Laboratory Report
Random variable generation

Francesco Pagano - s299266

1 Introduction and general strategy

In this laboratory session the main goal was to implement a random number generator for istances of the Binomial distribution, the Normal distribution and the Rice distribution.

1.1 RNG for Binomial Distribution

For what concerns the Binomial distribution, three methods were employed to implement the RNG:

- Convolution method
- Inverse-Transform method
- Empirical distributions estimation method

Among the techniques employed, as also shown in Fig.1, the Convolution method and the Empirical method resulted to be almost equivalent in performance, without considering the wide range of variation the parameters **n** and **p** were supposed to have.[1] For what concerns the Inverse-Transform method, this resulted in an overflow using Python as a programming language (probably any compiled language would have been able to handle the huge representation needed for $n = 1e^6$ and $p = 1e^{-5}$), nevertheless, either building the CDF manually or using specific routines, this specific method resulted wildly under-performing.

1.2 RNG for Normal Distribution

For the Standard-Normal distribution the acceptance/rejection method was employed, namely an instance of the r.v. was generated and it was confronted with a rectangle approximating the area of the pdf. Then, a simple confrontation was employed between empirical and theoretical mean and standard deviation.

[1] For this specific test, the generation of $k = 50$ istances of random variable distributed according to the Binomial distribution were used.

	method	n	p	time
0	'conv'	10	0.50000	0.000370
1	'inv'	10	0.50000	0.001001
2	'exp'	10	0.50000	0.001129
3	'conv'	100	0.01000	0.000270
4	'inv'	100	0.01000	0.008756
5	'exp'	100	0.01000	0.000426
6	'conv'	1000000	0.00001	0.000266
7	'inv'	1000000	0.00001	90.956383
8	'exp'	1000000	0.00001	0.001921

Figure 1: Small dataset containing experiments' info.

1.3 RNG for Rice Distribution

To generate an instance of the Rice distribution $R \sim Rice(\nu, \sigma)$ the following relations were exploited:

$$\text{Let} \quad N \sim Poisson(k) \quad \text{with} \quad k = \frac{\nu^2}{2\sigma^2} \tag{1}$$

$$\text{Let} \quad W \sim \chi(\hat{w}) \quad \text{with} \quad \hat{w} = 2N + 2 \tag{2}$$

$$\text{Then} \quad R = \sigma\sqrt{W} \quad \text{is} \qquad rician - \text{distributed}. \tag{3}$$

Here, the instance of the Poisson r.v. was generated via the Knuth algorithm and the corresponding instance of the ChiSquared r.v. was generated via the convolution method by summing $2N + 2$ instances of a standard normal distribution.

1.4 Code

The code used for this exercise can also be found at `https://www.github.com/wibox/casl` in the lab3 folder.