

Exercise 3.1.1

- (a) The set strings over $\Sigma = \{a, b, c\}$ containing at least one a & at least one b

$$(a+b+c)^* \cdot (a \cdot (a+b+c)^* \cdot b + b \cdot (a+b+c)^* \cdot a) \cdot (a+b+c)^*$$

- (b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

$$(0+1)^* \cdot 1 \cdot (0+1)^9$$

- (c) The set of strings of 0's and 1's with at most one pair of consecutive 1's

$$(0+10)^* \cdot (11 + \epsilon) \cdot (0+01)^*$$

Exercise 3.1.2

- (a) The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's

Membagi string menjadi 2 bagian. Bagian pertama tdk boleh ada "1" benjang agar "0" bersebelahan bisa di sbim "1" bersebelahan. Di bagian kedua tdk boleh ada "0" benjang agar kemunculan "0" bersebelahan muncul sbim "1" bersebelahan. Mk jwbnya:

$$(0+10)^* \cdot (1 + \epsilon) \cdot (0+01)^* \cdot (0 + \epsilon)$$

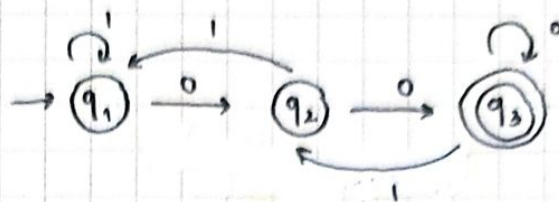
- (b) The set of strings of 0's and 1's whose number of 0's is divisible by five

$$(1^* \cdot 0 \cdot 1^* \cdot 0 \cdot 1^* \cdot 0 \cdot 1^* \cdot 0 \cdot 1^* \cdot 0 \cdot 1^*) + 1^*$$

Exercise 3.2.1

Table DFA

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2



a) Give all RE $R_{ij}^{[0]}$.

$$R_{11}^{[0]} = \epsilon + 1$$

$$R_{21}^{[0]} = 1$$

$$R_{31}^{[0]} = \emptyset$$

$$R_{12}^{[0]} = 0$$

$$R_{22}^{[0]} = \epsilon$$

$$R_{32}^{[0]} = 1$$

$$R_{13}^{[0]} = \emptyset$$

$$R_{23}^{[0]} = 0$$

$$R_{33}^{[0]} = \epsilon + 0$$

b) Give all RE $R_{ij}^{[1]}$

$$R_{ij}^{[1]} = R_{ij}^{[0]} + R_{i1}^{[0]}(R_{11}^{[0]})^* R_{1j}^{[0]}$$

$$\begin{aligned} R_{11}^{[1]} &= (\epsilon + 1) + (\epsilon + 1) \cdot (\epsilon + 1)^* \cdot (\epsilon + 1) \\ &= (\epsilon + 1)^* \\ &= 1^* \end{aligned}$$

$$\begin{aligned} R_{31}^{[1]} &= \emptyset + \emptyset \cdot (\epsilon + 1)^* \cdot (\epsilon + 1) \\ &= \emptyset \end{aligned}$$

$$\begin{aligned} R_{12}^{[1]} &= 0 + (\epsilon + 1) \cdot (\epsilon + 1)^* \cdot 0 \\ &= 0 + 1^* \cdot 0 \\ &= 1^* \cdot 0 \end{aligned}$$

$$\begin{aligned} R_{22}^{[1]} &= 1 + \emptyset \cdot (\epsilon + 1)^* \cdot 0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} R_{33}^{[1]} &= (\epsilon + 0) + \emptyset \cdot (\epsilon + 1)^* \cdot \emptyset \\ &= \epsilon + 0 \end{aligned}$$

$$\begin{aligned} R_{13}^{[1]} &= \emptyset + (\epsilon + 1) \cdot (\epsilon + 1)^* \cdot \emptyset \\ &= \emptyset \end{aligned}$$

$$\begin{aligned} R_{21}^{[1]} &= 1 + 1 \cdot (\epsilon + 1)^* \cdot (\epsilon + 1) \\ &= 1 + 1 \cdot 1^* \\ &= 1 \cdot 1^* = 1^+ \end{aligned}$$

$$\begin{aligned} R_{22}^{[1]} &= \epsilon + 1 \cdot (\epsilon + 1)^* \cdot 0 \\ &= \epsilon + 1 \cdot 1^* \cdot 0 \\ &= \epsilon + 1^+ \cdot 0 \end{aligned}$$

$$\begin{aligned} R_{23}^{[1]} &= 0 + 1 \cdot (\epsilon + 1)^* \cdot \emptyset \\ &= 0 \end{aligned}$$

c) GIVE all RE $R_{ij}^{[2]}$

$$R_{ij}^{[2]} = R_{ij}^{[1]} + R_{i2}^{[1]} \cdot (R_{22}^{[1]})^* \cdot R_{2j}^{[1]}$$

$$\begin{aligned} R_{11}^{[2]} &= 1^* + (1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0)^* \cdot 1^* \\ &= (1 + 01)^* \end{aligned}$$

$$\begin{aligned} R_{12}^{[2]} &= (1^* \cdot 0) + (1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0)^* \cdot (\epsilon + 1^* \cdot 0) \\ &= (1 + 01)^* \cdot 0 \end{aligned}$$

$$\begin{aligned} R_{13}^{[2]} &= \emptyset + (1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0)^* \cdot 0 \\ &= (1 + 01)^* \cdot 0 \cdot 0 \end{aligned}$$

