Causal Inference: Difference-in-differences

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Part 4

Recap

Strategies for estimating effects of treatments so far:

- Randomize treatment and take the DIGM
- Identify and control for confounding variables such that the CIA (Conditional Independence Assumption) holds

Now: Use observations at more than one point in time

Types of samples

- Cross section: observe many units at one point in time. So far!
- Time series: observe one unit at many points in time
- Repeated cross sections: observe many units at many points in time, and the units are (potentially) different over time
- Panel data: observe many units at many points in time, and the units are the same over time

Repeated Cross Sections

- At one point in time ($t = 1$) we take a random sample of n_1 individuals ($i = 1...n_1$)
- At another later point in time ($t = 2$) we take another random sample of potentially different n_2 individuals ($i = 1...n_2$)
- Every period we repeat the process of obtaining a cross section
- This data is not panel data because the individuals are not the same over time (or they are and we still do not treat them as such)
- Why would we want to have cross sections at different points in time? Larger N, study whether the slope estimate changes over time as a result of policy change (or shock in time)
- Often the case in surveys

Panel Data

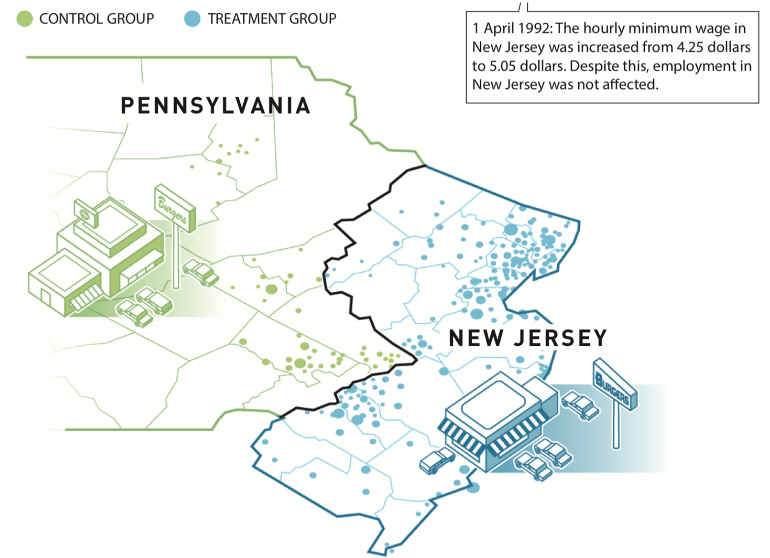
- At one point in time ($t = 1$) we take a random sample of n individuals/units ($i = 1...n$)
- At another later point in time ($t = 2$) we sample again the same n individuals/units ($i = 1...n$)
- Every period we repeat the process of **sampling the same individuals/units**: the same individuals/units are measured at different points in time
- If every individual appearing in $t = 1$ also appears in $t = 2, 3...$: **balanced panel**

Country ID	Year	Var1	Var 2
Afghanistan	2018	x1	y1
Afghanistan	2019	x1	y1
Afghanistan	2020	x1	y1
Croatia	2018	x2	y2
Croatia	2019	x2	y2
Croatia	2020	x2	y2
United Kingdom	2018	x3	y3
United Kingdom	2019	x3	y3
United Kingdom	2020	x3	y3

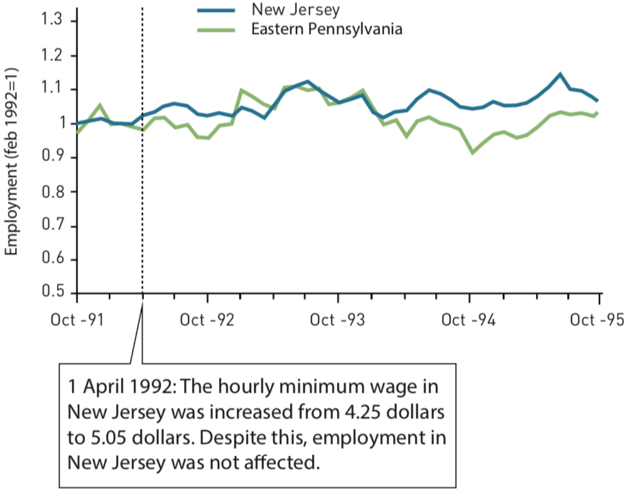
Diff-in-diff example: Minimum wage laws and employment

