(2)

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & -5 \end{pmatrix} \quad ; \quad A^{-1} ?$$

$$\left(\begin{array}{cccc|cccc} 2 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\downarrow R_1 \leftrightarrow R_2$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\downarrow R_2 - 2R_1$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & -8 & -24 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\downarrow R_2 / (-8)$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -1/8 & 1/4 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\downarrow R_4 + R_2$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -1/8 & 1/4 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -5 & -1/8 & 1/4 & 0 & 1 \end{array}\right)$$

$$\downarrow R_3 / 2$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -1/8 & 1/4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & -5 & -1/8 & 1/4 & 0 & 1 \end{array}\right)$$

$$\downarrow R_4 + R_3$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -1/8 & 1/4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -5 & -1/8 & 1/4 & 1/2 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -\tfrac{1}{8} & \tfrac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \tfrac{1}{40} & -\tfrac{1}{20} & -\tfrac{1}{10} & -\tfrac{1}{5} \end{array}\right)$$

↓ $R_1 - 2R_2$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 6 & 0 & \tfrac{1}{4} & \tfrac{1}{2} & 0 & 0 \\ 0 & 1 & 3 & 0 & -\tfrac{1}{8} & \tfrac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \tfrac{1}{40} & -\tfrac{1}{20} & -\tfrac{1}{10} & -\tfrac{1}{5} \end{array}\right)$$

$R_1 - 6R_3$

↓ $R_2 - 3R_3$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \tfrac{1}{4} & \tfrac{1}{2} & -3 & 0 \\ 0 & 1 & 0 & 0 & -\tfrac{1}{8} & \tfrac{1}{4} & -\tfrac{3}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \tfrac{1}{40} & -\tfrac{1}{20} & -\tfrac{1}{10} & -\tfrac{1}{5} \end{array}\right)$$

$$\therefore \quad A^{-1} = \left(\begin{array}{cccc} \tfrac{1}{4} & \tfrac{1}{2} & -3 & 0 \\ -\tfrac{1}{8} & \tfrac{1}{4} & -\tfrac{3}{2} & 0 \\ 0 & 0 & \tfrac{1}{2} & 0 \\ \tfrac{1}{40} & -\tfrac{1}{20} & -\tfrac{1}{10} & -\tfrac{1}{5} \end{array}\right) \quad \text{//}$$