

ICPC TEMPLATE

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1 动态规划

1.1 回退背包

$dp[i][j]$ 表示选择 i 个物品，体积为 j 的方案数 $f[i][j]$ 表示不考率其中某个物品，选择 i 个物品，体积为 j 的方案数 $dp[i][j] = f[i-1][j-w] + f[i][j] \Rightarrow f[i][j] = dp[i][j] - f[i-1][j-w]$

2 图论

2.1 最近公共祖先

```
1 // 倍增
2 int faz[N][20], dep[N];
3 void dfs(int u, int fa) {
4     faz[u][0] = fa;
5     dep[u] = dep[fa] + 1;
6     for (int i = 1; i < 20; i++) faz[u][i] = faz[faz[u][i-1]][i-1];
7     for (int v : G[u]) if (v != fa) {
8         dfs(v, u);
9     }
10 }
11 int LCA(int u, int v) {
12     if (dep[u] < dep[v]) swap(u, v);
13     int d = dep[u] - dep[v];
14     for (int i = 0; i < 20; i++) if ((d >> i) & 1) u = faz[u][i];
15     if (v == u) return u;
16     for (int i = 19; i >= 0; i--) if (faz[u][i] != faz[v][i])
17         u = faz[u][i], v = faz[v][i];
18     return faz[u][0];
19 }
20
21 // 树剖
22 int dfc, dfn[N], rnk[N], siz[N], top[N], dep[N], son[N], faz[N];
23 void dfs1(int u, int fa) {
24     dep[u] = dep[fa] + 1;
25     siz[u] = 1;
26     son[u] = -1;
27     faz[u] = fa;
28     for (int v : G[u]) {
29         if (v == fa) continue;
30         dfs1(v, u);
```

```

31     siz[u] += siz[v];
32     if (son[u] == -1 || siz[son[u]] < siz[v]) son[u] = v;
33 }
34 }
35 void dfs2(int u, int fa, int tp) {
36     dfn[u] = ++dfc;
37     rnk[dfc] = u;
38     top[u] = tp;
39     if (son[u] != -1) dfs2(son[u], u, tp);
40     for (int v : G[u]) {
41         if (v == fa || v == son[u]) continue;
42         dfs2(v, u, v);
43     }
44 }
45 int LCA(int u, int v) {
46     while (top[u] != top[v]) {
47         if (dep[top[u]] > dep[top[v]])
48             u = faz[top[u]];
49         else
50             v = faz[top[v]];
51     }
52     return dep[u] > dep[v] ? v : u;
53 }
54
55 // O(1) query
56
57 int dfn[N], faz[N], dep[N], rnk[N], dfc, st[N][20];
58 void dfs(int u, int fa) {
59     dfn[u] = ++dfc; faz[u] = fa; dep[u] = dep[fa] + 1; rnk[dfc] = u;
60     for (auto [v, w] : G[u]) if (v != fa) dfs(v, u);
61 }
62 int LCA(int u, int v) {
63     if (u == v) return u;
64     if (dfn[u] > dfn[v]) swap(u, v);
65     int l = dfn[u] + 1, r = dfn[v];
66     int k = __lg(r - l + 1);
67     return dep[st[l][k]] < dep[st[r - (1 << k) + 1][k]] ? faz[st[l][k]] : faz[
        st[r - (1 << k) + 1][k]];
68 }
69
70 int main() {
71     dfs(1, 0);
72     dep[0] = n + 1;
73     for (int i = 1; i <= n; i++) st[i][0] = rnk[i];

```

```

74     for (int j = 1; j < 20; j++) {
75         for (int i = 1; i <= n; i++) {
76             st[i][j] = dep[st[i][j - 1]] <= dep[st[min(n, i + (1 << (j - 1)))]
                        ][j - 1] ? st[i][j - 1] : st[min(n, i + (1 << (j - 1)))]
                        [j - 1];
77         }
78     }
79 }

```

2.2 2-SAT 前缀优化建图

1. 当前点选择说明之前的前缀都未选择

之前的前缀选择说明当前点被选择

2. 之前的前缀选择说明当前前缀被选择

当前前缀未选择说明之前前缀未选择

3. 当前点选择说明当前前缀选择

当前前缀未选择说明当前点未选择

2.3 Kruskal 重构树的性质

不难发现，原图中两个点之间的所有简单路径上最大边权的最小值 = 最小生成树上两个点之间的简单路径上的最大值 = Kruskal 重构树上两点之间的 LCA 的权值。

也就是说，到点 x 的简单路径上最大边权的最小值 $\leq val$ 的所有点 y 均在 Kruskal 重构树上的某一棵子树内，且恰好为该子树的所有叶子节点。

我们在 Kruskal 重构树上找到 x 到根的路径上权值 $\leq val$ 的最浅的节点。显然这就是所有满足条件的节点所在的子树的根节点。

如果要求原图中两个点之间的所有简单路径上最小边权的最大值，则在跑 Kruskal 的过程中按边权大到小的顺序加边。

2.4 广义圆方树

```

1 int stk[N], n, m, top, cnt, low[N], dfn[N], dfc;
2 bool vis[N];
3 vector<int> G[N], T[N];

```

```

4
5 void tarjan(int u) {
6     stk[++top] = u;
7     low[u] = dfn[u] = ++dfc;
8     for (int v : G[u]) {
9         if (!dfn[v]) {
10             tarjan(v);
11             low[u] = min(low[u], low[v]);
12             if (low[v] == dfn[u]) {
13                 cnt++;
14                 for (int x = 0; x != v; --top) {
15                     x = stk[top];
16                     T[cnt].push_back(x);
17                     T[x].push_back(cnt);
18                     val[cnt]++;
19                 }
20                 T[cnt].push_back(u);
21                 T[u].push_back(cnt);
22                 val[cnt]++;
23             }
24             } else low[u] = min(low[u], dfn[v]);
25     }
26 }
27 int main() {
28     cnt = n;
29     for (int i = 1; i <= n; i++) if (!dfn[i]) {
30         tarjan(i);
31         --top;
32     }
33 }

```

2.5 点分树

需要注意，点分树上的路径与原来的树完全没有关系。

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 using ll = long long;
4
5 const int mod = 998244353;
6 const int N = 2e5 + 5;
7
8 int n, q, val[N], rt, siz[N], SUM, mx[N];
9 int faz[N];

```

```

10 vector<int> T[N];
11 bool vis[N];
12
13 // 假设树剖已经写好
14
15 void getrt(int u, int fa) {
16     siz[u] = 1;
17     mx[u] = 0;
18     for (int v : t.G[u]) if (v != fa && !vis[v]) {
19         getrt(v, u);
20         siz[u] += siz[v];
21         mx[u] = max(mx[u], siz[v]);
22     }
23     mx[u] = max(mx[u], SUM - siz[u]);
24     if (mx[u] < mx[rt]) rt = u;
25 }
26 void Dfs(int u, int fa) {
27     vis[u] = 1;
28     for (int v : t.G[u]) if (v != fa && !vis[v]) {
29         rt = 0;
30         SUM = siz[v];
31         getrt(v, rt);
32         T[u].push_back(rt);
33         faz[rt] = u;
34         Dfs(rt, rt);
35     }
36 }
37
38 // 假设动态开点权值线段树已经写好
39
40 void dfs(int u, const int now) { // 预处理
41     add(rt1[now], 0, n, dist(faz[now], u), val[u]);
42     add(rt0[now], 0, n, dist(u, now), val[u]);
43     for (int v : T[u]) dfs(v, now);
44 }
45 void upd(int u, int d) {
46     int x = u;
47     while (true) {
48         add(rt1[u], 0, n, dist(x, faz[u]), d);
49         add(rt0[u], 0, n, dist(x, u), d);
50         if (u == root) break;
51         u = faz[u];
52     }
53 }

```



```

54 int qry(int u, int k) {
55     int ans = query(rt0[u], 0, n, 0, k), x = u;
56     while (u != root) {
57         if (k - dist(faz[u], x) >= 0) ans += query(rt0[faz[u]], 0, n, 0, k -
            dist(faz[u], x)) - query(rt1[u], 0, n, 0, k - dist(faz[u], x));
58         u = faz[u];
59     }
60     return ans;
61 }

```

2.6 二分图

最大匹配

```

1  int mch[maxn], vis[maxn];
2  std::vector<int> e[maxn];
3  bool dfs(const int u, const int tag) {
4      for (auto v : e[u]) {
5          if (vis[v] == tag) continue;
6          vis[v] = tag;
7          if (!mch[v] || dfs(mch[v], tag)) return mch[v] = u, 1;
8      }
9      return 0;
10 }
11 int main() {
12     int ans = 0;
13     for (int i = 1; i <= n; ++i) if (dfs(i, i)) ++ans;
14 }

```

2.7 有向图最小路径覆盖问题

```

1  int n, m;
2  bitset<N> f[N];
3  int vis[N], mch[N];
4
5  bool dfs(int u, int dfc) {
6      for (int v = 1; v <= n; v++) if (v != u && vis[v] != dfc && f[u][v]) {
7          vis[v] = dfc;
8          if (!mch[v] || dfs(mch[v], dfc)) return mch[v] = u, 1;
9      }
10     return 0;
11 }
12
13 void solve() {

```

```

14     memset(vis, 0, sizeof vis);
15     memset(mch, 0, sizeof mch);
16     for (int i = 1; i <= n; i++) f[i].reset();
17     for (int i = 1; i <= m; i++) {
18         int u, v;
19         scanf("%d %d", &u, &v);
20         f[u].set(v);
21     }
22     for (int k = 1; k <= n; k++) {
23         for (int i = 1; i <= n; i++) if (f[i][k]) f[i] |= f[k];
24     }
25     int res = n;
26     for (int i = 1; i <= n; i++) res -= dfs(i, i);
27     printf("%d\n", res);
28 }

```

2.8 网络流

2.8.1 Dinic 最大流

注意每次清空数组的范围是 s 到 t .