- Do higher minimum wages decrease low-wage employment?
- Card and Krueger (1994) exploit the change in New Jersey's 1992 minimum wage increase from $4.25 to $5.05 per hour to measure the effect of minimum wage on unemployment in the fast food industry
- New regulation only applies to NJ, which allows to have other States as control groups
- Compare employment in 410 fast-food restaurants in New Jersey and eastern Pennsylvania before and after the rise
- Survey data on wages and employment from two waves:
  - Wave 1: March 1992, one month before the minimum wage increase
  - Wave 2: December 1992, eight months after increase

Locations of Restaurants (Card and Krueger 2000)



Wages Before and After Rise in Minimum Wage



Sample Means: Minimum wage laws and employment

Variable	PA (i)	NJ (ii)	Difference, NJ – PA (iii)
	Stores by state		
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	–2.89 (1.44)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	–0.14 (1.07)
3. Change in mean FTE employment	–2.16 (1.25)	0.59 (0.54)	2.76 (1.36)

Is this a causal estimate? What selection bias is controlled for? What is remaining?

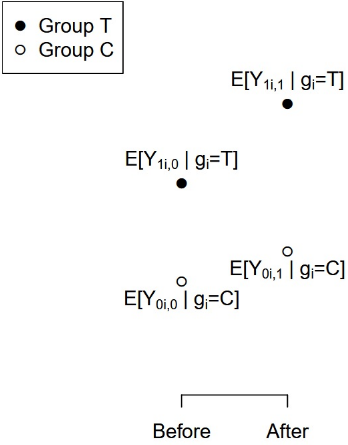
Difference-in-differences

- There are plenty of examples of **treatments that occur at a particular time.** We can see the world before the treatment is applied, and after. We want to know how much of the change in the world is due to that treatment.
- We are looking for how much more the treated group changed than the untreated group when going from before to after. The change in the untreated group represents how much change we would have expected in the treated group if no treatment had occurred - it serves as the *counterfactual*. So any additional change beyond that amount must be the effect of the treatment.
- Identification assumption: while the treated and control groups may vary in their characteristics over time, the selection bias into treatment must be **time-invariant**. This is the **parallel trends assumption**.

Difference-in-differences, II

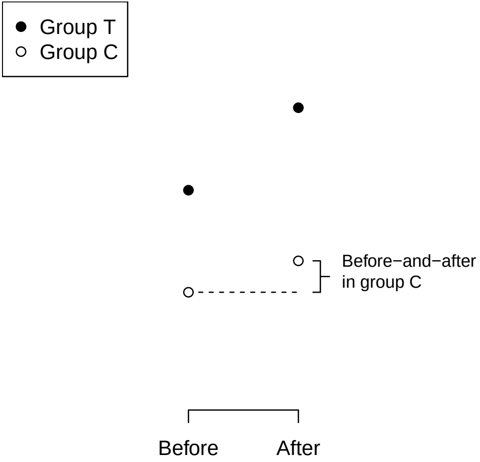
- **Simple case:** binary treatment, applied at one point in time (but not to everyone)
- **More general case:** general treatment, applied in any pattern
- Panel data requirements: multiple observations over time, with treatment varying within group or unit over time
- Estimation via a regression that controls for time period and group or unit **(fixed effects)**

Two groups, two time periods

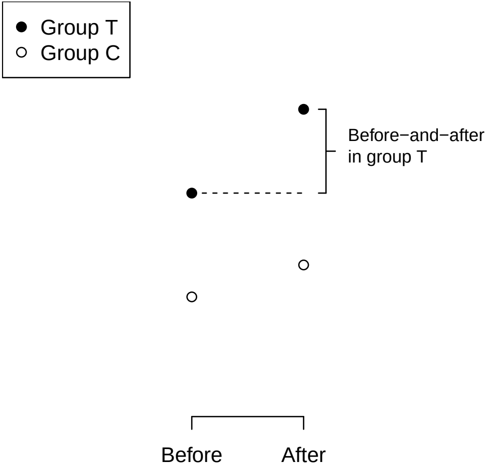


Where g_i denote i's group (treatment or control). For example, $E[Y_{1i,1} \mid g_i = T]$ is the average potential outcome under treatment in period 1 for units in group T.

