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13522053 - K1

PR 3

5.21 The surface of a circular dart board has a small center circle called the bull's-eye and 20 pie-shaped regions numbered from 1 to 20. Each of the pie-shaped regions is further divided into three parts such that a person throwing a dart that lands in a specific region scores the value of the number, double the number, or triple the number, depending on which of the three parts the dart hits. If a person hits the bull's-eye with probability 0.01, hits a double with probability 0.10, hits a triple with probability 0.05, and misses the dart board with probability 0.02, what is the probability that 7 throws will result in no bull's-eyes, no triples, a double twice, and a complete miss once?

$\to n = 7$

$x_1 = 0 \rightarrow$ no bull's eye ; $P_1 = 1 - 0{,}01 = 0{,}99$

$x_2 = 0 \rightarrow$ no triples ; $P_2 = 1 - 0{,}05 = 0{,}95$

$x_3 = 2 \rightarrow$ double twice ; $P_3 = 0{,}1$

$x_4 = 1 \rightarrow$ miss once ; $P_4 = 0{,}02$

$x_5 = 7 - 0 - 0 - 1 - 2 = 4$

↳ sisa ; $P_5 = 1 - 0{,}01 - 0{,}1 - 0{,}05 - 0{,}02 = 0{,}82$

$$\boxed{f(x_1, x_2, \ldots, x_k ; p_1, p_2, \ldots, p_k, n) = \binom{n}{x_1, x_2, \ldots, x_k} p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k}}$$

$$f(0,0,2,1,4 ; 0{,}99, 0{,}95, 0{,}1, 0{,}02, 0{,}82, 7) = \binom{7}{0,0,2,1,4} (0{,}99)^0 (0{,}95)^0 (0{,}1)^2 (0{,}02)^1 (0{,}82)^4$$

$$= \frac{7!}{0!\,0!\,2!\,1!\,4!} (1)(1)(0{,}01)(0{,}02)(0{,}82)^4$$

$$= 105\,(0{,}01)(0{,}02)(0{,}82)^4$$

multinomial distribution

$$\approx 0{,}00949456$$

5.35 A company is interested in evaluating its current inspection procedure for shipments of 50 identical items. The procedure is to take a sample of 5 and pass the shipment if no more than 2 are found to be defective. What proportion of shipments with 20% defectives will be accepted?

hypergeometric distribution

$n = 5$

$N = 50$

x = defective items

$k = 20\% \times 50 = 10$

$$\boxed{h(x; N, n, k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} \; ; \quad x = 0, 1, 2}$$

$$P(X=0) = h(0; 50, 5, 10) = \frac{\binom{10}{0}\binom{40}{5}}{\binom{50}{5}} \approx 0{,}3106$$

$$P(X=1) = h(1; 50, 5, 10) = \frac{\binom{10}{1}\binom{40}{4}}{\binom{50}{5}} \approx 0{,}4313$$

$$P(X=2) = h(2; 50, 5, 10) = \frac{\binom{10}{2}\binom{40}{3}}{\binom{50}{5}} \approx 0{,}2098$$

$\therefore P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0{,}3106 + 0{,}4313 + 0{,}2098$

$= 0{,}9517$

λt

5.57 On average, a textbook author makes two word-processing errors per page on the first draft of her textbook. What is the probability that on the next page she will make

(a) 4 or more errors?

(b) no errors?

$\lambda t = 2$

poisson distribution

$$\boxed{p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}}$$

a) $P(x \geq 4) = 1 - P(x \leq 3)$

$= 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3)$

$P(x=0) = \dfrac{e^{-2}(2)^0}{0!} = 0{,}1353$

$P(x=1) = \dfrac{e^{-2}(2)^1}{1!} = 0{,}2707$

$P(x=2) = \dfrac{e^{-2}(2)^2}{2!} = 0{,}2707$

$P(x=3) = \dfrac{e^{-2}(2)^3}{3!} = 0{,}1804$

$P(x \geq 4) = 1 - 0{,}1353 - 0{,}2707 - 0{,}2707 - 0{,}1804$

$\therefore P(x \geq 4) = 0{,}1429$

b) $P(x=0) = \dfrac{e^{-2}(2)^0}{0!} = 0{,}1353$

μ σ

6.13 A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are normally distributed with a standard deviation of 6.3 months, find the probability that a given mouse will live

(a) more than 32 months;

(b) less than 28 months;

(c) between 37 and 49 months.