```

1  int head[N], cur[N], ecnt, d[N];
2  struct Edge {
3      int nxt, v, flow, cap;
4  }e[];
5  void add_edge(int u, int v, int flow, int cap) {
6      e[ecnt] = {head[u], v, flow, cap}; head[u] = ecnt++;
7      e[ecnt] = {head[v], u, flow, 0}; head[v] = ecnt++;
8  }
9  bool bfs() {
10     memset(vis, 0, sizeof vis);
11     std::queue<int> q;
12     q.push(s);
13     vis[s] = 1;
14     d[s] = 0;
15     while (!q.empty()) {
16         int u = q.front();
17         q.pop();
18         for (int i = head[u]; i != -1; i = e[i].nxt) {
19             int v = e[i].v;
20             if (vis[v] || e[i].flow >= e[i].cap) continue;
21             d[v] = d[u] + 1;
22             vis[v] = 1;
23             q.push(v);

```

```

24     }
25 }
26 return vis[t];
27 }
28 int dfs(int u, int a) {
29     if (u == t || !a) return a;
30     int flow = 0, f;
31     for (int& i = cur[u]; i != -1; i = e[i].nxt) {
32         int v = e[i].v;
33         if (d[u] + 1 == d[v] && (f = dfs(v, std::min(a, e[i].cap - e[i].flow))
34             ) > 0) {
35             e[i].flow += f;
36             e[i ^ 1].flow -= f;
37             flow += f;
38             a -= f;
39             if (!a) break;
40         }
41     }
42     return flow;

```

2.8.2 最小费用最大流

```

1  const int inf = 1e9;
2  int head[N], cur[N], ecnt, dis[N], s, t, n, m, mincost;
3  bool vis[N];
4  struct Edge {
5      int nxt, v, flow, cap, w;
6  }e[100002];
7  void add_edge(int u, int v, int flow, int cap, int w) {
8      e[ecnt] = {head[u], v, flow, cap, w}; head[u] = ecnt++;
9      e[ecnt] = {head[v], u, flow, 0, -w}; head[v] = ecnt++;
10 }
11 bool spfa(int s, int t) {
12     std::fill(vis + s, vis + t + 1, 0);
13     std::fill(dis + s, dis + t + 1, inf);
14     std::queue<int> q;
15     q.push(s);
16     dis[s] = 0;
17     vis[s] = 1;
18     while (!q.empty()) {
19         int u = q.front();
20         q.pop();
21         vis[u] = 0;

```

```

22     for (int i = head[u]; i != -1; i = e[i].nxt) {
23         int v = e[i].v;
24         if (e[i].flow < e[i].cap && dis[u] + e[i].w < dis[v]) {
25             dis[v] = dis[u] + e[i].w;
26             if (!vis[v]) vis[v] = 1, q.push(v);
27         }
28     }
29 }
30 return dis[t] != inf;
31 }
32 int dfs(int u, int a) {
33     if (vis[u]) return 0;
34     if (u == t || !a) return a;
35     vis[u] = 1;
36     int flow = 0, f;
37     for (int i = cur[u]; i != -1; i = e[i].nxt) {
38         int v = e[i].v;
39         if (dis[u] + e[i].w == dis[v] && (f = dfs(v, std::min(a, e[i].cap - e[
40             i].flow))) > 0) {
41             e[i].flow += f;
42             e[i ^ 1].flow -= f;
43             flow += f;
44             mincost += e[i].w * f;
45             a -= f;
46             if (!a) break;
47         }
48     }
49     vis[u] = 0;
50     return flow;

```

2.8.3 最大闭权子图

正权点向 S 连边，负权点向 T 连边。边权为点权的绝对值。原图的边容量设为 INF。

则最大收益为 $\sum_{v>0} v - mincost$

在最大闭权子图中的点是残量网络中 S 能到达的点。

2.9 树哈希

```

1 const ull mask = chrono::steady_clock::now().time_since_epoch().count();
2
3 ull shift(ull x) {

```

```
4     x ^= mask;
5     x ^= x << 13;
6     x ^= x >> 7;
7     x ^= x << 17;
8     x ^= mask;
9     return x;
10 }
11 int n;
12 ull H[N];
13 vector<int> G[N];
14 set<ull> s;
15
16 void dfs(int u, int fa) {
17     H[u] = 1;
18     for (int v : G[u]) {
19         if (v == fa) continue;
20         dfs(v, u);
21         H[u] += shift(H[v]);
22     }
23     s.emplace(H[u]);
24 }
```

2.10 强联通分量

```
1 int n, dfc, dfn[N], low[N], stk[N], top, idx[N], in_stk[N], scc_cnt;
2 vector<int> G[N];
3
4 void tarjan(int u) {
5     low[u] = dfn[u] = ++dfc;
6     stk[++top] = u;
7     in_stk[u] = 1;
8     for (int v : G[u]) {
9         if (!dfn[v]) {
10             tarjan(v);
11             low[u] = min(low[u], low[v]);
12         } else if (in_stk[v]) low[u] = min(dfn[v], low[u]);
13     }
14     if (low[u] == dfn[u]) {
15         int x;
16         scc_cnt++;
17         do {
18             x = stk[top--];
19             idx[x] = scc_cnt;
20             in_stk[x] = 0;
```

```

21     } while (x != u);
22     }
23 }
24
25 // 多测清空
26 dfc = scc_cnt = top = 0;
27 for (int i = 1; i <= tot; i++) low[i] = dfn[i] = idx[i] = in_stk[i] = 0;

```

2.11 割点和桥

```

1  int dfn[N], low[N], dfs_clock;
2  bool iscut[N], vis[N];
3  void dfs(int u, int fa) {
4      dfn[u] = low[u] = ++dfs_clock;
5      vis[u] = 1;
6      int child = 0;
7      for (int v : e[u]) {
8          if (v == fa) continue;
9          if (!dfn[v]) {
10             dfs(v, u);
11             low[u] = min(low[u], low[v]);
12             child++;
13             if (low[v] >= dfn[u]) iscut[u] = 1;
14         } else if (dfn[u] > dfn[v] && v != fa) low[u] = min(low[u], dfn[v]);
15         if (fa == 0 && child == 1) iscut[u] = 0;
16     }
17 }

```

2.12 点双联通分量

```

1  int bccno[N], bcc_cnt, siz_e[N], siz_p[N], dfs_clock, low[N], dfn[N], top;
2  pair<int, int> stk[N];
3  void dfs(int u, int fa) {
4      low[u] = dfn[u] = ++dfs_clock;
5      for(int i = head[u]; i; i = e[i].nxt) {
6          int v = e[i].v;
7          if(v == fa) continue;
8          if(!dfn[v]) {
9              stk[++top] = make_pair(u, v);
10             dfs(v, u);
11             low[u] = min(low[u], low[v]);
12             if(low[v] >= dfn[u]) {
13                 bcc_cnt++;

```

```

14         while(true) {
15             int x = stk[top].first, y = stk[top].second;
16             top--;
17             siz_e[bcc_cnt]++;
18             if(bccno[x] != bcc_cnt) {bccno[x] = bcc_cnt; siz_p[bcc_cnt]++;}
19             if(bccno[y] != bcc_cnt) {bccno[y] = bcc_cnt; siz_p[bcc_cnt]++;}
20             if(x == u && y == v) break;
21         }
22     }
23     } else if(dfn[v] < dfn[u]) {stk[++top] = make_pair(u, v); low[u] = min
        (low[u], dfn[v]);}
24 }
25 }

```

2.13 边双联通分量

```

1  const int N = 5000 + 5;
2  int n, m, stk[N], top, ccno, sc[N];
3  int dfn[N], dfc, low[N];
4  int mp[N][N];
5  int in[N];
6  int head[N], ecnt;
7  struct Edge {
8      int nxt, v;
9  } e[N << 2];
10 void add_edge(int u, int v) {
11     e[ecnt] = {head[u], v}; head[u] = ecnt++;
12     e[ecnt] = {head[v], u}; head[v] = ecnt++;
13 }
14 void dfs(int u, int from) {
15     stk[++top] = u;
16     low[u] = dfn[u] = ++dfc;
17     for (int i = head[u]; i != -1; i = e[i].nxt) {
18         int v = e[i].v;
19         if (!dfn[v]) {
20             dfs(v, i);
21             low[u] = min(low[u], low[v]);
22         } else if ((i ^ 1) != from) low[u] = min(low[u], dfn[v]);
23     }
24     if (dfn[u] == low[u]) {
25         ccno++;
26         int x;

```