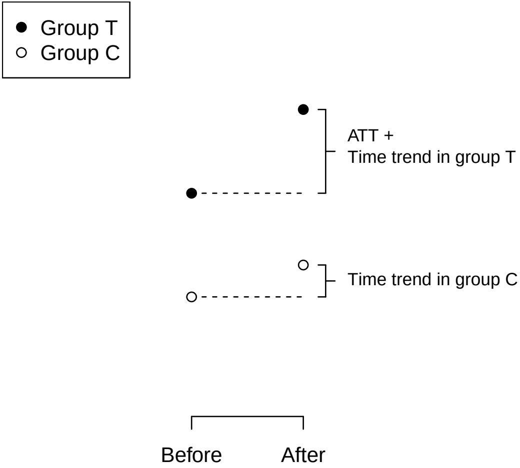
Before-and-after in group C



Before-and-after in group T

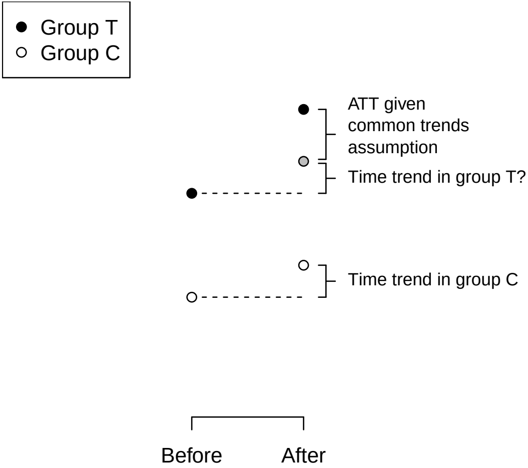


Before-and-after in both groups

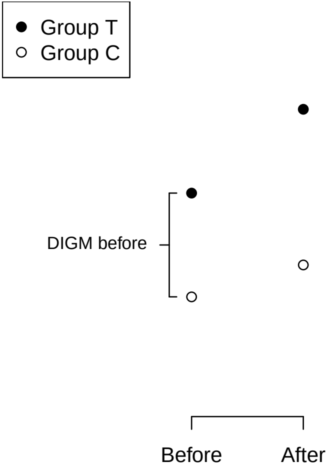


Identification assumption for ATT: common trend in group T and C.

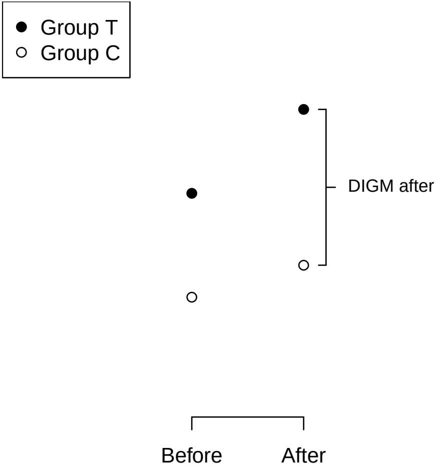
ATT given common trend assumption



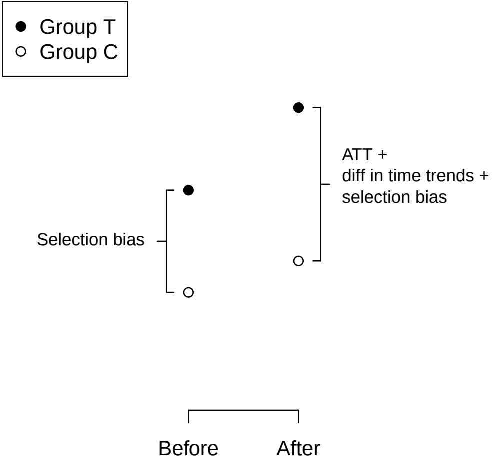
Difference in Group Means (DIGM) before



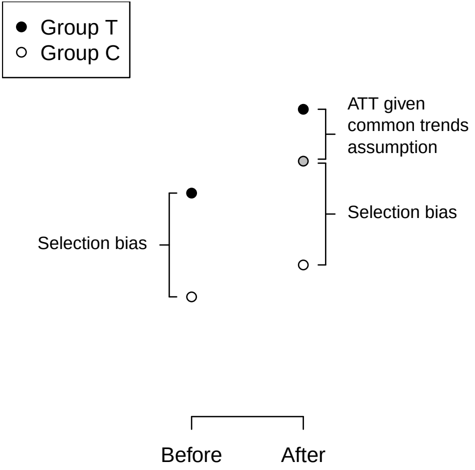
Difference in Group Means (DIGM) after



Both Difference in Group Means (DIGM)



ATT given common trends assumption



Can the common trends assumption be tested?

