EE2703: Applied Programming Lab
Assignment 5: The Laplace Transform

Purvam Jain EE20B101
Electrical Engineering

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Contents

1 Abstract

In this assignment, we will look at how to analyse "Linear Time-invariant Systems" with numerical tools in Python. LTI systems are what Electrical Engineers spend most of their time thinking about - linear circuit analysis or communication channels for example. The main part being:

- All the problems will be in "continuous time" and will use Laplace Transforms. Python has a Signals toolbox which is very useful and complete.

2 Program Structure

- Defining Transfer functions and performing convolution
- Computing Inverse Laplace Transform by computing unit step response
- Plotting Bode plots, frequency dependency and time evolution.

The program can be run from the command line and accepts inputs as:

```
python EE20B101.py
```

3 Single Spring System

3.1 Varying decay rate

We are given the equation of a forced oscillator:

$$\ddot{x} + 2.25x = f(t)$$

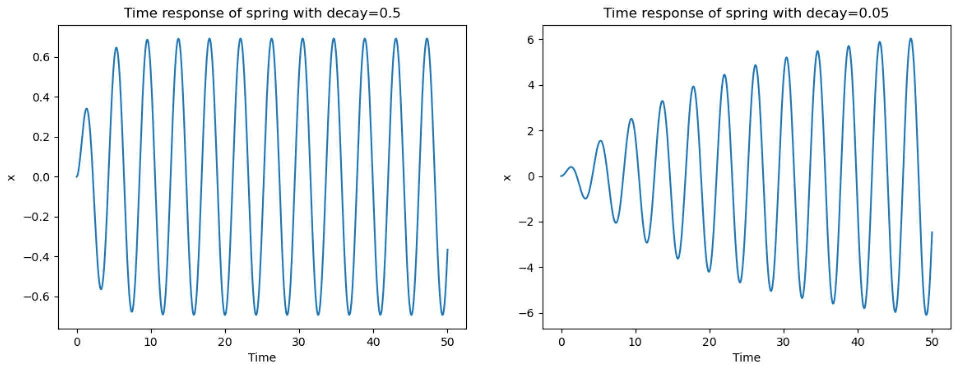
here,

$$f(t) = \cos(1.5t)e^{(-0.5t)}u_o(t)$$

On solving for $X(S)$ we get:

$$X(s) = \frac{s + 0.5}{(s^2 + 2.25)((s + 0.5)^2 + 2.25)}$$

We use *sp.impulse* to compute inverse laplace transform of the given function.



(a) Time domain response with $decay = 0.5$ (b) Time domain response with $decay = 0.05$

Figure 1: Time Domain response of forced oscillator with different decay rates

We observe that for decay rate of 0.5 amplitude quickly takes reaches its steady state value. While for the decay rate of 0.05 it take longer to reach a steady state value. Also we observe that the Amplitude is larger and initially increases for smaller decay rate due to slowed down decay rate.

3.2 Varying frequency

We plot time evolution of $X(s)/F(s)$ with different frequency inputs.

$$X(s) = \frac{1}{s^2 + (1.5)^2}$$

Comparing this equation with:

