Laboratory report 14
Galton-Watson process

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1 Introduction

The aim of this laboratory activity was to simulate a Galton-Watson process and studying it via the computation of generational extinction probabilities and their asymptotic behaviours. In particular, it was supposed that the number of children per generation is an instance of a Poisson-distributed random variable with varaible parameter λ.

2 Simulation's description

The simulation is built around various possible values of the Poisson's distribution parameter λ in order to experiment with different functional regimes for the system under simulation. In particular, the simulation works with **subcritical** and **supercritical** regimes ($\lambda < 1$ and $\lambda > 1$ respectively). Since the aim was studying the extinction behaviour of the GW-Process, the simulator stops the run if a particular generation contains 0 children or if its index exceeds the maximum number of generations allowed[1].

3 Results

Fig.1a shows the obtained results for a value of λ which brings the system in **subcritical regime**. While Fig.1b shows the experimental distribution governing the number of nodes per generation. For what concerns Fig.1b, the number of nodes in generation 0 is always 1 since there is always a first ancestor from which all the children are generated. The number of nodes in the first generation is almost equal to the chosen value of λ, which, theoretically, corresponds to the average number of nodes at the first generation. Eventually, the number of nodes decreases until it reaches 0 due the fact that the system is working in subcritical regime.

[1] It has been decided to use 70 as allowed number of generations per run since, according to theory, it the tree is destined to experience extinction then after $g \approx 50$ generation its extinction probability is almost 1

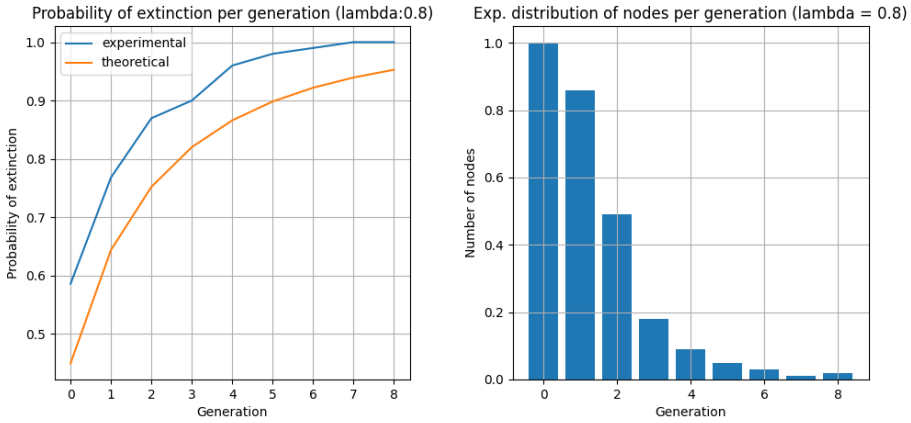

Figure 1: (a) Extinction behaviour (b) Experimental distribution of nodes per generation.