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PR 4

8.25 The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find

(a) the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;

(b) the value of x to the right of which 15% of the means computed from random samples of size 9 would fall.

$\mu = 7$
$\sigma = 1$

a) $n = 9$

$P(6{,}4 < \bar{x} < 7{,}2)$?

central limit theorem

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}}$$

$$z_1 = \frac{6{,}4 - 7}{1/\sqrt{9}} = -1{,}8 \qquad z_2 = \frac{7{,}2 - 7}{1/\sqrt{9}} = 0{,}6$$

$P(6{,}4 < \bar{x} < 7{,}2) = P(-1{,}8 < z < 0{,}6)$
$= P(z < 0{,}6) - P(z < -1{,}8)$
$= 0{,}7257 - 0{,}0359 \longrightarrow$ dr tabel A3
$= 0{,}6898$

b) $P(\bar{x} > x) = 0{,}15$

$P(\bar{x} < x) = 1 - 0{,}15 = 0{,}85$

cari z yg bernilai 0,85 $\longrightarrow$ $z = 1{,}04$

central limit theorem

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}}$$

$$1{,}04 = \frac{\bar{x} - 7}{1/\sqrt{9}}$$

$$\bar{x} = \frac{551}{75} = 7{,}346$$

8.31 Consider Case Study 8.2 on page 238. Suppose 18 specimens were used for each type of paint in an experiment and $\bar{x}_A - \bar{x}_B$, the actual difference in mean drying time, turned out to be 1.0.

(a) Does this seem to be a reasonable result if the two population mean drying times truly are equal? Make use of the result in the solution to Case Study 8.2.

(b) If someone did the experiment 10,000 times under the condition that $\mu_A = \mu_B$, in how many of those 10,000 experiments would there be a difference $\bar{x}_A - \bar{x}_B$ that was as large as (or larger than) 1.0?

central limit theorem

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 \text{ and } \sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}.$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$$

a) $n_A = n_B = 18$

$\mu_{\bar{x}_A} = \mu_{\bar{x}_B} = \mu$

$\sigma_{\bar{x}_A} = \sigma_{\bar{x}_B} = 1$

cari $P(\bar{x}_A - \bar{x}_B > 1)$

$\mu_{\bar{x}_A - \bar{x}_B} = \mu_{\bar{x}_A} - \mu_{\bar{x}_B}$
$= \mu - \mu = 0$

$$\sigma^2_{\bar{x}_A - \bar{x}_B} = \frac{\sigma_{\bar{x}_A}^2}{n_A} + \frac{\sigma_{\bar{x}_B}^2}{n_B} = \frac{1^2}{18} + \frac{1^2}{18} = \frac{1}{9}$$

untuk $\bar{x}_A - \bar{x}_B = 1$

$$z = \frac{(\bar{x}_A - \bar{x}_B) - (\mu_{\bar{x}_A} - \mu_{\bar{x}_B})}{\sqrt{\frac{\sigma_{\bar{x}_A}^2}{n_A} + \frac{\sigma_{\bar{x}_B}^2}{n_B}}} = \frac{1 - 0}{\sqrt{\frac{1}{9}}} = 3$$

shg $P(z > 3) = 1 - P(z \le 3)$
$= 1 - 0{,}9987 \longrightarrow$ dr tabel A3
$= 0{,}0013$

untuk $\bar{x}_A - \bar{x}_B > 1$

dr case study 8.2, didapat kemungkinan rata² waktu keringnya sama adlh sgt kecil yaitu 0,0013 shg tdk wajar

b) $P(x_A - x_B \geq 1) \cdot 10000$

$= (0{,}0013) \cdot 10000$

$= 13$ eksperimen

8.49 A normal population with unknown variance has a mean of 20. Is one likely to obtain a random sample of size 9 from this population with a mean of 24 and a standard deviation of 4.1? If not, what conclusion would you draw?

sample mean $\rightarrow \bar{x} = 24$

$n = 9$

$\mu = 20$

$s = 4,1$

$\rightarrow \sigma$ sample

t - distribution

$$\boxed{T = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

$$T = \frac{24 - 20}{4,1/\sqrt{9}} = 2,9268$$

$t_{0,01}$ dgn degree of freedom (v) $8 = 2,896$

∴ tdk bisa krn μ harus > 20

9.27 Consider the situation of Case Study 9.1 on page 281 with a larger sample of metal pieces. The diameters are as follows: 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 1.01, 1.03, 0.99, 1.00, 1.00, 0.99, 0.98, 1.01, 1.02, 0.99 centimeters. Once again the normality assumption may be made. Do the following and compare your results to those of the case study. Discuss how they are different and why.

