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# PR 5

**10.17** A new curing process developed for a certain type of cement results in a mean compressive strength of 5000 kilograms per square centimeter with a standard deviation of 120 kilograms. To test the hypothesis that $\mu = 5000$ against the alternative that $\mu < 5000$, a random sample of 50 pieces of cement is tested. The critical region is defined to be $\bar{x} < 4970$.

(a) Find the probability of committing a type I error when $H_0$ is true.

(b) Evaluate $\beta$ for the alternatives $\mu = 4970$ and $\mu = 4960$.

Tabel A.3

a) $\mu = 5000$

$\sigma = 120$

$\bar{x} = 4970$

$n = 50$

$$\boxed{z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}}$$

$$z = \frac{4970 - 5000}{120/\sqrt{50}}$$

$z = -1,767 \approx -1,77$

$\alpha = P(z < -1,77)$

$\therefore \alpha = 0,0384$

b) $\mu_1 = 4970$

$\mu_2 = 4960$

$$z_1 = \frac{4970 - 4970}{120/\sqrt{50}} = 0$$

$\beta_1 = P(z > 0) = 1 - P(z < 0)$

$\beta_1 = 1 - 0,5$

$\therefore \beta_1 = 0,5$

$$z_2 = \frac{4970 - 4960}{120/\sqrt{50}} = 0,589 \approx 0,59$$

$\beta_2 = P(z > 0,59) = 1 - P(z < 0,59)$

$\beta_2 = 1 - 0,7224$

$\therefore \beta_2 = 0,2776$

**10.21** An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a $P$-value in your answer.

$\mu = 800$

$\sigma = 40$

$n = 30$

$\bar{x} = 788$

Hipotesis

$H_0 : \mu = 800$

$H_1 : \mu \neq 800$ ($\mu < 800$ atau $\mu > 800$)

asumsi $\alpha = 0,05$

tes statistik $\boxed{z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}}$

Tabel A.3

$$z = \frac{788 - 800}{40/\sqrt{30}} = -1,64$$

nilai $p = 2^* \, P(z < -1,64)$

$= 2^* \, 0,0505 = 0,101$

nilai p $0,101 > \alpha$ ;

maka disimpulkan gagal menolak $H_0$

(rata² nya 800)

$z < -z_{\frac{\alpha}{2}}$ atau $z > z_{\frac{\alpha}{2}}$

$z_{\frac{\alpha}{2}} = z_{0,025} = 1,96$

$-1,64 \not< -1,96$ dan

$1,64 \not> 1,96$

**10.41** A study was conducted by the Department of Zoology at Virginia Tech to determine if there is a significant difference in the density of organisms at two different stations located on Cedar Run, a secondary stream in the Roanoke River drainage basin. Sewage from a sewage treatment plant and overflow from the Federal Mogul Corporation settling pond enter the stream near its headwaters. The following data give the density measurements, in number of organisms per square meter, at the two collecting stations:

**Number of Organisms per Square Meter**

| Station 1 | | Station 2 | |
|---|---|---|---|
| 5030 | 4980 | 2800 | 2810 |
| 13,700 | 11,910 | 4670 | 1330 |
| 10,730 | 8130 | 6890 | 3320 |
| 11,400 | 26,850 | 7720 | 1230 |
| 860 | 17,660 | 7030 | 2130 |
| 2200 | 22,800 | 7330 | 2190 |
| 4250 | 1130 | | |
| 15,040 | 1690 | | |

Can we conclude, at the 0.05 level of significance, that the average densities at the two stations are equal? Assume that the observations come from normal populations with different variances.

Statistik Deskriptif

$\alpha = 0,05$

$\bar{x}_1 = 9897,5$

$\bar{x}_2 = 4120,83$

$s_1^2 = 62005060$

$s_1 = \sqrt{62005060} = 7874,329$

$s_2 = 6,1477 \times 10^6$

$s_2 = \sqrt{6,1477 \times 10^6} = 2479,455$

Hipotesis

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$\sigma_1 \neq \sigma_2$ and unknown

Statistik uji t

$$\boxed{t' = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}} = \frac{(9897,5 - 4120,83) - 0}{\sqrt{\frac{6200506}{16} + \frac{6,1477 \times 10^6}{12}}} = 2,7578 \approx 2,76$$

$t' < -t_{\frac{\alpha}{2}}$ atau $t' > t_{\frac{\alpha}{2}}$

Derajat Kebebasan

$$\boxed{V = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}} = \frac{\left(\frac{6200506}{16} + \frac{6,1477 \times 10^6}{12}\right)^2}{\frac{\left(\frac{6200506}{16}\right)^2}{15} + \frac{\left(\frac{6,1477 \times 10^6}{12}\right)^2}{11}} = 19$$

$t' < -t_{0,025}$ atau $t' > t_{0,025}$

$t' < -2,093$ atau $t' > 2,093$

Tabel A.4

$t' = 2,76 > 2,093$

$\therefore$ tolak $H_0$ ($\mu_1 \neq \mu_2$)

**10.61** In a winter of an epidemic flu, the parents of 2000 babies were surveyed by researchers at a well-known pharmaceutical company to determine if the company's new medicine was effective after two days. Among 120 babies who had the flu and were given the medicine, 29 were cured within two days. Among 280 babies who had the flu but were not given the medicine, 56 recovered within two days. Is there any significant indication that supports the company's claim of the effectiveness of the medicine?