```

27     while (true) {
28         x = stk[top--];
29         sc[x] = ccno;
30         if (x == u) break;
31     }
32 }
33 }
34
35 void solve() {
36     memset(head, -1, sizeof head);
37     scanf("%d %d", &n, &m);
38     for (int i = 1; i <= m; i++) {
39         int u, v;
40         scanf("%d %d", &u, &v);
41         add_edge(u, v);
42     }
43     for (int i = 1; i <= n; i++) if (!dfn[i]) dfs(i, i);
44     for (int i = 1; i <= n; i++) {
45         for (int k = head[i]; k != -1; k = e[k].nxt) {
46             int j = e[k].v;
47             if (sc[i] != sc[j]) mp[sc[i]][sc[j]] = 1;
48         }
49     }
50
51     for (int i = 1; i <= ccno; i++) {
52         for (int j = 1; j <= ccno; j++) if (mp[i][j]) in[j]++;
53     }
54     int cnt = 0;
55     for (int i = 1; i <= ccno; i++) if (in[i] == 1) cnt++;
56     printf("%d\n", (cnt + 1) / 2);
57 }

```

2.14 2-SAT

$2 * u$ 代表不选择, $2 * u + 1$ 代表选择。

也可以求强连通分量。

如果对于一个 $*x*$ ‘sccno’ 比它的反状态 $*x*1$ 的 ‘sccno’ 要小, 那么我们用 $*x*$ 这个状态当做答案, 否则用它的反状态当做答案。

```

1 vector<int> G[N * 2];
2 bool mark[N * 2];
3 int stk[N], top;

```



```

4 void build_G() {
5     for (int i = 1; i <= n; i++) {
6         int u, v;
7         G[2 * u + 1].push_back(2 * v);
8         G[2 * v + 1].push_back(2 * u);
9     }
10 }
11 bool dfs(int u) {
12     if (mark[u ^ 1]) return false;
13     if (mark[u]) return true;
14     mark[u] = 1;
15     stk[++top] = u;
16     for (int v : G[u]) {
17         if (!dfs(v)) return false;
18     }
19     return true;
20 }
21 bool 2_sat() {
22     for (int i = 1; i <= n; i++) {
23         if (!mark[i * 2] && !mark[i * 2 + 1]) {
24             top = 0;
25             if (!dfs(2 * i)) {
26                 while (top) mark[stk[top--]] = 0;
27                 if (!dfs(2 * i + 1)) return 0;
28             }
29         }
30     }
31     return 1;
32 }

```

3 数据结构

3.1 Splay

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 using ll = long long;
4
5 #define rank abcdefg
6 const int mod = 998244353;
7 const int N = 1e5 + 5;
8
9 int tot, fa[N], tr[N][2], sz[N], cnt[N], val[N], rt;
10

```

```
11 void maintain(int x) {
12     sz[x] = sz[tr[x][0]] + sz[tr[x][1]] + cnt[x];
13 }
14 int getdir(int x) {
15     return tr[fa[x]][1] == x;
16 }
17 void clear(int x) {
18     fa[x] = sz[x] = cnt[x] = tr[x][0] = tr[x][1] = val[x] = 0;
19 }
20 int create(int v) {
21     ++tot;
22     val[tot] = v;
23     sz[tot] = cnt[tot] = 1;
24     return tot;
25 }
26 void rotate(int x) {
27     if (x == rt) return;
28     int y = fa[x], z = fa[y], d = getdir(x);
29     tr[y][d] = tr[x][d ^ 1];
30     if (tr[x][d ^ 1]) fa[tr[x][d ^ 1]] = y;
31     fa[y] = x;
32     tr[x][d ^ 1] = y;
33     fa[x] = z;
34     if (z) tr[z][y == tr[z][1]] = x;
35     maintain(y);
36     maintain(x);
37 }
38 void splay(int x) {
39     for (int f = fa[x]; f = fa[x], f; rotate(x)) {
40         if (fa[f]) rotate(getdir(f) == getdir(x) ? f : x);
41     }
42     rt = x;
43 }
44 void insert(int v) {
45     if (!rt) {
46         rt = create(v);
47         return;
48     }
49     int u = rt, f = 0;
50     while (true) {
51         if (val[u] == v) {
52             cnt[u]++;
53             maintain(u);
54             maintain(f);
```

```

55         splay(u);
56         return;
57     }
58     f = u, u = tr[u][v > val[u]];
59     if (u == 0) {
60         int id;
61         fa[id = create(v)] = f;
62         tr[f][v > val[f]] = id;
63         maintain(f);
64         splay(id);
65         return;
66     }
67 }
68 }
69
70 int rank(int v) {
71     int rk = 0;
72     int u = rt;
73     while (u) {
74         if (val[u] == v) {
75             rk += sz[tr[u][0]];
76             splay(u);
77             return rk + 1;
78         }
79         if (v < val[u]) {
80             u = tr[u][0];
81         } else {
82             rk += sz[tr[u][0]] + cnt[u];
83             u = tr[u][1];
84         }
85     }
86     return -1;
87 }
88
89 int kth(int x) {
90     int u = rt;
91     while (u) {
92         if (sz[tr[u][0]] + cnt[u] >= x && sz[tr[u][0]] < x) return val[u];
93         if (x <= sz[tr[u][0]]) {
94             u = tr[u][0];
95         } else {
96             x -= sz[tr[u][0]] + cnt[u];
97             u = tr[u][1];
98         }

```

```
99     }
100     return u ? val[u] : -1;
101 }
102 int pre() {
103     int u = tr[rt][0];
104     if (!u) return val[rt];
105     while (true) {
106         if (tr[u][1] == 0) return splay(u), val[u];
107         u = tr[u][1];
108     }
109     return 233;
110 }
111 int suf() {
112     int u = tr[rt][1];
113     if (!u) return val[rt];
114     while (true) {
115         if (tr[u][0] == 0) return splay(u), val[u];
116         u = tr[u][0];
117     }
118     return 233;
119 }
120 void del(int v) {
121     if (rank(v) == -1) return;
122     if (cnt[rt] > 1) {
123         cnt[rt]--;
124         return;
125     }
126     if (!tr[rt][1] && !tr[rt][0]) {
127         clear(rt), rt = 0;
128     } else if (!tr[rt][0]) {
129         int x = rt;
130         rt = tr[x][1];
131         fa[rt] = 0;
132         clear(x);
133     } else if (!tr[rt][1]) {
134         int x = rt;
135         rt = tr[x][0];
136         fa[rt] = 0;
137         clear(x);
138     } else {
139         int cur = rt, y = tr[cur][1];
140         pre();
141         tr[rt][1] = y;
142         fa[y] = rt;
```

```

143     clear(cur);
144     maintain(rt);
145 }
146 }
147
148 int main() {
149     int n, opt, x;
150
151     for (scanf("%d", &n); n; --n) {
152         scanf("%d%d", &opt, &x);
153
154         if (opt == 1)
155             insert(x);
156         else if (opt == 2)
157             del(x);
158         else if (opt == 3)
159             printf("%d\n", rank(x));
160         else if (opt == 4)
161             printf("%d\n", kth(x));
162         else if (opt == 5)
163             insert(x), printf("%d\n", pre()), del(x);
164         else
165             insert(x), printf("%d\n", suf()), del(x);
166     }
167
168     return 0;
169 }

```

3.2 李超线段树

```

1 struct Line {
2     ll k, b;
3 } lin[N];
4 int lcnt;
5 int add_line(ll k, ll b) {
6     lin[++lcnt] = {k, b};
7     return lcnt;
8 }
9 struct node {
10     int ls, rs, u;
11 } tr[N << 2];
12 int tot;
13 ll calc(int u, ll x) {
14     return lin[u].k * x + lin[u].b;

```

```

15 }
16 bool cmp(int u, int v, ll x) {
17     return calc(u, x) <= calc(v, x); // 如果要求最大值，只需要修改为大于等于
18 }
19 void pushdown(int &p, int l, int r, int v) {
20     if (!p) p = ++tot;
21     if (l == r) return;
22     int mid = (l + r) >> 1;
23     int &u = tr[p].u, b = cmp(v, u, mid);
24     if (b) swap(u, v);
25     int bl = cmp(v, u, l), br = cmp(v, u, r);
26     if (bl) pushdown(tr[p].ls, l, mid, v);
27     if (br) pushdown(tr[p].rs, mid + 1, r, v);
28 }
29 void update(int &p, int l, int r, int L, int R, int v) {
30     if (l > R || r < L) return;
31     if (!p) p = ++tot;
32     int mid = (l + r) >> 1;
33     if (l >= L && r <= R) return pushdown(p, l, r, v), void();
34     update(tr[p].ls, l, mid, L, R, v);
35     update(tr[p].rs, mid + 1, r, L, R, v);
36 }
37 ll query(int p, int l, int r, ll pos) {
38     if (!p) return 1e16;
39     ll res = calc(tr[p].u, pos);
40     int mid = (l + r) >> 1;
41     if (l == r) return res;
42     if (pos <= mid) {
43         res = min(res, query(tr[p].ls, l, mid, pos));
44     } else res = min(res, query(tr[p].rs, mid + 1, r, pos));
45     return res;
46 }
47
48 int main() {
49     lin[0].b = 1e16;
50     return 0;
51 }