(a) Compute a 99% confidence interval on the mean diameter.

(b) Compute a 99% prediction interval on the next diameter to be measured. $1-\gamma$

(c) Compute a 99% tolerance interval for coverage of the central 95% of the distribution of diameters.
$1-\alpha$

Case Study 9.1: Machine Quality: A machine produces metal pieces that are cylindrical in shape. A sample of these pieces is taken and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimeters. Use these data to calculate three interval types and draw interpretations that illustrate the distinction between them in the context of the system. For all computations, assume an approximately normal distribution. The sample mean and standard deviation for the given data are $\bar{x} = 1.0056$ and $s = 0.0246$.

(a) Find a 99% confidence interval on the mean diameter.

(b) Compute a 99% prediction interval on a measured diameter of a single metal piece taken from the machine.

(c) Find the 99% tolerance limits that will contain 95% of the metal pieces produced by this machine.

Solution: (a) The 99% confidence interval for the mean diameter is given by

$$\bar{x} \pm t_{0.005}s/\sqrt{n} = 1.0056 \pm (3.355)(0.0246/3) = 1.0056 \pm 0.0275.$$

Thus, the 99% confidence bounds are 0.9781 and 1.0331.

(b) The 99% prediction interval for a future observation is given by

$$\bar{x} \pm t_{0.005}s\sqrt{1+1/n} = 1.0056 \pm (3.355)(0.0246)\sqrt{1+1/9},$$

with the bounds being 0.9186 and 1.0926.

(c) From Table A.7, for $n = 9$, $1 - \gamma = 0.99$, and $1 - \alpha = 0.95$, we find $k = 4.550$ for two-sided limits. Hence, the 99% tolerance limits are given by

$$\bar{x} + ks = 1.0056 \pm (4.550)(0.0246),$$

with the bounds being 0.8937 and 1.1175. We are 99% confident that the tolerance interval from 0.8937 to 1.1175 will contain the central 95% of the distribution of diameters produced.

9.27

a) t - distribution

99 % on the mean diameter

$$\boxed{\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

$x_i = 1,01 + 0,97 + 1,03 + 1,04 + 0,99 + 0,98 + 1,01 + 1,03 + 0,99 + 1,00 + 0,99 + 0,98 + 1,01 + 1,02 + 0,99 = 16,04$

$n = 16$

$\bar{x} = \frac{1}{16} \cdot 16,04 = 1,0025$

$1 - \alpha = 0,99$

$\alpha = 0,01$

$$\boxed{s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}(x_i - \bar{x})^2\right)}$$

t - distribution

$s^2 = \frac{0,00609375}{15}$

$s = 0,02015 \approx 0,0202$

degree of freedom $= n - 1 = 15$

$t_{\frac{\alpha}{2}}$ utk degree of freedom 15

$= t_{0,005}$ utk degree of freedom 15 $= 2,947$

$$\bar{x} \pm t_{0,005}\left(\frac{s}{\sqrt{n}}\right) = 1,0025 \pm 2,947\left(\frac{0,0202}{\sqrt{16}}\right)$$

$= 1,0025 \pm 0,1488$

∴ (0,9876 , 1,0174) cm

b) 99 % on the next diameter

$$\bar{x} \pm t_{0,005}\left(s\sqrt{1+\frac{1}{n}}\right)$$

$$= 1,0025 \pm 2,947\left(0,0202\left(\sqrt{1+\frac{1}{16}}\right)\right)$$

$= 1,0025 \pm 0,06136$

∴ (0,9411 , 1,0639) cm

c) utk $n = 16 \longrightarrow 1 - \gamma = 0,99$; $1 - \alpha = 0,95$

$\gamma = 0,01$; $\alpha = 0,05$

cari k yg nilai $\alpha = 0,05$ dr tabel A7

$k = 3,421$

shg toleransinya

$\bar{x} \pm ks = 1,0025 \pm 3,421\,(0,0202)$

$= 1,0025 \pm 0,0691042$

∴ (0,9334 , 1,0716)

