Internal waves and other stratified flows in 2D

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Menu of the practical

- Interfacial wave
  - H vs NH
  - Stokes drift
  - Wave breaking
- Internal wave
  - Localized periodic forcing
  - Lee waves
- Kelvin-Helmholtz instability
- Horizontal convection

Transverse questions:

- mixing (tracer) and dissipation (energy)
- consequences of numerical errors

How are we gonna do that ?!

Boussinesq equations

advection terms

$$\partial_t u + J(\psi, u) = -\partial_x p$$
$$\epsilon \left[\partial_t w + J(\psi, w)\right] = -\partial_z p + b$$
$$\partial_t b + J(\psi, b) = 0$$
$$\partial_x u + \partial_z w = 0\,.$$

non-hydrostatic parameter

buoyancy

horizontal vorticity $\omega = \epsilon \partial_x w - \partial_z u$

stream function

$$\omega = \epsilon \partial^2_{xx}\psi + \partial^2_{zz}\psi$$

In vorticity formulation

torque of buoyancy force
= source of vorticity

$$\partial_t \omega + J(\psi, \omega) = b_x$$
$$\partial_t b + J(\psi, b) = 0$$
$$\omega = \epsilon \partial^2_{xx} \psi + \partial^2_{zz} \psi$$

In the hydrostatic limit

$$\omega = \partial^2_{zz} \psi$$

Decoupling in the horizontal

We see two bricks of CFD methods:

- advection
- Poisson equation

Fluid2d: a Python CFD code that solves a variety of 2D equations (including QG)

Internal waves
Linearizing around a mean stratification

perturbation

$$b = \bar{b}(z) + a\, b'$$

mean stratification

amplitude
(dimensionless)

Brünt-Vaisala frequency

source term on the buoyancy anomaly!
[it's easier to make sense of materially conserved quantities]

$$\partial_t b' + a\, J(\psi, b') = -J(\psi, \bar{b}) = w \partial_z \bar{b} \qquad N = \sqrt{\partial_z \bar{b}}$$

$$\partial_t \omega + a\, J(\psi, \omega)] = \partial_x b'$$

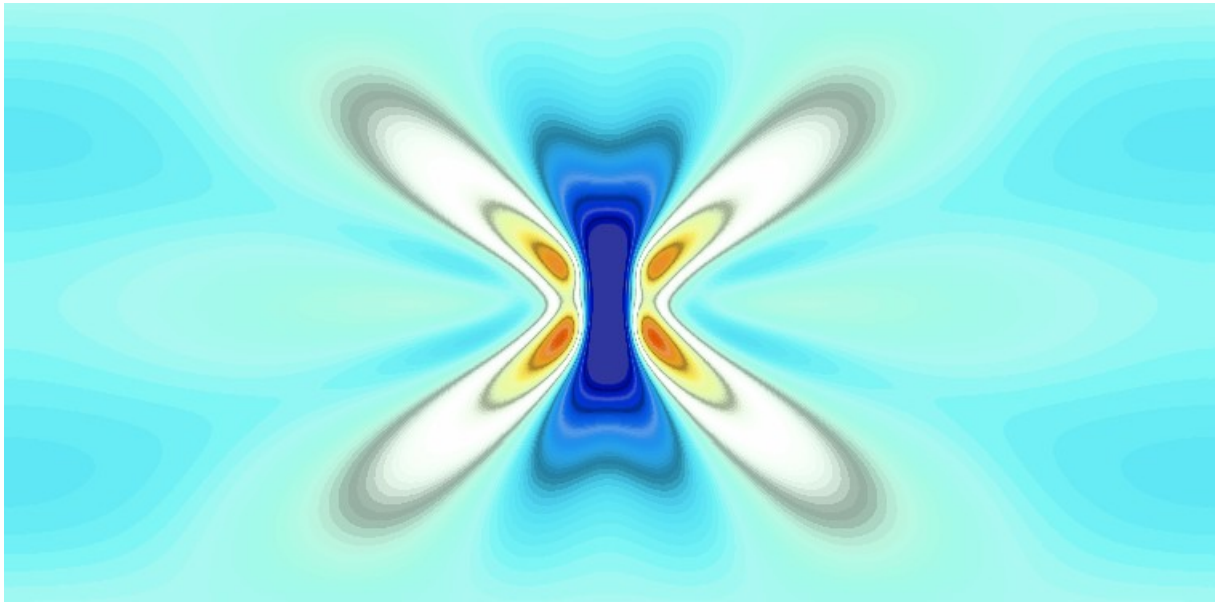
$$\omega = \partial^2_{xx}\psi + \partial^2_{zz}\psi\,.$$