```

3.3 Link Cut Tree

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 1e5 + 5;

```

```
5  int n, m, ch[N][2], f[N], s[N], r[N], v[N];
6
7  #define lc ch[x][0]
8  #define rc ch[x][1]
9
10 bool noroot(int x) {
11     return ch[f[x]][1] == x || ch[f[x]][0] == x;
12 }
13 void pushup(int x) {
14     s[x] = s[lc] ^ s[rc] ^ v[x];
15 }
16 void pushr(int x) {
17     swap(lc, rc);
18     r[x] ^= 1;
19 }
20 void pushdown(int x) {
21     if (r[x]) {
22         if (lc) pushr(lc);
23         if (rc) pushr(rc);
24         r[x] = 0;
25     }
26 }
27 void rotate(int x) {
28     int y = f[x], z = f[y], k = (ch[y][1] == x), w = ch[x][k ^ 1];
29     if (noroot(y)) ch[z][y == ch[z][1]] = x;
30     ch[x][k ^ 1] = y;
31     ch[y][k] = w;
32     f[w] = y;
33     f[y] = x;
34     f[x] = z;
35     pushup(y), pushup(x);
36 }
37 void update(int x) {
38     if (noroot(x)) update(f[x]);
39     pushdown(x);
40 }
41 bool get(int x) {
42     return ch[f[x]][1] == x;
43 }
44 void splay(int x) {
45     update(x);
46     for (int fa; fa = f[x], noroot(x); rotate(x)) {
47         if (noroot(fa)) rotate(get(x) == get(fa) ? fa : x);
48     }
```

```
49     pushup(x);
50 }
51 void access(int x) {
52     int p;
53     for (p = 0; x; p = x, x = f[x]) {
54         splay(x), ch[x][1] = p, pushup(x);
55     }
56 }
57 void makeroot(int x) {
58     access(x); splay(x);
59     pushr(x);
60 }
61 int findroot(int x) {
62     access(x);
63     splay(x);
64     while (lc) pushdown(x), x = lc;
65     splay(x);
66     return x;
67 }
68 void split(int x, int y) {
69     makeroot(x);
70     access(y); splay(y);
71 }
72 void link(int x, int y) {
73     makeroot(x);
74     if (findroot(y) != x) f[x] = y;
75 }
76 void cut(int x, int y) {
77     makeroot(x);
78     if (findroot(y) == x && f[y] == x && !ch[y][0]) {
79         f[y] = ch[x][1] = 0;
80         pushup(x);
81     }
82 }
83
84 int main() {
85     scanf("%d %d", &n, &m);
86     for (int i = 1; i <= n; i++) scanf("%d", &v[i]);
87     while (m--) {
88         int opt, x, y;
89         scanf("%d %d %d", &opt, &x, &y);
90         if (opt == 0) split(x, y), printf("%d\n", s[y]);
91         if (opt == 1) link(x, y);
92         if (opt == 2) cut(x, y);
```



```

93     if (opt == 3) splay(x), v[x] = y, pushup(x);
94 }
95 return 0;
96 }

```

3.4 兔队线段树

求有多少个严格前缀最大值。

线段树保存每个区间为子问题时右部分的答案 res (可以不需要信息可减), 和区间的最大值 mx 。

$calc$ 考虑一段区间之前有 x 大的数时, 区间此时前缀最大数的树目。

1. $x \geq val[lson]$, $ans = calc(rson)$
2. $x < val[lson]$, $ans = calc(lson) + res[p]$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  using ll = long long;
4
5  const int N = 1e5 + 5;
6  #define lson (p << 1)
7  #define rson ((p << 1) | 1)
8  #define mid ((l + r) >> 1)
9  int n, m;
10 struct node {
11     int s, a, b;
12 } tr[N << 2];
13 bool cmp(int a, int b, int c, int d) {
14     if (d == 0 && b == 0) return 0;
15     if (d == 0 && a == 0) return 0;
16     if (d == 0) return 1;
17     return a * 1ll * d > c * 1ll * b;
18 }
19 int calc(int p, int l, int r, int c, int d) {
20     if (l == r)
21         return cmp(tr[p].a, tr[p].b, c, d);
22     if (cmp(tr[lson].a, tr[lson].b, c, d)) {
23         return calc(lson, l, mid, c, d) + tr[p].s;
24     }
25     return calc(rson, mid + 1, r, c, d);
26 }
27 void modify(int p, int l, int r, int pos, int v) {
28     if (l == r) {

```

```

29     tr[p] = {0, v, pos};
30     return;
31 }
32 if (pos <= mid) modify(lson, l, mid, pos, v);
33 else modify(rson, mid + 1, r, pos, v);
34 if (cmp(tr[lson].a, tr[lson].b, tr[rson].a, tr[rson].b)) {
35     tr[p] = tr[lson];
36 } else tr[p] = tr[rson];
37 tr[p].s = calc(rson, mid + 1, r, tr[lson].a, tr[lson].b);
38 }
39
40 int main() {
41     scanf("%d %d", &n, &m);
42     while (m--) {
43         int x, y;
44         scanf("%d %d", &x, &y);
45         modify(1, 1, n, x, y);
46         printf("%d\n", calc(1, 1, n, 0, 0));
47     }
48     return 0;
49 }

```

3.5 线段树分治

有一个 n 个节点的图。

在 k 时间内有 m 条边会出现后消失。

要求出每一时间段内这个图是否是二分图。

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int N = 4e5 + 5, M = 4e6;
5  int tot, n, m, t, fa[N], d[N], u[N], v[N];
6  int stk[N], top, head[N];
7  bool fl[N];
8
9  struct E {
10     int nxt, id;
11 } e[M];
12
13 int find(int x) {
14     while (fa[x]) x = fa[x];
15     return x;

```

```

16 }
17 void merge(int x, int y) {
18     x = find(x), y = find(y);
19     if (x == y) return;
20     if (d[x] > d[y]) swap(x, y);
21     fa[x] = y;
22     stk[++top] = x;
23     d[y] += fl[top] = (d[x] == d[y]);
24 }
25
26 void upd(int p, int l, int r, int L, int R, const int &i) {
27     if (l == L && r == R) {
28         e[++tot] = (E){head[p], i};
29         head[p] = tot;
30         return ;
31     }
32     int mid = (l + r) >> 1;
33     if (R <= mid) upd(p << 1, l, mid, L, R, i);
34     else if (L > mid) upd(p << 1 | 1, mid + 1, r, L, R, i);
35     else upd(p << 1, l, mid, L, mid, i), upd(p << 1 | 1, mid + 1, r, mid + 1,
        R, i);
36 }
37 void solve(int p, int l, int r) {
38     int lst = top, mid = (l + r) >> 1;;
39     for (int i = head[p]; i; i = e[i].nxt) {
40         int x = u[e[i].id], y = v[e[i].id];
41         if (find(x) == find(y)) {
42             for (int i = l; i <= r; i++) puts("No");
43             goto che;
44         }
45         merge(x + n, y), merge(x, y + n);
46     }
47     if (l == r)
48         puts("Yes");
49     else {
50         solve(p << 1, l, mid);
51         solve(p << 1 | 1, mid + 1, r);
52     }
53     che : for (; top > lst; top--) d[fa[stk[top]]] -= fl[top], fa[stk[top]] =
        0;
54 }
55
56
57 int main() {

```

```

58     //freopen("2.in", "r", stdin);
59     scanf("%d %d %d", &n, &m, &t);
60     for (int i = 1; i <= m; i++) {
61         int l, r;
62         scanf("%d %d %d %d", &u[i], &v[i], &l, &r);
63         if (l == r) continue;
64         upd(1, 1, t, l + 1, r, i);
65     }
66     solve(1, 0, t - 1);
67     return 0;
68 }

```

4 字符串

4.1 哈希

4.1.1 最长回文子串

通过哈希同样可以 $O(n)$ 解决这个问题，具体方法就是记 R_i 表示以 i 作为结尾的最长回文的长度，那么答案就是 $\max_{i=1}^n R_i$ 。考虑到 $R_i \leq R_{i-1} + 2$ ，因此我们只需要暴力从 $R_{i-1} + 2$ 开始递减，直到找到第一个回文即可。记变量 z 表示当前枚举的 R_i ，初始时为 0，则 z 在每次 i 增大的时候都会增大 2，之后每次暴力循环都会减少 1，故暴力循环最多发生 $2n$ 次，总的时间复杂度为 $O(n)$ 。

4.2 字典树

4.3 维护异或和

```

1  const int N = 526010, MX = 22;
2  int ch[N * MX][2], tot, rt[N], w[N * MX], xorv[N * MX], val[N];
3  ll ans;
4
5  void pushup(int u) {
6      w[u] = xorv[u] = 0;
7      if (ch[u][0]) {
8          w[u] += w[ch[u][0]];
9          xorv[u] ^= (xorv[ch[u][0]] << 1);
10     }
11     if (ch[u][1]) {
12         w[u] += w[ch[u][1]];
13         xorv[u] ^= (xorv[ch[u][1]] << 1) | (w[ch[u][1]] & 1);

```

