACF of residuals are small and the p-value are significant for the previous. So we may get the same conclusion that the residual is a white noise.

Now we make forecast for this model in January 2011 to June and 95% prediction confidence intervals.



Forecast: DIFFE1_ANOF
Actual: DIFFE1_ANOMAL
Forecast sample: 1573 1578
Included observations: 5

Root Mean Squared Error	0.062979
Mean Absolute Error	0.058348
Mean Abs. Percent Error	96.27776
Theil Inequality Coefficient	0.529969
Bias Proportion	0.209948
Variance Proportion	0.039200
Covariance Proportion	0.750853

Since in Dec 2010 is 0.3838, hence we can obtain the forecast as follows:

Forecast in Jan 2011 to Jun 2011

Forecast	Real
0.5633	0.4731
0.4667	0.4046
0.4061	0.4373
0.5308	0.572
0.5205	0.4992
0.5075	0.5677

It’s easy to see that the ARMA model make a better forecast. So we choose the first model.

5. Conclusion

In conclusion, a look at the Partial Autocorrelation corrolegram suggests that global temperature follows a ARMA(1,2) model. Negative coefficients seem to suggest that a higher than average year will be followed by a lower than average year, and vice-versa. The difference of this two methods is not significant, since the coefficient for backward anomalies is very closed to 1. The result is interesting, that there is no unit root when we test the raw data, but in reality, maybe it contains a trend. However, I think the first one model is better, since the R^2 is very well. There does, however seem to be an overall drift that may continue into the future. It is important to point out that although this study was done with all data from 1880 through 2011, an inspection of the data may suggest that the pattern from 1960 and on seems different from before 1960, and it may be more appropriate to consider just data from 1960 and on when projecting future temperature patterns.