Dispersion relation for internal waves

$$\Omega = N \frac{k_x^2}{k_x^2 + k_z^2} = N \cos\theta$$

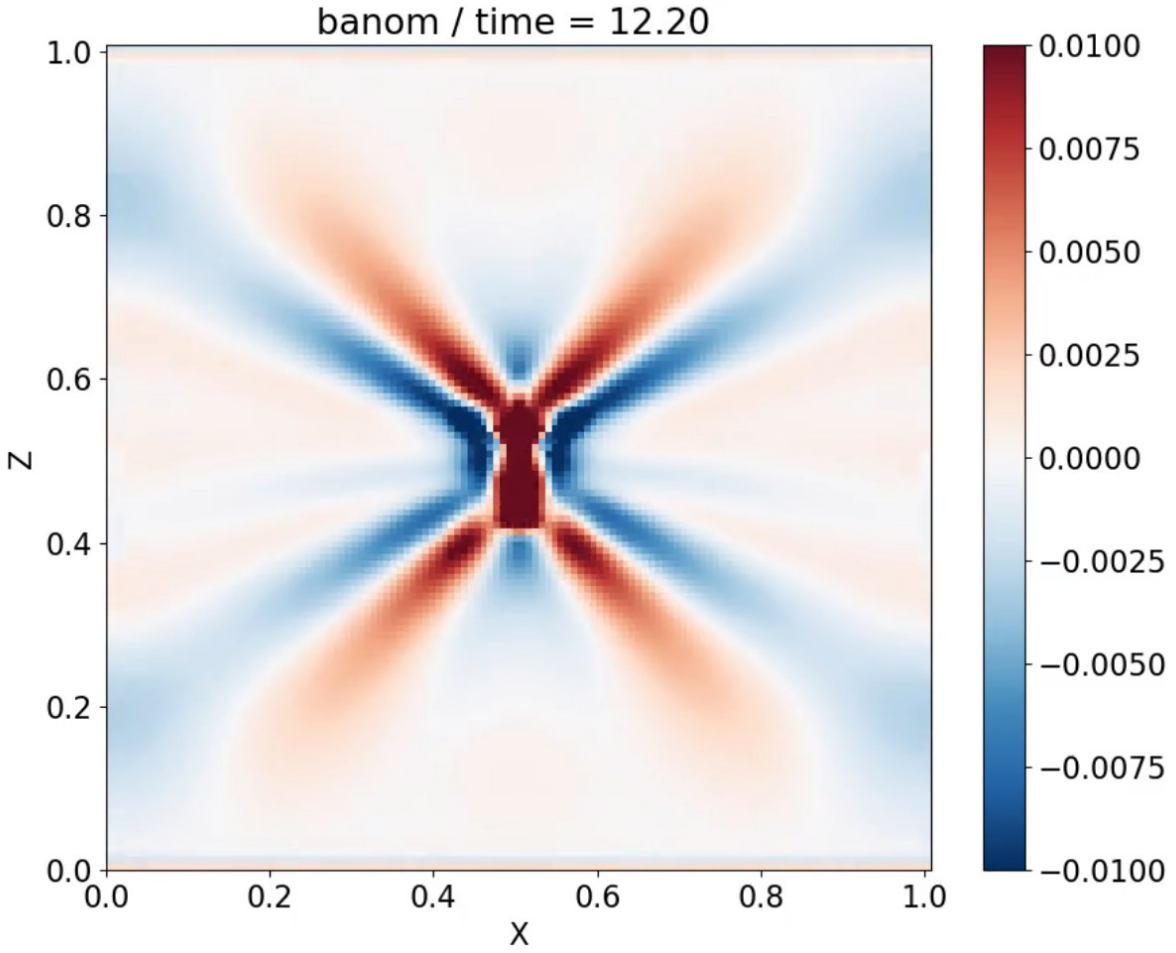
internalwave.py

- This is an initial value problem: gaussian perturbation of b’
- Integrate, look at u, v, b, banom, psi. Look at volume integrated quantities (conserved)
- Play with the wave amplitude
- Play with the hydroepsilon (0.2 is the minimum)
- Play with the size of the initial perturbation



