

Relational Logic: Syntax & Semantic

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Informatics Engineering Study Program
School of Electrical Engineering and Informatics ITB

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- ▶ Review
- ▶ Relational Logic \rightarrow Introduction

Review

- ▶ Reasoning: information \rightarrow conclusion
- ▶ Computational Logic
 - ▶ **Propositional Logic:**
 - ▶ Syntax \rightarrow Simple sentence, Compound Sentence
 - ▶ **[something] is [anything]: [something] \rightarrow [anything]**
 - ▶ Semantics \rightarrow interpretation, evaluation, reverse evaluation, types of compound sentence
 - ▶ Logical Entailment :
 - Semantic Reasoning \rightarrow Two tables, Validity Checking, Unsatisfiability Checking
 - Proof Method \rightarrow Rules of Inference, Axiom Schemata, Propositional Resolution
 - ▶ **Relational Logic \rightarrow Today**

Propositional Logic vs Relational Logic

- ▶ Constants refer to atomic propositions/ logical constants.

raining *snowing wet*

- ▶ Compound sentences capture relationships among propositions

raining \vee *snowing* \Rightarrow *wet*

- ▶ How to represent general concepts??

In Propositional Logic:

1. If Ali knows Budi, then Budi knows Ali ($a \rightarrow b$)
2. Ali knows Budi (a)
3. Conclusion by Modus Ponens : Budi knows Ali (b)

What if we want to say something more general, such as If person I knows person II, then person II knows person I

In Relational Logic: $\forall x,y (\text{knows}(x,y) \rightarrow \text{knows}(y,x))$

Relational Logic Syntax

▶ Two new vocabularies:

- ▶ Variables : begin with letters from the **end of the alphabet**
- ▶ Constants:
 - ▶ *begin with either alphabetic letters (other than **u, v, w, x, y, z**),*
 - ▶ *mathematical characters (+, -, etc.),* biasanya dipake jadi variabel
 - ▶ *or digits*
- ▶ Example:
 - ▶ $u, v, w, x, y, z \rightarrow ???$
 - ▶ $a, b, c, arthur, betty, cathy, 1, 2, \dots \rightarrow ???$

Relational Logic Syntax (2): Constants

- ▶ Object constants refer to objects in the universe of discourse.
- ▶ Function constants denote functions.
 - ▶ father, mother, age, plus, times
- ▶ Relation constants refer to relations.
 - ▶ person, happy, parent, loves
- ▶ There is no syntactic distinction between object constants, function constants, and relation constants. The type of each such word is determined from context

Relational Logic Syntax (3): Arity

- ▶ The arity of a function constant or a relation constant is the number of arguments it takes.
 - ▶ Unary Function constants: *father*₁, *mother*₁ menerima 1 masukan
 - ▶ Binary Function constants: *plus*₂, *times*₂ menerima 2 masukan
 - ▶ Ternary Function constants: *price*₃ menerima 3 masukan
 - ▶ Unary Relation constants: *person*₁, *happy*₁
 - ▶ Binary Relation constants: *parent*₂, *loves*₂
 - ▶ Ternary Relation constants: *between*₃
- ▶ The arity of a function constant or a relation constant is optionally notated as a subscript on the constant

Relational Logic Syntax (4): Term

- ▶ A term is either a variable, an object constant, or a functional term.
- ▶ Terms refer to items in the universe of discourse.
- ▶ Terms are analogous to noun phrases in natural language.

Functional Terms

- ▶ A functional term is an expression formed from an n-ary function constant and n terms enclosed in parentheses and separated by commas.
 - ▶ *father I (joe)*
 - ▶ *age I (joe)*
 - ▶ *plus2(x,2)*
- ▶ Functional terms are terms and, as such, can be nested.
 - ▶ *plus2(age I (father I (joe)),age I (mother I (joe)))*

Relational Logic Syntax (5): Sentences

- ▶ There are three types of sentences:
 - ▶ Relational sentences - analogous to the simple sentences in Propositional Logic
 - ▶ Logical sentences - analogous to the compound sentences in Propositional Logic
 - ▶ Quantified sentences - sentences that express the significance of variables

Relational Sentences

- ▶ A relational sentence is an expression formed from an n-ary relation constant and n terms enclosed in parentheses and separated by commas.

- ▶ $\text{happy}_1(\text{art})$ happy disini bukan fungsi, tp relasi

- ▶ $\text{loves}_2(\text{art}, \text{cathy})$ keluaran relasi selalu true/false
apakah art loves cathy?

- ▶ Relational sentences are not terms and cannot be nested in terms or relational sentences.

$\text{happy}_1(\text{person}_1(\text{joe}))$ - - > WRONG

- ▶ Should be written:

$\text{happy}_1(\text{joe})$ is joe happy?