No. But common trends in several pre-treatment periods is suggestive.

Dinas et al (2018) on political impact of refugees

- **Question**: Did the influx of refugees in Greece increase support for the right-wing Golden Dawn party in 2015?
- **Treatment**: Large number of refugees arriving in locality
- **Outcome**: Golden Dawn vote share in locality

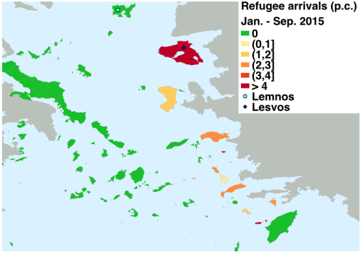
To consider:

- What about a cross-sectional approach? What covariates might help?
- How can we use variation over time in a diff-in-diff?

Dinas et al on the Golden Dawn (2)

Islands that received lots of refugees may vote differently even without the refugee influx.

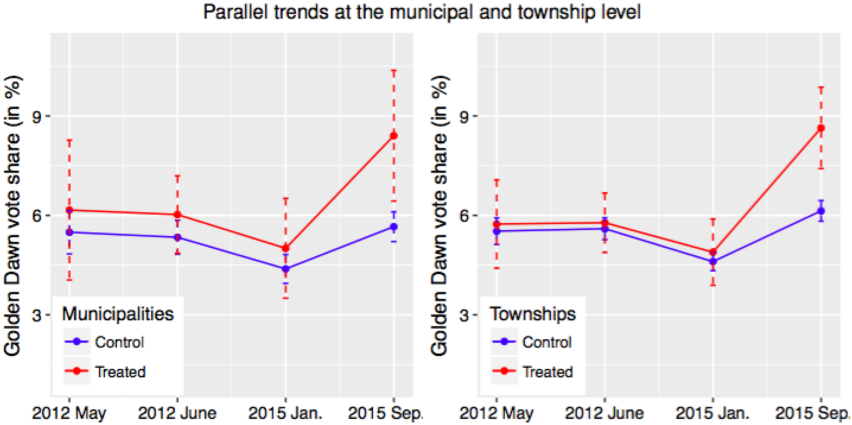
Maybe that difference is constant over time.



Common trends assumption: if they had not received refugees, islands that did receive refugee would have seen the same **change** in support for Golden Dawn as other islands.

To consider: are these other islands really *untreated*?

Dinas et al on the Golden Dawn (3)



Diff-in-diff implementation: method 1

Method 1: group-period interactions

- data structure: two rows for each municipality (elections of Jan. 2015, Sept. 2015)
- `evertr`: 1 for municipalities that received refugees
- `post`: 1 for election after the influx
- `gdper`: support for Golden Dawn

municipality	evertr	post	gdper
Αίγινας	0	0	6.363300
Αίγινας	0	1	7.617789
Αγίου Βασιλείου	0	0	2.714932
Αγίου Βασιλείου	0	1	3.694069
Αγίου Ευστρατίου	0	0	4.878048
Αγίου Ευστρατίου	0	1	5.988024
Αγίου Νικολάου	0	0	3.159049
Αγίου Νικολάου	0	1	4.604597
Αγαθονησίου	1	0	3.278688
Αγαθονησίου	1	1	5.000000
Αγκιστρίου	0	0	6.129032
Αγκιστρίου	0	1	9.981852
Αλοννήσου	0	0	5.727377
Αλοννήσου	0	1	5.976096

Estimating the linear regression:

$$\text{gdper}_{mt} = \beta_0 + \beta_1 \text{evertr}_m + \beta_2 \text{post}_t + \beta_3 \text{evertr}_m \times \text{post}_t + u_{mt}$$

Diff-in-diff implementation: method 2

Method 2: unit & time dummies and treatment indicator

We have controlled for group differences with a group dummy.