```
14     }
15     w[u] &= 1;
16 }
17 void insert(int &o, ll ux, int dep) {
18     if (!o) o = ++tot;
19     if (dep > MX) return (void)(w[o]++);
20     insert(ch[o][ux & 1], ux >> 1, dep + 1);
21     pushup(o);
22 }
23 void addall(int o) {
24     swap(ch[o][0], ch[o][1]);
25     if (ch[o][0]) addall(ch[o][0]);
26     pushup(o);
27 }
28 int merge(int a, int b) {
29     if (!b || !a) return a + b;
30     xorv[a] ^= xorv[b];
31     w[a] += w[b];
32     ch[a][0] = merge(ch[a][0], ch[b][0]);
33     ch[a][1] = merge(ch[a][1], ch[b][1]);
34     return a;
35 }
36
37 vector<int> G[N];
38 int read() {
39     int w = 0, f = 1; char ch = getchar();
40     while (ch > '9' || ch < '0') {
41         if (ch == '-') f = -1;
42         ch = getchar();
43     }
44     while (ch >= '0' && ch <= '9') {
45         w = w * 10 + ch - 48;
46         ch = getchar();
47     }
48     return w * f;
49 }
50
51 void dfs(int u) {
52     for (auto v : G[u]) {
53         dfs(v);
54         rt[u] = merge(rt[u], rt[v]);
55     }
56     addall(rt[u]);
57     insert(rt[u], val[u], 0);
```

```

58     ans += (ll)xorv[rt[u]];
59 }
60
61 int main() {
62     int n = read();
63     for (int i = 1; i <= n; i++) val[i] = read();
64     for (int i = 2; i <= n; i++) G[read()].push_back(i);
65     dfs(1);
66     printf("%lld\n", ans);
67     return 0;
68 }

```

4.4 KMP

```

1  int n = strlen(s + 1);
2  for (int i = 2; i <= n; i++) {
3      int j = k[i - 1];
4      while (j != 0 && s[i] != s[j + 1]) j = k[j];
5      if (s[i] == s[j + 1]) k[i] = j + 1;
6      else k[i] = 0;
7  }

```

4.4.1 字符串最小周期

设 border 长度为 r

则 $s[i] = s[n - r + i]$

$|T| = n - r$

4.4.2 每个前缀的出现次数

1. 统计每个前缀在自身的出现次数

```

1  vector<int> ans(n + 1);
2  for (int i = 1; i <= n; i++) ans[k[i]]++;
3  for (int i = n; i >= 1; i--) ans[k[i]] += ans[i];
4  for (int i = 1; i <= n; i++) ans[i]++;

```

2. 统计每个前缀在其他串的出现次数

我们应用来自 Knuth-Morris-Pratt 的技巧：构造一个字符串 $s + \# + t$ 并计算其前缀函数。与第一个问题唯一的不同之处在于，我们只关心与字符串 t 相关的前缀函数值，即 $i \geq n + 1$ 的 $\pi[i]$ 。

有了这些值之后，我们可以同样应用在第一个问题中的算法来解决该问题。

4.4.3 一个字符串中本质不同子串的数目

给定一个长度为 n 的字符串 s ，我们希望计算其本质不同子串的数目。

我们将迭代的解决该问题。换句话说，在知道了当前的本质不同子串的数目的情况下，我们要找出一种在 s 末尾添加一个字符后重新计算该数目的方法。

令 k 为当前 s 的本质不同子串数量。我们添加一个新的字符 c 至 s 。显然，会有一些新的子串以字符 c 结尾。我们希望对这些以该字符结尾且我们之前未曾遇到的子串计数。

构造字符串 $t = s + c$ 并将其反转得到字符串 t^{\sim} 。现在我们的任务变为计算有多少 t^{\sim} 的前缀未在 t^{\sim} 的其余任何地方出现。如果我们计算了 t^{\sim} 的前缀函数最大值 π_{\max} ，那么最长的出现在 s 中的前缀其长度为 π_{\max} 。自然的，所有更短的前缀也出现了。

因此，当添加了一个新字符后新出现的子串数目为 $|s| + 1 - \pi_{\max}$ 。

所以对于每个添加的字符，我们可以在 $O(n)$ 的时间内计算新子串的数目，故最终复杂度为 $O(n^2)$ 。

值得注意的是，我们也可以重新计算在头部添加一个字符，或者从尾或者头移除一个字符时的本质不同子串数目。

4.5 AC 自动机

```

1 namespace AC {
2     int ch[N][26], tot, fail[N], e[N];
3     void insert(const char *s) {
4         int u = 0, n = strlen(s + 1);
5         for (int i = 1; i <= n; i++) {
6             if (!ch[u][s[i] - 'a']) ch[u][s[i] - 'a'] = ++tot;
7             u = ch[u][s[i] - 'a'];
8         }
9         e[u] += 1;
10    }
11    void build() {
12        queue<int> q;
13        for (int i = 0; i <= 25; i++) if (ch[0][i]) q.push(ch[0][i]);
14        while (!q.empty()) {
15            int now = q.front(); q.pop();

```

```

16         for (int i = 0; i < 26; i++) {
17             if (ch[now][i]) fail[ch[now][i]] = ch[fail[now]][i], q.push(ch
                [now][i]);
18             else ch[now][i] = ch[fail[now]][i];
19         }
20     }
21 }
22 int query(const char *s) {
23     int u = 0, n = strlen(s + 1), res = 0;
24     for (int i = 1; i <= n; i++){
25         u = ch[u][s[i] - 'a'];
26         for (int j = u; j && e[j] != -1; j = fail[j]) {
27             res += e[j];
28             e[j] = -1;
29         }
30     }
31     return res;
32 }
33 }

```

4.6 后缀数组

```

1  const int N = 2e5 + 5;
2  int sa[N << 1], ork[N << 1], rk[N << 1], cnt[N], id[N << 1], M, n;
3  char s[N];
4
5  int main() {
6      scanf("%s", s + 1);
7      n = strlen(s + 1);
8      for (int i = n + 1; i <= (n << 1); i++) s[i] = s[i - n], M = max(M, (int)s
        [i]);
9      n <= 1;
10     for (int i = 1; i <= n; i++) if ((int)(s[i]) > M) M = (int)(s[i]);
11     for (int i = 1; i <= n; i++) cnt[rk[i] = s[i]]++;
12     for (int i = 0; i <= M; i++) cnt[i] += cnt[i - 1];
13     for (int i = n; i; i--) sa[cnt[rk[i]]--] = i;
14     for (int w = 1, p; w < n; w <= 1, M = p) {
15         p = 0;
16         for (int i = n; i > n - w; i--) id[++p] = i;
17         for (int i = 1; i <= n; i++) if (sa[i] > w) id[++p] = sa[i] - w;
18         for (int i = 0; i <= M; i++) cnt[i] = 0;
19         for (int i = 1; i <= n; i++) cnt[rk[i]]++;
20         for (int i = 1; i <= M; i++) cnt[i] += cnt[i - 1];
21         for (int i = n; i; i--) sa[cnt[rk[id[i]]]--] = id[i];

```



```

22     p = 0;
23     for (int i = 0; i <= n; i++) ork[i] = rk[i];
24     for (int i = 1; i <= n; i++) {
25         if (ork[sa[i]] == ork[sa[i - 1]] && ork[sa[i] + w] == ork[sa[i -
26             1] + w]) rk[sa[i]] = p;
27         else rk[sa[i]] = ++p;
28     }
29     if (p == n) break;
30 }
31 for (int i = 1, k = 0; i <= n; i++) {
32     if (rk[i] == 1) continue;
33     if (k) k--;
34     while (s[i + k] == s[sa[rk[i] - 1] + k]) k++;
35     h[rk[i]] = k;
36 }
37 return 0;
38 }

```

4.7 Manacher

对于第 i 个字符为对称轴:

1. 如果回文串长为奇数, $d[2 * i] / 2$ 是半径加上自己的长度
2. 如果长为偶数, $d[2 * i - 1] / 2$ 是半径的长度, 方向向右.

```

1  int n, d[N * 2];
2  char s[N];
3
4  for (int i = 1; i <= n; i++) t[i * 2] = s[i], t[i * 2 - 1] = '#';
5  t[n * 2 + 1] = '#';
6  m = n * 2 + 1;
7  for (int i = 1, l = 0, r = 0; i <= m; i++) {
8      int k = i <= r ? min(d[r - i + 1], r - i + 1) : 1;
9      while (i + k <= m && i - k >= 1 && t[i + k] == t[i - k]) k++;
10     d[i] = k--;
11     if (i + k > r) r = i + k, l = i - k;
12 }

```

4.8 Z 函数

$$z[i] = lcp(suf_1, suf_i)$$

```

1  for (int i = 2, l = 0, r = 0; i <= n; i++) {

```

```

2     if (r >= i && r - i + 1 > z[i - l + 1]) {
3         z[i] = z[i - l + 1];
4     } else {
5         z[i] = max(0, r - i + 1);
6         while (z[i] < n - i + 1 && s[z[i] + 1] == s[i + z[i]]) ++z[i];
7     }
8     if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
9 }

```

5 数学

5.1 基本算法

