

## Exercise 5.1.1

Design context-free grammar for the following languages.

- a) The set  $\{0^n 1^n \mid n \geq 1\}$ , that is, the set of all strings of one or more 0's followed by an equal number of 1's.

$$G = (\{S\}, \{0, 1\}, S, P)$$

dengan  $P$  adalah production rule :

$$S \rightarrow 0S1 \mid 01$$

- b) The set  $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$ , that is, the set of strings of  $a$ 's followed by  $b$ 's followed by  $c$ 's, such that there are either a different number of  $a$ 's &  $b$ 's or a different number of  $b$ 's &  $c$ 's, or both.

$$G = (\{S, A, B, C, D, E\}, \{a, b, c\}, S, P)$$

dengan  $P$  adalah production rule :

$$S \rightarrow AB \mid CD$$

$$A \rightarrow aA \mid \epsilon$$

$$B \rightarrow bBc \mid E \mid cD$$

$$C \rightarrow aCb \mid E \mid aA$$

$$D \rightarrow cD \mid \epsilon$$

$$E \rightarrow bE \mid b$$

- c) The set of all strings of  $a$ 's &  $b$ 's that are not of the form  $ww$ , that is, not equal to any string repeated.

$$G = (\{S, T, A, B\}, \{a, b\}, S, P)$$

dengan  $P$  adalah production rule :

$$S \rightarrow AB \mid BAIT$$

$$T \rightarrow aTb \mid bTa \mid aTa \mid bTb \mid a \mid b$$

$$A \rightarrow aAb \mid bAa \mid aAa \mid bAb \mid a$$

$$B \rightarrow aBb \mid bBa \mid aBa \mid bBb \mid b$$

- d) The set of all strings with twice as many 0's as 1's.

$$G = (\{S\}, \{0, 1\}, S, P)$$

dengan  $P$  adalah production rule :

$$S \rightarrow \epsilon \mid SS \mid 00S \mid 1S00 \mid 01S0$$



### Exercise 5.1.2

The following grammar generates the language of regular expression  $0^*1(0+1)^*$

$$S \rightarrow A1B$$

$$A \rightarrow 0A1 \mid \epsilon$$

$$B \rightarrow 0B1 \mid B \mid \epsilon$$

Give leftmost & rightmost derivations of the following strings:

a) 00101

$$\text{LM} : S \rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 001B \rightarrow 0010B \rightarrow 00101B \rightarrow 00101$$

$$\text{RM} : S \rightarrow A1B \rightarrow A10B \rightarrow A101B \rightarrow A101 \rightarrow 0A101 \rightarrow 00A101 \rightarrow 00101$$

b) 1001

$$\text{LM} : S \rightarrow A1B \rightarrow 1B \rightarrow 10B \rightarrow 100B \rightarrow 1001B \rightarrow 1001$$

$$\text{RM} : S \rightarrow A1B \rightarrow A10B \rightarrow A100B \rightarrow A1001B \rightarrow A1001 \rightarrow 1001$$

c) 00011

$$\text{LM} : S \rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 000A1B \rightarrow 0001B \rightarrow 00011B \rightarrow 00011$$

$$\text{RM} : S \rightarrow A1B \rightarrow 0A11B \rightarrow A11 \rightarrow 0A11 \rightarrow 00A11 \rightarrow 000A11 \rightarrow 00011$$

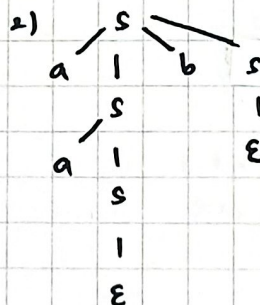
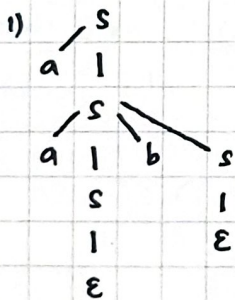
### Exercise 5.4.1

Consider the grammar

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

The grammar is ambiguous, show in particular that the string  $aab$  has two:

a) parse trees



b) Leftmost derivations

$$1) S \rightarrow aS \rightarrow aaaSbS \rightarrow aab$$

$$2) S \rightarrow aSbS \rightarrow aaSbS \rightarrow aabS \rightarrow aab$$

c) Rightmost derivations

$$1) S \rightarrow aS \rightarrow aaaSbS \rightarrow aaaSb \rightarrow aab$$

$$2) S \rightarrow aSbS \rightarrow aSb \rightarrow aaaSb \rightarrow aab$$



### Exercise 5.4.7

The following grammar generates prefix expressions with operands  $x$  and  $y$  and binary operators  $+$ ,  $-$ , and  $*$ :

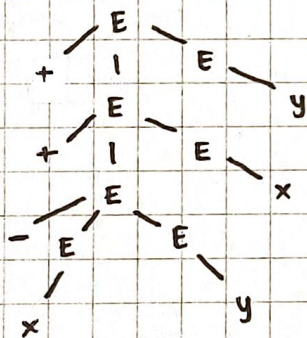
$$E \rightarrow +EE \mid *EE \mid -EE \mid x \mid y$$

- a) Find leftmost & rightmost derivations, and a derivation tree for the string  $+*-xyxy$

$$\begin{aligned} \text{LM: } E &\rightarrow +EE \rightarrow +*EEE \rightarrow +*-EEEE \rightarrow +*-xEEE \rightarrow +*-xyEE \\ &\rightarrow +*-xyxE \rightarrow +*-xyxy \end{aligned}$$

$$\begin{aligned} \text{RM: } E &\rightarrow +EE \rightarrow +Ey \rightarrow +*EEy \rightarrow +*Exy \rightarrow +*-EExy \\ &\rightarrow +*-Eyxxy \rightarrow +*-xyxy \end{aligned}$$

Derivation tree:



- b) Prove that grammar is unambiguous.

Pada grammar ini, setiap production rule memiliki awalan yg berbeda ( $+$ ,  $*$ ,  $-$ ,  $x$ ,  $y$ ). Utk string yg termasuk dlm EFL ini, apabila diambil leftmost derivation  $E$ , langkah berikutnya bergantung dgn simbol yg ingin dimunculkan dr  $w$ . Krn awalan tiap production rule berbeda, mk dijamin hanya terdapat satu rule yg dpt dipakai pd saat itu. Maka dijamin hanya ada satu leftmost derivation utk  $w$  dan grammar terbukti tdk ambigu.