$$s^2 + 2\zeta\omega_n^2 + \omega_n^2 = F(s)$$

We realize that the natural resonant frequency of this system is $1.5Hz$

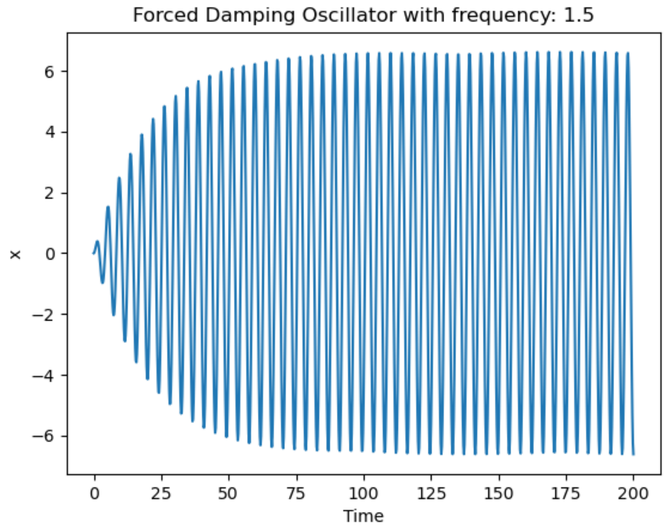
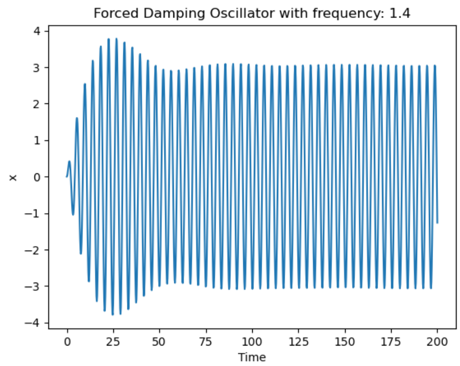
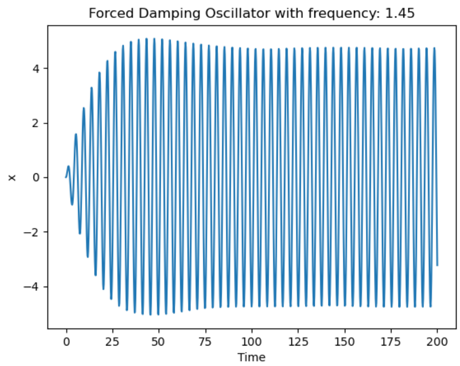


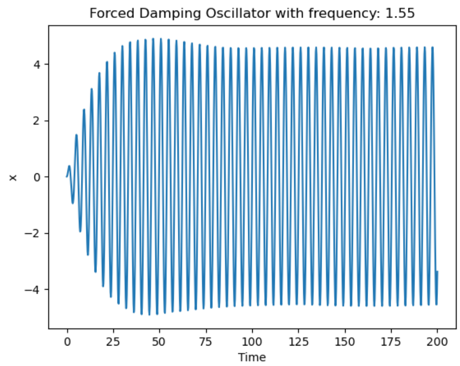
Figure 2: Plot of $x(t)$ with $frequency = 1.5Hz$



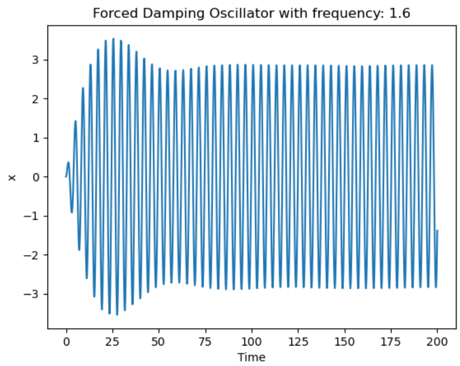
(a) Plot of $x(t)$ with $frequency = 1.4Hz$



(b) Plot of $x(t)$ with $frequency = 1.45Hz$



(a) Plot of $x(t)$ with $frequency = 1.55Hz$



(b) Plot of $x(t)$ with $frequency = 1.6Hz$

We observe that the largest steady state amplitude is attained for frequency input of $1.5Hz$ which is due to resonance with natural frequency of system. While for other frequency inputs the amplitude rises initially but then dies down to settle at a lower steady state value.The frequencies closer to natural frequency have comparatively higher steady state amplitude.

4 Double Spring System

We are given a coupled spring system:

$$\ddot{x} + (x - y) = 0$$

$$\ddot{y} + 2(y - x) = 0$$

Solving these we get,

$$X(s) = \frac{s^2 + 2}{s^3 + 3s}$$

$$Y(s) = \frac{2}{s^3 + 3s}$$

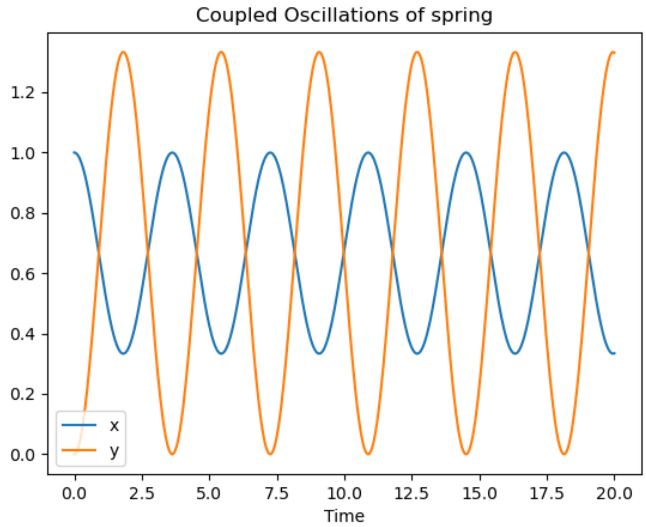


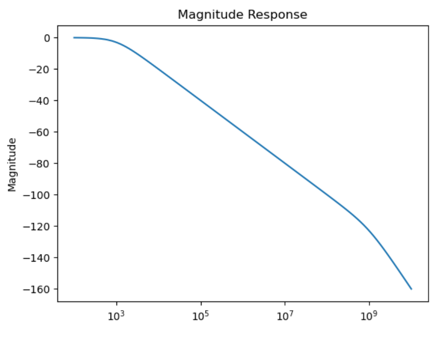
Figure 5: Plot of $x(t)$ and $y(t)$

5 Analysing LCR Circuit

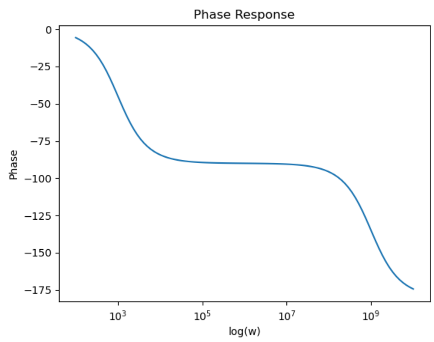
5.1 Steady State Transfer Function

On solving for Transfer function for the given two-port system we get:

$$V_c/V_{in} = H(s) = \frac{1}{LCs^2 + RCs + 1}$$



(a) Magnitude plot



(b) Phase plot

Figure 6: Bode Plot for $H(s)$

5.2 Response of two-port to given input

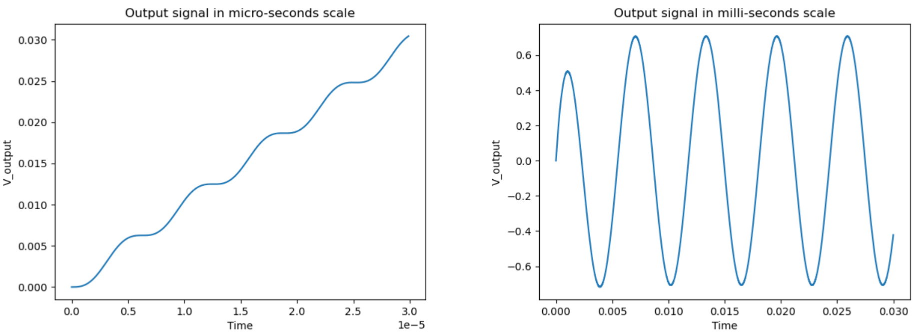
Given input,

$$V_i = \cos{(10^3 t)} u(t) - \cos{(10^6 t)} u(t)$$

The output $V_o(s) = H(s)V_i(s)$ The given low pass filter has the cut-off frequency given by:

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

For given circuit $f_c = 0.16MHz$



(a) Transient response

(b) Steady State Response

Figure 7: Bode Plot for $V_o(t)$

We plot the response for $t < 30\mu s$ to analyze transient response and for $0 < t < 30ms$ to analyze steady response. We observe that magnitude is increasing rapidly due to input voltage as capacitor charges up. In steady state response we observe that only low-frequency input is visible as the circuit acts as a low pass filter and the high frequency component is almost completely attenuated hence only low frequency input is visible in the output.