$\text{person}_1(\text{joe})$ is joe a person?

Logical Sentences

- ▶ Logical sentences in Relational Logic are analogous to those in Propositional Logic (Compound Sentences).
 - ▶ $\neg \text{loves}(\text{art}, \text{cathy})$ art doesn't love cathy
 - ▶ $(\text{loves}(\text{art}, \text{betty}) \wedge \text{loves}(\text{betty}, \text{art}))$ art loves betty and betty loves art
 - ▶ $(\text{loves}(\text{art}, \text{betty}) \vee \text{loves}(\text{art}, \text{cathy}))$
 - ▶ $(\text{loves}(x, y) \Rightarrow \text{loves}(y, x))$ x loves y then y loves x
 - ▶ $(\text{loves}(x, y) \Leftarrow \text{loves}(y, x))$
 - ▶ $(\text{loves}(x, y) \Leftrightarrow \text{loves}(y, x))$
- ▶ Parenthesization rules are the same as for Propositional Logic

Quantified Sentences

- ▶ Quantified sentences can be nested within other sentences.

for all $\forall x.(apple(x) \vee$ for some $\exists x.pear(x))$

$\forall x. \forall y.loves(x,y)$

- ▶ Universally quantified sentences:

- ▶ is used to assert that all objects have a certain property

- ▶ $\forall x.p(x)$ harus semua

- ▶ Existentially quantified sentences:

- ▶ is used to assert that some object has a certain property

- ▶ $\exists x.p(x)$ minimal 1

Other Sentences

- ▶ A sentence is **ground** if and only if it contains no **variables**
 - ▶ Example:
 - ▶ Ground Sentence: *human(joe)*
 - ▶ Not Ground Sentence: $\forall x.\text{human}(x)$
- ▶ A sentence is **open** if and only if it has free variables (iff it is not in the scope of a quantifier of that variable)
 - ▶ Example:
 - ▶ $p(y) \rightarrow \exists x.q(x, y)$ - - > Open sentence since *y* is a free variable
x -> bound y -> free
 - ▶ $\forall y.(p(y) \rightarrow \exists x.q(x, y))$ - - > Close sentence since *x* & *y* are bound
x dan y bound

Reminder

- ▶ Variables
- ▶ Constants: Object, [Functional, Relational] → has arguments (n-ary)
- ▶ Term: variables, object constants, functional terms → noun phrase in natural language
- ▶ Sentence: relational, logical, quantified
- ▶ Functional Term vs Relational Sentence:
 - ▶ Functional terms *may be used within other functional terms.*
 - ▶ Functional terms *may be used within relational sentences.*
 - ▶ Relational sentences *may not be used in functional terms.*
 - ▶ Relational sentences *may not be used in relational sentences.*

gabole ada relasi di dlm fungsi
gabole ada relasi di dlm relasi

Natural Language Representation

- ▶ $\forall x.P(x)$
For all x , x is P
- ▶ $\sim \forall x.P(x)$
Not all x are P .
Some x are P .
- ▶ $\forall x.\sim P(x)$
All x are not P .

Natural Language Representation(2)

- ▶ $\exists x.P(x)$
Some x is P
- ▶ $\exists x.\sim P(x)$
Some x is not P
- ▶ $\forall x.\exists y.P(x,y)$
For all x , there is y such that P

Natural Language Representation(3)

- ▶ $\forall x.(P(x) \rightarrow Q(x))$
For all x, all P are Q
All P is Q
- ▶ $\forall x.(P(x) \rightarrow \sim Q(x))$
For all x, no P are Q
No P is Q
- ▶ $\exists x.(P(x) \wedge Q(x))$
For some x, x are P and Q
Some P is Q
- ▶ $\exists x.(P(x) \wedge \sim Q(x))$
For some x, x are P and not Q
Some P is not Q

Examples

▶ **All human is mortal.**

- ▶ If a thing is human, then it is mortal.

$$\forall x.(\text{human}(x) \rightarrow \text{mortal}(x))$$

▶ **Purple mushrooms are poisonous.**

- ▶ If a thing is a purple mushroom, then it is poisonous.
- ▶ If a thing is mushroom and it is purple, then it is poisonous.

$$\forall x.(\text{mushroom}(x) \wedge \text{purple}(x) \rightarrow \text{poisonous}(x))$$

Exercise 1

► Given:

- Object Constants: art, betty, cathy
- Functional Constants: father₁, mother₁, age₁, plus₂, times₂
- Relational Constants: person₁, sad₁, reflexive₁, parent₂, loves₂, friends₂,

► Syntax Test:

1. friends(father(art),betty) true
2. friends(mother(art),friends(father(art),betty)) gabole relasi dalam relasi
3. sad(person(cathy)) gabole relasi di dalam relasi
4. loves(x,y) → loves(y,x) true
5. reflexive(z) → z(x,x) z hanya sebagai variabel, jadi salah

