

Forward problem

from elastic tensor to P and S
observables

stress strain

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$$

strain displacement

$$\varepsilon_{kl} = \frac{1}{2}(\partial_k u_l + \partial_l u_k)$$

momentum equation

$$\rho \ddot{u}_i = \partial_j \sigma_{ij}$$

plane wave displacement

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{a} e^{-i\omega(t - \mathbf{s} \cdot \mathbf{x})}$$

slowness vector \mathbf{s} is unit vector
divided by phase velocity c

$$\rho a_i = a_k C_{ijkl} s_j s_l$$

$$M_{ij} := \frac{1}{\rho} C_{ijkl} \hat{s}_j \hat{s}_l$$

3x3 symmetric eigenproblem

$$\mathbf{M} \cdot \mathbf{a} = c^2 \mathbf{a}$$

eigenvalues: phase velocities (P_n , P , SKS)
eigenvectors: polarizations (SKS , P_{pol})