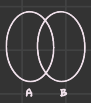
RANGKUMAN BAB 1-4

Aturan Penjumlahan



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cup B) = P(A) + P(B)$$

Saling Komplemen $\longrightarrow$ $P(A) + P(A^c) = 1$

Peluang Bersyarat

$P(B|A)$ = peluang B akibat A

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad ; \quad P(A) > 0$$

Saling lepar $\begin{cases} P(B|A) = P(B) \\ P(A|B) = P(A) \end{cases}$

Aturan Perkalian

$$P(A \cap B) = P(A) \cdot P(B|A)$$

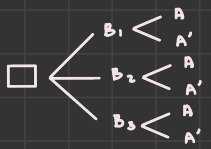
$P(A \cap B)$ → kejadian A dan B, biasanya memperhatikan urutan

saling bebar $\longrightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

Partisi Ruang Sampel

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) \quad \text{or} \quad P(A) = \sum_{i=1}^{k} P(B_i)\, P(A|B_i)$$

Aturan Bayes



$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)\, P(A|B_r)}{\sum_{i=1}^{k} P(B_i)\, P(A|B_i)} \quad ; \quad r = 1, 2, \ldots, k$$

Variabel Random

Ruang Sampel Diskrit → bil bulat

Berisi sejumlah hingga kemungkinan hasil / barisan tak hingga sebanyak elemen ≡ bil bulat

Ruang Sampel Kontinu → desimal

Berisi sejumlah tak hingga kemungkinan hasil

Distribusi Peluang Diskrit

1) $f(x) \geq 0$

2) $\sum_x f(x) = 1$

3) $P(X = x) = f(x)$ → bakal jd distribusi peluang

4) $F(x) = P(X \leq x) = \sum_{t \leq x} f(t)$; $-\infty < x < \infty$

Distribusi Peluang Kontinu

$$P(a < X \leq b) = P(a < X < b) + P(X = b) = P(a < x < b)$$

$$\Downarrow$$

$$P(a < x \leq b) = \int_a^b f(x)\,dx$$

1) $f(x) \geq 0$; $x \in R$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

3) $P(a < x < b) = \int_a^b f(x)\,dx$ → bakal jd distribusi peluang

Distribusi Kumulatif Kontinu

$$P(a < x < b) = F(b) - F(a)$$

$$f(x) = \frac{dF(x)}{dx}$$

Distribusi Peluang Gabungan

Diskrit $f(x,y) = P(X = x, Y = y)$

1) $f(x,y) \geq 0$ utk semua (x,y)

2) $\sum_x \sum_y f(x,y) = 1$

3) $P(X = x, Y = y) = f(x,y)$

untuk daerah sembarang A dlm bidang xy → $P[(x,y) \in A] = \sum_A \sum f(x,y)$

Kontinu

1) $f(x,y) \geq 0$ utk semua (x,y)

2) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

3) $[P(X < Y) \in A] = \iint f(x,y)\,dx\,dy$

Distribusi Marginal

Diskrit

$g(x) = \sum_y f(x,y)$

$h(y) = \sum_x f(x,y)$

Kontinu

$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$

$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

Distribusi Bersyarat

$$f(y|x) = \frac{f(x,y)}{g(x)} \qquad f(x|y) = \frac{f(x,y)}{h(y)}$$

syarat bebas statistik

$f(x,y) = g(x) \cdot h(y)$

Rataan / Nilai Ekspektasi

Diskrit

$$\mu = E(x) = \sum_x x f(x)$$

$$\mu_{g(x,y)} = E[g(X,Y)] = \sum_x \sum_y g(x,y) f(x,y)$$

Kontinu

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$\mu_{g(x,y)} = E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y) \, dx dy$$

Variansi (σ)

diskrit

$\sigma^2 = E[(x-\mu)^2] = \sum_x (x-\mu)^2 f(x)$

kontinu

$\sigma^2 = E[(x-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) \, dx$

* standar deviasi = $\sqrt{\sigma^2} = \sigma$
* Variansi dari var.random X

$\sigma^2 = E(x^2) - \mu^2$

Kovariansi (σ_{xy})

x, y var.random

diskrit

$\sigma_{xy} = E[(x-\mu_x)(y-\mu_y)] = \sum_x \sum_y (x-\mu_x)(y-\mu_y) f(x,y)$

→ distribusi peluang gabungan

kontinu

$\sigma_{xy} = E[(x-\mu_x)(y-\mu_y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-\mu_x)(y-\mu_y) f(x,y) \, dx dy$

* kovariansi dua var random X, Y dgn rataan μ_x dan μ_y

$\sigma_{xy} = E(XY) - \mu_x \mu_y$

Koefisien Korelasi

var.random (x, y)

$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$ → kovariansi / → simpangan baku

Rataan kombinasi linear var.random

* Teorema — a, b konstanta

$E(aX+b) = aE(X)+b$

$E[g(x) \pm h(x)] = E[g(x)] \pm E[h(x)]$

$E[g(x,y) \pm h(x,y)] = E[g(x,y)] \pm E[h(x,y)]$

* Teorema sifat rataan

misalkan x dan y var.random saling bebas

$E(XY) = E(x)E(y)$

* Teorema menghitung variansi — a, b konstanta; x, y var.random; f(x,y) distribusi peluang

$\sigma^2_{ax+b} = a^2 \sigma^2_x = a^2 \sigma^2$

$\sigma^2_{ax+by} = a^2 \sigma^2_x + b^2 \sigma^2_y + 2ab\sigma_{xy}$

Teorema Chebyshev

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

probabilitas — var. random — simpangan baku