```

1 int fpow(int a, int b) {
2     int res = 1;
3     for (; b; b >>= 1, a = a * 1ll * a % mod) if (b & 1) res = res * 1ll * a %
4         mod;
5     return res;
6 }
7 ll exgcd(ll a, ll b, ll &x, ll &y) {
8     if (b) {
9         ll d = exgcd(b, a % b, y, x);
10        return y -= a / b * x, d;
11    } else return x = 1, y = 0, a;
12 }
13 int getinv(int v) {
14     return fpow(v, mod - 2);
15     // ll x, y;
16     // exgcd(v, mod, x, y);
17     // return (x % mod + mod) % mod;
18 }
19 int fac[N], ifac[N];
20 void init_binom(int n) {
21     fac[0] = ifac[0] = 1;
22     for (int i = 1; i <= n; i++) fac[i] = fac[i - 1] * 1ll * i % mod;
23     ifac[n] = getinv(fac[n]);
24     for (int i = n; i > 1; i--) ifac[i - 1] = ifac[i] * 1ll * i % mod;
25 }
26 int binom(int a, int b) {
27     if (b < 0 || a < 0 || b > a) return 0;
28     return fac[a] * 1ll * ifac[b] % mod * ifac[a - b] % mod;
29 }
30 int getphi(int x) {
31     int res = 1;

```

```

31     for (int i = 2; i * i <= x; i++) {
32         if (x % i == 0) {
33             x /= i;
34             res *= (i - 1);
35             while (x % i == 0) {
36                 x /= i;
37                 res *= i;
38             }
39         }
40     }
41     if (x > 1) res *= (x - 1);
42     return res;
43 }
44 int prime[N], pcnt;
45 bool isp[N];
46 int get_prime(int n) {
47     for (int i = 2; i <= n; i++) {
48         if (!isp[i]) prime[++pcnt] = i;
49         for (int j = 1; j <= pcnt && i * prime[j] <= n; j++) {
50             isp[prime[j] * i] = 1;
51             if (i % prime[j] == 0) break;
52         }
53     }
54 }
55 pll get_up(ll a, ll b, ll x1, ll x2) {
56     //x2>=ax+b>=x1 a >= 0
57     if (a == 0) return (b >= x1 && b <= x2) ? (pll){-1e18, 1e18} : (pll){1,
58         0};
59     ll L, R;
60     ll l = (x1 - b) / a - 3;
61     for (L = l; L * a + b < x1; L++);
62     ll r = (x2 - b) / a + 3;
63     for (R = r; R * a + b > x2; R--);
64     return {L, R};
65 }
66 pll get_dn(ll a, ll b, ll x1, ll x2) {
67     //x2>=ax+b>=x1 a <= 0
68     if (a == 0) return (b >= x1 && b <= x2) ? (pll){-1e18, 1e18} : (pll){1,
69         0};
70     ll L, R;
71     ll l = (x2 - b) / a - 3;
72     for (L = l; L * a + b > x2; L++);
73     ll r = (x1 - b) / a + 3;
74     for (R = r; R * a + b < x1; R--);

```

```

73     return {L, R};
74 }

```

5.2 CRT

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long ll;
4  const int N = 100005;
5  ll n, m, a;
6  ll exgcd(ll a, ll b, ll &x, ll &y) {
7      if (b != 0) {
8          ll g = exgcd(b, a % b, y, x);
9          return y -= a / b * x, g;
10     } return x = 1, y = 0, a;
11 }
12 ll getinv(ll a, ll mod) {
13     ll x, y;
14     exgcd(a, mod, x, y);
15     x = (x % mod + mod) % mod;
16     return x;
17 }
18 int get(ll x) {
19     return x < 0 ? -1 : 1;
20 }
21 ll mul(ll a, ll b, ll mod) {
22     ll res = 0;
23     if (a == 0 || b == 0) return 0;
24     ll f = get(a) * get(b);
25     a = abs(a), b = abs(b);
26     for (; b; b >>= 1, a = (a + a) % mod) if (b & 1) res = (res + a) % mod;
27     res *= f;
28     if (res < 0) res += mod;
29     return res;
30 }
31 // m 互质
32 // int main() {
33 //     cin >> n;
34 //     ll phi = 1;
35 //     for (int i = 1; i <= n; i++) {
36 //         cin >> m[i] >> a[i];
37 //         phi *= m[i];
38 //     }
39 //     ll ans = 0;

```

```

40 //     for (int i = 1; i <= n; i++) {
41 //         ll p = phi / m[i], q = getinv(p, m[i]);
42 //         ans += mul(p, mul(q, a[i], phi), phi);
43 //         ans %= phi;
44 //     }
45 //     cout << ans << '\n';
46 // }
47 int main() {
48     cin >> n;
49     cin >> m >> a;
50     for (int i = 2; i <= n; i++) {
51         ll nm, na;
52         cin >> nm >> na;
53         ll x, y;
54         ll g = exgcd(m, -nm, x, y), d = (na - a) / g, md = abs(nm / g);
55         x = mul(x, d, md);
56         ll lc = abs(m / g);
57         lc *= nm;
58         a = (a + mul(m, x, lc)) % lc;
59         m = lc;
60     }
61     cout << a << '\n';
62 }

```

5.3 Lucas

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long ll;
4  const int N = 2e5;
5  int fac[N], ifac[N], mod;
6  ll exgcd(ll a, ll b, ll &x, ll &y) {
7      if (b != 0) {
8          ll g = exgcd(b, a % b, y, x);
9          return y -= a / b * x, g;
10     } return x = 1, y = 0, a;
11 }
12 int fpow(int a, int b) {
13     int res = 1;
14     for (; b; b >>= 1, a = a * 1ll * a % mod) if (b & 1) res = res * 1ll * a %
        mod;
15     return res;
16 }
17 ll getinv(ll a, ll mod) {

```

```

18     return fpow(a, mod - 2);
19     ll x, y;
20     exgcd(a, mod, x, y);
21     x = (x % mod + mod) % mod;
22     return x;
23 }
24 void init_binom(int n) {
25     fac[0] = ifac[0] = 1;
26     for (int i = 1; i <= n; i++) fac[i] = fac[i - 1] * 1ll * i % mod;
27     ifac[n] = getinv(fac[n], mod);
28     for (int i = n; i > 1; i--) ifac[i - 1] = ifac[i] * 1ll * i % mod;
29 }
30 int binom(int a, int b) {
31     if (b < 0 || a < 0 || b > a) return 0;
32     return fac[a] * 1ll * ifac[b] % mod * ifac[a - b] % mod;
33 }
34 int lucas(int a, int b) {
35     if (a < mod) return binom(a, b);
36     return lucas(a / mod, b / mod) * 1ll * binom(a % mod, b % mod) % mod;
37 }
38 int main() {
39     int T;
40     cin >> T;
41     while (T--) {
42         int n, m;
43         cin >> n >> m >> mod;
44         init_binom(mod - 1);
45         cout << lucas(n + m, m) << '\n';
46     }
47     return 0;
48 }

```

5.4 exLucas

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long ll;
4  const int N = 1e6;
5  ll a1, b1, mod;
6  ll m[N], a[N], pr[N], tot;
7  ll pre[N];
8  ll fpow(ll a, ll b, ll p) {
9     ll res = 1;
10    for (; b; b >>= 1, a = a * a % p) if (b & 1) res = res * a % p;

```

```
11     return res;
12 }
13 ll exgcd(ll a, ll b, ll &x, ll &y) {
14     if (b != 0) {
15         ll g = exgcd(b, a % b, y, x);
16         return y -= a / b * x, g;
17     } return x = 1, y = 0, a;
18 }
19 ll getinv(ll a, ll mod) {
20     ll x, y;
21     exgcd(a, mod, x, y);
22     x = (x % mod + mod) % mod;
23     return x;
24 }
25 ll F(ll n, int k) {
26     if (n == 0) return 1;
27     ll res = fpow(pre[m[k]], n / m[k], m[k]), rem = n % m[k];
28     res = res * pre[rem] % m[k];
29     return F(n / pr[k], k) * res % m[k];
30 }
31 int G(ll n, ll p) {
32     if (n < p) return 0;
33     return G(n / p, p) + n / p;
34 }
35 int get(ll x) {
36     return x < 0 ? -1 : 1;
37 }
38 ll mul(ll a, ll b, ll mod) {
39     ll res = 0;
40     if (a == 0 || b == 0) return 0;
41     ll f = get(a) * get(b);
42     a = abs(a), b = abs(b);
43     for (; b; b >>= 1, a = (a + a) % mod) if (b & 1) res = (res + a) % mod;
44     res *= f;
45     if (res < 0) res += mod;
46     return res;
47 }
48 int main() {
49     cin >> a1 >> b1 >> mod;
50     ll x = mod;
51     for (ll i = 2; i * i <= x; i++) {
52         if (x % i) continue;
53         pr[++tot] = i;
54         m[tot] = 1;
```

```

55     while (x % i == 0) x /= i, m[tot] *= i;
56 }
57 if (x != 1) pr[++tot] = x, m[tot] = x;
58 for (int k = 1; k <= tot; k++) {
59     pre[0] = 1;
60     for (int i = 1; i <= m[k]; i++) pre[i] = pre[i - 1] * (i % pr[k] == 0
        ? 1 : i) % m[k];
61     ll res = F(a1, k) * getinv(F(b1, k), m[k]) % m[k] * getinv(F(a1 - b1,
        k), m[k]) % m[k];
62     ll d = G(a1, pr[k]) - G(b1, pr[k]) - G(a1 - b1, pr[k]), r = (d < 0 ?
        getinv(fpow(pr[k], -d, m[k]), m[k]) : fpow(pr[k], d, m[k]));
63     res = res * r % mod;
64     a[k] = res;
65 }
66 ll ans = 0;
67 for (int i = 1; i <= tot; i++) {
68     ll p = mod / m[i], q = getinv(p, m[i]);
69     ans += mul(p, mul(q, a[i], mod), mod);
70     ans %= mod;
71 }
72 cout << ans << '\n';
73 return 0;
74 }