What about using *municipality* dummies instead?

municipality	evertr	election	treatment	gdper
Αίγινας	0	May12	0	7.9822884
Αίγινας	0	June12	0	7.2771678
Αίγινας	0	Jan15	0	6.3633003
Αίγινας	0	Sept15	0	7.6177893
Αγίου Βασιλείου	0	May12	0	2.5829175
Αγίου Βασιλείου	0	June12	0	4.2843981
Αγίου Βασιλείου	0	Jan15	0	2.7149322
Αγίου Βασιλείου	0	Sept15	0	3.6940687
Αγίου Ευστρατίου	0	May12	0	4.9549551
Αγίου Ευστρατίου	0	June12	0	4.7619047
Αγίου Ευστρατίου	0	Jan15	0	4.8780484
Αγίου Ευστρατίου	0	Sept15	0	5.9880238
Αγίου Νικολάου	0	May12	0	2.8652139
Αγίου Νικολάου	0	June12	0	3.0493212
Αγίου Νικολάου	0	Jan15	0	3.1590488
Αγίου Νικολάου	0	Sept15	0	4.6045966
Αγαθονησίου	1	May12	0	3.5714288
Αγαθονησίου	1	June12	0	4.6875000
Αγαθονησίου	1	Jan15	0	3.2786884
Αγαθονησίου	1	Sept15	1	5.0000000

Estimate the regression:

$$\text{gdper}_{mt} = \beta_1 \text{treatment}_{mt} + \alpha_m + \delta_t + u_{mt}$$

Diff-in-diff implementation: method 2

Method 2: unit & time dummies and a treatment indicator

Regression output:

```
Call:
lm(formula = gdper ~ treatment + as.factor(election) + as.factor(muni) -
    1, data = d[use, ])

Residuals:
    Min      1Q  Median      3Q     Max
-4.5855 -0.5236 -0.0003  0.4404  6.9990

Coefficients:
                                       Estimate Std. Error t value Pr(>|t|)
treatment                                2.0788     0.3948   5.265 2.79e-07 ***
as.factor(election)Sept15                7.7566     0.5635  13.764  < 2e-16 ***
as.factor(election)Jan15                 6.4612     0.5624  11.488  < 2e-16 ***
as.factor(election)June12                7.4365     0.5624  13.222  < 2e-16 ***
as.factor(election)May12                 7.5862     0.5624  13.489  < 2e-16 ***
as.factor(muni)Αγίου Βασιλείου          -3.9911     0.7829  -5.098 6.33e-07 ***
as.factor(muni)Αγίου Ευστρατίου         -2.1644     0.7829  -2.765 0.006078 **
as.factor(muni)Αγίου Νικολάου           -3.8906     0.7829  -4.969 1.17e-06 ***
as.factor(muni)Αγαθονησίου              -3.6954     0.7891  -4.683 4.41e-06 ***
as.factor(muni)Αγκιστρίου                4.2533     0.7829   5.433 1.20e-07 ***
as.factor(muni)Αλοννήσου                -2.1973     0.7829  -2.807 0.005357 **
as.factor(muni)Αμαρίου                  -4.5633     0.7829  -5.828 1.53e-08 ***
```

[result clipped]

Panel difference-in-difference

$$y_{it} = \beta_1 \text{treatment}_{it} + \alpha_i + \delta_t + u_{it}$$

Key points:

- β_1 estimated based on **variation in treatment over time within units**
- the only relevant confounders **vary with treatment over time within units**

Panel DiD regression as the **"within"** estimator.

Explaining panel DiD findings

Suppose the **data generating process (DGP)** is

$$Y_{it} = \beta_1 D_{it} + \eta \mathbf{X}_t + \zeta \mathbf{U}_i + \psi \mathbf{V}_{it} + \omega_{it}$$

- $\mathbf{X}_t$ are time-specific variables that affect outcomes for all units the same way (e.g. national economic indicators),
- $\mathbf{U}_i$ are unit-specific variables that are constant over time (e.g. urban/rural character),
- $\mathbf{V}_{it}$ are variables that may vary within units over time (e.g. presence of ambitious council member, local economic situation), and
- ω_{it} is random noise.

In panel-DiD analysis where we estimate $Y_{it} = \beta_1 D_{it} + \alpha_i + \delta_t + \epsilon_{it}$,

- time dummies (δ_t) control for all $\mathbf{X}_t$
- unit dummies (α_i) control for all $\mathbf{U}_i$

so the only possible confounders are $\mathbf{V}_{it}$.

Testing assumptions in panel DiD

Common practice is to visualise the parallel trends plot.

Figure: Ivandic, Kirchmaier and Machin, 2021