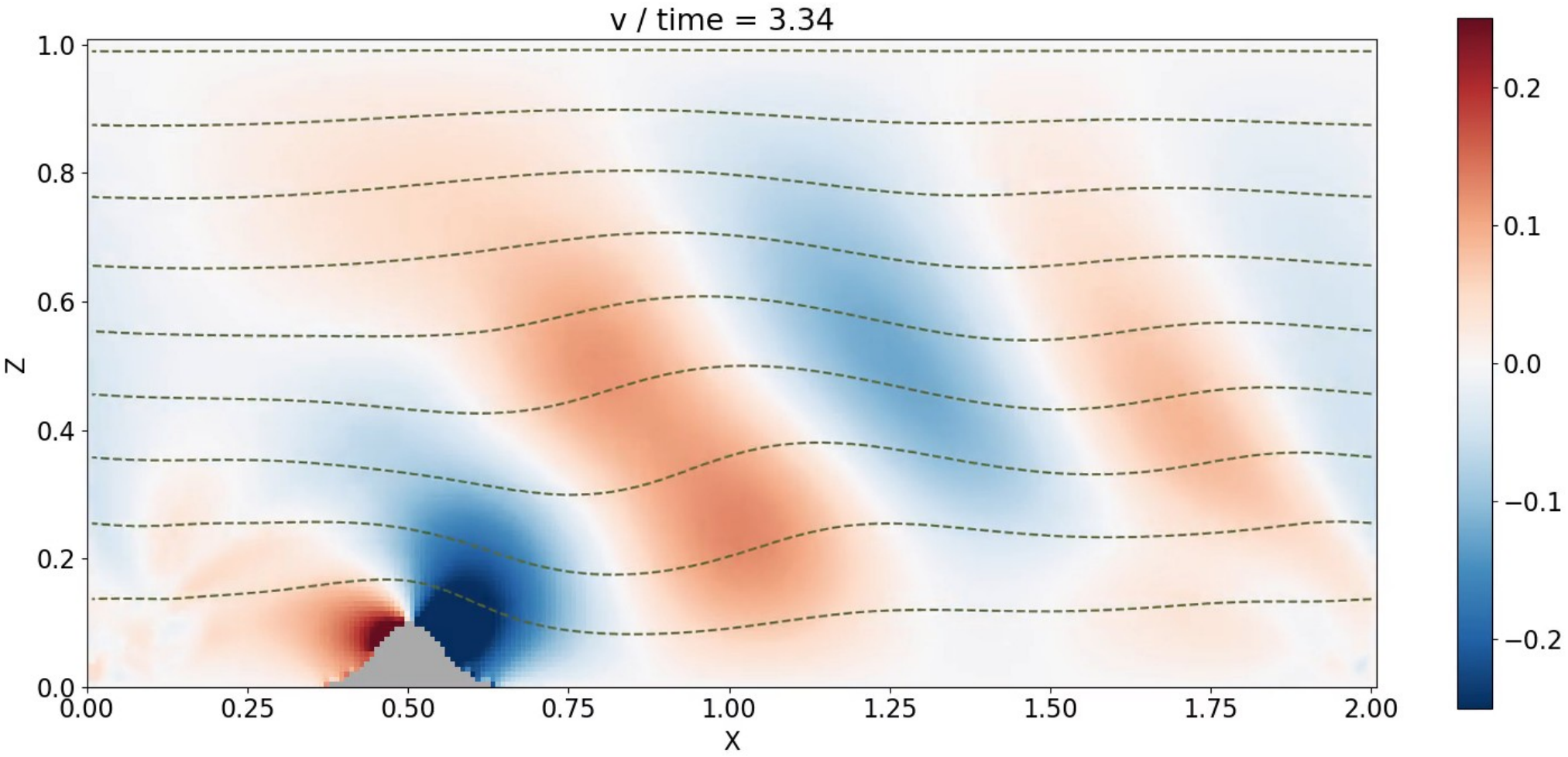
forced_wave.py + forcedinternal.py

- This is a localized periodic forcing
- Play with the forcing frequency (in forcedinternal.py)
- Generate evanescent waves



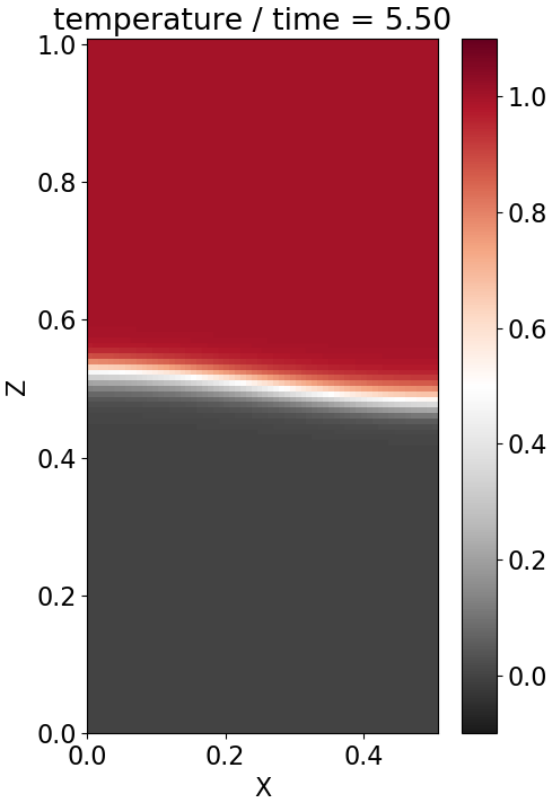
Lee wave

- A flow past a seamount
- See the wave setting up
- Increase N and observe mixing downstream