```

5.5 线性基

```

1 struct LinerBasis {
2     int a[20], pos[20];
3     void add(int v, int p) {
4         for (int i = 19; i >= 0; i--) if ((v >> i) & 1) {
5             if (a[i]) {
6                 if (p > pos[i]) {
7                     swap(p, pos[i]);
8                     swap(a[i], v);
9                 }
10                v ^= a[i];
11            } else {
12                a[i] = v;
13                pos[i] = p;
14                return;
15            }
16        }
17    }
18 } b[N];

```



```

19
20 LinerBasis operator + (LinerBasis a, LinerBasis b) {
21     for (int i = 19; i >= 0; i--) {
22         if (b.a[i]) a.add(b.a[i], b.pos[i]);
23     }
24     return a;
25 }

```

5.6 高斯消元

5.6.1 解线性方程组

```

1 void gauss() {
2     for (int i = 0; i < n; i++) {
3         int id = i;
4         for (int j = i + 1; j < n; j++) if (fabs(a[j][i]) > fabs(a[id][i])) id
            = j;
5         for (int j = i; j <= n; j++) swap(a[id][j], a[i][j]);
6         if (a[i][i] == 0) {
7             puts("No Solution");
8             return;
9         }
10        for (int j = 0; j < n; j++) {
11            if (j == i) continue;
12            double t = a[j][i] / a[i][i];
13            for (int k = i; k <= n; k++) a[j][k] -= a[i][k] * t;
14        }
15    }
16    for (int i = 0; i < n; i++) printf("%.2lf\n", a[i][n] / a[i][i]);
17 }

```

5.6.2 求行阶梯矩阵

```

1 bool gauss() {
2     int k = 1;
3     for (int i = 1; i <= m; i++) {
4         if (k > n) break;
5         if (a[k][i] == 0) {
6             for (int j = k + 1; j <= n; j++) if (a[j][i] != 0) {
7                 for (int l = 1; l <= m + 1; l++) swap(a[j][l], a[k][l]);
8                 break;
9             }
10        }
11        if (a[k][i] == 0) continue;

```

```

12     for (int j = k + 1; j <= n; j++) if (a[j][i] == 1) {
13         for (int l = i; l <= m + 1; l++) a[j][l] ^= a[k][l];
14     }
15     k++;
16 }
17 int flag = 1;
18 for (int i = k; i <= n; i++) if (a[i][m + 1] == 1) flag = 0;
19 return flag;
20 }

```

5.6.3 解不定方程

```

1  #define fi first
2  #define se second
3  typedef long long ll;
4  typedef pair<ll, ll> pll;
5  typedef long double ld;
6  //std::mt19937_64 rng(std::chrono::steady_clock::now().time_since_epoch().
    count());
7  #define y1 miku
8  const int mod = 998244353;
9  const int N = 1e5 + 5;
10 ll exgcd(ll a, ll b, ll &x, ll &y) {
11     if (b) {
12         ll d = exgcd(b, a % b, y, x);
13         return y -= a / b * x, d;
14     } return x = 1, y = 0, a;
15 }
16 pll get_up(ll a, ll b, ll x1, ll x2) {
17     //x2>=ax+b>=x1
18     if (a == 0) return (b >= x1 && b <= x2) ? (pll){0, min(n, m)} : (pll){1,
        0};
19     ll L, R;
20     ll l = (x1 - b) / a - 3;
21     for (L = l; L * a + b < x1; L++);
22     ll r = (x2 - b) / a + 3;
23     for (R = r; R * a + b > x2; R--);
24     return {L, R};
25 }
26 pll get_dn(ll a, ll b, ll x1, ll x2) {
27     //x2>=ax+b>=x1
28     if (a == 0) return (b >= x1 && b <= x2) ? (pll){0, min(n, m)} : (pll){1,
        0};
29     ll L, R;

```

```

30     ll l = (x2 - b) / a - 3;
31     for (L = l; L * a + b > x2; L++);
32     ll r = (x1 - b) / a + 3;
33     for (R = r; R * a + b < x1; R--);
34     return {L, R};
35 }
36 //ax+b+c=0 [x1,x2] [y1,y2]
37 ll solve(ll a, ll b, ll c, ll x1, ll x2, ll y1, ll y2) {
38     if (a == 0 && b == 0) return (c == 0) * (y2 - y1 + 1) * (x2 - x1 + 1);
39     ll x, y, d = exgcd(a, b, x, y);
40     if (c % d != 0) return 0;
41     x *= c / d, y *= c / d;
42     ll sx = b / d, sy = -a / d;
43     auto A = (sx > 0 ? get_up(sx, x, x1, x2) : get_dn(sx, x, x1, x2));
44     auto B = (sy > 0 ? get_up(sy, y, y1, y2) : get_dn(sy, y, y1, y2));
45     A.fi = max(A.fi, B.fi), A.se = min(A.se, B.se);
46     return max(0ll, A.se - A.fi + 1);
47 }

```

5.7 矩阵树定理

一、无向无环图

A 为邻接矩阵, $A[i][j] = i \rightarrow j$ 的边数

D 为度数矩阵, $D[i][i] = \sum_{j=1}^n A[i][j] = i$ 的度数, 其他位置为 0

基尔霍夫矩阵 $K = D - A$, 令 $K' = K$ 的去掉第 k 行第 k 列 (k 任意) 的 $n - 1$ 阶主子式

$\det(K') =$ 该无向图生成树个数

特别地, 完全图生成树个数是 n^{n-2}

二、加权

求所有生成树边权的乘积之和, 需要把邻接矩阵中边的条数改为为边权和

度数矩阵改为 $D[i][i] = \sum_{j=1}^n A[i][j]$

三、有向图

对于有根外向树, 需要把度数矩阵改为入度和, $D[i][i] = \sum_{j=1}^n A[j][i]$

对于有根内向树，需要把度数矩阵改为出度和， $D[i][i] = \sum_{j=1}^n A[i][j]$

类似地，求所有有向生成树边权的乘积之和，需要把邻接矩阵改为入/出边边权和

四、变形：边权和的和

求所有生成树边权和的和，给原先边权为 w 的边赋值为一次多项式 $wx + 1$ ，多项式乘法对 x^2

取模， $\prod(w_i x + 1)$ 的一次项系数即为 w_i 之和

```

1 struct P {
2     ll x,y; //x是一次项系数, y是常数项
3     P (ll x=0, ll y=0):x(x),y(y){}
4     friend P operator + (const P &u, const P &v) {
5         return P(add(u.x, v.x), add(u.y, v.y));
6     }
7     friend P operator - (const P &u, const P &v) {
8         return P(add(u.x, mod - v.x), add(u.y, mod - v.y));
9     }
10    friend P operator * (const P &u, const P &v) {
11        return P(add(mul(u.x, v.y), mul(u.y, v.x)), mul(u.y, v.y));
12    }
13    friend P operator / (const P &u, const P &v) {
14        ll inv=qpow(v.y, mod-2);
15        return P(add(mul(u.x, v.y), mod - mul(u.y, v.x)) * inv % mod * inv %
16            mod, mul(u.y, inv));
17    }
18 };
19 P g[N][N];
20 ll gauss(P g[N][N], int n){
21     P res(0,1);
22     for(int i=1; i<=n; ++i) {
23         int pos=-1;
24         for(int j=i; j<=n; ++j){
25             if(g[j][i].y){
26                 pos=j; break;
27             }
28         }
29         if(pos==-1) return 0;
30         swap(g[i], g[pos]);
31         if(pos!=i) res=res*P(0, mod-1);
32         res=res*g[i][i];
33         P inv=P(0,1)/g[i][i];
34         for(int j=i+1; j<=n; ++j){
35             P t=g[j][i]*inv;
36             for(int k=n; k>=i; --k) g[j][k]=g[j][k]-t*g[i][k];

```

```

36     }
37 }
38 return res.x;
39 }

```

5.8 多项式

5.8.1 FFT

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  typedef complex<double> cp;
5  const int N = 4e6 + 5;
6  const double pi = acos(-1.0);
7  int n, m, limit = 1, l, rev[N];
8  cp a[N], b[N];
9
10 void FFT(cp *a, int n, int inv) {
11     for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
12     for (int k = 1; k < n; k <= 1) {
13         cp wn(cos(pi / k), inv * sin(pi / k));
14         for (int i = 0; i < n; i = i + k + k) {
15             cp w(1, 0);
16             for (int j = 0; j < k; j++, w *= wn) {
17                 cp x = a[i + j], y = w * a[i + j + k];
18                 a[i + j] = x + y;
19                 a[i + j + k] = x - y;
20             }
21         }
22     }
23     if (inv < 0) for (int i = 0; i < n; i++) a[i] /= n;
24 }
25
26 int main() {
27     scanf("%d %d", &n, &m);
28     for (int i = 0; i <= n; i++) scanf("%lf", &a[i]);
29     for (int j = 0; j <= m; j++) scanf("%lf", &b[j]);
30     while (n + m >= limit) limit <= 1, l = l + 1;
31     for (int i = 1; i <= limit; i++) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) <<
        (l - 1));
32     FFT(a, limit, 1), FFT(b, limit, 1);
33     for (int i = 0; i < limit; i++) a[i] *= b[i];
34     FFT(a, limit, -1);

```

