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PR 2

3.11 A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X. Express the results graphically as a probability histogram.

defect = 2

total purchase = 3

number of defect = x

all probability = $\binom{7}{3}$

defect probability = $\binom{2}{x}$

non defect probability = $\binom{5}{3-x}$

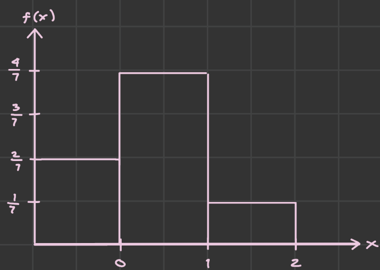
$$f(x) = \frac{\binom{2}{x}\binom{5}{3-x}}{\binom{7}{3}} \quad ; x = 0, 1, 2$$

$$f(0) = \frac{\binom{2}{0}\binom{5}{3}}{\binom{7}{3}} = \frac{1 \cdot 10}{35} = \frac{2}{7}$$

$$f(1) = \frac{\binom{2}{1}\binom{5}{2}}{\binom{7}{3}} = \frac{2 \cdot 10}{35} = \frac{4}{7}$$

$$f(2) = \frac{\binom{2}{2}\binom{5}{1}}{\binom{7}{3}} = \frac{1 \cdot 5}{35} = \frac{1}{7}$$

Histogram



3.27 The time to failure in hours of an important piece of electronic equipment used in a manufactured DVD player has the density function

$$f(x) = \begin{cases} \frac{1}{2000}\exp(-x/2000), & x \geq 0, \\ 0, & x < 0. \end{cases}$$

(a) Find $F(x)$.

(b) Determine the probability that the component (and thus the DVD player) lasts more than 1000 hours before the component needs to be replaced.

(c) Determine the probability that the component fails before 2000 hours.

a) $F(x) = \int_{-\infty}^{\infty} f(t)\,dt = \int_{-\infty}^{0} f(t)\,dt + \int_{0}^{\infty} f(t)\,dt$

$= 0 + \int_0^x \frac{1}{2000} e^{-\frac{t}{2000}}\,dt$

$-\frac{t}{2000} = u$

$-\frac{1}{2000} = \frac{du}{dt}$

$du = -\frac{dt}{2000}$

$= 0 + \int_0^{-\frac{x}{2000}} -e^u \, du$

$= 0 - \left[e^u\right]_0^{-\frac{x}{2000}}$

$= 1 - e^{-\frac{x}{2000}}$

$\therefore F(x) = \begin{cases} 0 & ; x < 0 \\ 1 - e^{-\frac{x}{2000}} & ; x \geq 0 \end{cases}$

b) $P(x > 1000) = 1 - F(1000)$

$= 1 - (1 - e^{-\frac{1000}{2000}})$

$= e^{-\frac{1}{2}} = 0,6065$

c) $P(x < 2000) = F(2000)$

$= 1 - e^{-\frac{2000}{2000}}$

$= 1 - e^{-1} = 0,6321$

3.49 Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

		x		
	$f(x,y)$	1	2	3
	1	0.05	0.05	0.10
y	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

(a) Evaluate the marginal distribution of X.
(b) Evaluate the marginal distribution of Y.
(c) Find $P(Y = 3 \mid X = 2)$.

a) $P(X=1) = g(1) = \Sigma f(1,y) = f(1,1) + f(1,3) + f(1,5)$
$= 0{,}05 + 0{,}05 + 0{,}00 = 0{,}10$

$P(X=2) = g(2) = \Sigma f(2,y) = f(2,1) + f(2,3) + f(2,5)$
$= 0{,}05 + 0{,}10 + 0{,}20 = 0{,}35$

$P(X=3) = g(3) = \Sigma f(3,y) = f(3,1) + f(3,3) + f(3,5)$
$= 0{,}10 + 0{,}35 + 0{,}10 = 0{,}55$

x	1	2	3
g(x)	0,10	0,35	0,55

b) $P(Y=1) = h(1) = \Sigma f(x,1) = f(1,1) + f(2,1) + f(3,1)$
$= 0{,}05 + 0{,}05 + 0{,}10 = 0{,}20$

$P(Y=3) = h(3) = \Sigma f(x,3) = f(1,3) + f(2,3) + f(3,3)$
$= 0{,}05 + 0{,}10 + 0{,}35 = 0{,}50$

$P(Y=5) = h(5) = \Sigma f(x,5) = f(1,5) + f(2,5) + f(3,5)$
$= 0{,}00 + 0{,}20 + 0{,}10 = 0{,}30$

y	1	3	5
h(y)	0, 20	0,50	0, 30

c) $P(Y=3 \mid X=2) = \dfrac{f(2,3)}{g(2)} = \dfrac{0{,}10}{0{,}35} = 0{,}2857$

4.51 For the random variables X and Y in Exercise 3.39 on page 105, determine the correlation coefficient between X and Y.