9.49 Two different brands of latex paint are being considered for use. Fifteen specimens of each type of paint were selected, and the drying times, in hours, were as follows:

Paint A					Paint B				
3.5	2.7	3.9	4.2	3.6	4.7	3.9	4.5	5.5	4.0
2.7	3.3	5.2	4.2	2.9	5.3	4.3	6.0	5.2	3.7
4.4	5.2	4.0	4.1	3.4	5.5	6.2	5.1	5.4	4.8

Assume the drying time is normally distributed with $\sigma_A = \sigma_B$. Find a 95% confidence interval on $\mu_B - \mu_A$, where μ_A and μ_B are the mean drying times.

$$\bar{x}_A = \frac{3,5 + 2,7 + 3,9 + 4,2 + 3,6 + 2,7 + 3,3 + 5,2 + 4,2 + 2,9 + 4,4 + 5,2 + 4,0 + 4,1 + 3,4}{15} = 3,82$$

$$\bar{x}_B = \frac{4,7 + 3,9 + 4,5 + 5,5 + 4,0 + 5,3 + 4,3 + 6,0 + 5,2 + 3,7 + 5,5 + 6,2 + 5,1 + 5,4 + 4,8}{15} = 4,94$$

$n_A = n_B = 15$

$$\boxed{s = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} (x_i - \bar{x})^2\right)}}$$

$$s_A = \sqrt{\frac{1}{14}\left(\sum_{i=1}^{n} (x_{i_A} - \bar{x}_A)^2\right)} = 0,7794$$

$$s_B = \sqrt{\frac{1}{14}\left(\sum_{i=1}^{n} (x_{i_B} - \bar{x}_B)^2\right)} = 0,7538$$

$$sp^2 = \frac{(n_A - 1) s_A^2 + (n_B - 1) s_B^2}{n_A + n_B - 2}$$

$$sp^2 = \frac{14(0,7794)^2 + 14(0,7538)^2}{28}$$

$$sp = \sqrt{0,587839} = 0,7667$$

$1 - \alpha = 0,95$

$\alpha = 0,05$

$\frac{\alpha}{2} = 0,025$

$t_{0,025}$ dgn degree of freedom 28 = 2,048

$$(\bar{x}_B - \bar{x}_A) \pm t_{0,025}\,(sp)\left(\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$= (4,94 - 3,82) \pm 2,048\,(0,7667)\left(\sqrt{\frac{1}{15} + \frac{1}{15}}\right)$$

$= 1,12 \pm 0,5733$

$= (0,5467 , 1,6933)$

$\therefore\ 0,5467 < \mu_B - \mu_A < 1,6933$

9.53 (a) A random sample of 200 voters in a town is selected, and 114 are found to support an annexation suit. Find the 96% confidence interval for the fraction of the voting population favoring the suit.

(b) What can we assert with 96% confidence about the possible size of our error if we estimate the fraction of voters favoring the annexation suit to be 0.57?

a) $n = 200$

$96\% = 100(1-\alpha)\%$

$1 - \alpha = 0,96$

$\alpha = 0,04$

$z_{0,02} = 2,05$

$$\hat{p} = \frac{114}{200} = 0,57$$

$$\hat{q} = \frac{200 - 114}{200} = 0,43$$

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0,57 \pm (2,05)\sqrt{\frac{(0,57)(0,43)}{200}}$$

$$= 0,57 \pm 0,072$$

$$\boxed{\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}}$$

$\therefore\ 0,498 < p < 0,642$

b) $1 - \alpha = 0,96$

$\alpha = 0,04$

$\frac{\alpha}{2} = 0,02 \longrightarrow z_{0,02} = 2,05$

Error won't exceed $z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$$\text{Error} \leq (2,05)\sqrt{\frac{(0,57)(0,43)}{200}}$$

Error $\leq 0,0717646$

$\approx$ Error $\leq 0,072$