Hipotesis

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$

$x_1 = 29$, $n_1 = 120$

$x_2 = 56$, $n_2 = 280$

dengan obat : $\hat{p}_1 = \frac{29}{120} \approx 0,2417$

tanpa obat : $\hat{p}_2 = \frac{56}{280} = 0,2$

gabung : $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{85}{400} = 0,2125$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\boxed{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}} = \frac{0,241667 - 0,2}{\sqrt{0,2125\,(1 - 0,2125)\left(\frac{1}{120} + \frac{1}{280}\right)}} = 0,933$$

Standard Error

nilai p

$p(z > 0,93) = 1 - P(z < 0,93)$

$= 1 - 0,8238$

$= 0,1762$

Tabel A.3

asumsi $\alpha = 0,05$

nilai p $= 0,1762 > \alpha$

$\therefore$ tolak $H_0$, tidak ada bukti yg mendukung obat baru lbh efektif menyembuhkan flu dlm 2 hari dibanding tanpa obat

**10.77** An experiment was conducted to compare the alcohol content of soy sauce on two different production lines. Production was monitored eight times a day. The data are shown here.

Production line 1:

0.48 0.39 0.42 0.52 0.40 0.48 0.52 0.52

Production line 2:

0.38 0.37 0.39 0.41 0.38 0.39 0.40 0.39

Assume both populations are normal. It is suspected that production line 1 is not producing as consistently as production line 2 in terms of alcohol content. Test the hypothesis that $\sigma_1 = \sigma_2$ against the alternative that $\sigma_1 \neq \sigma_2$. Use a $P$-value.

$\bar{x}_1 = 0,46625$

$\bar{x}_2 = 0,38875$

$s_1^2 = 0,00305809$

$s_1 = 0,0553$

$s_2^2 = 0,00015625$

$s_2 = 0,0125$

Hipotesis

$H_0 : \sigma_1 = \sigma_2$

$H_1 : \sigma_1 \neq \sigma_2$

Distribusi f

$$f = \frac{s_1^2}{s_2^2} = \frac{(0,0553)^2}{(0,0125)^2} = 19,67$$

nilai p $= 2 * P(f > 19,67)$

$= 2 * (0,0004)$

$= 0,0008$

$v_1 = n_1 - 1 = 7$

$v_2 = n_2 - 1 = 7$

critical region $f < f_{1-\frac{\alpha}{2}}(v_1, v_2)$ atau $f > f_{\frac{\alpha}{2}}(v_1, v_2)$

asumsi $\alpha = 0,05$

$f < f_{0,975}(7,7)$

$\therefore$ Tolak Ho, krn $0,0008 \leq \alpha$ utk 7 degree of freedom.

Terdapat indikasi signifikan bhw $\sigma_1 \neq \sigma_2$. Produksi 1 tidak menghasilkan kandungan alkohol se-konsisten produksi 2

**10.93** To determine current attitudes about prayer in public schools, a survey was conducted in four Virginia counties. The following table gives the attitudes of 200 parents from Craig County, 150 parents from Giles County, 100 parents from Franklin County, and 100 parents from Montgomery County:

| | County | | | | |
|---|---|---|---|---|---|
| **Attitude** | **Craig** | **Giles** | **Franklin** | **Mont.** | Total |
| Favor | 65 | 66 | 40 | 34 | 205 |
| Oppose | 42 | 30 | 33 | 42 | 147 |
| No opinion | 93 | 54 | 27 | 24 | 198 |
| Total | 200 | 150 | 100 | 100 | 550 |

Test for homogeneity of attitudes among the four counties concerning prayer in the public schools. Use a $P$-value in your conclusion.

Hipotesis

$H_0$ : attitudes homogen

$H_1$ : attitudes not homogen

Tabel e

| | craig | giles | franklin | montgomery |
|---|---|---|---|---|
| favor | $\frac{200}{550} \times 205 = 74,5$ | $\frac{150}{550} \times 205 = 55,9$ | $\frac{100}{550} \times 205 = 37,3$ | $\frac{100}{550} \times 205 = 37,3$ |
| oppose | $\frac{200}{550} \times 147 = 53,5$ | $\frac{150}{550} \times 147 = 40,1$ | $\frac{100}{550} \times 147 = 26,7$ | $\frac{100}{550} \times 147 = 26,7$ |
| no opinion | $\frac{200}{550} \times 198 = 72,0$ | $\frac{150}{550} \times 198 = 54,0$ | $\frac{100}{550} \times 198 = 36,0$ | $\frac{100}{550} \times 198 = 36,0$ |

$$\boxed{X^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}}$$

$$= \frac{(65-74,5)^2}{74,5} + \frac{(66-55,9)^2}{55,9} + \frac{(40-37,3)^2}{37,3} + \frac{(34-37,3)^2}{37,3}$$

$$+ \frac{(42-53,5)^2}{53,5} + \frac{(30-40,1)^2}{40,1} + \frac{(33-26,7)^2}{26,7} + \frac{(42-26,7)^2}{26,7}$$

$$+ \frac{(93-72,0)^2}{72,0} + \frac{(54-54,0)^2}{54,0} + \frac{(27-36,0)^2}{36,0} + \frac{(24-36,0)^2}{36,0}$$

$$= 31,17$$

$V = (r-1)(c-1)$

$= (3-1)(4-1) = 6$ $\longrightarrow$ utk $v = 6$, tdk ada $\alpha$ yg lbh besar dr 31,17

$\alpha$ yg menghasilkan nilai terbesar

$P(X^2 > 31,17) < 0,001$

$\therefore$ Tolak $H_0$, krn $P(X^2 > 31,17) < 0,001$ dgn 6 degree of freedom, disimpulkan bhw attitudes tdk homogen