I will formalize a proof of L'Hôpital's theorem, using only the definitions used in assignment 5. Many new definitions will be required, such as a formalization of the derivative using only definitions from assignment 5, which I am working on at the moment.

Theorem: Given $f, g : \mathbb{R} \mapsto \mathbb{R} \in C^2$,

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Proof: It suffices to show that

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f'(c)}{g'(c)}, \tag{1}$$

since

$$\frac{f'(c)}{g'(c)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

by definition of continuity. Then,

$$\begin{aligned}
\lim_{x \to c} \frac{f(x)}{g(x)} &= \lim_{x \to c} \frac{f(x) - 0}{g(x) - 0} \\
&= \lim_{x \to c} \frac{f(x) - f(c)}{g(x) - g(c)} \\
&= \lim_{x \to c} \frac{\frac{f(x) - f(c)}{x - c}}{\frac{g(x) - g(c)}{x - c}} \\
&= \lim_{x \to c} \frac{\lim_{x \to c} \frac{f(x) - f(c)}{x - c}}{\lim_{x \to c} \frac{g(x) - g(c)}{x - c}} \\
&= \frac{f'(c)}{g'(c)},
\end{aligned}$$

proving (1), as desired.