Exercise 2

- ▶ Apakah ekspresi logika relasional di bawah ini legal atau tidak ? Jika tidak, jelaskan dimana letak kesalahannya dan mengapa, dengan ketentuan variabel dan konstanta sebagai berikut:
- ▶ *variables: x, y, z*
- ▶ *object constants: patrick, joe, kevinKW, PR, cemilan*
- ▶ *function constants: mother, anak, plus*
- ▶ *relational constants dengan aritas satu: hantu, biru, ramah, senang, sepupu*
- ▶ *relational constants dengan aritas dua: takutpada, suka, mengerjakan, teman*

Exercise 2 (2)

- (a) $\sim \text{suka}(\text{joe}, \text{PR}) \vee \text{mengerjakan}(\text{joe}, \text{PR})$ legal
- (b) $\text{plus}(\text{joe}, \text{z}) \rightarrow \text{suka}(\text{mother}(\text{kevinKW}), \text{joe})$ ilegal
- (c) $\text{hantu}(\text{x}) \wedge \text{ramah}(\text{x}) \rightarrow \text{patrick}(\text{x})$ ilegal
- (d) $\sim \text{senang}(\text{z}) \wedge \text{takutpada}(\text{kevinKW}, \text{anak}(\text{sepupu}(\text{patrick})))$ ilegal
- (e) $\text{biru}(\text{mother}(\text{anak}(\text{patrick}))) \vee \text{ramah}(\text{anak}(\text{mother}(\text{kevinKW})))$
 $\Leftrightarrow \text{senang}(\text{joe})$ legal
- (f) $\text{suka}(\text{joe}, \text{cemilan}) \wedge \text{suka}(\text{kevinKW}, \text{cemilan}) \rightarrow \text{teman}(\text{kevinKW})$ ilegal
harusnya isinya 2
- (g) $\text{takutpada}(\text{joe}, \text{sepupu}(\text{patrick}))$ ilegal, gabole relasi di dalam relasi
- (h) $\text{teman}(\text{patrick}, \text{joe}) \wedge \sim \text{takutpada}(\text{kevinKW}, \text{hantu}(\text{patrick}))$ ilegal karena ada relasi dalam relasi
- (i) $\text{suka}(\text{kevinKW}, \text{mother}(\text{x})) \wedge \text{teman}(\text{kevinKW}, \text{x}) \rightarrow \text{hantu}(\text{x}) \wedge \text{biru}(\text{y})$ ilegal
- (j) $\text{plus}(\text{mother}(\text{anak}(\text{z})), \text{anak}(\text{anak}(\text{anak}(\text{cemilan})))) \wedge \text{plus}(\text{joe}, \text{kevinKW})$

Relational Logic Semantics

- ▶ Review Propositional Logic Semantic:

A Propositional logic *interpretation* is an association between the propositional constants in a propositional language and the truth values T or F

- ▶ The big question: what is a relational logic interpretation?
- ▶ There are no propositional constants, just object constants, function constants, and relation constants.
- ▶ To what do they refer?

Introduction to Relational Logic Semantic

- ▶ **Modeling the World**
 - ▶ Objects, Functions, Relations
 - ▶ Data
 - ▶ Models
- ▶ **Semantics of Relational Logic**
 - ▶ Atomic Sentences
 - ▶ Logical Sentences
 - ▶ Quantified Sentences

Exercise 3: Translate into FOL

Relation Constants:

- ▶ person(x)
- ▶ femur(x)
- ▶ leg(x)
- ▶ eye(x)
- ▶ part_of(x, y): x part of y
- ▶ has(x, y)
- ▶ heart(x)
- ▶ sinus_rhythm(x)
- ▶ seeing(x)
- ▶ living(x)
- ▶ rhythm(x)
- ▶ regular(x)
- ▶ differ(x, y): x and y are different

- a) **All femurs are part of some legs**
- b) **All living hearts have a rhythm**
- c) **Not all eyes are seeing**
- d) **All people have two eyes**

Exercise 4: Translate into FOL

Relation Constants:

- ▶ $\text{person}(x)$
- ▶ $\text{child}(x)$
- ▶ $\text{parent}(x,y)$: x is the parent of y
- ▶ $\text{male}(x)$
- ▶ $\text{female}(x)$
- ▶ $\text{ancestor}(x,y)$: x is the ancestor of y
- ▶ $\text{sibling}(x,y)$
- ▶ $\text{differ}(x,y)$: x and y are different

- a) **All people have two parents**
- b) **No person is both male and female**
- c) **All people have one male parent and one female parent**
- d) **One child is a sibling of another if they both have the same two parents**



THANK YOU