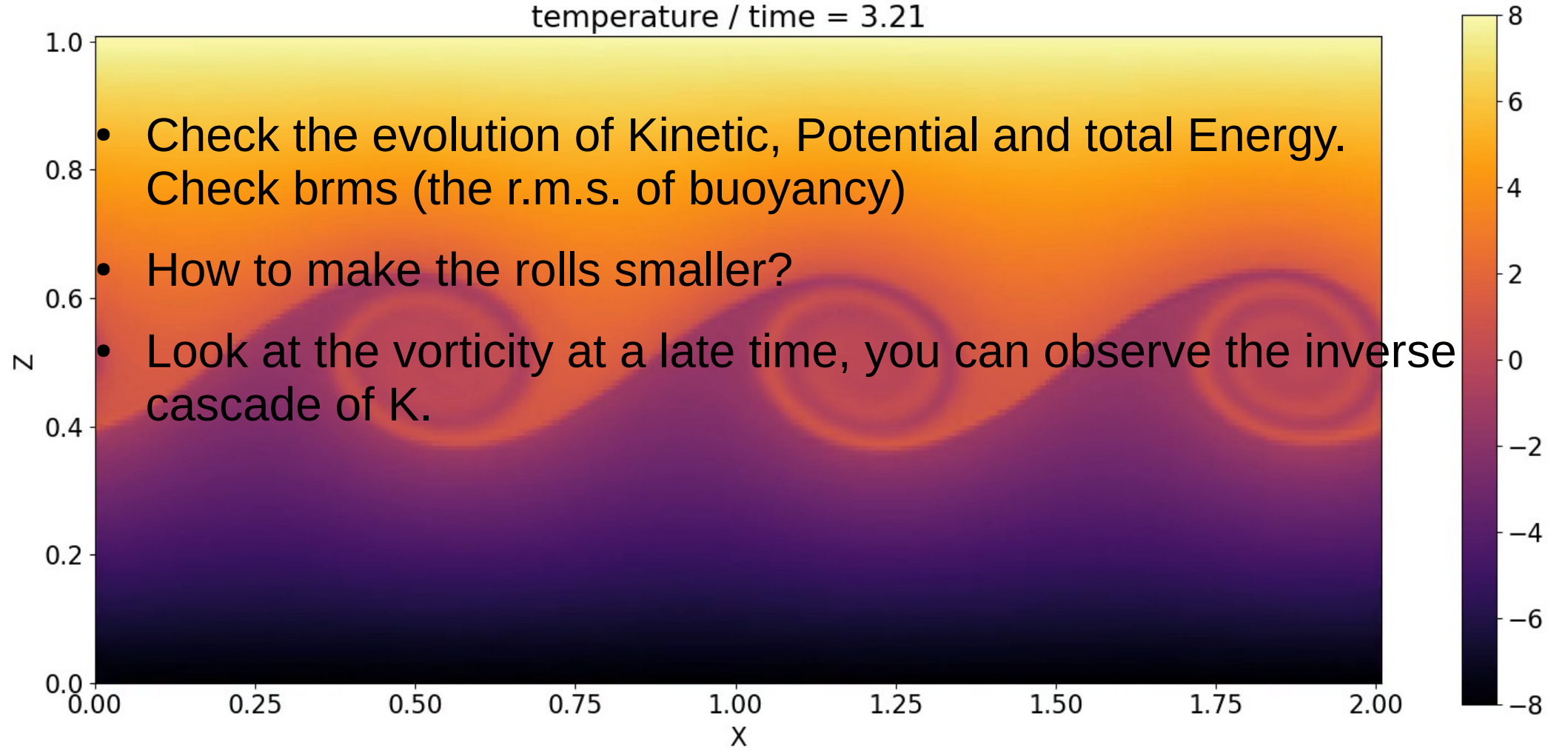
Interfacial wave

- Either
  - 1) a sine wave in a tank (stationary wave) or
  - 2) a localized perturbation on the left (propagating wave)
- In 2) look at how the Stokes drift deforms the tracer field
- In 1) play with hydroepsilon (not smaller than 0.2), look at the structure of the velocity field
- In 1) increase the amplitude until you trigger KHI and wave breaking