```

35     for (int i = 0; i <= n + m; i++) printf("%d ", (int)(a[i].real() + 0.5));
36     return 0;
37 }

```

5.8.2 NTT

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int N = 4e6 + 5, mod = 998244353;
5  int n, m, r[N], lim, a[N], b[N];
6
7  int fpow(int a, int b) {
8      int res = 1;
9      for (; b; b >>= 1, a = a * 1ll * a % mod) if (b & 1) res = res * 1ll * a %
      mod;
10     return res;
11 }
12 void ntt(int *x, int lim, int op) {
13     int i, j, k, m, gn, g, tmp;
14     for (int i = 0; i < lim; i++) if (r[i] < i) swap(x[i], x[r[i]]);
15     for (m = 2; m <= lim; m <= 1) {
16         k = m >> 1;
17         gn = fpow(3, (mod - 1) / m);
18         for (i = 0; i < lim; i += m) {
19             g = 1;
20             for (j = 0; j < k; j++, g = g * 1ll * gn % mod) {
21                 tmp = x[i + j + k] * 1ll * g % mod;
22                 x[i + j + k] = (x[i + j] - tmp + mod) % mod;
23                 x[i + j] = (x[i + j] + tmp) % mod;
24             }
25         }
26     }
27     if (op == -1) {
28         reverse(x + 1, x + lim);
29         int inv = fpow(lim, mod - 2);
30         for (int i = 0; i < lim; i++) x[i] = x[i] * 1ll * inv % mod;
31     }
32 }
33
34 int main() {
35     scanf("%d %d", &n, &m);
36     for (int i = 0; i <= n; i++) scanf("%d", &a[i]);
37     for (int i = 0; i <= m; i++) scanf("%d", &b[i]);

```

```

38     lim = 1;
39     while (lim < (n + m) << 1) lim <=< 1;
40     for (int i = 0; i < lim; i++) r[i] = (i & 1) * (lim >> 1) + (r[i >> 1] >>
        1);
41     ntt(a, lim, 1), ntt(b, lim, 1);
42     for (int i = 0; i < lim; i++) a[i] = a[i] * 1ll * b[i] % mod;
43     ntt(a, lim, -1);
44     for (int i = 0; i <= n + m; i++) printf("%d ", a[i]);
45     return puts(""), 0;
46 }

```

5.9 组合数学

5.9.1 小球放盒

第二类斯特林数（斯特林子集数） $\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$ ，也可记做 $S(n, k)$ ，表示将 n 个两两不同的元素，划分为 k 个互不区分的非空子集的方案数。

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \left\{ \begin{matrix} n-1 \\ k-1 \end{matrix} \right\} + k \left\{ \begin{matrix} n-1 \\ k \end{matrix} \right\}$$

边界是 $\left\{ \begin{matrix} n \\ 0 \end{matrix} \right\} = [n = 0]$ 。

假设小球个数为 n ，盒子个数为 m

1. 小球无标号，盒子有标号，不允许空盒。

即求解方程 $\sum_{i=1}^m x_i = n$ 解的个数

即 $\binom{n-1}{m-1}$

2. 小球无标号，盒子有标号，允许空盒。

令 $y_i = x_i + 1$

即求解方程 $\sum_{i=1}^m y_i = n$ 解的个数

即 $\binom{n+m-1}{m-1}$

3. 小球有标号，盒子有标号，允许空盒。

即 m^n

4. 小球有标号，盒子有标号，不允许空盒。

$$m! \times \begin{Bmatrix} n \\ m \end{Bmatrix}$$

5. 小球有标号，盒子无标号，不允许空盒。

$$\begin{Bmatrix} n \\ m \end{Bmatrix}$$

6. 小球有标号，盒子无标号，允许空盒。

$$\sum_{i=1}^m \begin{Bmatrix} n \\ i \end{Bmatrix}$$

7. 小球无标号，盒子无标号，允许空盒。

设 $f[i][j]$ 表示 i 个球放入 j 个盒子的方案数。

1. $i = 0$ 或者 $j = 1$, 方案数为 1

2. $i < j$, $f[i][j] = f[i][i]$

3. $i \geq j$, $f[i][j] = f[i-j][j] + f[i][j-1]$

8. 小球无标号，盒子无标号，不允许空盒。

用 7 的结论，提前在每个盒子放 1 个球。

方案数就是 $f[n-m][m]$

6 MISC

6.1 完全平方数判断

```

1  typedef unsigned long long ull;
2  int sqrt1(ull x) {
3      ull y = sqrt(x);
4      return y * y == x;
5  }
6  constexpr ull calc_table(int k) {
7      ull table = 0;
8      for (int i = 0; i < 64; i++)
9          table |= 1ull << (i * i % (1 << k));
10     return table;
11 }
12 int sqrt4(ull x) {
13     constexpr int k = 6;

```



```

14     constexpr auto table = calc_table(k);
15     ull y = x % (1 << k);
16     if ((table >> y) & 1) return sqrt1(x);
17     return 0;
18 }

```

6.2 维护区间 GCD 值

```

1  int main() {
2      for (int i = 1; i <= n; i++) {
3          v.push_back({i, a[i]});
4          for (int j = (int)(v.size()) - 2; j >= 0; j--) {
5              v[j].second = gcd(v[j].second, a[i]);
6              if (v[j].second == v[j + 1].second) v.erase(v.begin() + j + 1);
7          }
8          mp[v[(int)(v.size()) - 1].second] += i - v[(int)(v.size()) - 1].first
          + 1;
9          for (int j = (int)(v.size()) - 2; j >= 0; j--) {
10             mp[v[j].second] += v[j + 1].first - v[j].first;
11         }
12     }
13 }

```

6.3 FastIO

```

1  //https://github.com/huanghaox1212/FastIO/
2  struct control{
3      int ct,val;
4      control(int Ct,int Val=-1):ct(Ct),val(Val){}
5      inline control operator()(int Val){
6          return control(ct,Val);
7      }
8  }_endl(0),_prs(1),_setprecision(2);
9  struct FastIO{
10     #define IOSIZE 1000000
11     char in[IOSIZE],*p,*pp,out[IOSIZE],*q,*qq,ch[20],*t,b,K,prs;
12     FastIO():p(in),pp(in),q(out),qq(out+IOSIZE),t(ch),b(1),K(6){}
13     ~FastIO(){fwrite(out,1,q-out,stdout);}
14     inline char getch(){
15         return p==pp&&(pp=(p=in)+fread(in,1,IOSIZE,stdin),p==pp)?b=0,EOF:*p++;
16     }
17     inline void putch(char x){
18         q==qq&&(fwrite(out,1,q-out,stdout),q=out),*q++=x;

```

```

19     }
20     inline void puts(const char str[]){fwrite(out,1,q-out,stdout),fwrite(str
    ,1,strlen(str),stdout),q=out;}
21     inline void getline(string& s){
22         s="";
23         for( char ch;(ch=getch())!='\n'&&b;)s+=ch;
24     }
25     #define indef(T) inline FastIO& operator>>(T& x){\
26         x=0; char f=0,ch;\
27         while(!isdigit(ch=getch())&&b)f|=ch=='-';\
28         while(isdigit(ch))x=(x<<1)+(x<<3)+(ch^48),ch=getch();\
29         return x=f?-x:x,*this;\
30     }
31     indef(int)
32     indef(long long)
33     inline FastIO& operator>>(char& ch){return ch=getch(),*this;}
34     inline FastIO& operator>>(string& s){
35         s=""; char ch;
36         while(isspace(ch=getch())&&b);
37         while(!isspace(ch)&&b)s+=ch,ch=getch();
38         return *this;
39     }
40     inline FastIO& operator>>(double& x){
41         x=0; char f=0,ch;
42         double d=0.1;
43         while(!isdigit(ch=getch())&&b)f|=(ch=='-');
44         while(isdigit(ch))x=x*10+(ch^48),ch=getch();
45         if(ch=='.')while(isdigit(ch=getch()))x+=d*(ch^48),d*=0.1;
46         return x=f?-x:x,*this;
47     }
48     #define outdef(_T) inline FastIO& operator<<(_T x){\
49         !x&&(putch('0'),0),x<0&&(putch('-'),x=-x);\
50         while(x)*t++=x%10+48,x/=10;\
51         while(t!=ch)*q++=*--t;\
52         return *this;\
53     }
54     outdef(int)
55     outdef(long long)
56     inline FastIO& operator<<(char ch){return putch(ch),*this;}
57     inline FastIO& operator<<(const char str[]){return puts(str),*this;}
58     inline FastIO& operator<<(const string& s){return puts(s.c_str()),*this;}
59     inline FastIO& operator<<(double x){
60         int k=0;
61         this->operator<<(int(x));

```

```
62     putchar(' ');
63     x-=int(x);
64     prs&&(x+=5*pow(10,-K-1));
65     while(k<K)putch(int(x*=10)^48),x-=int(x),++k;
66     return *this;
67 }
68 inline FastIO& operator<<(const control& cl){
69     switch(cl.ct){
70         case 0:putch('\n');break;
71         case 1:prs=cl.val;break;
72         case 2:K=cl.val;break;
73     }
74 }
75 inline operator bool(){return b;}
76 }io;
```