$\mu = 40$

$\sigma = 6{,}3$

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

nilai z dr tabel A.3

a) $x > 32$

$z = \dfrac{32-40}{6{,}3} \approx -1{,}27$

$P(z > -1{,}27) = 1 - P(z < -1{,}27)$

$= 1 - 0{,}1020$

$= 0{,}898$

b) $x < 28$

$z = \dfrac{28-40}{6{,}3} \approx -1{,}90$

$P(z < -1{,}90) = 0{,}0287$

c) $37 < x < 49$

$z_1 = \dfrac{37-40}{6{,}3} \approx -0{,}48$

$z_2 = \dfrac{49-40}{6{,}3} \approx 1{,}43$

$P(-0{,}48 < z < 1{,}43) = P(z < 1{,}43) - P(z < -0{,}48)$

$= 0{,}9236 - 0{,}3156$

$= 0{,}608$

n

6.35 A company produces component parts for an engine. Parts specifications suggest that 95% of items meet specifications. The parts are shipped to customers in lots of 100.

(a) What is the probability that more than 2 items in a given lot will be defective?

(b) What is the probability that more than 10 items in a lot will be defective?

p meet spec

n = 100

p meet spec = 0,95

$$\boxed{\begin{aligned} \mu &= np \\ \sigma^2 &= npq \end{aligned}}$$

a) p = not spec

$= 1 - 0,95 = 0,05$

$x = 2 + 0,5 = 2,5$

aproksimasi (krn >)

$\mu = np = 100 \cdot 0,05 = 5$

$\sigma = \sqrt{npq} = \sqrt{100 \cdot 0,05 \cdot 0,95} \approx 2,1794$

$$z = \frac{x - \mu}{\sigma} = \frac{2,5 - 5}{2,1794} \approx -1,147$$

$\therefore P(x = 2) \approx P(z \geq -1,147) = 1 - P(z \leq -1,15)$

$= 1 - 0,1251 = 0,8749$

b) $x = 10 + 0,5 = 10,5$

aproksimasi (krn >)

$$z = \frac{x - \mu}{\sigma} = \frac{10,5 - 5}{2,1794} \approx 2,524$$

$\therefore P(x = 10) \approx P(z \geq 2,524) = 1 - P(z \leq 2,52)$

$= 1 - 0,9941 = 0,0059$

6.49 Suppose the random variable X follows a beta distribution with $\alpha = 1$ and $\beta = 3$.

(a) Determine the mean and median of X.

(b) Determine the variance of X.

(c) Find the probability that $X > 1/3$.

a) $\boxed{\mu = \frac{\alpha}{\alpha + \beta}} = \frac{1}{1+3} = \frac{1}{4}$ → mean

density function beta distribution

$$\boxed{f(x) = \frac{1}{\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx}\, x^{\alpha-1}(1-x)^{\beta-1}}$$

$$= \frac{1}{\int_0^1 x^{1-1}(1-x)^{3-1}\,dx}\, x^{1-1}(1-x)^{3-1}$$

$$= \frac{1}{\int_0^1 (1-x)^2\,dx}\,(1-x)^2$$

$$\int_0^1 1 - 2x + x^2\,dx$$

$$= \left[x - x^2 + \frac{x^3}{3}\right]_0^1$$

$$= 1 - 1 + \frac{1}{3} = \frac{1}{3}$$

$$= \frac{1}{1/3}(1-x)^2 = 3(1-x)^2 \quad ; \text{ utk } 0 < x < 1$$

median → nilai x saat cdf

$F(x) = \frac{1}{2}$; $f(x) = 3(1-x)^2$

cari cdf dr $f(x)$

$I_x(a,b) = 1 - I_{1-x}(b,a)$

$I_x(1,b) = 1 - (1-x)^b$

$I_x(\alpha,\beta) \longrightarrow$ cdf

$I_x(1,3) = 1 - (1-x)^3$

$F(x) = 1 - (1-x)^3$

$\frac{1}{2} = 1 - (1-x)^3$

$(1-x)^3 = \frac{1}{2}$

$1 - x = 0,7937$ → median

$\therefore x = 0,2063$

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b) $$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$= \frac{1.3}{(1+3)^2(1+3+1)}$$

$$= \frac{3}{16.5} = 0{,}0375$$

c) $P(x > \frac{1}{3})$

cara 1

$$z = \frac{x-\mu}{\sigma} = \frac{\frac{1}{3}-\frac{1}{4}}{\sqrt{0{,}0375}} = 0{,}4303$$

$$P(x > \tfrac{1}{3}) \approx P(z > 0{,}4303)$$
$$= 1 - P(z \leq 0{,}43)$$
$$= 1 - 0{,}6664$$
$$= 0{,}3336$$

$$I_x(1,3) = 1-(1-x)^3$$

$$P(x) = 1 - I_x(1,3)$$

$$P(\tfrac{1}{3}) = 1 - \left(1 - (1-\tfrac{1}{3})^3\right)$$

$$= 0{,}296$$

cara 2

$$P(x > \tfrac{1}{3}) = \int_{1/3}^{1} 3(1-x)^2\,dx$$

$$= 3\left[x - x^2 + \frac{x^3}{3}\right]_{1/3}^{1}$$

$$= 0{,}2963$$