Kelvin Helmholtz instability

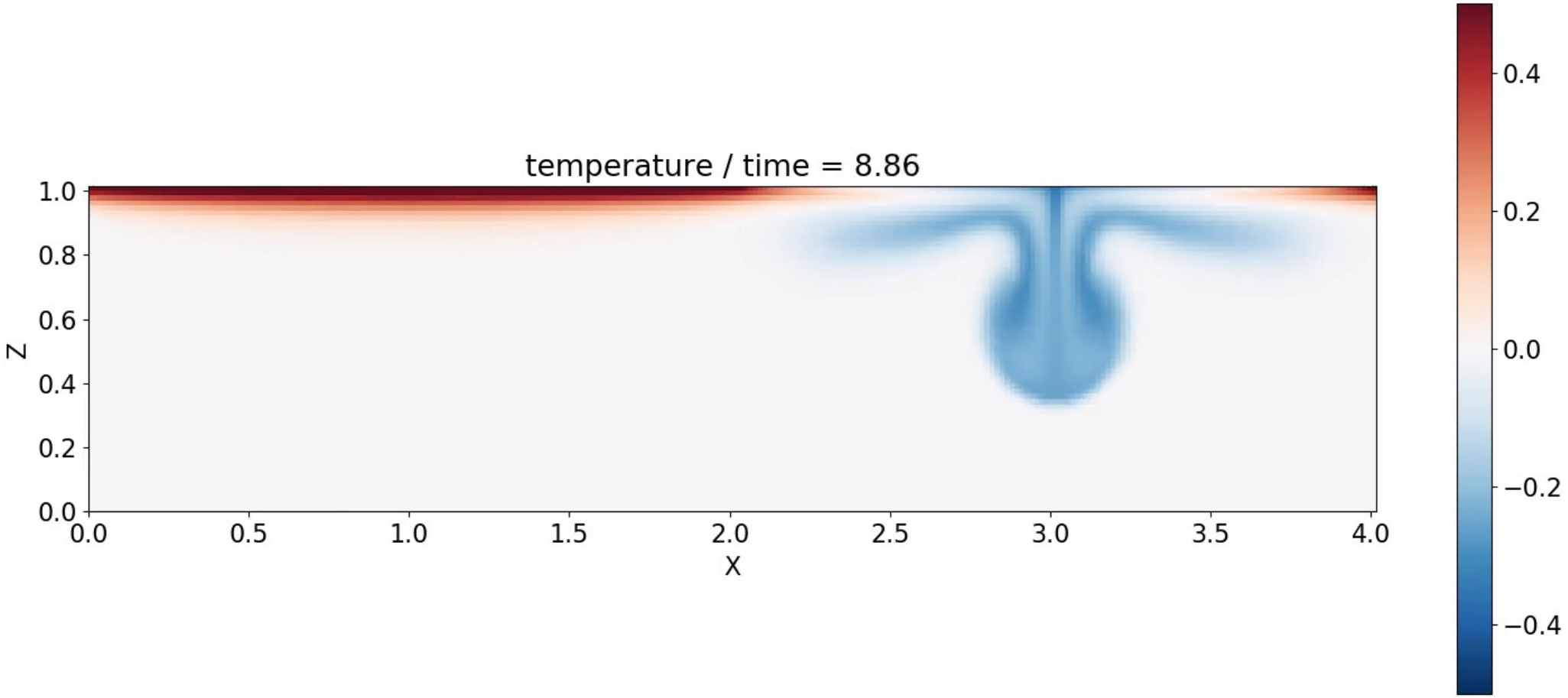
- Look at the initial velocity profile
- Check the condition for instability $S^2/N^2 < 0.25$ (S=shear)
- Look at the exponential growth (on v or banom)



- Check the evolution of Kinetic, Potential and total Energy. Check brms (the r.m.s. of buoyancy)
- How to make the rolls smaller?
- Look at the vorticity at a late time, you can observe the inverse cascade of K.

Horizontal convection

- To continue stimulating discussions with Bill Dewar!



Howto?

- param.colorscheme = ‘max’ (auto-adjust), ‘imposed’ (by param.cax)
- param.plot_var = name of the variable to be animated (u,v,vorticity,psi,buoyancy,banom) and tracer (if any)
- param.expname = change it to not overwrite your previous results
- Change the size of the box tweak param.nx, param.ny, param.Lx and param.Ly. Watch out you should have dx=dy!!!
- Change the advection scheme: it’s a volume flux discretization in space, param.order select the order (1st, 3rd and 5th are upwinded). Several time schemes are available ‘LFAM3’ (ROMS/CROCO) or ‘RK3_SSP’ also ‘Heun’, ‘AB2’, ‘AB3’ ..
- Results are stored in a ‘myexp_his.nc’ history file and a ‘myexp_diag.nc’ diagnostic NetCDF files. Use ncview to browse them.

- Ask me