

③

$$A = \begin{pmatrix} 1 & -1 & 1 & 6 \\ 0 & 0 & -3 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 3 & 2 \end{pmatrix}$$

a) A^{-1} ?

b) Lösung $AU = b$

b.1) $b = (1, 0, 0, 0)$

b.2) $b = (2, -1, -1, 3)$

a) A^{-1}

$$\left(\begin{array}{cccc|cccc} 1 & -1 & 1 & 6 & 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 3 & 2 & 0 & 0 & 0 & 1 \end{array} \right)$$

↓ $R_1 / 4$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 3 & 2 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} R_3 - 4R_1 \\ \downarrow R_4 - 4R_1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & -6 & -1 & 0 & 1 & 0 \\ 0 & 1 & 2 & -4 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\downarrow R_2 \leftrightarrow R_3$$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 2 & -1 & -6 & -1 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -4 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\downarrow R_2 / 2$$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -3 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -4 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\downarrow R_4 - R_2$$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -3 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 5/2 & -1 & -1/2 & 0 & -1/2 & 1 \end{array} \right)$$

$$\downarrow R_3 / (-3)$$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -3 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 5/2 & -1 & -1/2 & 0 & -1/2 & 1 \end{array} \right)$$

$$\downarrow R_4 - \frac{5}{2} R_3$$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -3 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1/2 & 5/6 & -1/2 & 1 \end{array} \right)$$

$$\downarrow R_4 (-1)$$

$$\left(\begin{array}{cccc|cccc} 1 & -1/4 & 1/4 & 3/2 & 1/4 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -3 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/2 & -5/6 & 1/2 & -1 \end{array} \right)$$

$$\downarrow R_1 + \frac{1}{4} R_2$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 1/8 & 3/4 & 1/8 & 0 & 1/8 & 0 \\ 0 & 1 & -1/2 & -3 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/2 & -5/6 & 1/2 & -1 \end{array} \right)$$

$$R_1 - \frac{1}{8} R_3 \quad \downarrow \quad R_2 + \frac{1}{2} R_3$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 3/4 & 1/8 & 1/24 & 1/8 & 0 \\ 0 & 1 & 0 & -3 & -1/2 & -1/6 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/2 & -5/6 & 1/2 & -1 \end{array} \right)$$

$$R_1 - \frac{3}{4} R_4 \quad \downarrow \quad R_2 + 3R_4$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1/4 & 2/3 & -1/4 & 3/4 \\ 0 & 1 & 0 & 0 & 1 & -8/3 & 2 & -3 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/2 & -5/6 & 1/2 & -1 \end{array} \right)$$

$$\therefore A^{-1} = \begin{pmatrix} -1/4 & 2/3 & -1/4 & 3/4 \\ 1 & -8/3 & 2 & -3 \\ 0 & -1/3 & 0 & 0 \\ 1/2 & -5/6 & 1/2 & -1 \end{pmatrix} //$$

b) 1) $Au = b$
 $u = A^{-1}b$; $b = (1, 0, 0, 0)$

$$u = A^{-1}b = \begin{pmatrix} -1/4 & 2/3 & -1/4 & 3/4 \\ 1 & -8/3 & 2 & -3 \\ 0 & -1/3 & 0 & 0 \\ 1/2 & -5/6 & 1/2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/4 \\ 1 \\ 0 \\ 1/2 \end{pmatrix} //$$

2) $b = (2, -1, -1, 3)$

$$u = A^{-1}b = \begin{pmatrix} -1/4 & 2/3 & -1/4 & 3/4 \\ 1 & -8/3 & 2 & -3 \\ 0 & -1/3 & 0 & 0 \\ 1/2 & -5/6 & 1/2 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4/3 \\ -19/3 \\ 1/3 \\ -5/3 \end{pmatrix} //$$