$$\begin{aligned} R_{21}^{[2]} &= 1^* + (\epsilon + 1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0)^* \cdot 1^* \\ &= 1^+ \cdot (\epsilon + 01^+) \end{aligned}$$

$$\begin{aligned} R_{22}^{[2]} &= (\epsilon + 1^* \cdot 0) + (\epsilon + 1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0) \\ &= (\epsilon + 1^* \cdot 0)^* \\ &= (1^+ \cdot 0)^* \end{aligned}$$

$$\begin{aligned} R_{23}^{[2]} &= 0 + (\epsilon + 1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0) \cdot 0 \\ &= (\epsilon + 1^* \cdot 0)^* \cdot 0 \\ &= (1^+ \cdot 0)^* \cdot 0 \end{aligned}$$

$$\begin{aligned} R_{31}^{[2]} &= \emptyset + 1 \cdot (\epsilon + 1^* \cdot 0) \cdot 1^* \\ &= \emptyset + 1 \cdot (\epsilon + 1^* \cdot 0)^* \cdot 1^* \\ &= 1 \cdot (1^+ \cdot 0)^* \cdot 1^+ \end{aligned}$$

$$\begin{aligned} R_{32}^{[2]} &= 1 + 1 \cdot (\epsilon + 1^* \cdot 0) \cdot (\epsilon + 1^* \cdot 0) \\ &= 1 + 1 \cdot (\epsilon + 1^* \cdot 0)^* \\ &= 1 \cdot (1^+ \cdot 0)^* \end{aligned}$$

$$\begin{aligned} R_{33}^{[2]} &= (\epsilon + 0) + 1 \cdot (\epsilon + 1^* \cdot 0) \cdot 0 \\ &= (0 + \epsilon) + 1 \cdot (\epsilon + 1^* \cdot 0)^* \cdot 0 \\ &= 0 + 1 \cdot (1^+ \cdot 0)^* \cdot 0 + \epsilon \end{aligned}$$

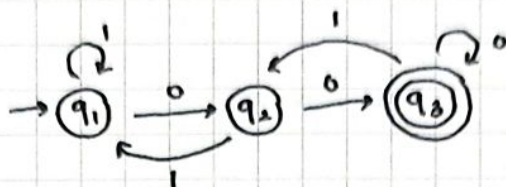
d) Give a RE for language of the automaton

$$\text{Language} = R_{13}^{[3]}$$

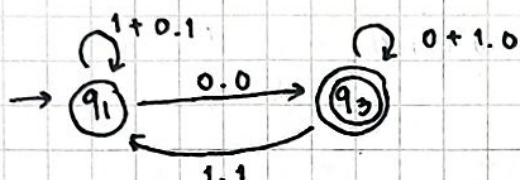
$$\begin{aligned} R_{13}^{[3]} &= R_{13}^{[2]} + R_{13}^{[2]} \cdot (R_{33}^{[2]})^* \cdot R_{33}^{[2]} \\ &= (1 + 0 \cdot 1)^* \cdot 0 \cdot 0 + ((1 + 0 \cdot 1)^* \cdot 0 \cdot 0) \cdot (0 + 1 \cdot (1^* \cdot 0)^* \cdot 0 + \epsilon)^* \cdot (0 + 1 \cdot (1^* \cdot 0)^* \cdot 0 + \epsilon) \\ &= (1 + 0 \cdot 1)^* \cdot 0 \cdot 0 \cdot (0 + 1 \cdot (1^* \cdot 0)^* \cdot 0 + \epsilon)^* \\ &= (1 + 0 \cdot 1)^* \cdot 0 \cdot 0 \cdot (0 + 1 \cdot (1^+ \cdot 0)^* \cdot 0)^* \end{aligned}$$

e) Construct the transition diagram for the DFA & give a regular expression for its language by eliminating state q_2 .

DFA Awal :



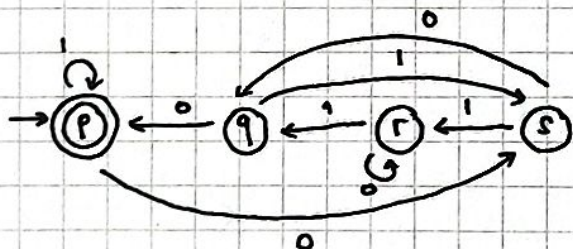
DFA setelah q_2 dihilangkan :



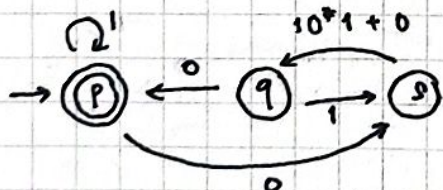
$$RE : [1+01+00(0+10)^*1]^* 00(0+10)^*$$

Exercise 3.2.3

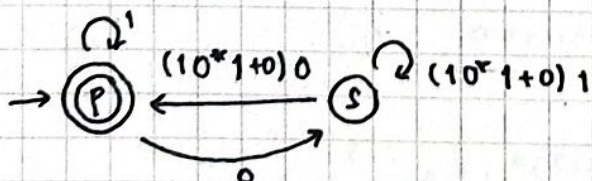
	0	1
$\rightarrow^* p$	s	p
q	p	s
r	r	q
s	q	r



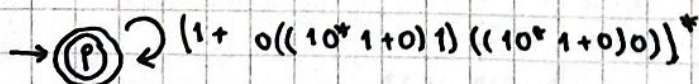
DFA Awal



Eliminasi r



Eliminasi q



Eliminasi s

$$\therefore RE : (1 + 0(01 + 10^*11)^* (00 + 10^*10))^*$$