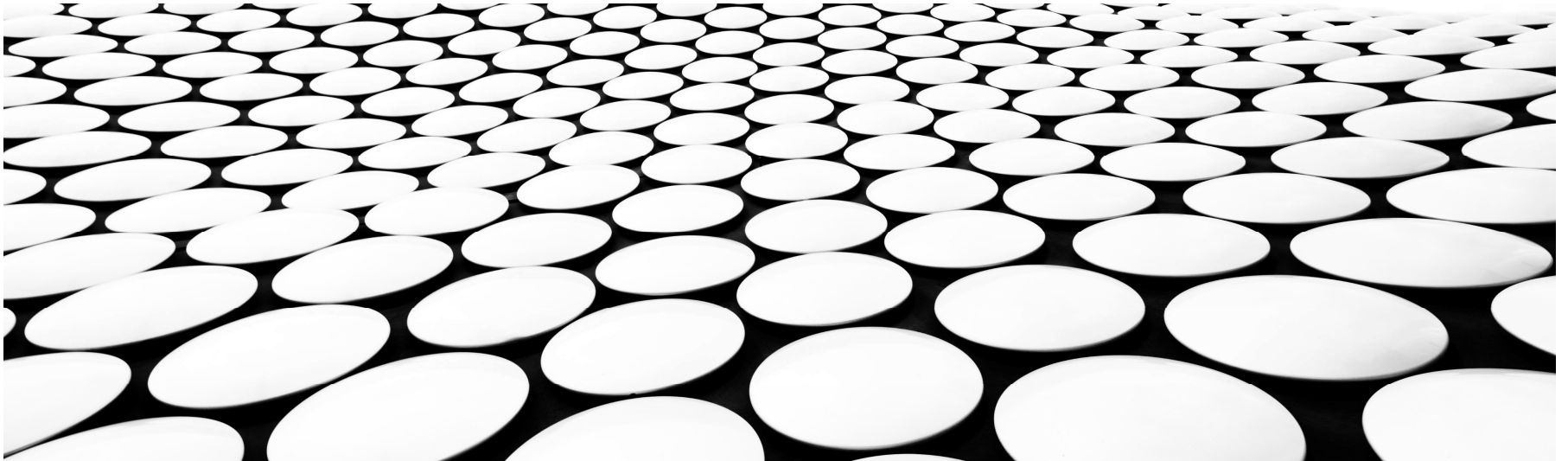


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# FINITE AUTOMATA

IF 2124 TEORI BAHASA FORMAL OTOMATA

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## Motivation

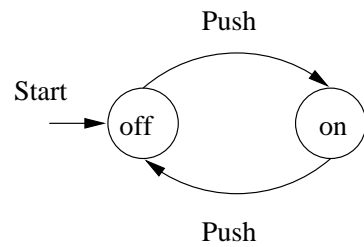
- Automata = abstract computing devices
- Turing studied Turing Machines (= computers) before there were any real computers
- We will also look at simpler devices than Turing machines (Finite State Automata, Push-down Automata, . . . ), and specification means, such as grammars and regular expressions.
- NP-hardness = what cannot be efficiently computed

## **Finite Automata**

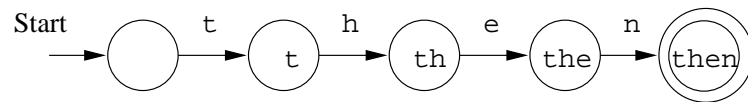
Finite Automata are used as a model for

- Software for designing digital circuits
- Lexical analyzer of a compiler
- Searching for keywords in a file or on the web.
- Software for verifying finite state systems, such as communication protocols.

- Example: Finite Automaton modelling an on/off switch



- Example: Finite Automaton recognizing the string then



## Structural Representations

These are alternative ways of specifying a machine

**Grammars:** A rule like  $E \Rightarrow E + E$  specifies an arithmetic expression

- $Lineup \Rightarrow Person.Lineup$

says that a lineup is a person in front of a lineup.

**Regular Expressions:** Denote structure of data, e.g.

'[A-Z][a-z]\*[ ][A-Z][A-Z]'

matches Ithaca NY

does not match Palo Alto CA

**Question:** What expression would match  
Palo Alto CA

## Central Concepts

**Alphabet:** Finite, nonempty set of symbols

Example:  $\Sigma = \{0, 1\}$  binary alphabet

Example:  $\Sigma = \{a, b, c, \dots, z\}$  the set of all lower case letters

Example: The set of all ASCII characters

**Strings:** Finite sequence of symbols from an alphabet  $\Sigma$ , e.g. 0011001

**Empty String:** The string with zero occurrences of symbols from  $\Sigma$

- The empty string is denoted  $\epsilon$

**Length of String:** Number of positions for symbols in the string.

$|w|$  denotes the length of string  $w$

$$|0110| = 4, |\epsilon| = 0$$

**Powers of an Alphabet:**  $\Sigma^k$  = the set of strings of length  $k$  with symbols from  $\Sigma$

Example:  $\Sigma = \{0, 1\}$

$$\Sigma^1 = \{0, 1\}$$

$$\Sigma^2 = \{00, 01, 10, 11\}$$

$$\Sigma^0 = \{\epsilon\}$$

**Question:** How many strings are there in  $\Sigma^3$

The set of all strings over  $\Sigma$  is denoted  $\Sigma^*$

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$$

Also:

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$$

$$\Sigma^* = \Sigma^+ \cup \{\epsilon\}$$

**Concatenation:** If  $x$  and  $y$  are strings, then  $xy$  is the string obtained by placing a copy of  $y$  immediately after a copy of  $x$

$$x = a_1a_2 \dots a_i, y = b_1b_2 \dots b_j$$

$$xy = a_1a_2 \dots a_ib_1b_2 \dots b_j$$

Example:  $x = 01101, y = 110, xy = 01101110$

**Note:** For any string  $x$

$$x\epsilon = \epsilon x = x$$



## Languages:

If  $\Sigma$  is an alphabet, and  $L \subseteq \Sigma^*$   
then  $L$  is a language

Examples of languages:

- The set of legal English words
- The set of legal C programs
- The set of strings consisting of  $n$  0's followed by  $n$  1's

$$\{\epsilon, 01, 0011, 000111, \dots\}$$

- The set of strings with equal number of 0's and 1's

$$\{\epsilon, 01, 10, 0011, 0101, 1001, \dots\}$$

- $L_P$  = the set of binary numbers whose value is prime

$$\{10, 11, 101, 111, 1011, \dots\}$$

- The empty language  $\emptyset$
- The language  $\{\epsilon\}$  consisting of the empty string

**Note:**  $\emptyset \neq \{\epsilon\}$

**Note2:** The underlying alphabet  $\Sigma$  is always finite

**Problem:** Is a given string  $w$  a member of a language  $L$ ?

Example: Is a binary number prime = is it a member in  $L_P$

Is  $11101 \in L_P$ ? What computational resources are needed to answer the question.

Usually we think of problems not as a yes/no decision, but as something that transforms an input into an output.

Example: Parse a C-program = check if the program is correct, and if it is, produce a parse tree.

Let  $L_X$  be the set of all valid programs in programming language  $X$ . If we can show that determining membership in  $L_X$  is hard, then parsing programs written in  $X$  cannot be easier.

**Question:** Why?

## Finite Automata Informally

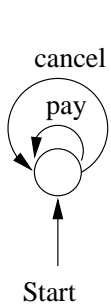
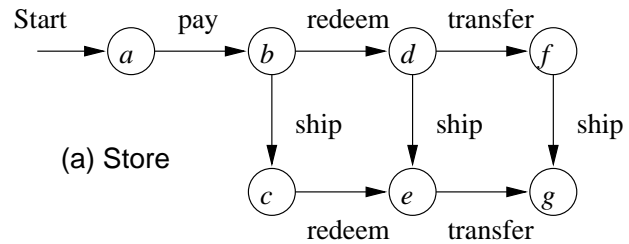
Protocol for e-commerce using e-money

### Allowed events:

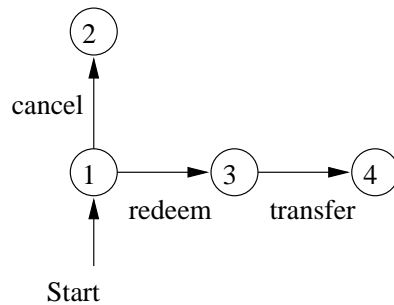
1. The customer can *pay* the store (=send the money-file to the store)
2. The customer can *cancel* the money (like putting a stop on a check)
3. The store can *ship* the goods to the customer
4. The store can *redeem* the money (=cash the check)
5. The bank can *transfer* the money to the store

## e-commerce

The protocol for each participant:

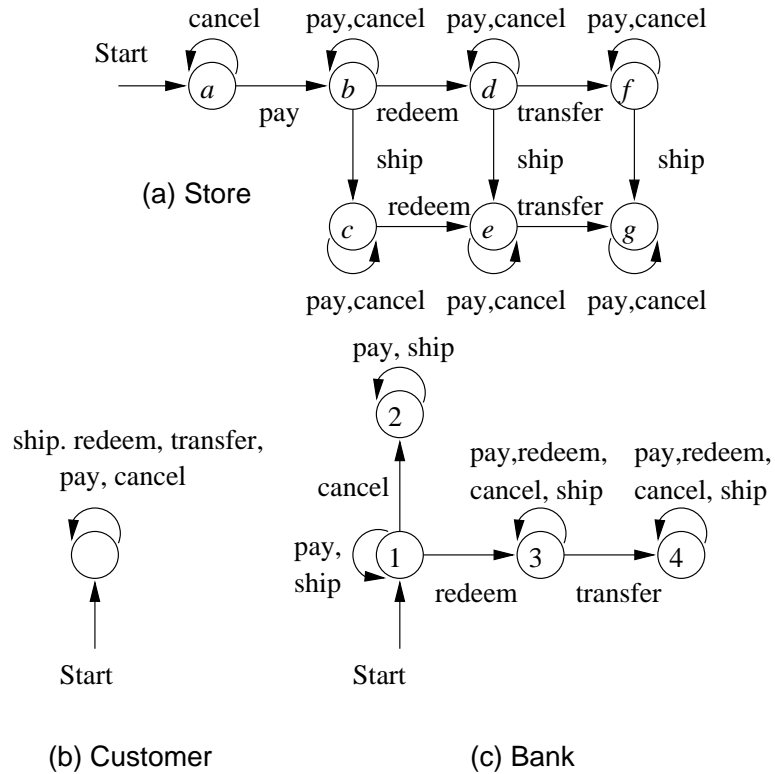


(b) Customer



(c) Bank

Completed protocols:



The entire system as an Automaton:

