Interpolation of phonons

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1 Transformation of phonon green's function

The phonon green's function in terms of phonon creation and annihilation operator can be written as

$$G(qvv';t) = \langle i|\mathcal{T}[A_{qv}(t)A^{\dagger}_{qv'}(0)]|i\rangle \tag{1}$$

where $A_{qv} = a_{qv} + a^{\dagger}_{-qv}$. The atomic displacement operator is connected with the phonon operator by transformation

$$\eta_{lb} = \sum_{qv}\left(\frac{\hbar}{2N\omega_{qv}m_b}\right)^{\frac{1}{2}} e^{b}_{qv}e^{iql}A_{qv} \tag{2}$$

$$A_{qv} = \sum_{lb}\left(\frac{2\omega_{qv}m_b}{N\hbar}\right)^{\frac{1}{2}} e^{b*}_{qv}e^{-iql}\eta_{lb} \tag{3}$$

The phonon green's function in terms of atomic displacement operator is written as:

$$D_{ll'bb'}(t) = \langle i|\mathcal{T}[\eta_{lb}(t)\eta^{\dagger}_{l'b'}(0)]|i\rangle \tag{4}$$

substituting Eq.2, we obtain the relationship:

$$D_{ll'bb'}(t) = \sum_{qq'vv'}\langle i|\mathcal{T}[\left(\frac{\hbar}{2N\omega_{qv}m_b}\right)^{\frac{1}{2}} e^{b}_{qv}e^{iql}A_{qv}(t)\left(\frac{\hbar}{2N\omega_{q'v'}m'_b}\right)^{\frac{1}{2}} e^{b'*}_{q'v'}e^{-iq'l'}A^{\dagger}_{q'v'}(0)]|i\rangle \tag{5}$$

$$= \frac{\hbar}{2N}\sum_{qq'vv'}(\omega_{qv}m_b\omega_{q'v'}m'_b)^{-\frac{1}{2}}\langle i|\mathcal{T}[e^{b}_{qv}e^{iql}A_{qv}(t)e^{b'*}_{q'v'}e^{-iq'l}A^{\dagger}_{q'v'}(0)]|i\rangle \tag{6}$$

$$= \frac{\hbar}{2}\sum_{qvv'}(\omega_{qv}m_b\omega_{q'v'}m'_b)^{-\frac{1}{2}}e^{iq(l-l')}e^{b}_{qv}e^{b'*}_{q'v'}\langle i|\mathcal{T}[A_{qv}(t)A^{\dagger}_{q'v'}(0)]|i\rangle \tag{7}$$

$$\tag{8}$$

where we used the result $\sum_{qq'} e^{i(ql-q'l')} = N\delta_{qq'}e^{iq(l-l')}$. The reverse of the above relationship is:

$$G_{qvv'}(t) = \sum_{ll'bb'}\frac{2}{N\hbar}(\omega_{qv}\omega_{qv'}m_bm'_b)^{\frac{1}{2}}e^{b*}_{qv}e^{b'}_{qv'}e^{iq(l-l')}D_{ll'bb'}(t) \tag{9}$$

2 Form of the phonon green's function