3.39 From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

(a) the joint probability distribution of X and Y;
(b) $P[(X,Y) \in A]$, where A is the region that is given by $\{(x,y) \mid x + y \leq 2\}$.

y \ x	0	1	2	3	f(y)
0	0	3/70	9/70	3/70	3/14
1	1/35	9/35	9/35	1/35	4/7
2	3/70	9/70	3/70	0	3/14
f(x)	1/14	3/7	3/7	1/14	

$E(x) = 0(1/14) + 1(3/7) + 2(3/7) + 3(1/14) = \frac{21}{14} = \frac{3}{2} = \mu_x$

$E(x^2) = 0(f(0)) + 1(f(1)) + 4(f(2)) + 9(f(3))$
$= 0 + \frac{3}{7} + \frac{12}{7} + \frac{9}{14} = \frac{39}{14}$

$\sigma_x^2 = E(x^2) - \mu_x^2$
$= \frac{39}{14} - \left(\frac{3}{2}\right)^2 = \frac{15}{28} \rightarrow \sigma_x = \pm\sqrt{\frac{15}{28}}$

$E(Y) = 0(3/14) + 1(4/7) + 2(3/14) = \frac{14}{14} = 1 = \mu_Y$

$E(Y^2) = 0(f(0)) + 1(f(1)) + 4(f(2))$
$= 0 + \frac{4}{7} + \frac{12}{14} = \frac{10}{7}$

$\sigma_Y^2 = E(Y^2) - \mu_Y^2$
$= \frac{10}{7} - 1 = \frac{3}{7} \rightarrow \sigma_Y = \pm\sqrt{\frac{3}{7}}$

$E(XY) = \sum_{x=0}^{3}\sum_{y=0}^{2} xy\, f(x,y)$
$= (0)(0)f(0,0) + (0)(1)f(0,1) + (0)(2)f(0,2) + (1)(0)f(1,0) +$
$(1)(1)f(1,1) + (1)(2)f(1,2) + (2)(0)f(2,0) + (2)(1)f(2,1) +$
$(2)(2)f(2,2) + (3)(0)f(3,0) + (3)(1)f(3,1) + (3)(2)f(3,2)$
$= 0 + 0 + 0 + 0 + 1(9/35) + 2(9/70) +$
$0 + 2(9/35) + 4(3/70) + 0 + 3(1/35) + 0$
$= \frac{90}{70} = \frac{9}{7} = \mu_{xy}$

$\sigma_{XY} = E(XY) - \mu_X \cdot \mu_Y$
$= \frac{9}{7} - \frac{3}{2} = -\frac{3}{14}$

$\rho_{XY} = \dfrac{\sigma_{XY}}{\sigma_X \cdot \sigma_Y} = \dfrac{-3/14}{\sqrt{15/28} \cdot \sqrt{3/7}} = -\dfrac{1}{\sqrt{5}}$

4.67 If the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{2}{7}(x+2y), & 0<x<1,\ 1<y<2, \\ 0, & \text{elsewhere,} \end{cases}$$

find the expected value of $g(X,Y) = \frac{X}{Y^3} + X^2Y$.

$$E\left(\frac{X}{Y^3}\right) = \int_1^2\int_0^1 \frac{2x(x+2y)}{7y^3}\,dx\,dy$$

$$= \int_1^2 \left[\frac{\frac{2}{3}x^3 + 2x^2y}{7y^3}\right]_0^1 dy$$

$$= \frac{2}{7}\int_1^2 \left(\frac{1}{3y^3} + \frac{1}{y^2}\right) dy$$

$$= \frac{2}{7}\left[-\frac{1}{6x^2} - \frac{1}{x}\right]_1^2$$

$$= \frac{2}{7}\left(-\frac{1}{24} - \frac{1}{2} + \frac{1}{6} + 1\right)$$

$$= \frac{2}{7}\cdot\frac{15}{24} = \frac{5}{28}$$

$$E(X^2Y) = \int_1^2\int_0^1 \frac{2x^2y(x+2y)}{7}\,dx\,dy$$

$$= \int_1^2 \left[\frac{\frac{1}{2}x^4y + \frac{4}{3}x^3y^2}{7}\right]_0^1 dy$$

$$= \frac{2}{7}\int_1^2 \left(\frac{1}{4}y + \frac{2}{3}y^2\right) dy$$

$$= \frac{2}{7}\left[\frac{1}{8}y^2 + \frac{2}{9}y^3\right]_1^2$$

$$= \frac{2}{7}\left(\frac{4}{8} + \frac{16}{9} - \frac{1}{8} - \frac{2}{9}\right)$$

$$= \frac{2}{7}\cdot\frac{139}{72} = \frac{139}{252}$$

$$E[g(X,Y)] = E\left(\frac{X}{Y^3} + X^2Y\right)$$

$$= E\left(\frac{X}{Y^3}\right) + E(X^2Y)$$

$$= \frac{5}{28} + \frac{139}{252} = \frac{46}{63}$$

4.75 An electrical firm manufactures a 100-watt light bulb, which, according to specifications written on the package, has a mean life of 900 hours with a standard deviation of 50 hours. At most, what percentage of the bulbs fail to last even 700 hours? Assume that the distribution is symmetric about the mean.

mean life $\rightarrow$ $\mu = 900$ hours

standard deviation $\rightarrow$ $\sigma = 50$ hours

fail to last even 700 hours :

$\mu - k\sigma = 700$

$900 - 50k = 700$

$50k = 200$

$k = 4$

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$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

$P(700 < X < 1100) \geq 1 - \frac{1}{16}$

$\therefore P(700 < X < 1100) \geq 0,9375$

$P(x \leq 700) \leq \frac{1}{k^2}$

$P(x \leq 700) \leq \frac{0,0625}{2}$ symmetric

$\therefore P(x \leq 700) \leq 0,03125$